Relativistic spatial distribution of charge and magnetization

Based on **[C.L., PRL125 (2020) 232002]**
[C.L., Wang, PRD105 (2022) 9, 096032]
[Chen, C.L., PRD106 (2022) 11, 116024]
[Chen, C.L., 2302.04672 [hep-ph] to appear in PRD]

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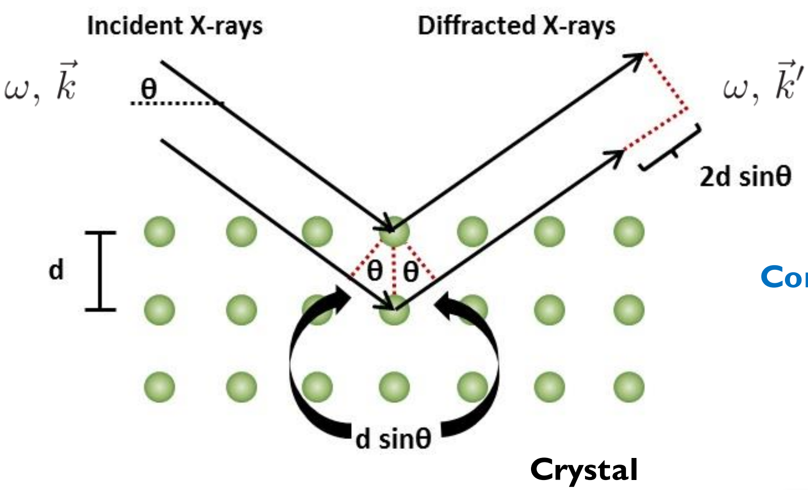
May 4, Principia Institute, Sao Paulo, Brazil

Outline

- Elastic scattering
- Relativistic interpretation of electromagnetic form factors
- Phase-space formalism
- Frame dependence of spatial distributions
- Electric and magnetic polarizations

Spatial structure through elastic scattering

Example: X-ray diffraction

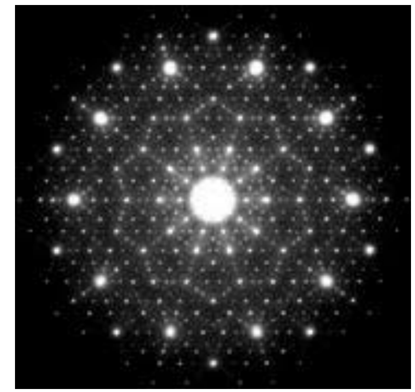


Constructive interference
(Bragg's law)

$$2d\,\sin\theta = n\lambda$$

$$\boxed{\lambda \sim d}$$

Diffraction pattern



$$\propto |A_{\mathrm{scatt}}|^2$$

Scattered amplitude

$$A_{\mathrm{scatt}} \propto F(\vec{q}) = \int \mathrm{d}^3 r\, e^{i\vec{q}\cdot\vec{r}}\, \rho(\vec{r}) \qquad \vec{q} = \vec{k} - \vec{k}'$$

Form factor

Scatterer distribution

Nuclear elastic scattering

Crystals & atoms	$d \approx 10^{-10}\,\mathrm{m}$	$\Rightarrow$	$\hbar\omega \approx 10^{4}\,\mathrm{eV}$	$\rightarrow$	**X-rays**
Nuclei & nucleons	$d \approx 10^{-15}\,\mathrm{m}$	$\Rightarrow$	$\hbar\omega \approx 10^{9}\,\mathrm{eV}$	$\rightarrow$	**High-energy electron beams**

⚠ **Large recoil for light nuclei!**

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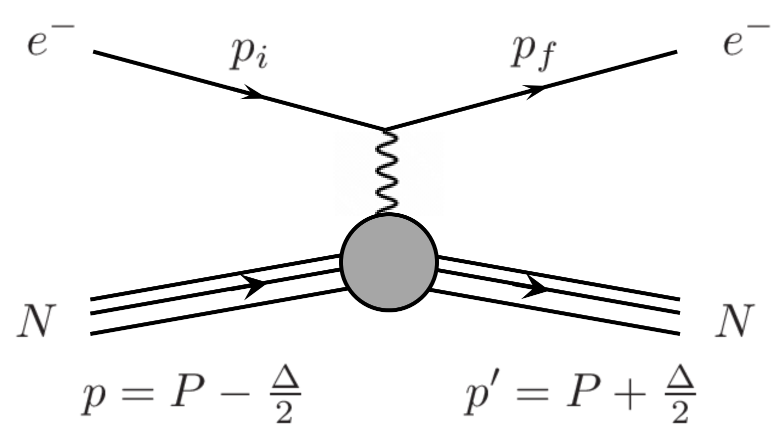
Relativistic treatment
in Born approximation

Spin-0 target

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\Big/\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\Big|_{\mathrm{pointlike}} = [F(Q^2)]^2$$



$Q^2 = -\Delta^2$

[Rosenbluth, PR79 (1950) 615]
[Hofstadter, RMP28 (1956) 214]
[Yennie, Lévy, Ravenhall, RMP29 (1957) 144]

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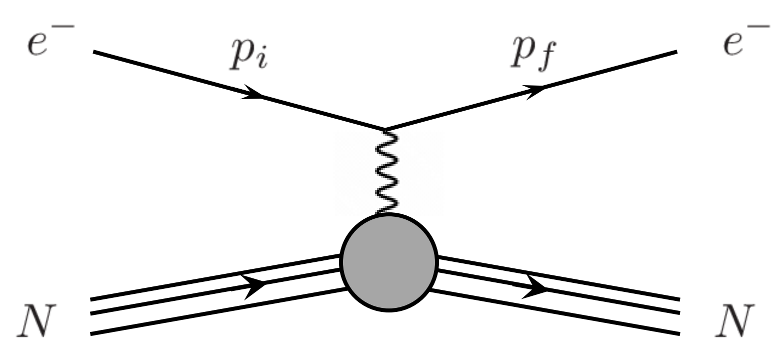

$p = P - \frac{\Delta}{2}$ $\quad p' = P + \frac{\Delta}{2}$

Spin-0 target

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \Big/ \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\Big|_{\mathrm{pointlike}} = [F(Q^2)]^2$$

Spin-1/2 target

$$= \left\{ \left[G_E(Q^2)\right]^2 + \frac{\tau}{\epsilon}\left[G_M(Q^2)\right]^2 \right\} \frac{1}{1+\tau}$$

Electric form factor — **Magnetic** form factor

$Q^2 = -\Delta^2$

$\tau = Q^2/4M_N^2$

$\epsilon = (1 + 2(1+\tau)\tan^2\frac{\theta_e}{2})^{-1}$

[Rosenbluth, PR79 (1950) 615]
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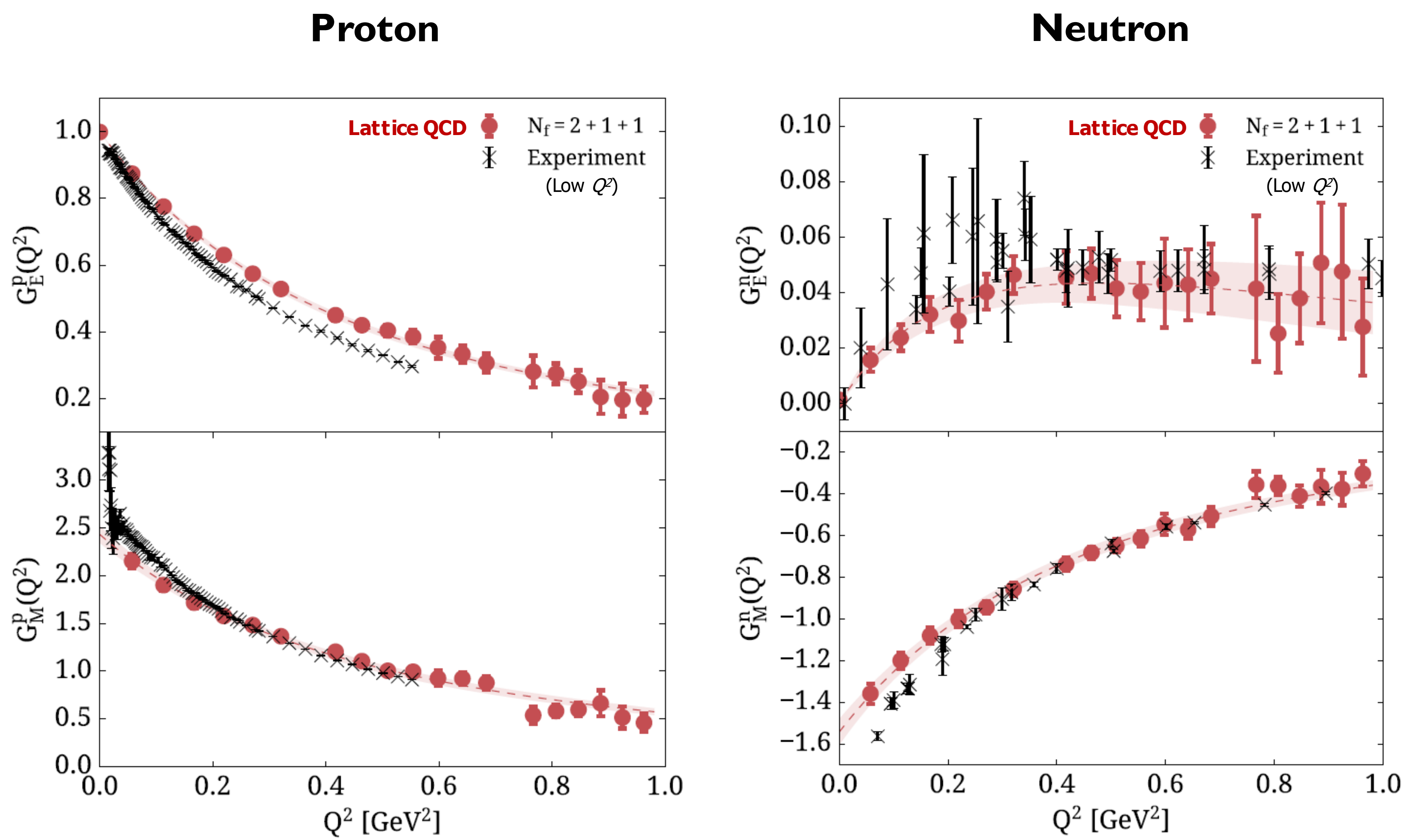
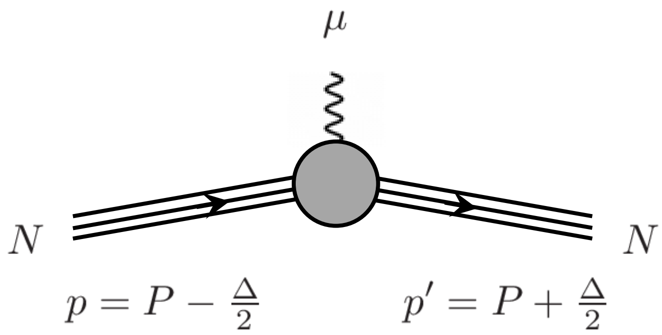
Nucleon form factors



[Alexandrou *et al.*, PRD100 (2019) 014509]

Electromagnetic current matrix elements

Normalization $\langle p'|p\rangle = (2\pi)^3 2p^0\,\delta^{(3)}(\vec{p}\,' - \vec{p})$



$$\langle p', s'|J^\mu(0)|p, s\rangle = \overline{u}(p', s')\Gamma^\mu(P, \Delta)u(p, s)$$

$$\Gamma^\mu(P, \Delta) = \gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M_N} F_2(Q^2)$$

Dirac form factor — **Pauli form factor**

$$F_1(0) = q_N, \qquad F_2(0) = \kappa_N$$

Electric charge — **Anomalous magnetic moment**

Sachs form factors

$$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

$Q^2 = -\Delta^2$

$\tau = Q^2/4M_N^2$

[Foldy, PR87 (1952) 688]
[Ernst, Sachs, Wali, PR119 (1960) 1105]
[Sachs, PR126 (1962) 2256]

Non-relativistic interpretation

Localized states

Normalization $\langle\vec{p}\,'|\vec{p}\rangle = (2\pi)^3\,\delta^{(3)}(\vec{p}\,' - \vec{p})$

$$\langle\vec{x}\,'|\rho(\vec{r})|\vec{x}\rangle = \int \frac{\mathrm{d}^3P}{(2\pi)^3}\,\frac{\mathrm{d}^3\Delta}{(2\pi)^3}\,e^{i\vec{P}\cdot(\vec{x}\,'-\vec{x})}e^{-i\vec{\Delta}\cdot(\vec{r}-\frac{\vec{x}\,'+\vec{x}}{2})}\,\langle\vec{P}+\tfrac{\vec{\Delta}}{2}|\rho(\vec{0})|\vec{P}-\tfrac{\vec{\Delta}}{2}\rangle$$

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Galilean symmetry $\boxed{\langle\vec{P}+\tfrac{\vec{\Delta}}{2}|\rho(\vec{0})|\vec{P}-\tfrac{\vec{\Delta}}{2}\rangle = \langle\tfrac{\vec{\Delta}}{2}|\rho(\vec{0})|-\tfrac{\vec{\Delta}}{2}\rangle}$

$$\langle \vec{x}\,'|\rho(\vec{r})|\vec{x}\rangle = \delta^{(3)}(\vec{x}\,'-\vec{x})\,\underbrace{\rho(\vec{r}-\vec{x})}$$

