Azimuthal correlations in hadronic collisions from instabilities of the initial state

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1 Introduction
   Elliptic flow
   Scaling of $\nu_2$

2 The GLR-MQ evolution equation

3 Results

4 Conclusions
A disclaimer

I don’t really believe what I will say here has much chance of being correct. It is an attempt to answer some questions which I do believe deserves answering, and is based on an attempt at answering them. and it gives a definite experimental signature at the EiC! if this is found I’ll give many talks like this, if not I’ll forget it!
A very short detour into philosophy...

Science is done via two basic mechanisms...

Puzzle-solving within a paradigm using accepted assumptions to draw conclusions (Nowadays, just drop system into a ML/Bayesian code and wait!)

Paradigm shifts questioning the assumptions and trying to look for new ones (Humans are still useful here!)

Switching typically happens when the ”weight of the puzzles” becomes too much and someone finds a set of assumptions that makes them go away
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The paradigm in question

Cover of PRL!!!!

BBC! SPACE DAILY!

RHIC found the perfect fluid!
Elliptic flow $\nu_2$ (Harmonic flow $\nu_n$)

Elliptic flow is parametrized as the $n = 2$ Fourier component in the $p_T$ distribution of the produced particles:

$$\frac{dN}{dp_T dy d\phi} = \frac{dN}{dp_T dy} \left[ 1 + \sum_{n=1}^{\infty} 2\nu_n(p_T) \cos(\phi - \phi_{0n}) \right]$$

(1)

Figure: A geometrical view of elliptic flow.
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Observable:

$$\frac{dN}{p_T dp_T dy d\phi} = \frac{dN}{p_T dp_T dy} \left[ 1 + 2\nu_n(p_T, y) \cos \left( n \left( \phi - \phi_0(n, p_T, y) \right) \right) \right]$$

"Collectivity" $\nu_n$ appears in $\forall$ n-particle correlations,

$$\left\langle \frac{dN}{d\phi_1} \frac{dN}{d\phi_2} \ldots \right\rangle$$
Hydro works well

Data points to a viscosity not much bigger than $\eta/s = 1/4\pi$. ”Lowest possible for a fluid”, comparable to string theory prediction

Led to a lot of theoretical development in relativistic fluid dynamics
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**Calculations using ideal hydrodynamics**


P.Romatschke, PRL99:172301, 2007

B.Schenke, QM2014

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**People like this description because...**

**It fits quite a lot of data** with reasonable precision (but also a lot of parameters: EoS, transport, initial conditions, ....)

**The interpretation is reasonable**, connects to fundamental science consistently, people "expect it" in some limit

**It’s considered as a given.** Details are now best sorted out via Bayesian fitting/ML
My issue is that scalings in energy, rapidity and system size of \( v_n \) look suspiciously simple compared to the Hydrodynamical picture.
Buckingham’s theorem (How to do hydro, circa 19th century, before ML!)

Any quantitative law of nature expressible as a formula

\[ f(x_1, x_2, \ldots, x_n) = 0 \]

can be expressed as a dimensionless formula

\[ F(\pi_1, \pi_2, \ldots, \pi_{n-k}) = 0 \]

where

\[ \pi_i = \prod x_i^{\lambda_i}, \quad \sum \lambda_i = 0 \]

Widely applied within hydrodynamics in the 19th century: Knudsen’s number, Reynolds number, Rayleigh’s number, etc.

Since we are varying a whole slew of experimental \((y, p_T, N_{\text{part}}, \sqrt{s}, A)\) And theoretical \((T, \mu, \eta, s, \hat{q}, \tau_0, \tau_{\text{life}})\) parameters it would be nice to represent heavy ion observables this way
How should $v_2$ scale in hydrodynamics?

