Nonlinear Dynamics, Complex Networks, Information Theory and Machine Learning in Neuroscience

ORGANIZERS

LECTURERS

Hilda Cerdeira
(ICTP-SAIFR, Brazil)

Jesus Gomez-Gardeñes
(Universidad de Zaragoza, España)

Cristina Masoller
(Universitat Politécnica de Catalunya, España)

Ana Amador
(Universidad de Buenos Aires, Argentina)

Osvaldo Rosso
(Universidade Federal de Alagoas, Brazil)

Jordi Soriano
(Universitat de Barcelona, España)

ICTP SAIFR
International Centre for Theoretical Physics
South American Institute for Fundamental Research

Support: Humberto, Thiago and Jandira

Time	Monday, day 22	Tuesday, day 23	Wednesday, day 24	Thursday, day 25	Friday, day 26
8 :30 - 9 :30	Registration				
9 :30 - 10 :30	Course 1.1. Cristina Masoller: Introduction to time-series analysis	Course 2.2. Ana Amador: Introduction to nonlinear dynamics and excitable systems	Course 4.2. Jordi Soriano: Introduction to neuronal cultures: experimental and data analysis techniques	Course 1.3. Cristina Masoller: Introduction to time-series analysis	Course 4.2. Jordi Soriano: Introduction to neuronal cultures: experimental and data analysis techniques
10 :30 - 11 :00	BREAK	BREAK	BREAK	BREAK	BREAK
11 :00 - 12 :00	Course 2.1. Ana Amador: Introduction to nonlinear dynamics and excitable systems	Course 1.2. Cristina Masoller: Introduction to time-series analysis	Course 3.3. Jesús Gomez-Gardenes: Introduction to synchronization phenomena and the Kuramoto model	Course 5.2. Osvaldo Rosso: Introduction to information theory and complexity measures	Course 5.3. Osvaldo Rosso: Introduction to information theory and complexity measures
12 :00 - 13 :00	Course 3.1. Jesús Gomez-Gardenes: Introduction to synchronization phenomena and the Kuramoto model	Course 3.2. Jesús Gomez-Gardenes: Introduction to synchronization phenomena and the Kuramoto model	Course 5.1. Osvaldo Rosso: Introduction to information theory and complexity measures	Course 2.3. Ana Amador: Introduction to nonlinear dynamics and excitable systems	Hands on
13 :00 - 14 :30	LUNCH	LUNCH	LUNCH	LUNCH	LUNCH
14 :30 - 15 :30	Course 4.1. Jordi Soriano: Introduction to neuronal cultures: experimental and data analysis techniques	Hands on	IFT-COLLOQUIUM. Jesús Gomez-Gardenes (at 14: 00)	Hands on	Hands on
15 :30 - 17 :00	STUDENTS´ PRESENTATION	Hands on	Hands on	Hands on	PRESENTATION PROJECTS

Colloquium

Network epidemiology: A complex systems' approach towards epidemic control

School on Nonlinear Dynamics, Complex Networks, Information Theory and Machine Learning in Neuroscience, 22-26 May 2023

Nonlinear time series analysis

Cristina Masoller

Departamento de Fisica

Universitat Politecnica de Catalunya

Class 1: From dynamical systems to complex systems

Class 2: Univariate time series analysis

Class 3: Bivariate and multivariate analysis

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Campus d'Excel·lència Internacional

ICTP SAIFR International Centre for Theoretical Physics South American Institute for Fundamental Research

cristina.masoller@upc.edu @cristinamasoll1

Presentation

- Originally from Montevideo, Uruguay.
- Bachelor and Master degrees from Facultad de Ciencias, Universidad de la Republica, Uruguay.
- PhD in physics (Bryn Mawr College, USA).
- Professor of Physics, Universitat Politecnica de Catalunya.
- Research group: Dynamics, Nonlinear Optics and Lasers

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BRYN MAWR COLLEGE

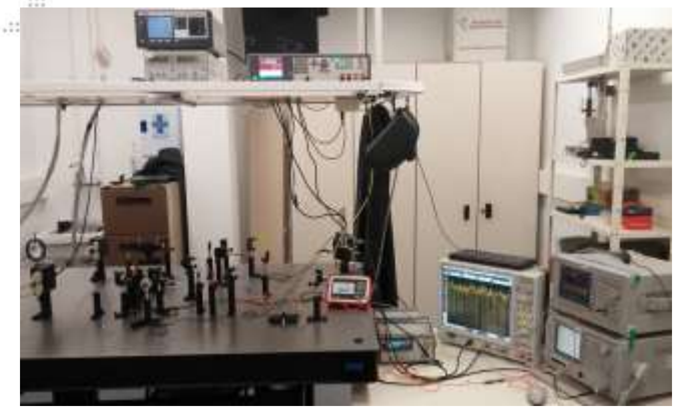
UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH

cristina.masoller@upc.edu @cristinamasoll1

Where are we? UPC Campus Terrassa


Viernes, 25 de septiembre de 2009 Diari de Terrassa

El edificio Gaia centraliza grupos científicos consolidados y emergentes.



Laser lab in Gaia Building,
UPC Campus Terrassa

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Research lines

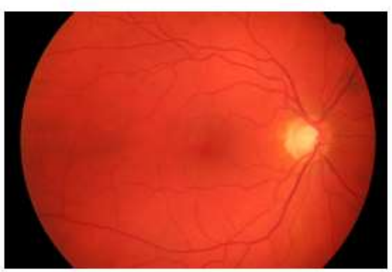
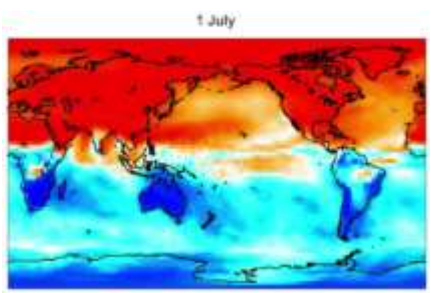
Nonlinear dynamics and complex systems

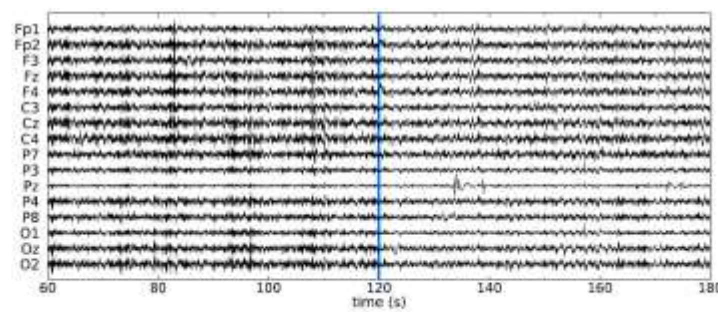
Data analysis techniques

Applications





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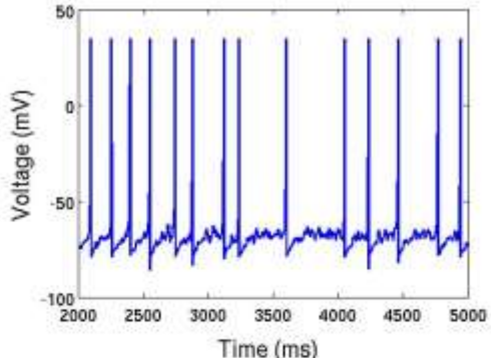
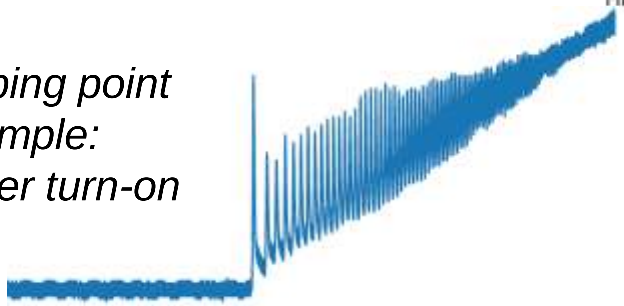
Lasers, neurons, climate, complex systems?

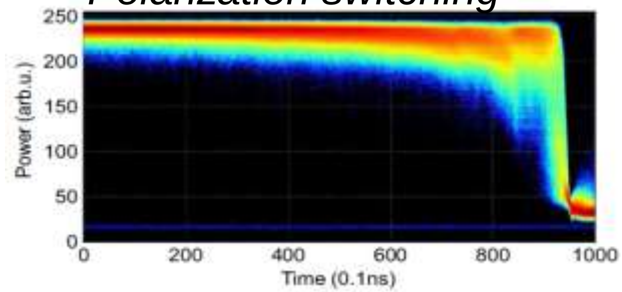
Laser & neuronal spikes



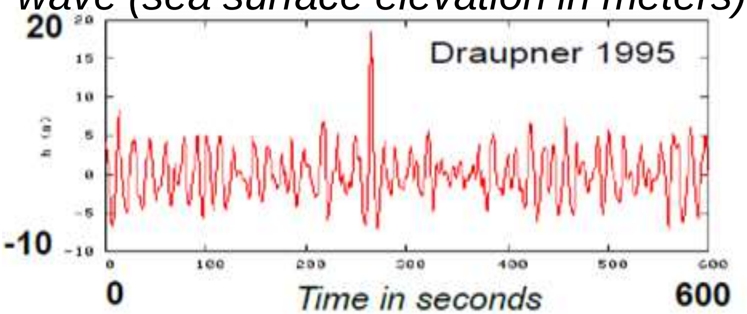
Tipping point example: Laser turn-on



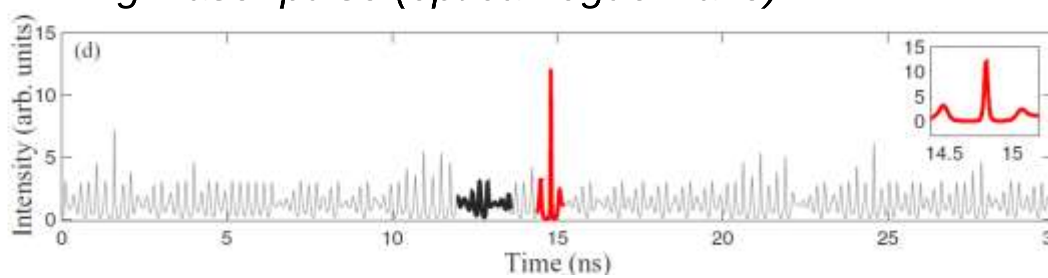
Polarization switching



Extreme event example: ocean rogue wave (sea surface elevation in meters)



High laser pulse (optical rogue wave)



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Outline

Class 1: From dynamical systems to complex systems

- Dynamical systems
- The Logistic map
- Chaotic attractors
- Synchronization
- The Kuramoto model
- Complex networks
- Machine learning and data science

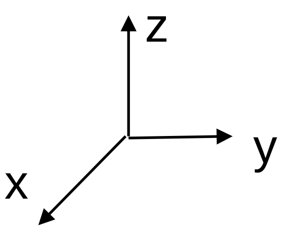
Class 2: Univariate time series analysis

Class 3: Bivariate and Multivariate analysis

cristina.masoller@upc.edu @cristinamasoll1

The start of dynamical systems: Newton & Poincare

- Mid-1600s: Newtonian mechanics
- Analytic planetary orbits (the “two-body” problem).
- No analytic solution of the “three-body” problem.
- Late 1800s: Poincare’s phase space and recurrence theorem



Certain systems will, after a sufficiently long but finite time, return to a state very close to the initial state.

