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INCLUYENTE, INTEGRADA Y PARTICIPATIVA

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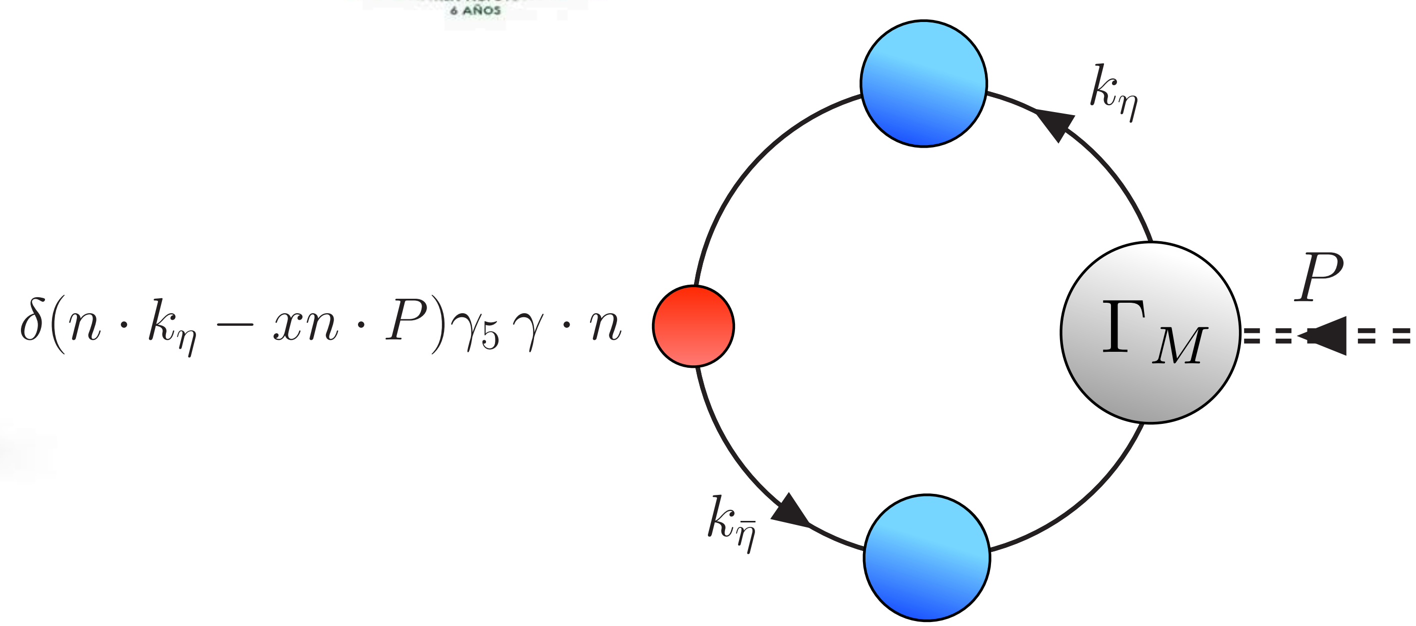
Resolución MEN No. 015153 de 2022 6 AÑOS

$$\phi_M(x,\mu)$$



Meson Distribution Amplitudes from Bethe-Salpeter Wave Functions

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Universidad de Sucre, Sincelejo - Colombia

Physics Opportunities at an the Electron-Ion Collider (POETIC)

May 04, 2023

"WE COLLABORATE. I'M AN EXPERT, BUT NOT AN AUTHORITY, AND DR. GELPIS IS AN AUTHORITY, BUT NOT AN EXPERT."

Work in collaboration with

- Bruno El-Bennich, Universidade Cidade de São Paulo, Brazil.
- Roberto Correa Da Silveira, Universidade Cidade de São Paulo, Brazil.
- Eduardo Rojas, Universidad de Nariño, Colombia
- Jesús Javier Cobos, Universidad de Sonora, México.

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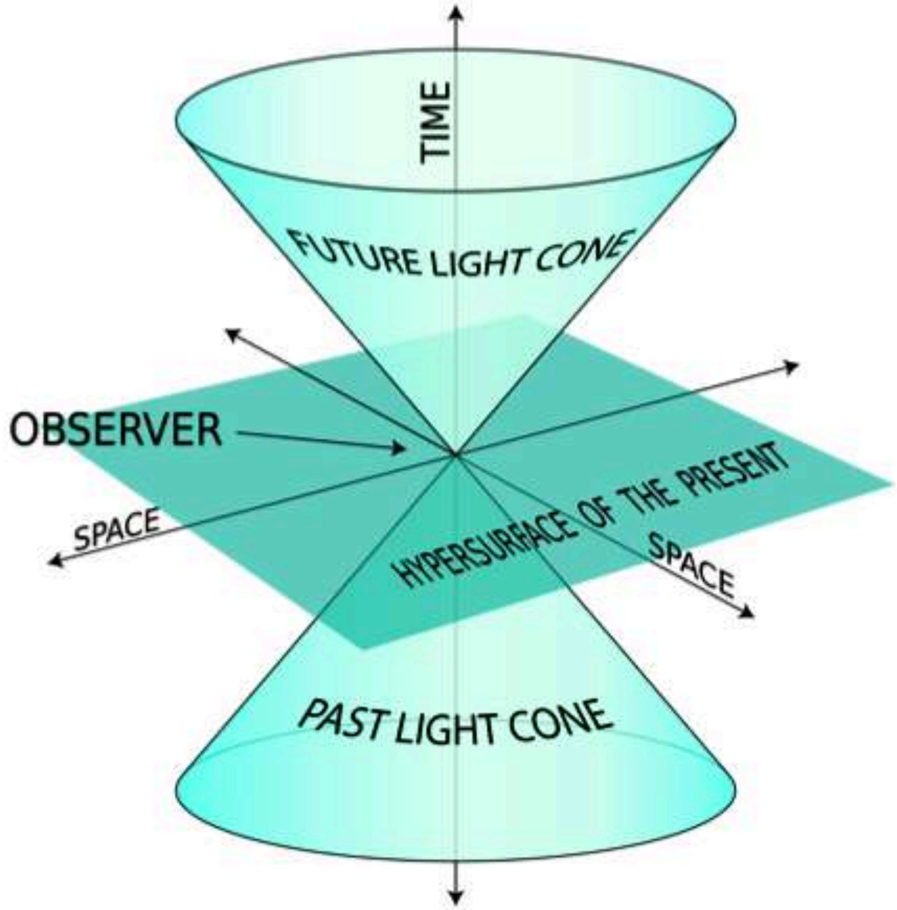
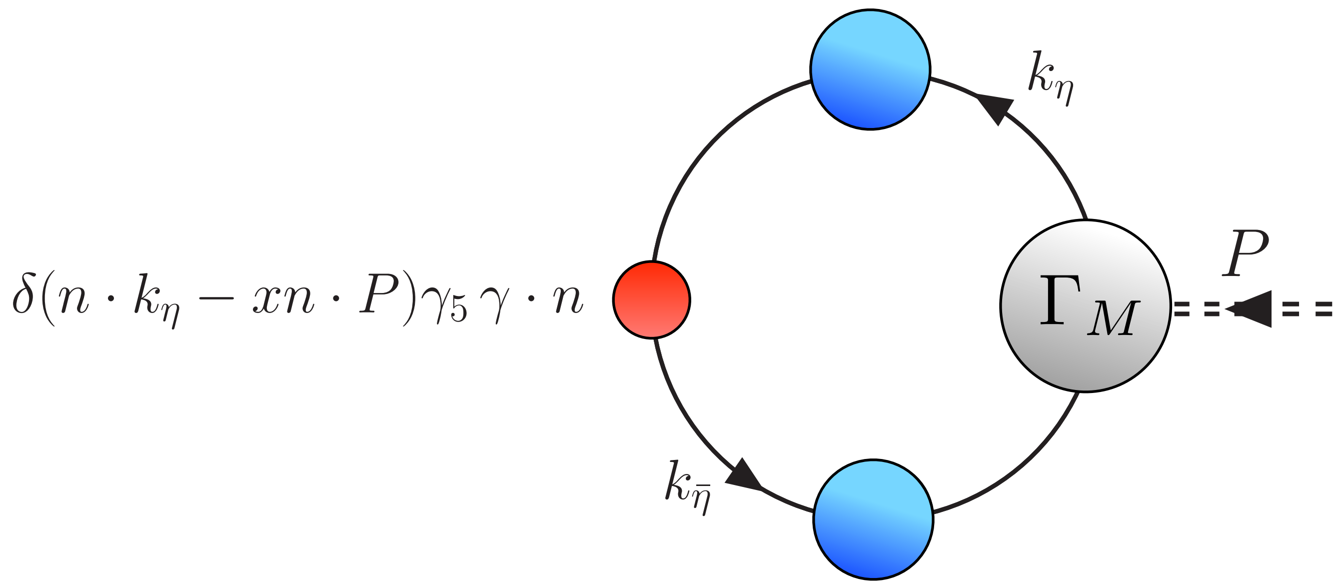




Light-Cone Distribution Amplitudes

$$\boxed{\phi_M(x, \mu)}$$

Light-Cone Distribution Amplitudes

$$\phi_M(x,\mu)$$

- Hadronic light-cone distribution amplitudes (LCDAs) were introduced four decades ago in the context of QCD description of hard exclusive reactions.

- QCD Factorization involves matrix elements which are convolution integrals

$$\langle \pi^+\pi^- | (\bar{u}b)_{\mathrm{V-A}}(\bar{u}d)_{\mathrm{V-A}} | \bar{B}_d \rangle \to \int_0^1 d\xi\, du\, dv\, \phi_B(\xi)\phi_\pi(u)\phi_\pi(v)\, T(\xi,u,v;m_b)$$

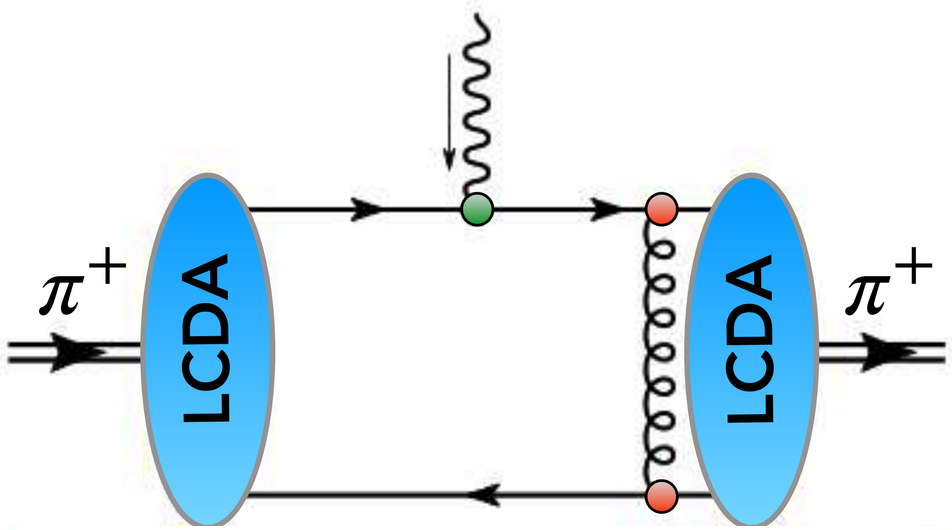
$T(\xi,u,v;m_b)$ ➡ **Hard - scattering kernels (perturbative)**

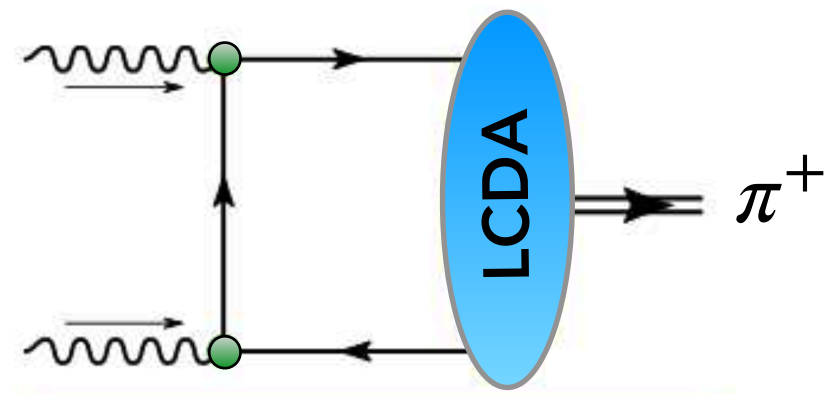
$\phi_B(u),\ \phi_\pi(v)$ ➡ **Meson's Light-front distribution Amplitudes (LCDAS) (nonperturbative)**

$$\phi_M(x) \neq \phi^{\mathrm{asy}}(x) = 6x(1-x)$$

Light-Cone Distribution Amplitudes

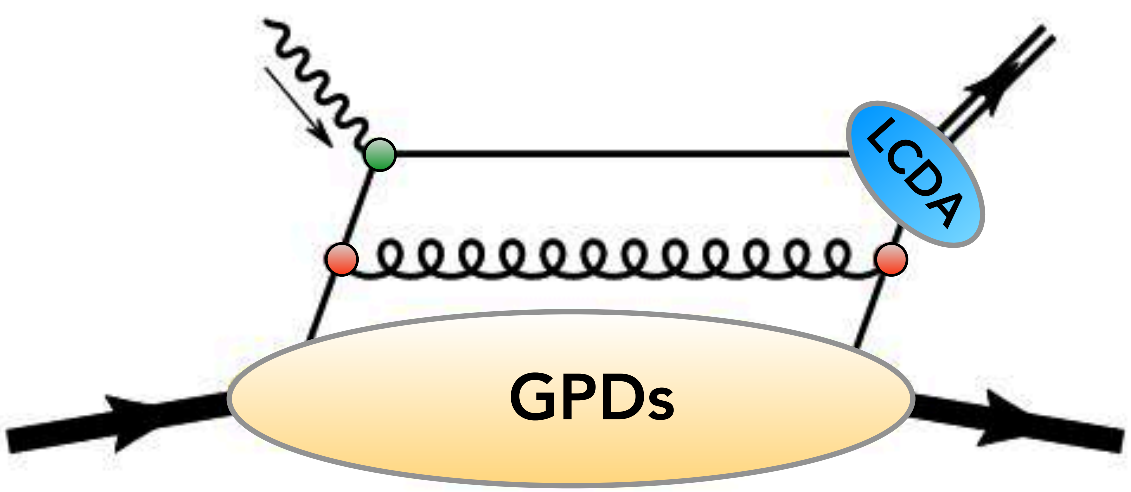
- Hard exclusive scattering processes.

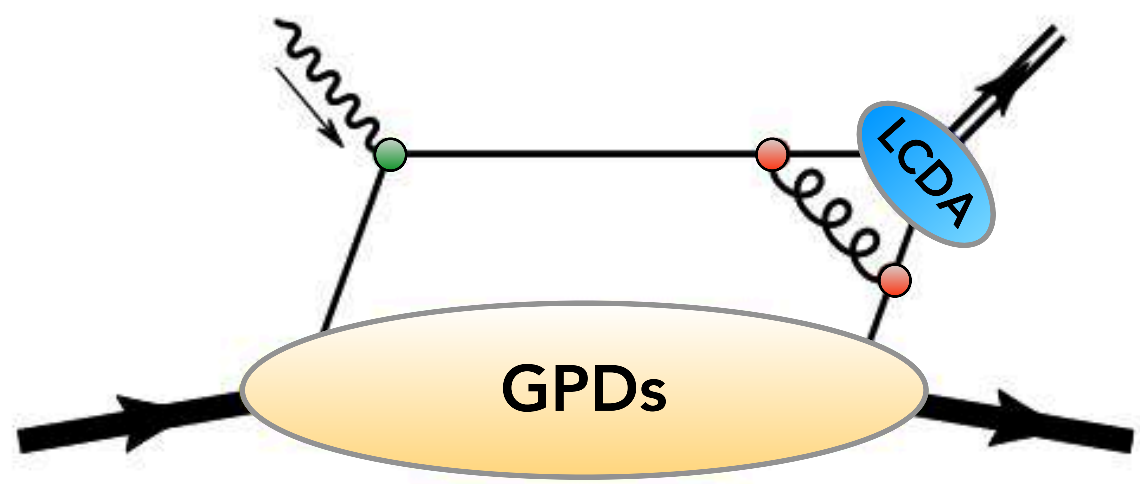




$$Q^2 F_\pi(Q^2) \to 16\pi f_\pi^2 \alpha_s(Q^2)$$

$$Q^2 F_{\gamma^*\gamma\pi}(Q^2) \to 2 f_\pi$$





Light-Cone Distribution Amplitudes

- The LCDAs are scale-dependent functions can be understood as the closest relative of quantum mechanical wave functions in quantum field theory

- $\phi_M(x,\mu)$ expresses the light-front fraction of the hadron's momentum carried by a valence quark. Allows for a probability interpretation of partons.

- x is the light-front momentum fraction: $x = k^+/P^+$ and μ the renormalization scale.

- **Leading-twist LCDA for pseudoscalar meson**

$$
\langle 0|\bar{q}_f(y_2 n)\mathcal{W}\,[y_2 n, y_1 n]\,\gamma\cdot n\,\gamma_5\,q_g(y_1 n)|M(P)\rangle
= i f_M\, n\cdot P\int_0^1 dx\, e^{-in\cdot P(y_1 x + y_2\bar{x})}\phi_M(x,\mu)
$$

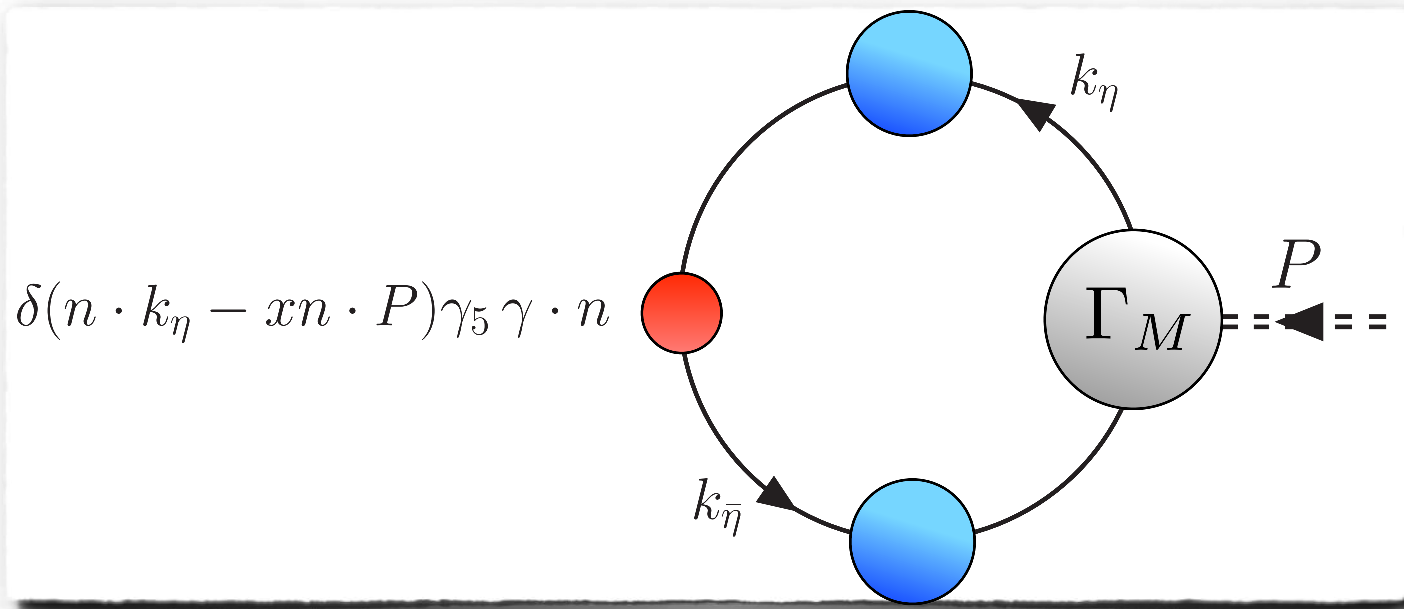
f_M = **weak decay constant**

$n^2 = 0$

$n\cdot P = -m_M$

Light-Cone Distribution Amplitudes

Light-cone projection of the Bethe-Salpeter wave function



$$f_M \phi_M(x, \mu) = \frac{\mathcal{Z}_2 N_c}{\sqrt{2}} \mathrm{Tr}_D \int^\Lambda \frac{d^4k}{(2\pi)^4} \delta(n \cdot k_\eta - xn \cdot P)\gamma_5 \gamma \cdot n \, \chi_M(k_\eta, k_{\bar{\eta}})$$