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Internal distribution

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Internal distribution

Generic expectation value $\langle\psi|\psi\rangle = 1$

$$\Rightarrow\quad \langle\rho\rangle_\psi(\vec{r}) = \langle\psi|\rho(\vec{r})|\psi\rangle = \int \mathrm{d}^3x\,|\psi(\vec{x})|^2\,\rho(\vec{r}-\vec{x})$$

Probabilistic interpretation

Relativistic interpretation (Sachs approach)

Generic expectation value $\langle\psi|\psi\rangle = 1$

Normalization $\langle\vec{p}\,'|\vec{p}\rangle = (2\pi)^3\,\delta^{(3)}(\vec{p}\,'-\vec{p})$

Wave packet $\tilde{\psi}(\vec{p}) = \langle\vec{p}|\psi\rangle$

$$\langle\psi|O(x)|\psi\rangle = \int \frac{\mathrm{d}^3P}{(2\pi)^3}\,\frac{\mathrm{d}^3\Delta}{(2\pi)^3}\,\tilde{\psi}^*(\vec{P}+\tfrac{\vec{\Delta}}{2})\tilde{\psi}(\vec{P}-\tfrac{\vec{\Delta}}{2})\,\langle\vec{P}+\tfrac{\vec{\Delta}}{2}|O(x)|\vec{P}-\tfrac{\vec{\Delta}}{2}\rangle$$

[Sachs, PR126 (1962) 2256]
[Burkardt, PRD62 (2000) 071503]
[Belitsky, Ji, Yuan, PRD69 (2004) 074014]

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Crucial assumption $\tilde{\psi}(\vec{P}\pm\tfrac{\vec{\Delta}}{2}) \approx \tilde{\psi}(\vec{P})$ $\quad\Delta^0 \approx 0$

$$\langle\psi|O(x)|\psi\rangle \approx \int \frac{\mathrm{d}^3P}{(2\pi)^3}\,|\tilde{\psi}(\vec{P})|^2 \underbrace{\int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\,e^{-i\vec{\Delta}\cdot\vec{x}}\,\langle\vec{P}+\tfrac{\vec{\Delta}}{2}|O(0)|\vec{P}-\tfrac{\vec{\Delta}}{2}\rangle}_{\text{Internal distribution}}$$

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Probabilistic interpretation

Validity domain $1/D \ll |\vec{\Delta}| \ll |\delta\vec{p}| \ll P^0$

Hydrogen $M_H D_H \approx 10^5$

Nucleon $M_N D_N \approx 4$ ⚠

[Sachs, PR126 (1962) 2256]
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Rest frame $|\vec{P}| = 0 \quad \Rightarrow \quad P^0 \approx M$

$|\psi(\vec{P})|^2 \to (2\pi)^3\delta^{(3)}(\vec{P})$ ⚠ **Clash with** $\tilde{\psi}(\vec{P}\pm\tfrac{\vec{\Delta}}{2}) \approx \tilde{\psi}(\vec{P})$ **!**

[Sachs, PR126 (1962) 2256]
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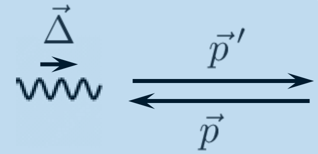
Relativistic interpretation (Sachs approach)

Breit (aka brick-wall) **frame**



$$\vec{P} = \vec{0} \quad \Rightarrow \quad \Delta^0 = \frac{\vec{P}\cdot\vec{\Delta}}{P^0} = 0$$

[Sachs, PR126 (1962) 2256]
[Friar, Negele, In *Adv. Nucl. Phys., Vol.8* (1975) 219]

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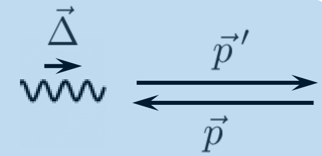

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Same structure as in non-relativistic case !

$$Q^2\big|_{\mathrm{BF}} = \vec{\Delta}^2$$

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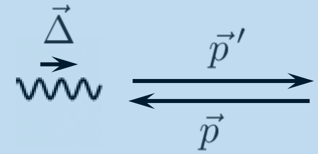

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3D charge distribution

$$\boxed{\rho_E^{\rm BF}(\vec{r}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{r}}\, \frac{G_E(Q^2)}{\sqrt{1+\tau}}}$$

Relativistic recoil corrections ?

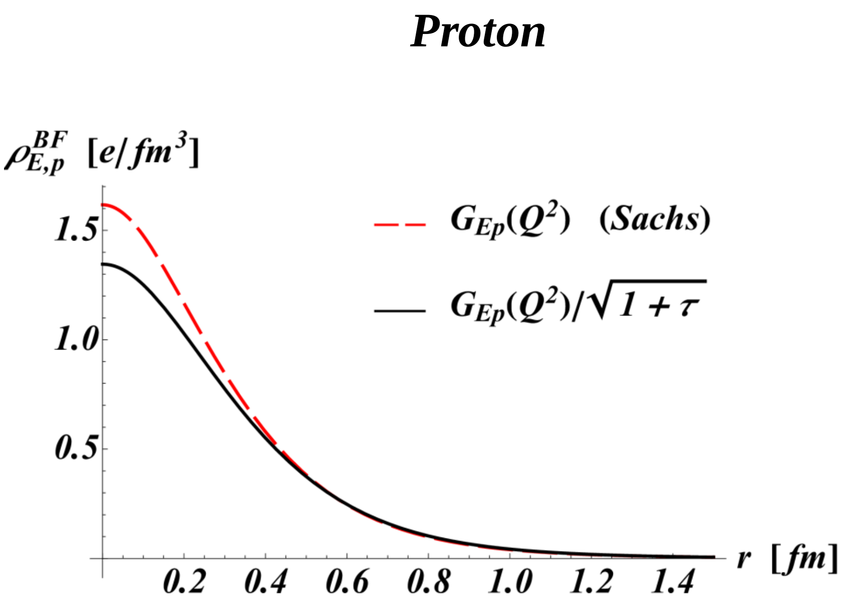
$$P^0\big|_{\rm BF} = M_N\sqrt{1+\tau}$$

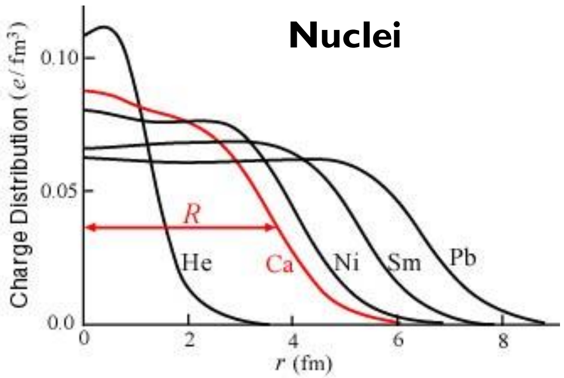
responsible for the Darwin term in the non-relativistic expansion

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\Big/\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\Big|_{\rm pointlike} = \left\{\left[G_E(Q^2)\right]^2 + \frac{\tau}{\epsilon}\left[G_M(Q^2)\right]^2\right\}\frac{1}{1+\tau}$$

[Sachs, PR126 (1962) 2256]
[Friar, Negele, In *Adv. Nucl. Phys., Vol.8* (1975) 219]

Breit frame distributions

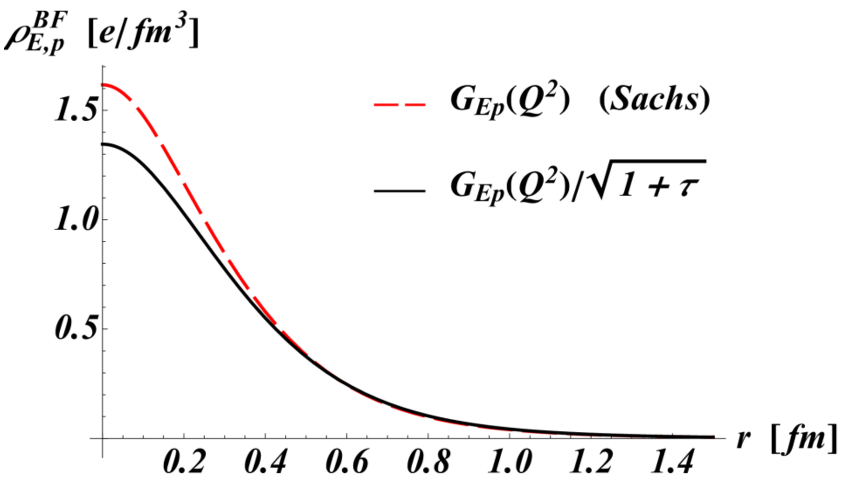




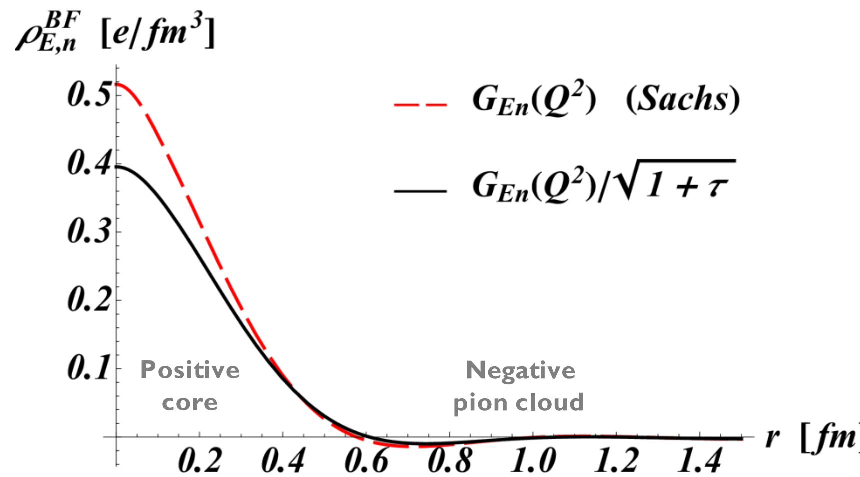
[C.L., PRL125 (2020) 232002]

Breit frame distributions

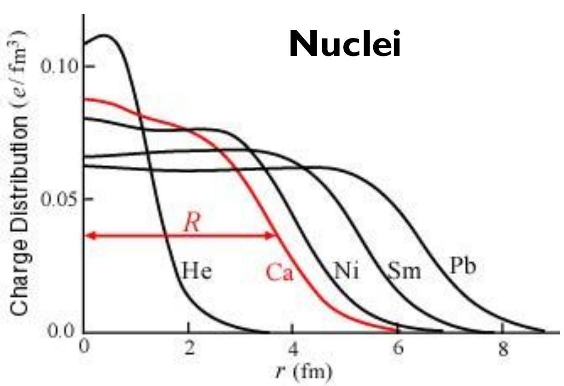
Proton



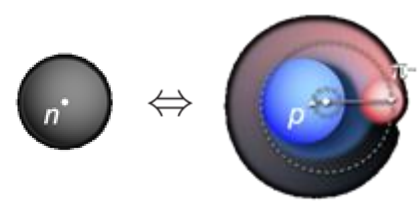
Neutron



Nuclei



Proton-pion fluctuation



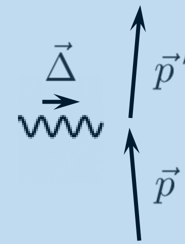
[C.L., PRL125 (2020) 232002]

Relativistic interpretation (IMF approach)

Probabilistic interpretation

Validity domain $1/D \ll |\vec{\Delta}| \ll |\delta\vec{p}| \ll P^0$

Infinite-momentum frame



$$P_z \to \infty \quad \Rightarrow \quad \Delta^0 \approx \Delta_z \ll P^0$$

[Bouchiat, Fayet, Meyer, NPB34 (1971) 157]
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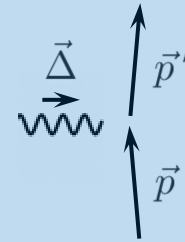

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$$\langle p', \lambda' | J^0(0) | p, \lambda \rangle \Big|_{\text{IMF}} = 2P^0 \left[\delta_{\lambda'\lambda} \, F_1(Q^2) + \frac{i(\vec{\sigma}_{\lambda'\lambda} \times \vec{\Delta})_z}{2M_N} \, F_2(Q^2) \right] \qquad Q^2\big|_{\text{IMF}} = \vec{\Delta}_\perp^2$$

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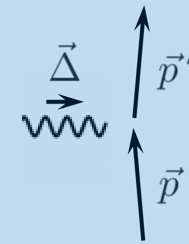

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2D charge distribution

$$\boxed{\rho_E^{\mathrm{IMF}}(\vec{b}_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp}\, F_1(Q^2) - \frac{(\vec{S} \times \vec{\nabla})_z}{M_N} \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp}\, F_2(Q^2)}$$

