One-loop corrections to single and double inclusive hadron production in DIS at small x

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OUTLINE

Gluon saturation: a brief introduction

a dense system of gluons

The Color Glass Condensate: a framework

classical fields

renormalization group

Toward precision CGC

Next to Leading Order

single and double inclusive hadron production

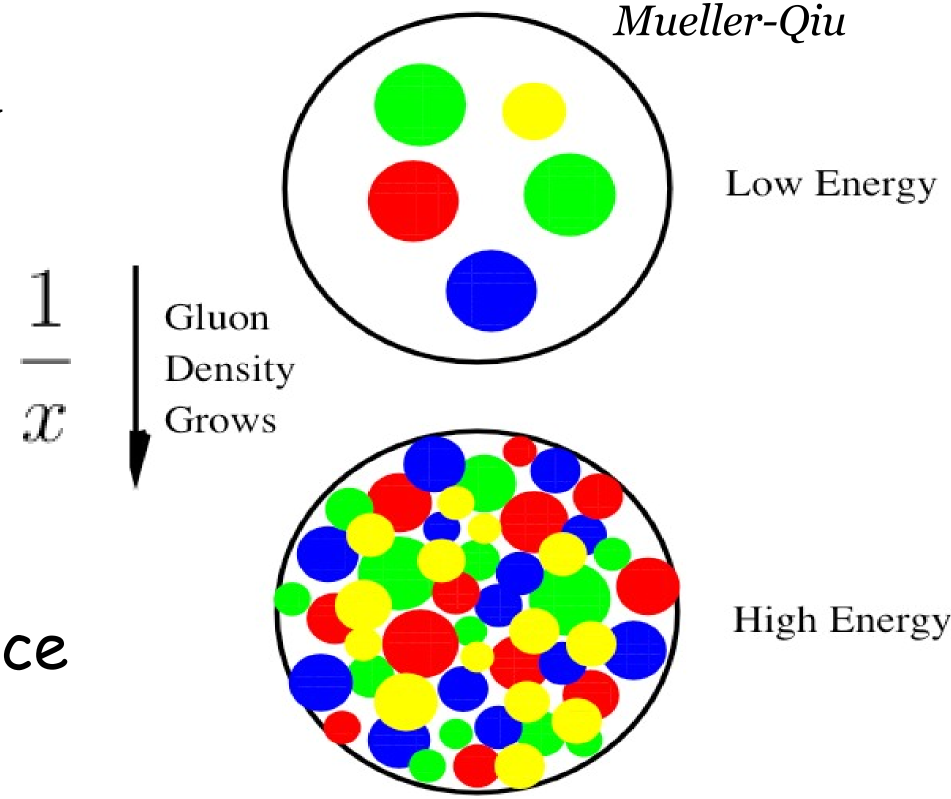
sub-eikonal corrections

Utilizing the full kinematics of EIC

transition from large to small x

A hadron at high energy

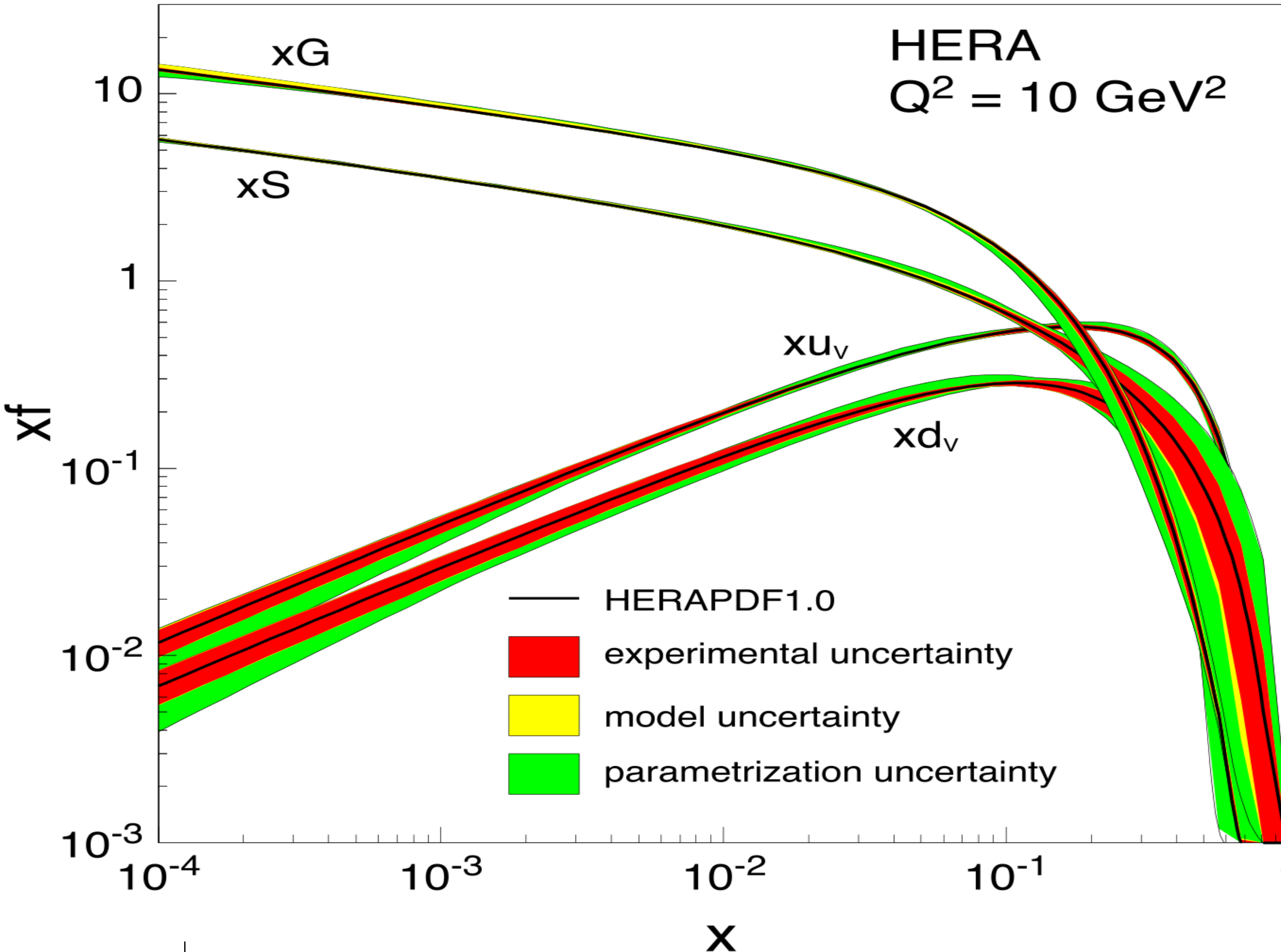
Gribov-Levin-Ryskin
Mueller-Qiu



radiated gluons have the same size ($1/Q^2$) - the number of partons increase due to the increased longitudinal phase space

hadron/nucleus becomes a dense system of gluons: concept of a quasi-free parton is not useful

can reach the same dense state in a <u>nucleus at lower energy</u>



$x = \frac{p^+}{P^+}$ ***x** is the fraction of hadron energy carried by a parton*

What drives the growth of parton distributions?

Splitting functions at leading order $O(\alpha_S{}^0)$ $(x \neq 1)$

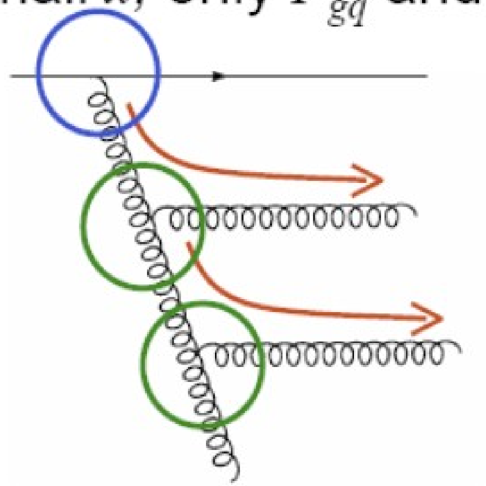
$$P_{qq}^{(0)}(x) = C_F \frac{1+x^2}{1-x}$$

$$P_{qg}^{(0)}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]$$

$$P_{gq}^{(0)}(x) = C_F \frac{1+(1-x)^2}{x}$$

$$P_{gg}^{(0)}(x) = 2C_A \left[\frac{x}{1-x} + \frac{1-x}{x} + x(1-x)\right]$$

At small x, only P_{gq} and P_{gg} are relevant.



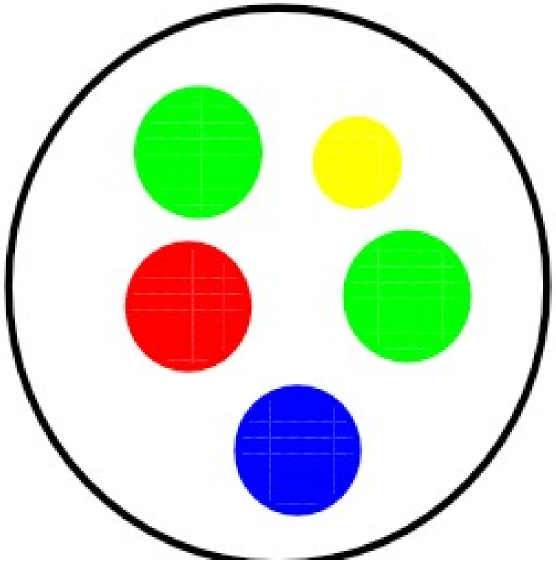
→ Gluon dominant at small *x*!

The double log approximation (DLA) of DGLAP is easily solved.

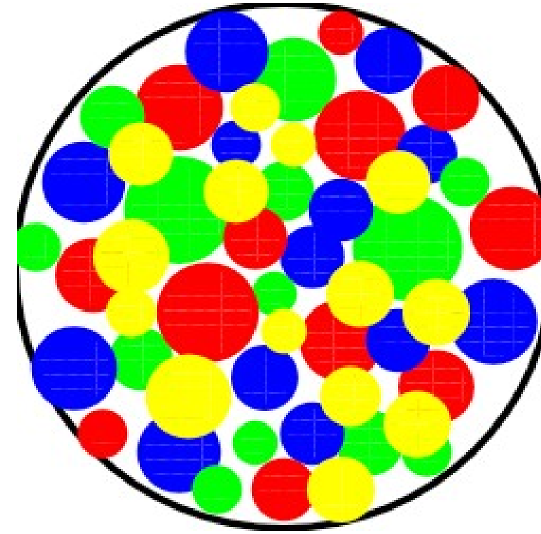
-- increase of gluon distribution at small x

$$\mathbf{xg(x, Q^2) \sim e^{\sqrt{\alpha_s\,(\log 1/x)\,(\log Q^2)}}}$$

new QCD dynamics at small x ?



energy ~ 1/x

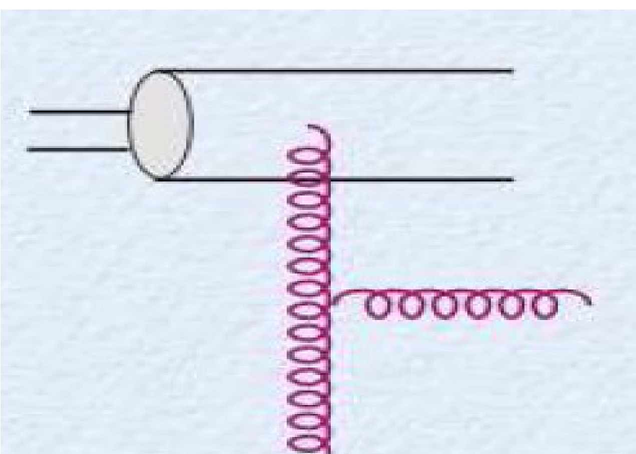


$$\frac{\alpha_s}{Q^2}\,\frac{xG(x,Q^2)}{\pi r^2} \sim 1$$

saturation scale

$$Q_s^2(x, b_t, A) \sim A^{1/3}\,(\frac{1}{x})^{0.3}$$

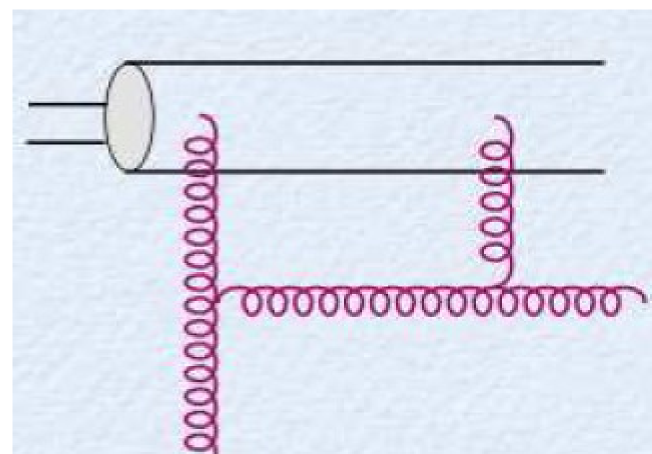
"attractive" bremsstrahlung *vs.* *"repulsive" recombination*



included in pQCD

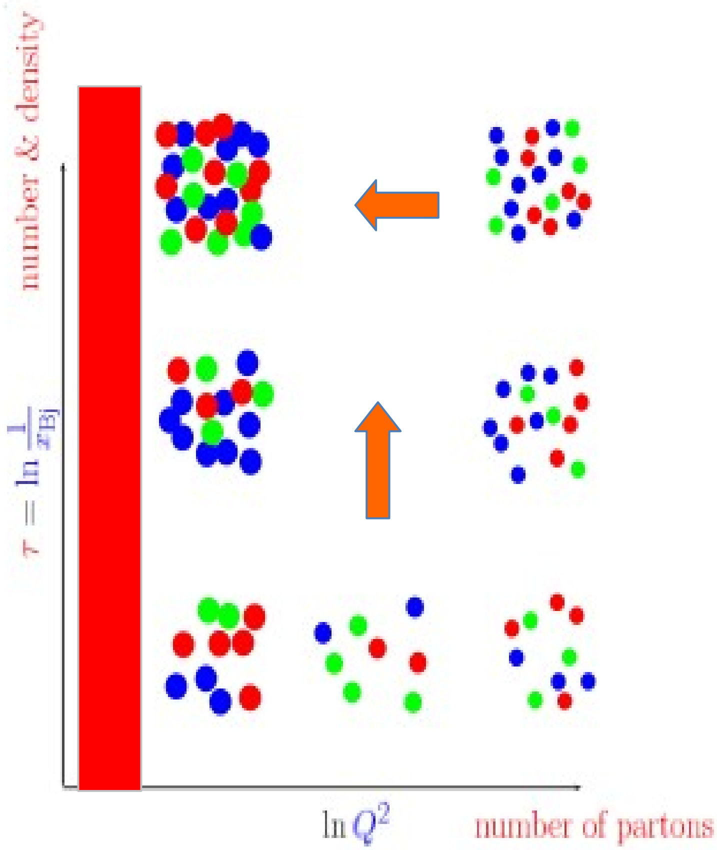
$$S \to \infty,\;\; Q^2 \;\; fixed$$

$$x_{Bj} \equiv \frac{Q^2}{S} \to 0$$



not included in pQCD

Small x QCD:
many-body dynamics of universal gluonic matter (**CGC**)



How does this happen ?