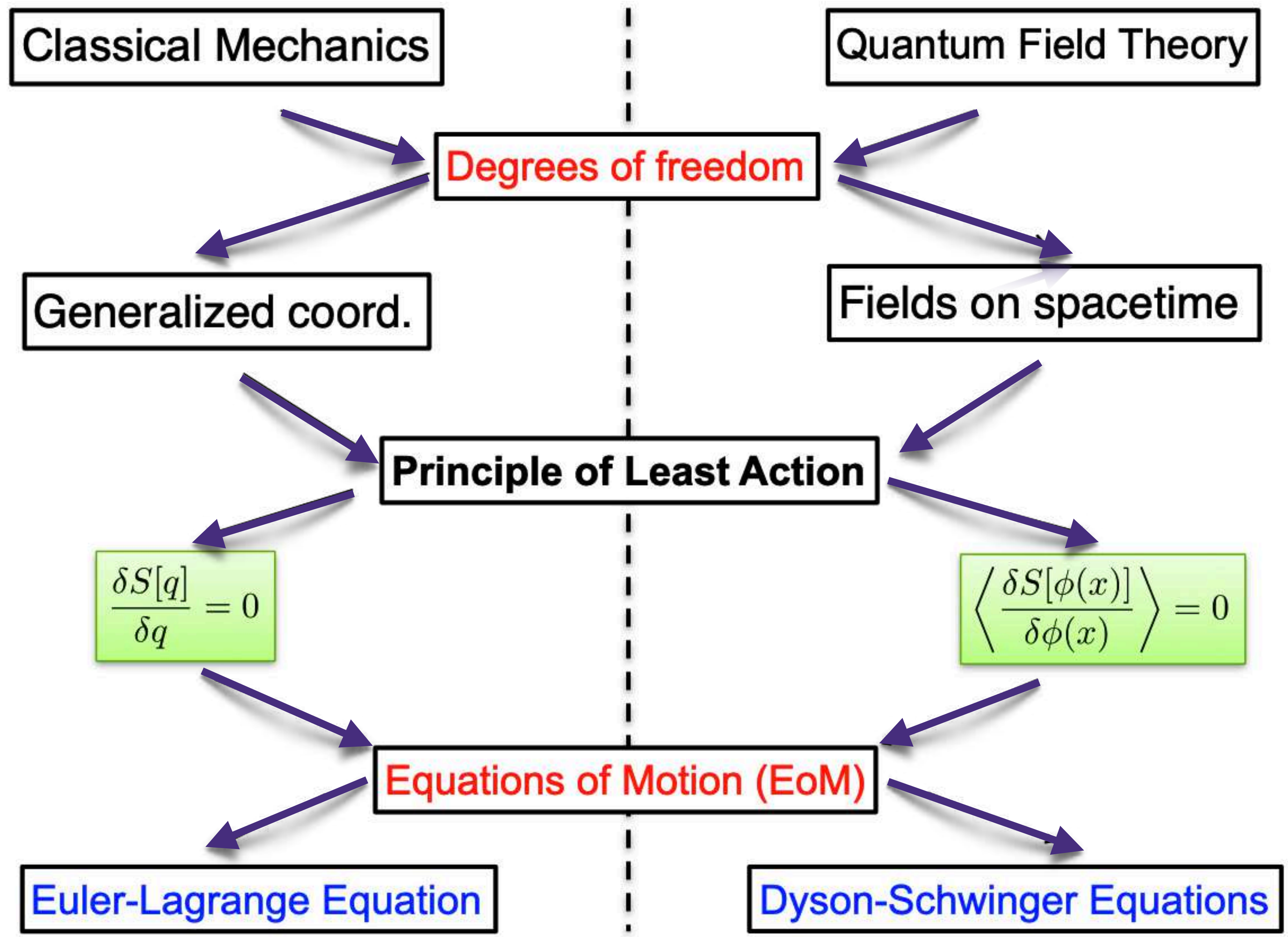
$\chi_M(k_\eta, k_{\bar{\eta}}) := S(k_\eta)\Gamma_M(k, P)S(k_{\bar{\eta}})$ ➡ Meson's Bethe-Salpeter wave function (BSWF)

$\Gamma_M(k, P)$ ➡ Meson's Bethe-Salpeter Amplitude

$S(k_\eta)$ ➡ Quark propagator

L. Chang, I.C. Cloët, J.J. Cobos-Martínez, C.D. Roberts, S.M.Schmidt, P. C. Tandy, Phys. Rev. Lett. 110 (2013)
J. Segovia, L. Chang, I. C. Cloët, C. D. Roberts, S. M. Schmidt, Phys. Lett. B 731 (2014)

Non-perturbative continuum tools for QCD

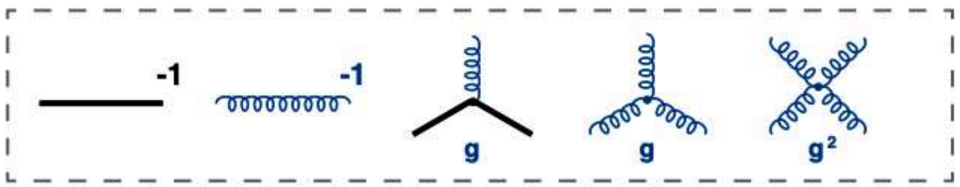


QCD's Dyson-Schwinger Equations (DSEs)

QCD's classical action:

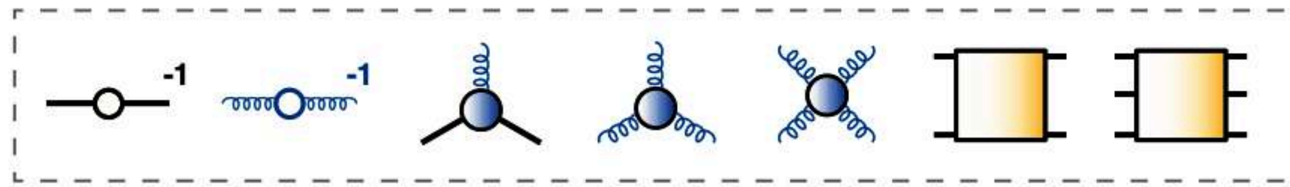
$$S = \int d^4x \left[\bar{\psi}\,(\not{\partial} + ig\not{A} + m)\,\psi + \tfrac{1}{4} F^{a}_{\mu\nu} F^{\mu\nu}_{a}\right]$$

=



Quantum "effective action":

$$\int \mathcal{D}[\psi, \bar{\psi}, A]\, e^{-S} = e^{-\Gamma}$$



DSE = Quantum equation of motion: obtained from path integral, relate n-point functions

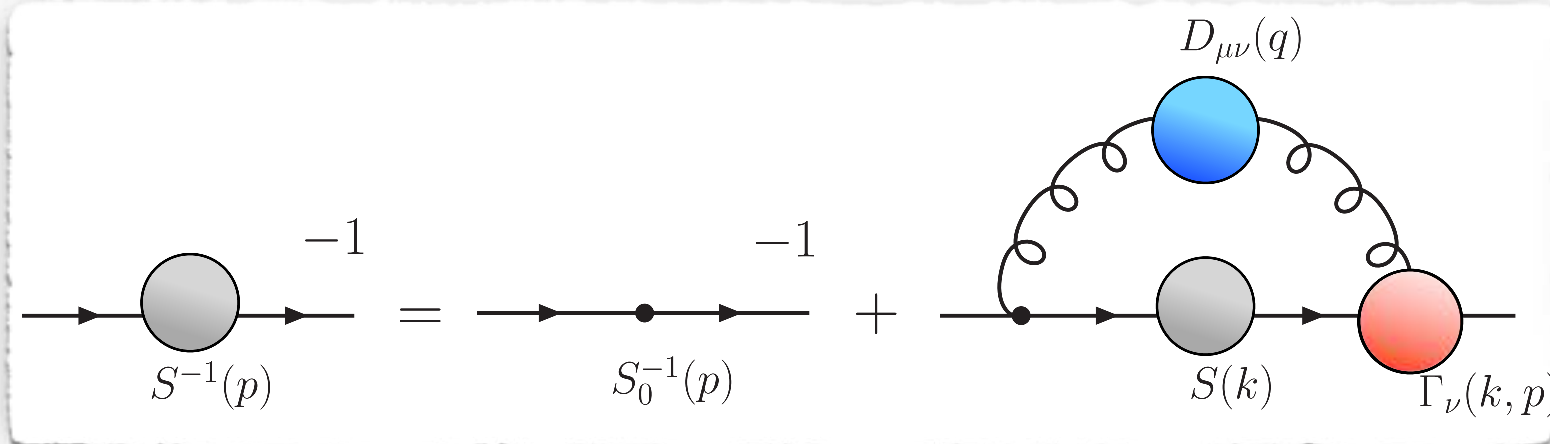
- Infinitely many coupled equations
- Continuum methods: Reproduce perturbation theory, but non-perturbative
- Systematic truncations: neglect higher n-point functions to obtain **closed system**

Capture two emergent phenomena:

- **Dynamical chiral symmetry breaking**
- **Confinement**

QCD's Dyson-Schwinger Equations (DSEs)

Quark Dyson-Schwinger Equation



Quark mass function

$$M_f(p^2) = \frac{B_f(p^2,\mu^2)}{A_f(p^2,\mu^2)}$$

$$S_f^{-1}(p) = Z_2^f\,(i\gamma\cdot p + m_f^{\rm bm}) + Z_1^f g^2 \int^\Lambda \frac{d^4k}{(2\pi)^4} D_{\mu\nu}^{ab}(q)\frac{\lambda^a}{2}\gamma_\mu S_f(k)\Gamma^b_{\nu,f}(k,p)$$

Quark propagator

$$S_f^{-1}(p) = i\gamma\cdot pA_f(p^2,\mu) + B_f(p^2,\mu)$$
$$= A_f(p^2,\mu)[i\gamma\cdot p + M_f(p^2)]$$

- $D_{\mu\nu}^{ab}(q)$ = Dressed gluon propagator
- $\Gamma_\nu^b(k,p)$ = Dressed quark-gluon vertex
- Z_2 = Quark wave function renormalization constant
- Z_4 = Quark-gluon vertex renormalization constant

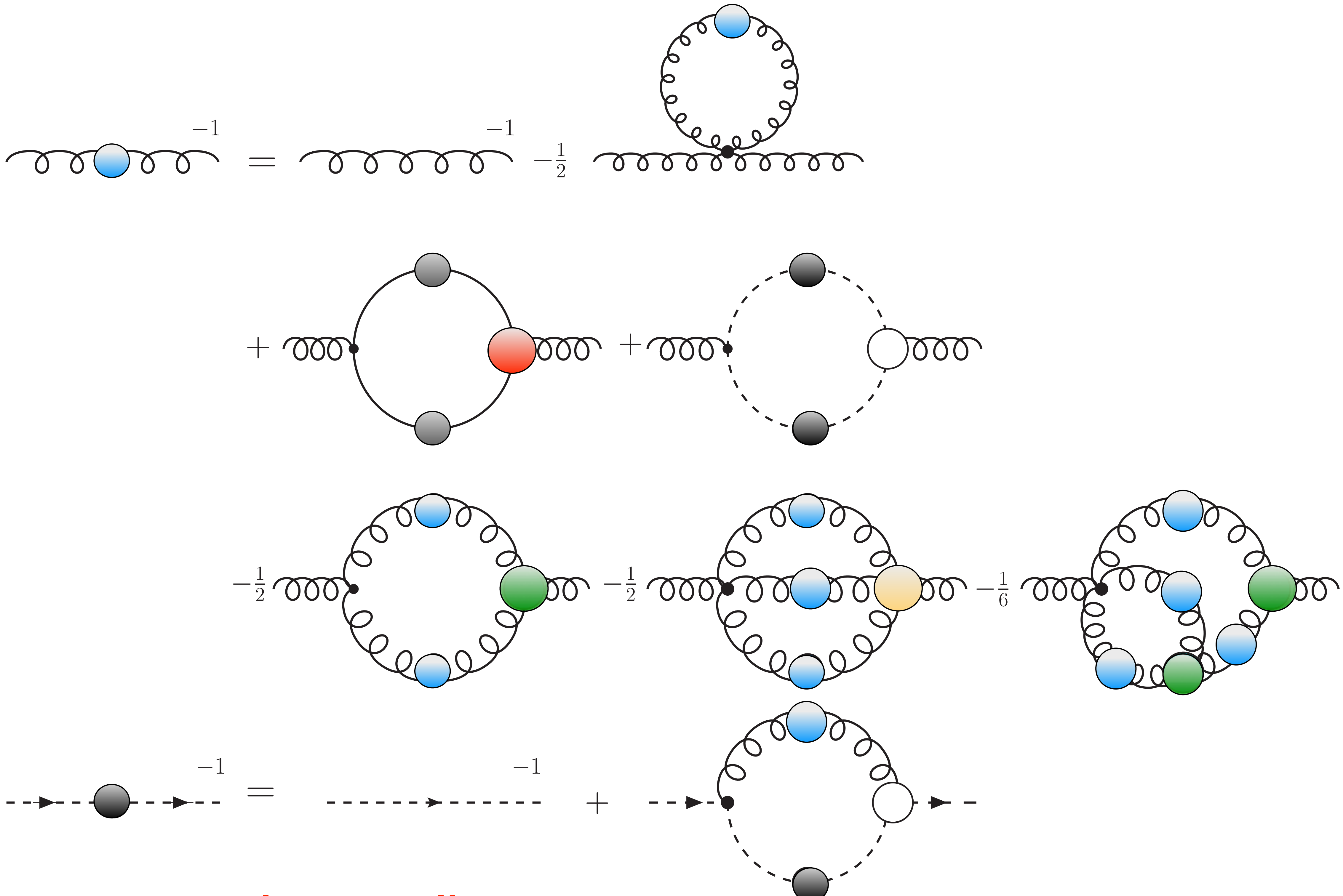
Renormalization condition:

$$A_f(p^2) = A_f(p^2)\big|_{p^2=\mu^2} = 1 \qquad S_f^{-1}(p)\,\big|_{p^2=\mu^2} = i\gamma\cdot p + m_f(\mu)$$

QCD's Dyson-Schwinger Equations (DSEs)

Gluon

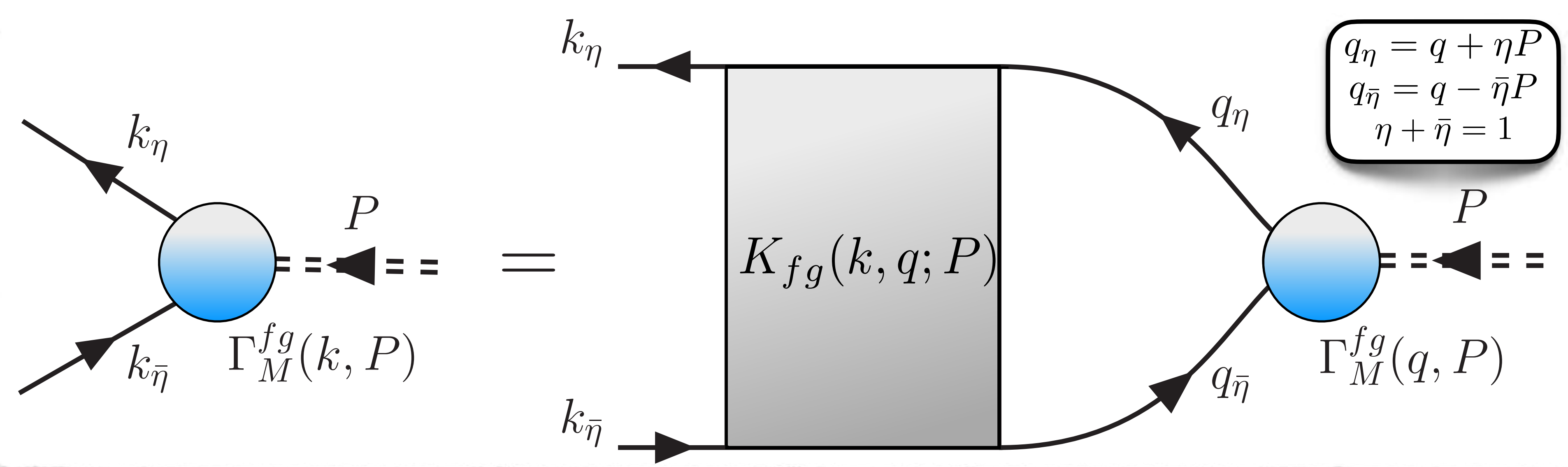
Ghost



See Arlene's Talk

Bethe-Salpeter Equations for QCD bound-states

BSE = Bound-state equation for meson



$$\Gamma_M^{fg}(k,P) = \int^\Lambda \frac{d^4q}{(2\pi)^4} K_{fg}(k,q;P) S_f(q_\eta) \Gamma_M^{fg}(q,P) S_g(q_{\bar\eta})$$

- $K_{fg}(q,k;P)$ = Quark-antiquark scattering kernel
- $S_f(q_\eta)$ = Dressed quark propagator
- $\Gamma_M^{fg}(k,P)$ = Meson's Bethe-Salpeter Amplitude (BSA)

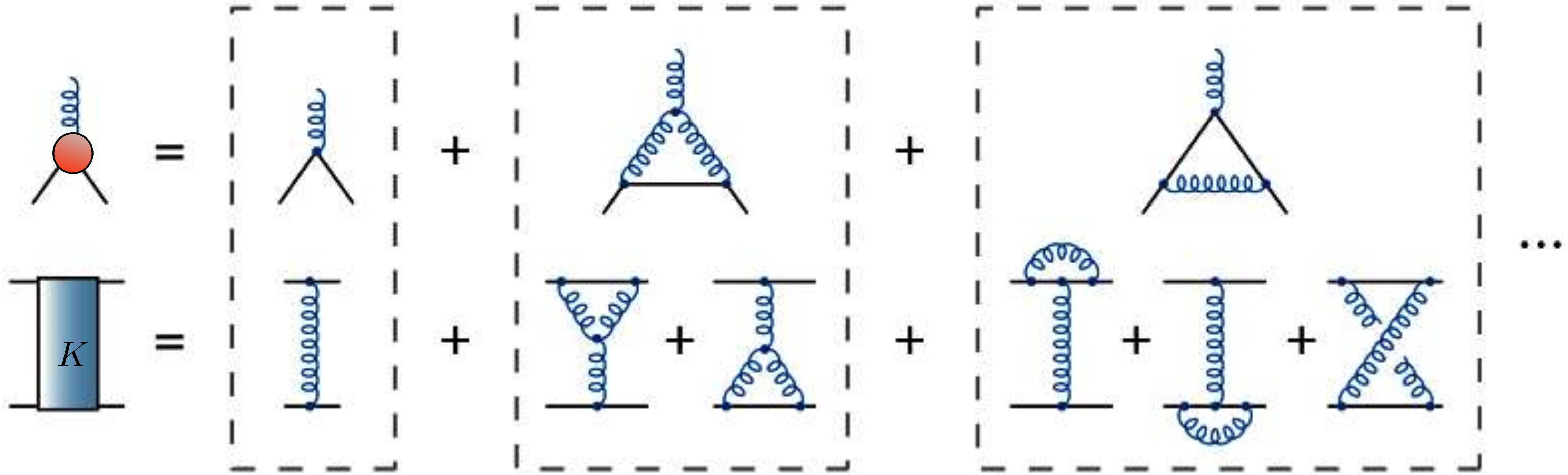
$$\Gamma_M(k,P) = \sum_{i=1}^{N} T_M^i(k,P) F_i(k,P)$$

General solution for Poincaré invariant pseudoscalar BSA →

$$\Gamma_M^{fg}(k,P) = \gamma_5 \Big[iE_M^{fg}(k,P) + \gamma\cdot P\, F_M^{fg}(k,P) + \gamma\cdot k\; k\cdot P\, G_M^{fg}(k,P) + \sigma_{\mu\nu} k_\mu P_\nu\, H_M^{fg}(k,P) \Big]$$

Truncation schemes and symmetries

- **DSE/BSE:** Kernel can be derived in accordance with chiral symmetry



Truncation must preserve **AV-WTI**, which ensures that we will have massless pions in the chiral limit.