(Buckingam’s theorem vs $v_2$)

- Approximately it $\propto \epsilon$ since $v_2(\epsilon = 0) = 0$ and $\epsilon$ small and dimensionless
- Approximately it $\propto c_s$ since it is sensitive to EoS, $v_2(c_s = 0) = 0$ and $c_s$ small, dimensionless
- It is maximum for ideal hydro. Since $Kn$ small and dimensionless, $v_2 \sim v_2^{\text{ideal}}(1 - Kn)$ . The Knudsen number is $Kn \sim \eta/(sTR)$
- $v_2^{\text{ideal}} \sim v_2(\tau \to \infty) \times f(\tau_f/\tau_0) . f(\ldots)$ a monotonically increasing saturating, $
\sim f(\langle p_T \rangle) \text{tanh}(\ldots)$ in Cooper-Frye . $\tau_f$ is the freezeout time.
- $\tau_f/\tau_0 \sim (e_0/e_f)^{4\alpha} \sim (T_0/T_f)^{\alpha}$ , with $1/3 < \alpha < 4/3$
- $T_0/T_f \sim ((1/(\tau_0 T_f S))(dN/dy))^{1/3}$ For constant $T_f$ Heiselberg-Levy scaling recovered
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Figure: Elliptic flow $v_2$ vs. rapidity [2,3].

$v_2$ response in region where temperature dramatically changes remarkably smooth, follows $dN/dy$ exactly (as far as we can tell). EoS, $\eta/s$ shouldnt.
size effects also remarkably absent, down to pp. Remember that hydro expansion around small Knudsen number, $Kn \sim \eta/(R \times s \times T)$. we should scan this, but we dont seem to!
Especially when you consider cumulants, “hydrodynamics” seems remarkably indepenent of number of constituents. Even if one does not consider mean free path, what about thermodynamic fluctuations?
Furthermore, rise in $v_2$ seems entirely due to rise in $\langle p_T \rangle$!

$v_2(p_T)$ nearly constant
In rapidity, $v_2$ of small systems above hydro prediction! Perhaps limiting fragmentation together with $p_T$ scaling with multiplicity could explain the trend. But this is not hydrodynamics, nor an explanation.
Cooper-Frye

\[ \nu_2(p_T) = \int d\phi \cos(2\phi) \left( E - p_T \left( \frac{dt}{dr} + \Delta \frac{dt}{dr}(\phi) \right) \right) e^{-\gamma(E-p_T(u_T+\delta u_T(\phi)))} \]

\[ \simeq \int d\phi \cos^2(2\phi) \left[ e^{-\frac{\gamma(E-p_T u_T)}{T}} - p_T \Delta \frac{dt}{dr} + \frac{\gamma \delta u_T(\phi) p_T}{T} + O(\epsilon^2, Kn) \right] \]

As long as \( \frac{\delta v_T}{T} \sim \epsilon f(R, \sqrt{s}) \) deviations \( \sim p_T \), more prominent at high \( p_T \)
Putting everything together we have

\[ v_n(p_T) \sim O(1) \epsilon_n F(p_T) \text{ universal}, \quad \langle v_n \rangle \sim \epsilon_n F(\langle p_T \rangle) \]

\[ \langle p_T \rangle \sim \frac{1}{S} \frac{dN}{dy} \]

For a non-linear theory such as hydrodynamics we do not expect matrix below to be sparse.

\[
\begin{pmatrix}
\frac{dN}{dy} \\
\langle p_T \rangle \\
v_n
\end{pmatrix} =
\begin{pmatrix}
... & ... & ...
\end{pmatrix}
\times
\begin{pmatrix}
T_{initial} \\
L \\
\epsilon_n
\end{pmatrix}
\]

\[ \eta/s, c_s, \tau_{\pi}, ... \]

\[ \rightarrow N_{\text{part}}, A, \sqrt{s} \]

So \( v_2(A, \sqrt{s}, N_{\text{part}}, y, ...) \) non-separable!

Analytical solutions (Hatta, Noronha, Xiao, GT) confirm this
Particle species dependence is also strange

Low energy scan
STAR SQM11
$p_T$

Does this scaling hold by SPECIES?

