1 - Vector mesons to probe quark axial current
2- Quark-meson mixings: flavor symmetry breaking/sea quarks

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Talk based on:

* F.L.B., Constituent quark axial current couplings to light vector mesons in the vacuum and with a weak magnetic field, Phys. Rev. D105, 054009 (2022).

* F.L.B., Strangeness content of the pion in the U(3) Nambu Jona Lasinio model, J. Phys. G: Nucl. Part. Phys. 49, 055101 (2022);
* F.L.B., Flavor-dependent corrections for the U(3) NJL coupling constant, Phys. Rev. D 103, 094028 (2021),
* F.L.B. Quark-antiquark states of the lightest scalar mesons within the Nambu-Jona-Lasinio model with flavor-dependent coupling constants, arXiv:2212.06616.
* W.F.de S., F.L. B., Charm and beauty content of the pion and kaon in the Flavor U(5) Nambu-Jona-Lasinio model, arXiv:2301.10128

INCT-FNA INSTITUTO NACIONAL DE CIÊNCIA E TECNOLOGIA FÍSICA NUCLEAR E APLICAÇÕES CNPq

Principia Institute + ICTP-SAIFR (workshop)

Presentation Overview

1. Motivations/context

2. Vector meson coupling to constituent quark axial current
   - Quark-antiquark interaction- dynamical calculation
   - Relation to g_A and form factor

3. Quark-antiquark mesons + sea-quarks in improved NJL model
   - Quark polarization in the NJLmodel - FSB
   - A calculation on U(5) NJL
   - Pion strangeness content leading to Meson Mixing
   - Quark-antiquark states of light scalars

4. Summary

I: Hadrons and NJL model: valence + sea quarks

(Low energy) QCD effective models: **global hadron properties**
* Dynamical Chiral Symmetry Breaking: $< \bar{q}q >$ masses/couplings

* Some models "Near-exausted" (?) resources:
still phenomenology and test-model

Nambu-Jona-Lasinio (NJL) model: low energy QCD/Quark model
$\sim$ punctual interactions $G_0 \sim 1/M_G^2$ or $1/\Lambda^2$ valence quarks
Usually improvements rely on further free parameters

Strangeness content of nucleon (electromag. $\sim 5\%$)
Charm content of nucleon: Brodsky, Hoyer, Peterson/many (1%)
LHCb-NNPDF: evidence 3σ c.l. (?)

Outcome $\rightarrow$ quarks/meson mixings: sea quarks
* Flavor symmetry breaking (FSB)

Motivations - II: Vector mesons probe axial current

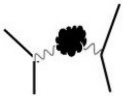
* Spin content of the nucleon (hadrons) from axial current
* Pion and axial mesons (unstables) to nucleon: axial charge (nucleon or constituent quark)
* Roughly: **axial mesons** as chiral partners ($\rho - A_1$ and $\omega - f_1$)
* Non-central collisions: vector mesons production

- Straightforward dynamical (one-loop polarization) calculation of **leading meson couplings to constituent quarks**

F.L.B., Phys. Rev. D105, 054009 (2022); Phys. Rev. (2019); E (2023) Journ. of Phys. G47, 115102 (2020); Phys. Rev. D97, 0140022 (2018) ; D101, 039902(E) (2020)

* Vector mesons probe/couples to axial current
* If yes, Even a photon could probe the axial current (by VMD)

Vector mesons couplings to axial current (dynamically generated)



$$Z[\eta,\bar\eta] = N\int \mathcal{D}[\bar\psi,\psi]$$

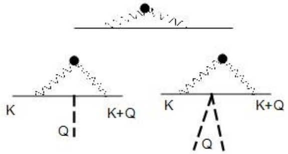
$$\exp\; i \int d^4x \left[\bar\psi\left(i\not{D}-m\right)\psi - \frac{g^2}{2}\int_y j^\beta_\mu(x)\tilde{R}^{\mu\nu}_{\beta\alpha}(x-y)j^\alpha_\nu(y) + \bar\psi\eta + \bar\eta\psi\right]$$

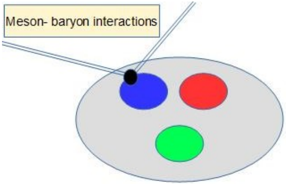
color quark current $j^\mu_\alpha = \bar\psi\lambda_\alpha\gamma^\mu\psi$,
$i,j,k = 0,...(N_f^2-1)$ for U($N_f=2$), $\alpha,\beta... = 1,...(N_c^2-1)$

Fierz transformation $\to$ all flavor-Dirac channels
Auxiliary fields: suitable for quark-antiquark states

Leading couplings: meson- constituent quarks

Expansion of quark determinant (some ambiguities-symmetries)





Leading meson-constituent quark couplings (form factors)