AV-WTI: $$P_\mu \Gamma_{5\mu}^{fg}(k;P) = S_f^{-1}(k_\eta)\, i\gamma_5 + i\gamma_5 S_g^{-1}(k_{\bar{\eta}})$$

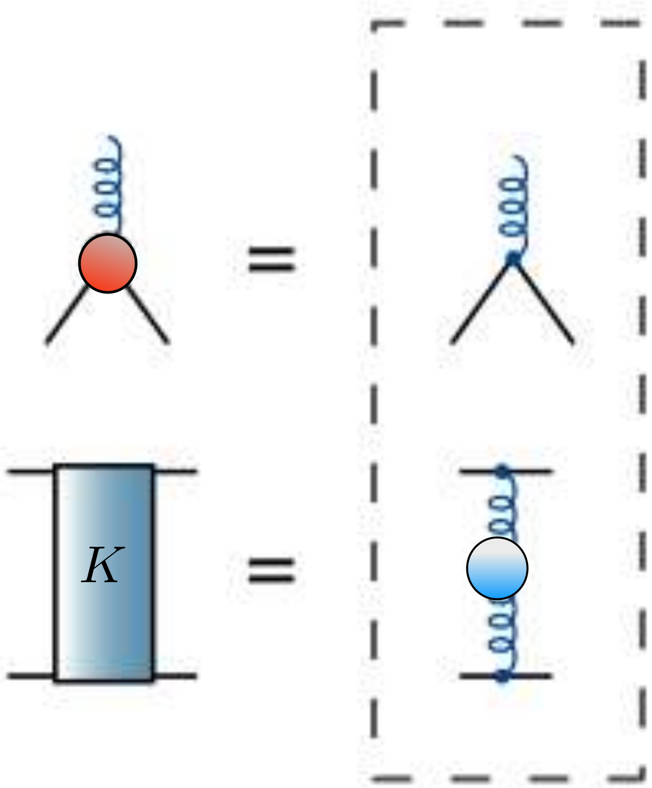
Axial vector-vertex:

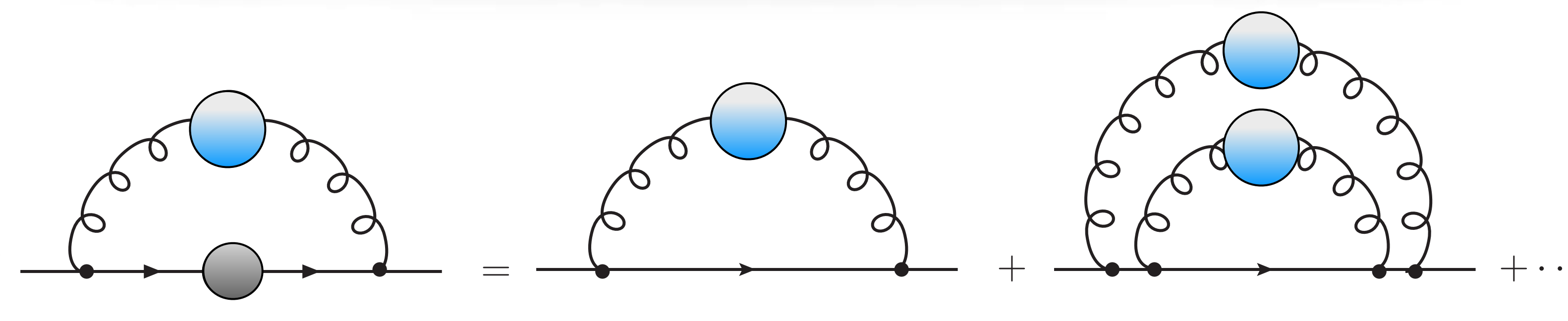
$$\Gamma_{5\mu}^{fg}(k;P) = Z_2^f \gamma_5\gamma_\mu + \int^\Lambda \frac{d^4q}{(2\pi)^4} K_{fg}(q,k;P)\, S_f(q_\eta)\Gamma_{5\mu}^{fg}(q;P) S_g(q_{\bar{\eta}})$$

Rainbow-Ladder truncation

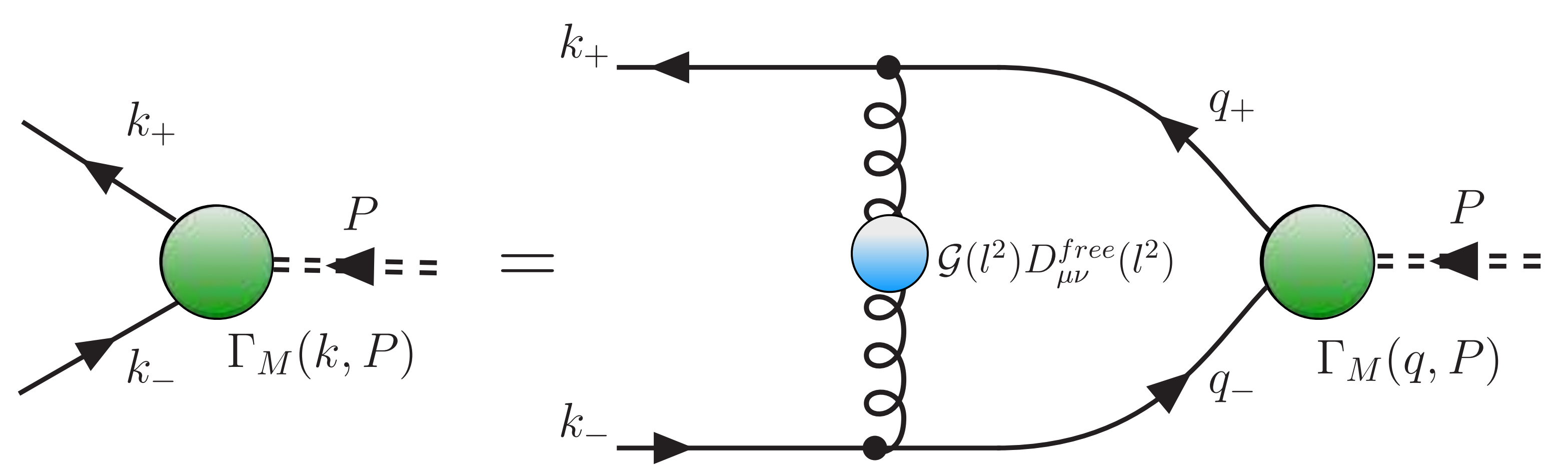
- **Leading truncation**

$$Z_1^f g^2 D_{\mu\nu}(q)\Gamma_{\nu,f}(k,p) = \left(Z_2^f\right)^2 \mathcal{G}(q^2) D_{\mu\nu}^{\text{free}}(q)\frac{\lambda^a}{2}\gamma_\nu$$





$$K(q,k;P) = -Z_2^2\,\mathcal{G}\left(l^2\right)\,D_{\mu\nu}^{\text{free}}(l)\,\gamma_\mu\frac{\lambda^a}{2}\gamma_\nu\frac{\lambda^a}{2}$$



$\mathscr{G}(q)$: Effective gluon interaction

Gluon interaction model

An interaction ansatz for $\mathcal{G}(q^2)$ that has proven its merits in meson and baryon phenomenology can be decomposed as

$$\frac{\mathcal{G}_f(q^2)}{q^2} = \mathcal{G}_f^{\mathrm{IR}}(q^2) + 4\pi\tilde{\alpha}_{\mathrm{PT}}(q^2)$$

- Infrared part: Two-Models

MT-Model: $\mathcal{G}_f^{\mathrm{IR}}(q^2) = \frac{4\pi^2}{\omega_f^6} q^2 D_f\, e^{-q^2/\omega_f^2}$

QC-Model: $\mathcal{G}_f^{\mathrm{IR}}(q^2) = \frac{8\pi^2}{\omega_f^4} D_f\, e^{-q^2/\omega_f^2}$ ✔

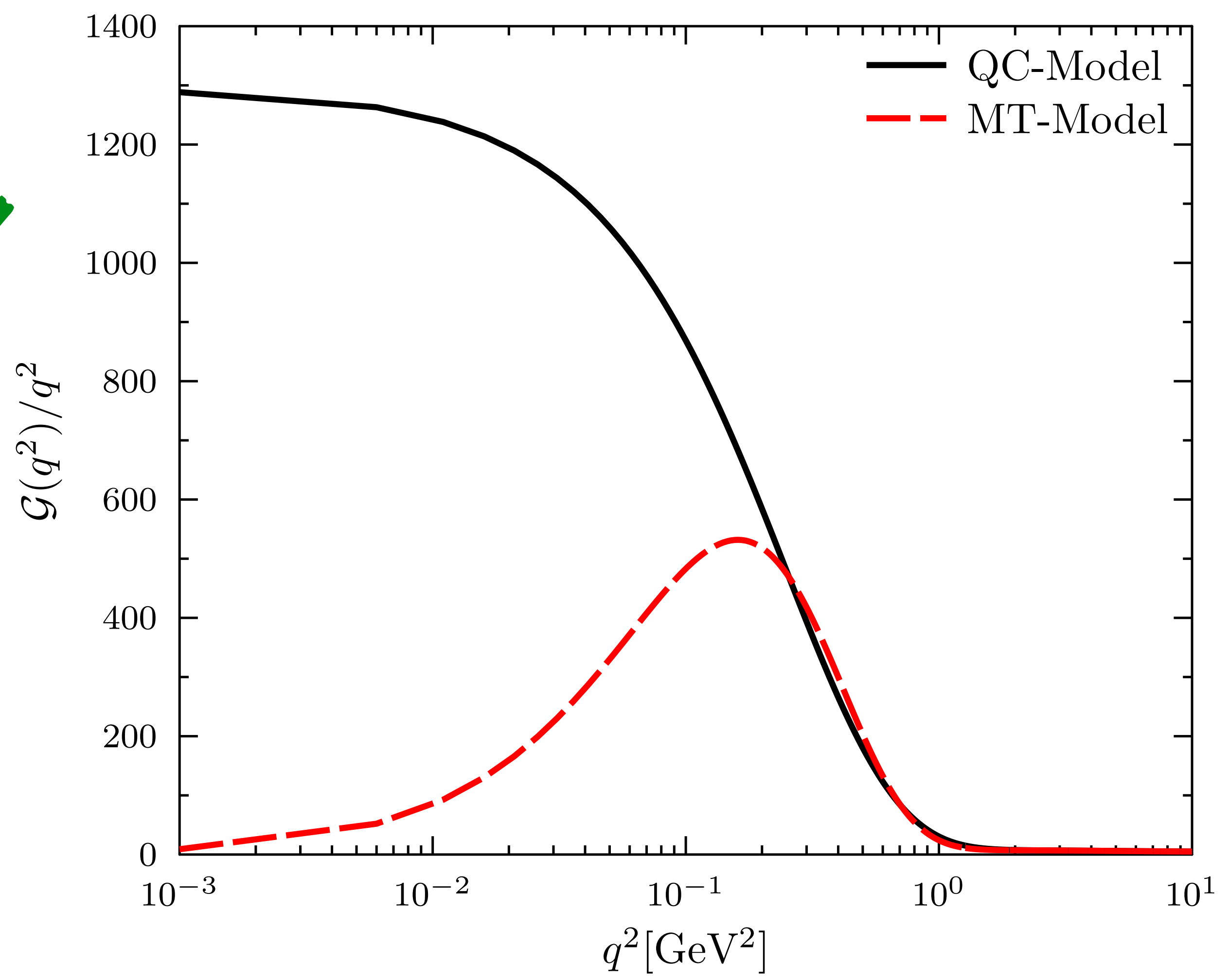
- Ultraviolet part: Model

$$4\pi\tilde{\alpha}_{\mathrm{PT}}(q^2) = \frac{8\pi^2\gamma_m \mathcal{F}(q^2)}{\ln\left[\tau + \left(1 + q^2/\Lambda^2_{\mathrm{QCD}}\right)^2\right]}$$

$$\gamma_m = 12/(33 - 2N_f),\ N_f = 4,\ \Lambda_{\mathrm{QCD}} = 0.234\,\mathrm{GeV}$$

$$\mathcal{F}(q^2) = [1 - \exp(-q^2/4m_t^2)]/q^2,\ \ m_t = 0.5\ \mathrm{GeV}$$

$$\tau = e^2 - 1$$



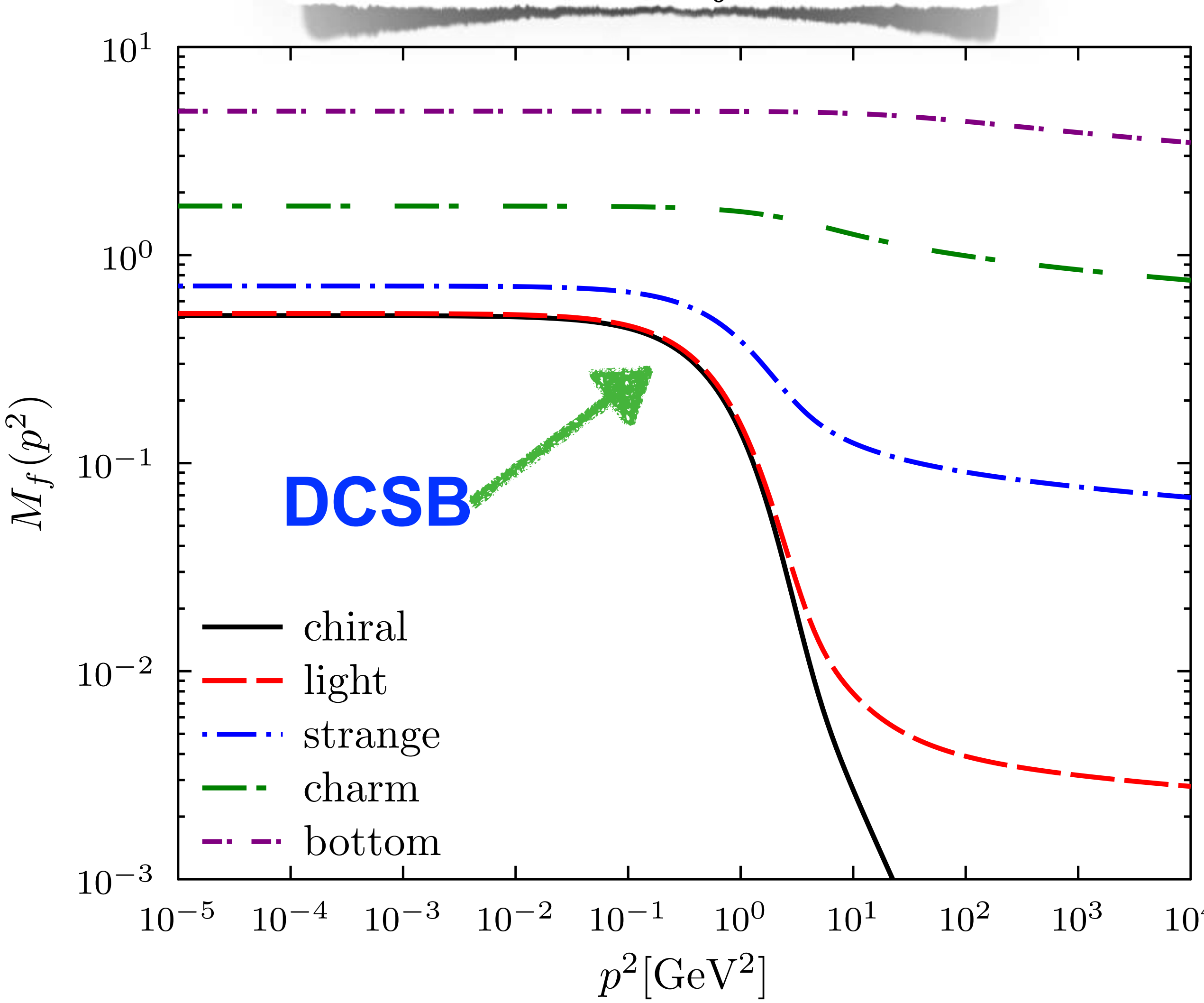
P. Maris and P. C. Tandy, Phys. Rev. C 60, 055214 (1999)
S. X. Qin, L. Chang, Y. X. Liu, C. D. Roberts and D. J. Wilson, Phys. Rev. C 84, 042202 (2011)

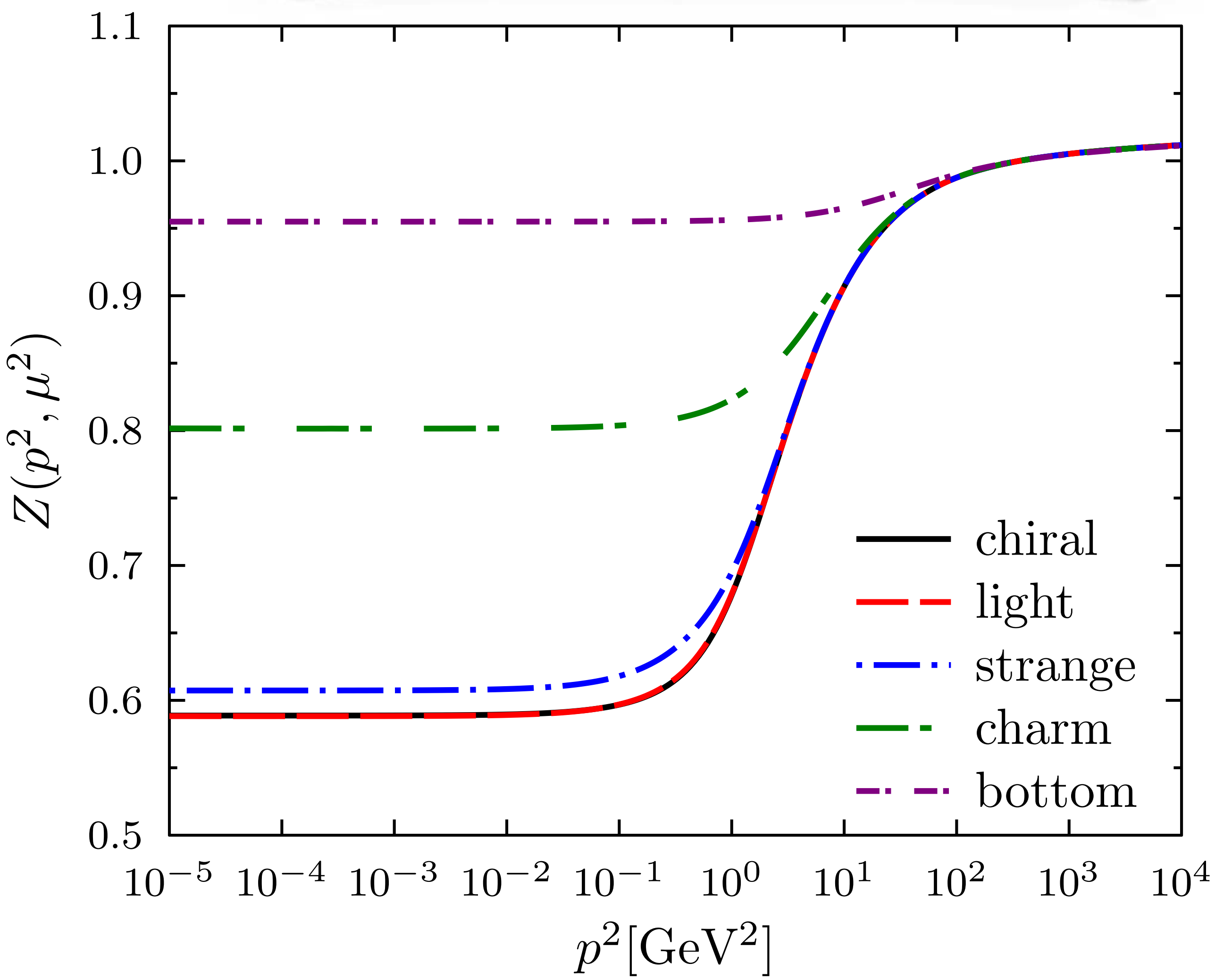
Numerical solutions: Quark DSE

REAL AXIS SOLUTION

$$M_f(p^2) = \frac{B_f(p^2,\mu^2)}{A_f(p^2,\mu^2)}$$

$$Z_f(p^2,\mu^2) = 1/A_f(p^2,\mu^2)$$

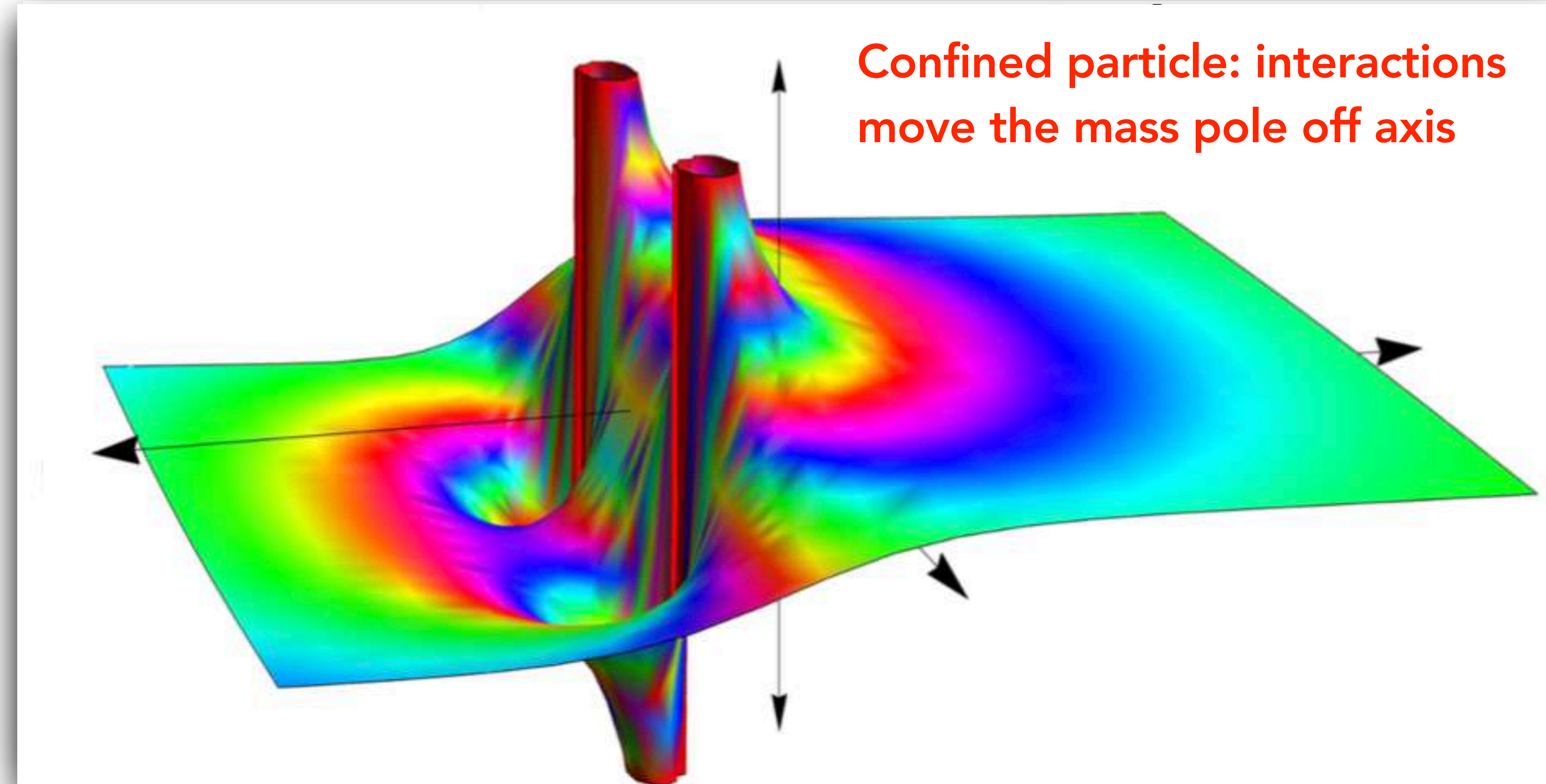




Quark Propagator: Dynamical chiral symmetry breaking generates 'constituent-quark masses'.