Note that a lot of these effects do not arise by particle species but only when all species are counted. But

$$v_2 = \sum_i \frac{v_{2i}(T, m)n_i(T, \mu)}{\sum_i n_i(T, \mu)}$$

Why would this cancellation occur? $\mu$ and $m$ independent!
Photon vs hadron $v_n$

**Figure**: Photon $v_3$ vs. $p_T$ (red) and Proton $v_3$ vs. $p_T$ (black) [6]. Direct photon $v_2$ similar! Why are they the same at low $p_T$?
All these puzzles have (satisfactory?) explanations within the "standard model"

- photons contaminated by final-state decays, boosted by magnetic field based mechanisms (But why same as hadrons?)
- small systems origin of $v_n$ in small, large systems different (CGC/antennae) but why small and large systems scale?
- $v_2(p_T)$ vs $\sqrt{s}$ many effects cancel out

All these are plausible, but not so elegant!
All of these scalings really remind me of the scalings that imposed pQCD/partons over the then popular “bootstrap” models! no reason within bootstrap for the scaling!
Parton distributions

Let’s see Deep Inelastic Scattering

\[ d\frac{N}{d^3 p} = \int f(Q_1, x_1, \theta_1) f(Q_2, x_2, \theta_2) \sigma_{gg \rightarrow j} (x_{Q_1} - x_{Q_2}, \theta_1 - \theta_2) D_{j \rightarrow i}(z) [x_{Q_1} - x_{Q_2}]^2 dx_{1,2} dQ_{1,2} dz \]

The probability that the struck parton carries a fraction \( x_{Bj} \) of the proton momentum is called parton distribution function \( f(x, Q) \). Same in eA,AA collisions. This is initial state (all reinteractions “renormalized”)

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Bjorken scaling

Structure functions (PDFs, eventually GPDs) depend on the scale they are measured; i.e. $x$ and $Q^2$. In the perturbative limit dependence on $Q^2$ is subleading.

As $p_T \sim Q$ and $\eta \sim \ln(\frac{1}{x})$, then scaling of elliptic flow in HIC may resemble Bjorken scaling when adding an angular dependence on the structure functions.
Let us entertain a crazy idea

What if parton distribution functions became azimuthally asymmetric, but still kept the running we expect from QCD???

ν_2 of Photons as expected, ν_n would be an initial state effect!

Scaling in x, Q exactly as expected from Bjorken-like running

Particle species protected by unitarity of the fragmentation function

But there is a reason I called it crazy: PDFs are universal and QCD is azimuthally symmetric!
Could structure functions be azimuthally asymmetric?

- Sivers functions (spin difference gives you an asymmetry) But uncorrelated with geometry, special role for $v_2$ so unlikely
- "Color antennae" and such (CGC models, Kovner et al, Gyulassy, Biro, ...) Since antenna point in random directions, effect always goes away for large systems ("many antennae") I think scaling implies Same origin for pA, AA
Could structure functions be azimuthally asymmetric?

The running of $f(x, Q)$ is really an RG equation, $f(x, Q)$ probe dependent at subleading order in $\alpha_s$. At $\mathcal{O}(\alpha_s^2 \epsilon_n) \ll \nu_n$ (2nd and higher Twist) they should generally be azimuthally asymmetric for extended probes. Can this small effect be amplified?
RG, symmetry breaking and nucleon maps

M.Diehl
1512.01328

Usually $f(x)$ is thought of as averaging transverse information, but it is a quantum operator, subject to symmetry breaking.
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\[
\frac{Q}{2} \frac{\partial}{\partial Q} \left( \frac{\partial x G(x, Q^2)}{\partial \ln(1/x)} \right) = \frac{\alpha_s N_c}{\pi} x G(x, Q^2)
\]

\( G(x, Q) \) evolve according to renormalization-group type linear operator evolution equations (DGLAP in \( Q \), BFKL in \( x \)). But in \( x \) evolution blows up. This evolution breaks Froissart’s bound (unitarity in hadron-hadron scattering) at low \( x \). In order to correct this, a non-linear term is added.
The GLR-MQ evolution equation