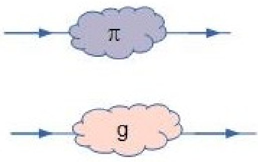
$$\begin{aligned}
\mathcal{L}_{j_A} &= \left[G_A(Q,K) Q_\mu \pi^i(Q) + G_{\bar{A}}(Q,K) \bar{A}^i_\mu(Q) \right] j^\mu_{A,i}(K,Q), \\
\mathcal{L}_{v-q} &= g_{r1}(Q,K) V^\mu_i(Q) j^{V,i}_\mu(K,Q) + g_{A1}(Q,K) \bar{A}^\mu_i(Q) j^{A,i}_\mu(K,Q) \\
&+ g_{v1}(Q,K) V^\mu(Q) j_\mu(K,Q) + g_{f1}(Q,K) \bar{A}_\mu(Q) j^\mu_A(K,Q), \qquad (1)
\end{aligned}$$

$G_A(Q,K), g_{r1}(Q,K), g_{A1}(Q,K), g_{f1}(Q,K)$ are one loop integrals
Coupling constants ($K = Q = 0$) or ($Q^2 = M^2_\pi$) ..
Numerically: correct order of magnitude (renormalization=1-fit)

Quark determinant: Emergence of Gluon and pion clouds

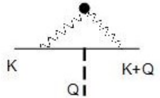
* Gluon cloud: dressing to quark currents $\rightarrow$ constituent quarks

* Pion cloud from the *Goldstone boson* couplings
to (all) quark currents



* Last part of the talk: Flavor symmetry breaking
$\rightarrow$ emergence of diverse sea quark "cloud"

Next leading terms



Note that:
Meson-quark momenta
Transversal to each other and
Transversal meson polarization

For isosinglet V_μ and isotriplet V^i_μ mesons

$$\begin{aligned} \mathcal{L}_{vja} &= i\delta_{ij}\epsilon^{\sigma\rho\mu\nu} F^{vja}(K,Q) K_\sigma \mathcal{F}^i_{\rho\mu}(Q) j^{A,j}_\nu(K,K+Q) \\ &+ i\epsilon^{\sigma\rho\mu\nu} F^{vja}(K,Q) K_\sigma \mathcal{F}_{\rho\mu}(Q) j^A_\nu(K,K+Q), \end{aligned} \tag{2}$$

$j^{A,i}_\mu(K,K+Q) = \bar{\psi}(K+Q)\gamma_\mu\gamma_5\sigma^i\psi(K)$ and
$j^A_\mu(K,K+Q) = \bar{\psi}(K+Q)\gamma_\mu\gamma_5\psi(K)$.

* Polarized vector meson, transversal directions in $\epsilon^{\sigma\rho\mu\nu}$

$$\mathcal{F}^i_{\rho\mu}(Q) = Q_\rho V^i_\mu(Q) - Q_\mu V^i_\rho(Q), \qquad \mathcal{F}_{\rho\mu}(Q) = Q_\rho V_\mu(Q) - Q_\mu V_\rho(Q).$$

Chiral partners: axial mesons-vector current

For isosinglet $\bar{A}_\mu$ and isotriplet $\bar{A}^i_\mu$ mesons

$$\begin{aligned} \mathcal{L}_{vja-A} &= i\epsilon^{\sigma\rho\mu\nu} F^{vja}(K,Q) K_\sigma \mathcal{G}^i_{\rho\mu}(Q) j^{V,i}_\nu(K,K+Q) \\ &+ i\epsilon^{\sigma\rho\mu\nu} F^{vja}(K,Q) K_\sigma \mathcal{G}_{\rho\mu}(Q) j^{V}_\nu(K,K+Q), \end{aligned} \quad (3)$$

$j^{V,i}_\mu(K,K+Q) = \bar{\psi}(K+Q)\gamma_\mu \sigma^i \psi(K)$
$j^{V}_\mu(K,K+Q) = \bar{\psi}(K+Q)\gamma_\mu \psi(K)$.

$$\mathcal{G}^i_{\mu\nu} = \partial_\mu \bar{A}^i_\nu - \partial_\nu \bar{A}^i_\mu, \qquad \mathcal{G}_{\mu\nu} = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu. \quad (4)$$

Axial pion coupling and the ρ coupling

Couplings to the axial current (out of 8 structures Ball-Chiu)

$$\mathcal{L}_{j_A} = \left[G_A Q_\mu \pi^i(Q) + G_{\bar{A}} \bar{A}^i_\mu(Q) + i\, F_{vja} \epsilon_{\mu\nu\rho\sigma} K^\nu Q^\rho V_i^\sigma(Q) \right] \times j^\mu_{A,i}(K,Q), \quad (5)$$

From the same method:

$$\frac{F_{vja}(K,Q)}{G_A(K,Q)} = \frac{1}{4M^* F} = \text{constant}. \quad (6)$$

Renormalization condition can be $G_A \sim 1$.
(Relativistic Consituent quark model - S.Weinberg + GT relation)

$$\left.\frac{F_{vja}(K,Q) \times |K||Q|}{G_V(K,Q)}\right|_{Q \sim K \sim 200-500 MeV} \sim 0.1. \quad (7)$$

What happens at high energies?
Can a Photon probe the axial current (Vector Meson Dominance)?