Numerical solutions: Quark DSE

- Dramatic change in the in the analytic structure of the quark propagator:



Complex conjugate singularities characterized by a dynamically generated mass scale.

Numerical solutions: Quark DSE

Quark Propagator: In order to obtain bound-states masses, we need to know the quark propagator in a parabolic region in the complex plane.

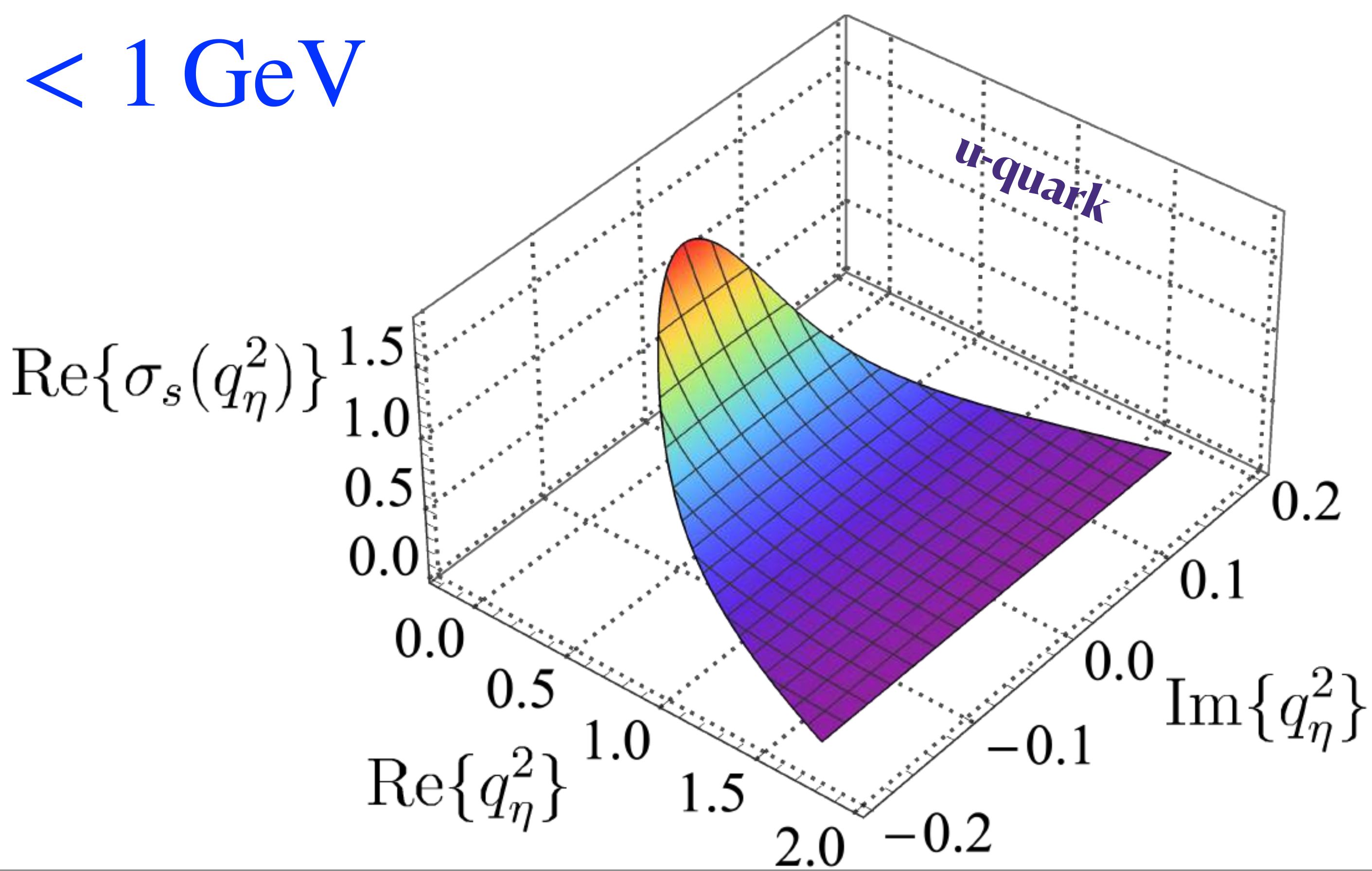
COMPLEX PLANE SOLUTION

$$S(q_\eta) = -i\gamma \cdot q_\eta \, \sigma_{\rm v}(q_\eta^2) + \sigma_{\rm s}(q_\eta^2)$$
$$q_\eta^2 = q^2 - \eta^2 m_M^2 + 2i\eta \, m_M |q| z_q$$

$$\sigma_{\rm v}^f(q_\eta^2) = \frac{A_f(q_\eta^2)}{q_\eta^2 A_f^2(q_\eta^2) + B_f^2(q_\eta^2)}$$
$$\sigma_{\rm s}^f(q_\eta^2) = \frac{B_f(q_\eta^2)}{q_\eta^2 A_f^2(q_\eta^2) + B_f^2(q_\eta^2)}$$

- **Light meson:** For instant, the pion with $\eta = 1/2$.

$$m_M = m_\pi < 1\,\text{GeV}$$



Numerical solutions: Quark DSE

Quark Propagator: In order to obtain bound-states masses, we need to know the quark propagator in a parabolic region in the complex plane.

COMPLEX PLANE SOLUTION

$$S(q_\eta) = -i\gamma \cdot q_\eta \, \sigma_{\rm v}(q_\eta^2) + \sigma_{\rm s}(q_\eta^2)$$
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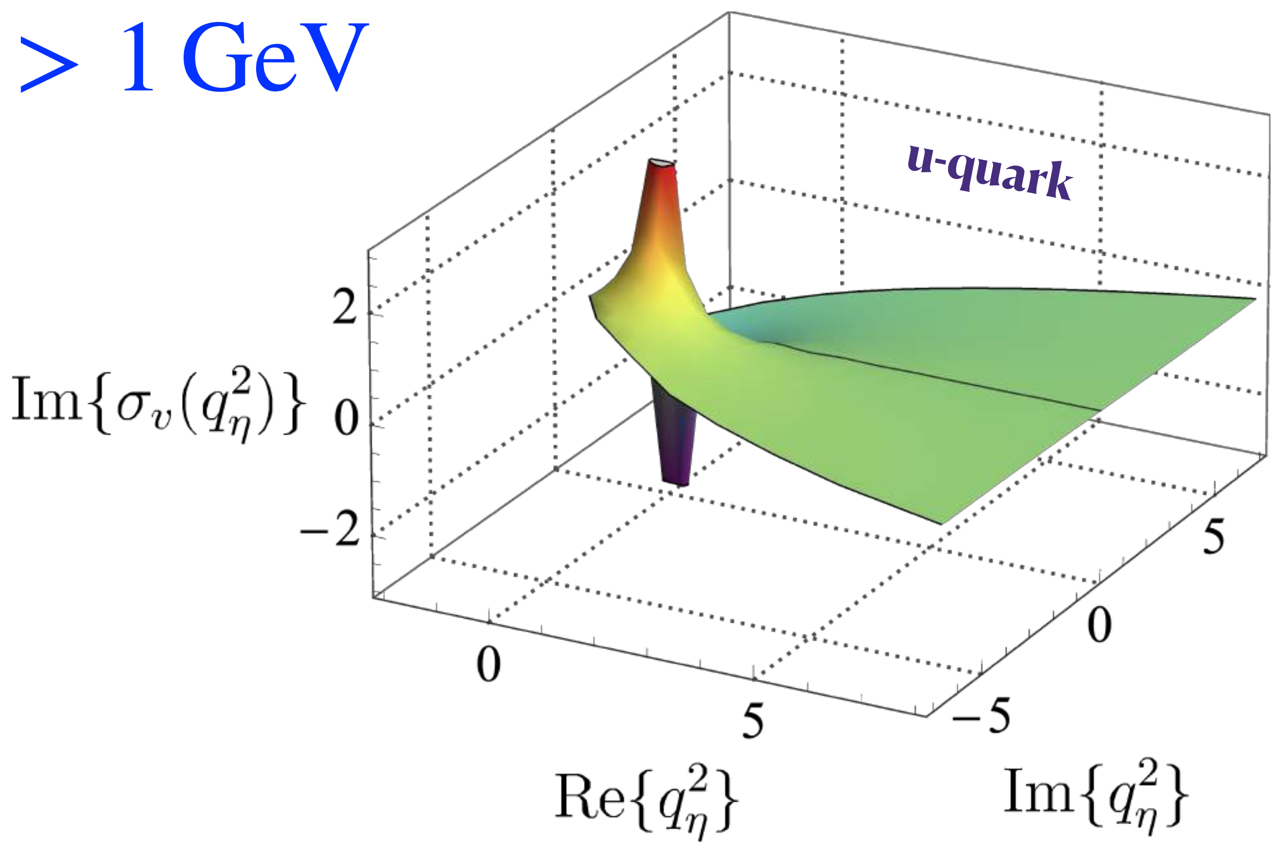
$$\sigma_{\rm s}^f(q_\eta^2) = \frac{B_f(q_\eta^2)}{q_\eta^2 A_f^2(q_\eta^2) + B_f^2(q_\eta^2)}$$

- **Heavy meson:** For instant, the D meson with ($\eta = 0.5$

$$m_M = m_D > 1\,\text{GeV}$$



Numerical solutions: Quark DSE

Quark Propagator: In order to obtain bound-states masses, we need to know the quark propagator in a parabolic region in the complex plane.

COMPLEX PLANE SOLUTION

$$S(q_\eta) = -i\gamma \cdot q_\eta \, \sigma_{\rm v}(q_\eta^2) + \sigma_{\rm s}(q_\eta^2)$$

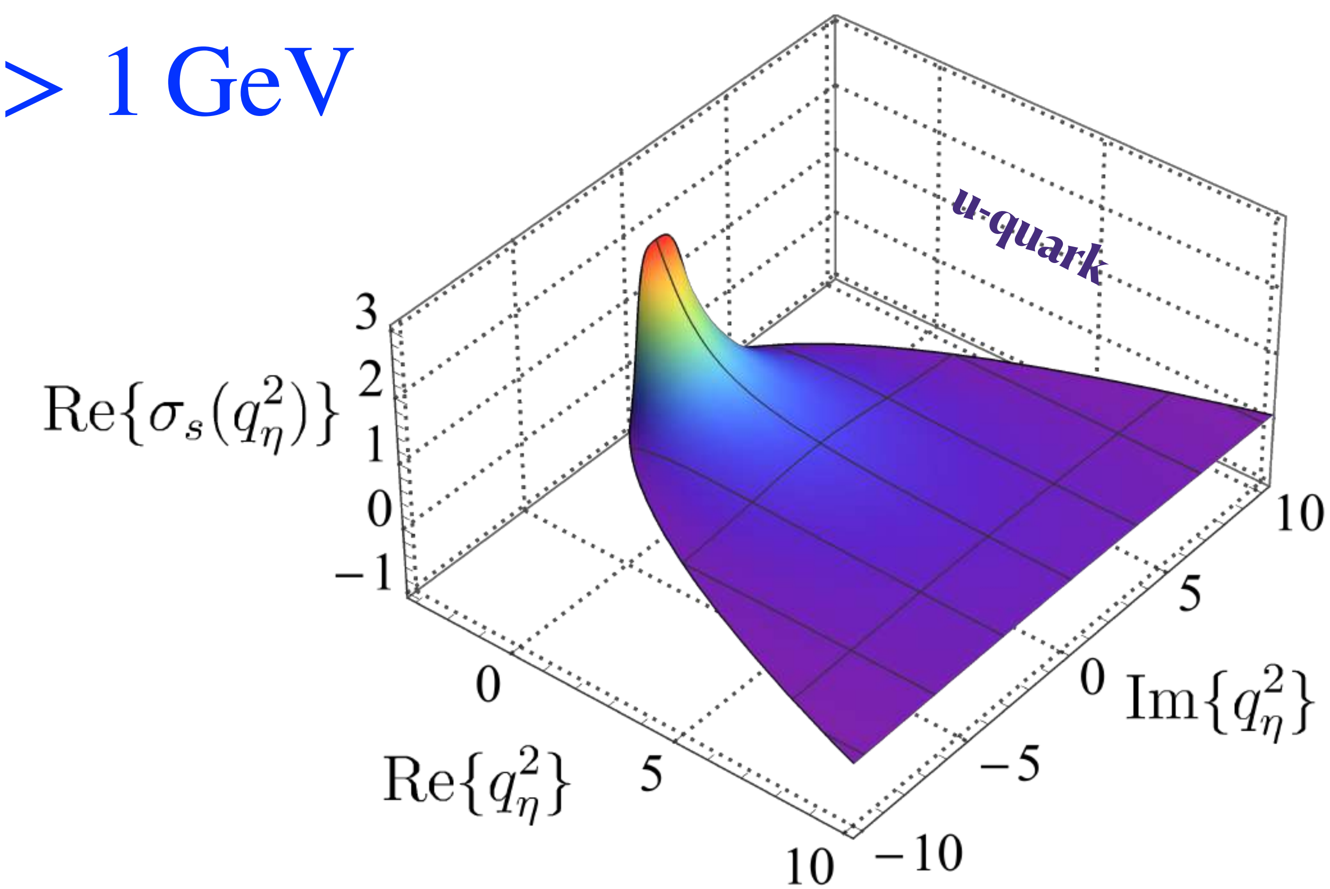
$$q_\eta^2 = q^2 - \eta^2 m_M^2 + 2i\eta \, m_M |q| z_q$$

$$\sigma_{\rm v}^f(q_\eta^2) = \frac{A_f(q_\eta^2)}{q_\eta^2 A_f^2(q_\eta^2) + B_f^2(q_\eta^2)}$$

$$\sigma_{\rm s}^f(q_\eta^2) = \frac{B_f(q_\eta^2)}{q_\eta^2 A_f^2(q_\eta^2) + B_f^2(q_\eta^2)}$$

- **Heavy meson:** For instant, the D meson with $\eta = 0.2$

$$m_M = m_D > 1\,\mathrm{GeV}$$



Numerical solutions: BSE

For the bound state masses we solve an artificial eigenvalue problem:

$$\lambda_n(P^2)\Gamma_M^{fg}(k,P) = \int^{\Lambda} \frac{d^4q}{(2\pi)^4} K_{fg}(k,q;P) S_f(q_\eta)\Gamma_M^{fg}(q,P)S_g(q_{\bar\eta})$$

$$\Gamma_M(k,P) = \sum_{i=1}^{N} T_M^i(k,P) F_i(k^2, z_k, P^2), \qquad z_k = k\cdot P/|k||P$$

Chebyschev polynomials

$$\mathcal{F}_i(k,P) = \sum_{m=0}^{\infty} {}^m\mathcal{F}_i(k^2,P^2)U_m(z_k)$$

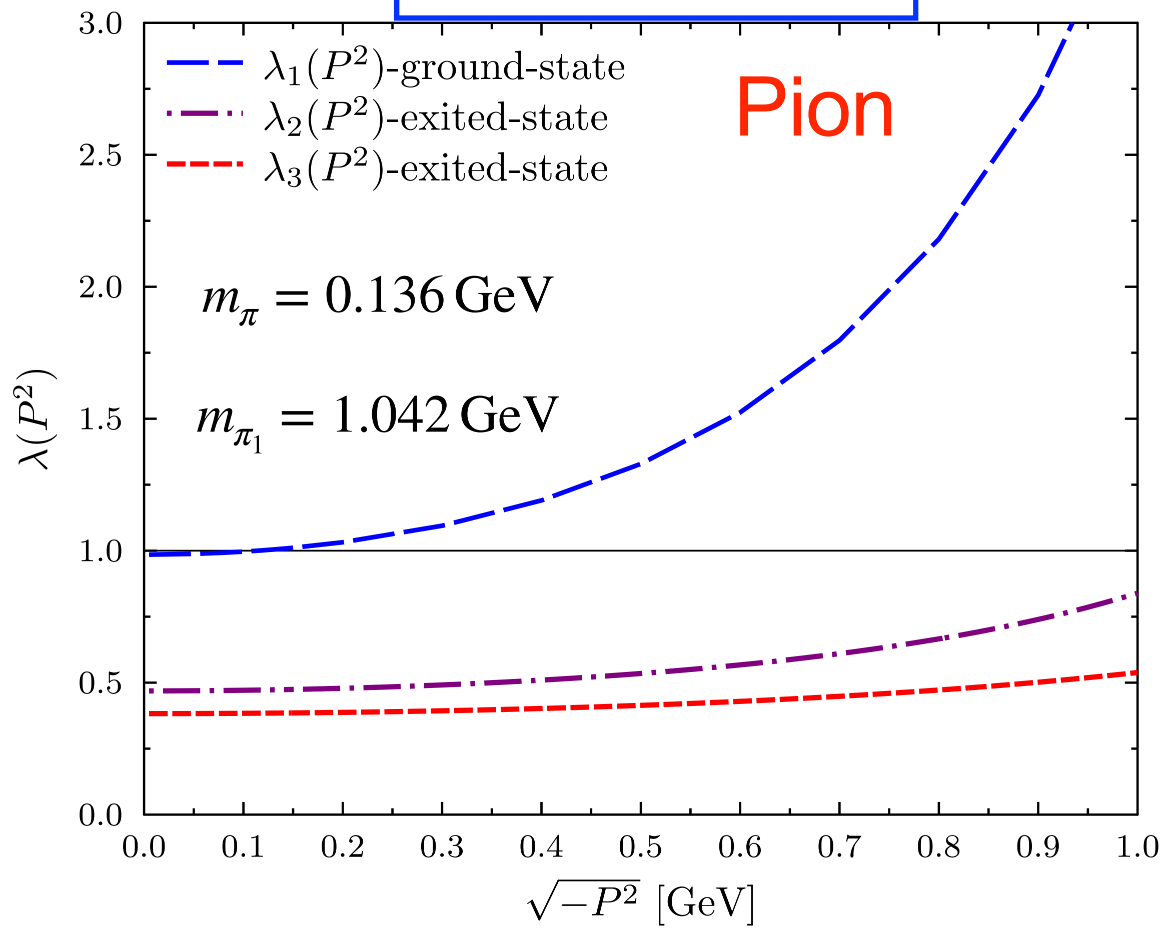
Chebyschev's Moments

$$\frac{2}{\pi}\int_0^1 \sqrt{1-x^2}\,U_m(x)U_n(x) = \delta_{mn}$$

$$\lambda(P^2)\,{}^m\mathcal{F}^{\alpha i}(P) = (\mathbf{K}^{\alpha}_{\beta})^{mi}_{n\,j}(P)\,{}^n\mathcal{F}^{\beta j}(P)$$

Basically to obtain the meson mass we have to find the root of $\lambda(P^2) - 1 = 0$.