In the dense parton limit, the equation that governs the evolution of parton distribution functions inside hadrons is thought to be given by

$$\frac{Q}{2} \frac{\partial}{\partial Q} \frac{\partial}{\partial \ln(1/x)} G(x, Q^2) = \frac{\alpha_s N_c}{\pi} xG(x, Q^2) - \frac{\alpha_s^2 N_c \pi}{2 C_F S_\perp} \frac{1}{Q^2} [xG(x, Q^2)]^2$$

(2)

(It is a high $Q$ limit of an integro-differential (GLR) equation).

Balancing the linear and the non-linear term defines the saturation scale $Q_s$, assuming azimuthal symmetry.
Saturation together with an RG picture for saturation generates JIMWLK action, CGC (JIMWLK/CGC result: Azimuthally symmetric action, asymmetric boundary conditions)

But non-linear 2+1 differential equation can have instabilities breaking the underlying symmetry!
Our proposal

Adding an angular dependence the GLR-MQ equation and keeping the same limits modify the equations the following way

\[
\frac{xQ}{2} \left( \frac{\partial}{\partial Q} + \frac{1}{Q} \frac{\partial}{\partial \phi} \right) \frac{\partial}{\partial x} [xG(x, Q^2, \phi)] = \frac{\alpha_s N_c}{\pi} xG(x, Q^2, \phi) - \frac{\alpha_s^2 N_c \pi}{2 C_F S_{\perp} Q^2} [xG(x, Q^2, \phi)]^2
\]

\textbf{(NB: angular ladder effects neglected as a first attempt, will modify this qualitative estimate)}
As a solution, we try

$$G(x, Q^2, \phi) = G_0(x, Q^2) \left(1 + \sum_{n=1}^{\infty} u_n(x, Q^2) \cos(n\phi + \beta_n)\right),$$

$$G_0(x, Q^2)$$ is the azimuthally symmetric solution (i.e. saturation)
Azimuthal symmetry as a broken symmetry

High $Q, -\ln(1/x)$

$$u_n \sim \varepsilon_n \alpha_s^2$$

lower $Q, -\ln(1/x)$

$$u_n \gg \varepsilon_n \alpha_s^2$$

Small geometry-driven anisotropies at higher $x, Q$ amplified by evolution
Azimuthal symmetry as a broken symmetry

Figure: Elliptic flow $\nu_2$ vs. rapidity [2,3].

Arbitrary small tilt (tiny gradients at high $x$) produce large effects at low $x$. Different from CGC effects since lagrangian acquires a $\theta$ dependence (which will need to be added to JIMWLK equation)
Non-linear evolution can break underlying symmetries

If non-linearities are strong enough, azimuthal symmetries broken dynamically. In hydrodynamics this effect is well-known but exists in most 2+1 non-linear systems.
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2+1 non-linear evolution equation

For unintegrated in $x_\perp$ General Parton distribution functions we could have: ”Inverse cascade”: Instabilities go from high frequency (local in transverse space) to low frequency as $x$ evolves. No ”many antennae” problem.
Equations for the Fourier coefficients

Working on the limiting case $Q << Q_s(x)$, we insert the solution with azimuthal perturbations into eq. fully asymmetric GLR-MQ equation and get three linear equations for our Fourier coefficients.

1. An infinite set of equations equation that relate the Fourier coefficients with the phases.

$$
\sum_k u^2_k(x, Q^2) \cos(2\beta_k) = 0 \quad (3)
$$
An infinite set of equations regarding only the derivative with respect to x.