Witten's procedure: quantization

$\mathcal{L}_{vja}$ as a 5dim closed surface (Stoke's theorem)

$$n\,\Gamma = -\epsilon^{\sigma\rho\mu\nu}\frac{i}{240\pi^2}\int d^4K\, d^4Q\, F^{vja}(K,Q)K_\sigma \mathcal{F}^i_{\rho\mu}(Q) j^{A,i}_\nu(K,K+Q), \quad (8)$$

n is an integer: $\Gamma = \epsilon_{\sigma\rho\mu\nu}\Gamma^{\sigma\rho\mu\nu}$

Quantized integrals (Sum over $\mu\nu\rho\sigma$) contain integrals of the type

$$\Gamma_{(xyz0)} = -\frac{i}{240\pi^2}\int d^4K\, d^4Q\, F^{vja}(K,Q)K_x Q_y$$
$$\times \left[\rho^-_z(Q)\bar{u}(K+Q)\gamma_0\gamma_5 d(K) + \rho^+_z(Q)\bar{d}(K+Q)\gamma_0\gamma_5 u(K)\right] (9)$$

* $\rho^\pm_z(Q)$ = z-polarization component

From the last slide (one loop - rainbow ladder):

$$\frac{F_{vja}(K,Q)}{G_A(K,Q)} = \frac{1}{4M^* F}. \quad (10)$$

* Sum of $\Gamma_{\sigma\rho\mu\nu} \to$ "sum rule"?

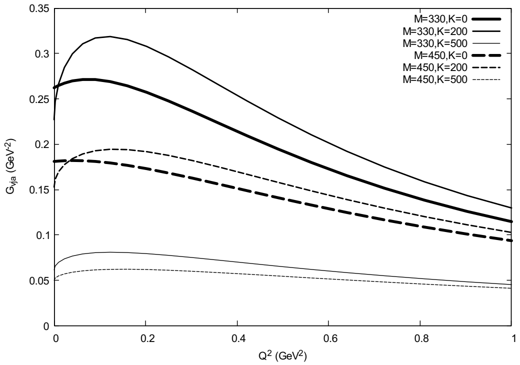


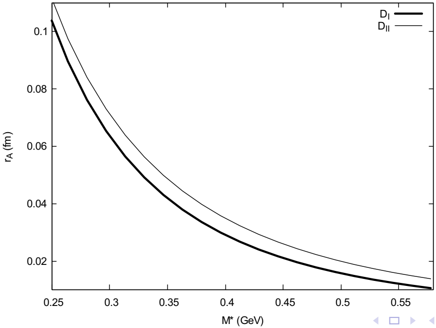
Figure: Form factor $G_{vja}(K, Q)$ for effective gluon propagator (Tandy-Maris) as a function Q^2 for different values of K. Two effective masses $M^* = 0.33$GeV and $M^* = 0.45$ GeV.

Vector meson Axial radius

$$\Delta_A < r_\rho^2 >= -6 \left. \frac{d\bar{G}_{vja}}{dQ^2} \right|_{Q=0} , \qquad < r_\rho^2 > \simeq 0.28 - 0.56 fm^2$$

Bhagwat etal, Krutov etal,H.Roberts etal, Ballon-Bayona etal, F.L.B.

$$\sqrt{\Delta_A < r_\rho^2 >} \sim \frac{\sqrt{< r^2 >_\rho}}{10}$$



II Improved- NJL model

NJL model and quark-antiquark Mesons

* Nambu-Jona-Lasinio model:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m_f)\psi + \frac{G_0}{2}[(\bar{\psi}\lambda_i\psi)^2 + (\bar{\psi}i\gamma_5\lambda_i\psi)^2]$$

* Gluon exchange(s) and dynamics $G_0 \sim \frac{1}{M_G^2}, \frac{1}{\Lambda^2}$ (flavorless)

* Current light quark masses m_f and generation of mass $M_f = m_f + G_0 < \bar{q}q >_f$

* Light meson multiplets (pseudoscalar, vector) reasonably well

* $\eta - \eta'$ puzzle - $U_A(1)$ anomaly - 't Hooft interaction
* Vacuum polarization also generates U(3) 'tHooft int. for NJLmodel (without instantons) A.P.J., F.L.B., PRD90, 014049 (2014)

Quark model pseudoscalar mesons nonet

Non degenerate quarks: $|u>, |d>, |s>$

$$\frac{P_a \lambda_a}{\sqrt{2}} = \begin{pmatrix} \frac{P_u}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{P_d}{\sqrt{2}} & K^0 \\ K^- & \bar{K}_0 & \frac{P_s}{\sqrt{2}} \end{pmatrix}$$

$$\begin{aligned} P_{1,2,3} &\rightarrow \text{pions} \\ P_{4,5,6,7} &\rightarrow \text{kaons} \\ P_8 &= \pi_8 = \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d - 2\bar{s}s) \rightarrow \eta \\ P_0 &= \pi_0 = \frac{\sqrt{2}}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s) \rightarrow \eta' \end{aligned} \tag{11}$$

Improving NJL model: sea quarks manifest

Two reasons for improved NJL model (flavor symmetry breaking)