Eigenvalue trajectory



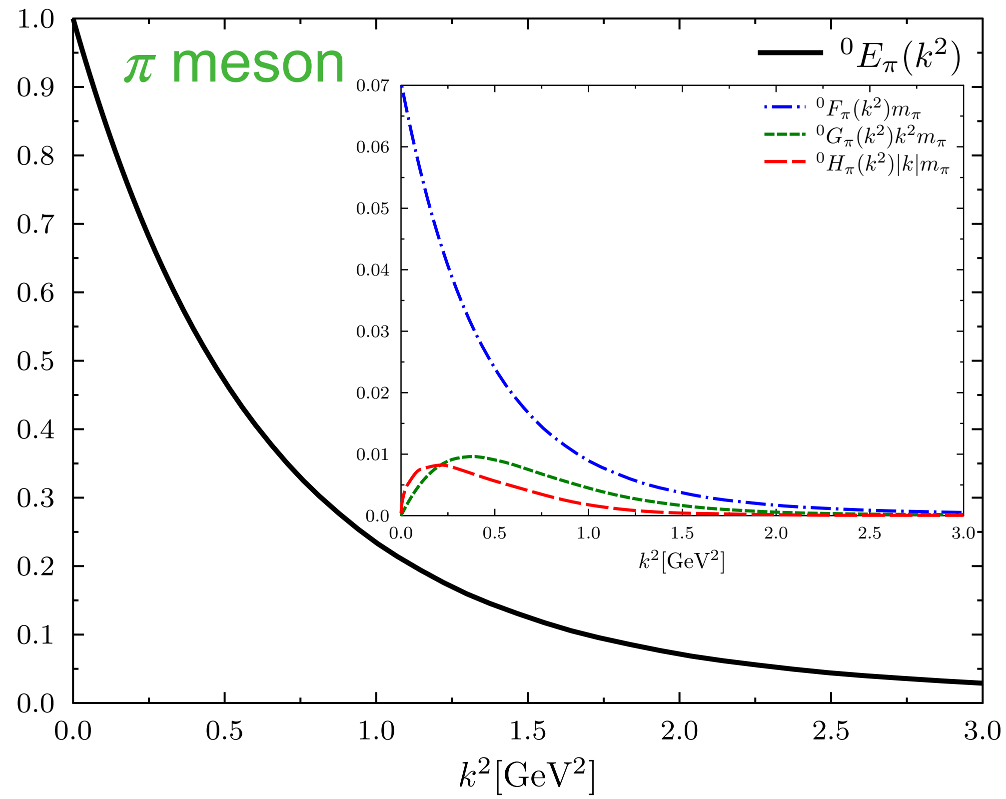
Pseudoscalar mesons

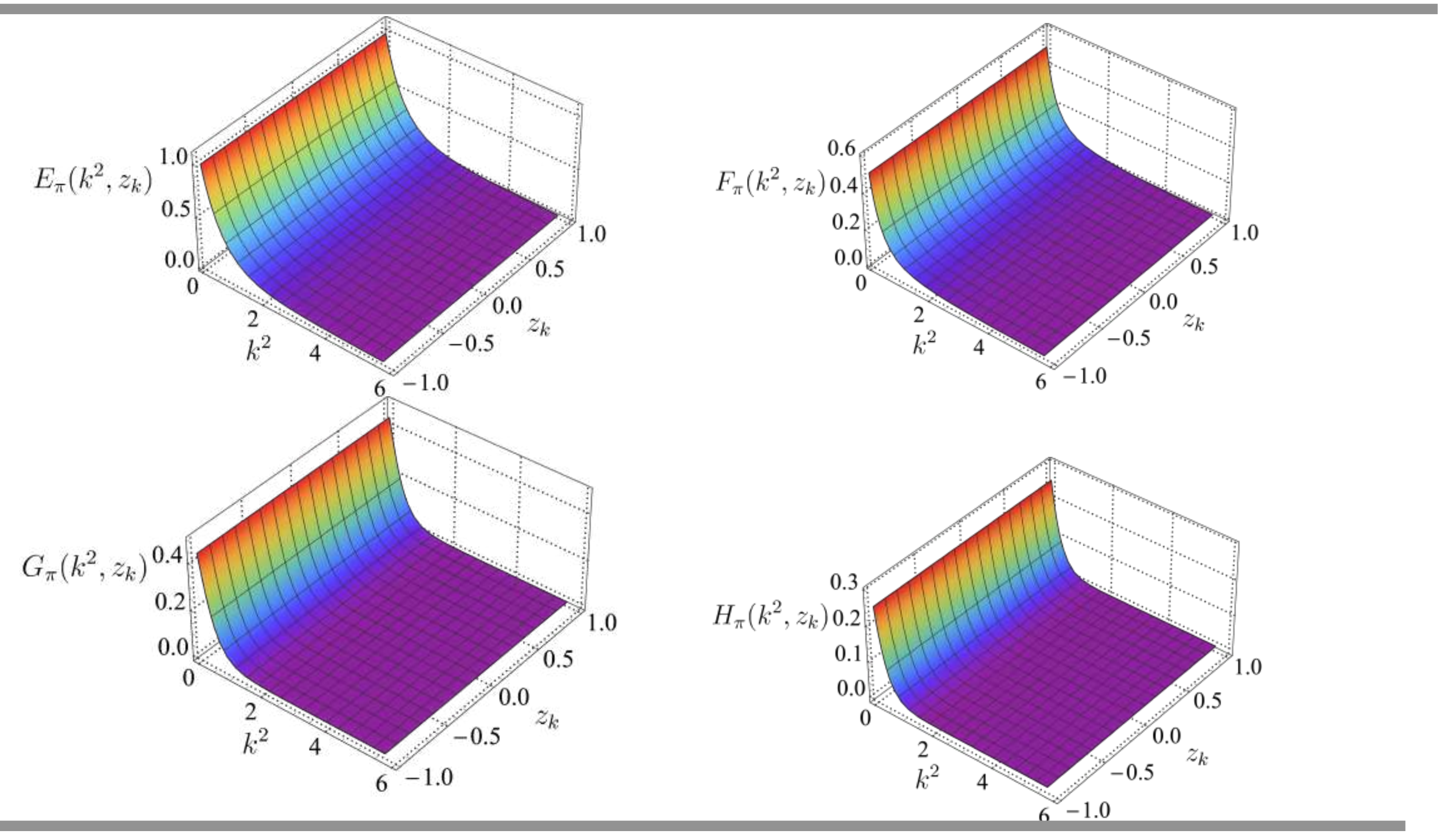
	M_P	M_P^{exp}	ϵ_{M_P} [%]	f_P	$f_P^{\text{exp/lQCD}}$	ϵ_{f_P} [%]
$\pi(u\bar{d})$	0.140	0.138	1.45	$0.094^{+0.001}_{-0.001}$	0.092(1)	2.17
$K(u\bar{s})$	0.494	0.494	0	$0.110^{+0.001}_{-0.001}$	0.110(2)	0
$D(c\bar{d})$	$1.867^{+0.008}_{-0.004}$	1.864	0.11	$0.144^{+0.001}_{-0.001}$	0.150 (0.5)	4.00
$D_s(c\bar{s})$	$2.015^{+0.021}_{-0.018}$	1.968	2.39	$0.179^{+0.004}_{-0.003}$	0.177(0.4)	1.13
$\eta_c(c\bar{c})$	$3.012^{+0.003}_{-0.039}$	2.984	0.94	$0.270^{+0.002}_{-0.005}$	0.279(17)	3.23
$\eta_b(b\bar{b})$	$9.392^{+0.005}_{-0.004}$	9.398	0.06	$0.491^{+0.009}_{-0.009}$	0.472(4)	4.03
$B(u\bar{b})$	$5.277^{+0.008}_{-0.005}$	5.279	0.04	$0.132^{+0.004}_{-0.002}$	0.134(1)	4.35
$B_s(s\bar{b})$	$5.383^{+0.037}_{-0.039}$	5.367	0.30	$0.128^{+0.002}_{-0.003}$	0.162(1)	20.5
$B_c(c\bar{b})$	$6.282^{+0.020}_{-0.024}$	6.274	0.13	$0.280^{+0.005}_{-0.002}$	0.302(2)	10.17

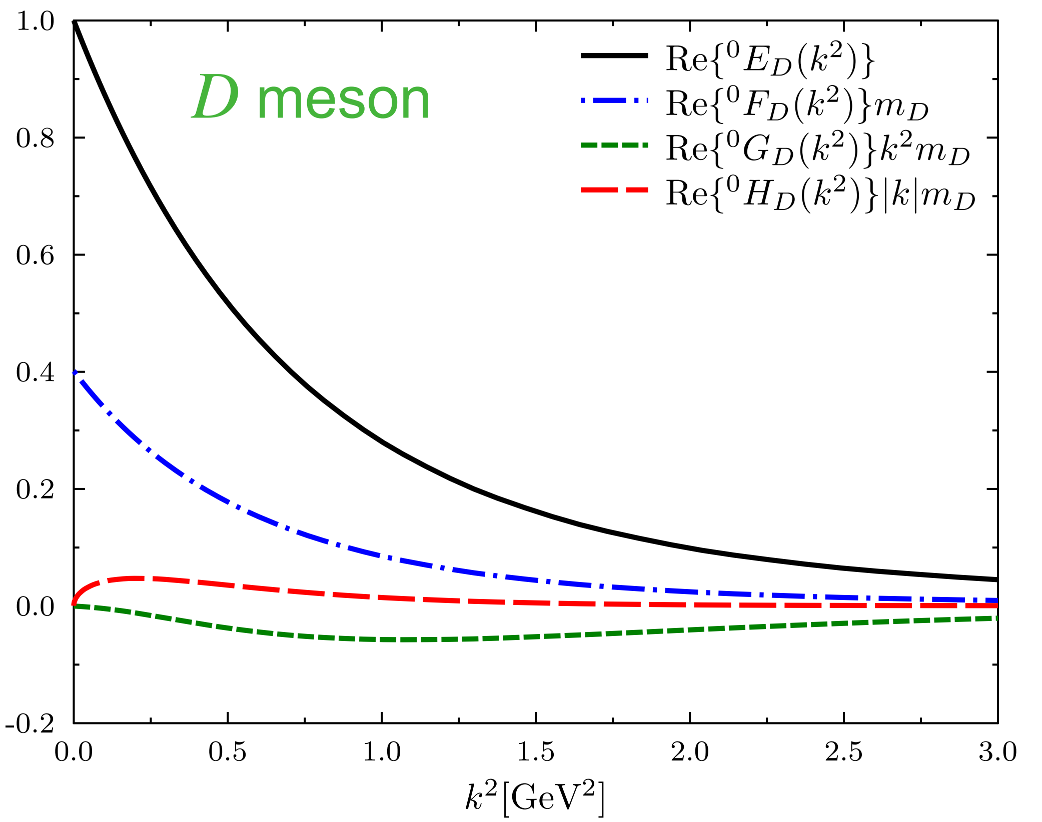
$$f_P P_\mu = \frac{\mathcal{Z}_2 N_c}{\sqrt{2}} \int^\Lambda \frac{d^4k}{(2\pi)^4} \mathrm{Tr_D}\left[i\gamma_5\gamma_\mu S_f(k_\eta)\Gamma_P^{fg}(k,P)S_g(k_{\bar{\eta}})\right]$$

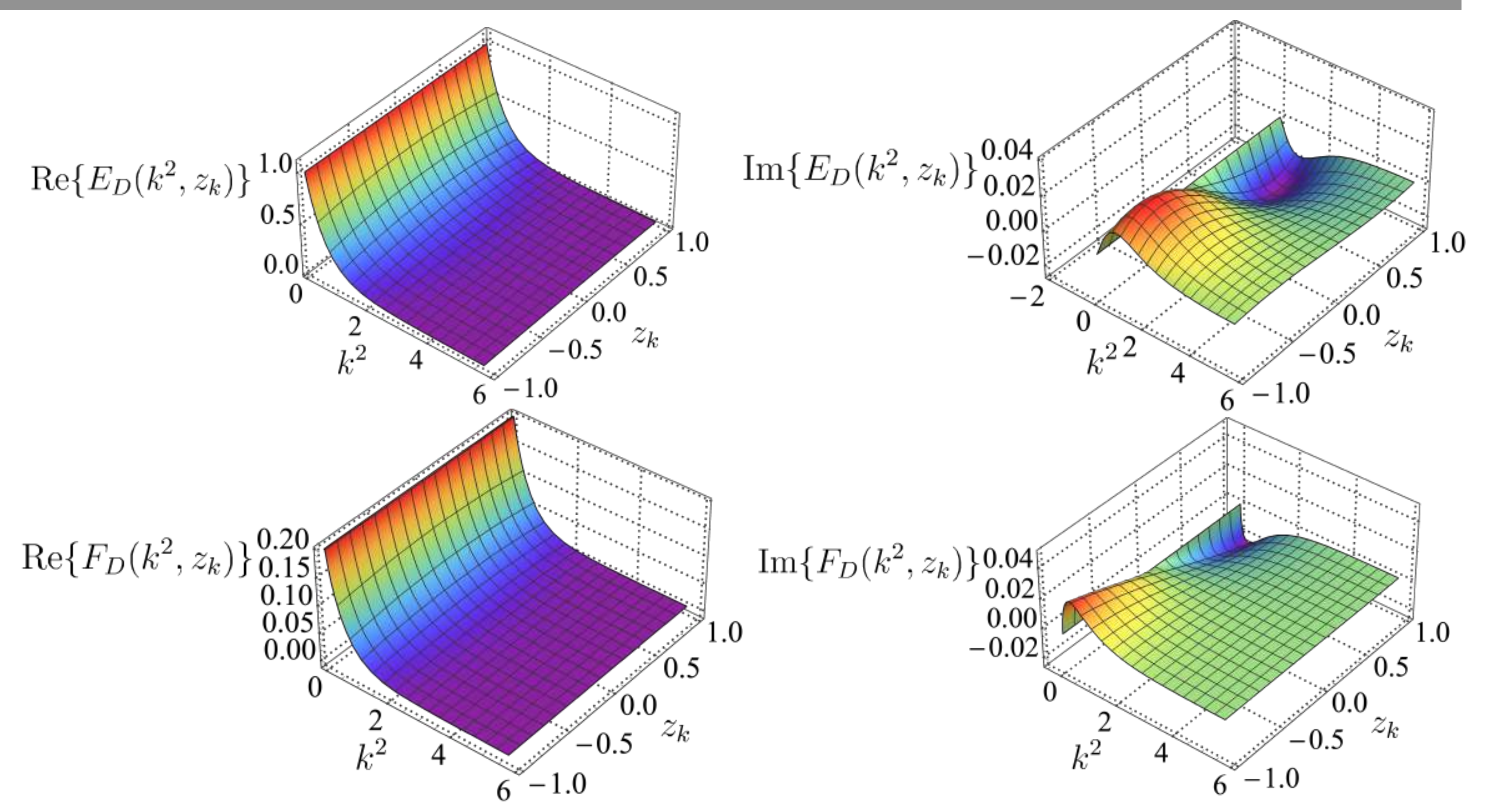
F. E. S, R. C. da Silveira, J. Cobos-Martínez, B. El-Bennich and E. Rojas, Eur. Phys. J. C 80, 955 (2020).

Pseudoscalar mesons









Extracting LCDAs

$$\phi_M(x,\mu)$$

- Projection of BS wave-function onto the light front : $\chi_M(k_\eta, k_{\bar{\eta}}) := S(k_\eta)\Gamma_M(k,P)S(k_{\bar{\eta}})$

$$f_M \phi_M(x,\mu) = \frac{\mathcal{Z}_2 N_c}{\sqrt{2}} \mathrm{Tr}_D \int^\Lambda \frac{d^4k}{(2\pi)^4} \delta(n\cdot k_\eta - x n\cdot P)\gamma_5\,\gamma\cdot n\;\chi_M(k_\eta, k_{\bar{\eta}})$$

Not accessible in Euclidean space

- Computing Mellin moments

$$\langle x^m \rangle = \int_0^1 dx\, x^m \phi_M(x,\mu) \qquad \langle x^0 \rangle = \int_0^1 dx \phi_M(x,\mu) = 1$$

$$\langle x^m \rangle = \frac{\mathcal{Z}_2 N_c}{\sqrt{2} f_M} \mathrm{Tr}_D \int^\Lambda \frac{d^4k}{(2\pi)^4} \frac{(n\cdot k_\eta)^m}{(n\cdot P)^{m+1}}\,\gamma_5\,\gamma\cdot n\;\chi_M(k_\eta, k_{\bar{\eta}})$$

- Interpolation of quark propagator and Bethe-Salpeter amplitudes

$$S_f(q) = \sum_{k=1}^{N}\left[\frac{z_k^f}{i\gamma\cdot q + m_k^f} + \frac{(z_k^f)^*}{i\gamma\cdot q + (m_k^f)^*}\right] \qquad \mathcal{F}_i(k,P) = \sum_{j=0}^{2}\int_{-1}^{1} dz\,\rho_{\nu_j}(z)\frac{\mathcal{U}_j \Lambda_{\mathcal{F}_i}^{2n_j}}{(k^2 + z\,k\cdot P + \Lambda_{\mathcal{F}_i}^2)^{n_j}}$$

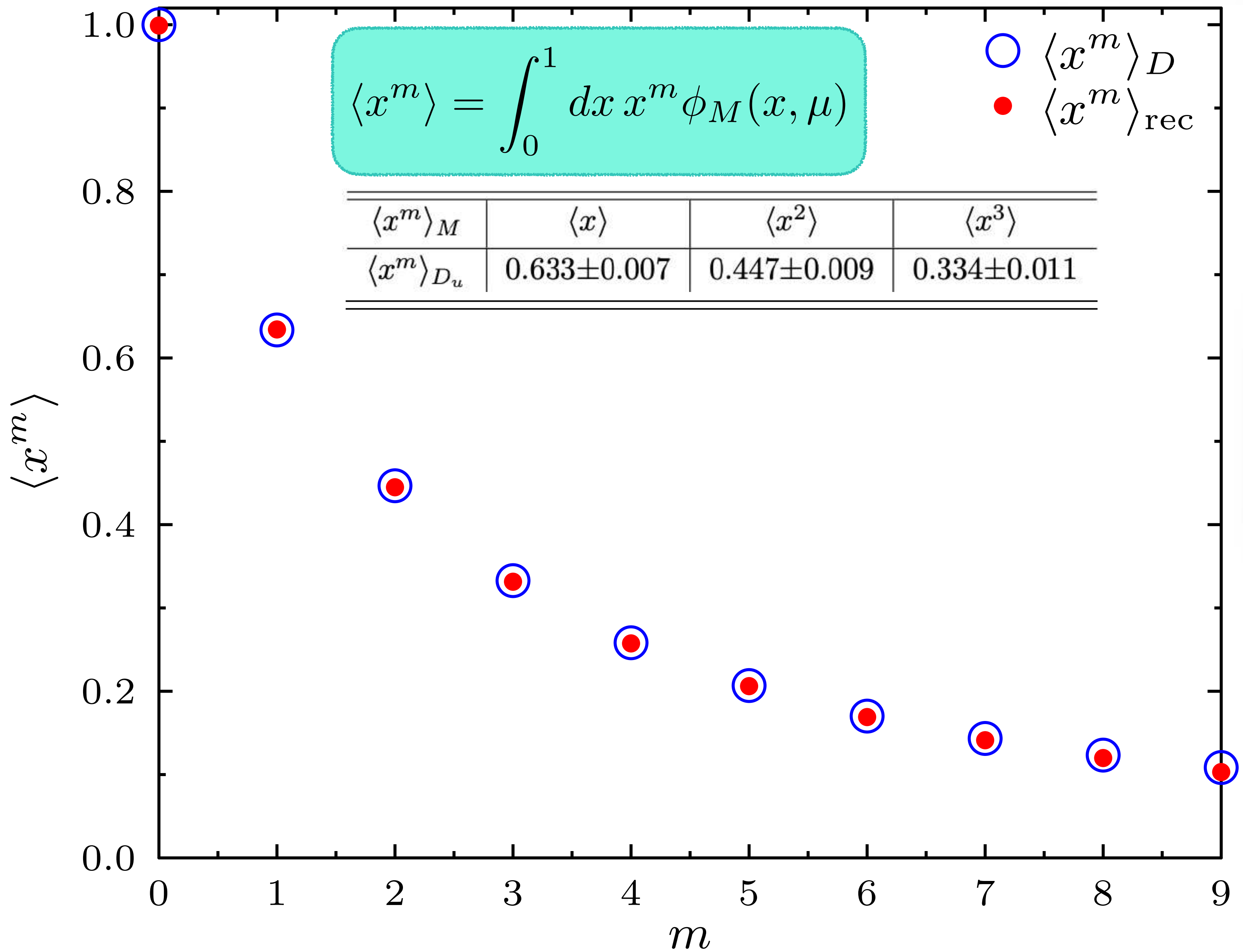
Extracting LCDAs

$$\phi_M(x,\mu)$$

- **Reconstructing LCDAs**

$$\phi_\pi^{\text{rec.}}(x,\mu) = \mathcal{N}(\alpha)\,[x\bar{x}]^{\alpha-1/2}\,[1 + a_2 C_2^{\alpha}(2x-1)]$$

$$\phi_H^{\text{rec.}}(x,\mu) = \mathcal{N}(\alpha,\beta)\,4x\bar{x}\,e^{4\,\alpha x\bar{x}+\beta(x-\bar{x})}$$

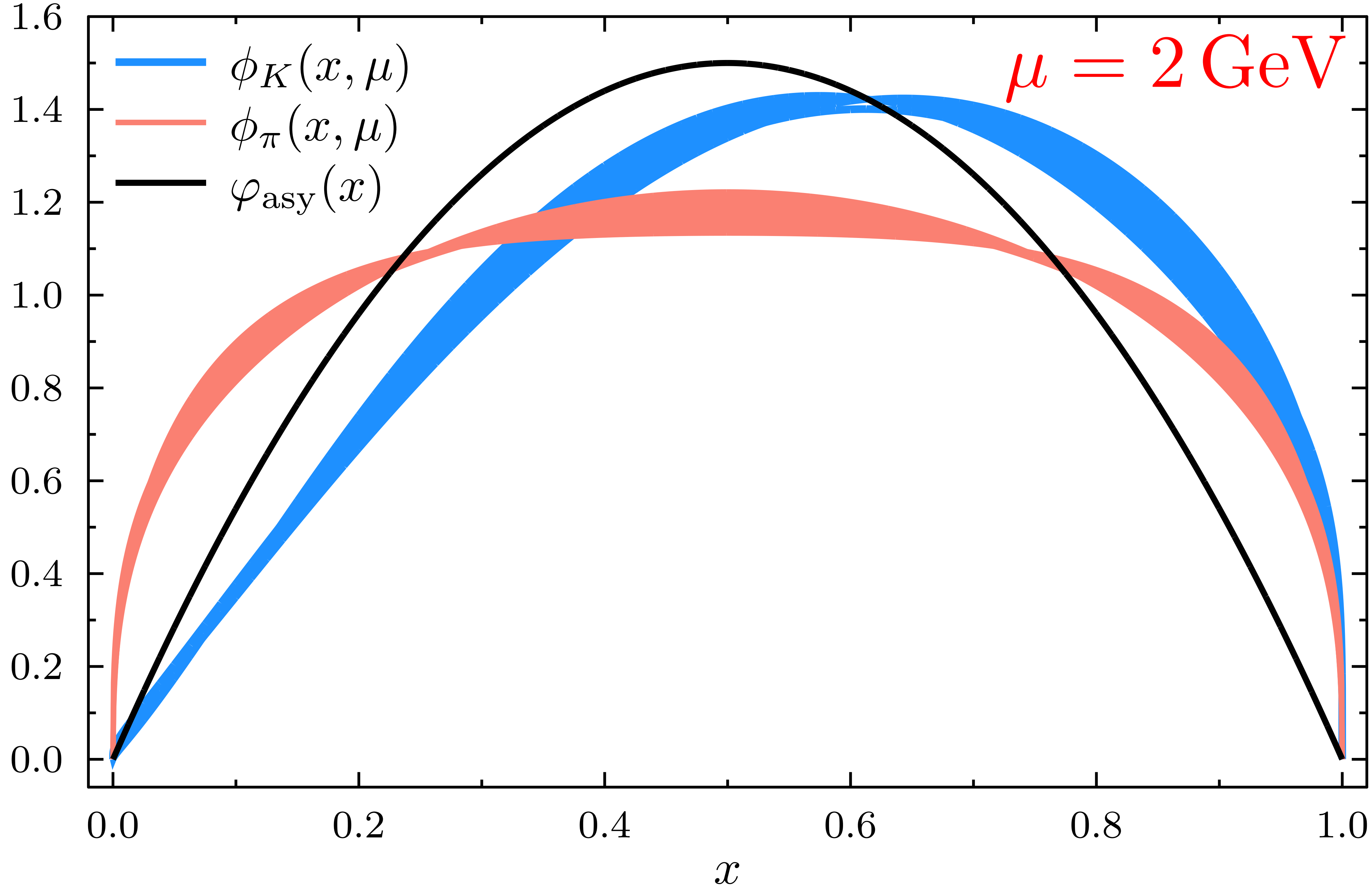


$\langle x^m \rangle_M$	$\langle x \rangle$	$\langle x^2 \rangle$	$\langle x^3 \rangle$
$\langle x^m \rangle_{D_u}$	0.633±0.007	0.447±0.009	0.334±0.011

$$\epsilon(\alpha,\beta) = \sum_{m=1}^{m_{\max}} \left| \frac{\langle x^m \rangle_{\text{rec.}}}{\langle x^m \rangle_H} - 1 \right|$$