Galilean symmetry under finite boosts $\Rightarrow$ **No recoil correction !**

[Bouchiat, Fayet, Meyer, NPB34 (1971) 157]
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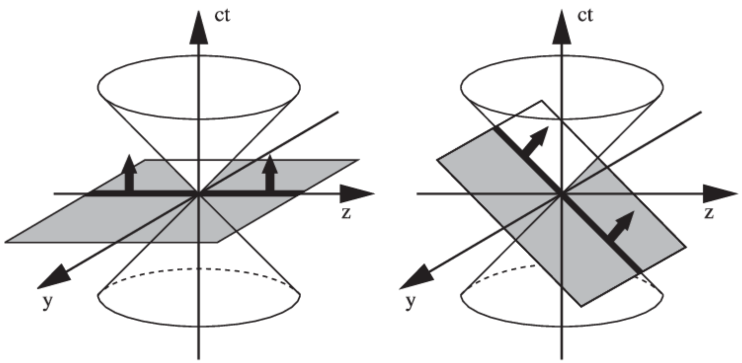
Other approaches with similar results

Light-front coordinates (no need to consider IMF)

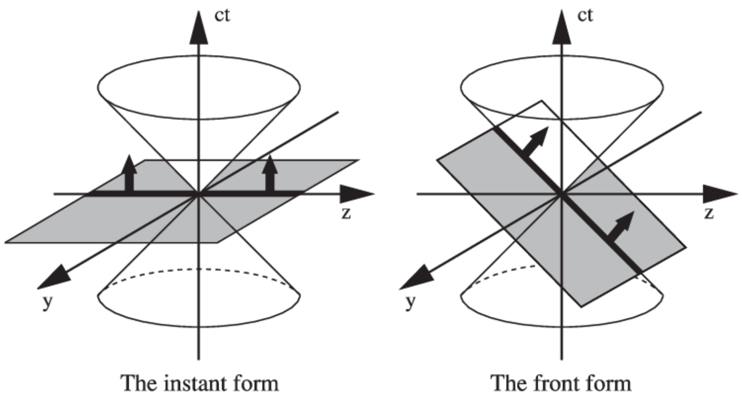


The instant form   The front form

[Ralston, Jain, Buniy, AIP Conf. Proc. 549 (2000) 1, 302]
[Burkardt, IJMPA 18 (2003) 2, 173]
[Miller, PRL99 (2007) 11200]
[Carlson, Vanderhaeghen, PRL100 (2008) 032004]

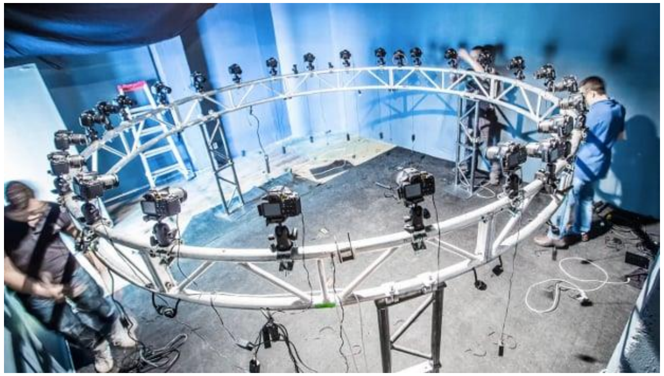
Other approaches with similar results

Light-front coordinates (no need to consider IMF)



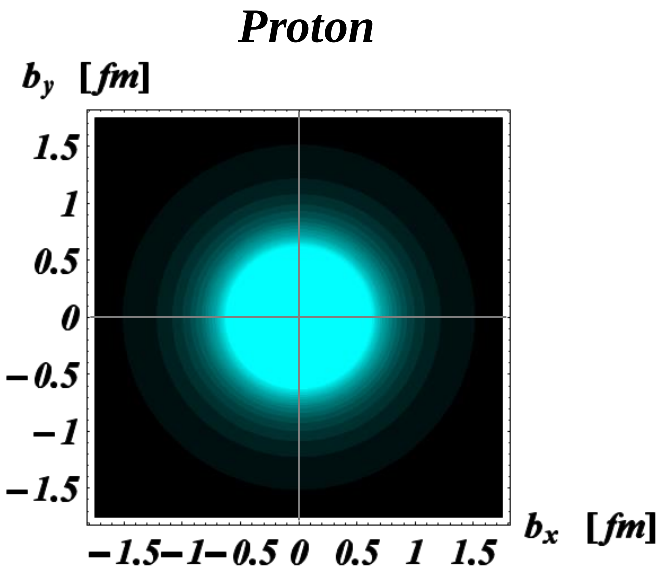
[Ralston, Jain, Buniy, AIP Conf. Proc. 549 (2000) 1, 302]
[Burkardt, IJMPA 18 (2003) 2, 173]
[Miller, PRL99 (2007) 11200]
[Carlson, Vanderhaeghen, PRL100 (2008) 032004]

Method of dimensional counting (IMF averaged over all directions)

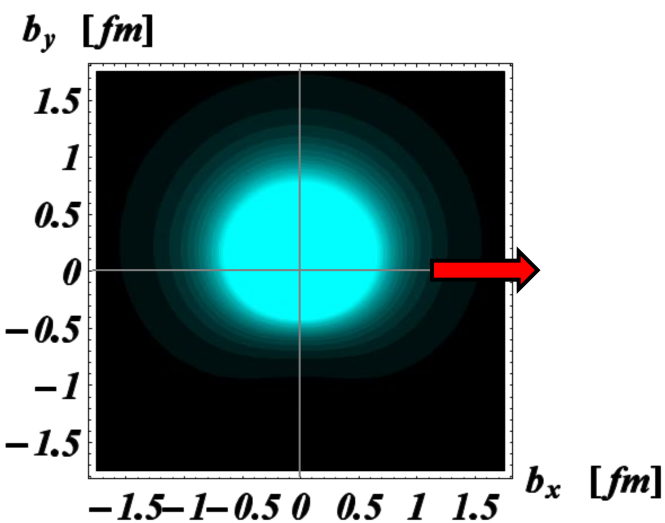


[Fleming, In *Phys. Reality & Math. Descrip.* (1974) 357]
[Epelbaum, Gegelia, Lange, Meissner, Polyakov, PRL129 (2022) 012001]
[Panteleeva, Epelbaum, Gegelia, Meissner, PRD106 (2022) 5, 056019]

IMF distributions



$$\vec{S} = \frac{\hbar}{2}\,\vec{e}_z$$



$$\vec{S} = \frac{\hbar}{2}\,\vec{e}_x$$

[Miller, PRL99 (2007) 11200]
[Carlson, Vanderhaeghen, PRL100 (2008) 032004]

IMF distributions

$\vec{S} = \frac{\hbar}{2}\,\vec{e}_z$

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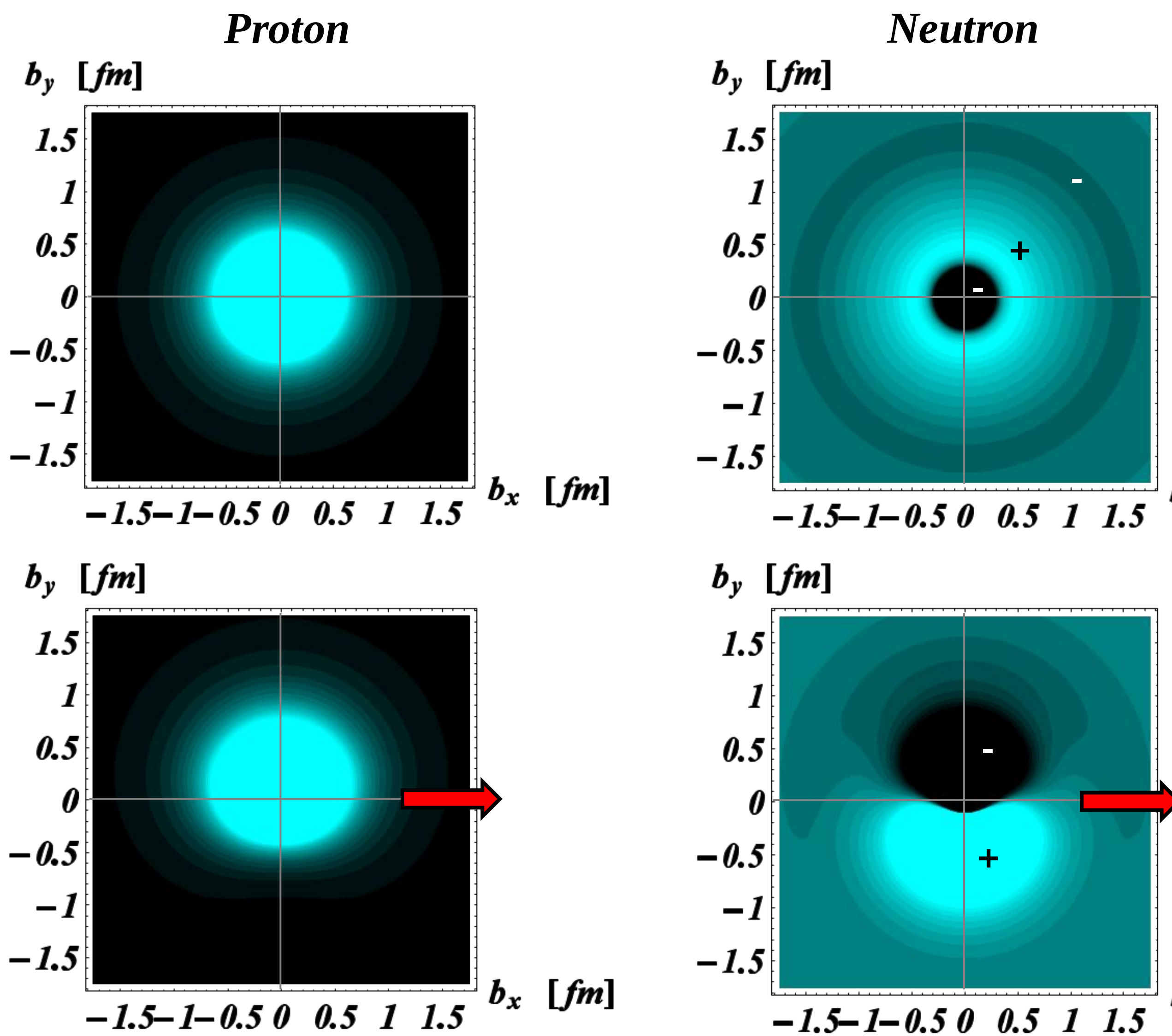


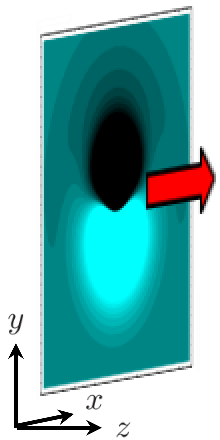
Negative core !?!

Electric dipole moment !?!

[Miller, PRL99 (2007) 11200]
[Carlson, Vanderhaeghen, PRL100 (2008) 032004]

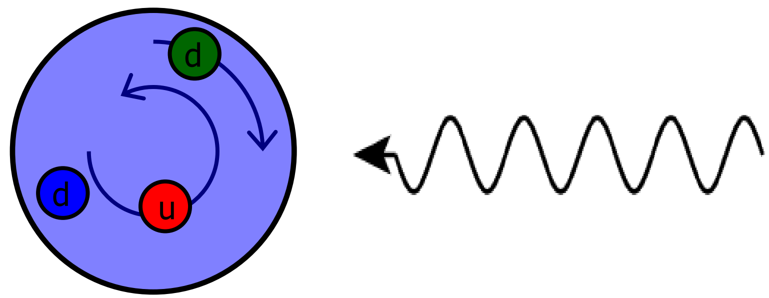
IMF artifacts – component mixing

$$J^0_{\mathrm{IMF}} \propto J^0_{\mathrm{rest}} + J^3_{\mathrm{rest}}$$



Neutron at rest

$J^3_{\mathrm{rest}} < 0$



$J^3_{\mathrm{rest}} > 0$

$\vec{S} \otimes$ $\quad \kappa_n = F_2^n(0) = -1.91$

Anomalous magnetic moment

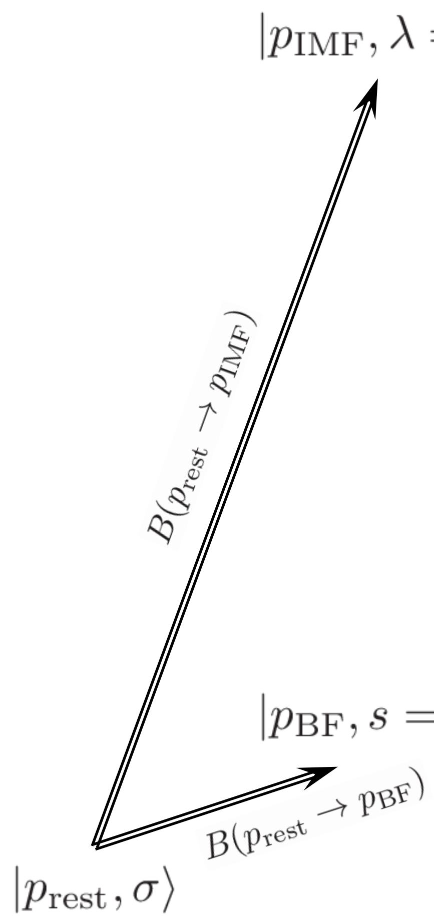
$$\vec{E}' = \gamma(\vec{E} + \vec{v} \times \vec{B}) \quad \Rightarrow \quad \vec{d}' = \gamma \vec{v} \times \vec{\mu}$$

Induced electric dipole moment

[Burkardt, IJMPA18 (2003) 173]
[Carlson, Vanderhaeghen, PRL100 (2008) 032004]