How do correlation functions of these evolve ?

Are there scaling laws ?

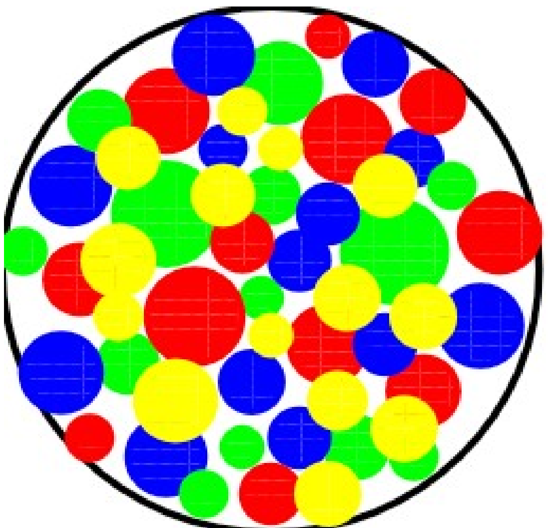
Can CGC explain aspects of HIC ?

Initial conditions for hydro?
Thermalization ?
Long range rapidity correlations ?
Azimuthal angular correlations ?
Nuclear modification factor ?

Collinear factorization breaks down at small x

<u>Need a new formalism for this high density state</u>

A very large nucleus at high energy: MV model

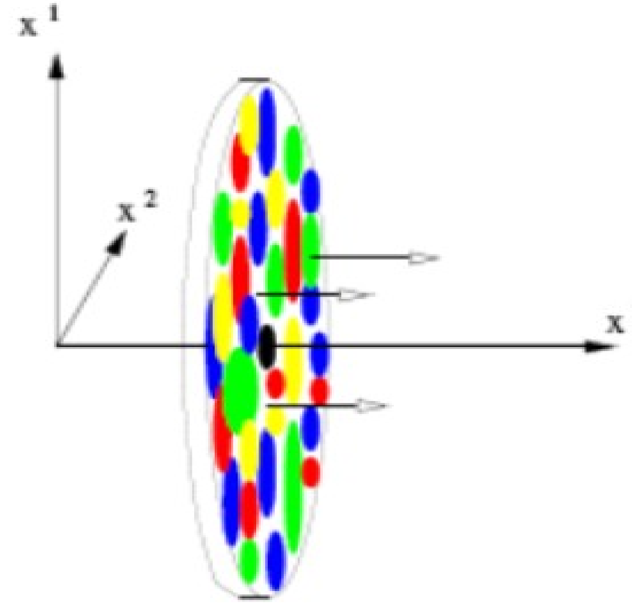


boost

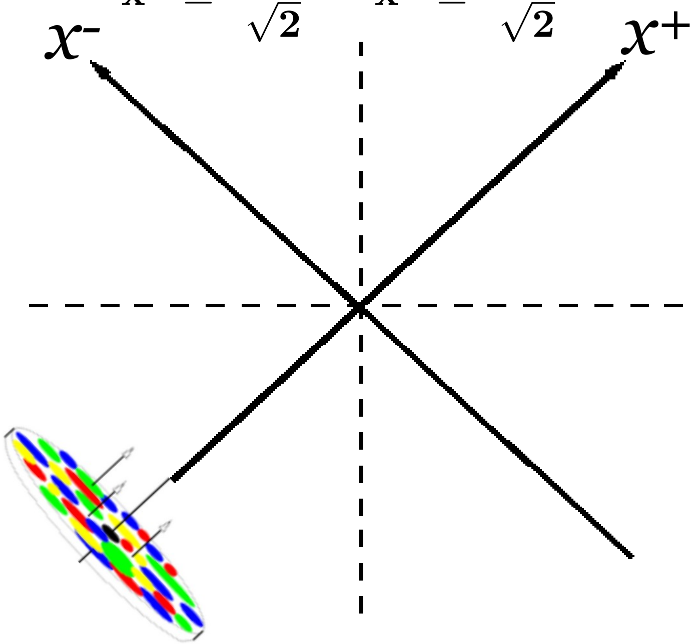
$$R \to \frac{R}{\gamma}$$

$\gamma \sim 100$ RHIC

$\gamma \sim 2500$ LHC



$$\mathbf{x}^{+} \equiv \frac{\mathbf{t}+\mathbf{z}}{\sqrt{2}} \qquad \mathbf{x}^{-} \equiv \frac{\mathbf{t}-\mathbf{z}}{\sqrt{2}}$$



sheet of color charge moving along x^{+} and sitting at $x^{-} = 0$

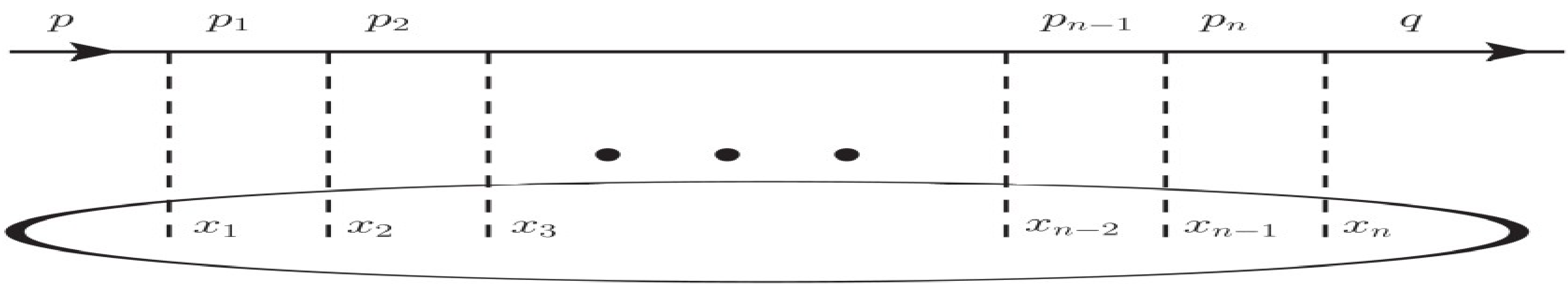
$$\boxed{\mathbf{J}^{\mu}_{\mathbf{a}}(\mathbf{x}) \equiv \delta^{\mu +}\, \delta(\mathbf{x}^{-})\, \rho_{\mathbf{a}}(\mathbf{x_t})}$$

color current *color charge*

$$\mathbf{A}^{\mathbf{a}}_{\mathbf{i}}(\mathbf{x}^{-}, \mathbf{x_t}) = \theta(\mathbf{x}^{-})\, \alpha^{\mathbf{a}}_{\mathbf{i}}(\mathbf{x_t})$$

with $\quad \partial_i\, \alpha^a_i = g\, \rho^a$

Dense proton/nucleus: multiple eikonal scatterings



sum over all scatterings $\qquad i\mathcal{M} = \sum_n i\,\mathcal{M}_n$

$$i\mathcal{M}(p,q) = 2\pi\delta(p^+ - q^+)\,\bar{u}(q)\,\not{n} \int d^2x_t\, e^{-i(q_t - p_t)\cdot x_t}\,\left[V(x_t) - 1\right]\,u(p)$$

with

$$V(x_t) \equiv \hat{P}\,\exp\left\{ig\int_{-\infty}^{+\infty} dx^+\, n^-\, S_a(x^+, x_t)\, t_a\right\}$$

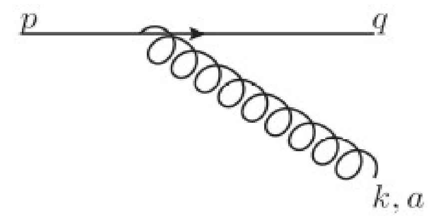
Wilson lines: effective degrees of freedom that contain all the target information

$$\frac{d\,\sigma^{q\,T\to q\,X}}{d^2p_t\,dy} \sim |i\mathcal{M}|^2 \sim\ F.T.\ < Tr\, V(x_t)\, V^\dagger(y_t) >$$

dipole

One-loop correction: energy dependence

basic ingredient: soft radiation vertex (LC gauge)



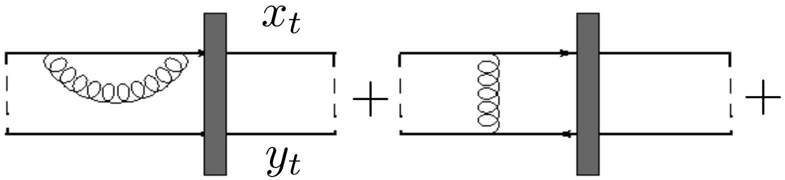
$$g\,\bar{u}(q)\,t^a\,\gamma_\mu\,u(p)\,\epsilon^{\mu}_{(\lambda)}(k) \longrightarrow 2\,g\,t^a\,\frac{\epsilon_{(\lambda)}\cdot k_t}{k_t^2}$$

coordinate space:

$$\int \frac{d^2k_t}{(2\pi)^2} e^{ik_t\cdot(x_t-z_t)} 2\,g\,t^a\,\frac{\epsilon_{(\lambda)}\cdot k_t}{k_t^2} = \frac{2\,i\,g}{2\pi}\,t^a\,\frac{\epsilon_{(\lambda)}\cdot(x_t-z_t)}{(x_t-z_t)^2}$$

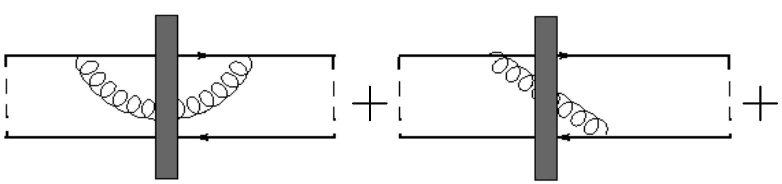
x_t, z_t are transverse coordinates of the quark and gluon

<u>virtual corrections</u>:



$$\longrightarrow Tr\,V(x_t)\,V^{\dagger}(y_t)$$ *a dipole*

<u>real corrections</u>:



$$\longrightarrow Tr\,V(x_t)\,V^{\dagger}(z_t)\,Tr\,V(z_t)\,V^{\dagger}(y_t)$$

$$\frac{1}{(x_t-z_t)^2} \qquad \frac{(x_t-z_t)\cdot(y_t-z_t)}{(x_t-z_t)^2(y_t-z_t)^2}$$

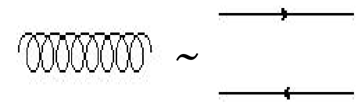
the S matrix

$$S(x_t,y_t) \equiv \frac{1}{N_c} Tr\,V(x_t)\,V^{\dagger}(y_t)$$

One-loop corrections: BK-JIMWLK eq.

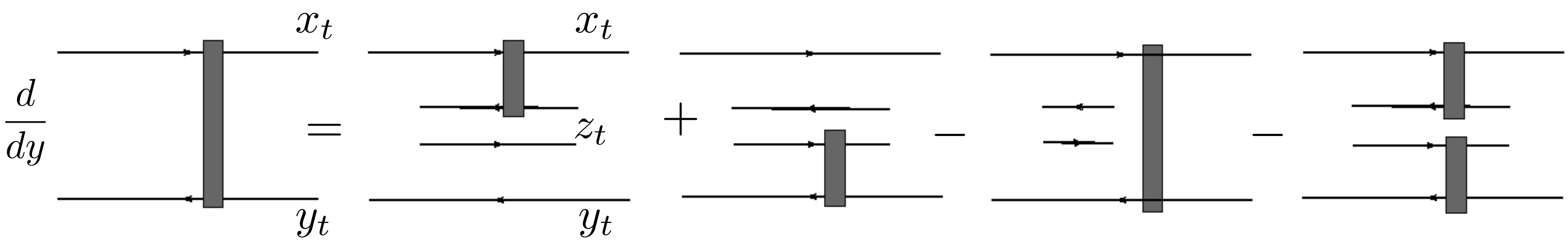
at large N_c

$3 \otimes \bar{3} = 8 \oplus 1 \simeq 8$



$$\frac{d}{dy} T(x_t, y_t) = \frac{N_c \alpha_s}{2\pi^2} \int d^2 z_t \frac{(x_t - y_t)^2}{(x_t - z_t)^2 (y_t - z_t)^2} \left[T(x_t, z_t) + T(z_t, y_t) - T(x_t, y_t) - T(x_t, z_t) T(z_t, y_t) \right]$$

$$T \equiv 1 - S$$



$$\tilde{T}(p_t) \sim \frac{1}{p_t^2} \left[\frac{Q_s^2}{p_t^2} \right] \qquad Q_s^2 \ll p_t^2$$

$$\tilde{T}(p_t) \sim \log \left[\frac{Q_s^2}{p_t^2} \right] \qquad Q_s^2 \gg p_t^2$$

$$\tilde{T}(p_t) \sim \frac{1}{p_t^2} \left[\frac{Q_s^2}{p_t^2} \right]^\gamma \qquad Q_s^2 < p_t^2$$

nuclear modification factor

$$R_{pA} \equiv \frac{\frac{d\sigma^{pA}}{d^2 p_t \, dy}}{A^{1/3} \frac{d\sigma^{pp}}{d^2 p_t \, dy}}$$

suppression of p_t spectra

nuclear shadowing

centrality dependence

....