** Usually mean field NJL model for fixed G_0:
1) Gap equations for DChSB, one-loop
1) Mesons from Bound state equations, one-loop

** QCD Lagrangian: flavor symmetry breaking in m_f
2) in a "GOOD" effective model, this flavor breaking SHOULD be present in all parameters.. (EFT, Weinberg "theorem" 1979)

* So, one step further **
- One loop level for the coupling constant - calculated
- NJL coupling constant with flavor symmetry breaking

Quark-polarization: fundamental and NJL model

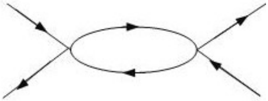


Figure: Polarization in the NJL model, solid lines are quarks, $P = 0$

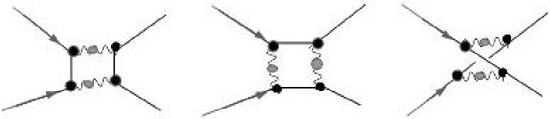


Figure: Wiggly lines with a dot = (dressed) gluon propagator.
* The dots in the vertices = running quark-gluon coupling constant.
* Need to (re)normalize resulting strength of interactions..

Resulting interaction G_{ij}: flavor-dependent

Gap/Bound state equations/$G \to$ coupled equations

One obtains (that plugs into the BSE) $\quad i, j = 0, ...N_f^2 - 1$:

$$G_{ij} = G_{ij}(M_u^*, M_d^*, M_s^*). \tag{12}$$

Standard NJL gap equations $\quad f = u, d, s$ (U(3) flavor)

$$(G_0) \quad M_f - m_f = G_0 Tr(S_{0,f}(0)) \tag{13}$$

By neglecting ALL mixing interactions $G_{i \neq j}$ and $G_{f_1 \neq f_2}$

$$(G_{ij}) \quad M_f^* - m_f = G_{ff} Tr(S_{0,f}(0)). \tag{14}$$

with $S_{0f}(k) = 1/(\not{k} - M^*)$
Chiral condensates $\to$ sea quark-antiquark degrees of freedom

Coupled equations: G_{ij} and M_f^* perturbatively/self consistently

To fix parameters of the model (to fit observables), meson masses

First and second mixings: Coupling constants

Coupling constants in the fundamental representation (quarks) G_{ff}
Different from the ones of adjoint representation (mesons) G_{ij}

$$\begin{aligned} 2G_{uu} &= 2\frac{G_{00}}{3} + G_{33} + \frac{G_{88}}{3}, \\ 2G_{dd} &= 2\frac{G_{00}}{3} + G_{33} + \frac{G_{88}}{3}, \\ 2G_{ss} &= 2\frac{G_{00}}{3} + 4\frac{G_{88}}{3}, \end{aligned} \tag{15}$$

where $G_{88}(M_f^*), G_{00}(M_f^*)$ (for $f = u, d, s$)
Emergence of sea quarks in GAP eqs. (1-quark mixings)
Screening in coupling constants G_{ff} above

And resulting mixing interations (2- meson mixing interactions)

$$G_{i \neq j} \propto (M_{f_1} - M_{f_2})^{n=1,2} \qquad G_{f_1 \neq f_2} \propto (M_{f_1} - M_{f_2})^{n=1,2}$$

Mesons Bound State Eqs.: Bethe-Salpeter-Born

By neglecting ALL mixing interactions $G_{i \neq j}$ - uncoupled equations:

$$1 - 2G_{ij} I^{ij}_{f_1 f_2}(P_0^2 = -M_\phi^2, \vec{P}^2 = 0) = 0, \quad (16)$$

rest frame of meson ϕ, eg, pseudoscalar mesons (NG)

G_{ij}: defines the meson structure in the adjoint representation Eg.

G_{11}, G_{22}, G_{33}: pion structure
$G_{44}, G_{55}, \ldots$ kaon structure

$$I^{ij}_{f_1 f_2}(P_0, \vec{P}) = Tr_{D,F,C} \int \frac{d^4k}{(2\pi)^4} \lambda_i i\gamma_5 S_{0,f_1}(k + P/2) \lambda_j i\gamma_5 S_{0,f_2}(k - P/2), \quad (17)$$

* Since $G_{ij}(M_u, M_d, M_s ...)$ strange/heavier quark-antiquark states (sea) contribute for the pion...

* Both fundamental and adjoint representations

* F.L.B., Phys. Rev. D 103, 094028 (2021); J.Phys. G 49, 055101 (2022); arXiv:2212.06616;
W.F.S.+F.L.B. arxiv: 2301.05695 arXiv:2301.10128

"Third" flavor mixings - Flavor U(3)

The polarization tensor

$$\Pi_{ij} \sim \int \frac{d^4k}{(2\pi)^4} \lambda_i \Gamma^D S_{0,\alpha} \lambda_j \Gamma^D S_{0,\beta} \quad (18)$$

where the Dirac Γ^D matrices
By writing the quark propagator in the adjoint representation

$$S_{0f} = \lambda_0 S_{0f,0} + \lambda_3 S_{0f,3} + \lambda_8 S_{0f,8}$$

we obtain combinations of the following integrals:

$$I_{f_1 f_2}(P^2) = iN_c\, Tr_D \int_k S_{0,f_1}\left(k + \frac{P}{2}\right) S_{0,f_2}\left(k - \frac{P}{2}\right) , \quad (19)$$