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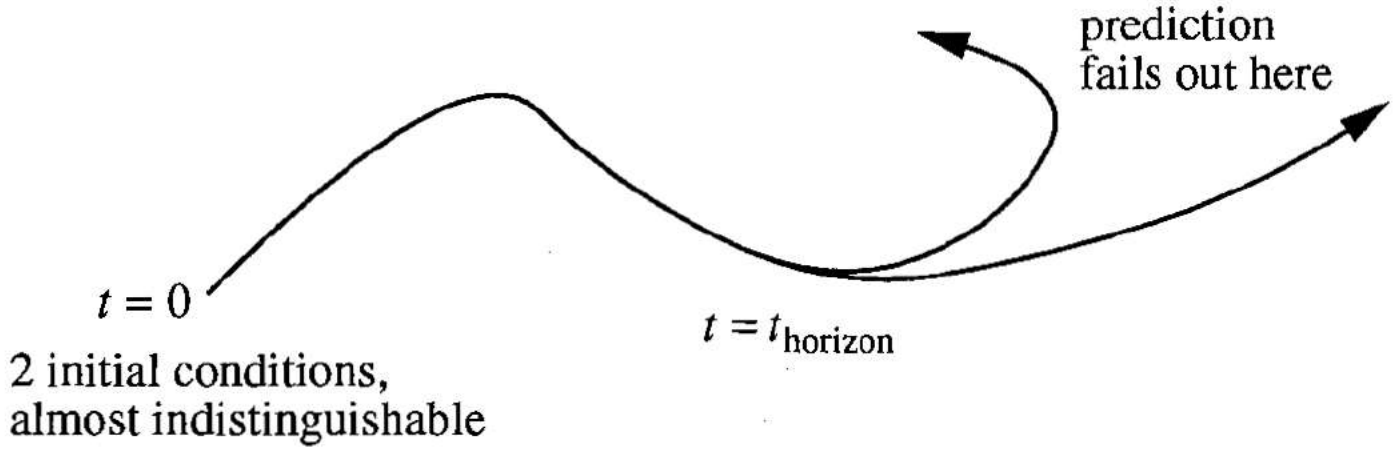
Poincare also had the intuition of the possibility of chaos

*"The evolution of a **deterministic** system can be aperiodic, unpredictable, and strongly depends on the initial conditions".*



How to determine the prediction horizon?
How to estimate the uncertainty?

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1950-60s: computer simulations

- Huge advance in the field of "*Dynamical Systems*".
- 1960s: **Eduard Lorenz**: simple model of convection rolls in the atmosphere.

$$\frac{dx}{dt} = -\sigma x + \sigma y,$$
$$\frac{dy}{dt} = -xz + rx - y,$$
$$\frac{dz}{dt} = xy - bz.$$

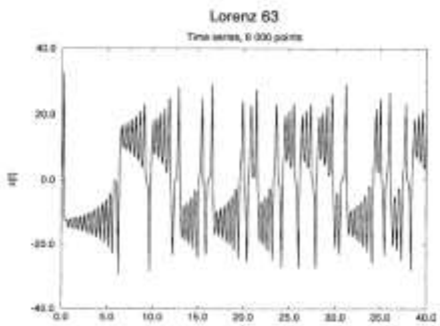
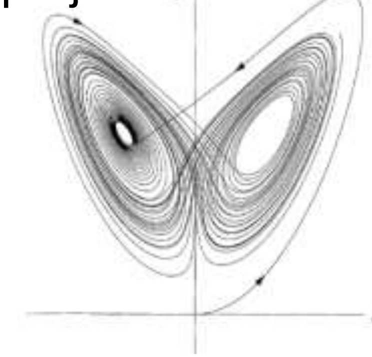


FIG. 1. Chaotic time series $x(t)$ produced by Lorenz (1963) equations (11) with parameter values $r=45.92$, $b=4.0$, $\sigma=16.0$.

2D projection of 3D attractor



- Most famous **chaotic** attractor.

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Can we observe chaos experimentally?

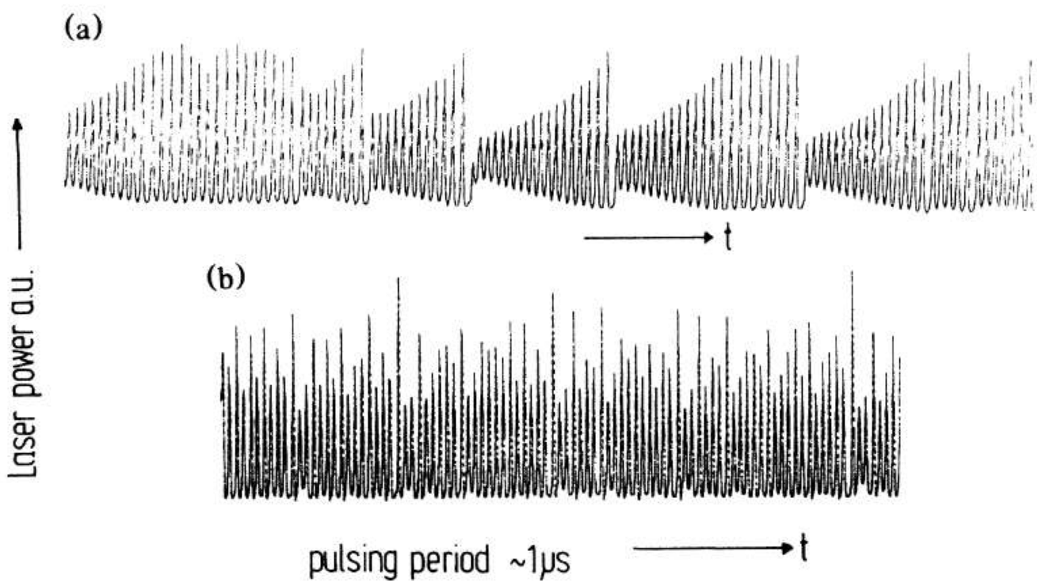
VOLUME 57, NUMBER 22 PHYSICAL REVIEW LETTERS 1 DECEMBER 1986

Evidence for Lorenz-Type Chaos in a Laser

C. O. Weiss and J. Brock[a]

Physikalisch-Technische Bundesanstalt, D-3300 Braunschweig, Federal Republic of Germany

(Received 18 April 1986)



optically pumped NH_3 laser

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The 1970s

- **Robert May** : "Simple mathematical models with very complicated dynamics", *Nature* (1976).

$$x_{t+1} = f(x_t)$$

A classical example: **The Logistic map** $f(x) = r\,x(1-x)$

$x \in (0,1)$, $r \in (0,4)$

- Difference equations ("iterated maps"), in spite of being simple and deterministic, can exhibit: **stable points**, **stable cycles**, and **apparently random fluctuations**.

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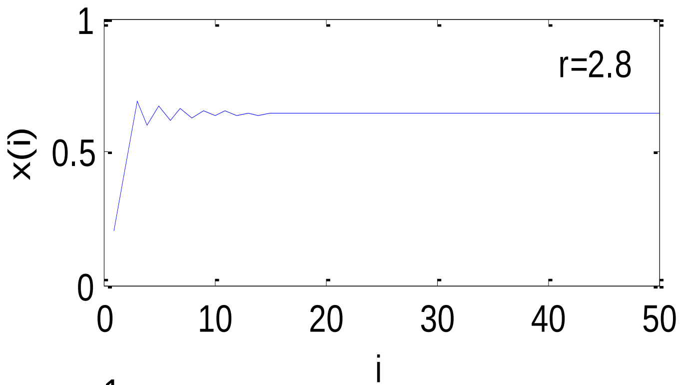
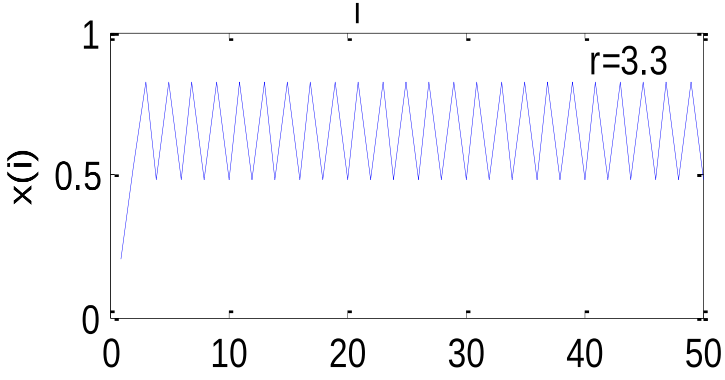
The logistic map: $x(i+1) = r\, x(i)[1 - x(i)]$ $x \in (0,1)$, $r \in (0,4)$



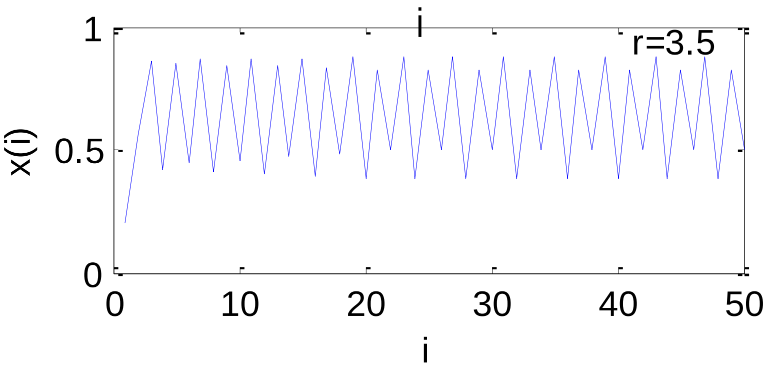
r=2.8, Initial condition: x(1) = 0.2

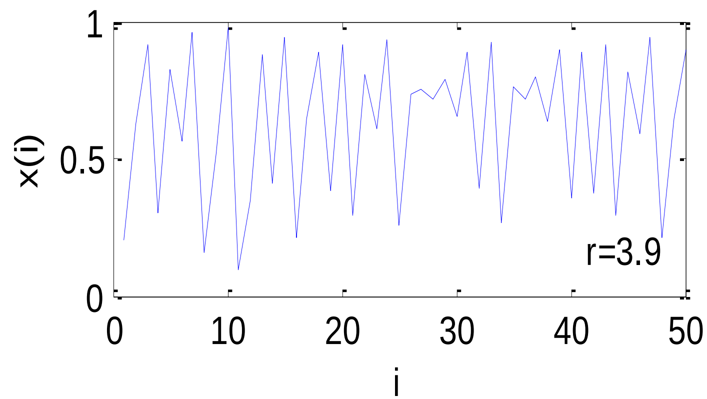
Transient relaxation → long-term stability

The fixed point is the solution of: $x = r\,x\,(1-x) \Rightarrow x = 1 - 1/r$



Transient dynamics → oscillations (regular or irregular)





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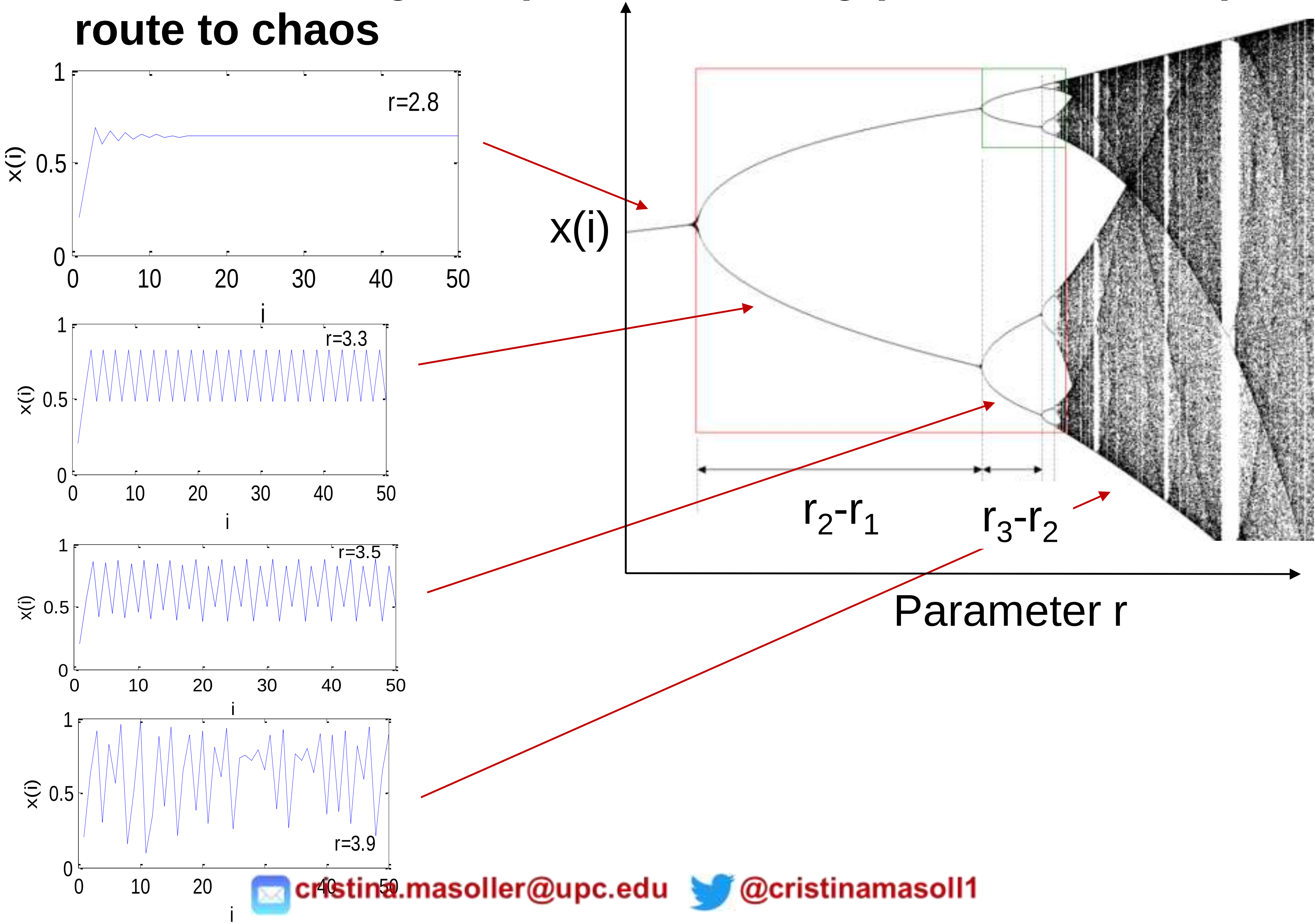
Bifurcation diagram: period-doubling (or subharmonic) route to chaos