Extracting LCDAs

$\phi_M(x,\mu)$



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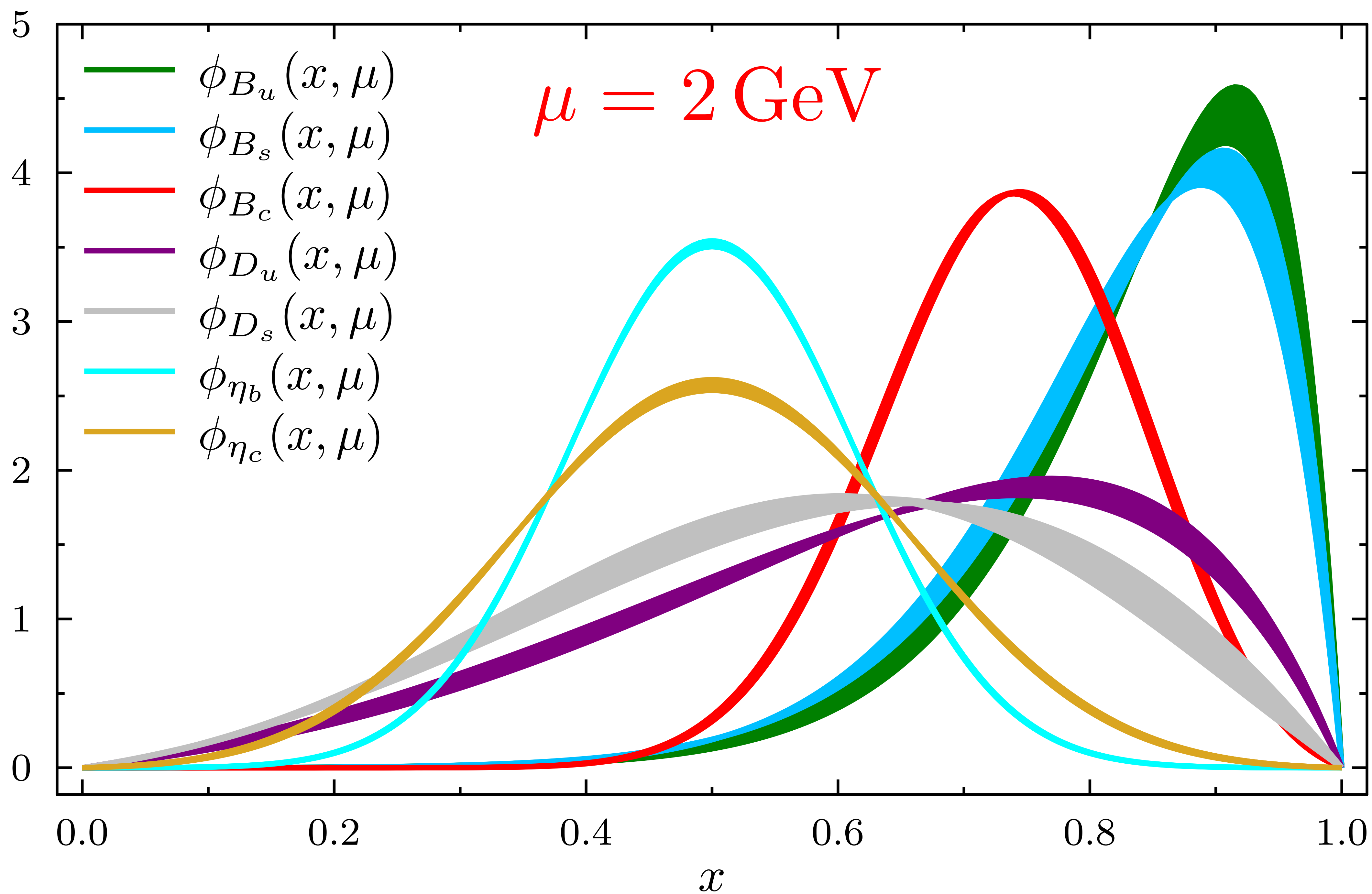
F. E. S, R. C. da Silveira, J. Cobos-Martínez, B. El-Bennich and E. Rojas, Eur. Phys. J. C 80, 955 (2020)

Extracting LCDAs

$\phi_M(x,\mu)$



F. E. S, R. C. da Silveira, J. Cobos-Martínez, B. El-Bennich and E. Rojas, Eur. Phys. J. C 80, 955 (2020)

Extracting LCDAs

LCDAs for vector mesons:

PHYSICAL REVIEW D **106**, L091504 (2022)

Letter

D^* and D_s^* distribution amplitudes from Bethe-Salpeter wave functions

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We report on the first calculation of the longitudinal and transverse light front distribution amplitudes of the D^* and D_s^* mesons and their first four moments. As a by-product, we also obtain these distribution amplitudes for the ρ, ϕ, K^*, and J/Ψ mesons and confirm a prediction of lattice QCD for the vector kaon: while the longitudinal distribution amplitude is almost symmetric, the transverse one is oblique implying that the strange quark carries more momentum.

DOI: 10.1103/PhysRevD.106.L091504

Extracting LCDAs

Bound-state equation for vector mesons

$$\Gamma_\mu(k;P) = \int^\Lambda \frac{d^4q}{(2\pi)^4} K(k,q;P) S_f(q_+) \Gamma_\mu(q;P) S_g(q_-)$$

$$\Gamma_\mu(k;P) = \sum_{i=1}^{8} T^i_\mu(k,P) F_i(k^2, z_k, P^2)$$

$$\begin{aligned}
T^1_\mu(k,P) &= i\gamma^T_\mu, \\
T^2_\mu(k,P) &= i[3k^T_\mu(\gamma\cdot k^T) - \gamma^T_\mu(k^T)^2], \\
T^3_\mu(k,P) &= i(k\cdot P)k^T_\mu \gamma\cdot P, \\
T^4_\mu(k,P) &= i[\gamma^T_\mu \gamma\cdot P(\gamma\cdot k^T) + k^T_\mu \gamma\cdot P],
\end{aligned}$$

$$\begin{aligned}
T^5_\mu(k,P) &= k^T_\mu, \\
T^6_\mu(k,P) &= (k\cdot P)[\gamma^T_\mu(\gamma\cdot k^T) - (\gamma\cdot k^T)\gamma^T_\mu], \\
T^7_\mu(k,P) &= \gamma^T_\mu \gamma\cdot P - \gamma\cdot P \gamma^T_\nu - 2T^8_\mu(k,P), \\
T^8_\mu(k,P) &= \hat{k}^T_\mu(\gamma\cdot\hat{k}^T)\gamma\cdot P,
\end{aligned}$$

where $a^T_\mu = T_{\mu\nu}a_\nu$, with $T_{\mu\nu} = \delta_{\mu\nu} - P_\mu P_\nu/P^2$, $P\cdot a^T = 0$ for any four-vector a_μ and $\hat{k}^T\cdot\hat{k}^T = 1$.

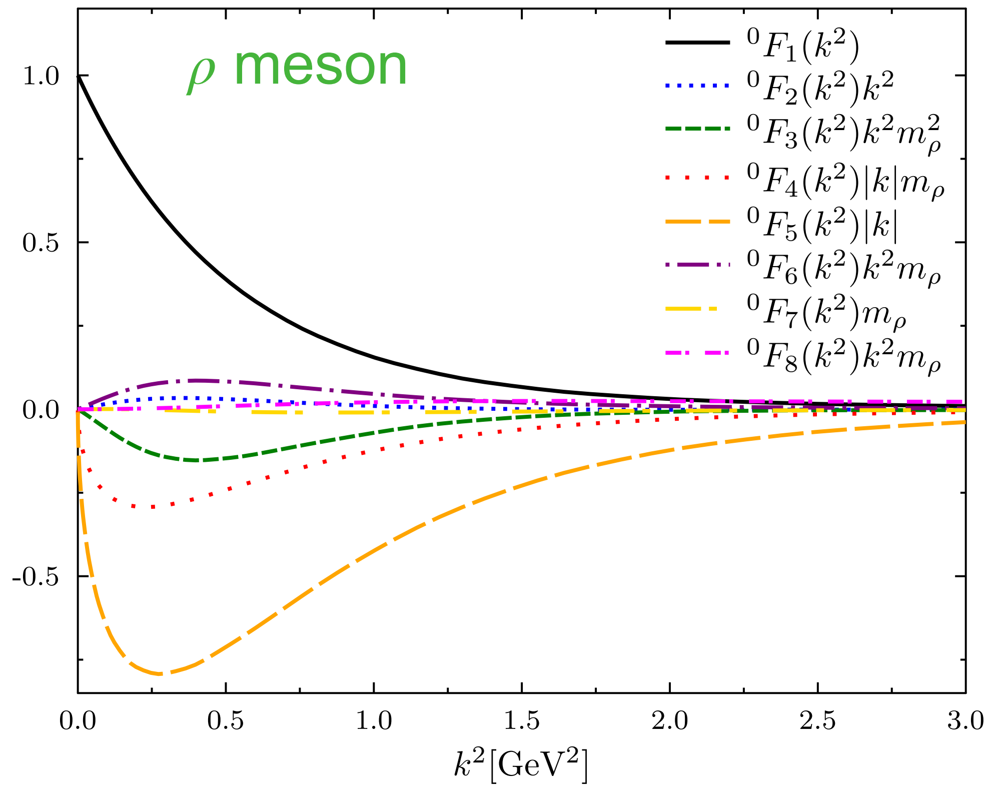
Extracting LCDAs

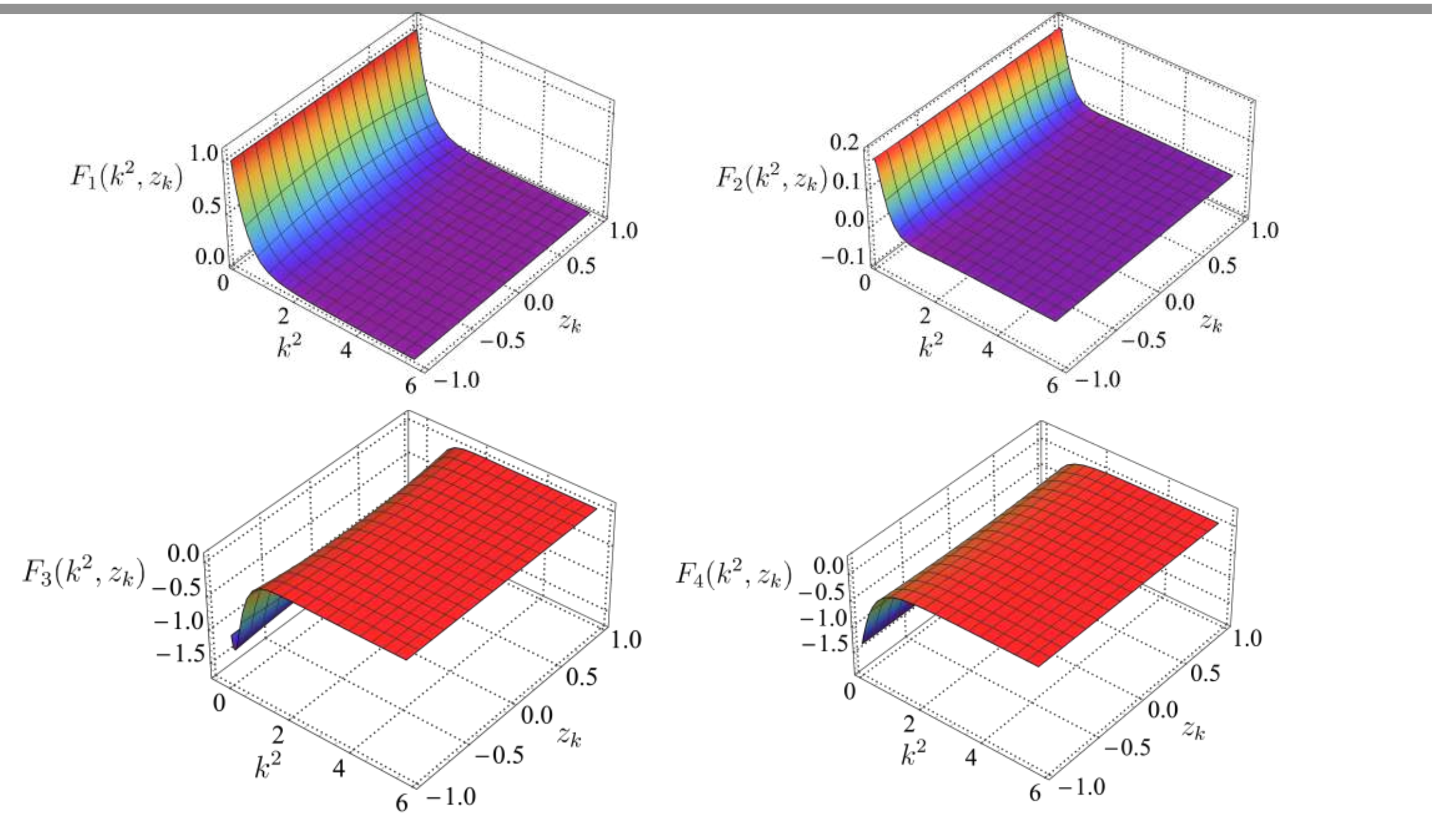
- **Bound-state equation for vector mesons**

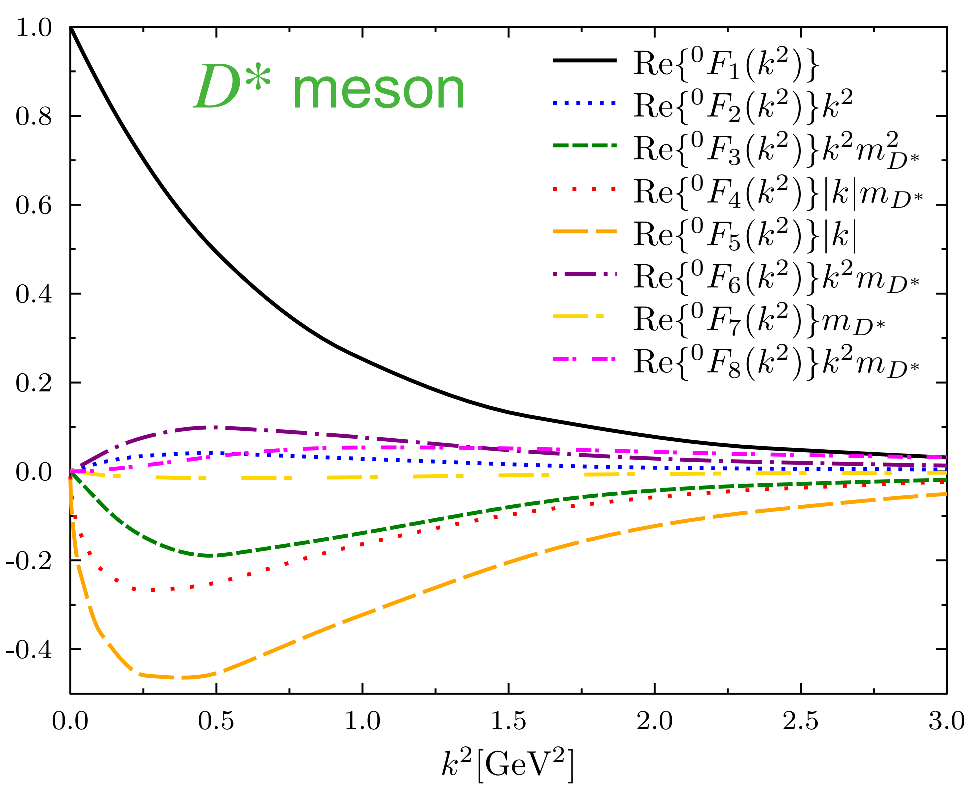
	M_V	$M_V^{\exp}$	ϵ_{M_V} [%]	f_V	$f_V^{\exp/\mathrm{lQCD}}$	ϵ_{f_V} [%]
$\rho(u\bar{u})$	0.730	0.775	5.81	0.145	0.153(1)	5.23
$\phi(s\bar{s})$	1.070	1.019	5.20	0.187	0.168(1)	11.31
$K^*(u\bar{s})$	0.942	0.896	5.13	0.177	0.159(1)	11.32
$D^*(c\bar{d})$	2.021	2.009	0.60	0.165	0.158(6)	4.43
$D_s^*(c\bar{s})$	2.169	2.112	2.70	0.205	0.190(5)	7.90
$J/\psi(c\bar{c})$	3.124	3.097	0.87	0.277	0.294(5)	5.78
$\Upsilon(b\bar{b})$	9.411	9.460	0.52	0.594	0.505(4)	17.62

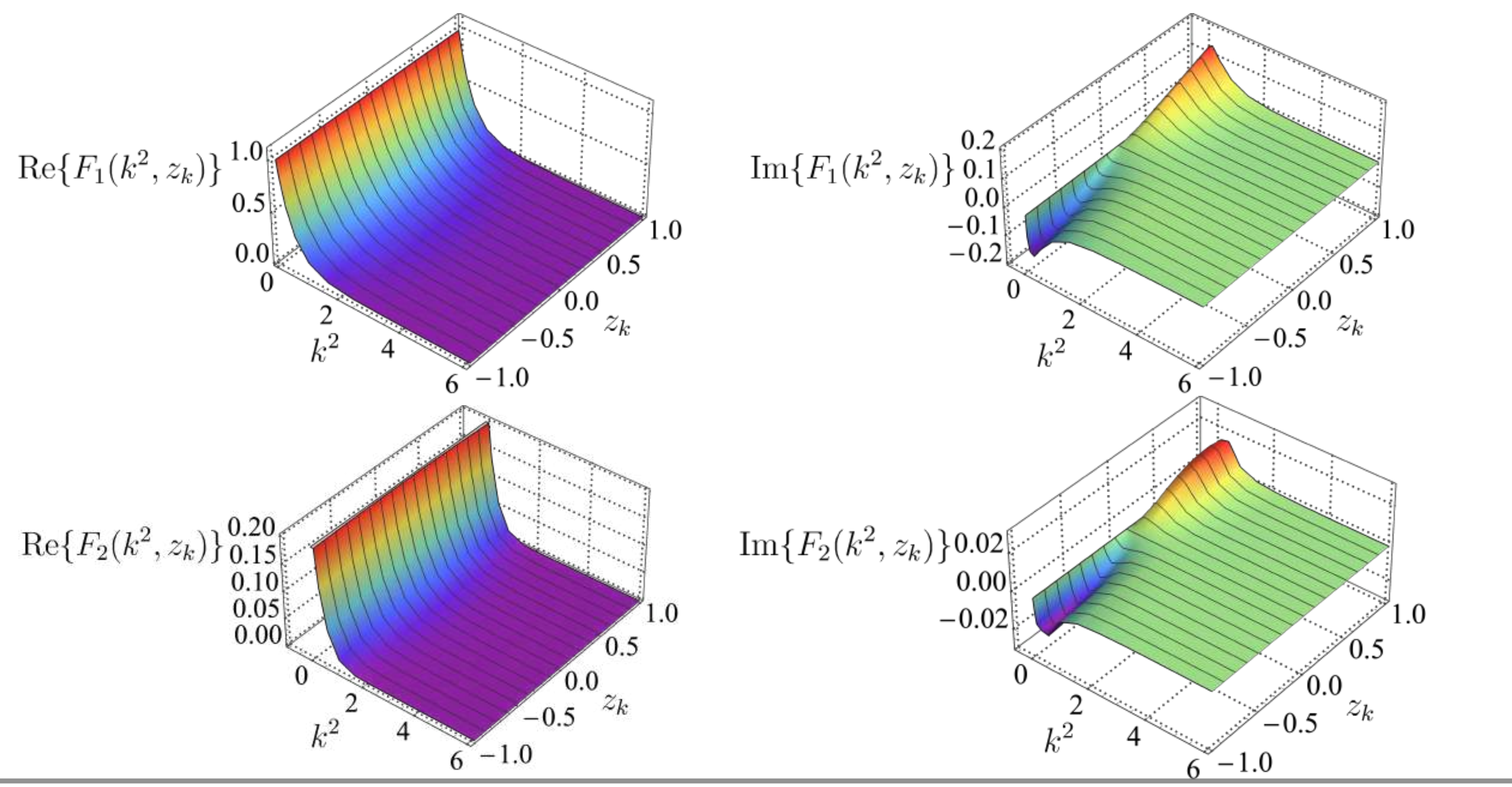
$$f_V M_V = \frac{\mathcal{Z}_2 N_c}{3\sqrt{2}} \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \mathrm{Tr_D}\left[\gamma_\mu S_f(k_\eta)\, \Gamma^{fg}_{V\mu}(k,P)\, S_g(k_{\bar{\eta}})\right]$$

