Azimuthal correlations in hadronic collisions from instabilities of the initial state

UNIVERSIDADE ESTADUAL DE CAMPINAS

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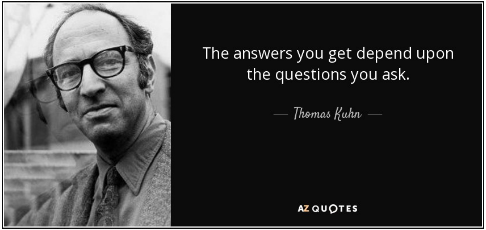
G.Torrieri

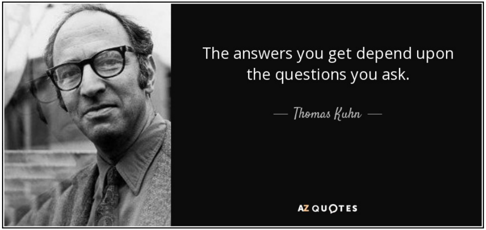
1. Introduction
   - Elliptic flow
   - Scaling of ν_2
2. The GLR-MQ evolution equation
3. Results
4. Conclusions

A disclaimer



I don't really believe what I will say here has much chance of being correct. It is an attempt to answer some questions which I do believe deserves answering, and is based on an attempt at answering them. and it gives a definite experimental signature at the EiC! if this is found Ill give many talks like this, if not Ill forget it!

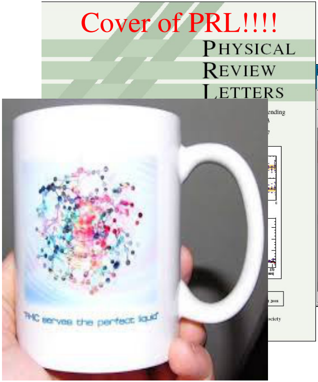
A very short detour into philosophy...



Science is done via two basic mechanisms...
Puzzle-solving within a paradigm using accepted assumptions to draw conclusions (Nowadays, just drop system into a ML/Bayesian code and wait!)
Paradigm shifts questioning the assumptions and trying to look for new ones (Humans are still useful here!)
Switching typically happens when the "weight of the puzzles" becomes too much and someone finds a set of assumptions that makes them go away

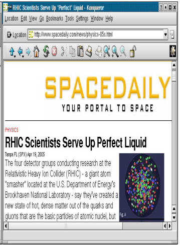
The paradigm in question

Cover of PRL!!!!



BBC!

SPACE DAILY!



RHIC found the perfect fluid!

Elliptic flow ν_2 (Harmonic flow ν_n)

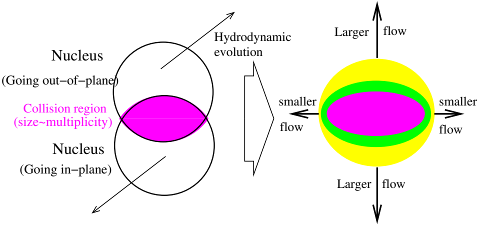
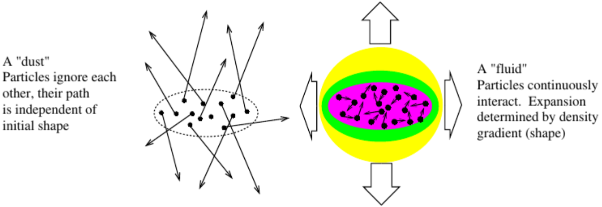


Figure: A geometrical view of elliptic flow.

Elliptic flow is parametrized as the $n = 2$ Fourier component in the p_T distribution of the produced particles:

$$\frac{dN}{dp_T dy d\phi} = \frac{dN}{dp_T dy}\left[1 + \sum_{n=1}^{\infty} 2\nu_n(p_T) cos(\phi - \phi_{0n})\right] \tag{1}$$



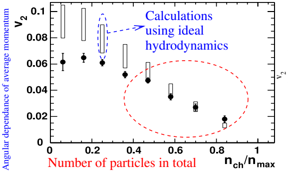
Observable:

$$\frac{dN}{p_T dp_T dy d\phi} = \frac{dN}{p_T dp_T dy}\left[1 + 2v_n(p_T, y)\cos\left(n\left(\phi - \phi_0\left(n, p_T, y\right)\right)\right)\right]$$

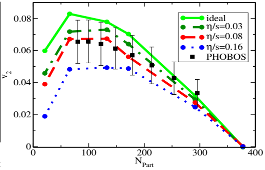
"Collectivity" Same v_n appears in $\forall$ n-particle correlations , $\left\langle \frac{dN}{d\phi_1}\frac{dN}{d\phi_2}\ldots \right\rangle$

Hydro works well

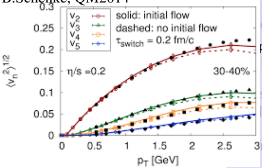
P.Kolb and U.Heinz,Nucl.Phys.A702:269,2002.