1) $M_s \neq M_u, M_d$ mixing strangeness contributions of the type pionS

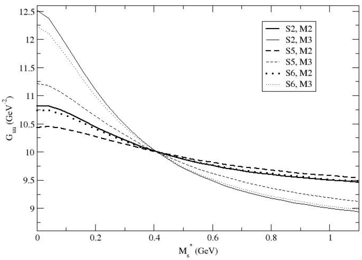
$$I_{8i8j} = (2\delta_{ij=8} + \frac{2}{3}\delta_{ij=1,2,3} - \frac{4}{3}\delta_{ij=4,5,6,7}) I_{88}(P^2)$$

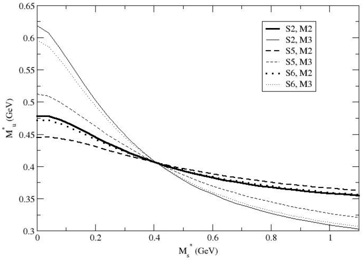
2) Mixing terms in the diagonal polarization tensor:
$\Pi_{ii}(P^2)$ ($i = 3, 8, 0$) contains $I_{ud}(P^2), I_{us}(P^2), I_{ds}(P^2) \to \pi^0, \eta, \eta'$

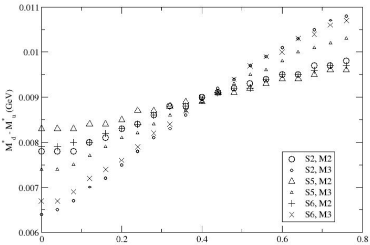
Table: Sets of parameters: Lagrangian quark masses, ultraviolet cutoff and the quark effective masses obtained from an initial NJL-gap equation $G_0 = 10\text{GeV}^{-2} \rightarrow$ fitting procedure neutral π^0, K^0

set of parameters	m_u MeV	m_d MeV	m_s MeV	Λ MeV	M_u MeV	M_d MeV	M_s MeV
S	3	7	133	680	405	415	612
V	3	7	133	685	422	431	625

By varying freely M_s^* in up-down gap equations

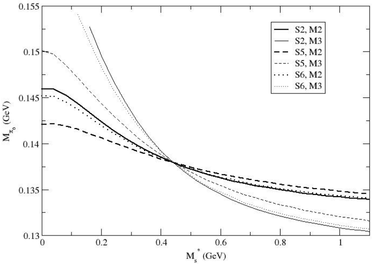


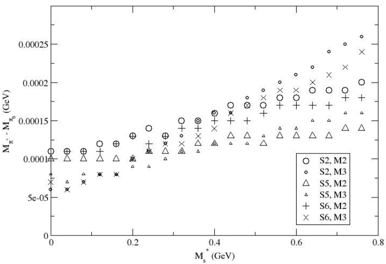




BSE of neutral pion: Strangeness in pions

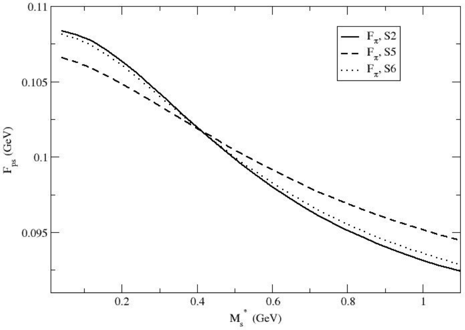
* Normalization point at nearly $M_s^* \sim 450$ MeV $G_{ij} \to G_0$
* "Physical point" $M_s^* \sim 550$ MeV





Strangeness in F_π

$F_\pi \simeq$ 102 MeV at the "physical point"
(value obtained from the fixed parameters of the model)



U(5) NJL-model and cutoffs: mesons masses

* NJL model not expected to work for heavy hadrons,
still, we did some calculation

* Vector interaction Λ_f (Bashir et al, Serna et al,others)

* $< \bar{c}c >, < \bar{b}b >$ non zero: NJL-type model with DChSB

W.F.de S., F.L. B., arXiv:2301.10128 $\rightarrow$ yes: meson masses
Non-covariant ultraviolet cutoff improves improves interpretation

$$|\vec{k}| \leq \Lambda_u \simeq \Lambda_s \simeq \Lambda_c \simeq \Lambda_b \sim 0.5 GeV$$

1) heavy quarks: non covariant (non relativistic) anyway
2) light quarks: results similar to other regularizations

* 5 parameters $\rightarrow$ (7+4) or (21+4) PS mesons, (5 or 18) S meson
Masses $m_u = m_d$ within $\sim$ 6% and 10%

$$\mathcal{J}_{f_1 f_2}(P^2) \quad = \quad iN_c \; Tr_D \int_k S_{0,f_1}\left(k+\frac{P}{2}\right) \; S_{0,f_2}\left(k-\frac{P}{2}\right) \; , \qquad (20)$$