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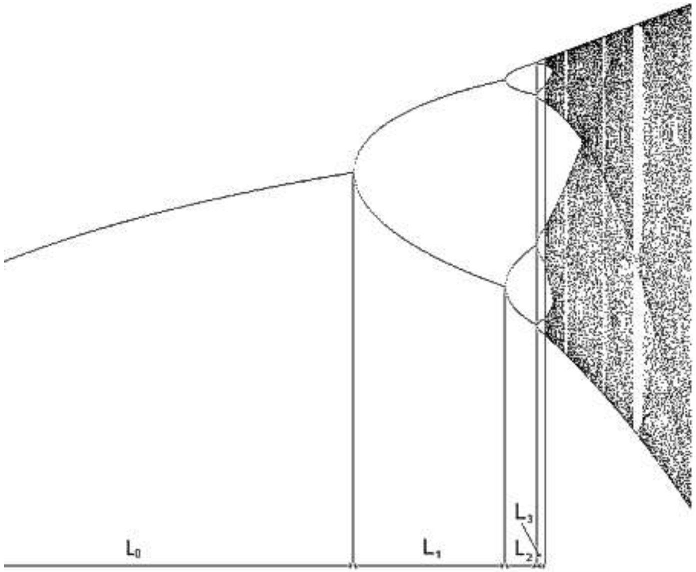
Order within chaos (1975)

M. Feigenbaum, using a small HP-65 programmable calculator, discovered "hidden" order in the route to chaos: the scaling of the bifurcation points of the Logistic map.



$$\delta = \lim \frac{L_i}{L_{i+1}} = 4.669201...$$

HP-65 calculator: the first magnetic card-programmable handheld calculator

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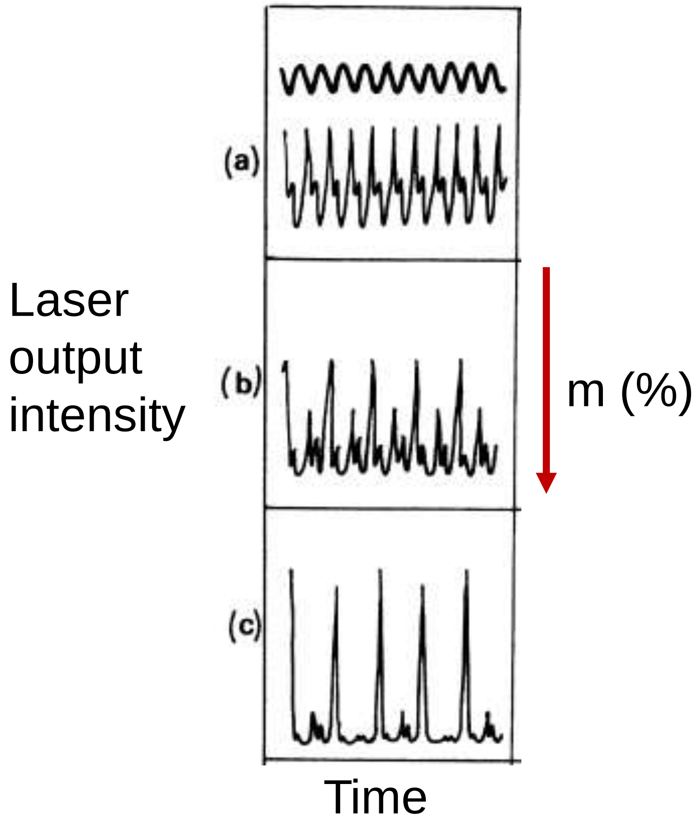
A universal law

Feigenbaum demonstrated that the same behavior, with the same mathematical constant (δ=4.6692…), occurs for a wide class of functions. $x_{t+1} = f(x_t)$

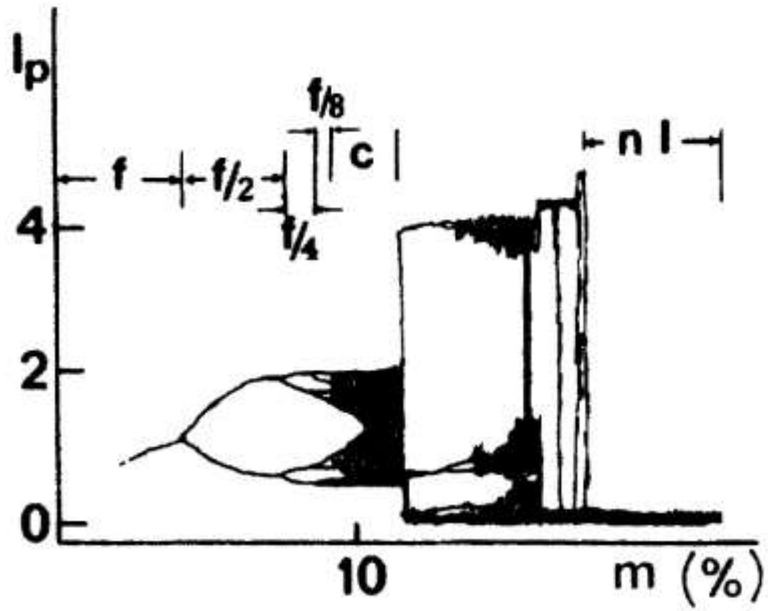
$\Rightarrow$ Very different systems (in chemistry, biology, physics, etc.) go to chaos in the same way, quantitatively.

Can we observe the period doubling route experimentally?

(about 10 years later) With a modulated laser, keeping constant the modulation frequency and increasing modulation amplitude.



J. R. Tredicce et al,
Phys. Rev. A 34, 2073 (1986).



Problems:

- *How to identify an approaching bifurcation point (tipping point)?*
- *How to distinguish transient from non-transient behavior?*

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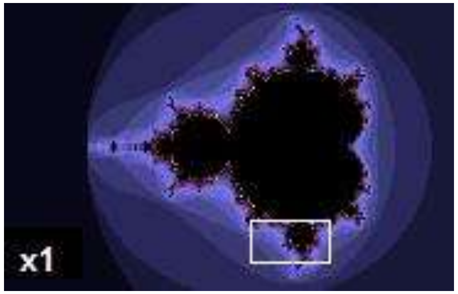
The late 1970s

- **Benoit B. Mandelbrot** (Polish-born, French and American mathematician 1924-2010): "self-similarity" and **fractal objects:**

  each part of the object is like the whole object but smaller.

- Because of his access to IBM's computers, Mandelbrot was one of the first to use **computer graphics** to create and display fractal geometric images.



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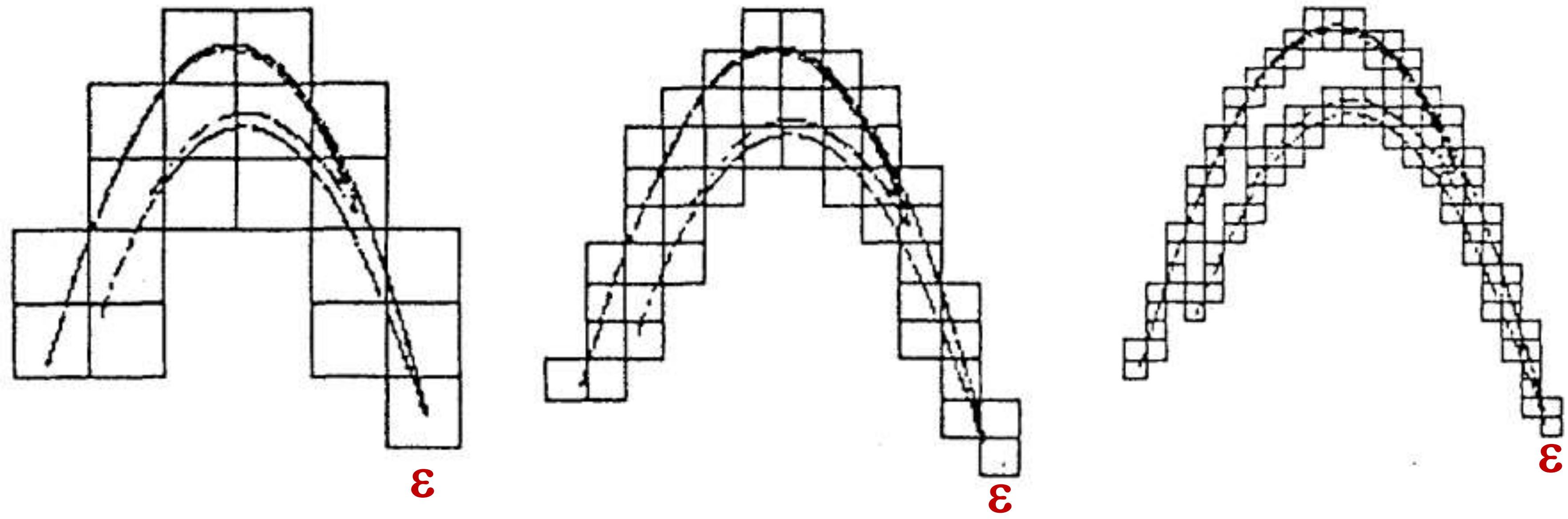
How to estimate the dimension of a fractal?



Box counting: number of occupied boxes scales as $(1/\varepsilon)^D$

Abarbanel et al, Reviews of Modern Physics 65, 1331 (1993).

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Examples of fractal objects in nature

Broccoli D=2.66

Human lung D=2.97

Coastline of Ireland D=1.22

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Patterns in nature: how "self-organization" emerges?




- **Ilya Prigogine** (Belgium, born in Moscow, Nobel Prize in Chemistry 1977).
- Studied chemical systems far from equilibrium.
- Discovered that the interplay of (external) **input of energy** and **dissipation** can lead to "self-organized" patterns.


cristina.masoller@upc.edu @cristinamasoll1

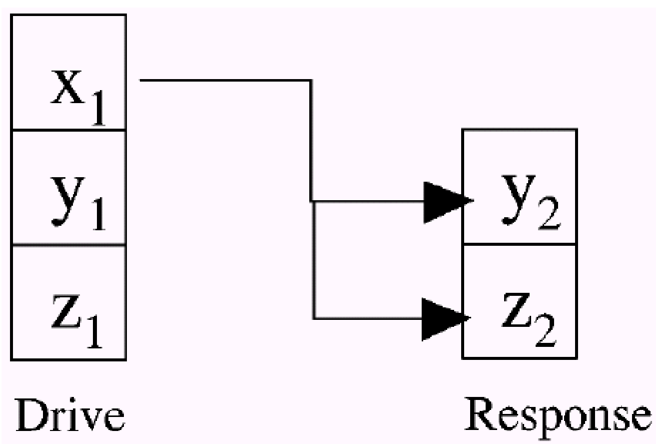
The 1990s: can two chaotic systems synchronize?