P.Romatschke,PRL99:172301,2007



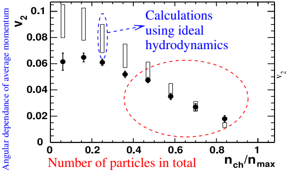
B.Schenke, QM2014



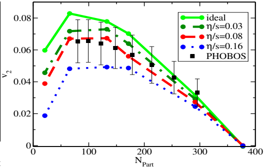
Data points to a viscosity not much bigger than $\eta/s = 1/4\pi$. "Lowest possible for a fluid", comparable to string theory prediction

Led to a lot of theoretical development in relativistic fluid dynamics

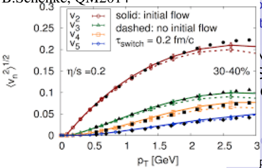
P.Kolb and U.Heinz,Nucl.Phys.A702:269,2002.



P.Romatschke,PRL99:172301,2007



B.Schenke, QM2014

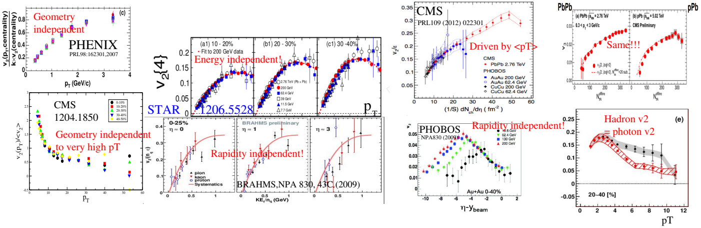


People like this description because...

It fits quite a lot of data with reasonable precision (but also a lot of parameters:EoS, transport,initial conditions,....

The interpretation is reasonable , connects to fundamental science consistently, people “expect it” in some limit

It’s considered as a given. Details are now best sorted out via Bayesian fitting/ML



My issue is that scalings in energy,rapidity ans system size of v_n look suspiciously simple compared to the Hydrodinamical picture.

Buckingam's theorem (How to do hydro, circa 19th century,before ML!)

Any quantitative law of nature expressible as a formula

$$f(x_1, x_2, ..., x_n) = 0$$

can be expressed as a dimensionless formula

$$F(\pi_1, \pi_2, ..., \pi_{n-k}) = 0$$

where

$$\pi_i = \prod x_i^{\lambda_i} \quad , \quad \sum \lambda_i = 0$$

Widely applied within hydrodynamics in the 19th century: Knudsen's number, Reynolds number, Rayleigh's number, etc.

Since we are varying a whole slew of experimental $(y, p_T, N_{part}, \sqrt{s}, A)$ And theoretical $(T, \mu, \eta, s, \hat{q}, \tau_0, \tau_{life})$ parameters it would be nice to represent heavy ion observables this way

How should v_2 scale in hydrodynamics?

(Buckingam's theorem vs v_2)

- Approximately it $\propto \epsilon$ since $v_2(\epsilon = 0) = 0$ and ϵ small and dimensionless
- Approximately it $\propto c_s$ since it is sensitive to EoS, $v_2(c_s = 0) = 0$ and c_s small, dimensionless
- It is maximum for ideal hydro. Since Kn small and dimensionless, $v_2 \sim v_2^{ideal}(1 - Kn)$. The Knudsen number is $Kn \sim \eta/(sTR)$
- $v_2^{ideal} \sim v_2(\tau \to \infty) \times f(\tau_f/\tau_0)$. $f(...)$ a monotonically increasing saturating, $\sim f(\langle p_T \rangle) \tanh(...)$ in Cooper-Frye . τ_f is the freezeout time.
- $\tau_f/\tau_0 \sim (e_0/e_f)^{4\alpha} \sim (T_0/T_f)^{\alpha}$, with $\frac{1}{3}\Big|_{bjorken} < \alpha < \frac{4}{3}\Big|_{hubble}$ $T_0/T_f \sim ((1/(\tau_0 T_f S))(dN/dy))^{1/3}$ For constant T_f Heiselberg-Levy scaling recovered