Probing saturation in high energy collisions

need quite a bit of modeling

"nucleus-nucleus" (dense-dense)

"proton-nucleus" (dilute-dense)

DIS

much less modeling

structure functions (diffraction)
NLO *dihadron/dijet correlations*
3-hadron/jet angular correlations

signatures in production spectra

multiple scattering via Wilson lines:

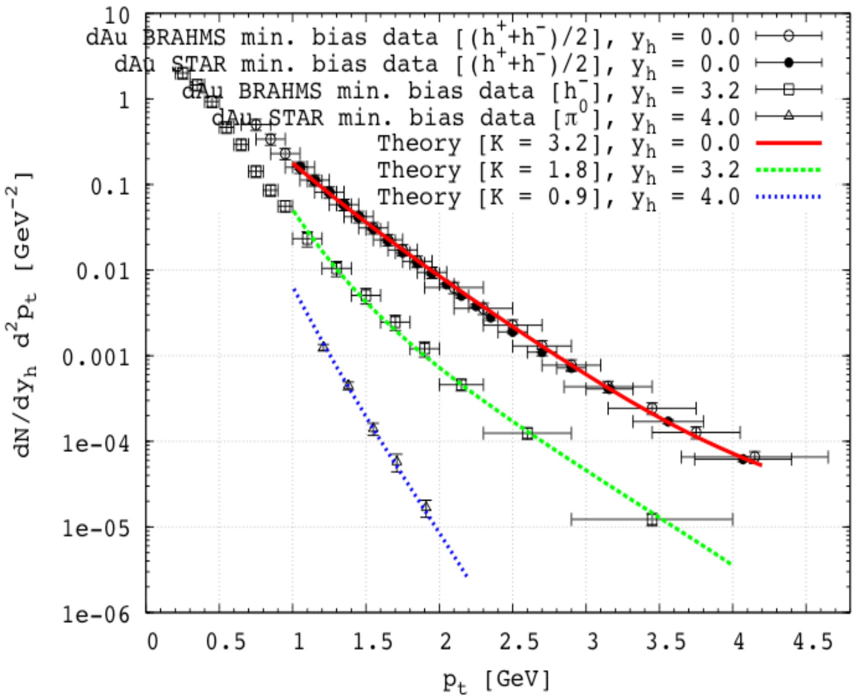
p_t broadening

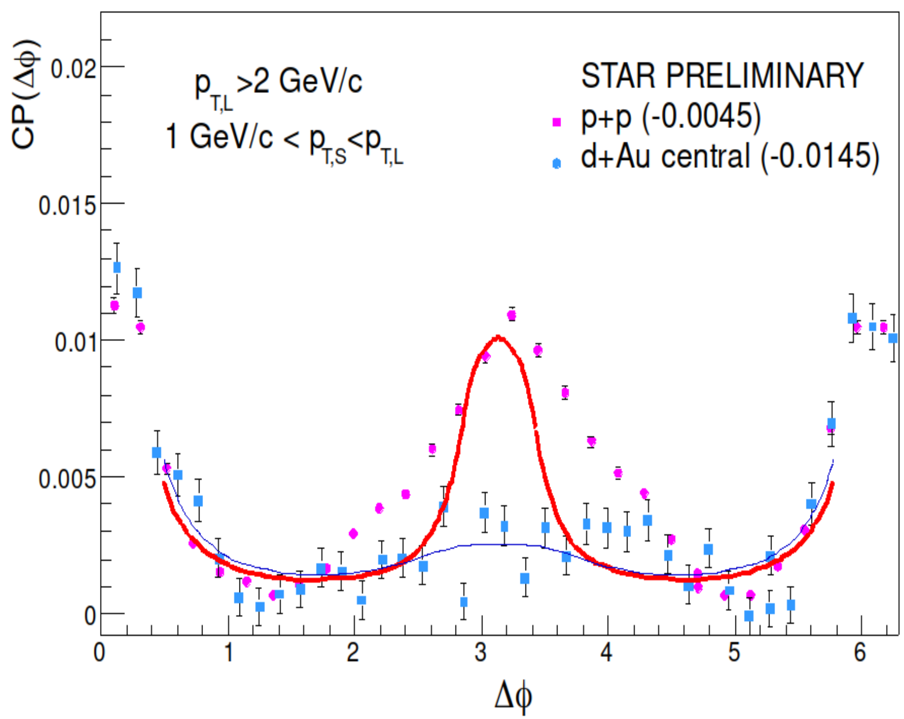
evolution with x (energy) via JIMWLK:

suppression of spectra/away side peaks

CGC at RHIC

Single and double inclusive hadron production in dA collisions



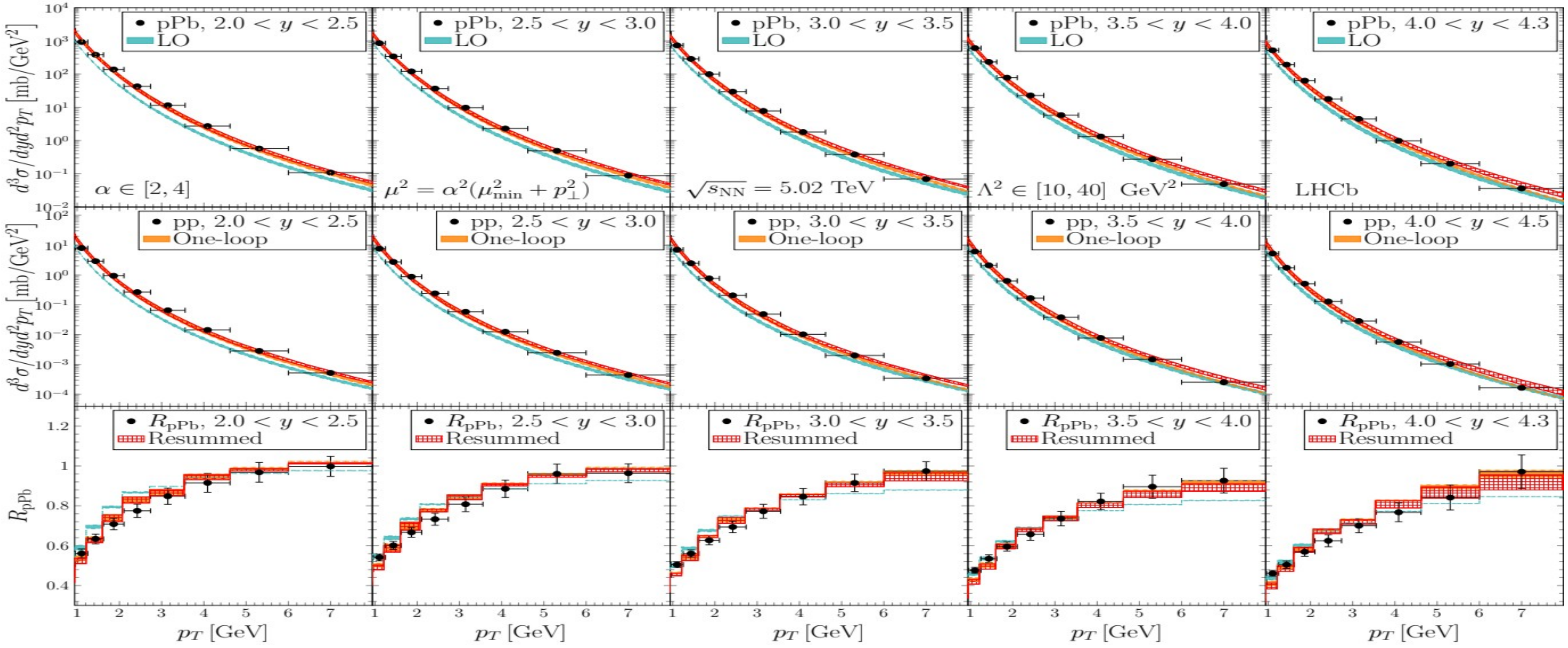


Dumitru,Hayashigaki, JJM, NPA770 (2006) 57

Albacete, Marquet, PRL105 (2010) 162301

CGC at NLO

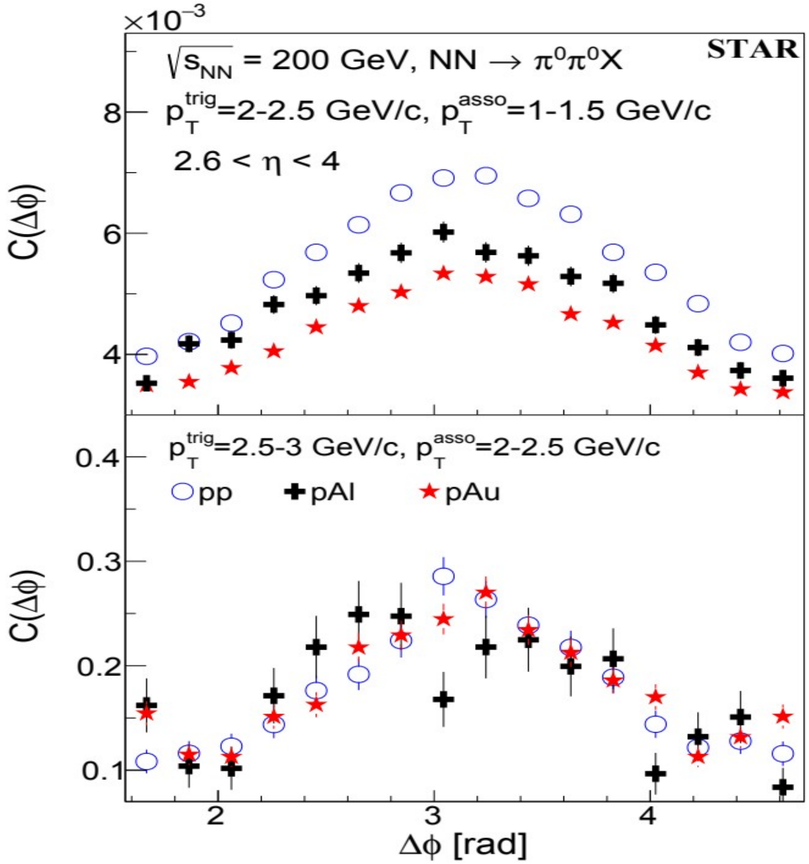
Single inclusive hadron production in pA collisions: LHCb

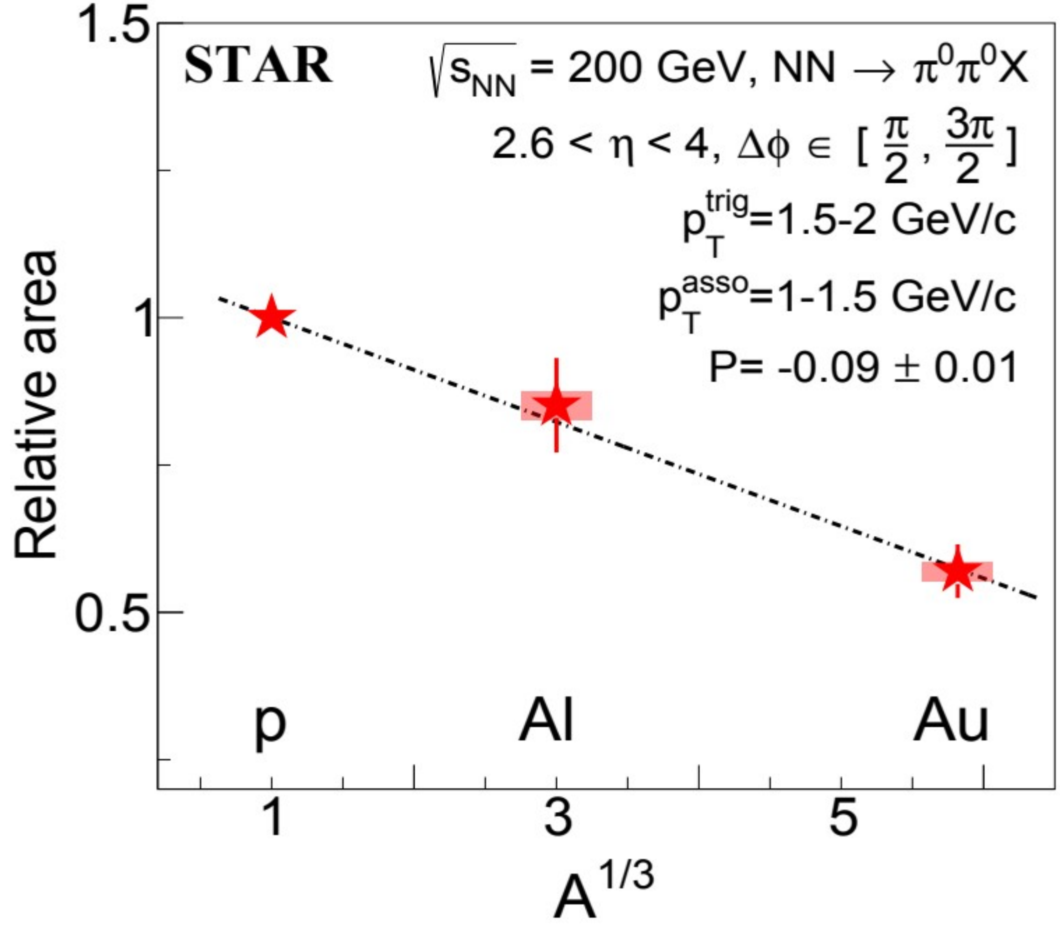


Shi, Wang, Wei, Xiao, arXiv:2112.06975

Back to back hadron production in pA collisions: forward rapidity

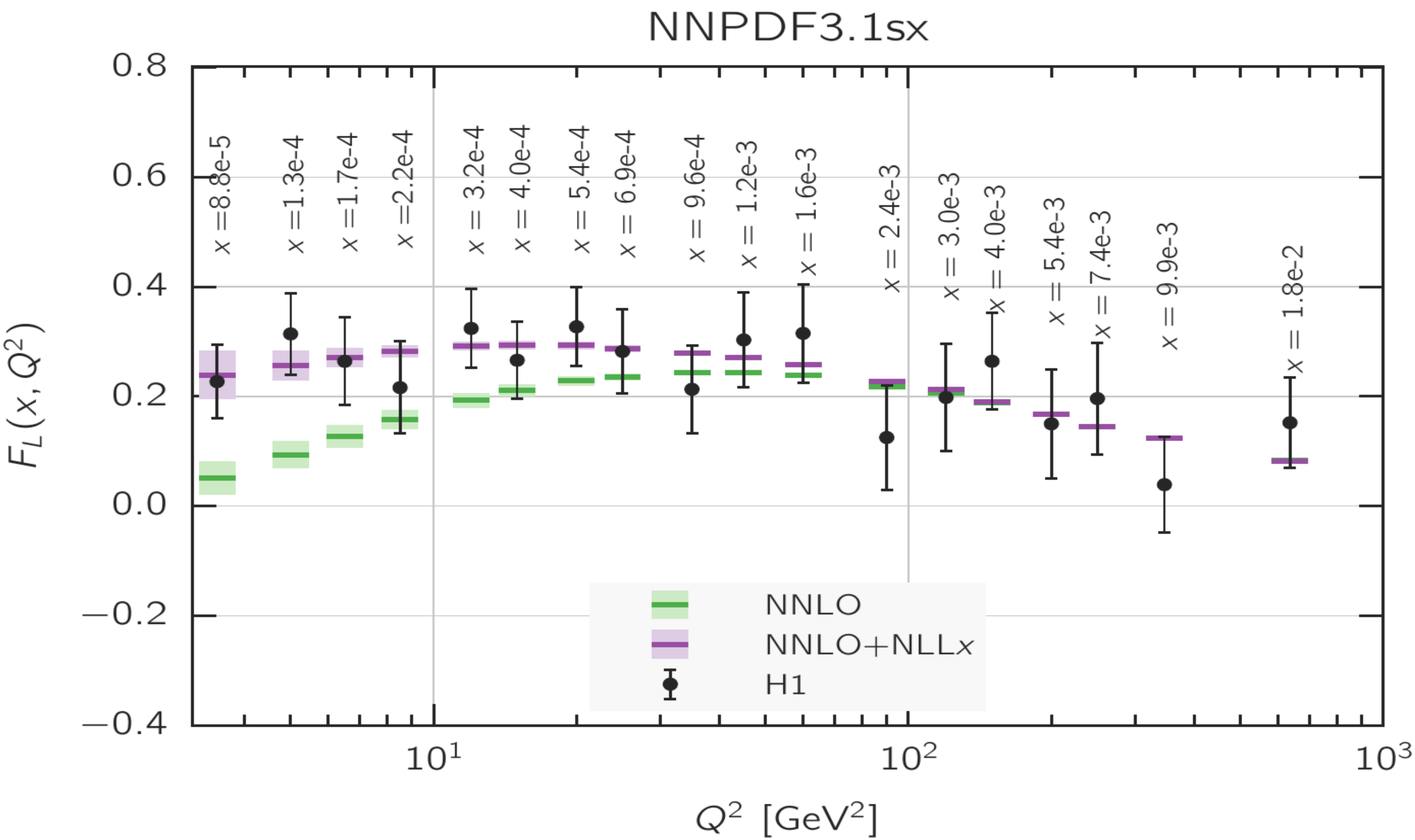
STAR collaboration(2021)
arXiv:2111.10396





A challenge for Gribov-Galuber model of shadowing!