IMF artifacts – spin rotation

Relativistic boosts do not commute! $[K^i, K^j] = -i\epsilon^{ijk} J^k$



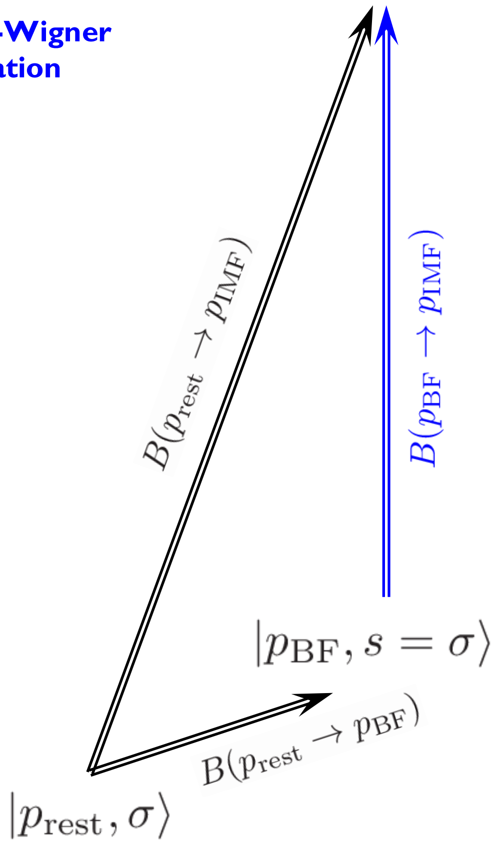
[Melosh, PRD9 (1974) 1095]
[Chung *et al.*, PRC37 (1988) 2000]
[Rinehimer, Miller, PRC80 (2009) 015201]

IMF artifacts – spin rotation

Relativistic boosts do not commute! $[K^i, K^j] = -i\epsilon^{ijk} J^k$

$$\sum_\lambda [R(p_{\rm BF} \to p_{\rm IMF})]_{\lambda'\lambda}\, |p_{\rm IMF}, \lambda = \sigma\rangle$$

Melosh-Wigner rotation



[Melosh, PRD9 (1974) 1095]
[Chung *et al.*, PRC37 (1988) 2000]
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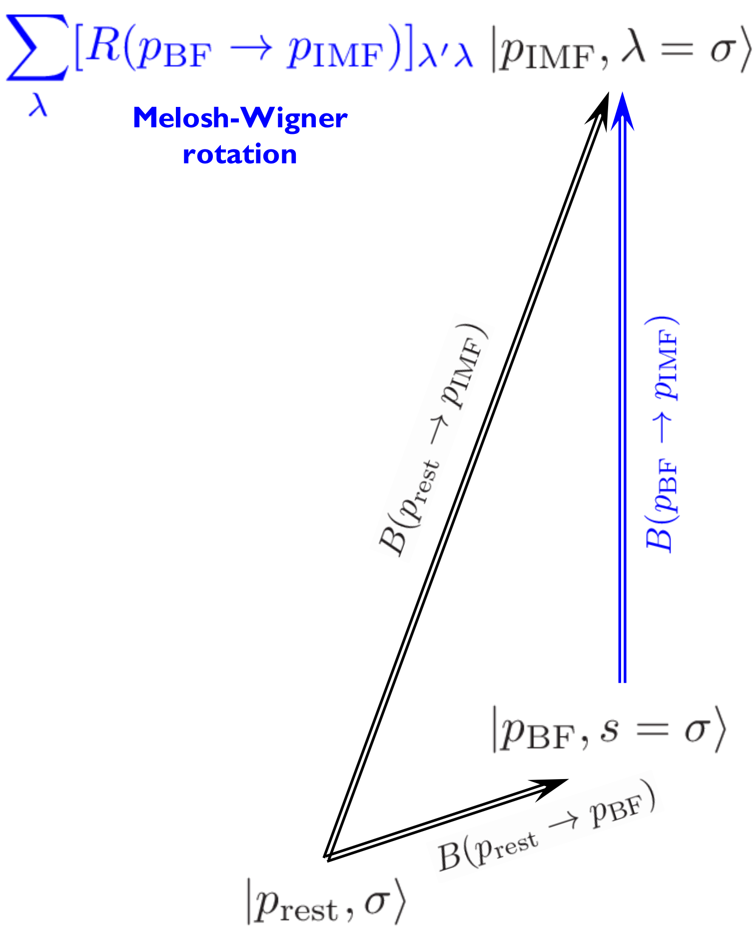

	Spin independent	*Spin dependent*
BF	G_E	G_M
IMF	$F_1 = \dfrac{G_E + \tau G_M}{1+\tau}$	$F_2 = \dfrac{G_M - G_E}{1+\tau}$

Which set is the « physical » one ?

[Melosh, PRD9 (1974) 1095]
[Chung *et al.*, PRC37 (1988) 2000]
[Rinehimer, Miller, PRC80 (2009) 015201]

Fundamental problems

1) **The notion of spatial distribution relies on simultaneity**
2) **A probabilistic interpretation requires that inertia does not depend on momentum**

[C.L., EPJC78 (2018) 9, 785]
[C.L., PRL125 (2020) 232002]

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… but not Lorentz symmetry !

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***Traditional perspective*:** **maintain strict probabilistic interpretation by**

- **neglecting recoil corrections (Sachs approach)**
- **restricting to Galilean subgroup (IMF approach)**

[C.L., EPJC78 (2018) 9, 785]
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***<u>Traditional perspective</u>*: maintain strict probabilistic interpretation by**

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- **restricting to Galilean subgroup (IMF approach)**

***<u>New perspective</u>*: relax probabilistic interpretation**
but fully account for frame dependence !

[C.L., EPJC78 (2018) 9, 785]
[C.L., PRL125 (2020) 232002]

Phase-space approach

Phase-space representation

$$\langle\psi|O(x)|\psi\rangle = \int \frac{\mathrm{d}^3P}{(2\pi)^3}\,\mathrm{d}^3R\,\rho_\psi(\vec{R},\vec{P})\,\langle O\rangle_{\vec{R},\vec{P}}(x)$$

Nucleon Wigner distribution

$$\rho_\psi(\vec{R},\vec{P}) = \int \mathrm{d}^3z\, e^{-i\vec{P}\cdot\vec{z}}\,\psi^*(\vec{R}-\tfrac{\vec{z}}{2})\psi(\vec{R}+\tfrac{\vec{z}}{2})$$

$$= \int \frac{\mathrm{d}^3q}{(2\pi)^3}\, e^{-i\vec{q}\cdot\vec{R}}\,\tilde{\psi}^*(\vec{P}+\tfrac{\vec{q}}{2})\tilde{\psi}(\vec{P}-\tfrac{\vec{q}}{2})$$

$$\psi(\vec{r}) = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\, e^{-i\vec{p}\cdot\vec{r}}\,\tilde{\psi}(\vec{p})$$

[Wigner, PR40 (1932) 749]
[Hillery, O'Connell, Scully, Wigner, PR106 (1984) 121]
[Bialynicki-Birula, Gornicki, Rafelski, PRD 44 (1991) 1825]

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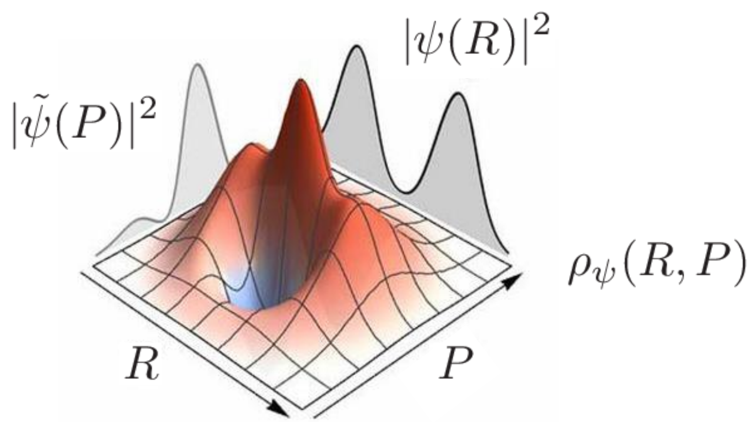
$$\psi(\vec{r}) = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\,e^{-i\vec{p}\cdot\vec{r}}\,\tilde{\psi}(\vec{p})$$

Quasi-probabilistic interpretation

$$\int \mathrm{d}^3R\,\rho_\psi(\vec{R},\vec{P}) = |\tilde{\psi}(\vec{P})|^2$$
$$\int \frac{\mathrm{d}^3P}{(2\pi)^3}\,\rho_\psi(\vec{R},\vec{P}) = |\psi(\vec{R})|^2$$



[Wigner, PR40 (1932) 749]
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[Bialynicki-Birula, Gornicki, Rafelski, PRD 44 (1991) 1825]

Relativistic spatial distributions

Internal distribution (for a state « localized » in phase-space)

$$\langle O\rangle_{\vec{R},\vec{P}}(\vec{x}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot(\vec{x}-\vec{R})}\, \langle \vec{P}+\tfrac{\vec{\Delta}}{2}|O(0)|\vec{P}-\tfrac{\vec{\Delta}}{2}\rangle$$

$$= \langle O\rangle_{\vec{0},\vec{P}}(\vec{r}), \qquad \vec{r} = \vec{x}-\vec{R}$$

[C.L., Mantovani, Pasquini, PLB776 (2018) 38]
[C.L., EPJC78 (2018) 9, 785]
[C.L., Moutarde, Trawinski, EPJC79 (2019) 89]

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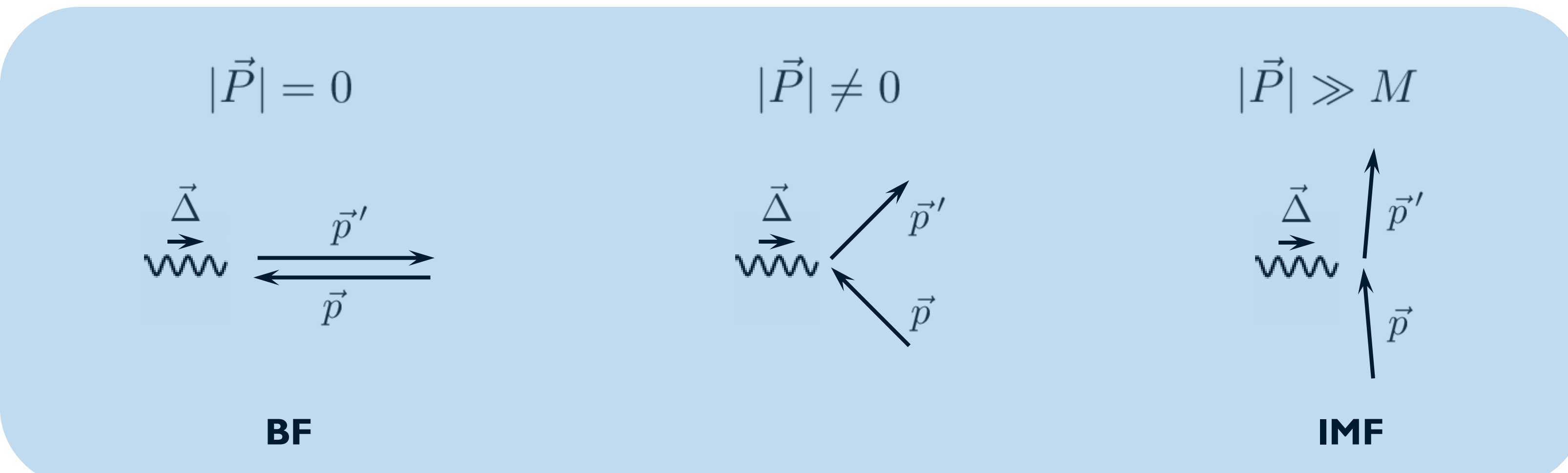
$$= \langle O\rangle_{\vec 0,\vec P}(\vec r), \qquad \vec r = \vec x - \vec R$$

Elastic frames $\Delta^0 = \frac{\vec P\cdot\vec\Delta}{P^0} \stackrel{!}{=} 0$ (no energy transfer ➡ same initial and final boost factor)



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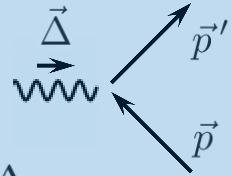
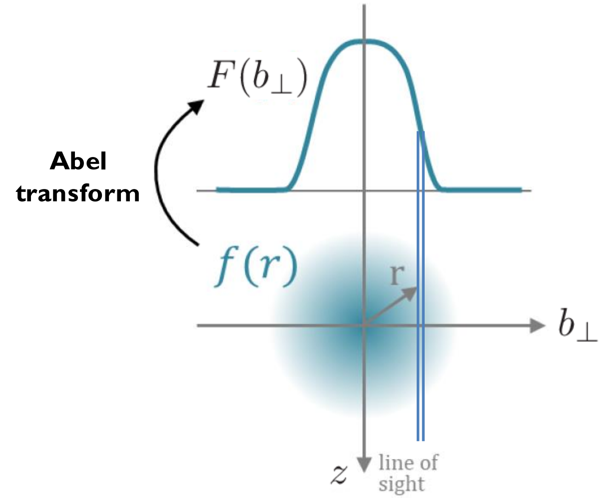
Relativistic spatial distributions