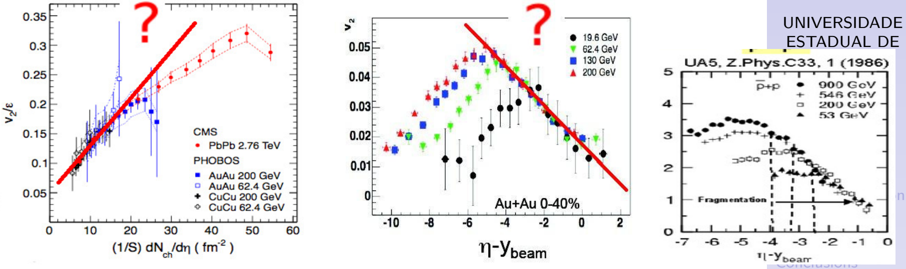
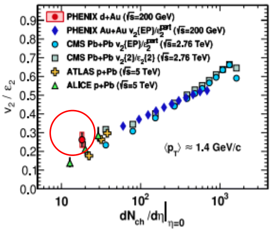
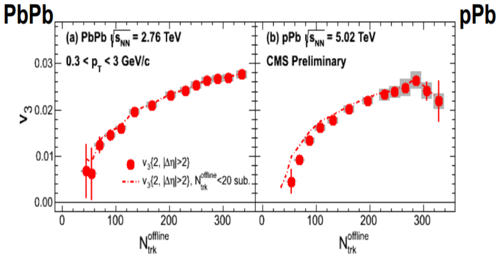


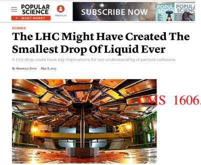
Figure: Elliptic flow ν_2 vs. rapidity [2,3].

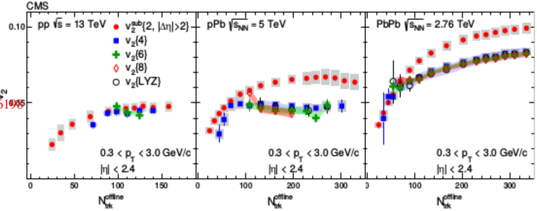
v_2 response in region where temperature dramatically changes remarkably smooth, follows dN/dy exactly (as far as we can tell). EoS,η/s shouldnt.





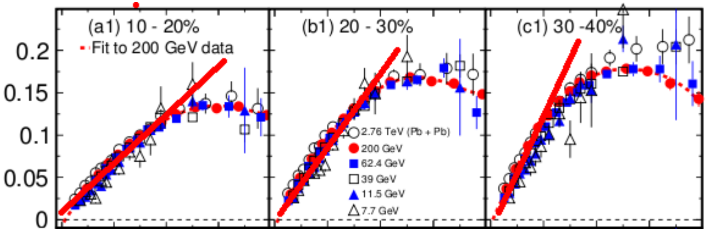
size effects also remarkably absent, down to *pp* .
Remember that hydro expansion around small Knudsen number, $Kn \sim \eta/(R \times s \times T)$.
we should scan this, but we dont seem to!



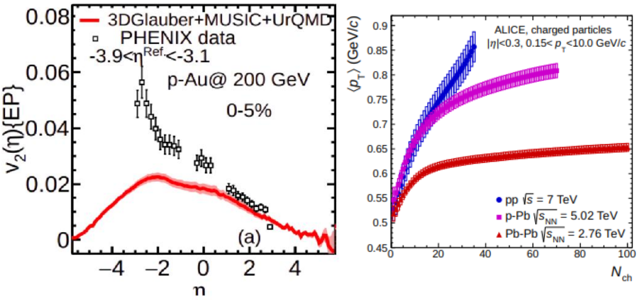


CMS 1606.06198

Especially when you consider cumulants, "hydrodynamics" seems remarkably indepenent of number of constituents. Even if one does not consider mean free path, what about thermodynamic fluctuations?



Furthermore, rise in v_2 seems entirely due to rise in $\langle p_T \rangle$
! $v_2(p_T)$ nearly constant



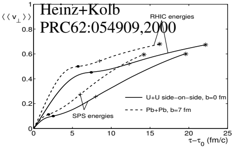
In rapidity, v_2 of small systems above hydro prediction! Perhaps limiting fragmentation together with p_T scaling with multiplicity could explain the trend. But this is not hydrodynamics,nor an explanation.

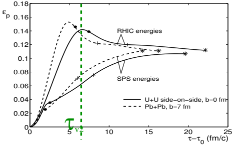
Cooper-Frye

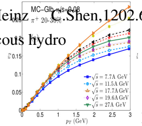
$$v_2(p_T) = \int d\phi \cos(2\phi) \left(E - p_T \left(\frac{dt}{dr} + \Delta \frac{dt}{dr}(\phi) \right) \right) e^{\left(- \frac{\gamma(E - p_T(u_T + \delta u_T(\phi)))}{T} \right)}$$

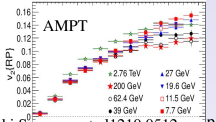
$$\simeq \int d\phi \cos^2(2\phi) \left[\underbrace{e^{-\frac{\gamma(E - p_T u_T)}{T}}}_{=0} - \underbrace{p_T \Delta \frac{dt}{dr}}_{\epsilon p_T} + \underbrace{\frac{\gamma \delta u_T(\phi) p_T}{T}}_{\sim \frac{\delta v_T}{T} p_T \sim \epsilon p_T / T} + \mathcal{O}\left(\epsilon^2, Kn\right) \right]$$