F_L at HERA



arXiv:1710.05935

Toward precision CGC at small x: DIS

NLO corrections to DIS structure functions:

Beuf (2017)
Beuf, Lappi, Paatelainen (2022)
………………

NLO corrections to single inclusive hadron production in DIS:

Bergabo, JJM (2023)

NLO corrections to inclusive two-particle production in DIS:

Bergabo, JJM (2022, 2023)
Taels, Altinoluk, Beuf, Marquet (2022)
Caucal, Salazar, Schenke, Venugopalan (2022)
Caucal, Salazar, Venugopalan (2021)
……………………

Sub-eikonal corrections at small x:

Altinoluk, Armesto, Beuf (2023)
Altinoluk, Beuf, Czajka, Tymowska (2021, 2022)
……………

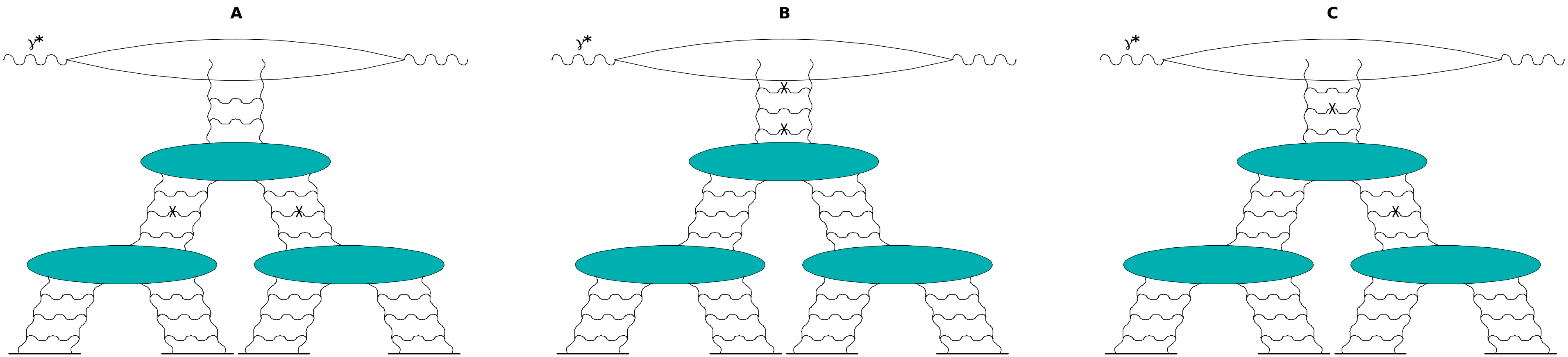
Diffraction, spin,…

Inclusive dihadron production in DIS at small x:

central vs forward rapidity

Inclusive dihadron production in midrapidity: LO

JJM, Yu. Kovchegov
PRD70 (2004) 114017



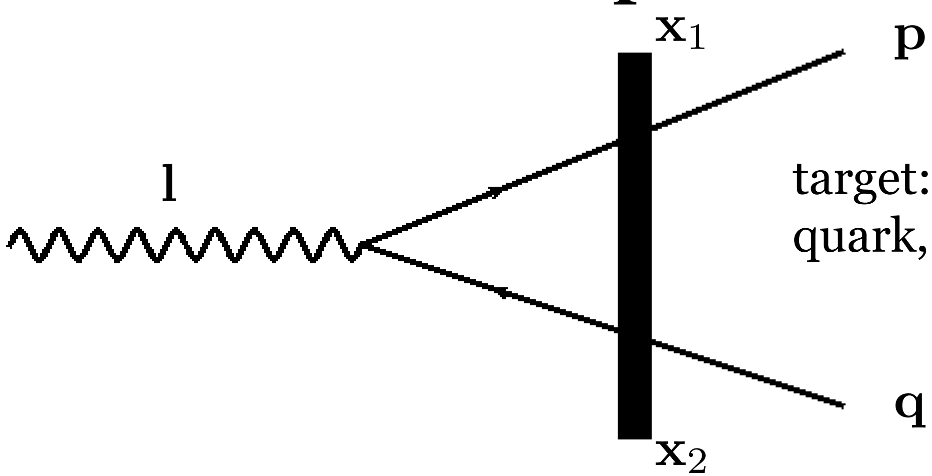
need a very large rapidity window

target is treated as a classical color field $\mathbf{A}_a^\mu = \delta^{\mu -}\, n^\mu\, S_a(x^+, \mathbf{x})$

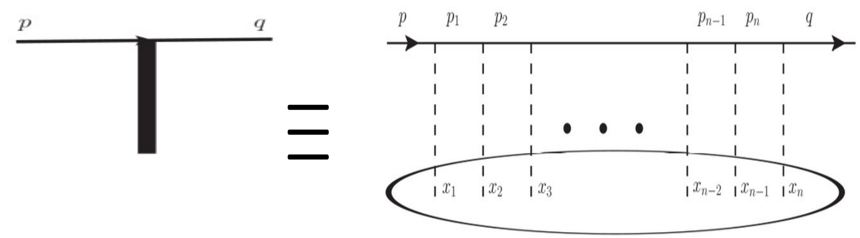
scatterings of gluons on the target encoded in Wilson lines $\mathbf{U}\ (\mathbf{x}_1), U^\dagger(\mathbf{x}_2)$

leading log evolution included

Inclusive dihadron production in forward rapidity: LO



target: a classical color field
quark, antiquark multiply scatter on the target



$$\frac{d\sigma^{\gamma^* A\to q\bar{q}X}}{d^2p\, d^2q\, dy_1\, dy_2} = \frac{e^2Q^2(z_1z_2)^2N_c}{(2\pi)^7}\delta(1-z_1-z_2)$$

$$\int d^8x_\perp e^{ip\cdot(x_1'-x_1)}e^{iq\cdot(x_2'-x_2)}\left[S_{122'1'}-S_{12}-S_{1'2'}+1\right]$$

with

$$\Big\{4z_1z_2K_0(|x_{12}|Q_1)K_0(|x_{1'2'}|Q_1)+$$

$$(z_1^2+z_2^2)\,\frac{x_{12}\cdot x_{1'2'}}{|x_{12}||x_{1'2'}|}\,K_1(|x_{12}|Q_1)K_1(|x_{1'2'}|Q_1)\Big\}$$

dipole $\mathbf{S}_{12}\equiv\frac{1}{N_c}Tr\,V(x_1)\,V^\dagger(x_2)$

$$\mathbf{x}_{12}\equiv\mathbf{x}_1-\mathbf{x}_2$$

quadrupole

$$\mathrm{S}_{122'1'}\equiv\frac{1}{N_c}Tr\,V(\mathbf{x}_1)\,V^\dagger(\mathbf{x}_2)\,V(\mathbf{x}_{2'})\,V^\dagger(\mathbf{x}_{1'})$$

Dominguez,Marquet,Xiao,Yuan, PRD 83 (2011) 105005

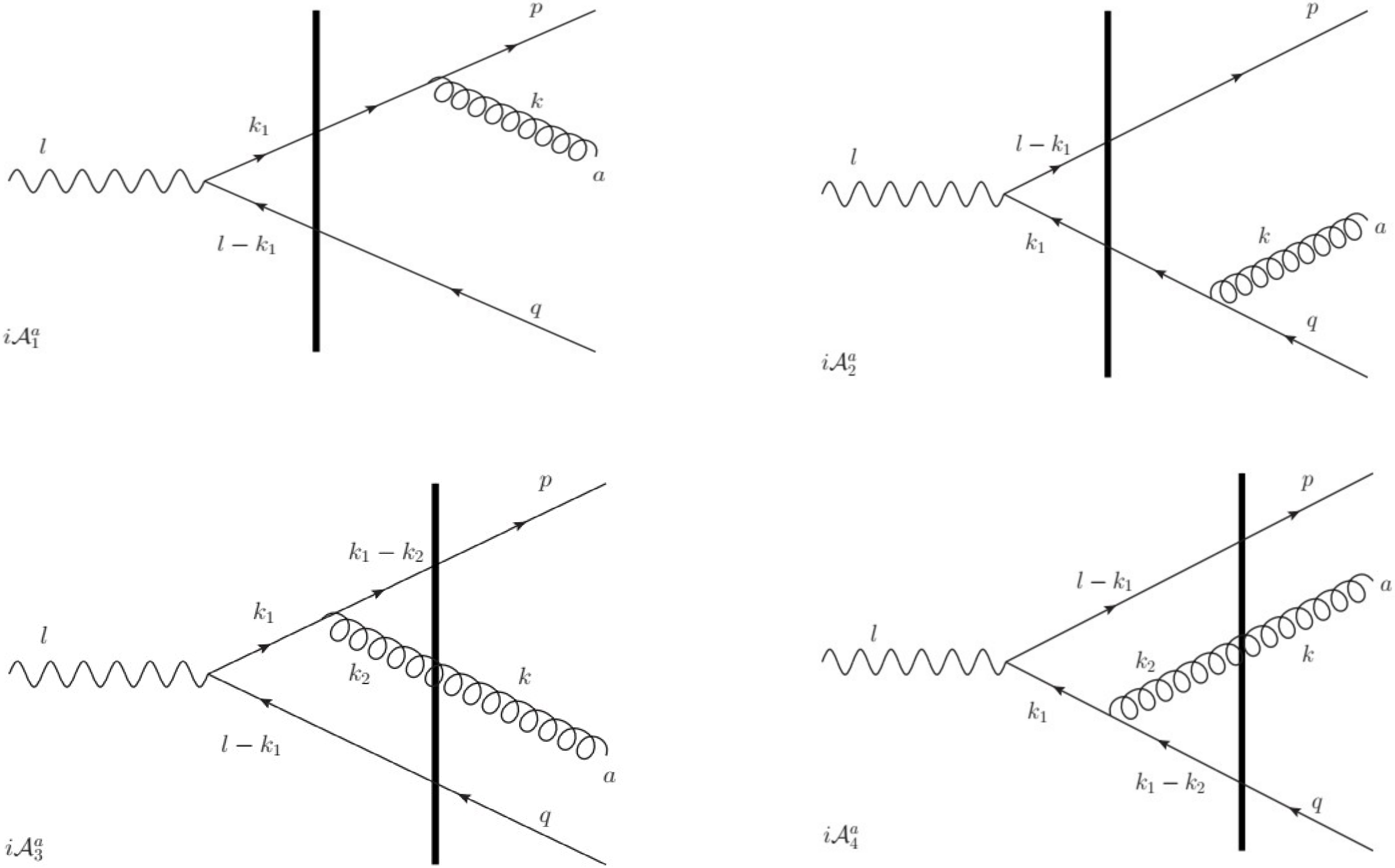
Inclusive dihadron production in forward rapidity: NLO

Based on F. Bergabo and JJM:

PRD 107 (2023) 5, 054036
JHEP 01 (2023) 095
NPA 1018 (2022) 122358
PRD 106 (2022) 5, 054035

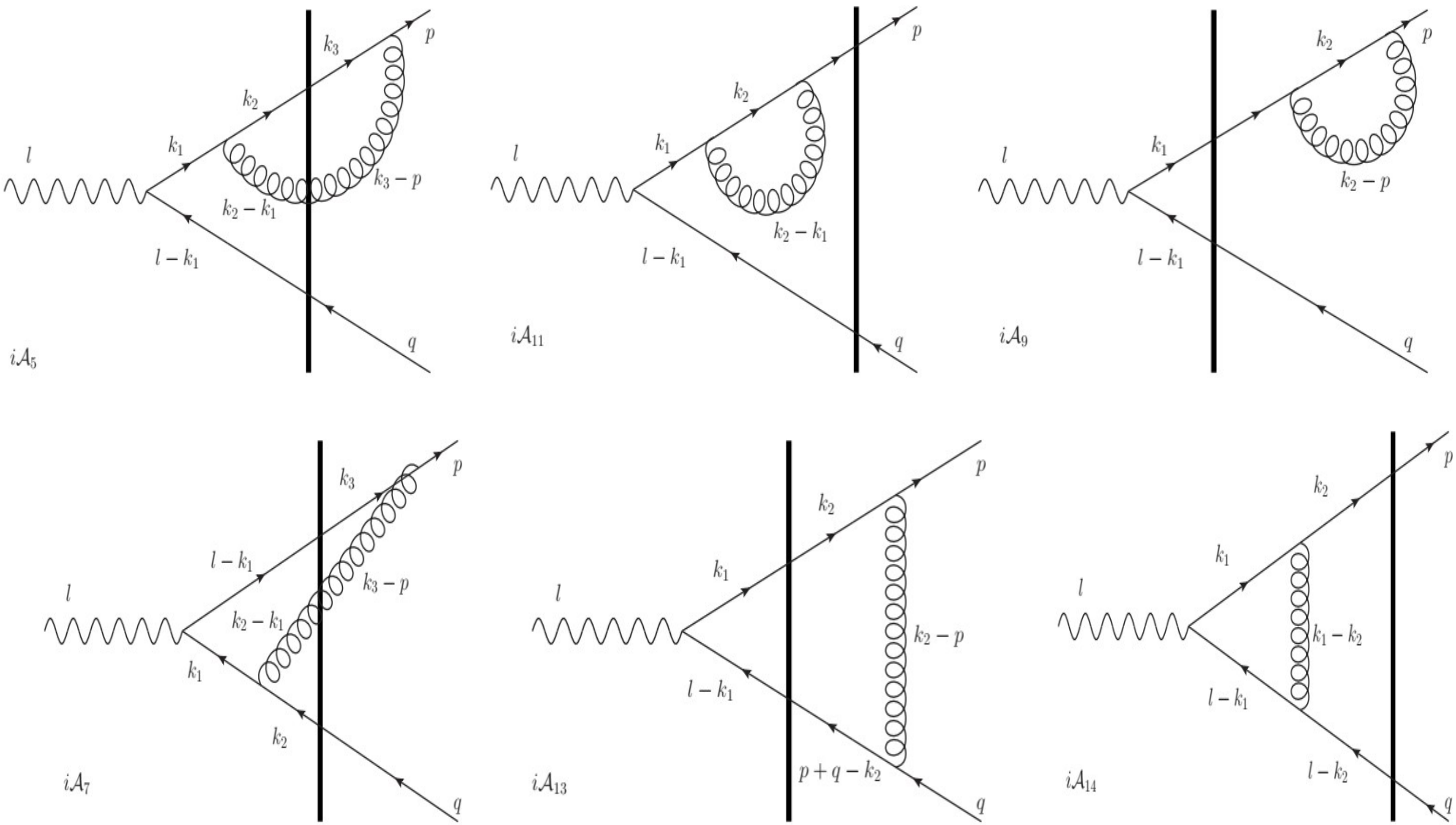
NLO dijets (+Sudakov): F. Salazar, Thursday

One loop corrections - real diagrams



3-parton production: Ayala, Hentschinski, JJM, Tejeda-Yeomans
PLB 761 (2016) 229 and NPB 920 (2017) 232