Note mixing terms in the diagonal polarization tensor

$i=j$	Π_{ij}
1-3	$\mathcal{J}_{uu}$
4-7	$\mathcal{J}_{us}$
8	$\frac{11}{45}\mathcal{J}_{uu} + \frac{26}{45}\mathcal{J}_{ss} + \frac{8}{45}\mathcal{J}_{us}$
9-12	$\mathcal{J}_{uc}$
13-14	$\mathcal{J}_{sc}$
15	$\frac{17}{180}\mathcal{J}_{uu} + \frac{17}{720}\mathcal{J}_{ss} + \frac{57}{80}\mathcal{J}_{cc} + \frac{17}{180}\mathcal{J}_{us} + \frac{1}{20}\mathcal{J}_{uc} + \frac{1}{40}\mathcal{J}_{sc}$
16-19	$\mathcal{J}_{ub}$
20-21	$\mathcal{J}_{sb}$
22-23	$\mathcal{J}_{cb}$
24	$\frac{1}{20}\mathcal{J}_{uu} + \frac{1}{80}\mathcal{J}_{ss} + \frac{1}{80}\mathcal{J}_{cc} + \frac{4}{5}\mathcal{J}_{bb} + \frac{1}{20}\mathcal{J}_{us} + \frac{1}{20}\mathcal{J}_{uc} + \frac{1}{40}\mathcal{J}_{sc}$
0	$\frac{4}{25}\mathcal{J}_{uu} + \frac{1}{25}\mathcal{J}_{ss} + \frac{1}{25}\mathcal{J}_{cc} + \frac{1}{25}\mathcal{J}_{bb} + \frac{4}{25}\mathcal{J}_{us} + \frac{4}{25}\mathcal{J}_{uc} + \frac{4}{25}\mathcal{J}_{ub} + \frac{2}{25}\mathcal{J}_{sc}\cdots$

Set	2	3	Exp.
M_π (MeV)	165(**140**)[-]	**147**(118)[-] { 147(118) }	$137^\dagger$
M_K (MeV)	505(**494**)[475]	**512**(501)[481] { 512(501) }	495
M_D (MeV)	1870(**1863**)[1869]	**1868**(1869)[1873] { 1310(1378) }	1870
M_{D_s} (MeV)	2011(**1985**)[2005]	**2018**(2000)[2019] { 1469(1515) }	1968
M_B (MeV)	5294(**5275**)[5288]	**5279**(5274)[5283] { 4740(4831) }	5280
M_{B_s} (MeV)	5427(**5392**)[5418]	**5421**(5397)[5421] { 4882(4954) }	5367
M_{B_c} (MeV)	6542(**6460**)[6504]	**6539**(6477)[6516] { 5491(5595) }	6275

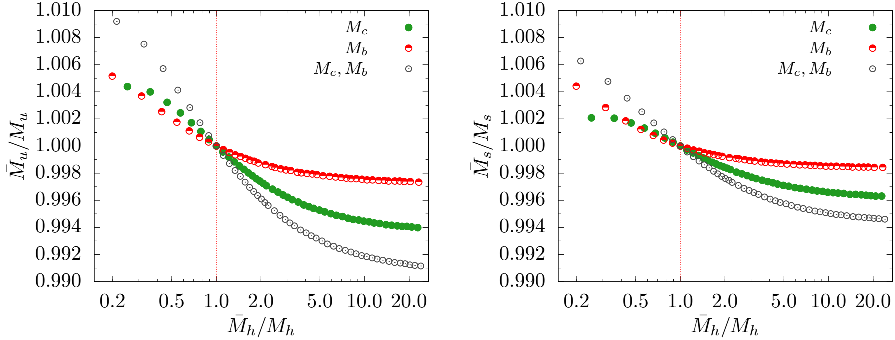
$G_0 \quad (G_{ij}) \quad [G_{i\neq j} = 0] \quad \{\bar{S}_c = \bar{S}_b = 0\}$

Table: Probabilities of a meson with valence quark-antiquark structure to develop other types of sea quark/antiquark components from $G_{psqq} = Z_{ps}^2$

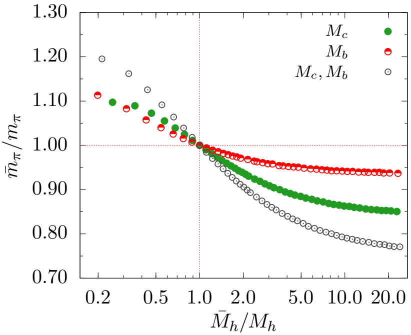
Set	1	2	3	4
$Pr(\pi)$	2 %	2 %	2 %	2 %
$Pr(K)$	3 %	3 %	3 %	5 %
$Pr(D)$	4 %	7 %	7 %	7 %
$Pr(D_s)$	5 %	4 %	5 %	5 %
$Pr(B)$	9 %	8 %	9 %	8 %
$Pr(B_s)$	11 %	7 %	7%	7 %
$Pr(B_c)$	6 %	6 %	6 %	5 %