Elastic frame



$$\vec{P} = P_z \vec{e}_z \quad \Rightarrow \quad \Delta^0 = \frac{P_z \Delta_z}{P^0}$$

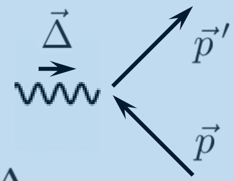
$$\Delta^0 = 0 \quad \Rightarrow \quad \Delta_z = 0 \quad \Leftrightarrow \quad \int \mathrm{d}z$$



[C.L., Mantovani, Pasquini, PLB776 (2018) 38]
[C.L., PRL125 (2020) 232002]
[Panteleeva, Polyakov, PRD104 (2021) 1, 014008]
[Kim, Kim, PRD104 (2021) 7, 074003]

Relativistic spatial distributions

Elastic frame



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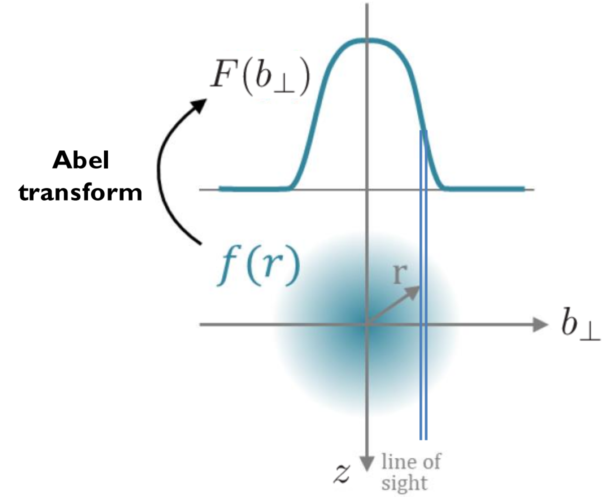

2D charge distribution

$$\boxed{\begin{aligned} \rho_E^{\mathrm{EF}}(\vec{b}_\perp; P_z) &\equiv \int \mathrm{d}z \, \langle J^0(r) \rangle_{\vec{R}, P_z \vec{e}_z} \\ &= \int \frac{\mathrm{d}^2 \Delta_\perp}{(2\pi)^2} \, e^{-i \vec{\Delta}_\perp \cdot \vec{b}_\perp} \, \frac{\langle p', s' | J^0(0) | p, s \rangle}{2P^0} \bigg|_{\mathrm{EF}} \end{aligned}}$$

Interpolates between BF and IMF

$$\rho_E^{\mathrm{EF}}(\vec{b}_\perp; 0) = \int \mathrm{d}z \, \rho_E^{\mathrm{BF}}(\vec{r})$$

$$\rho_E^{\mathrm{EF}}(\vec{b}_\perp; \infty) = \rho_E^{\mathrm{IMF}}(\vec{b}_\perp)$$

$$\vec{b}_\perp = \vec{r}_\perp - \vec{R}_\perp$$

$$\langle p', s' | p, s \rangle = (2\pi)^3 2p^0 \, \delta^{(3)}(\vec{p}\,' - \vec{p}) \, \delta_{s's}$$

[C.L., Mantovani, Pasquini, PLB776 (2018) 38]
[C.L., PRL125 (2020) 232002]
[Panteleeva, Polyakov, PRD104 (2021) 1, 014008]
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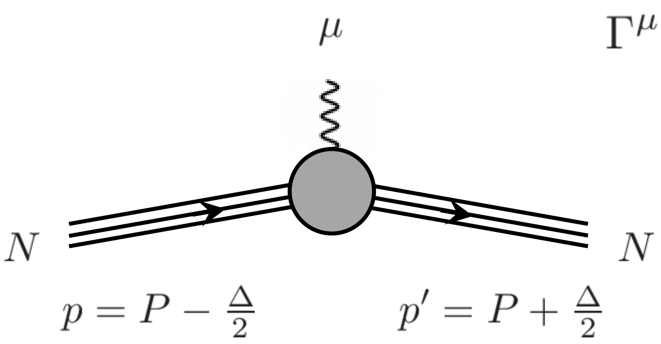
EF charge distributions



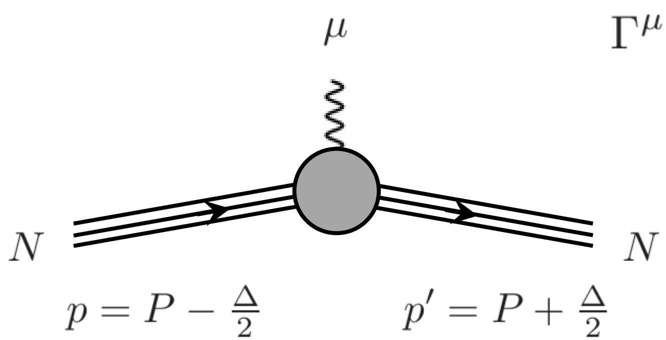
$$\Gamma^\mu(P,\Delta) = \gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M_N} F_2(Q^2)$$

$$= \frac{MP^\mu}{P^2} G_E(Q^2) + \frac{i\epsilon^{\mu\alpha\beta\lambda}\Delta_\alpha P_\beta \gamma_\lambda \gamma_5}{2P^2} G_M(Q^2)$$

Reminiscent of $\vec{J} = \rho\vec{v} + \vec{\nabla} \times \vec{M}$ **!**

[C.L., PRL125 (2020) 232002]

EF charge distributions



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Reminiscent of $\vec{J} = \rho\vec{v} + \vec{\nabla} \times \vec{M}$!

$$\boxed{\rho_E^{\mathrm{EF}}(b;P_z) = \int_0^\infty \frac{\mathrm{d}Q}{2\pi}\, Q\, J_0(Qb)\, [\tilde{\rho}_E^{\mathrm{conv}}(Q;P_z) + \tilde{\rho}_E^{\mathrm{magn}}(Q;P_z)]}$$

Longitudinal polarization

$$\tilde{\rho}_E^{\mathrm{conv}}(Q;P_z) = \frac{P^0 + M(1+\tau)}{(P^0+M)(1+\tau)} G_E(Q^2)$$

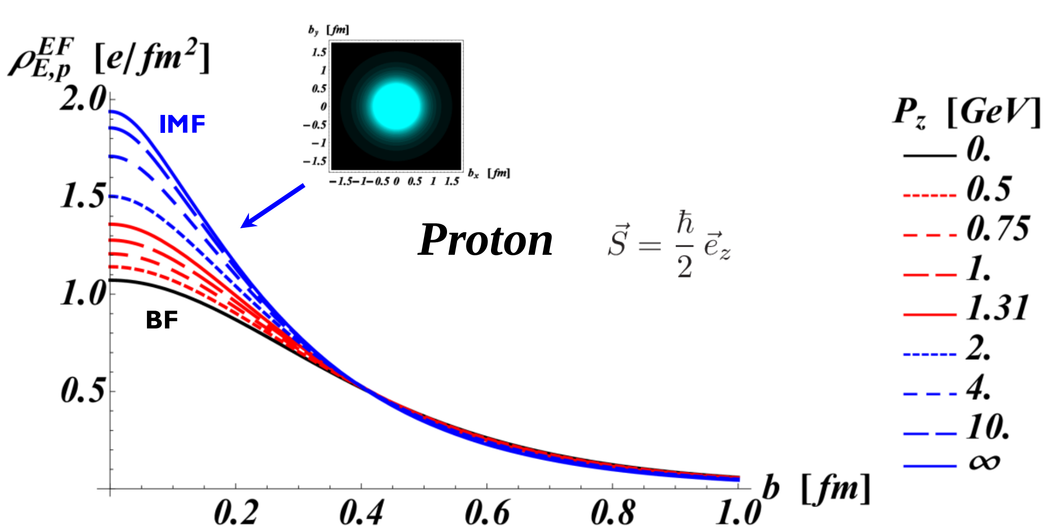
$$\tilde{\rho}_E^{\mathrm{magn}}(Q;P_z) = \frac{\tau P_z^2}{P^0(P^0+M)(1+\tau)} G_M(Q^2)$$

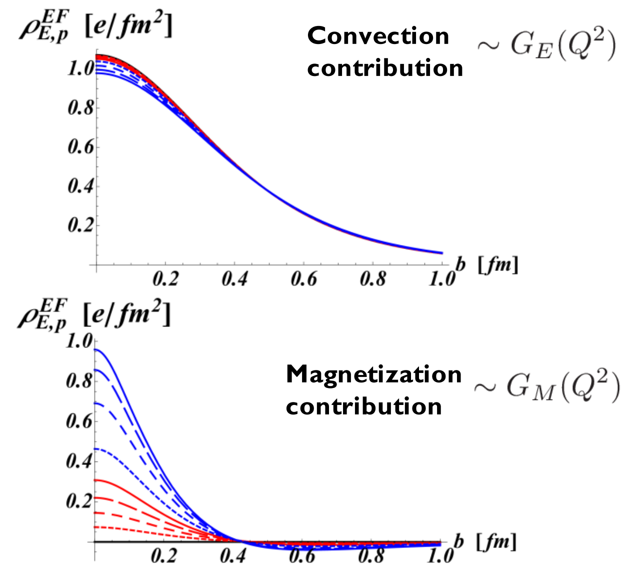
BF	IMF
$\tilde{\rho}_E^{\mathrm{conv}}(Q;0) = \frac{G_E(Q^2)}{\sqrt{1+\tau}}$	$\tilde{\rho}_E^{\mathrm{conv}}(Q;\infty) = \frac{G_E(Q^2)}{1+\tau}$
$\tilde{\rho}_E^{\mathrm{magn}}(Q;0) = 0$	$\tilde{\rho}_E^{\mathrm{magn}}(Q;\infty) = \frac{\tau G_M(Q^2)}{1+\tau}$

$$Q^2 = -\Delta^2$$
$$\tau = Q^2/4M_N^2$$
$$P^0 = \sqrt{M^2(1+\tau) + P_z^2}$$

[C.L., PRL125 (2020) 232002]

EF charge distributions (longitudinal polarization)





[C.L., PRL125 (2020) 232002]

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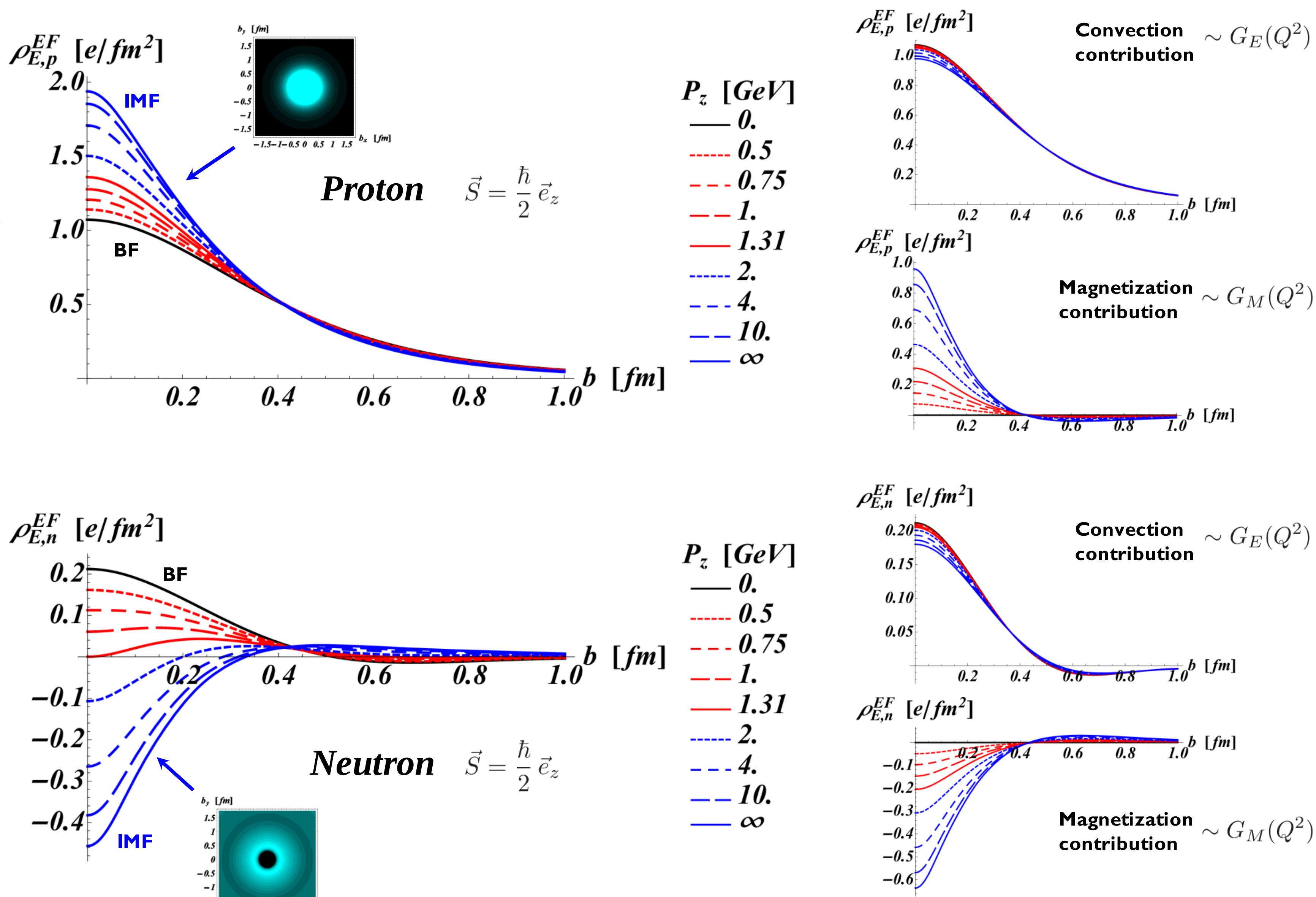
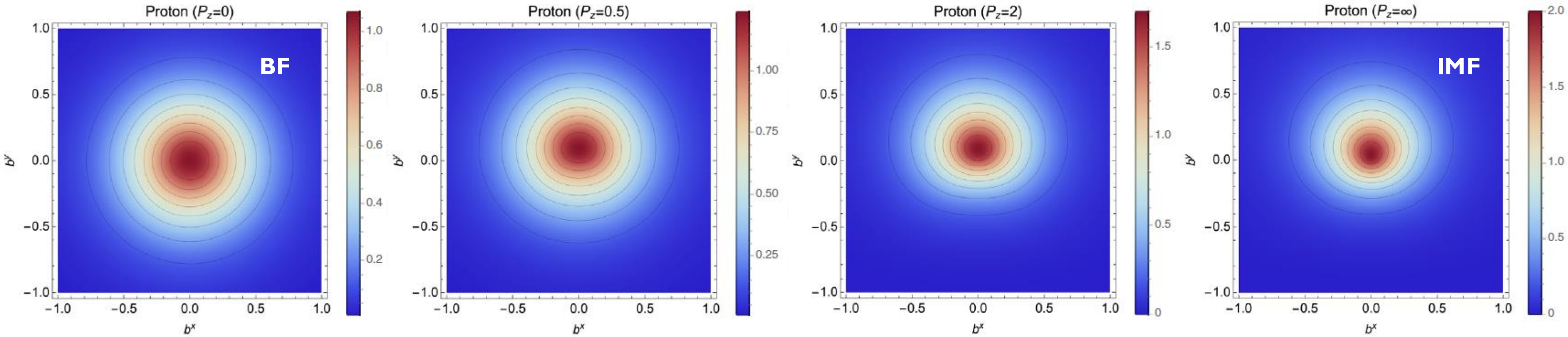