One loop corrections – virtual diagrams



<u>F. Bergabo</u> and JJM, dihadrons, 2207.03606
P. Taels et al., dijets, 2204.11650
P. Caucal et al., dijets, 2108.06347

$$\frac{\sigma^{\text{Real},L}_{1-1}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{2e^2g^2Q^2N_c^2z_2^3(1-z_2)^2(z_1^2+(1-z_2)^2)}{(2\pi)^{10}z_1}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(|\mathbf{x}_{12}|Q_2)K_0(|\mathbf{x}_{1'2'}|Q_2)\Delta^{(3)}_{11'}$$
$$[S_{122'1'}-S_{12}-S_{1'2'}+1]\,e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}e^{i\frac{z}{z_1}\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}\delta(1-z_1-z_2-z).$$
$$\frac{\sigma^{\text{Real},L}_{2-2}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{2e^2g^2Q^2N_c^2z_1^3(1-z_1)^2(z_2^2+(1-z_1)^2)}{(2\pi)^{10}z_2}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(|\mathbf{x}_{12}|Q_1)K_0(|\mathbf{x}_{1'2'}|Q_1)\Delta^{(3)}_{22'}$$
$$[S_{122'1'}-S_{12}-S_{1'2'}+1]e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\frac{z}{z_2}\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}\delta(1-z_1-z_2-z).$$
$$\frac{\sigma^{\text{Real},L}_{1-2}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{-2e^2g^2Q^2N_c^2z_1z_2(1-z_1)(1-z_2)(z_1(1-z_1)+z_2(1-z_2))}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(|\mathbf{x}_{12}|Q_2)K_0(|\mathbf{x}_{1'2'}|Q_1)$$
$$\Delta^{(3)}_{12'}[S_{12}S_{1'2'}-S_{12}-S_{1'2'}+1]e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}e^{i\frac{z}{z_1}\mathbf{p}\cdot(\mathbf{x}'_2-\mathbf{x}_3)}e^{i\frac{z}{z_2}\mathbf{q}\cdot(\mathbf{x}_3-\mathbf{x}_1)}\delta(1-z_1-z_2-z).$$
$$\frac{\sigma^{\text{Real},L}_{3-3}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{2e^2g^2Q^2N_c^2z_1z_2^3(z_1^2+(1-z_2)^2)}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(QX)K_0(QX')\Delta^{(3)}_{11'}$$
$$[S_{11'}S_{22'}-S_{13}S_{23}-S_{1'3}S_{2'3}+1]e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}\delta(1-z_1-z_2-z).$$
$$\frac{\sigma^{\text{Real},L}_{4-4}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{2e^2g^2Q^2N_c^2z_1^3z_2(z_2^2+(1-z_1)^2)}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(QX)K_0(QX')\Delta^{(3)}_{22'}$$
$$[S_{11'}S_{22'}-S_{13}S_{23}-S_{1'3}S_{2'3}+1]e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}\delta(1-z_1-z_2-z).$$
$$\frac{\sigma^{\text{Real},L}_{3-4}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{-2e^2g^2Q^2N_c^2z_1^2z_2^2(z_1(1-z_1)+z_2(1-z_2))}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(QX)K_0(QX')\Delta^{(3)}_{12'}$$
$$[S_{11'}S_{22'}-S_{13}S_{23}-S_{1'3}S_{2'3}+1]e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}\delta(1-z_1-z_2-z).$$
$$\frac{\sigma^{\text{Real},L}_{1-3}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{-2e^2g^2Q^2N_c^2z_2^3(1-z_2)(z_1^2+(1-z_2)^2)}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}\,K_0(|\mathbf{x}_{12}|Q_2)K_0(QX')\Delta^{(3)}_{11'}$$
$$[S_{122'3}S_{1'3}-S_{1'3}S_{2'3}-S_{12}+1]e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}e^{i\frac{z}{z_1}\mathbf{p}\cdot(\mathbf{x}_3-\mathbf{x}_1)}\delta(1-z_1-z_2-z).$$
$$\frac{\sigma^{\text{Real},L}_{1-4}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{2e^2g^2Q^2N_c^2z_1z_2^2(1-z_2)(z_1(1-z_1)+z_2(1-z_2))}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}K_0(|\mathbf{x}_{12}|Q_2)K_0(QX')\Delta^{(3)}_{12'}$$
$$[S_{122'3}S_{1'3}-S_{1'3}S_{2'3}-S_{12}+1]e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}e^{i\frac{z}{z_1}\mathbf{p}\cdot(\mathbf{x}_3-\mathbf{x}_1)}\delta(1-z_1-z_2-z).$$
$$\frac{\sigma^{\text{Real},L}_{2-3}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{2e^2g^2Q^2N_c^2z_1^2z_2(1-z_1)(z_1(1-z_1)+z_2(1-z_2))}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}K_0(|\mathbf{x}_{12}|Q_1)K_0(QX')\Delta^{(3)}_{21'}$$
$$[S_{1231'}S_{2'3}-S_{1'3}S_{2'3}-S_{12}+1]e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}e^{i\frac{z}{z_2}\mathbf{q}\cdot(\mathbf{x}_3-\mathbf{x}_2)}\delta(1-z_1-z_2-z).$$
$$\frac{\sigma^{\text{Real},L}_{2-4}}{d^2\mathbf{p}\,d^2\mathbf{q}\,dy_1\,dy_2} = \frac{-2e^2g^2Q^2N_c^2z_1^3(1-z_1)(z_2^2+(1-z_1)^2)}{(2\pi)^{10}}\int\frac{z}{z}\int d^{10}\mathbf{x}K_0(|\mathbf{x}_{12}|Q_1)K_0(QX')\Delta^{(3)}_{22'}$$
$$[S_{1231'}S_{2'3}-S_{1'3}S_{2'3}-S_{12}+1]e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)}e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)}e^{i\frac{z}{z_2}\mathbf{q}\cdot(\mathbf{x}_3-\mathbf{x}_2)}\delta(1-z_1-z_2-z).$$

Spinor helicity formalism: helicity amplitudes

Numerator	$\lambda_\gamma; \lambda_q, \lambda_g$	$N_i^{\lambda_\gamma;\lambda_q,\lambda_g}$
N_1	$L; +, +$	$-Q(z_1 z_2)^{3/2}(1-z_2)\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\epsilon]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}$
	$L; +, -$	$-Q(z_2)^{3/2}\sqrt{z_1}(1-z_2)^2\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\epsilon]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}$
	$+; +, +$	$-(z_1)^{3/2}\sqrt{z_2}(1-z_2)\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\epsilon]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}\,(\mathbf{k}_1\cdot\epsilon)$
	$+; +, -$	$-\sqrt{z_1 z_2}(1-z_2)^2\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\epsilon^*]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}\,(\mathbf{k}_1\cdot\epsilon)$
	$+; -, +$	$(z_2)^{3/2}\sqrt{z_1}(1-z_2)\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\epsilon]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}\,(\mathbf{k}_1\cdot\epsilon)$
	$+, -, -$	$(z_1 z_2)^{3/2}\frac{[(z_1\mathbf{k}-z_3\mathbf{p})\cdot\epsilon^*]}{(z_1\mathbf{k}-z_3\mathbf{p})^2}\,(\mathbf{k}_1\cdot\epsilon)$
N_2	$L; +, +$	$Q(z_1)^{3/2}\sqrt{z_2}(1-z_1)^2\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\epsilon]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}$
	$L; +, -$	$Q(z_1 z_2)^{3/2}(1-z_1)\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\epsilon^*]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}$
	$+; +, +$	$-(z_1)^{3/2}\sqrt{z_2}(1-z_1)\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\epsilon]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}\,(\mathbf{k}_1\cdot\epsilon)$
	$+; +, -$	$-(z_1 z_2)^{3/2}\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\epsilon^*]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}\,(\mathbf{k}_1\cdot\epsilon)$
	$+; -, +$	$(z_2)^{3/2}\sqrt{z_1}(1-z_1)\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\epsilon]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}\,(\mathbf{k}_1\cdot\epsilon)$
	$+, -, -$	$\sqrt{z_1 z_2}(1-z_1)^2\frac{[(z_2\mathbf{k}-z_3\mathbf{q})\cdot\epsilon^*]}{(z_2\mathbf{k}-z_3\mathbf{q})^2}\,(\mathbf{k}_1\cdot\epsilon)$
N_3	$L; +, +$	$Q(z_1 z_2)^{3/2}(1-z_2)\left(\frac{\mathbf{k}_2\cdot\epsilon}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon}{1-z_2}\right)$
	$L; +, -$	$Q(z_2)^{3/2}\sqrt{z_1}(1-z_2)^2\left(\frac{\mathbf{k}_2\cdot\epsilon^*}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon^*}{1-z_2}\right)$
	$+; +, +$	$(z_1)^{3/2}\sqrt{z_2}(1-z_2)\left(\frac{\mathbf{k}_2\cdot\epsilon}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon}{1-z_2}\right)\mathbf{k}_1\cdot\epsilon$
	$+; +, -$	$\sqrt{z_1 z_2}(1-z_2)^2\left(\frac{\mathbf{k}_2\cdot\epsilon^*}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon^*}{1-z_2}\right)\mathbf{k}_1\cdot\epsilon$
	$+; -, +$	$-(z_2)^{3/2}\sqrt{z_1}(1-z_2)\left(\frac{\mathbf{k}_2\cdot\epsilon}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon}{1-z_2}\right)\mathbf{k}_1\cdot\epsilon$
	$+, -, -$	$-(z_1 z_2)^{3/2}\left[\left(\frac{\mathbf{k}_2\cdot\epsilon^*}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon^*}{(1-z_2)}\right)\mathbf{k}_1\cdot\epsilon+\frac{\mathbf{k}_1^2+z_2(1-z_2)Q^2}{2z_2(1-z_2)}\right]$
N_4	$L; +, +$	$-Q(z_1)^{3/2}\sqrt{z_2}(1-z_1)^2\left(\frac{\mathbf{k}_2\cdot\epsilon}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon}{1-z_1}\right)$
	$L; +, -$	$-Q(z_1 z_2)^{3/2}(1-z_1)\left(\frac{\mathbf{k}_2\cdot\epsilon^*}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon^*}{1-z_1}\right)$
	$+; +, +$	$(z_1)^{3/2}\sqrt{z_2}(1-z_1)\left(\frac{\mathbf{k}_2\cdot\epsilon}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon}{1-z_1}\right)\mathbf{k}_1\cdot\epsilon$
	$+; +, -$	$(z_1 z_2)^{3/2}\left[\left(\frac{\mathbf{k}_2\cdot\epsilon^*}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon^*}{(1-z_1)}\right)\mathbf{k}_1\cdot\epsilon+\frac{\mathbf{k}_1^2+z_1(1-z_1)Q^2}{2z_1(1-z_1)}\right]$
	$+; -, +$	$-(z_2)^{3/2}\sqrt{z_1}(1-z_1)\left(\frac{\mathbf{k}_2\cdot\epsilon}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon}{1-z_1}\right)\mathbf{k}_1\cdot\epsilon$
	$+, -, -$	$-\sqrt{z_1 z_2}(1-z_1)^2\left(\frac{\mathbf{k}_2\cdot\epsilon^*}{z_3}-\frac{\mathbf{k}_1\cdot\epsilon^*}{1-z_1}\right)\mathbf{k}_1\cdot\epsilon$

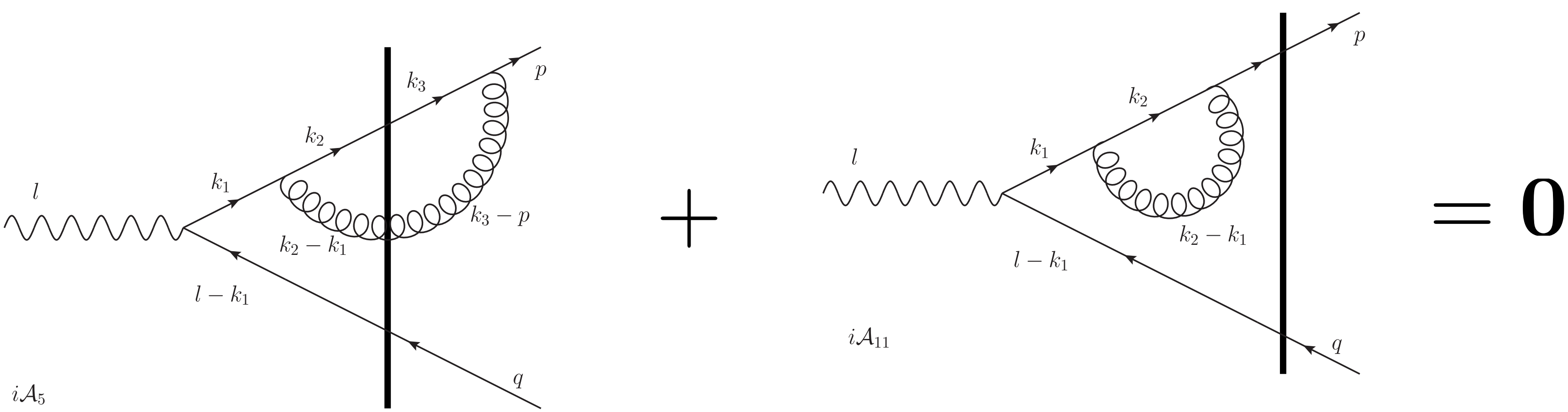
divergences

• ***Ultraviolet:***

Real corrections are UV finite

UV divergences cancel among virtual corrections $\mathbf{k \to \infty}$ **or** $\mathbf{x_3 \to x_i}$

$$\left(d\sigma_5 + d\sigma_{11}\right)_{UV} = 0$$
$$\left(d\sigma_6 + d\sigma_{12}\right)_{UV} = 0$$
$$\left(d\sigma_9 + d\sigma_{10} + d\sigma_{14(1)} + d\sigma_{14(2)}\right)_{UV} = 0$$



divergences

- ***Soft:*** $\mathbf{k \to 0\,(x_3 \to \infty)} \quad \textbf{AND} \quad \mathbf{z \to 0}$

Soft divergences cancel between real and virtual corrections

$$(d\sigma_{1-1} + d\sigma_9)_{soft} = 0,$$

$$\left(d\sigma_{1-2} + d\sigma_{13}^{(1)} + d\sigma_{13}^{(2)}\right)_{soft} = 0$$