As long as $\frac{\delta v_T}{T} \sim \epsilon f(R, \sqrt{s}$ deviations $\sim p_T$, more prominent at @high p_T









Solanki,Sorensen et al1210.0512

Putting everything together we have

$$v_n(p_T) \simeq \mathcal{O}(1)\, \epsilon_n \underbrace{F(p_T)}_{universal} \quad , \quad \langle v_n \rangle \sim \epsilon_n \underbrace{F(\langle p_T \rangle)}_{\langle p_T \rangle \sim \frac{1}{S}\frac{dN}{dy}}$$

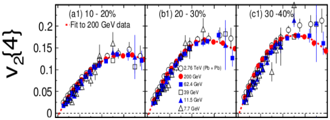
For a non-linear theory such as hydrodynamics we do not expect matrix below to be sparse.

$$\begin{pmatrix} dN/dy \\ \langle p_T \rangle \\ v_n \end{pmatrix} = \underbrace{\begin{pmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{pmatrix}}_{\eta/s, c_s, \tau_\pi, \dots} \times \underbrace{\begin{pmatrix} T_{initial} \\ L \\ \epsilon_n \end{pmatrix}}_{\to N_{part}, A, \sqrt{s}}$$

So $v_2(A, \sqrt{s}, N_{part}, y, \dots)$ non-separable !

Analytical solutions (Hatta,Noronha,Xiao,GT) confirm this

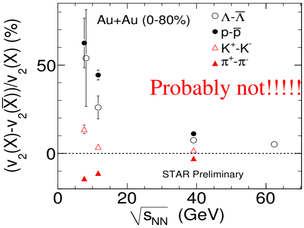
Particle species dependence is also strange



Low energy scan STAR SQM11

p_T

Does this scaling hold by SPECIES?



Probably not!!!!!!

Note that a lot of these effects do not arise by particle species but only when all species are counted. But

$$v_2 = \frac{\sum_i v_{2i}(T, m) n_i(T, \mu)}{\sum_i n_i(T, \mu)}$$

Why would this cancellation occur? μ and m independent!

Photon vs hadron v_n

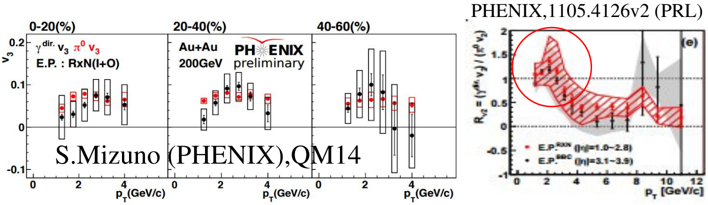


Figure: Photon ν_3 vs. p_T (red) and Proton ν_3 vs. p_T (black) [6]. Direct photon v_2 similar! Why are they the same at low p_T ?

All these puzzles have (satisfactory?) explanations within the "standard model"

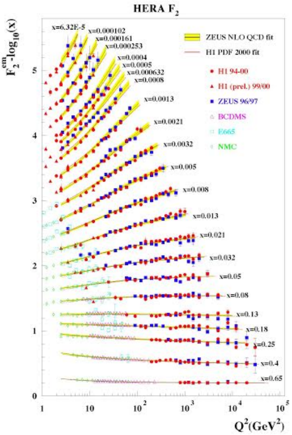
photons contaminated by final-state decays, boosted by magnetic field based mechanisms (But why <u>same</u> as hadrons?

small systems origin of v_n in small,large systems different (CGC/antennae) but why small and large systems scale?

$v_2(p_T)$ vs $\sqrt{s}$ many effects cancel out

All these are plausible, but not so elegant!

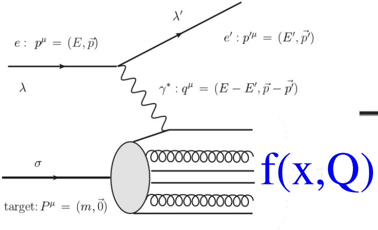
What I find really funny!

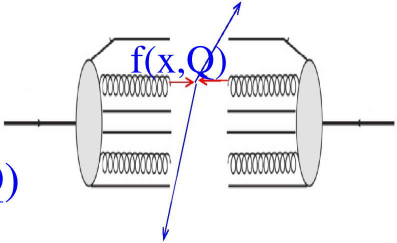


<u>All</u> of these scalings <u>really</u> remind me of the scalings that imposed pQCD/partons over the then popular "bootstrap" models! no reason within boostrap for the scaling!