Extracting LCDAs









Extracting LCDAs

LCDAs for vector mesons:

- For a longitudinally polarized vector meson

$$f_{\rm v}(n\cdot P)\phi_{||}(z,\mu) = m_{\rm v}{\rm Tr}_{\rm CD} Z_2 \int_q \delta(n\cdot q_+ - xn\cdot P)\gamma\cdot n n_\nu \chi_\nu(q;P)$$

- For a transversely polarized vector meson

$$f_{\rm v}^\perp m_{\rm v}^2 \phi_\perp(z,\mu) = (n\cdot P){\rm Tr}_{\rm CD} Z_T \int_q \delta(n\cdot q_+ - xn\cdot P)\sigma_{\mu\nu}P_\mu \chi_\nu(q;P)$$

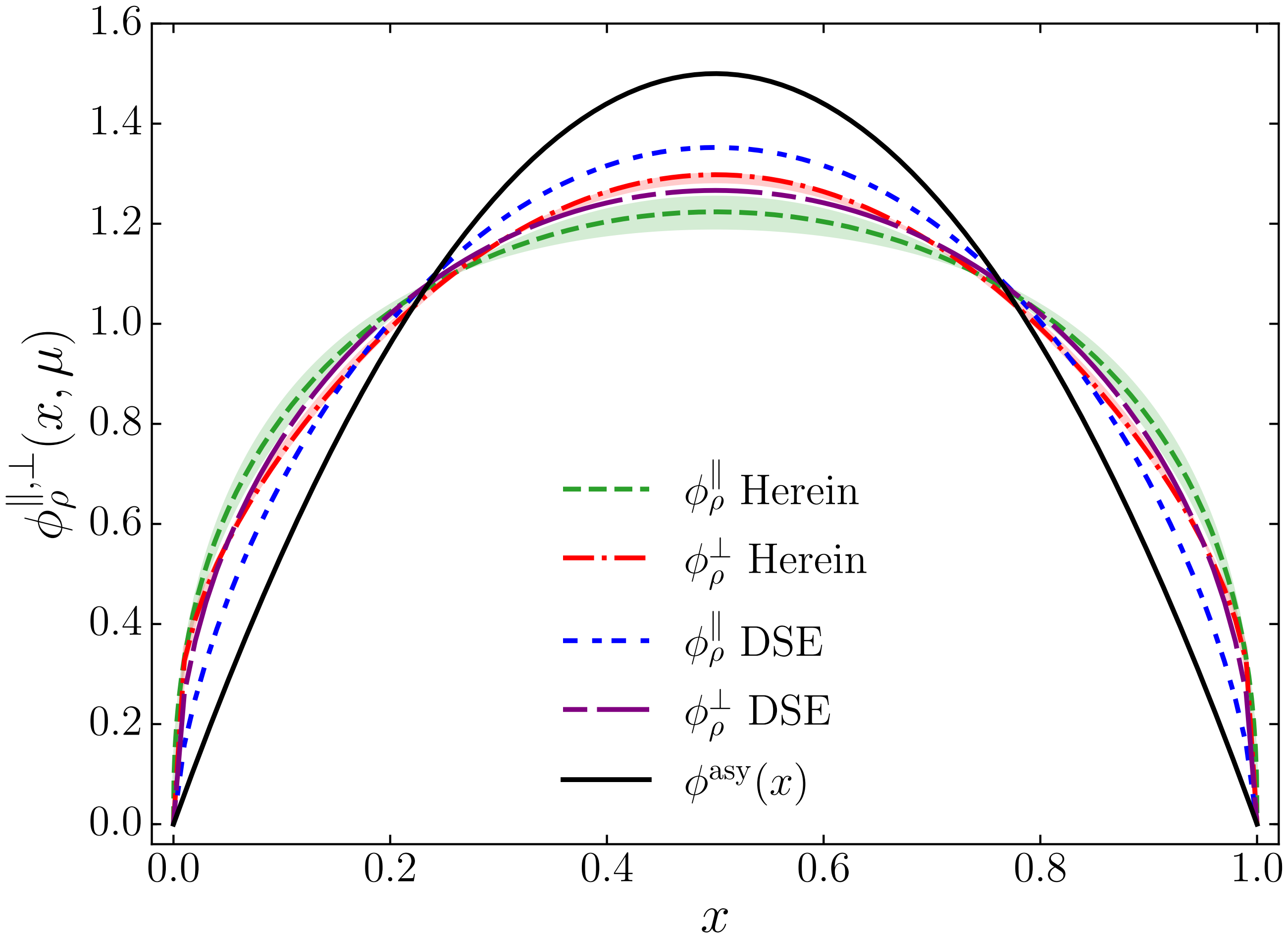
$$\chi_\nu(q;P) = S_f(q_+)\Gamma_\nu(q;P)S_g(q_-),$$

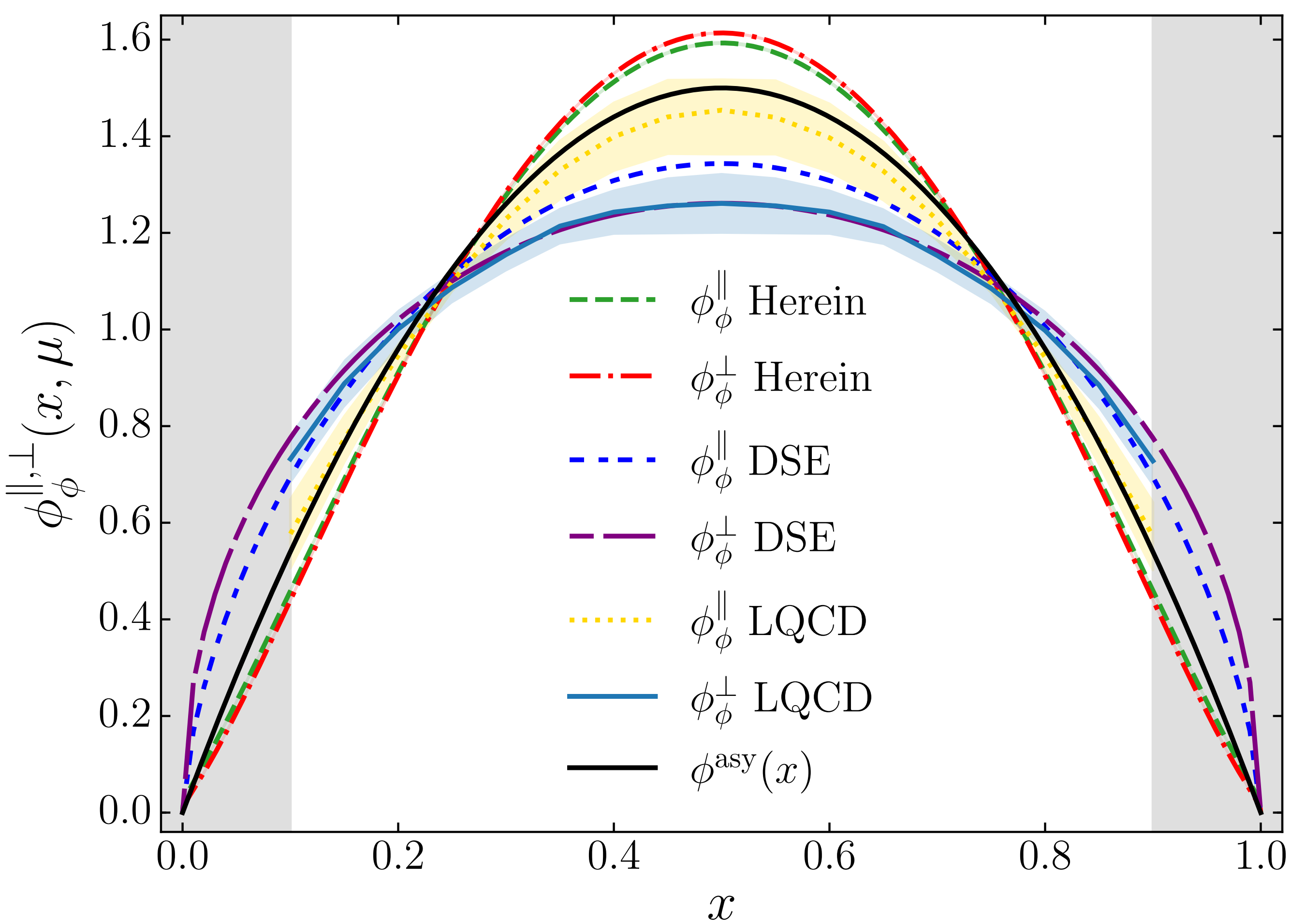
Extracting LCDAs

$\phi_M(x,\mu)$

Longitudinal and transverse LCDAs (ρ and ϕ mesons)

$$\phi^{\|,\perp}_{V\text{rec}}(x,\mu) = \mathcal{N}(\alpha)[x\bar{x}]^{\alpha-\frac{1}{2}}\left[1+\sum_{n=1}^{N} a_n C_n^{\alpha}(2x-1)\right]$$





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Lattice QCD: Lattice Parton Collaboration, PRL 127 (2021)

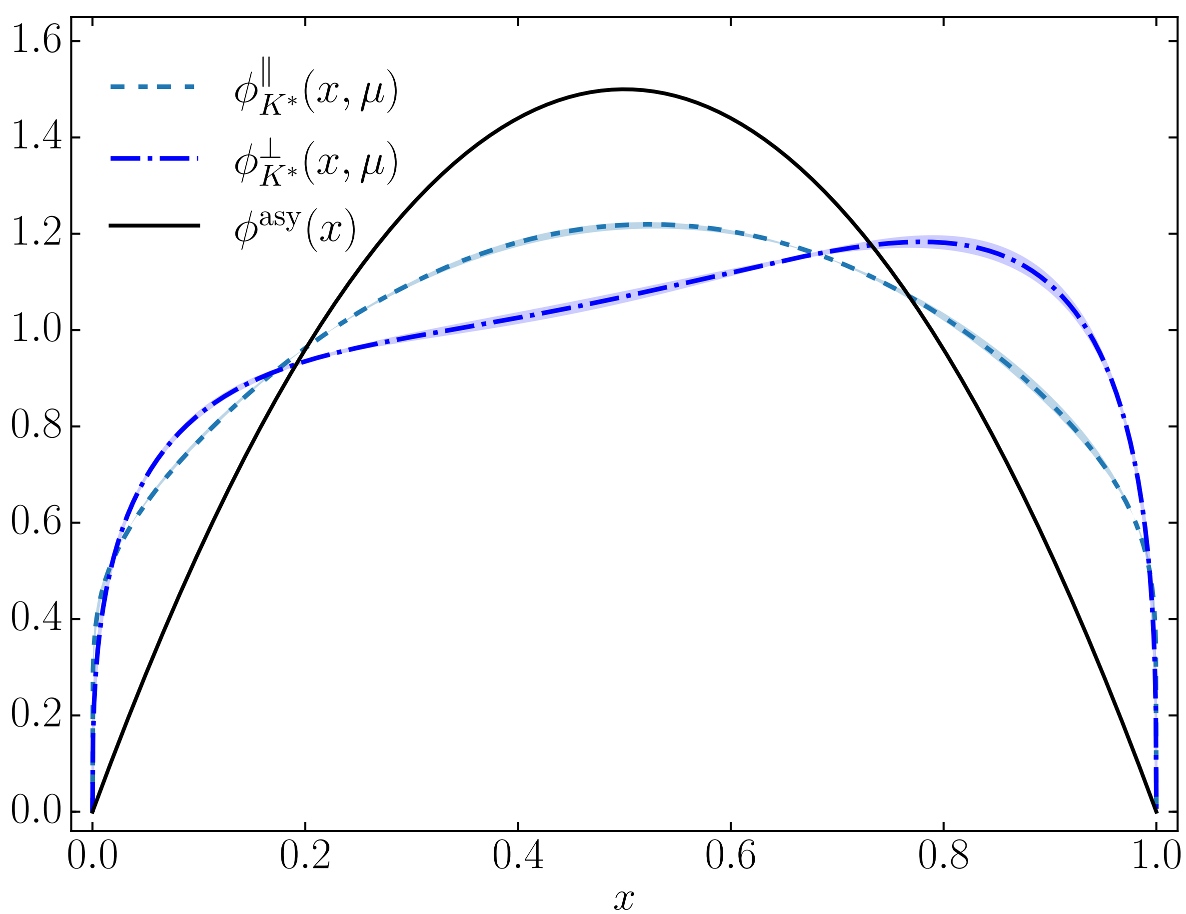
F. E. S, R. C. da Silveira, B. El-Bennich, Phys. Rev. D 106, L091504 (2022).

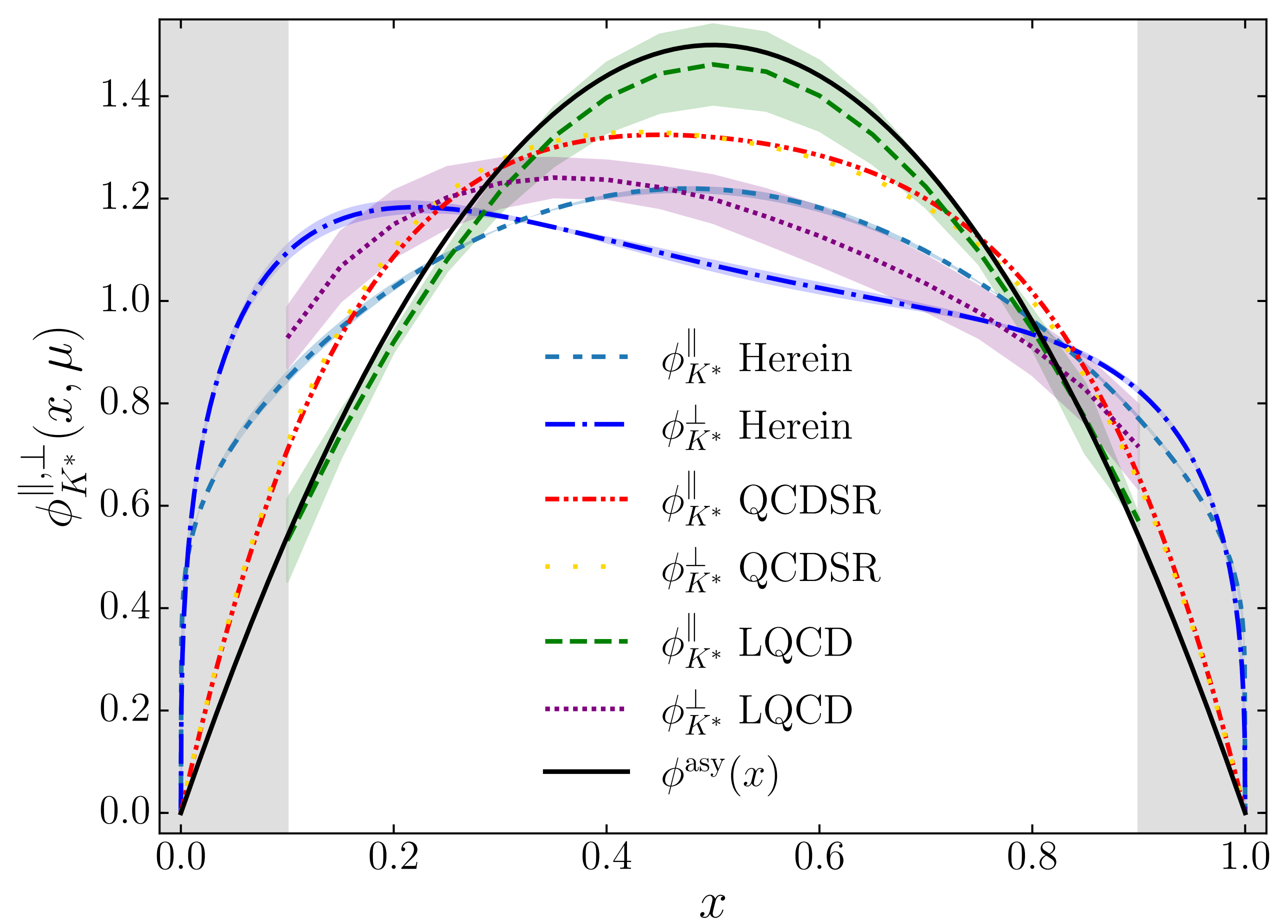
Extracting LCDAs

$$\phi_M(x,\mu)$$

Longitudinal and transverse LCDAs (K^* meson)

$$\phi^{\parallel,\perp}_{V\mathrm{rec}}(x,\mu) = \mathcal{N}(\alpha)[x\bar{x}]^{\alpha-\frac{1}{2}}\left[1+\sum_{n=1}^{N} a_n C_n^{\alpha}(2x-1)\right]$$





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Lattice QCD: Lattice Parton Collaboration, PRL 127 (2021)

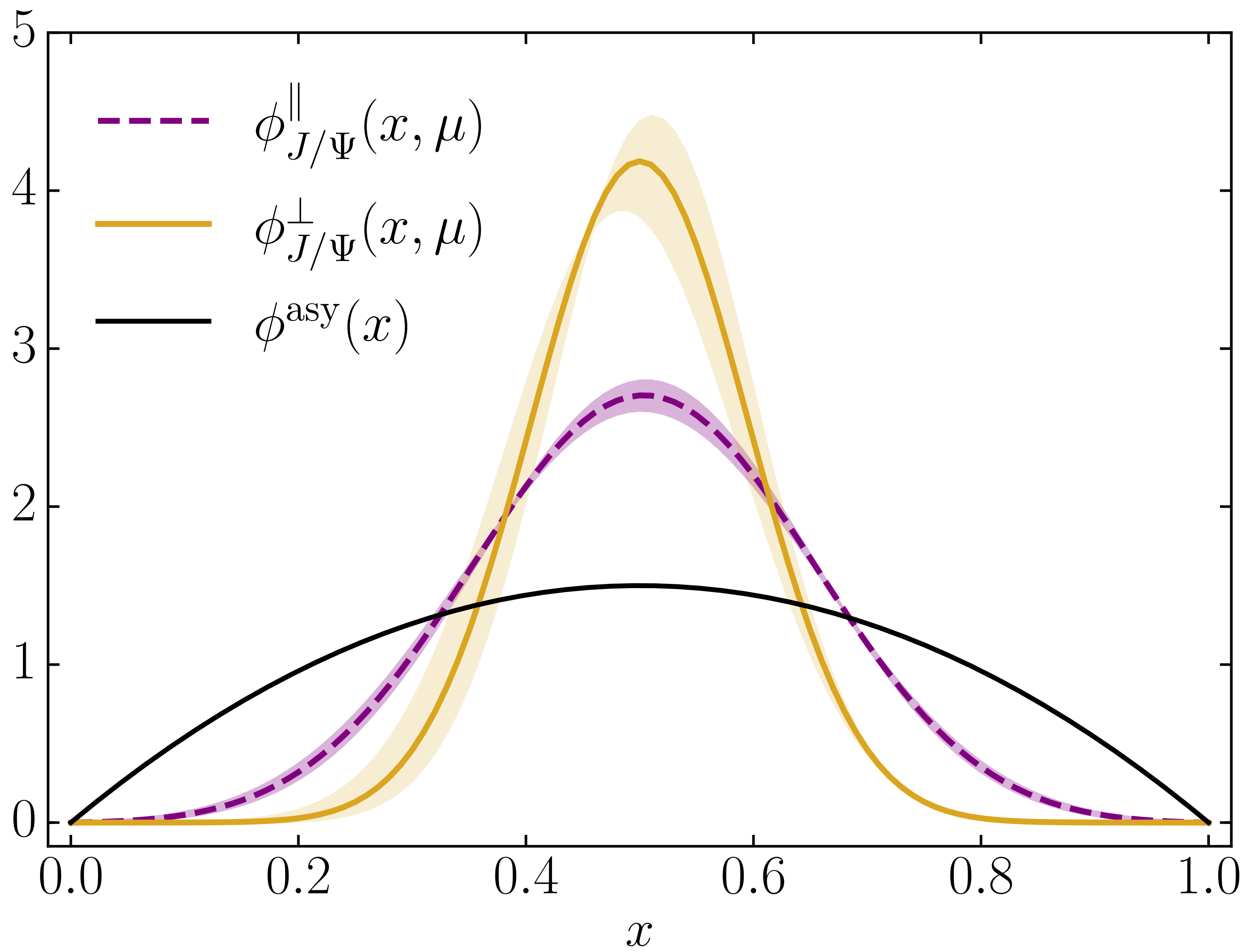
QCD Sum Rule: P. Ball, V. M. Braun and A. Lenz, JHEP 08 (2007)