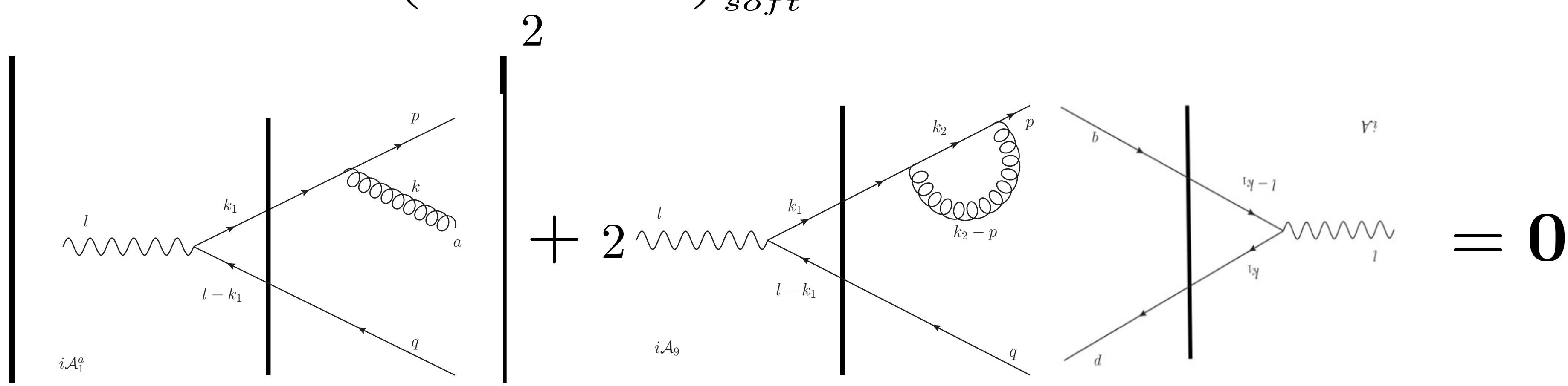
$$(d\sigma_{3-3} + d\sigma_{4-4} + d\sigma_{3-4})_{soft} = 0$$

$$(d\sigma_{1-3} + d\sigma_{1-4})_{soft} = 0$$

$$(d\sigma_{2-3} + d\sigma_{2-4})_{soft} = 0$$

$$(d\sigma_5 + d\sigma_7)_{soft} = 0$$

$$\left(d\sigma_{11} + d\sigma_{14}^{(1)}\right)_{soft} = 0$$

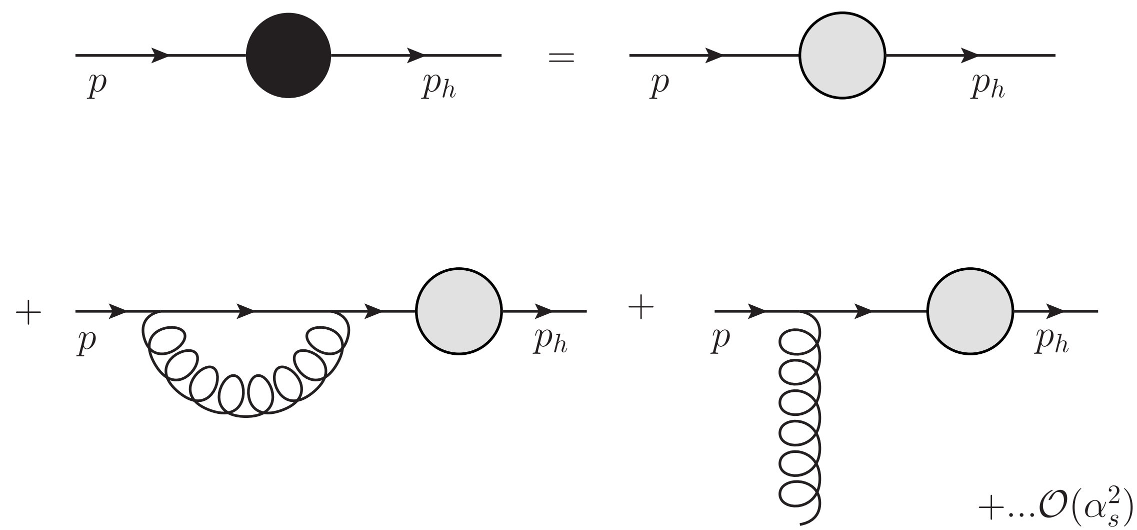


divergences

- ***Collinear:***

$$\frac{1}{(p+k)^2} = \frac{1}{|\overrightarrow{p}||\overrightarrow{k}|(1-cos\,\theta)} \longrightarrow \infty \quad as \quad \theta \to 0$$

Collinear divergences are absorbed into evolution of parton-hadron fragmentation functions



$$D_{h_1/q}(z_{h1},\mu^2) = \int_{z_{h1}}^{1} \frac{d\xi}{\xi} D^0_{h_1/q}\left(\frac{z_{h1}}{\xi}\right) \left[\delta(1-\xi) + \frac{\alpha_s}{2\pi} P_{qq}(\xi)\,\log\left(\frac{\mu^2}{\Lambda^2}\right)\right]$$

divergences

• ***Rapidity:*** $\mathbf{z} \to 0$ ***, but finite*** $\boldsymbol{k_t}$

$$\int_0^1 \frac{dz}{z} = \int_0^{z_f} \frac{dz}{z} + \int_{z_f}^1 \frac{dz}{z}$$

Rapidity divergences are absorbed into JIMWLK evolution of dipoles and quadrupoles

$$\frac{d\sigma^L_{\rm NLO}}{d^2\mathbf{p}\, d^2\mathbf{q}\, dy_1\, y_2} = \frac{2e^2 g^2 Q^2 N_c^2 (z_1 z_2)^3}{(2\pi)^{10}}\, \delta(1-z_1-z_2) \int_0^{z_f} \frac{dz}{z} \int d^{10}\mathbf{x}\, K_0(|\mathbf{x}_{12}|Q_1) K_0(|\mathbf{x}_{1'2'}|Q_1)$$
$$e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}} e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}} \Big\{ \left(\tilde\Delta_{12} + \tilde\Delta_{22'} - \tilde\Delta_{12'}\right) S_{132'1'} S_{23} + \left(\tilde\Delta_{1'2'} + \tilde\Delta_{22'} - \tilde\Delta_{21'}\right) S_{1'321} S_{2'3}$$
$$+ \left(\tilde\Delta_{12} + \tilde\Delta_{11'} - \tilde\Delta_{21'}\right) S_{322'1'} S_{13} + \left(\tilde\Delta_{1'2'} + \tilde\Delta_{11'} - \tilde\Delta_{12'}\right) S_{32'21} S_{1'3}$$
$$- \left(\tilde\Delta_{11'} + \tilde\Delta_{22'} + \tilde\Delta_{12} + \tilde\Delta_{1'2'}\right) S_{122'1'} - \left(\tilde\Delta_{12} + \tilde\Delta_{1'2'} - \tilde\Delta_{12'} - \tilde\Delta_{21'}\right) S_{12} S_{1'2'}$$
$$- \left(\tilde\Delta_{11'} + \tilde\Delta_{22'} - \tilde\Delta_{12'} - \tilde\Delta_{21'}\right) S_{11'} S_{22'} - 2\tilde\Delta_{12}\left(S_{13}S_{23} - S_{12}\right) - 2\tilde\Delta_{1'2'}\left(S_{1'3}S_{2'3} - S_{1'2'}\right) \Big\}$$

JIMWLK evolution of quadrupoles

JIMWLK evolution of dipoles

Divergences

•*Ultraviolet*

Real corrections are UV finite
UV divergences cancel among virtual corrections

•*Soft*

Soft divergences cancel between real and virtual corrections

•*Collinear*

Collinear divergences are absorbed into hadron fragmentation functions

•*Rapidity*

rapidity divergences are absorbed into JIMWLK evolution of dipoles, quadrupoles

$$\sigma^{\gamma^* A \to h_1 h_2 X} = \sigma_{LO} \otimes \mathrm{JIMWLK} + \sigma_{LO} \otimes D_{h_1/q}(z_1, \mu^2) D_{h_2/\bar{q}}(z_2, \mu^2) + \sigma_{NLO}^{\mathrm{finite}}$$

Single inclusive hadron production in DIS at small x: NLO

Larger kinematic phase space at EIC

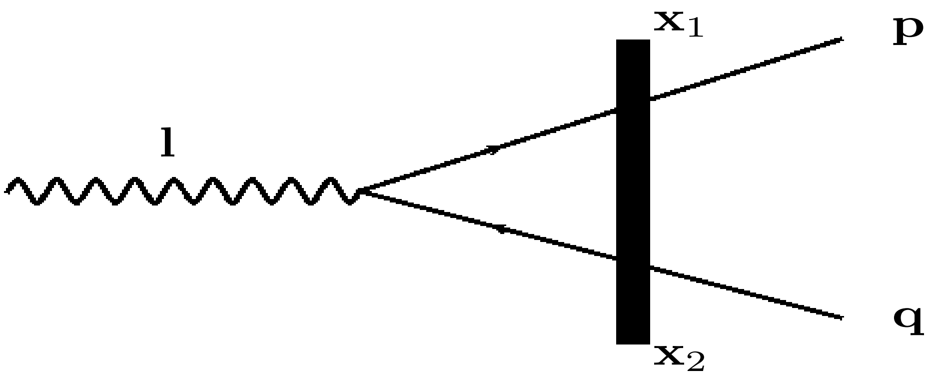
No Sudakov

Dipoles only

Forward rapidity: quark or antiquark production

F. Bergabo and JJM: arXiv:2210.03208

LO: antiquark (quark) production in DIS at small x



integrate out quark (antiquark)

$$\frac{d\sigma^{\gamma^* A\to\bar{q}X}}{d^2\mathbf{q}\,dy_2} = \frac{e^2 Q^2 z_2^2 (1-z_2) N_c}{(2\pi)^5} \int d^6\mathbf{x}\,[S_{22'} - S_{12} - S_{12'} + 1]$$

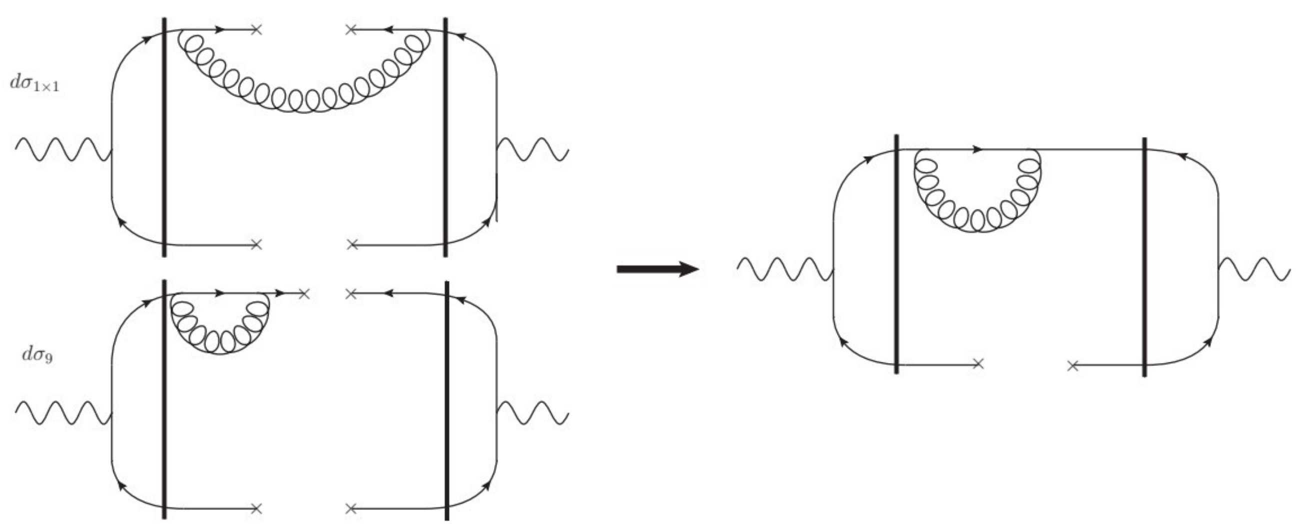
$$e^{i\mathbf{q}\cdot(\mathbf{x}_2' - \mathbf{x}_2)} \Big[4 z_2 (1-z_2) K_0(|\mathbf{x}_{12}|Q_2) K_0(|\mathbf{x}_{12'}|Q_2) +$$

$$\left[z_2^2 + (1-z_2)^2\right] \frac{\mathbf{x}_{12}\cdot\mathbf{x}_{12'}}{|\mathbf{x}_{12}||\mathbf{x}_{12'}|} K_1(|\mathbf{x}_{12}|Q_2) K_1(|\mathbf{x}_{12'}|Q_2) \Big]$$

NLO corrections

start with NLO corrections to dihadron production and integrate out quark

cancellations among diagrams



Single inclusive hadron production in DIS at small x: NLO

all terms with quadrupoles cancel; only <u>dipoles</u> contribute to the cross section

cancellations of divergences as before

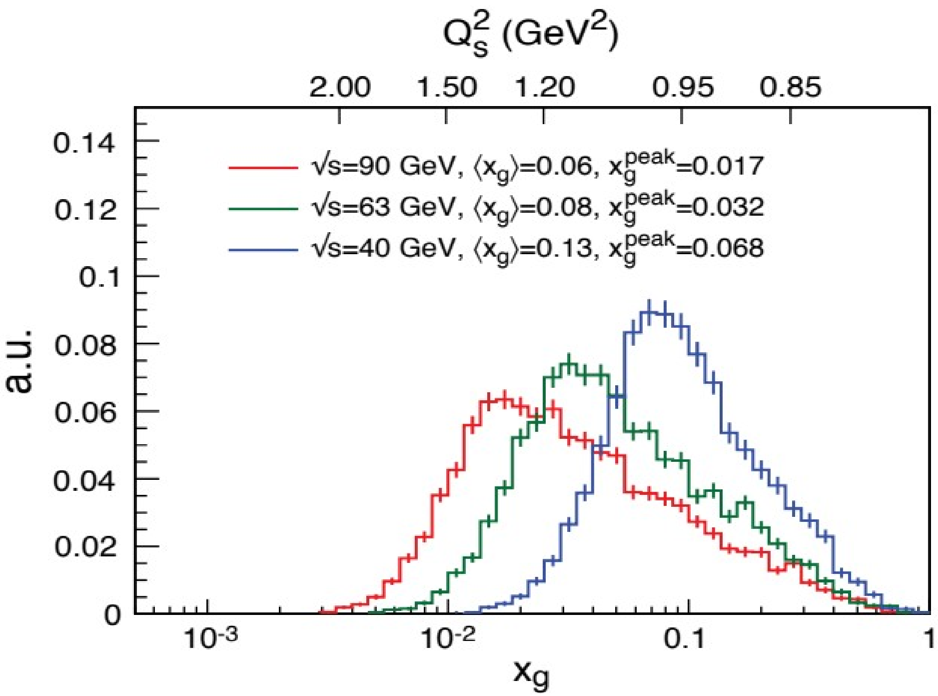
$$\sigma^{\gamma^* A \to hX} = \sigma_{LO} \otimes \mathrm{JIMWLK} + \sigma_{LO} \otimes D_{h/\bar{q}}(z_h, \mu^2) + \sigma_{NLO}^{\mathrm{finite}}$$

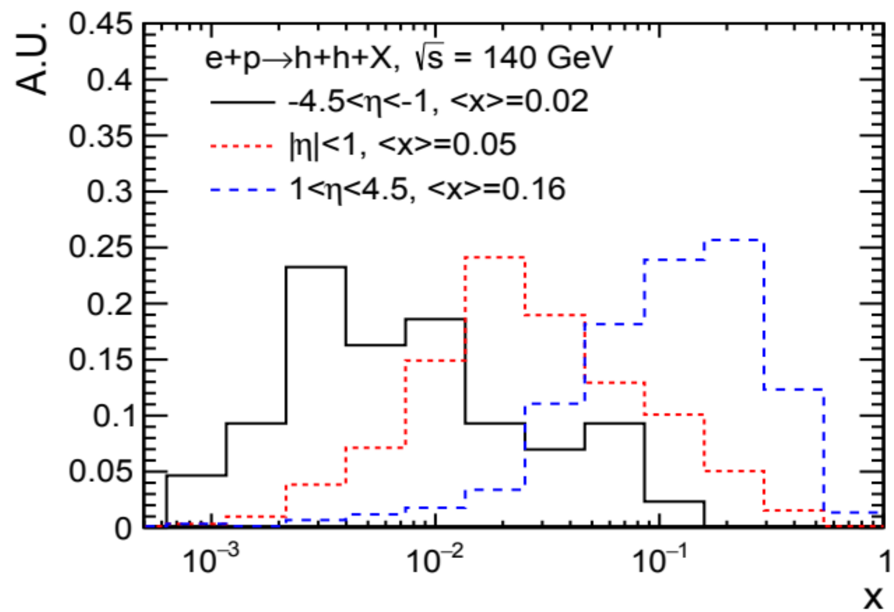
phenomenology: need to consider hadronization of any of the 3 partons

relation to TMD,...