Parton distributions

Let's see Deep Inelastic Scattering





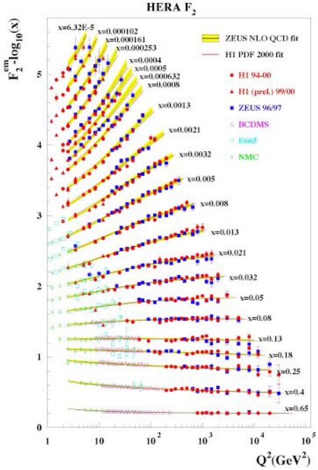
$$\frac{dN}{d^3p} = \int f(Q_1, x_1, \theta_1) f(Q_2, x_2, \theta_2) \sigma_{gg\to j}(xQ_1 - xQ_2, \theta_1 - \theta_2) D_{j\to i}(z) [xQ_1 - xQ_2]^2 dx_{1,2} dQ_{1,2} dz$$

The *probability* that the struck parton carries a fraction x_{Bj} of the proton momentum is called *parton distribution function* $f(x, Q)$. Same in eA,AA collisions . THis is initial state (all reinteractions "renormalized")

Bjorken scaling

Structure functions (PDFs,eventually GPDs) depend on the scale they are measured; i.e. x and Q^2. In the perturbative limit dependence on Q^2 is subleading.

As $p_T \sim Q$ and $\eta \sim ln(\frac{1}{x})$, then scaling of elliptic flow in HIC may resemble Bjorken scaling when adding an angular dependence on the structure functions.



Let us entertain a crazy idea

What if parton distribution functions became azimuthally asymmetric, but still kept the running we expect from QCD???

v_2 of Photons: as expected, v_n would be an initial state effect!

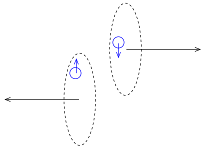
Scaling in x, Q: exactly as expected from Bjorken-like running

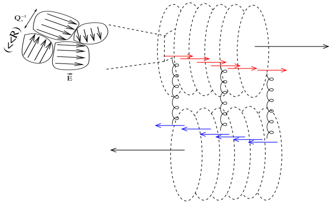
Particle species: protected by unitarity of the fragmentation function

But there is a reason I called it crazy: PDFs are universal and QCD is azimuthally symmetric!

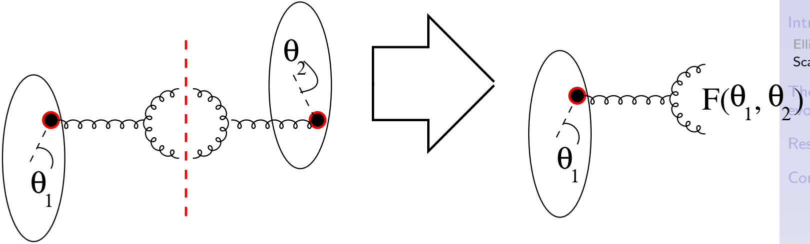
Could structure functions be azimuthally asymmetric?

- Sivers functions (spin difference gives you an asymmetry) But uncorrelated with geometry, special role for v_2 so unlikely
- "Color antennae" and such (CGC models, Kovner et al,Gyulassy,Biro,...) Since antenna point in random directions, effect always goes away for large systems ("many antennae") I think scaling implies Same origin for pA,AA





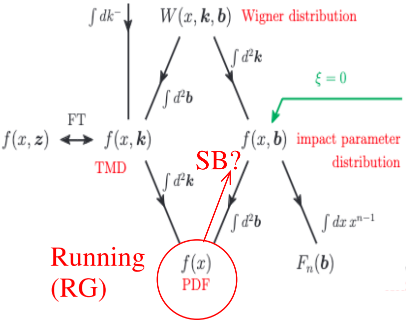
Could structure functions be azimuthally asymmetric?



The running of $f(x, Q)$ is really an RG equation, $f(x, Q)$ probe dependent at subleading order in α_s . At $\mathcal{O}\left(\alpha_s^2 \epsilon_n\right) \ll v_n$ (2nd and higher Twist) they should generally be azimuthally asymmetric for extended probes. Can this small effect be amplified?

RG, symmetry breaking and nucleon maps

M.Diehl
1512.01328

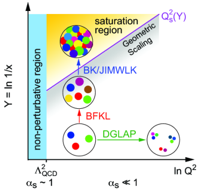


Usually $f(x)$ is thought of as averaging transverse information, but it is a quantum operator, subject to symmetry breaking

$$\frac{Q}{2}\frac{\partial}{\partial Q}\frac{\partial xG(x,Q^2)}{\partial ln(1/x)} = \frac{\alpha_s N_c}{\pi} xG(x,Q^2)$$

$G(x,Q)$ evolve according to renormalization-group type linear operator evolution equations (DGLAP in Q,BFKL in x) But in x evolution blows up. This evolution breaks Froissart's bound (unitarity in hadron-hadron scattering) at low x.

In order to correct this, a non-linear term is added.