VOLUME 64, NUMBER 8 PHYSICAL REVIEW LETTERS 19 FEBRUARY 1990

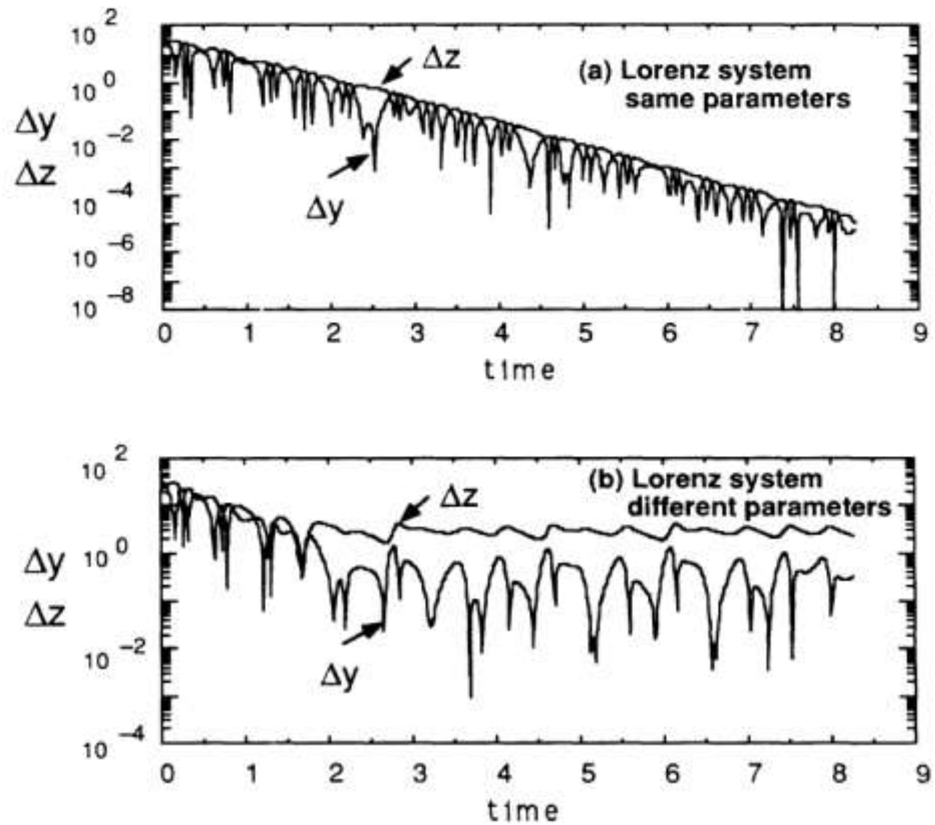
Synchronization in Chaotic Systems

Louis M. Pecora and Thomas L. Carroll
Code 6341, Naval Research Laboratory, Washington, D.C. 20375
(Received 20 December 1989)

Coupled Lorenz systems



$$t \to \infty \quad |y_2 - y_1| \to 0, \; |z_2 - z_1| \to 0$$



Can we observe the synchronization of two chaotic systems?

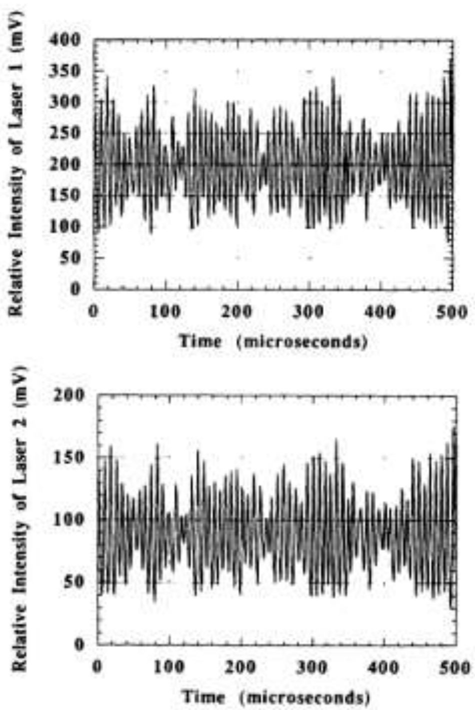
VOLUME 72, NUMBER 13 PHYSICAL REVIEW LETTERS 28 MARCH 1994

Experimental Synchronization of Chaotic Lasers

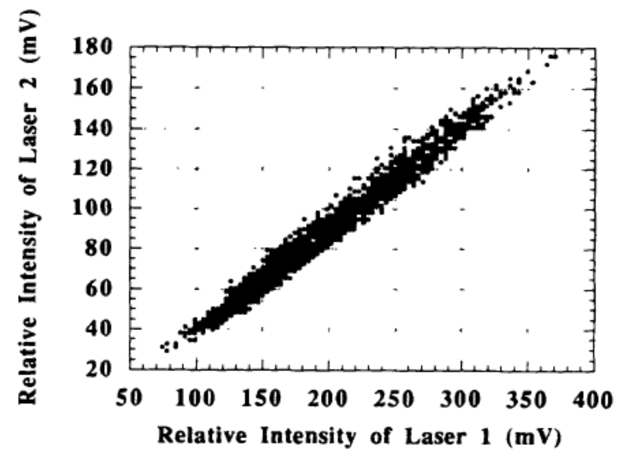
Rajarshi Roy and K. Scott Thornburg, Jr.
School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332
(Received 30 August 1993)

We report the observation of synchronization of the chaotic intensity fluctuations of two Nd:YAG lasers when one or both the lasers are driven chaotic by periodic modulation of their pump beams.

(a)



(b)



A problem of time series analysis: How to quantify synchronization?

cristina.masoller@upc.edu @cristinamasoll1

In fact, the first observation of synchronization was done much earlier: mutual *entrainment* of two pendulum clocks

Mid-1600s **Christiaan Huygens**: two pendulum clocks mounted on a common board synchronized and oscillated in opposite directions (in-phase also possible).

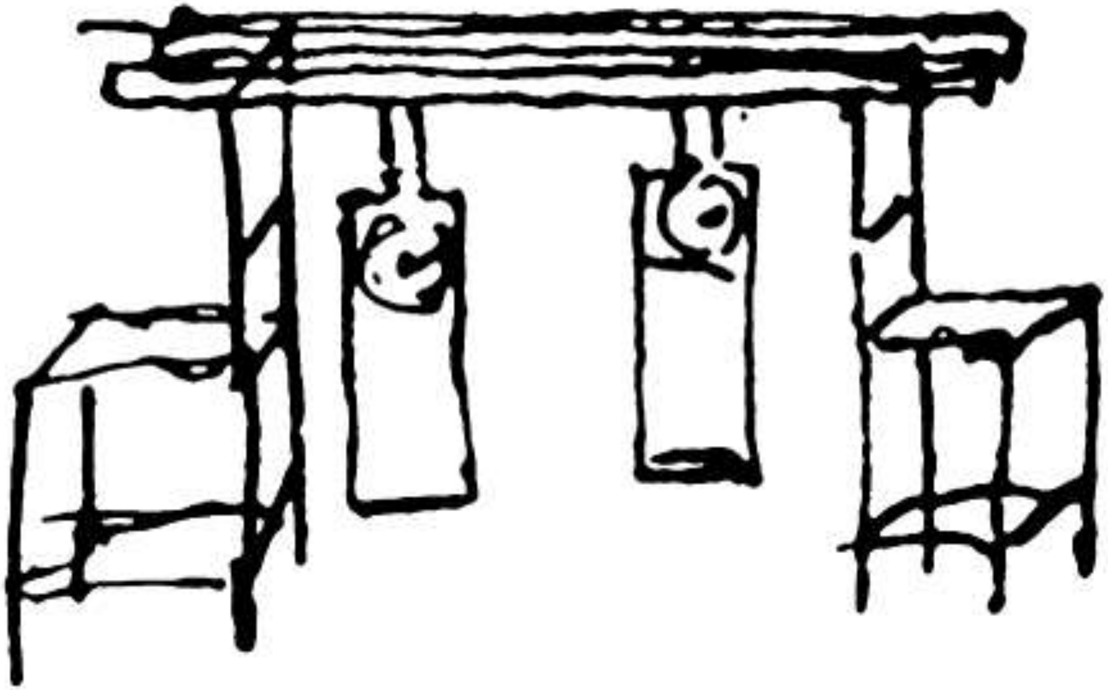


Figure 1.2. Original drawing of Christiaan Huygens illustrating his experiments with two pendulum clocks placed on a common support.

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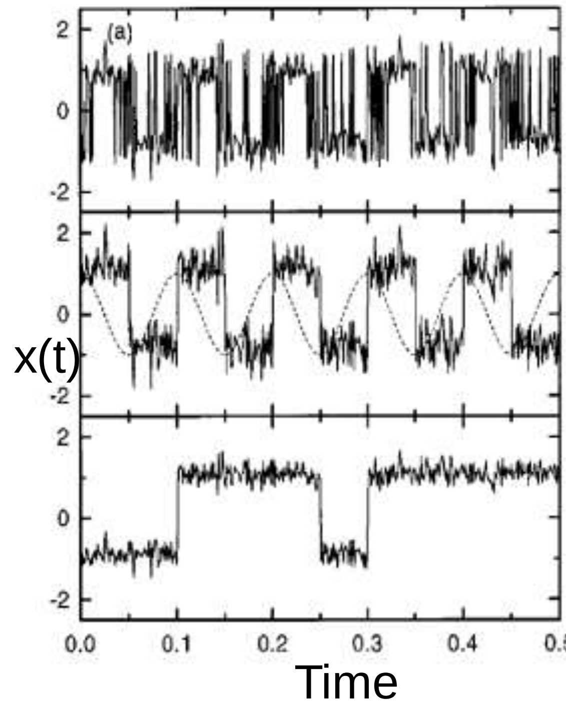
Effect of noise in nonlinear systems? (late 80' and 90')

Stochastic resonance: an optimal level of noise can, in some **bistable** systems, enhance the detection of a weak signal, improving the performance of the system.

Bistable system — Periodic signal — Noise

$$\dot{x}(t) = -V'(x) + A_0 \cos(\Omega t + \varphi) + \xi(t)$$

$$V(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4$$



Gammaitoni, Hanggi et al, Rev. Mod. Phys. 70, 223 (1998).

cristina.masoller@upc.edu @cristinamasoll1

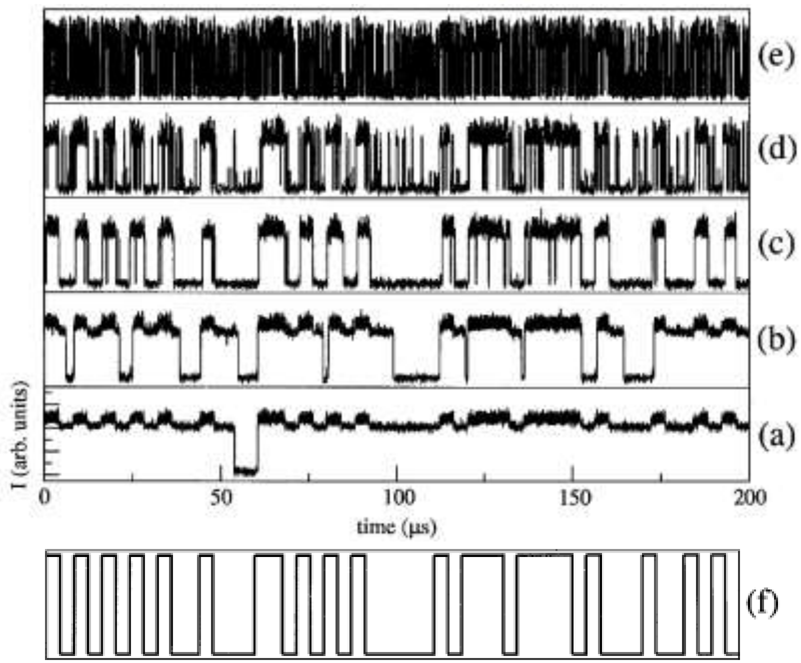
Can we observe the stochastic resonance phenomenon?