EIC

kinematics of inclusive dihadron production



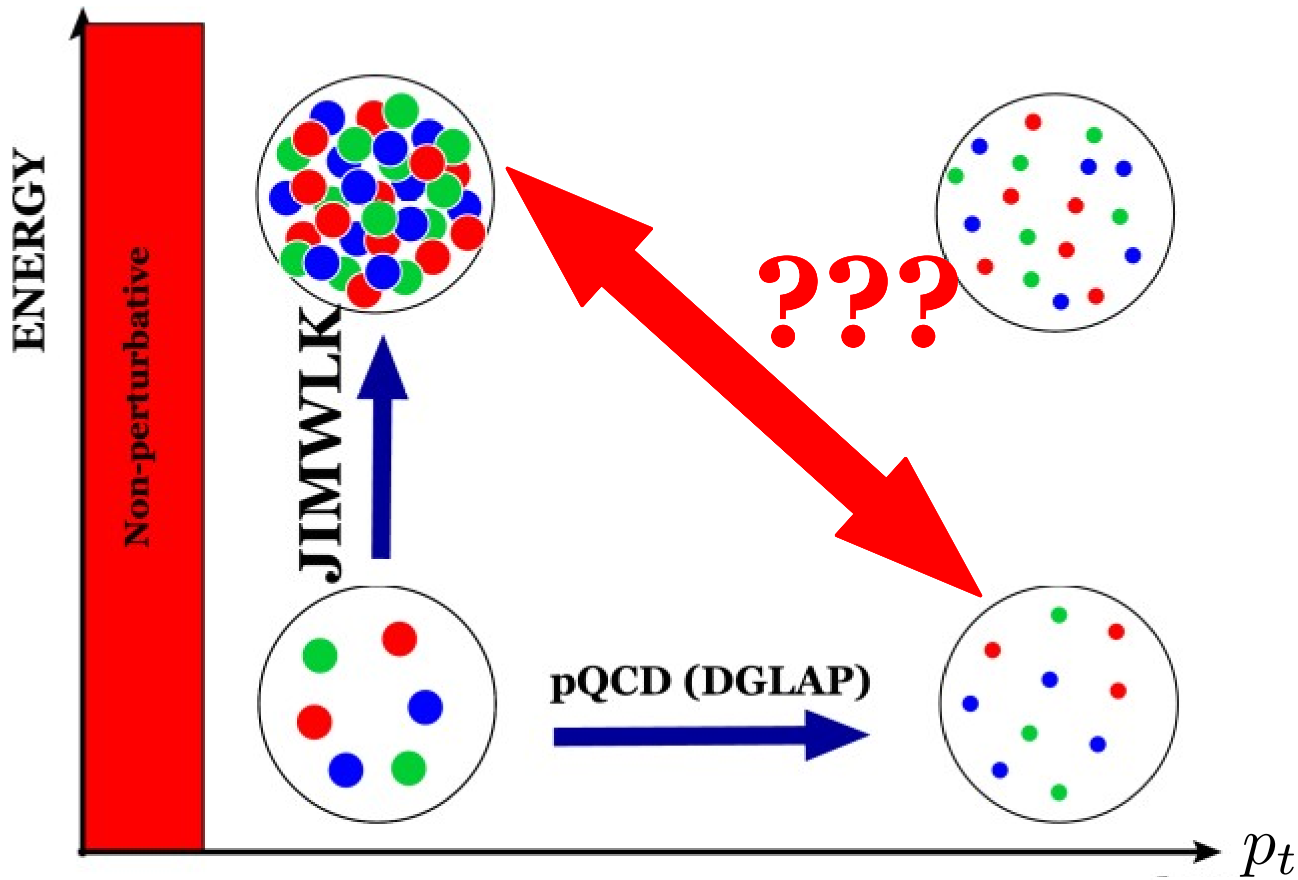


Aschenauer et al. arXiv:1708.01527

Fig. courtesy of Xiaoxuan Chu

transition region: from large x to small x

QCD kinematic phase space



Unifying saturation with large x (pQCD) physics?

jet physics
partially/fully coherent energy loss
interactions of UHE neutrinos, ...

How to unify collinear factorization (pQCD) with CGC?

collinear factorization:

tree level partonic cross section: one-parton exchange
quantum corrections: DGLAP evolution of parton dist.,frag. functions,
resum large logs of Q^2

CGC:

classical fields: multiple soft scatterings
quantum corrections: JIMWLK evolution of Wilson line correlators,
resum large logs of $1/x$

Toward unifying small and large x: pA collisions

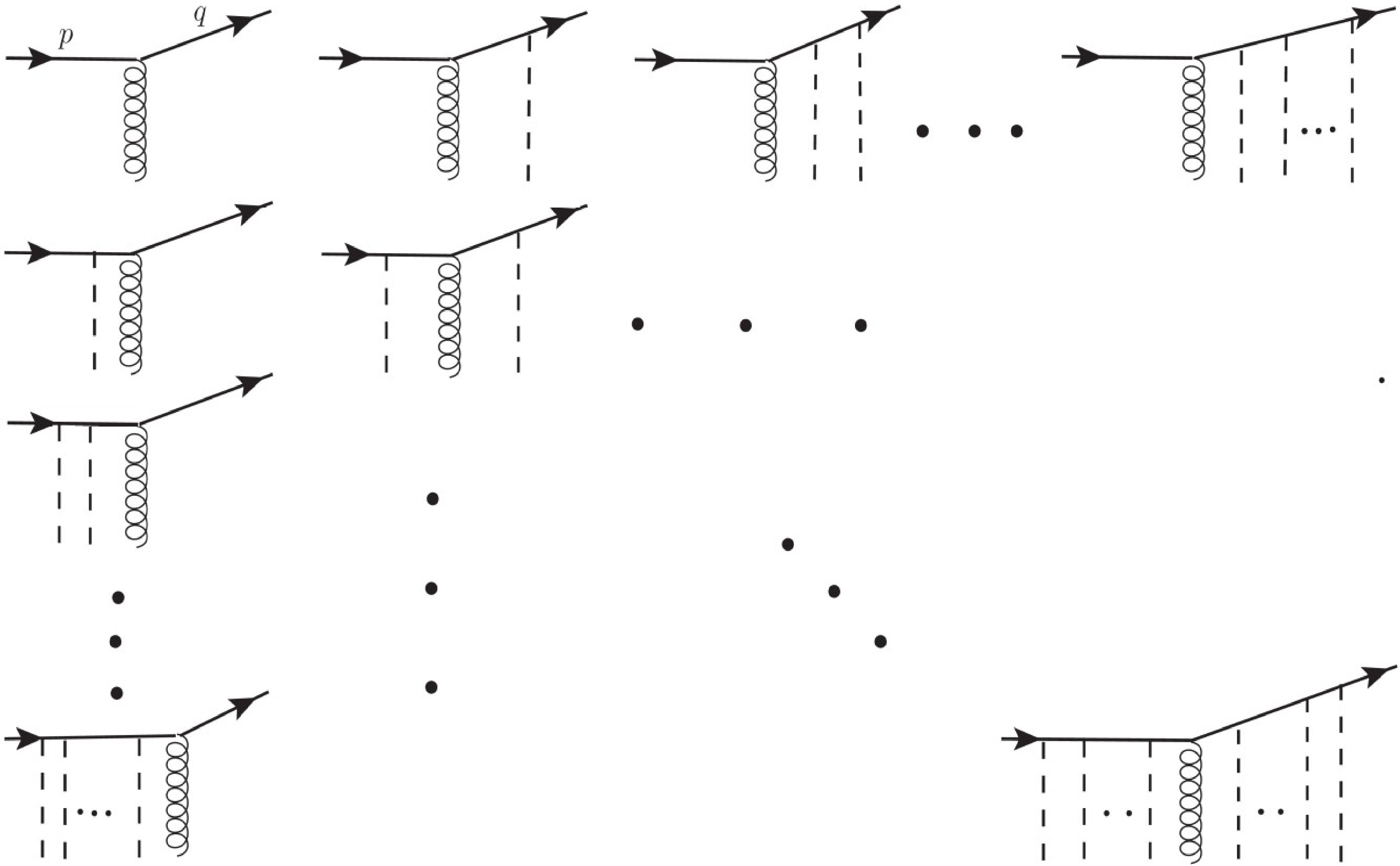
scattering from small x modes of the target field $A^- \equiv n^- S$ involves only small transverse momenta exchange (small angle deflection)

$$\begin{aligned} p^\mu &= (p^+ \sim \sqrt{s},\, p^- = 0, p_t = 0) \\ S &= S(p^+ \sim 0, p^-/P^- \ll 1, p_t) \end{aligned}$$

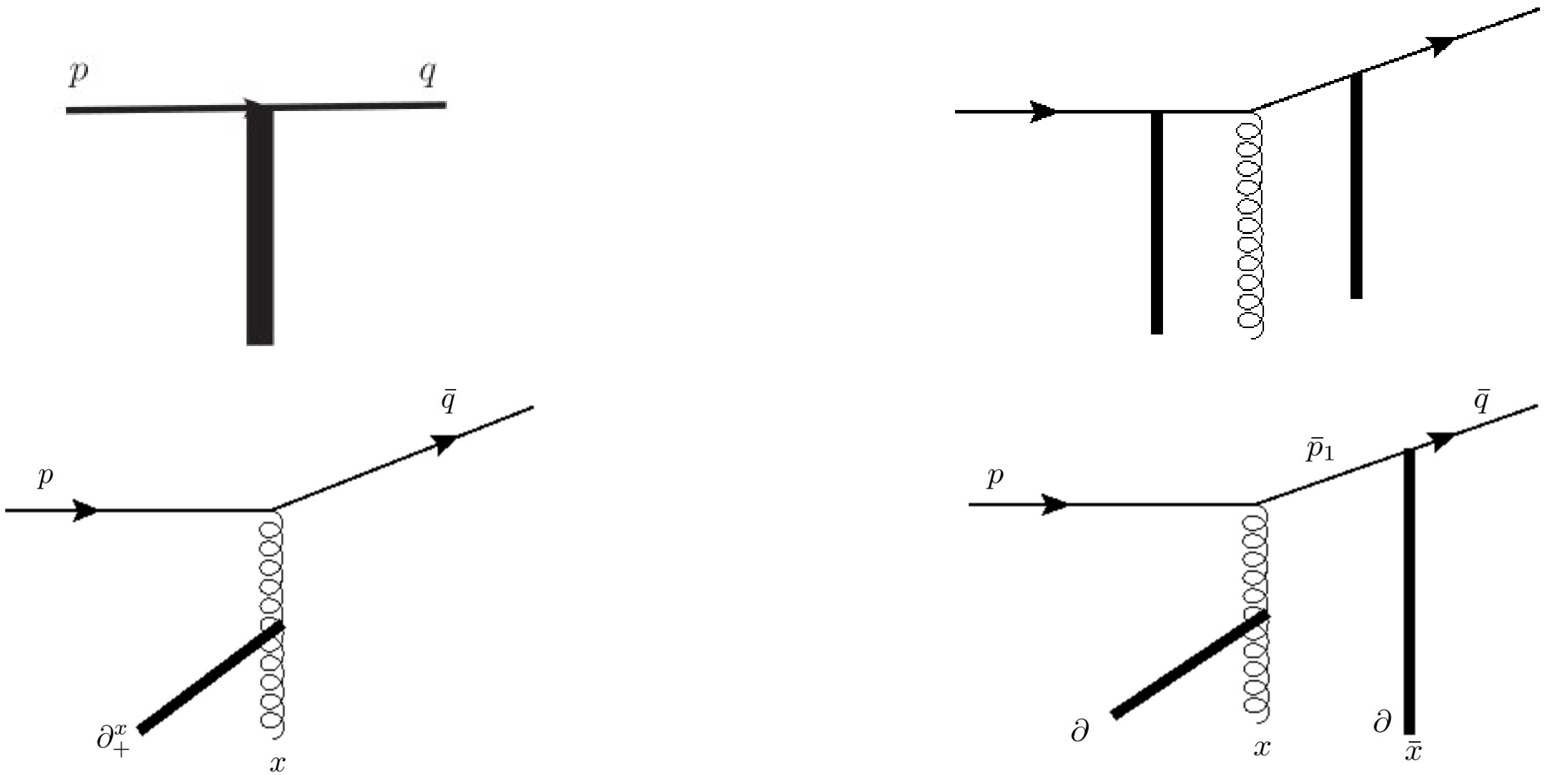
allow hard scattering by including one all x field during which there is large momenta exchanged and quark can get deflected by a large angle.

$A_a^\mu(x^+, x^-, x_t)$

include eikonal multiple scattering before and after (along a different direction) the hard scattering


p
q

Full amplitude: $\mathbf{i}\mathcal{M} = \mathbf{i}\mathcal{M}_{\mathbf{eik}} + \mathbf{i}\mathcal{M}_{\mathbf{1}} + \mathbf{i}\mathcal{M}_{\mathbf{2}} + \mathbf{i}\mathcal{M}_{\mathbf{3}}$



soft (eikonal) limit: $A^\mu(x) \to n^- \, S(x^+, x_t)$ $\quad n \cdot \bar{q} \to n \cdot p$

$$i\mathcal{M} \longrightarrow i\mathcal{M}_{eik}$$

Including large x partons of the target leads to:

longitudinal double spin asymmetries (A_{LL})

baryon transport (beam rapidity loss), ...

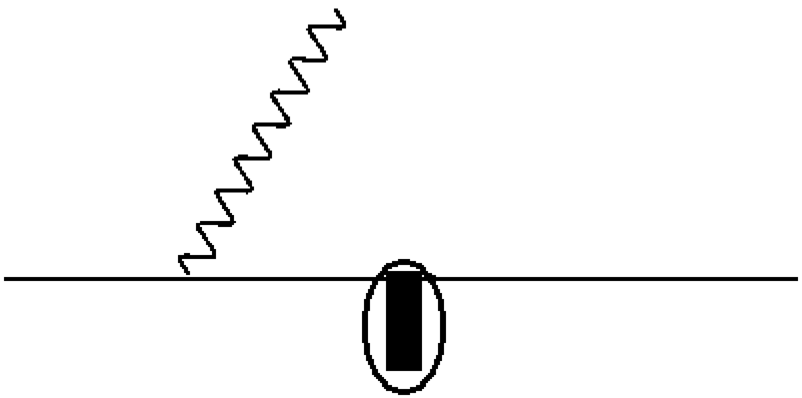
Photon production at all x

photon-hadron correlations:

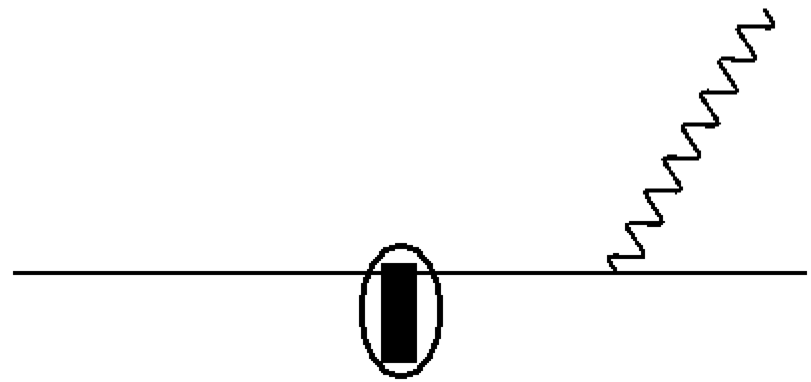
azimuthal angular correlations from low to high p_t

forward-backward rapidity correlations

photon production at small x: eikonal approx.