Set	2	3	Exp. - PDG
$M_{PS,0}$ ($M_{\eta'}$) (MeV)	932(**921**)	**927**(920)	958
		{ 914(922) }	
$M_{PS,8}$ (M_{η}) (MeV)	544(**537**)	**550**(544)	548
		{ 550(544) }	
$M_{PS,15}$ (M_{η_c}) (MeV)	3403(**3288**)	**3410**(3311)	2984
		{ 2428(2451) }	
$M_{PS,24}$ (M_{η_b}) (MeV)	10024(**9882**)	**10010**(9891)	9399
		{ 8940(9013) }	
$M_{S,0}$ ($M_{f_0(980)}$) (MeV)	995(**995**)	**992**(990)	990
		{ 897(928) }	
$M_{S,8}$ ($M_{f_0(500)}$) (MeV)	784(**754**)	**791**(760)	500
		{ 791(760) }	
$M_{S,15}$ ($M_{\chi_{c0}}$) (MeV)	3718(**3629**)	**3730**(3651)	3415
		{ 2083(2082) }	
$M_{S,24}$ ($M_{\chi_{b0}}$) (MeV)	10523(**10413**)	**10508**(10410)	9859
		{ 9440(9440) }	

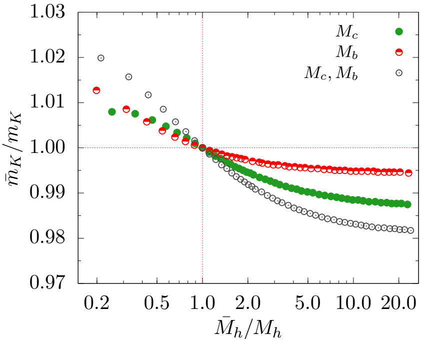
$G_0 \quad (G_{ij}) \quad [G_{i\neq j} = 0] \quad \{\bar{S}_c = \bar{S}_b = 0\}$



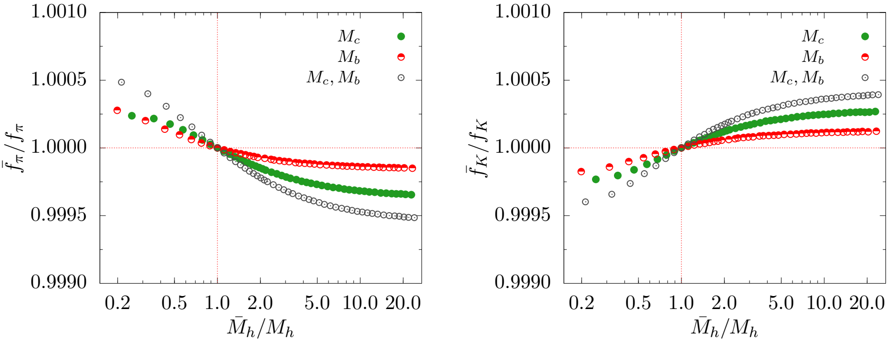
1.010
1.008
1.006
1.004
1.002
1.000
0.998
0.996
0.994
0.992
0.990
$\bar{M}_u/M_u$
0.2
0.5
1.0
2.0
5.0
10.0
20.0
$\bar{M}_h/M_h$
M_c
M_b
M_c, M_b
1.010
1.008
1.006
1.004
1.002
1.000
0.998
0.996
0.994
0.992
0.990
$\bar{M}_s/M_s$
0.2
0.5
1.0
2.0
5.0
10.0
20.0
$\bar{M}_h/M_h$
M_c
M_b
M_c, M_b



1.30
1.20
1.10
1.00
0.90
0.80
0.70
$\bar{m}_\pi/m_\pi$
M_c
M_b
M_c, M_b
0.2
0.5
1.0
2.0
5.0
10.0
20.0
$\bar{M}_h/M_h$



1.03
1.02
1.01
1.00
0.99
0.98
0.97
$\bar{m}_K/m_K$
M_c
M_b
M_c, M_b
0.2
0.5
1.0
2.0
5.0
10.0
20.0
$\bar{M}_h/M_h$



$$\frac{\Delta_{c,b} M_u}{M_u} > \frac{\Delta_{c,b} M_s}{M_s},$$

Pion strangeness content leading to Meson Mixing

Mixing matrix (Kroll, Feldmann et al)

$$\begin{pmatrix} \pi^0 \\ \eta \\ \eta' \end{pmatrix} = M \begin{pmatrix} P_3 \\ P_8 \\ P_0 \end{pmatrix} .$$

Leading mixings:

$$\begin{aligned} |\eta > &= \cos\theta_{ps}|P_8 > - \sin\theta_{ps}|P_0 >, \\ |\eta' > &= \sin\theta_{ps}|P_8 > + \cos\theta_{ps}|P_0 > . \end{aligned} \tag{21}$$

$$\theta_{ps} = \frac{1}{2} \arcsin \left(\frac{4 G_{08}^{n} \bar{G}_{08}}{(M_\eta^2 - M_{\eta'}^2)} \right) . \tag{22}$$

$$\begin{aligned} |\eta > &= -(\epsilon_2 + \epsilon_1 \cos(\phi_{08})|P_3 > + \sqrt{\frac{2}{3}} \cos(\phi_{08})|P_8 >, \\ |\pi_0 > &= |P_3 > + \left(\sqrt{\frac{2}{3}} (\epsilon_1 + \epsilon_2 \cos(\phi_{08})) - \frac{\epsilon_2 S_\psi}{\sqrt{3}} \right) |P_8 > . \end{aligned}$$