\[
 x \frac{\partial u_n(x, Q^2)}{\partial x} = -(2\lambda + 1)u_n(x, Q^2)
\]

\[
 + \frac{N_c \pi}{2 C_F S_\perp \alpha_s^2} \frac{1}{Q^2} x^{2\lambda + 1} \frac{1}{n} \left[ \sum_{k}^{n-1} u_k(x, Q^2) u_{n-k}(x, Q^2) \sin(\beta_n - \beta_k + \beta_{n-k}) - 2 \sum_{k} u_k(x, Q^2) u_{n+k}(x, Q^2) \sin(\beta_n + \beta_k - \beta_{n+k}) \right] (4)
\]
3 An infinite set of equations that regards derivatives with respect to Q and mixed terms.

\[
(2\lambda+1)\frac{Q}{2} \frac{\partial u_n(x, Q^2)}{\partial Q} + \frac{Q}{2} x \frac{\partial^2 u_n(x, Q^2)}{\partial Q \partial x} = \frac{\alpha_s N_c}{\pi} u_n(x, Q^2) \]

\[
+ \frac{N_c \pi}{2 C_F S_\perp \alpha_s^2} \frac{1}{Q^2} x^{2\lambda+1} \left[ 2u_n(x, Q^2) \right] \]

\[
+ \frac{1}{2} \sum_{k}^{n-1} u_k(x, Q^2) u_{n-k}(x, Q^2) \cos(\beta_n - \beta_k - \beta_{n-k}) \]

\[
+ \sum_{k} u_k(x, Q^2) u_{n+k}(x, Q^2) \cos(\beta_n + \beta_k - \beta_{n+k}) \]

(5)
As an ansatz we propose

\[ u_n(x, Q^2) = \delta_{n,2} \sum_{k=0}^{\infty} A_k \frac{(Bx^C)^k}{k!} Q^{D-2k} \]  

then solve the equation linearized in \( u_k \) from initial conditions

\[ u_n(\ln x^{-1} \to 0, Q) \sim \epsilon_n \alpha_s^2 \]  

High \( Q, -\ln(1/x) \)

\[ u_n \sim \epsilon_n \alpha_s^2 \]

lower \( Q, -\ln(1/x) \)

\[ u_n \gg \epsilon_n \alpha_s^2 \]
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Preliminary results

\[ Q = 20 \Lambda_{QCD} \]
\[ Q = 10 \Lambda_{QCD} \]
\[ Q = 5 \Lambda_{QCD} \]
Preliminary results

\[ \frac{u_2(x, Q^2)}{\epsilon} \]

\[ x = 10^{-3}, 10^{-4}, 10^{-5} \]

\[ b = R, R/2, R/4 \]

\[ Q/Q_{QCD} \]

\[ Q/Q_{QCD} \]
Preliminary results: very encouraging

- Near independence of $u_n(Q, x)$ on $x$ (all dependence on $G_0(Q, x)$ which in turn depends weakly on $Q$. Just like $v_2$
- Near linear dependence on $\epsilon_n$ Just like $v_2$
- Near decoupling of fourier modes

Forthcoming: A phenomenological study including factorization and fragmentation
What if were right?

Relation between $v_n$ non-linearities could be more predictive than hydro models, fewer parameters so easier to falsify

Photon correlations Correlations between high rapidity photons and mid-rapidity hadrons, pA and AA

And the ultimate signature is...
Ridges/$v_n$ at the EIC?

HERA looked for flow and failed

UrEIC could dramatically show if something like what I'm advocating here could be true
Alternatively, we are wrong and hydro valid to smallest scales (Poster, G. Soares)

Hydro in small systems could lead to “classical spin measurement”
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Spin dependent nucleon shape changes $\nu_2$ in polarized pA collisions. ultimate small system hydrodynamics?
Spin dependent nucleon shape changes $v_2$ in polarized pA collisions. Ultimate small system hydrodynamics?
Conclusions

- $\nu_2$ scaling similar to scaling of parton distribution functions. Could they be azimuthally asymmetric?
- Instabilities in the non-linear regime?
- Work in progress to develop this hypothesis to quantitative test level

References

[4] STAR collaboration, 1206.5528
[6] PHENIX Collaboration - Observation of direct-photon collective flow in Au+Au collisions at $\sqrt{s_{NN}}= 200$ GeV.