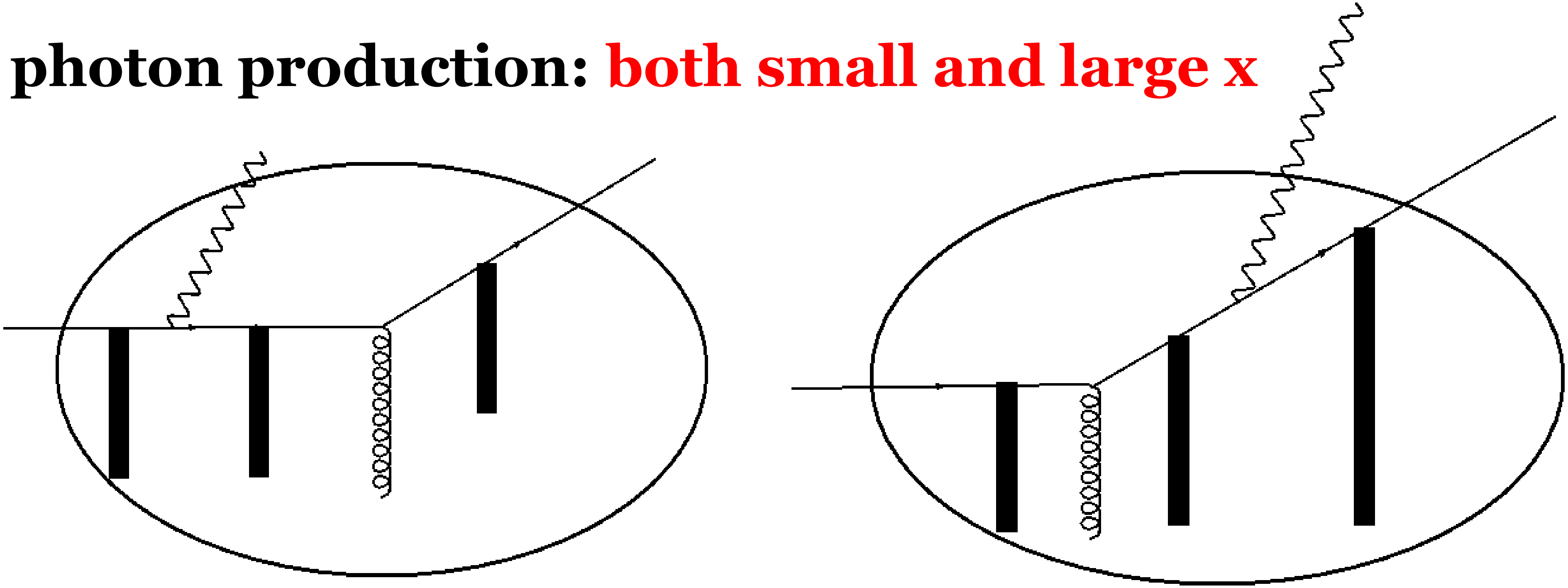
before quark scatters on the target



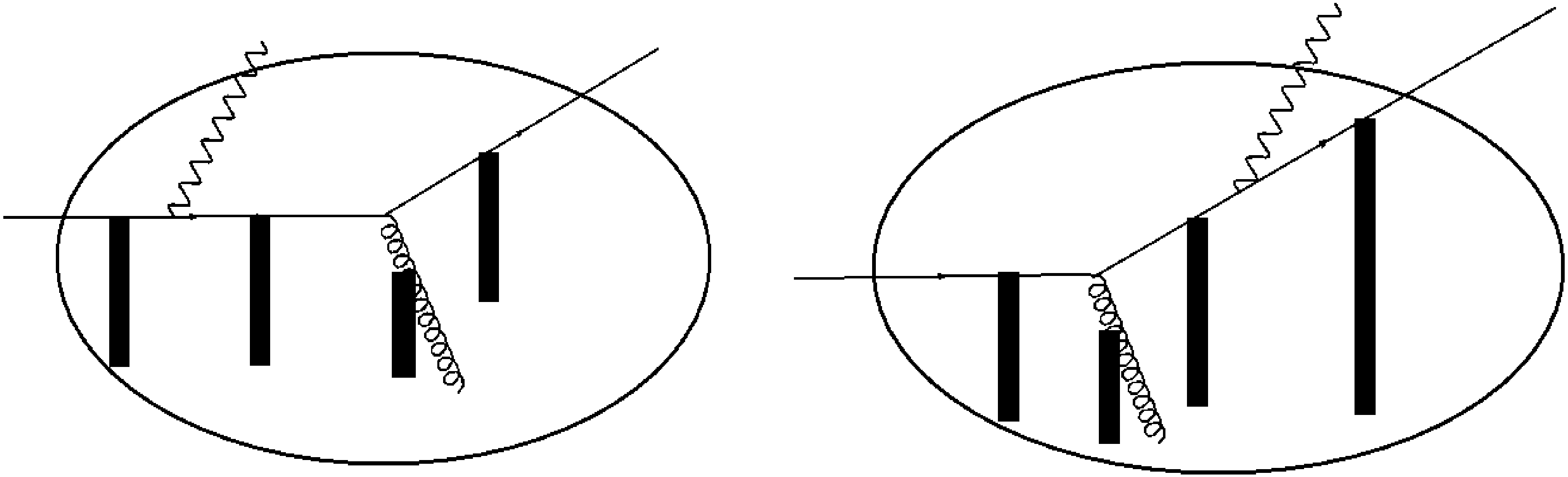
after quark scatters on the target

Eikonal approximation: no radiation inside the target

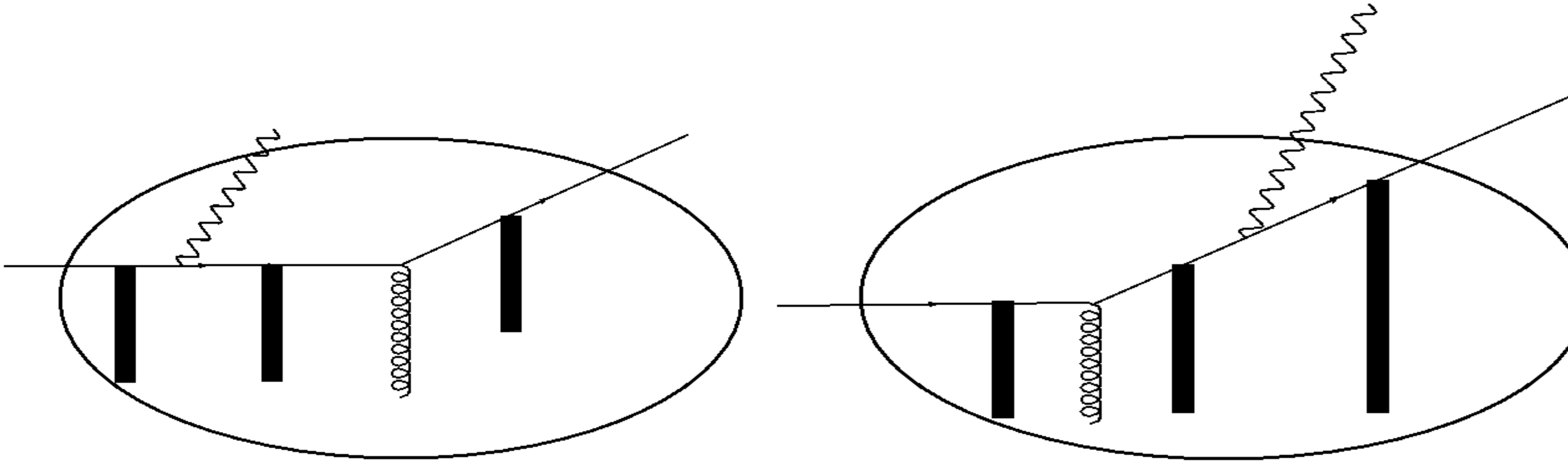
photon production: both small and large x



before hard scattering after hard scattering



pQCD limit (large x: gluon PDF X partonic cross section):



V = U = 1

tree level so far, how about quantum corrections (evolution)?

What would the evolution equation(s) look like ?

JIMWLK at small x

DGLAP (collinear factorization) at large x

try something "simpler":

DGLAP evolution of gluon distribution function in light cone gauge with classical background fields

work in progress with T. Altinoluk and G. Beuf

SUMMARY

pQCD and collinear factorization at high p_t

breaks down at small x (low p_t)

CGC is a systematic approach to high energy collisions

strong hints from RHIC, LHC,...

to be probed precisely at EIC

toward precision: NLO, sub-eikonal corrections, ...

CGC breaks down at large x (high p_t)

a significant part of EIC/RHIC/LHC phase space is at large x

transition from large x physics (pQCD) to small x (CGC)

Toward inclusion of large x physics:

spin asymmetries

beam rapidity loss

particle production in both small and large p_t kinematics

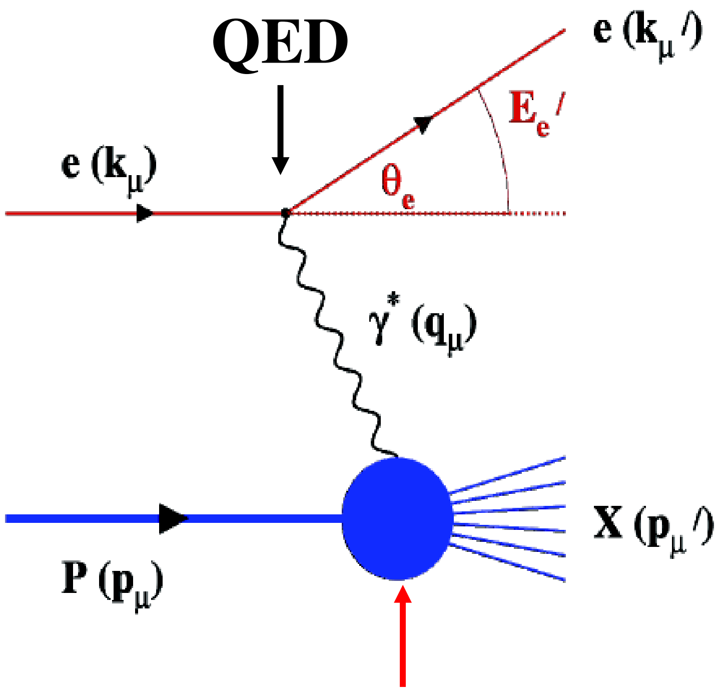
two-particle correlations: from forward-forward to forward-backward

one-loop correction: both collinear and CGC factorization limits

<u>*need to clarify/understand: gauge invariance, initial conditions, ...*</u>

Deep Inelastic Scattering (DIS)

probing hadron structure



QCD

(structure functions: F_2, F_L)

Kinematic Invariants

$$Q^2 = -q^2 = -(k_\mu - k'_\mu)^2$$

$$Q^2 = 4E_e E'_e \sin^2\left(\frac{\theta'_e}{2}\right)$$

Measure of resolution power

$$y = \frac{pq}{pk} = 1 - \frac{E'_e}{E_e}\cos^2\left(\frac{\theta'_e}{2}\right)$$

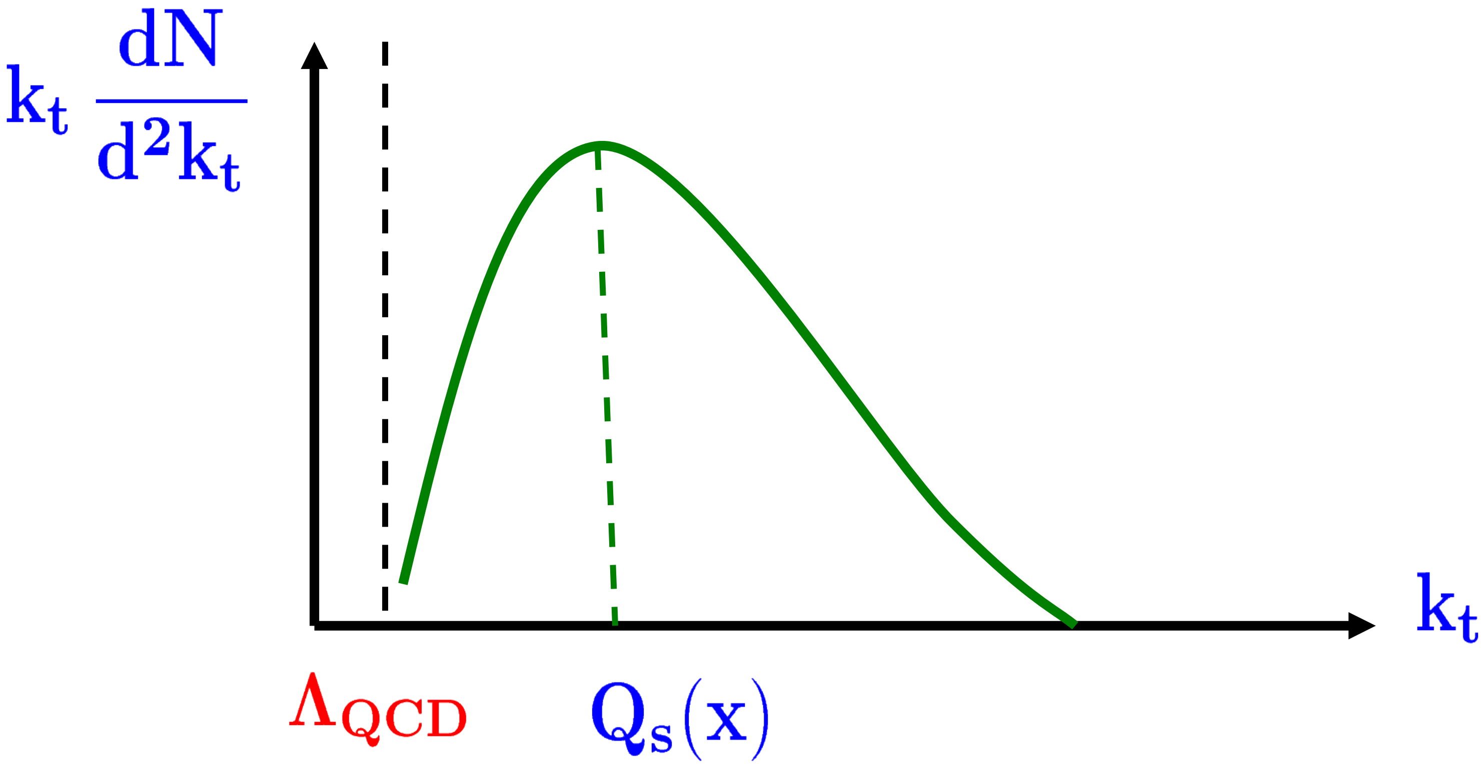
Measure of inelasticity

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

Measure of momentum fraction of struck quark

$$\mathbf{s} \equiv (\mathbf{p} + \mathbf{k})^2$$

hadron at high energy: Color Glass Condensate



most gluons in the wave function of a hadron have momentum Q_s

$$Q_s^2(x, b_t, A) \sim Q_0^2 \frac{A^{1/3}}{x^{0.3}} \gg \Lambda_{QCD}^2$$