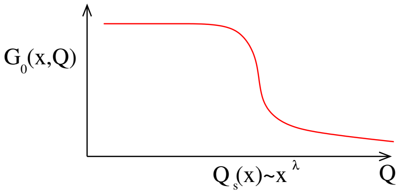


The GLR-MQ evolution equation

In the dense parton limit, the equation that governs the evolution of parton distribution functions inside hadrons is thought to be given by

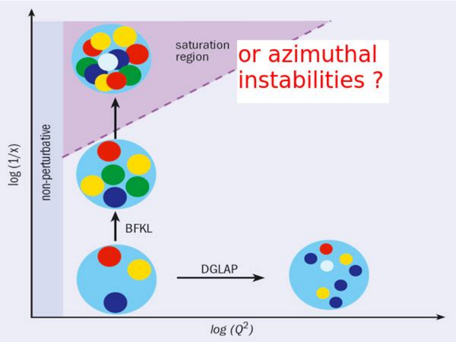
$$\frac{Q}{2}\frac{\partial}{\partial Q}\frac{\partial xG(x,Q^2)}{\partial ln(1/x)} = \frac{\alpha_s N_c}{\pi} xG(x,Q^2) - \frac{\alpha_s^2 N_c \pi}{2 C_F S_\perp}\frac{1}{Q^2}[xG(x,Q^2)]^2 \quad (2)$$

(It is a high Q limit of an integro-differential (GLR) equation).



Balancing the linear and the non-linear term defines the saturation scale Q_s, assuming azimuthal symmetry

Saturation together with an RG picture for saturation generates JIMWLK action, CGC (JIMWLK/CGC result : Azimuthally symmetric action, asymmetric boundary conditions



But non-linear 2+1 differential equation *can have instabilities* breaking the underlying symmetry!

Our proposal

Adding an angular dependence the GLR-MQ equation and keeping the same limits modify the equations the following way

$$\frac{xQ}{2}\left(\frac{\partial}{\partial Q}+\frac{1}{Q}\frac{\partial}{\partial\phi}\right)\frac{\partial}{\partial x}[xG(x,Q^2,\phi)] = \frac{\alpha_s N_c}{\pi}xG(x,Q^2,\phi)$$

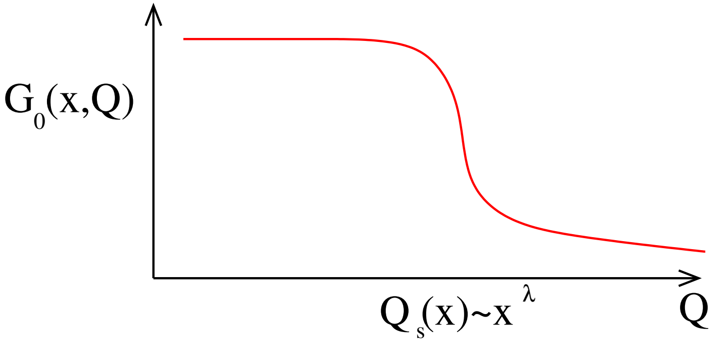
$$-\frac{\alpha_s^2 N_c \pi}{2C_F S_\perp}\frac{1}{Q^2}[xG(x,Q^2,\phi)]^2$$

(NB: angular ladder effects neglected as a first attempt,willmodify this qualittive estimate)

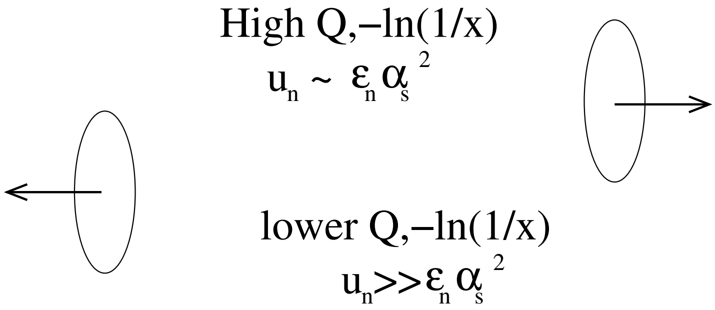
As a solution, we try

$$G(x,Q^2,\phi) = G_0(x,Q^2)\left(1+\sum_{n=1}^{\infty} u_n(x,Q^2)cos(n\phi+\beta_n)\right),$$

$G_0(x,Q^2)$ is the azimuthally symmetric solution (i.e. saturation)



Azimuthal symmetry as a broken symmetry



Small geometry-driven anisotropies at higher x, Q amplified by evolution

Azimuthal symmetry as a broken symmetry

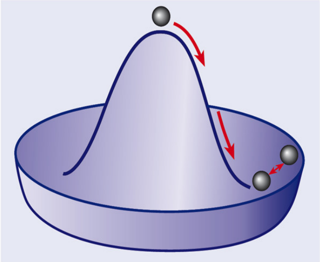


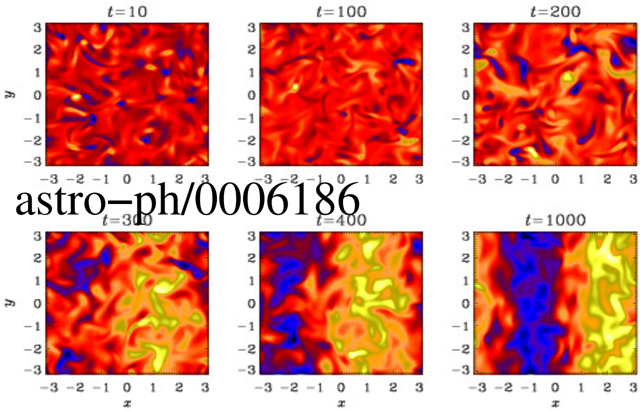
Figure: Elliptic flow ν_2 vs. rapidity [2,3].