VOLUME 85, NUMBER 22 PHYSICAL REVIEW LETTERS 27 NOVEMBER 2000

Experimental Evidence of Binary Aperiodic Stochastic Resonance

Sylvain Barbay,[1] Giovanni Giacomelli,[1,3,*] and Francesco Marin[2,3]
[1]*Istituto Nazionale di Ottica Applicata, Largo E. Fermi 6, 50125 Firenze, Italy*
[2]*Dipartimento di Fisica, Università di Firenze, and Laboratorio Europeo di Spettroscopia Nonlineare, Largo E. Fermi 2, 50125 Firenze, Italy*
[3]*Istituto Nazionale di Fisica della Materia, unità di Firenze, Italy*
(Received 14 March 2000)

(using a bistable laser that emits in two orthogonal polarizations)



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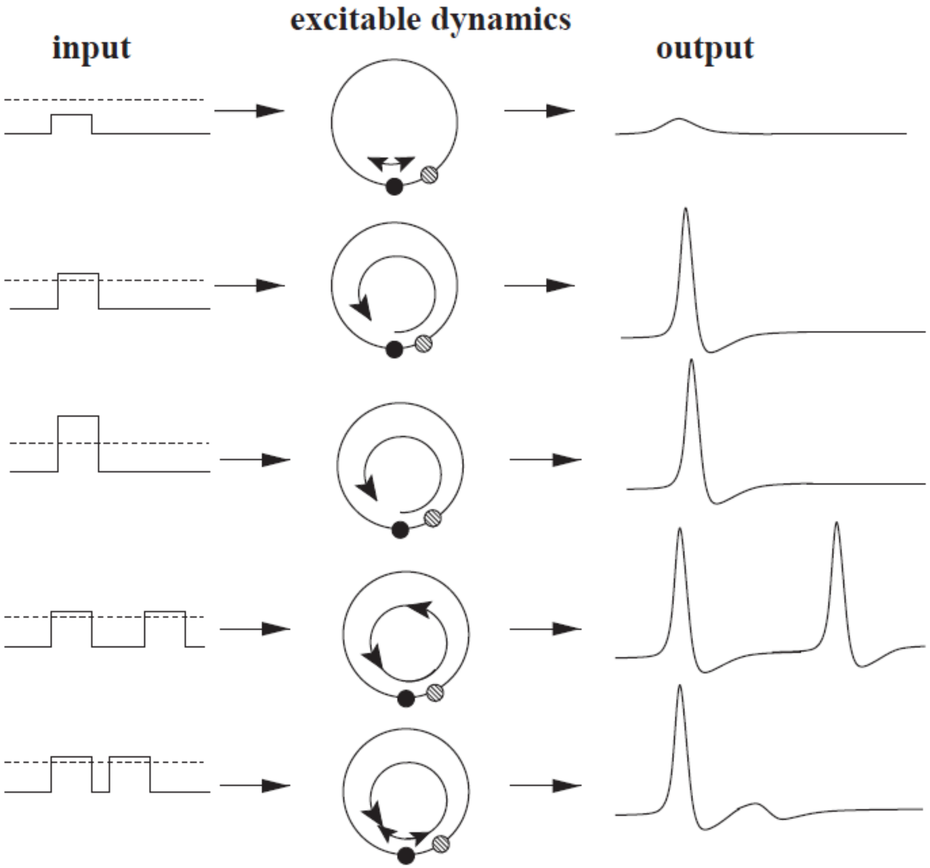
Effect of noise in excitable systems?



B. Lindner et al., Phys. Rep. 392, 321 (2004).

cristina.masoller@upc.edu @cristinamasoll1

VOLUME 78, NUMBER 5 PHYSICAL REVIEW LETTERS 3 FEBRUARY 1997

Coherence Resonance in a Noise-Driven Excitable System

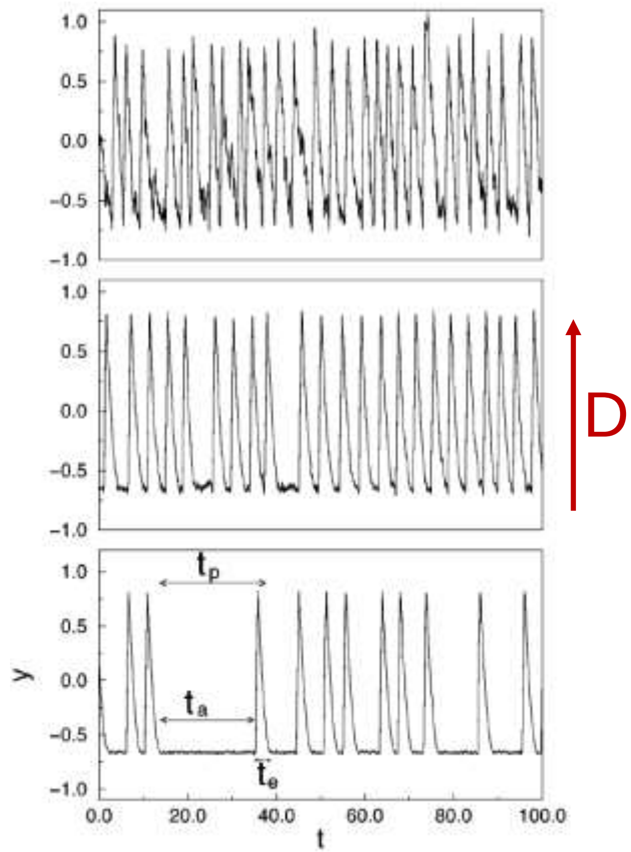
Arkady S. Pikovsky* and Jürgen Kurths*
Max–Planck–Arbeitsgruppe "Nichtlineare Dynamik" an der Universität Potsdam Am Neuen Palais 19, PF 601553, D-14415, Potsdam, Germany
(Received 9 August 1996)

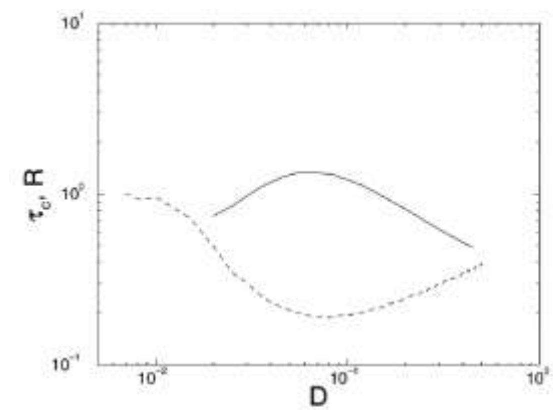
Fitz Hugh–Nagumo model

$$\varepsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y\,,$$

$$\frac{dy}{dt} = x + a + D\xi(t)$$

D=0: stable behavior





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Observation of coherence and stochastic resonance in excitable lasers

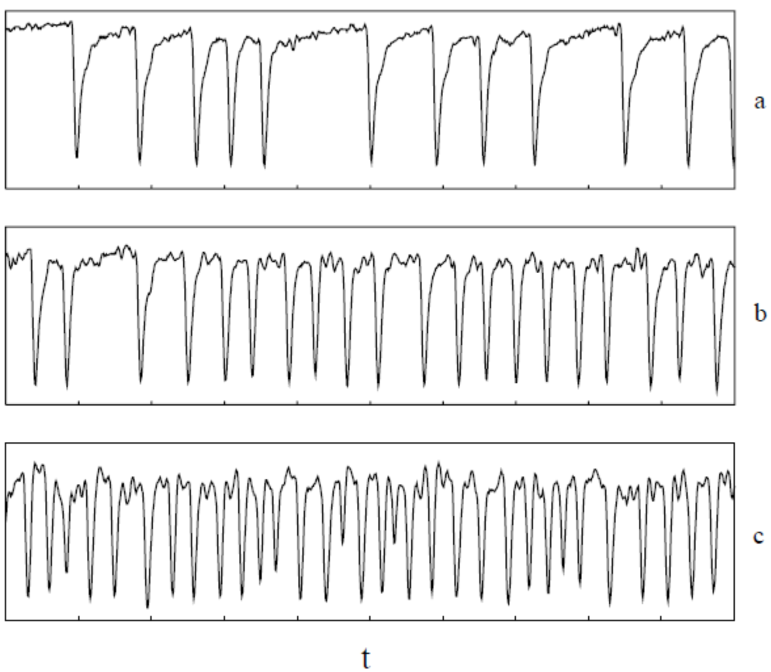
VOLUME 84, NUMBER 15 PHYSICAL REVIEW LETTERS 10 APRIL 2000

Experimental Evidence of Coherence Resonance in an Optical System

Giovanni Giacomelli
Istituto Nazionale di Ottica, Largo E. Fermi 6, 50125 Firenze, Italy

Massimo Giudici and Salvador Balle
Departamento de Fisica Interdisciplinar, Instituto Mediterraneo de Estudios Avanzados (CSIC-UIB), 07071 Palma de Mallorca, Spain

Jorge R. Tredicce
Institut Non-Linéaire de Nice, UMR 6618 Centre National de la Recherche Scientifique-Université de Nice Sophia-Antipolis, 06560 Valbonne, France

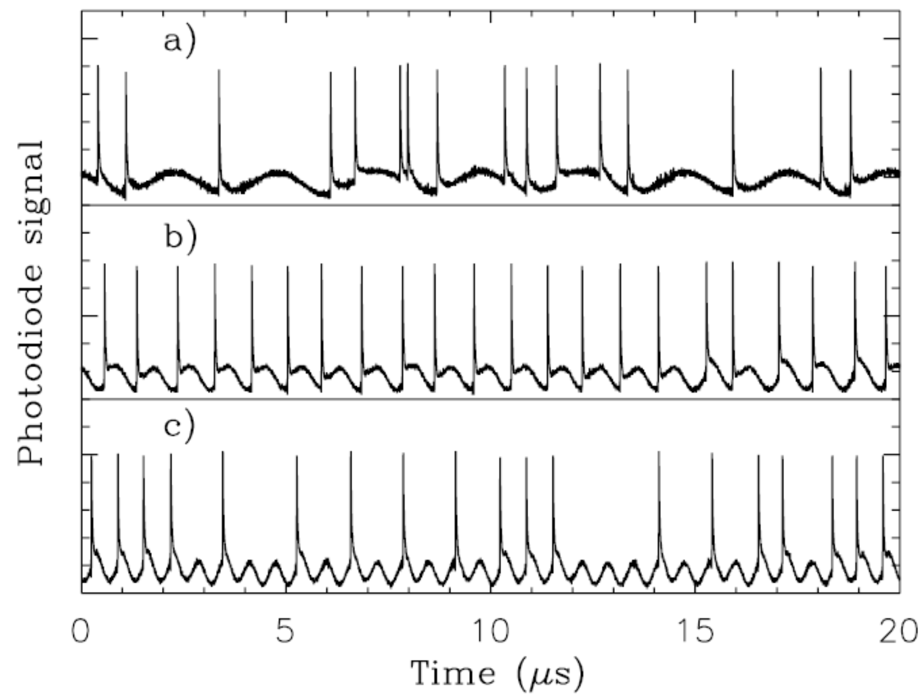


(varying the level of noise)

VOLUME 88, NUMBER 4 PHYSICAL REVIEW LETTERS 28 JANUARY 2002

Experimental Evidence of Stochastic Resonance in an Excitable Optical System

Francesco Marino, Massimo Giudici,* Stéphane Barland,† and Salvador Balle
Department de Física Interdisciplinar, Institut Mediterrani de Estudios Avanzados (CSIC-UIB), C/ Miquel Marqués 21, E-07190 Esporles, Spain
(Received 1 August 2001; published 10 January 2002)



(varying the frequency of the signal)

cristina.masoller@upc.edu @cristinamasoll1

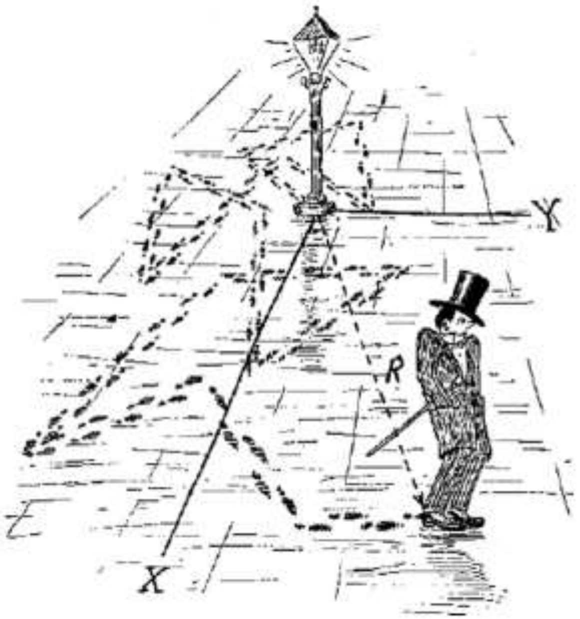
And in neural systems?