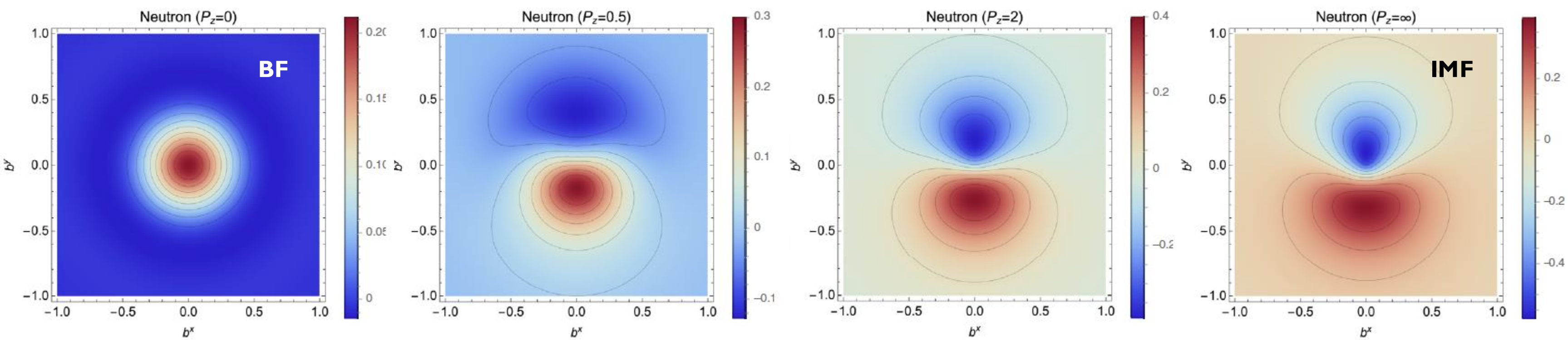
[C.L., PRL125 (2020) 232002]

EF charge distributions (transverse polarization)

Proton $\vec{S} = \frac{\hbar}{2}\,\vec{e}_x$



Neutron $\vec{S} = \frac{\hbar}{2}\,\vec{e}_x$



[Kim, Kim, PRD104 (2021) 7, 074003]

Four-current amplitude

Expected Lorentz transformation of an off-forward amplitude

$$\langle p', s' | J^\mu(0) | p, s \rangle = \sum_{s'_B, s_B} D^{*(j)}_{s'_B s'}(p'_B, \Lambda) D^{(j)}_{s_B s}(p_B, \Lambda)\, \Lambda^\mu{}_\nu \, \langle p'_B, s'_B | J^\nu(0) | p_B, s_B \rangle$$

[Durand, De Celles, Marr, PR126 (1962) 1882]

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[Durand, De Celles, Marr, PR126 (1962) 1882]

Confirmation by explicit calculation

$$J^0_{\mathrm{EF}}(\boldsymbol{b}_\perp; P_z) = e\int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left[\delta_{s's}\cos\theta + \frac{(\boldsymbol{\sigma}_{s's}\times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}}\sin\theta\right]\frac{G_E(\boldsymbol{\Delta}^2_\perp)}{\sqrt{1+\tau}} + e\int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\frac{P_z}{P^0}\left[-\delta_{s's}\sin\theta + \frac{(\boldsymbol{\sigma}_{s's}\times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}}\cos\theta\right]\frac{\sqrt{\tau}G_M(\boldsymbol{\Delta}^2_\perp)}{\sqrt{1+\tau}}$$

$$J_{z,\mathrm{EF}}(\boldsymbol{b}_\perp; P_z) = e\int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\frac{P_z}{P^0}\left[\delta_{s's}\cos\theta + \frac{(\boldsymbol{\sigma}_{s's}\times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}}\sin\theta\right]\frac{G_E(\boldsymbol{\Delta}^2_\perp)}{\sqrt{1+\tau}} + e\int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\left[-\delta_{s's}\sin\theta + \frac{(\boldsymbol{\sigma}_{s's}\times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}}\cos\theta\right]\frac{\sqrt{\tau}G_M(\boldsymbol{\Delta}^2_\perp)}{\sqrt{1+\tau}}$$

$$\boldsymbol{J}_{\perp,\mathrm{EF}}(\boldsymbol{b}_\perp; P_z) = e(\sigma_z)_{s's}\int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\frac{(\boldsymbol{e}_z\times i\boldsymbol{\Delta})_\perp}{2P^0} G_M(\boldsymbol{\Delta}^2_\perp)$$

Wigner rotation

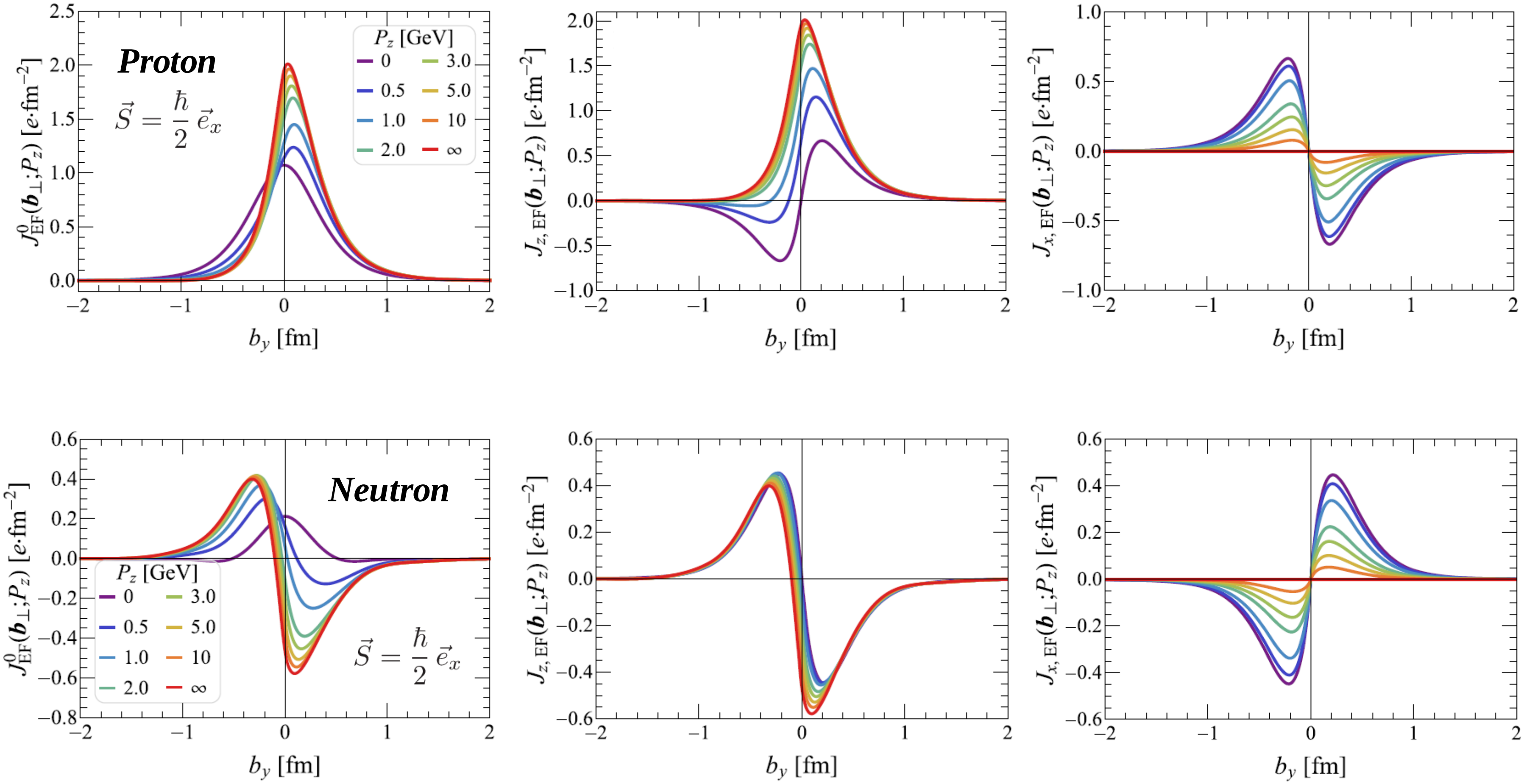
$$\cos\theta = \frac{P^0 + M(1+\tau)}{(P^0+M)\sqrt{1+\tau}}$$

$$\sin\theta = -\frac{\sqrt{\tau}P_z}{(P^0+M)\sqrt{1+\tau}}$$

[C.L., Wang, PRD105 (2022) 9, 096032]
[Chen, C.L., PRD106 (2022) 11, 116024]

Four-current distributions (transverse polarization)

Charge distribution | **Longitudinal current** | **Transverse current**



[Chen, C.L., PRD106 (2022) 11, 116024]

Convection and polarization

Polarized medium

$$J^\mu = J_c^\mu + \partial_\alpha P^{\alpha\mu}$$

Convection current **Polarization** current

$$J^0 = \rho_c - \vec{\nabla} \cdot \vec{\mathcal{P}}$$

$$\vec{J} = \rho_c \vec{v} + \vec{\nabla} \times \vec{M} + \partial_t \vec{\mathcal{P}}$$

Electric polarization

$$P^{\mu\nu} = \begin{pmatrix} 0 & \mathcal{P}_x & \mathcal{P}_y & \mathcal{P}_z \\ -\mathcal{P}_x & 0 & -M_z & M_y \\ -\mathcal{P}_y & M_z & 0 & -M_x \\ -\mathcal{P}_z & -M_y & M_x & 0 \end{pmatrix}$$

Magnetic polarization
(or magnetization)

[Chen, C.L., 2302.04672 [hep-ph] to appear in PRD]

Convection and polarization

Polarized medium

$$J^\mu = J^\mu_c + \partial_\alpha P^{\alpha\mu}$$

Convection current

Polarization current

Electric polarization

$$P^{\mu\nu} = \begin{pmatrix} 0 & \mathcal{P}_x & \mathcal{P}_y & \mathcal{P}_z \\ -\mathcal{P}_x & 0 & -M_z & M_y \\ -\mathcal{P}_y & M_z & 0 & -M_x \\ -\mathcal{P}_z & -M_y & M_x & 0 \end{pmatrix}$$

Magnetic polarization (or magnetization)

$$J^0 = \rho_c - \vec{\nabla}\cdot\vec{\mathcal{P}}$$

$$\vec{J} = \rho_c\vec{v} + \vec{\nabla}\times\vec{M} + \partial_t\vec{\mathcal{P}}$$

General spin ½ target

$$\langle p', s'|J^\mu(0)|p, s\rangle = \overline{u}(p', s')\Gamma^\mu(P, \Delta)u(p, s)$$

$$\Gamma^\mu(P, \Delta) = \gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M_N}F_2(Q^2)$$

$\Rightarrow \quad \Gamma^{\mu\nu}_P = \;?$

[Chen, C.L., 2302.04672 [hep-ph] to appear in PRD]