Arbitrary small tilt (tiny gradients at high x) produce large effects at low x. Different from CGC effects since lagrangian aquires a θ dependence (which will need to be added to JIMWLK equation)

Non-linear evolution can break underlying symmetries

If non-linearities are strong enough, azimuthal symmetries broken dynamically. In hydrodynamics this effect is well-known but exists in most 2+1 non-linear systems

2+1 non-linear evolution equation



For unintegrated in $x_\perp$ General Parton distribution functions we could have: "Inverse cascade": Instabilities go from high frequency (local in transverse space) to low frequency as x evolves. No "many antennae" problem.

Equations for the Fourier coefficients

Working on the limiting case $Q << Q_s(x)$, we insert the solution with azimuthal perturbations into eq. fully asymmetric GLR-MQ equation and get three linear equations for our Fourier coefficients.

1. An infinite set of equations equation that relate the Fourier coefficients with the phases.

$$\sum_k u_k^2(x, Q^2) cos(2\beta_k) = 0 \tag{3}$$

2 An infinite set of equations regarding only the derivative with respect to x.

$$x\frac{\partial u_n(x,Q^2)}{\partial x} = -(2\lambda+1)u_n(x,Q^2)$$

$$+\frac{N_c\pi}{2C_F S_\perp \alpha_s^2}\frac{1}{Q^2}x^{2\lambda+1}\frac{1}{n}\left[\sum_k^{n-1} u_k(x,Q^2)u_{n-k}(x,Q^2)sin(\beta_n-\beta_k-\beta_{n-k})\right.$$

$$\left.+2\sum_k u_k(x,Q^2)u_{n+k}(x,Q^2)sin(\beta_n+\beta_k-\beta_{n+k})\right] \quad (4)$$

3 An infinite set of equations that regards derivatives with respect to Q and mixed terms.

$$(2\lambda+1)\frac{Q}{2}\frac{\partial u_n(x,Q^2)}{\partial Q}+\frac{Q}{2}x\frac{\partial^2 u_n(x,Q^2)}{\partial Q\partial x}=\frac{\alpha_s N_c}{\pi}u_n(x,Q^2)$$

$$+\frac{N_c\pi}{2C_F S_\perp \alpha_s^2}\frac{1}{Q^2}x^{2\lambda+1}\Big[2u_n(x,Q^2)$$

$$+\frac{1}{2}\sum_k^{n-1}u_k(x,Q^2)u_{n-k}(x,Q^2)cos(\beta_n-\beta_k-\beta_{n-k})$$

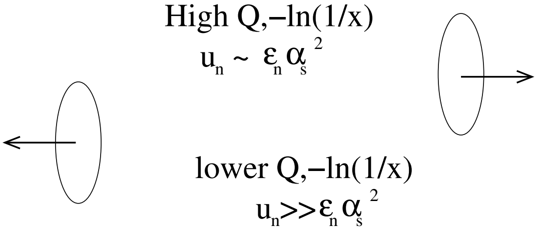
$$+\sum_k u_k(x,Q^2)u_{n+k}(x,Q^2)cos(\beta_n+\beta_k-\beta_{n+k})\Big] \quad (5)$$

As an ansatz we propose

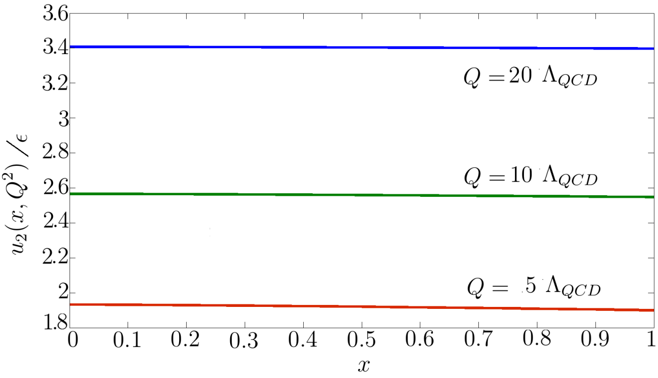
$$u_n(x, Q^2) = \delta_{n,2} \sum_{k=0}^{\infty} A_k \frac{(Bx^C)^k}{k!} Q^{D-2k} \qquad (6)$$