- Douglass et al., “*Noise enhancement of information-transfer in crayfish mechanoreceptors by stochastic resonance*”, Nature 365, 337 (1993).
- Levin and Miller, “*Broadband neural encoding in the cricket cercal sensory system enhanced by stochastic resonance*”, Nature 380, 165 (1996).
- Moss et al., “*Stochastic resonance and sensory information processing: a tutorial and review of application*”, Clinical Neurophysiology 115, 267 (2004).
- McDonnell and Lawrence, “*The benefits of noise in neural systems: Bridging theory and experiment”*, Nat. Rev. Neurosci. 12, 415 (2011).

cristina.masoller@upc.edu @cristinamasoll1

However, what is “noise”? “neural noise”?

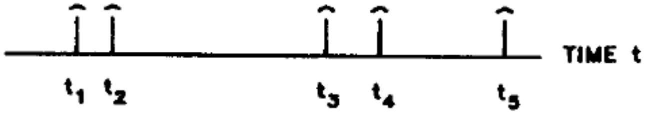
Someone's noise is another one's signal
(example: for a climatologist “weather” is noise).



*2D **random walk** or drunkard’s walk*
(The Viking Press, New York,1955)

A main problem in time series analysis: How to “find the signal”?

How to filter out noise?
How to define a “point process”?



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Late 90s, early 2000s: synchronization of a large number of dynamical systems

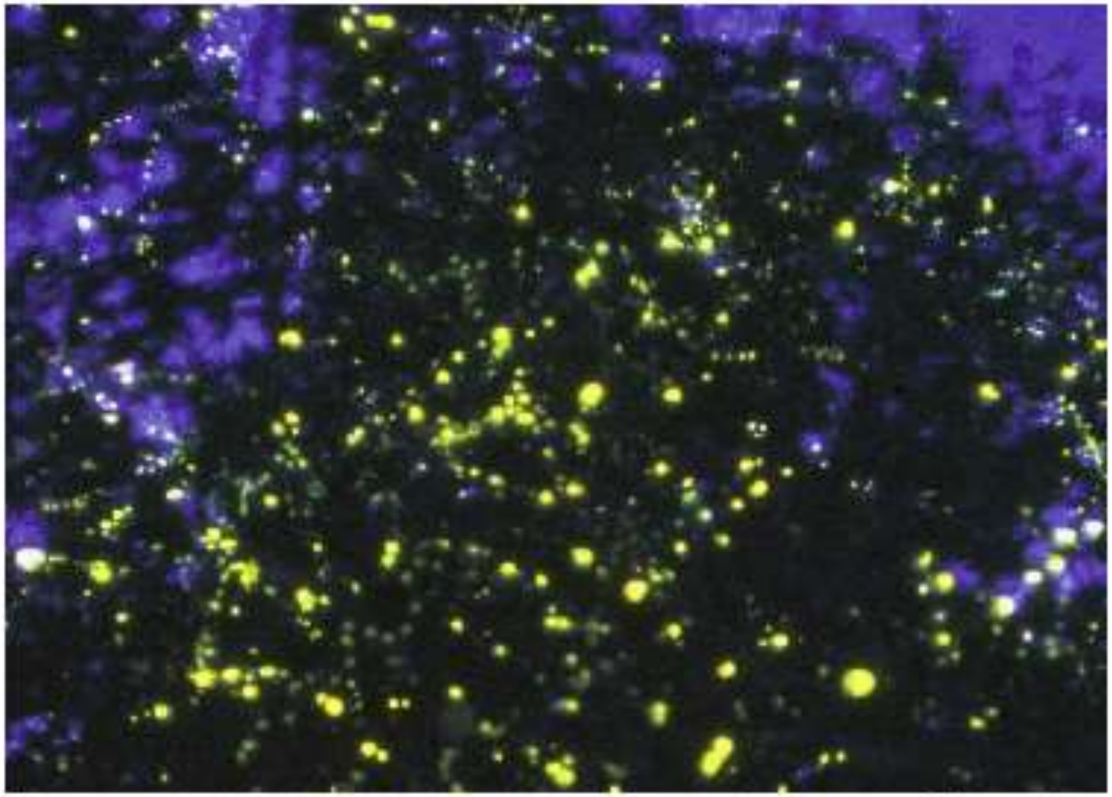


Figure 1 | Fireflies, fireflies burning bright. In the forests of the night, certain species of firefly flash in perfect synchrony — here *Pteroptyx malaccae* in a mangrove apple tree in Malaysia. Kaka *et al.*[2] and Mancoff *et al.*[3] show that the same principle can be applied to oscillators at the nanoscale.

cristina.masoller@upc.edu @cristinamasoll1

Another example of synchronization: the opening of the London Millennium Bridge, June 10, 2000

Source: BBC

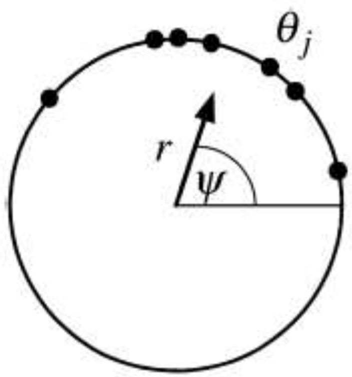
Crowd synchrony on the Millennium Bridge, Strogatz et al, Nature 438, 43 (2005)

cristina.masoller@upc.edu @cristinamasoll1

The Kuramoto model (Japanese physicist, 1975)

Model of **all-to-all** coupled **phase oscillators**.

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i) + \xi_i, \qquad i = 1 \ldots N$$



K = coupling strength, ξ_i = stochastic term (noise)

Describes the emergence of collective behavior

How to quantify?

With the **order parameter**:

$$re^{i\psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

r =0 incoherent state (oscillators scattered in the unit circle)

r =1 all oscillators are in phase ($\theta_i = \theta_j \ \forall$ i,j)

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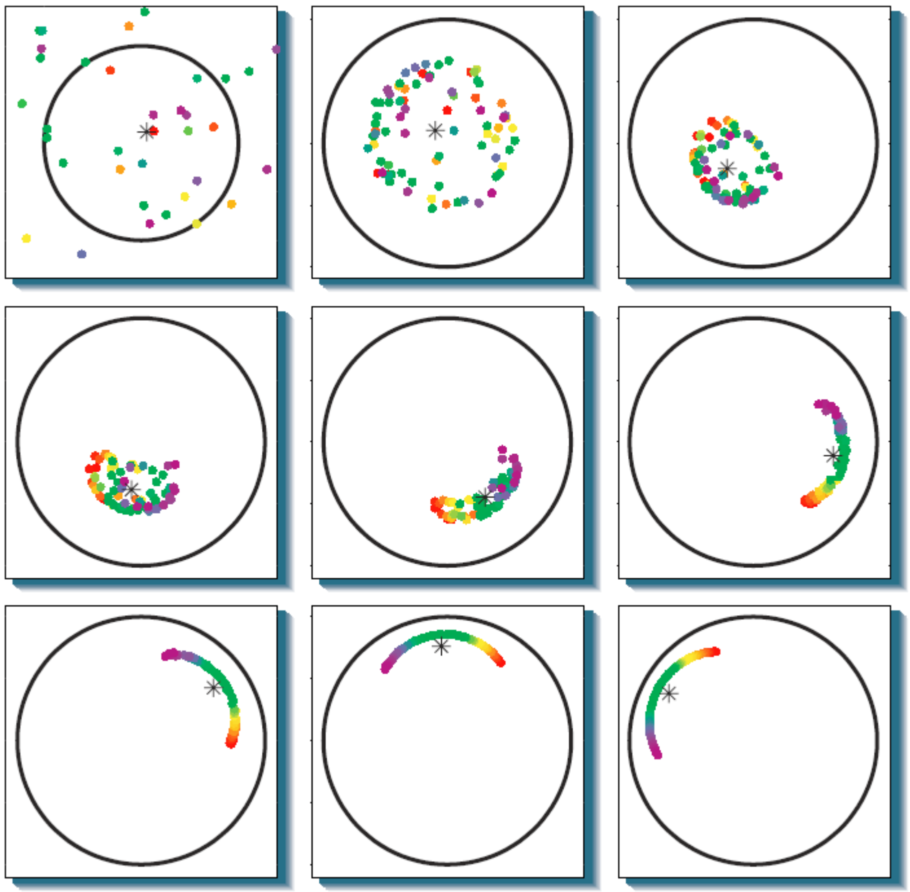
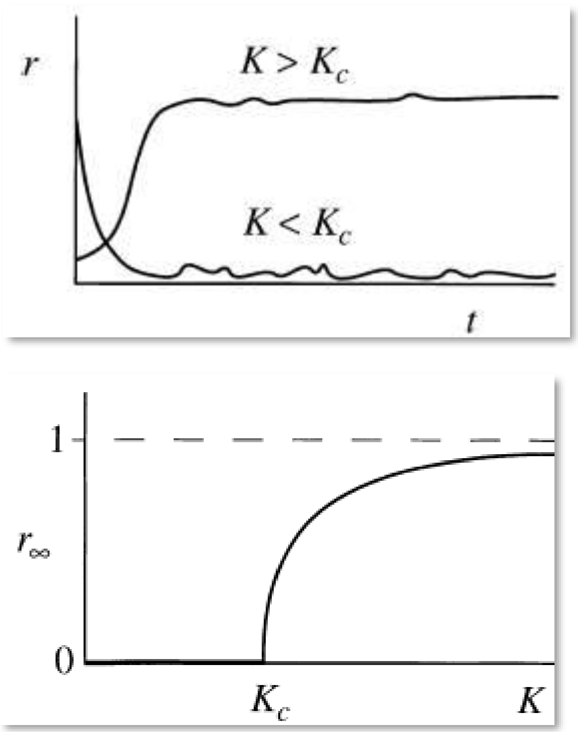
Synchronization transition as the coupling strength increases





Strogatz, Nature 2001

Video: https://www.ted.com/talks/steven_strogatz_on_sync

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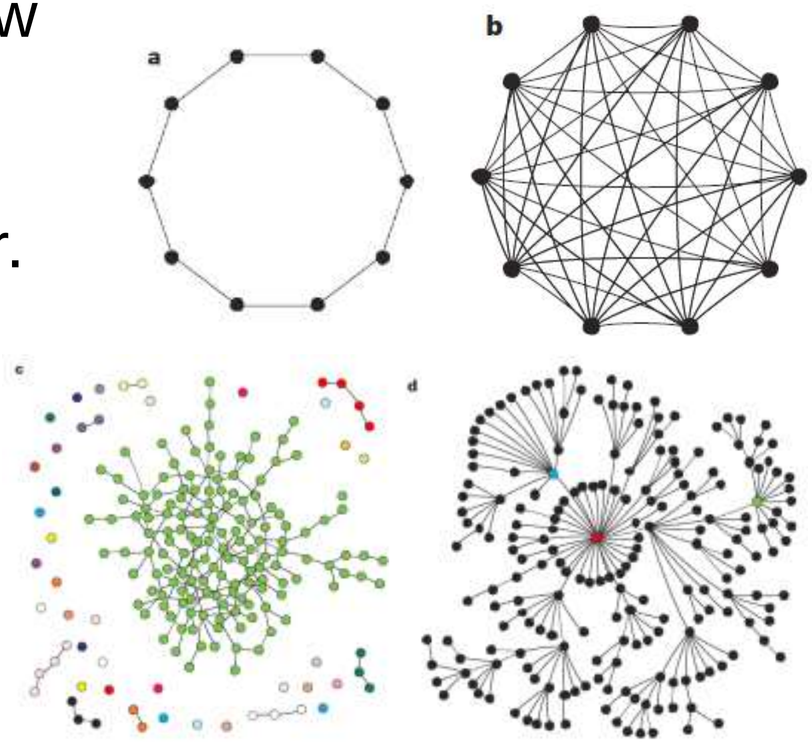
2000s to present: from chaotic systems to complex systems

- Large number of interacting elements
- The elements and/or their interactions are **nonlinear**.
- Main difference with linear systems: a "reductionist" approach does not work.
- <u>The behavior of complex system can not be predicted from the behavior of the individual units</u>.

cristina.masoller@upc.edu @cristinamasoll1

Complexity science

- **Networks** (or **graphs**) are used for mathematical modelling of complex systems.
- Emergent properties, not present in the individual elements.
- The challenge: to understand how the **structure** of the network and the **dynamics** of individual units determine the collective behavior.
- Applications
  - Communication networks
  - Transport networks
  - Epidemic and rumor spreading
  - Neuroscience
  - Physiology
  - Etc.