Convection and polarization

Polarized medium

$$J^\mu = J^\mu_c + \partial_\alpha P^{\alpha\mu}$$

Convection current — **Polarization** current

Electric polarization

$$P^{\mu\nu} = \begin{pmatrix} 0 & \mathcal{P}_x & \mathcal{P}_y & \mathcal{P}_z \\ -\mathcal{P}_x & 0 & -M_z & M_y \\ -\mathcal{P}_y & M_z & 0 & -M_x \\ -\mathcal{P}_z & -M_y & M_x & 0 \end{pmatrix}$$

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(or magnetization)

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$$\Gamma^\mu(P,\Delta) = \gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M_N}F_2(Q^2) \quad \Rightarrow \quad \Gamma^{\mu\nu}_P = \; ?$$

$$= \frac{MP^\mu}{P^2}G_E(Q^2) + \frac{i\epsilon^{\mu\alpha\beta\lambda}\Delta_\alpha P_\beta\gamma_\lambda\gamma_5}{2P^2}G_M(Q^2) \quad \Rightarrow \quad \Gamma^{\mu\nu}_P = -\frac{e}{2M}\frac{M\epsilon^{\mu\nu\beta\lambda}P_\beta\gamma_\lambda\gamma_5}{P^2}G_M(Q^2)$$

[Chen, C.L., 2302.04672 [hep-ph] to appear in PRD]

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Polarized medium

$$J^\mu = J_c^\mu + \partial_\alpha P^{\alpha\mu}$$

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$$\Gamma^\mu(P,\Delta) = \gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M_N}F_2(Q^2) \quad\Rightarrow\quad \Gamma_P^{\mu\nu} = \ ?$$

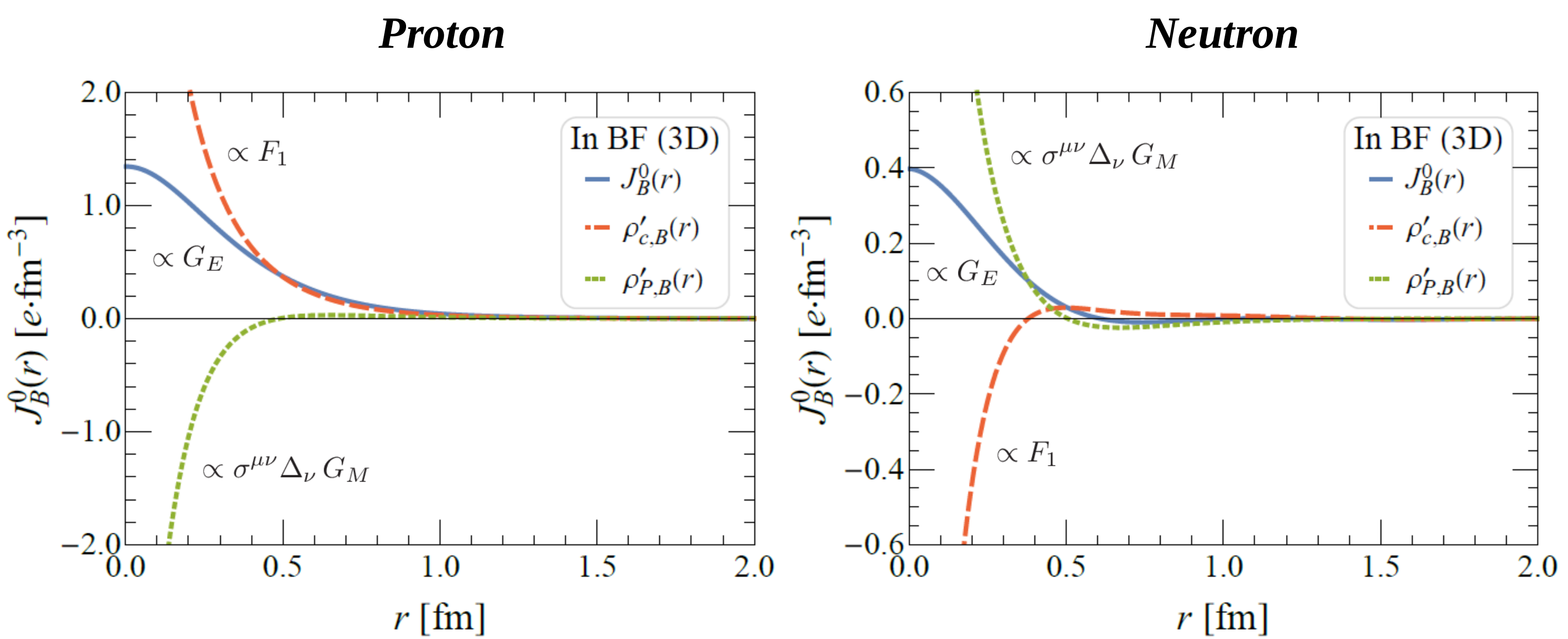
$$= \frac{MP^\mu}{P^2}G_E(Q^2) + \frac{i\epsilon^{\mu\alpha\beta\lambda}\Delta_\alpha P_\beta\gamma_\lambda\gamma_5}{2P^2}G_M(Q^2) \quad\Rightarrow\quad \Gamma_P^{\mu\nu} = -\frac{e}{2M}\frac{M\epsilon^{\mu\nu\beta\lambda}P_\beta\gamma_\lambda\gamma_5}{P^2}G_M(Q^2)$$

$$= \frac{P^\mu}{M}F_1(Q^2) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}G_M(Q^2) \quad\Rightarrow\quad \Gamma_P^{\mu\nu} = -\frac{e}{2M}\sigma^{\mu\nu}G_M(Q^2)$$

[Chen, C.L., 2302.04672 [hep-ph] to appear in PRD]

Convection and polarization

Breit frame charge distributions



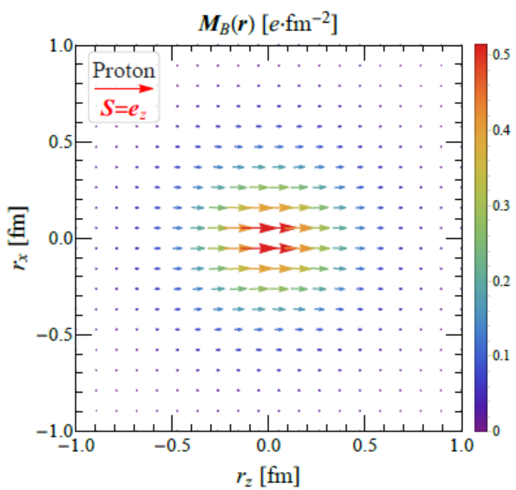
$$\boxed{\Gamma_P^{\mu\nu} = -\frac{e}{2M}\frac{M\epsilon^{\mu\nu\beta\lambda}P_\beta\gamma_\lambda\gamma_5}{P^2}G_M(Q^2)}$$

is the most natural definition !

[Chen, C.L., 2302.04672 [hep-ph] to appear in PRD]

BF magnetization distributions

$$\vec{M}_B = \frac{e}{2M}\int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{r}}\left[\vec{\sigma} - \frac{\vec{\Delta}(\vec{\Delta}\cdot\vec{\sigma})}{4P_B^0(P_B^0+M)}\right]\frac{G_M(Q^2)}{\sqrt{1+\tau}}$$



Genuine magnetization distribution

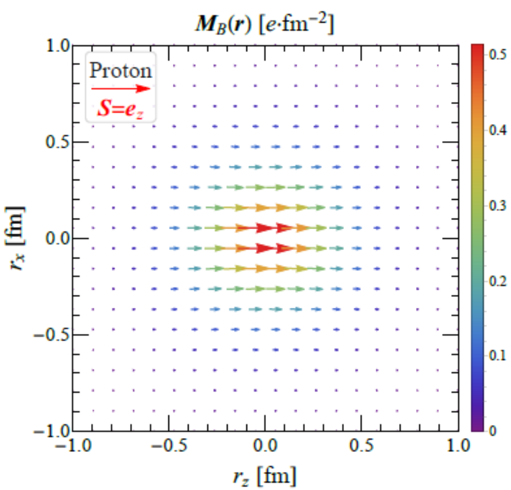
$$\vec{\mu}_B = \int \mathrm{d}^3r\, \vec{M}_B$$

[Chen, C.L., 2302.04672 [hep-ph] to appear in PRD]

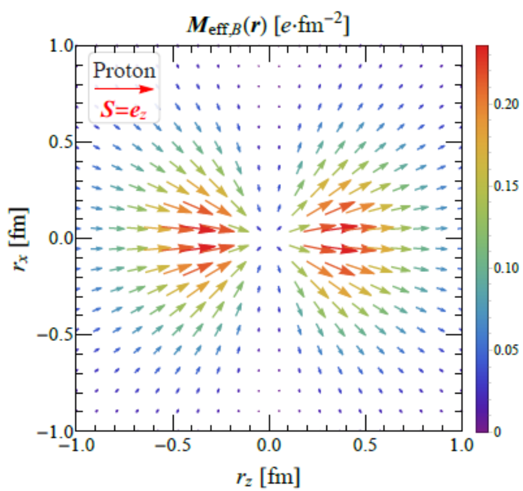
BF magnetization distributions

$$\vec{M}_B = \frac{e}{2M}\int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{r}}\left[\vec{\sigma} - \frac{\vec{\Delta}(\vec{\Delta}\cdot\vec{\sigma})}{4P_B^0(P_B^0+M)}\right]\frac{G_M(Q^2)}{\sqrt{1+\tau}}$$

$$\vec{M}_{\text{eff},B} = \vec{r}\rho_{M,B}, \qquad \rho_{M,B} = -\vec{\nabla}\cdot\vec{M}_B$$



Genuine magnetization distribution



Contributions to the MDM sitting at the origin from effective magnetic density

$$\vec{\mu}_B = \int \mathrm{d}^3r\,\vec{M}_B = \int \mathrm{d}^3r\,\vec{M}_{\text{eff},B}$$

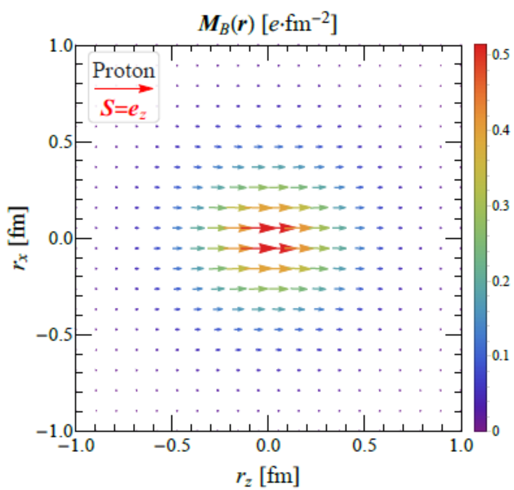
[Chen, C.L., 2302.04672 [hep-ph] to appear in PRD]

BF magnetization distributions

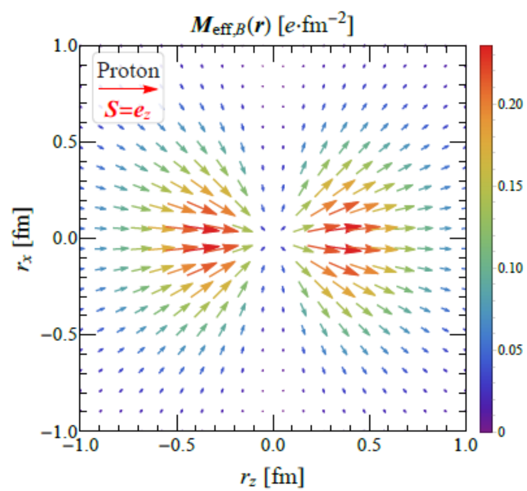
$$\vec{M}_B = \frac{e}{2M}\int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{r}}\left[\vec{\sigma} - \frac{\vec{\Delta}(\vec{\Delta}\cdot\vec{\sigma})}{4P_B^0(P_B^0+M)}\right]\frac{G_M(Q^2)}{\sqrt{1+\tau}}$$

$$\vec{M}_{\mathrm{eff},B} = \vec{r}\,\rho_{M,B}, \qquad \rho_{M,B} = -\vec{\nabla}\cdot\vec{M}_B$$

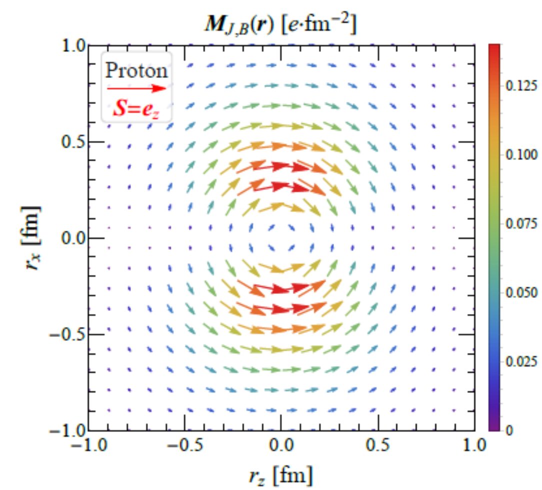
$$\vec{M}_{J,B} = \frac{\vec{r}\times\vec{J}_B}{2}, \qquad \vec{J}_B = \vec{\nabla}\times\vec{M}_B$$