$\eta - \eta'$ mixing: usual basis

$\pi^0 - \eta$ mixing:

$$|\pi^0> \;=\; \frac{1}{\sqrt{2}}\left[1 + a_l\right]|\bar{u}u> - \frac{1}{\sqrt{2}}\left[1 - a_l\right]|\bar{d}d> - 2a_s|\bar{s}s>,$$

* Contributions for Up and down are different
* $< \bar{q}q|\hat{H}|\bar{q}q > \simeq 2M_q \sim 900$ MeV.

$$\Delta_\eta m_{\pi^0} \simeq 4a^2 M_s \;\sim\; 1 - 5 \text{ MeV} \qquad\qquad \Delta_\eta M^*_{u,d} \sim \frac{3}{4}\Delta_\eta m_{\pi^0}.$$

* Meson -constituent quark couplings with mixings - on going

* Similarly: ChPT - eg Kaiser, (2007)- $\Delta_s M_\pi \sim 9 - 19$MeV

Pion c-b content leading to mixing to η_c, η_b

Flavor eigenstates $P_3 = \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d) \to \pi^0$

$$\begin{aligned}
P_0 &= \sqrt{\frac{2}{5}}(\bar{u}u + \bar{d}d + \bar{s}s + \bar{c}c + \bar{b}b) \to \eta'(958) \\
P_8 &= \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d - 2\bar{s}s) \to \eta(548) \\
P_{15} &= \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d + \bar{s}s - 3\bar{c}c) \to \eta_c(3415) \\
P_{24} &= \frac{1}{\sqrt{10}}(\bar{u}u + \bar{d}d + \bar{s}s + \bar{c}c - 4\bar{b}b) \to \eta_b(9859)
\end{aligned} \tag{23}$$

From the mixing: $\pi^0 \sim P_3 + G_{30}P_0 + G_{38}P_8 + G_{3,15}P_{15} + G_{3,24}P_{24}$
such that mixing amplitude:

$$< \eta_c|\pi^0 > \sim G_{3,15}, \qquad < \eta_b|\pi^0 > \sim G_{3,24}$$

Problem to identify: $M_{\pi^0} << M_{\eta_c} < M_{\eta_b}$
η_c, η_b at rest $\to$ pions $K_{\pi^0} \sim (3375MeV)$ $(9719MeV)$??

Quark-antiquark states of light scalars

Strong consequences of strangeness

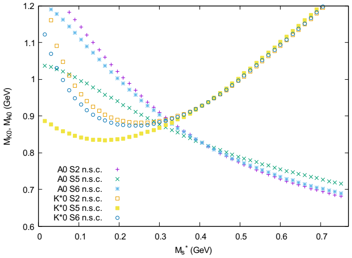


Figure: Mesons A_0 and κ: inversion of hierarchy

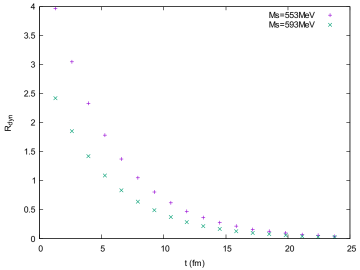
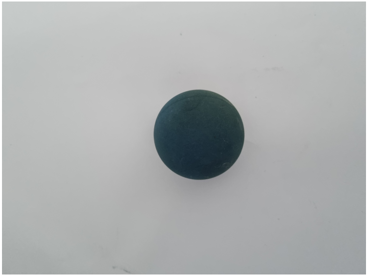


Figure: Ratio of mixings $A_0 - f_0$ BESS-III: 0.4 or 0.97

$$R_{a0f0} \equiv \frac{A_0^0(980) \to S_8 \to f_0(980)}{f_0(980) \to S_3 \to A_0^0(980)} \sim \frac{\left|\frac{a_{0,(8)}}{a_0}\right|^2}{\left|\frac{f_{0,(3)}}{f_0}\right|^2} \equiv \left|\frac{A_{8,a_0}}{A_{3,f_0}}\right|^2 .$$

One quark, one antiquark, one meson



Summary

- Vector mesons may probe axial quark current ($< r_\rho >^2_A$)
- FSB $\rightarrow$ Mixing effects $\rightarrow$ sea quark structures
- Meson mixings and strangeness/charm/bottom content

On going/planned:

- Flavor symmetry breaking/mixing effects in punctual vector effective interaction with B.El Bennich+F.Serna
- Flavor symmetry breaking/mixing effects in couplings, e.g. pion-constituent quarks and $j_{A,\mu}$
- Behavior of FSB with increasing energies/momenta (GPDs,PDFs)
- ...

Thank you for your attention!