S. Strogatz, Nature 2001

cristina.masoller@upc.edu @cristinamasoll1

Multilayer networks

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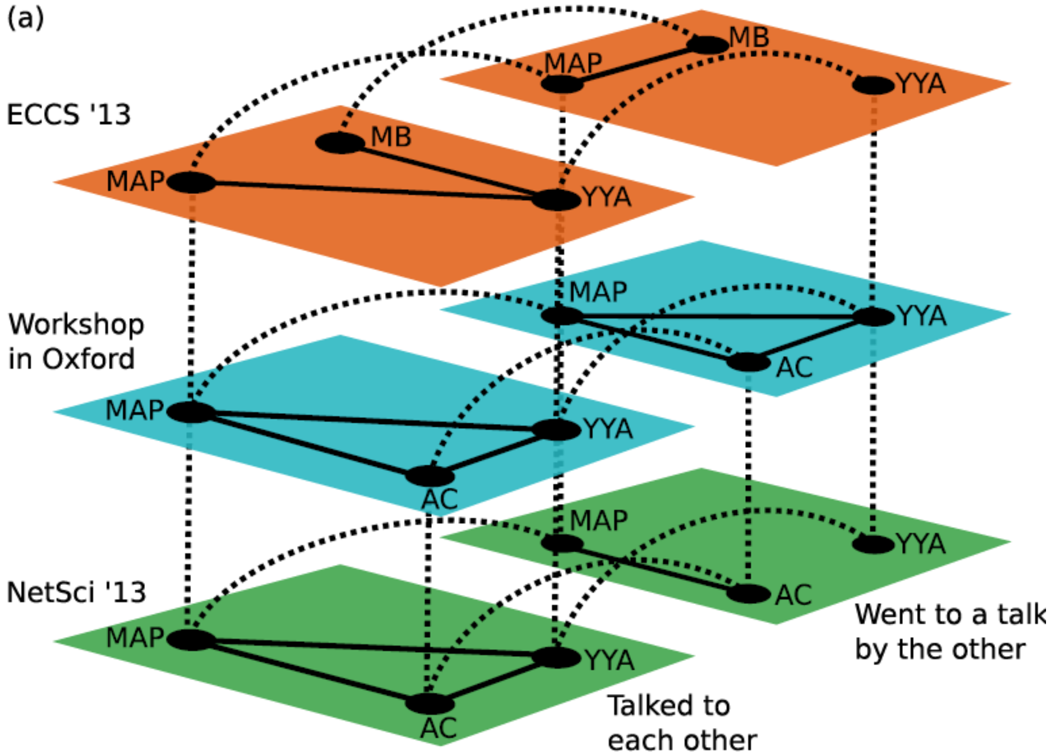
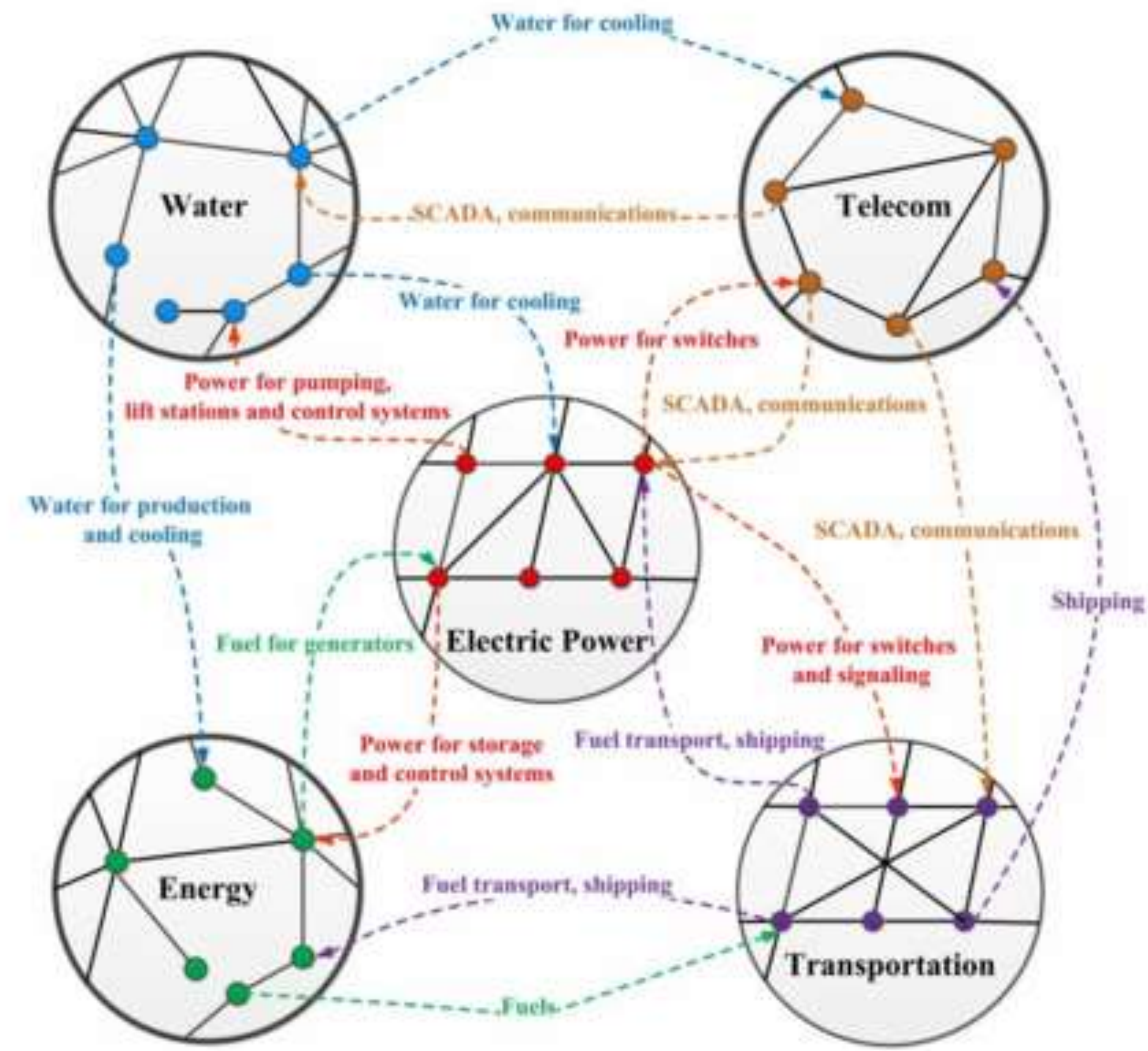


Kivela et al, J. Complex Netw. 2, 203 (2014).

cristina.masoller@upc.edu @cristinamasoll1

Networks of networks

Can we predict the effect of a critical (or extreme) event in one network? Cascade of failures?



Source: Wikipedia

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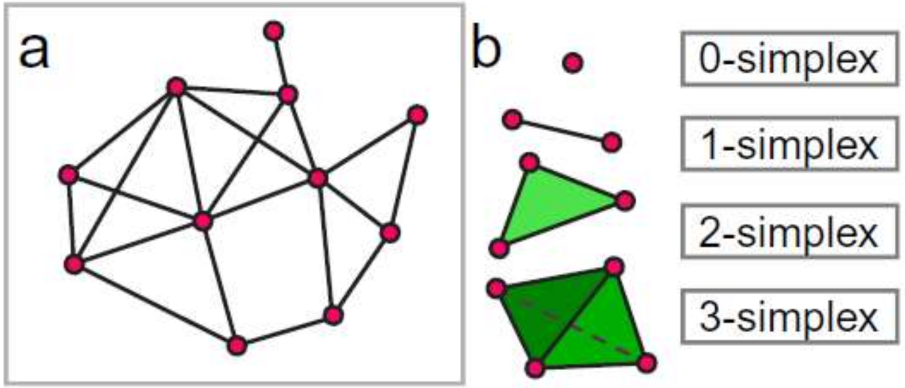
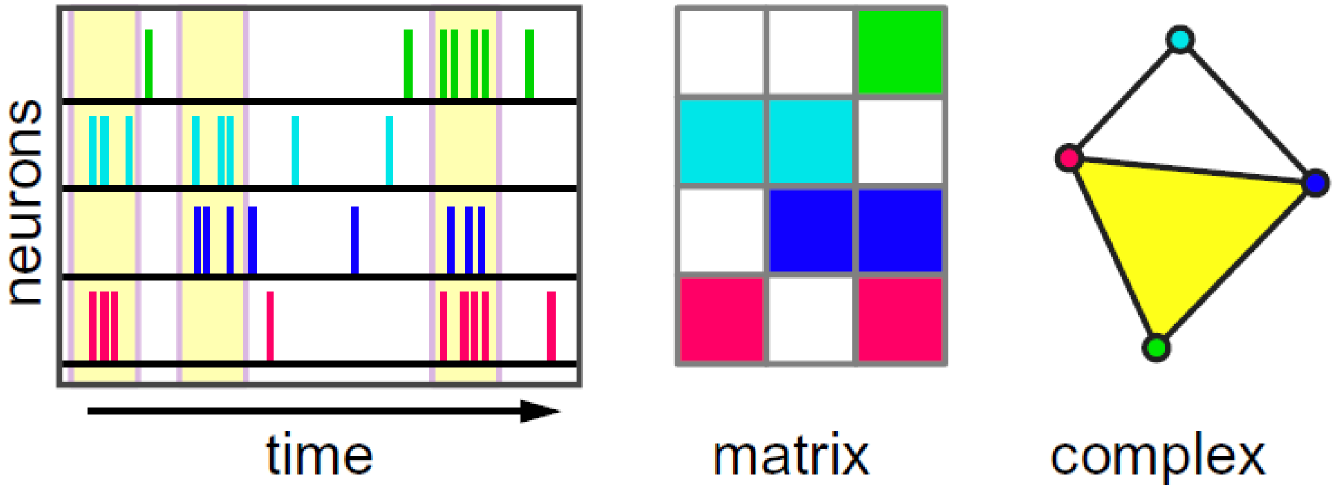
Interactions among several elements: simplicial complexes