Genuine magnetization distribution



Contributions to the MDM sitting at the origin from effective magnetic density

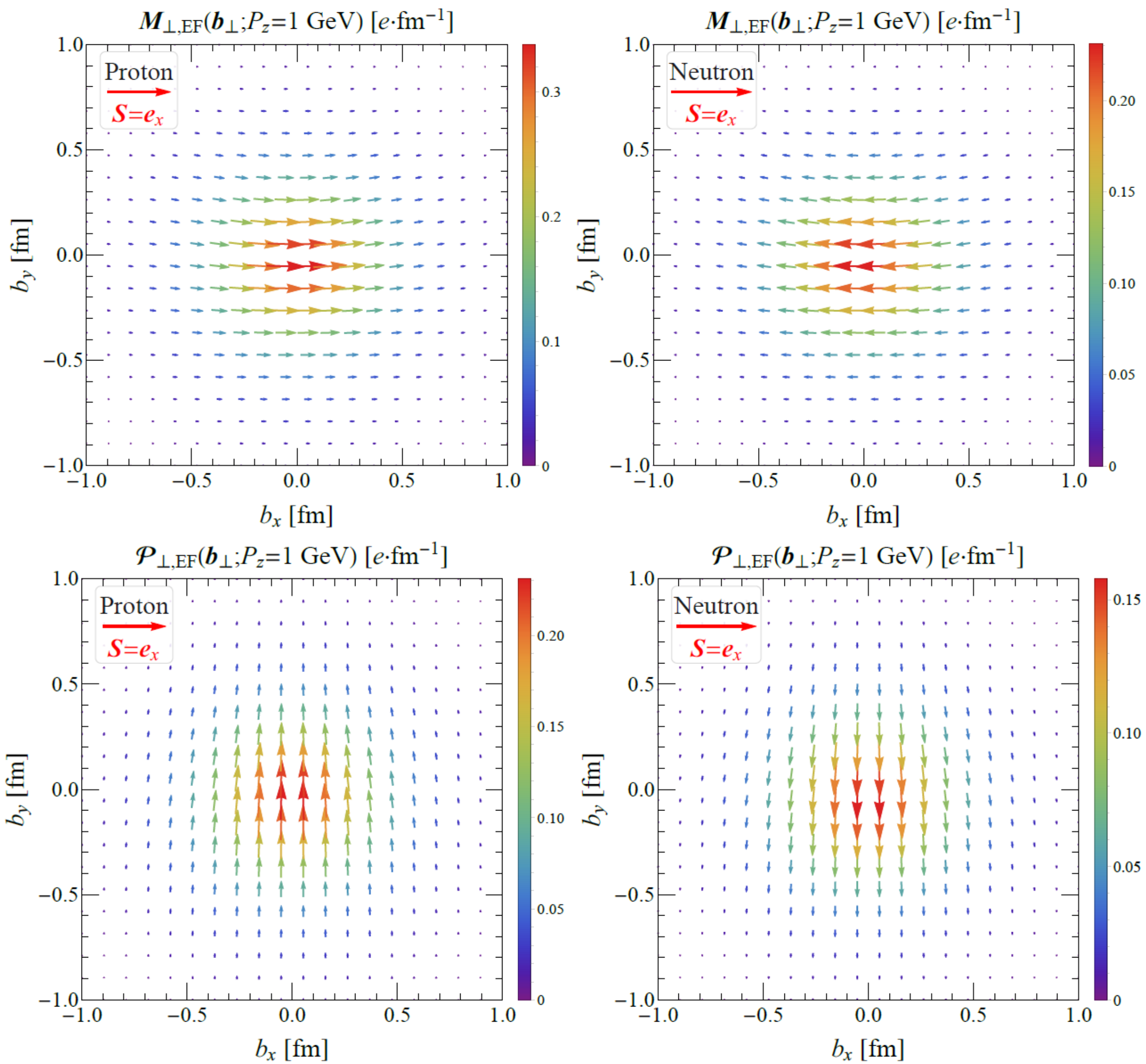


Contributions to the MDM sitting at the origin from charge current density

$$\vec{\mu}_B = \int \mathrm{d}^3r\,\vec{M}_B = \int \mathrm{d}^3r\,\vec{M}_{\mathrm{eff},B} = \int \mathrm{d}^3r\,\vec{M}_{J,B}$$

[Chen, C.L., 2302.04672 [hep-ph] to appear in PRD]

EF polarization and magnetization distributions

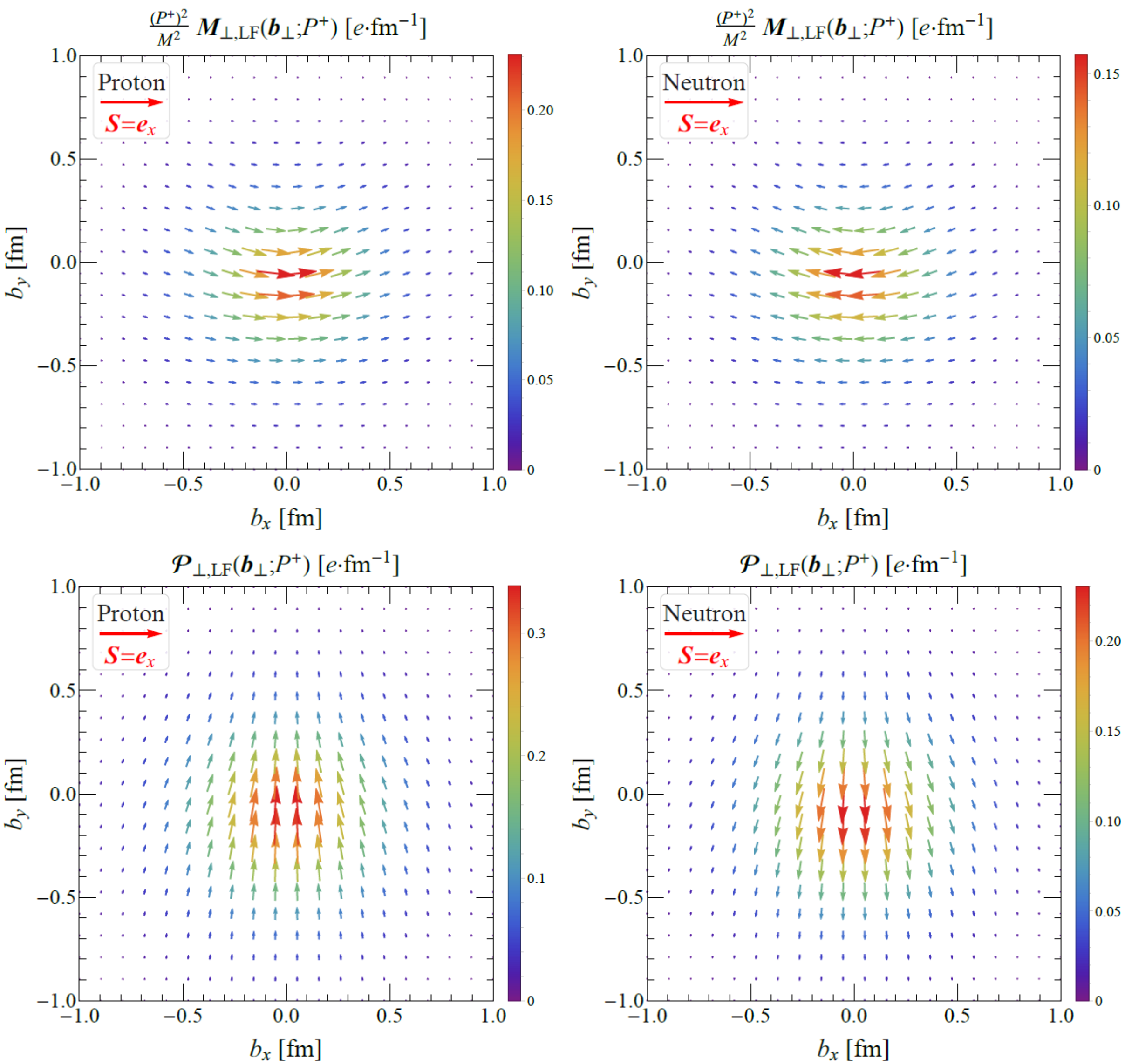


Induced electric polatization

$$\vec{\mathcal{P}}_{\perp,\mathrm{EF}} = \vec{v} \times \vec{M}_{\perp,\mathrm{EF}}$$

[Chen, C.L., 2302.04672 [hep-ph] to appear in PRD]

LF polarization and magnetization distributions

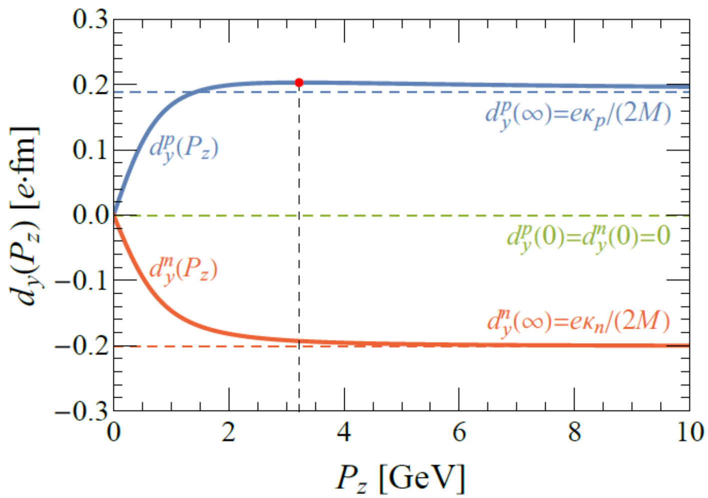


$$M^i_{\perp,\mathrm{LF}} = -\epsilon^{ij}_\perp P^{-j} = \frac{M^i_\perp - \epsilon^{ij}_\perp \mathcal{P}^j_\perp}{\sqrt{2}}$$

$$\mathcal{P}^i_{\perp,\mathrm{LF}} = P^{+i} = \frac{\mathcal{P}^i_\perp - \epsilon^{ij}_\perp M^j_\perp}{\sqrt{2}}$$

[Chen, C.L., 2302.04672 [hep-ph] to appear in PRD]

Induced electric dipole moment

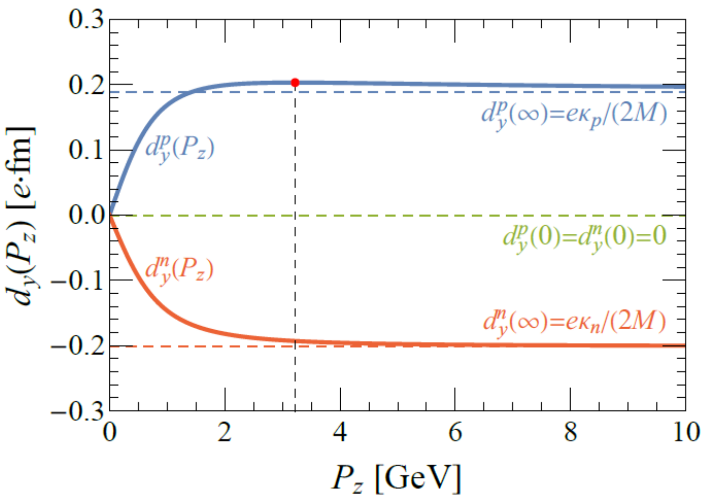


Contribution due to shift between center of spin and center of mass

$$\vec{d}_{\perp,\mathrm{EF}}(P_z) = (\vec{e}_z \times \vec{\sigma})_\perp \underbrace{\frac{P_z}{E_P}\left[G_M(0) - \boxed{\frac{E_P}{E_P+M}\,G_E(0)}\right]}_{\xrightarrow[P_z\to\infty]{} F_2(0)=\kappa} \frac{e}{2M}$$

[C.L., PRD79 (2009) 113011]
[Kim, Kim, PRD104 (2021) 7, 074003]
[C.L., Wang, PRD105 (2022) 9, 096032]
[Chen, C.L., 2302.04672 [hep-ph] to appear in PRD]

Induced electric dipole moment



Contribution due to shift between center of spin and center of mass

$$\vec{d}_{\perp,\mathrm{EF}}(P_z) = (\vec{e}_z\times\vec{\sigma})_\perp \underbrace{\frac{P_z}{E_P}\left[G_M(0) - \boxed{\frac{E_P}{E_P+M}\,G_E(0)}\right]}_{\xrightarrow[P_z\to\infty]{} F_2(0)=\kappa}\frac{e}{2M}$$

Higher-spin targets

$$\vec{d}^{(j)}_{\perp,\mathrm{EF}}(P_z) = (\vec{e}_z\times\vec{\Sigma})_\perp \underbrace{\frac{P_z}{E_P}\left[G_{M1}(0) - \frac{E_P}{E_P+M}\,2j\,G_{E0}(0)\right]}_{\xrightarrow[P_z\to\infty]{} G_{M1}(0)-2j\,G_{E0}(0)}\frac{e}{2M}$$

agrees with $g=2$ for pointlike particles !

[C.L., PRD79 (2009) 113011]
[Kim, Kim, PRD104 (2021) 7, 074003]
[C.L., Wang, PRD105 (2022) 9, 096032]
[Chen, C.L., 2302.04672 [hep-ph] to appear in PRD]

Conclusions

- Clash between probabilistic interpretation and Lorentz symmetry
- Phase-space approach provides a general perspective interpolating between different pictures (BF, IMF, …)
- Wigner spin rotations are not intuitive but are key to understanding the frame dependence of spatial distributions
- Sachs form factors (G_E, G_M) lead to a much simpler general picture than Dirac and Pauli (F_1, F_2) form factors

Centers of a relativistic system