then solve the equation linearized in u_k from initial conditions

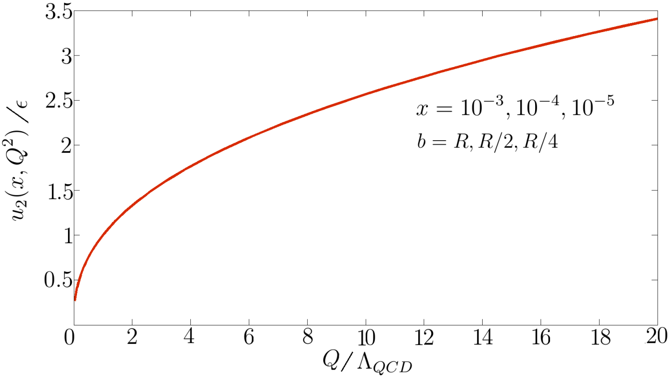
$$u_n(\ln x^{-1} \to 0, Q) \sim \epsilon_n \alpha_s^2$$



Preliminary results



Preliminary results



Preliminary results:very encouraging

- Near independence of $u_n(Q,x)$ on x (all dependence on $G_0(Q,x)$ which in turn depends weakly on Q . Just like v_2
- Near linear dependence on ϵ_n Just like v_2
- near decoupling of fourier modes

Forthcoming: A phenomenological study including factorization and fragmentation

What if were right?

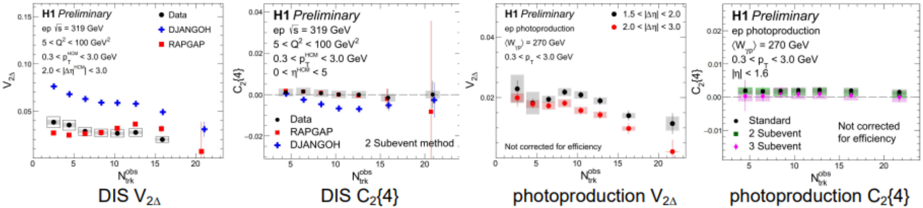
Relation between v_n non-linearities could be more predictive than hydro models, fewer parameters so easier to falsify

Photon correlations Correlations between high rapidity photons and mid-rapidity hadrons, pA and AA

And the ultimate signature is...

Ridges/v_n at the EIC?

HERA looked for flow and failed

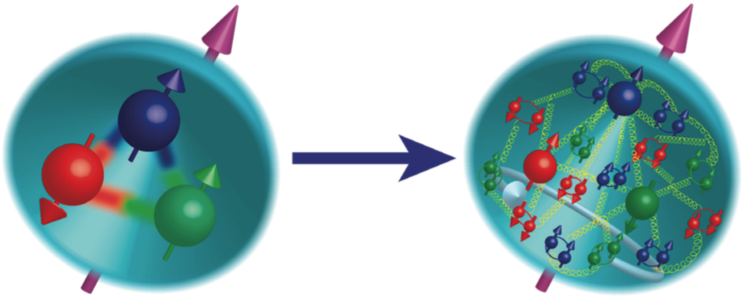


QM 2022, Chuan Sun

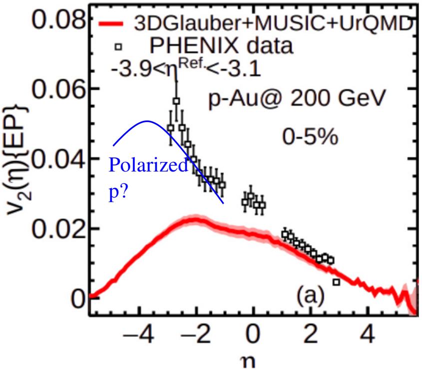
UrEIC could dramatically show if something like what Im advocating here could be true

Alternatively, we are wrong and hydro valid to smallest scales (Poster,G.Soares)

Hydro in small systems could lead to "classical spin measurement"



Spin dependent nucleon shape changes v_2 in polarized pA collisions. ultimate small system hydrodynamics?



Spin dependent nucleon shape changes v_2 in polarized pA collisions. ultimate small system hydrodynamics?

Conclusions

- ν_2 scaling similar to scaling of parton distribution functions. Could they be azimuthally asymmetric?
- Instabilities in the non-linear regime?
- Work in progress to develop this hypothesis to quantiative test level

References

[1] Torrieri G., Betz B. and Gyulassy M. - Scaling of elliptic flow in heavy-ion collisions.

[2] CMS collaboration, PRL 109 202301(2012)

[3] PHOBOS collaboration, Nucl. Phys. A830 (2009)

[4] STAR collaboration, 1206.5528

[5] BRAHMS collaboration, Nucl. Phys. A 830 43C (2009)

[6] PHENIX Collaboration - Observation of direct-photon collective flow in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.