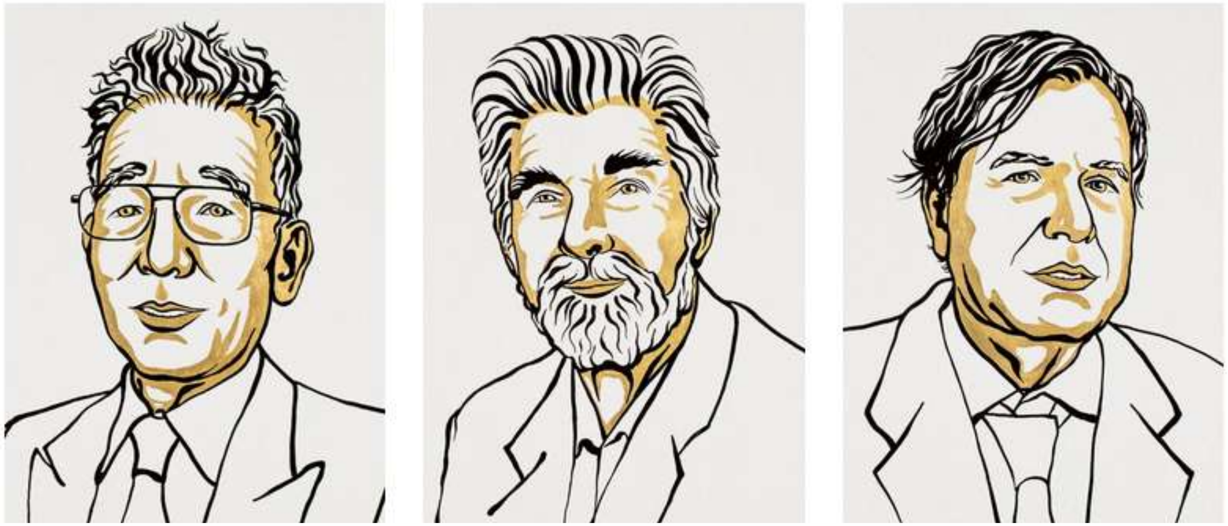
Example:



Giusti et al., J Comput Neurosci 41, 1 (2016).
Battiston et al., Phys. Rep. 874, 1–92 (2020).

cristina.masoller@upc.edu @cristinamasoll1

The Nobel Prize in Physics 2021



for groundbreaking contributions to our understanding of **complex systems**

½ Syukuro Manabe and Klaus Hasselmann "*for the physical modelling of Earth's climate, quantifying variability and reliably predicting global warming*"

½ Giorgio Parisi "*for the discovery of the interplay of disorder and fluctuations in physical systems from atomic to planetary scales*."

cristina.masoller@upc.edu @cristinamasoll1

Which systems are "complex"?

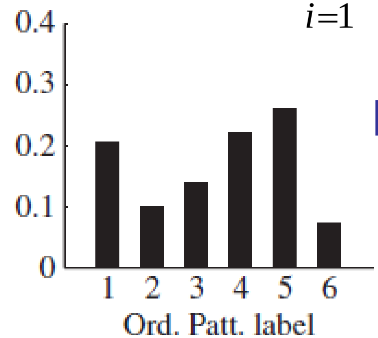
- Systems formed by a large number of elements / subsystems that have nonlinear behavior.
- The elements / subsystems interact with each other in a non-linear way (multiple spatial and/or temporal scales).
- The structure of the system is heterogeneous (neither regular nor completely random).
- The response of the system to a change or to a perturbation is often unexpected, contra intuitive (adaptation).
- **A large linear system is complicated but not complex**.

cristina.masoller@upc.edu @cristinamasoll1

Time series analysis: extracts "features" from the output signals of complex systems

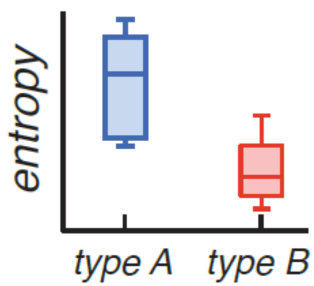
Time series

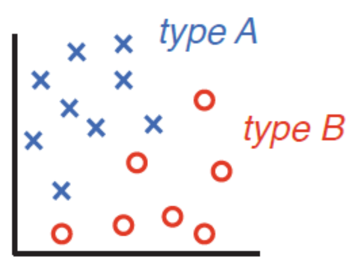
Probabilities $\sum_{i=1}^{N} p_i = 1$



Entropy

$$H = -\sum_{i=1}^{N} p_i \ln p_i$$





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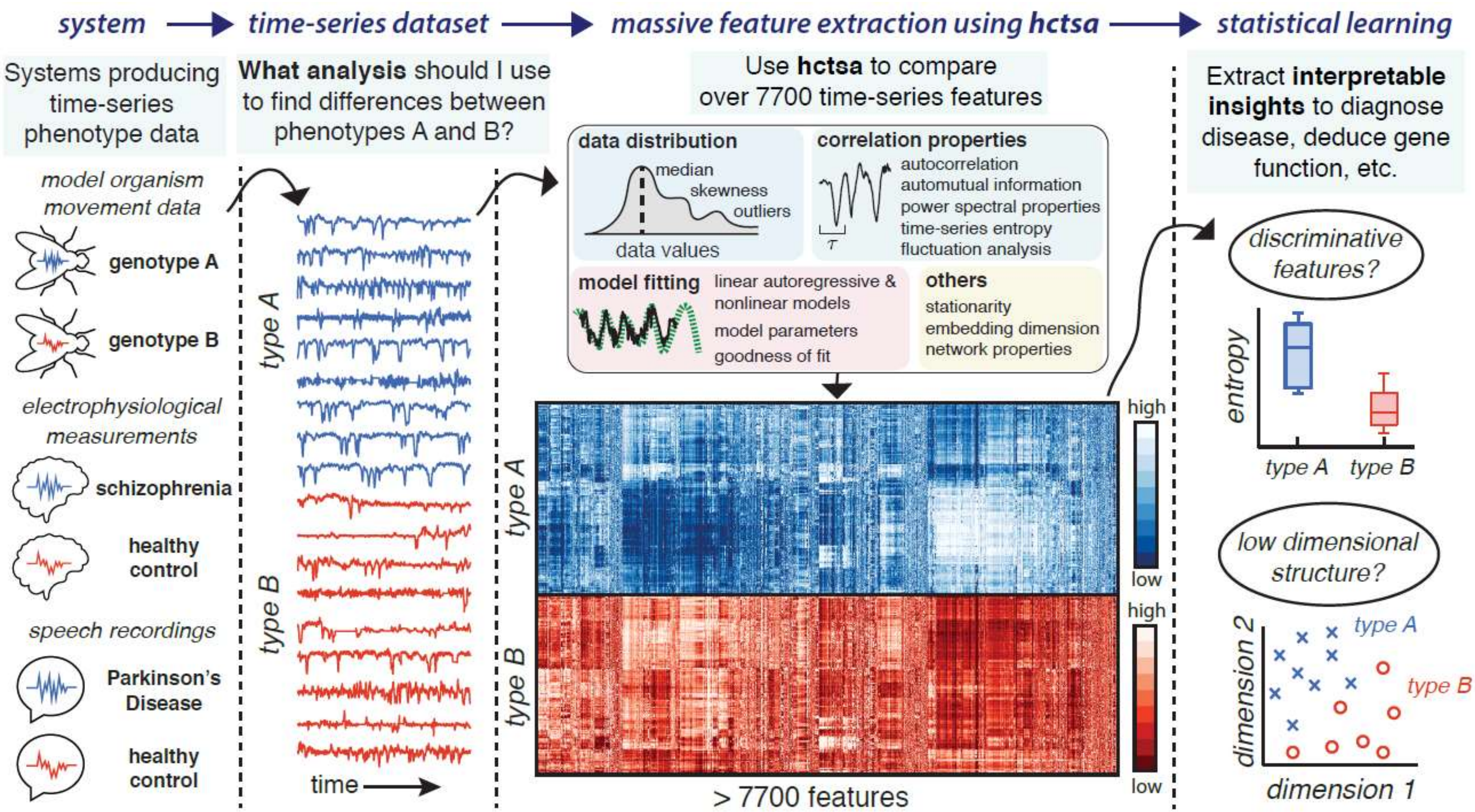
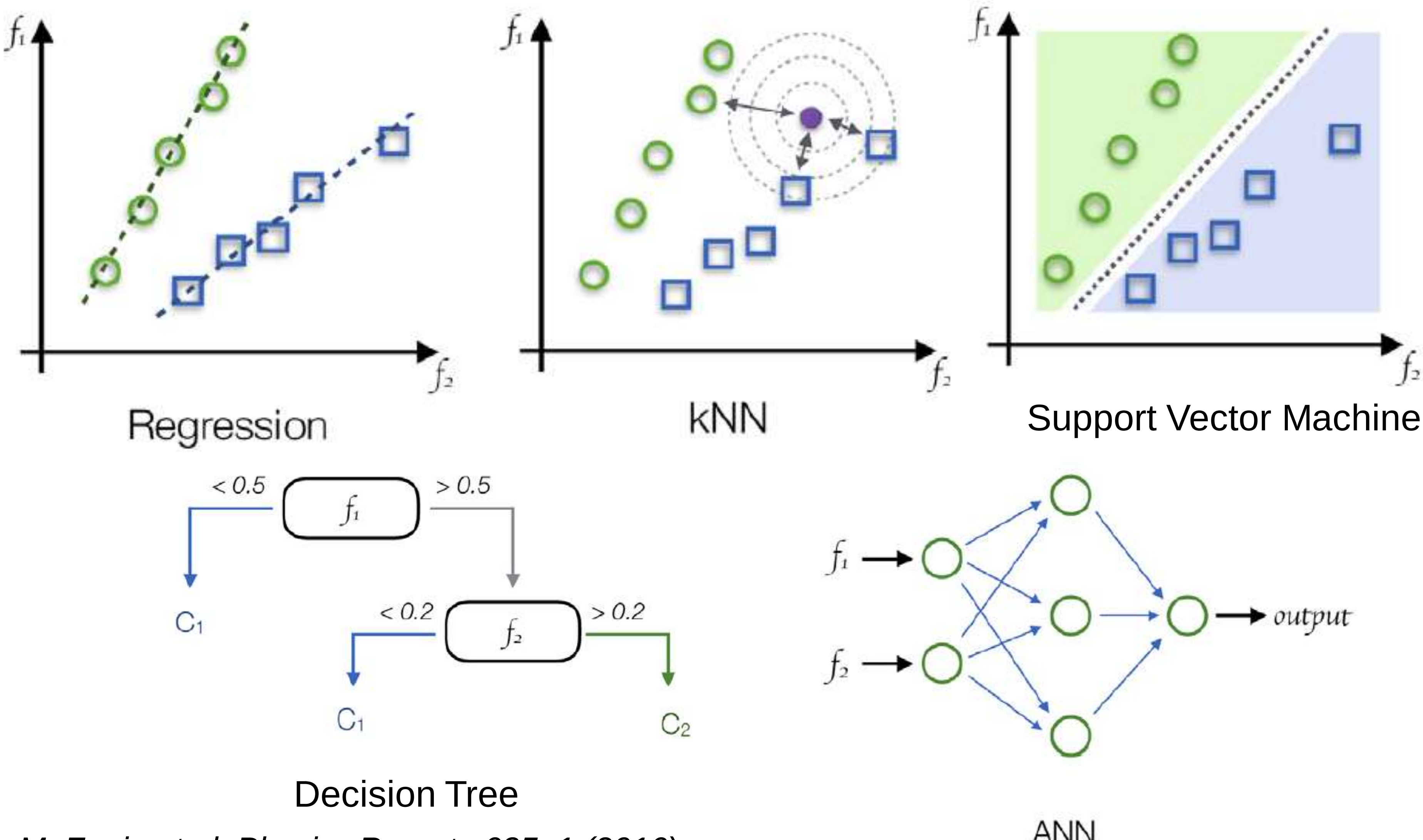
Algorithms allow massive feature extraction from data



*Fulcher & Jones, hctsa: A Computational Framework for Automated Time-Series Phenotyping Using Massive Feature Extraction. Cell Systems, **5**, 527–531 (2017).*

cristina.masoller@upc.edu @cristinamasoll1

Machine learning classification algorithms



M. Zanin et al, Physics Reports 635, 1 (2016).

cristina.masoller@upc.edu @cristinamasoll1

From dynamical systems to complex systems & data science

- Dynamical systems theory (bifurcations, low-dimensional attractors) allows to
  - uncover "order within chaos",
  - uncover universal characteristics
- Synchronization emerges in interacting systems
- Complexity science: study "emergent" phenomena in large sets of nonlinear interacting units (tipping points, critical transitions).
- Time series analysis allows to characterize signals and to "obtain features" that encapsulate properties of the signals.
- Data science: feature selection, classification, forecasting.

cristina.masoller@upc.edu @cristinamasoll1