Extracting LCDAs

$$\phi_M(x,\mu)$$

Longitudinal and transverse LCDAs

$$\phi^{\|,\perp}_{V\text{rec}}(x,\mu) = \mathcal{N}(\alpha,\beta)4x\bar{x}e^{4\alpha x\bar{x}+\beta(x-\bar{x})}$$



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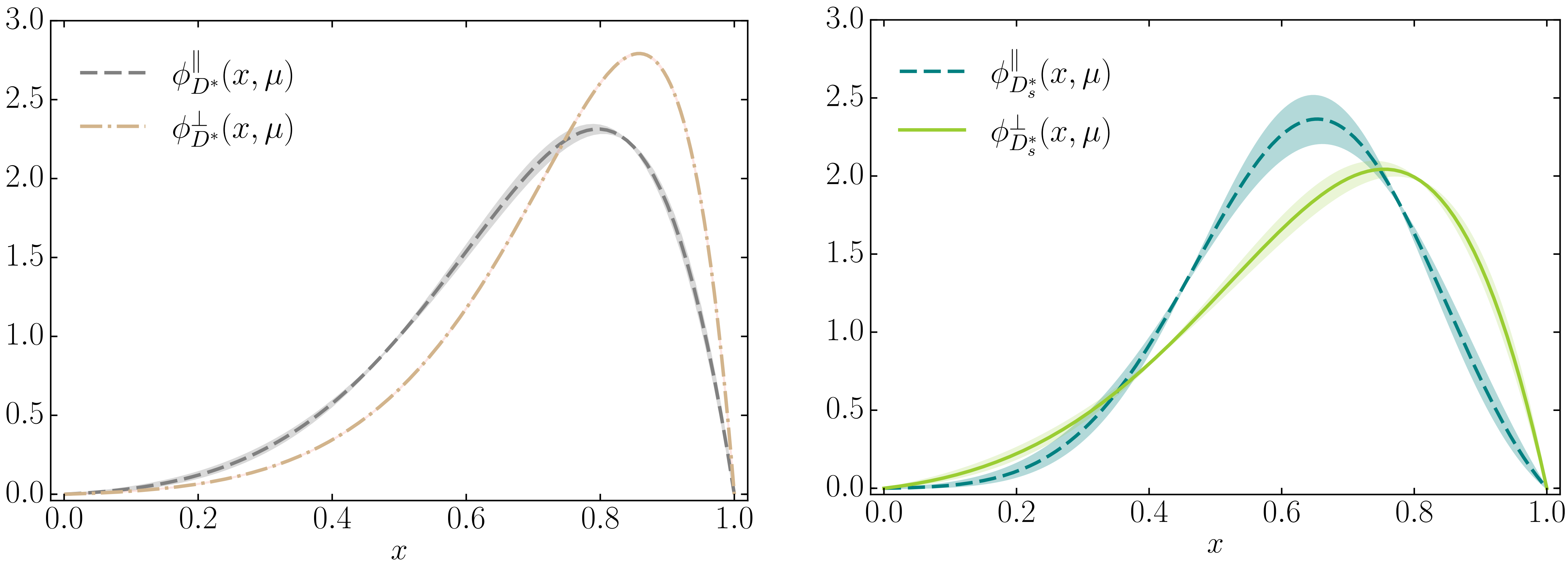
Extracting LCDAs

$\phi_M(x,\mu)$

Longitudinal and transverse LCDAs

$$\phi^{\parallel,\perp}_{V\mathrm{rec}}(x,\mu) = \mathcal{N}(\alpha,\beta)4x\bar{x}e^{4\alpha x\bar{x}+\beta(x-\bar{x})}$$



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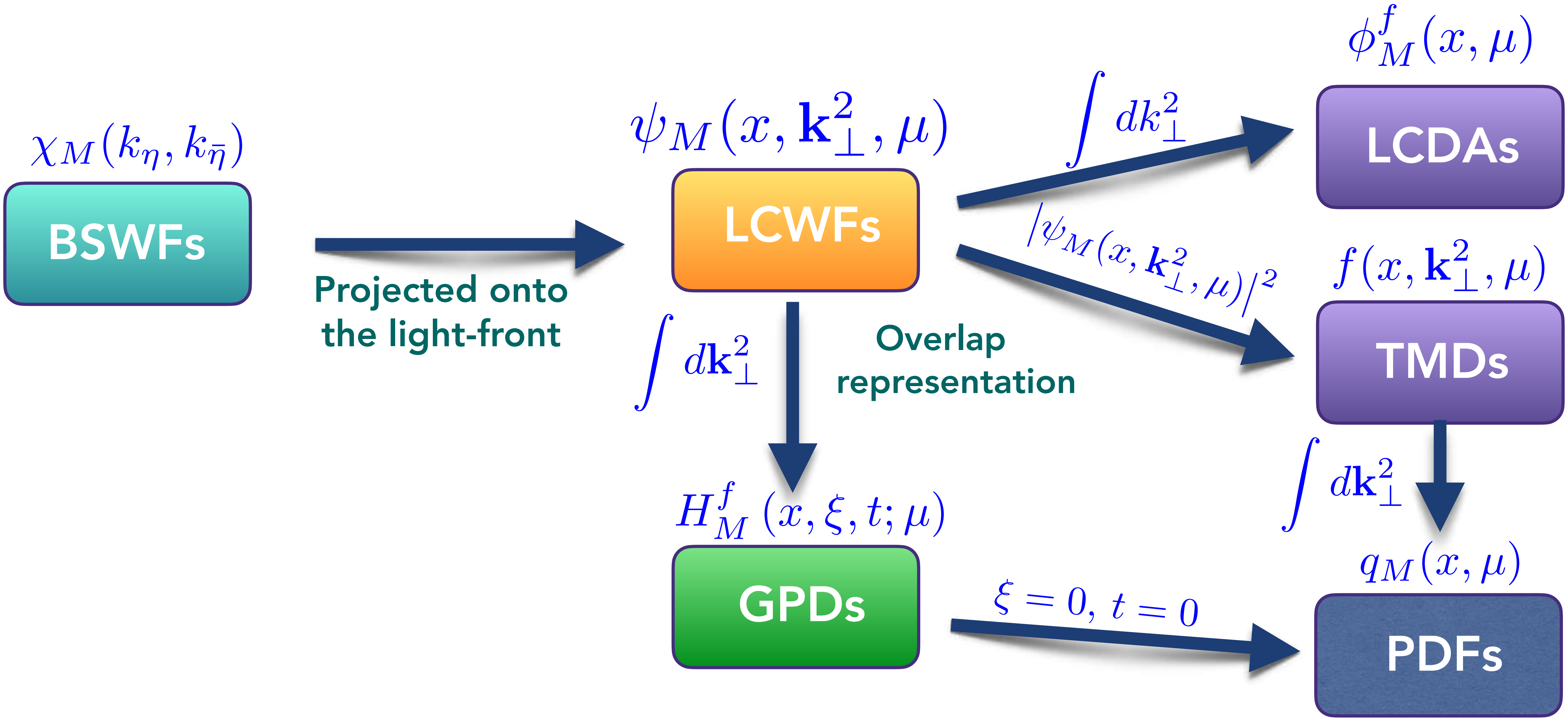
Light-cone wave functions and quantum information

- Bruno El-Bennich, Universidade Cidade de São Paulo, Brazil.
- Gastão Krein, IFT-UNESP, Brazil.
- Ian Cloët, Argonne National Lab, US.

Light-Cone Wave Functions (LCWFs)

- From the **LCWFs** we can access to structure functions:



- **The inputs:** **Solutions** from **DSE** and **Bound-State equations.**

See Khépani's Talk

Light-Cone Wave Functions (LCWFs)

- The **LCWFs** are obtained from the **BSWF** via the light front projections:

$$\psi_M^{\uparrow\downarrow}(x,\mathbf{k}_\perp^2) = \sqrt{3}i\int\frac{dk^+dk^-}{\pi}\delta(xP^+-k^+)\mathrm{Tr}_D[\gamma^+\gamma_5\chi(k,P)],$$

$$\psi_M^{\uparrow\uparrow}(x,\mathbf{k}_\perp^2) = -\frac{\sqrt{3}i}{\mathbf{k}_\perp^2}\int\frac{dk^+dk^-}{\pi}\delta(xP^+-k^+)\mathrm{Tr}_D[i\sigma_{+i}k_T^i\gamma_5\chi(k,P)]$$

See Tobias's Talk

- With the **LCWF** one can readily derive two distributions:
  - The leading-twist **TMD**

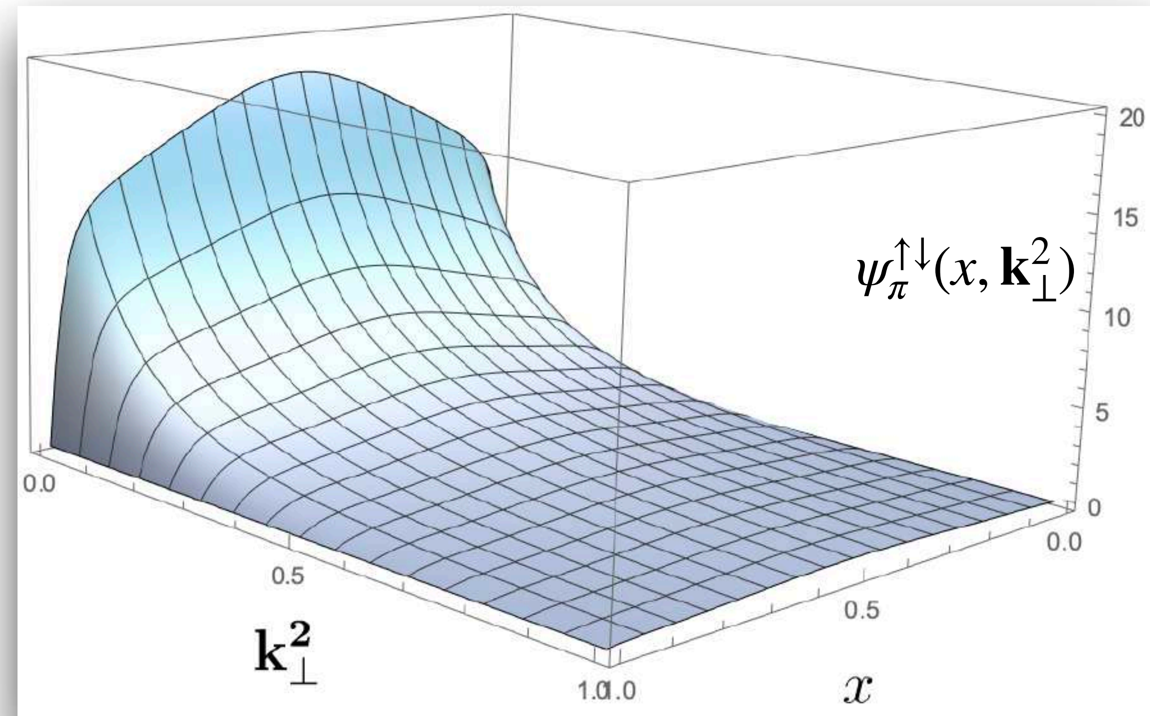
$$f_M(x,\mathbf{k}_\perp^2,\mu) = \frac{1}{(2\pi)^3}\left|\psi_M^{\uparrow\downarrow}(x,\mathbf{k}_\perp^2,\mu)+\mathbf{k}_\perp^2\psi_M^{\uparrow\downarrow}(x,\mathbf{k}_\perp^2,\mu)\right|^2$$

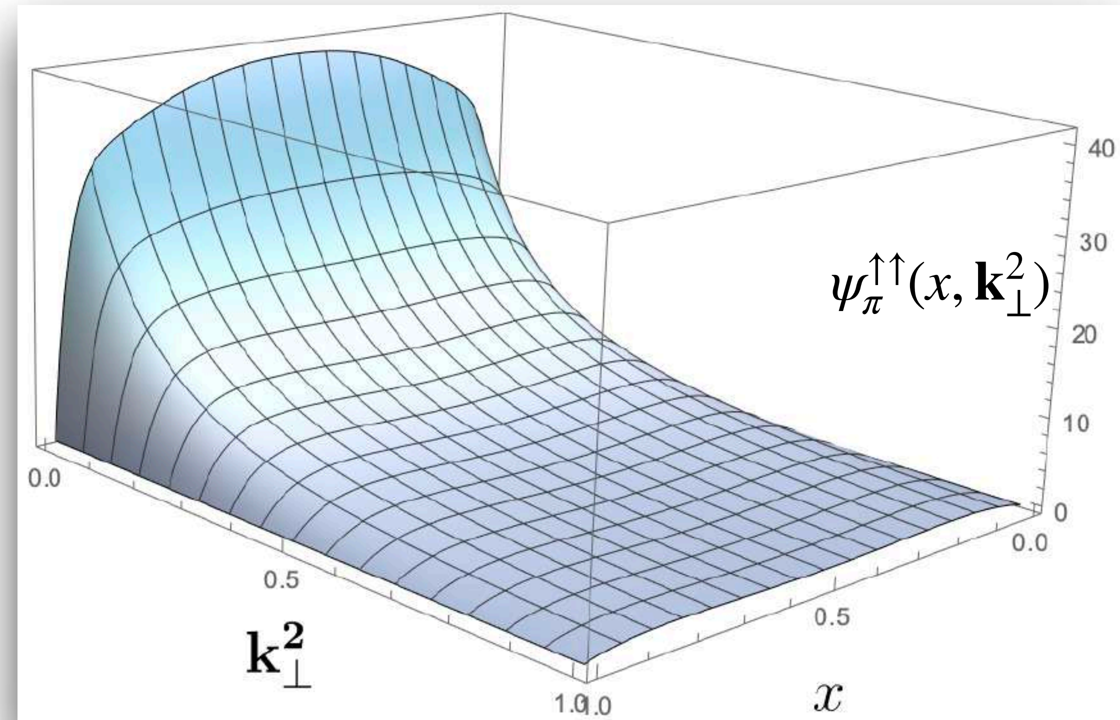
  - The **PDF**

$$q_M(x,\mu) = \int d\mathbf{k}_\perp^2 f_M(x,\mathbf{k}_\perp^2,\mu)$$

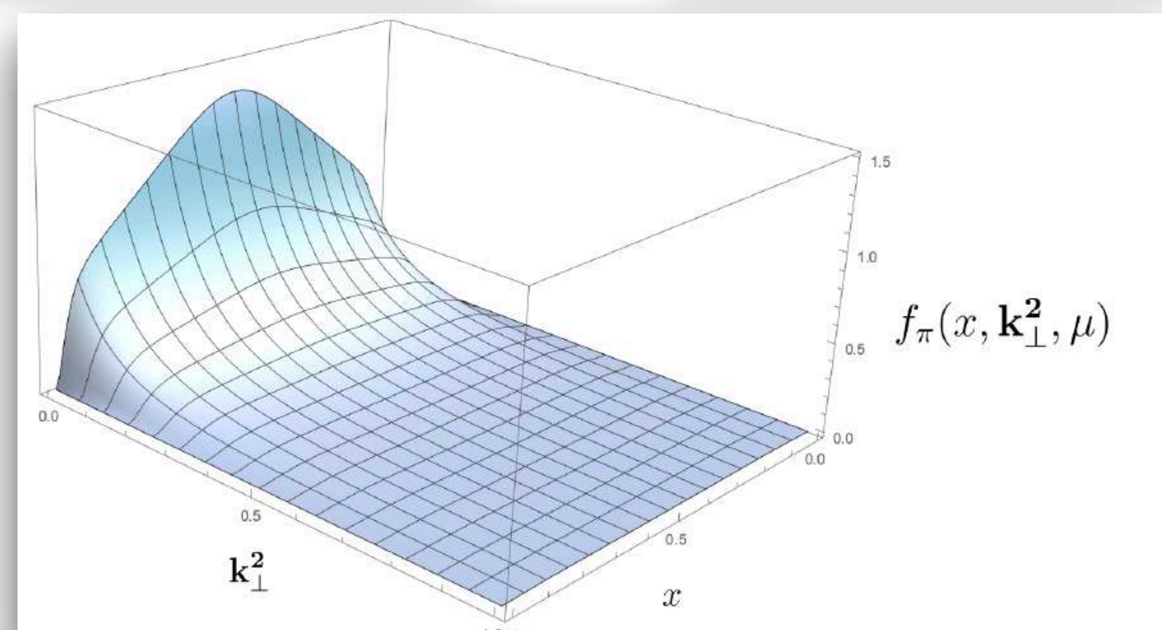
Light-Cone Wave Functions (LCWFs)

- Pion **LCWF**





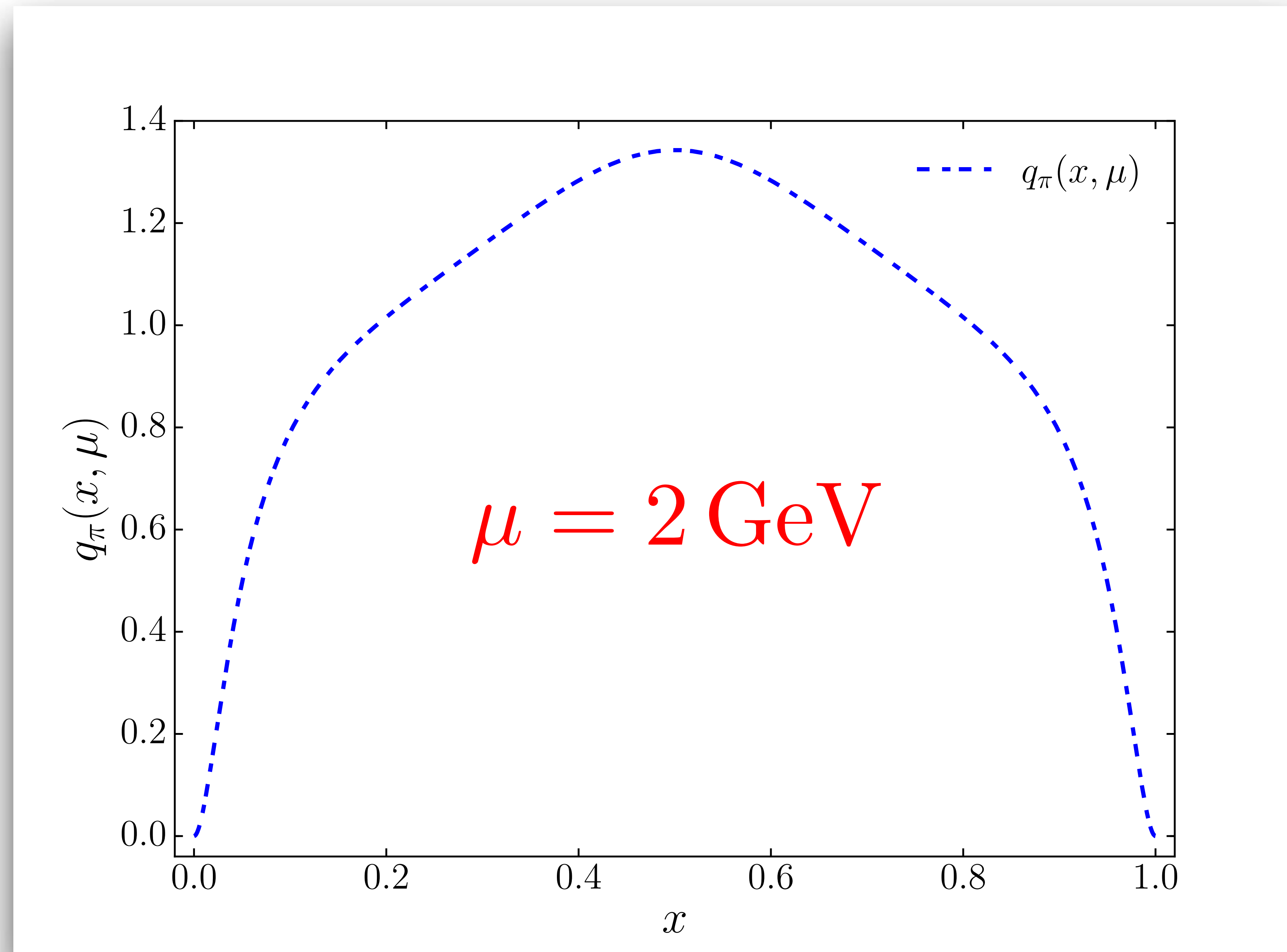
- Pion **TMD**



Light-Cone Wave Functions (LCWFs)

- Pion **PDF**

$$q_M(x,\mu) = \int d\mathbf{k}_\perp^2 f_M(x,\mathbf{k}_\perp^2,\mu)$$



Entanglement entropy of constituent quarks in the pion LCWF

- Entropy of the valence distribution of the valence quark distribution in the pion.

$$s(\mu^2) = -\int_0^1 \left\{ q_\pi^u(x,\mu^2)\log[q_\pi^u(x,\mu^2)] + q_\pi^d(x,\mu^2)\log[q_\pi^d(x,\mu^2)] \right\} dx$$

- This definition is related to the **Von Neumann entropy** for a quantum object in a non-pure state.

- Explore how this entropy evolves as a function of μ^2.

- Explore how the entropy behaves when the coupling constant in the gap and bound- state equations decreases or increases.

Entanglement entropy of constituent quarks in the pion LCWF

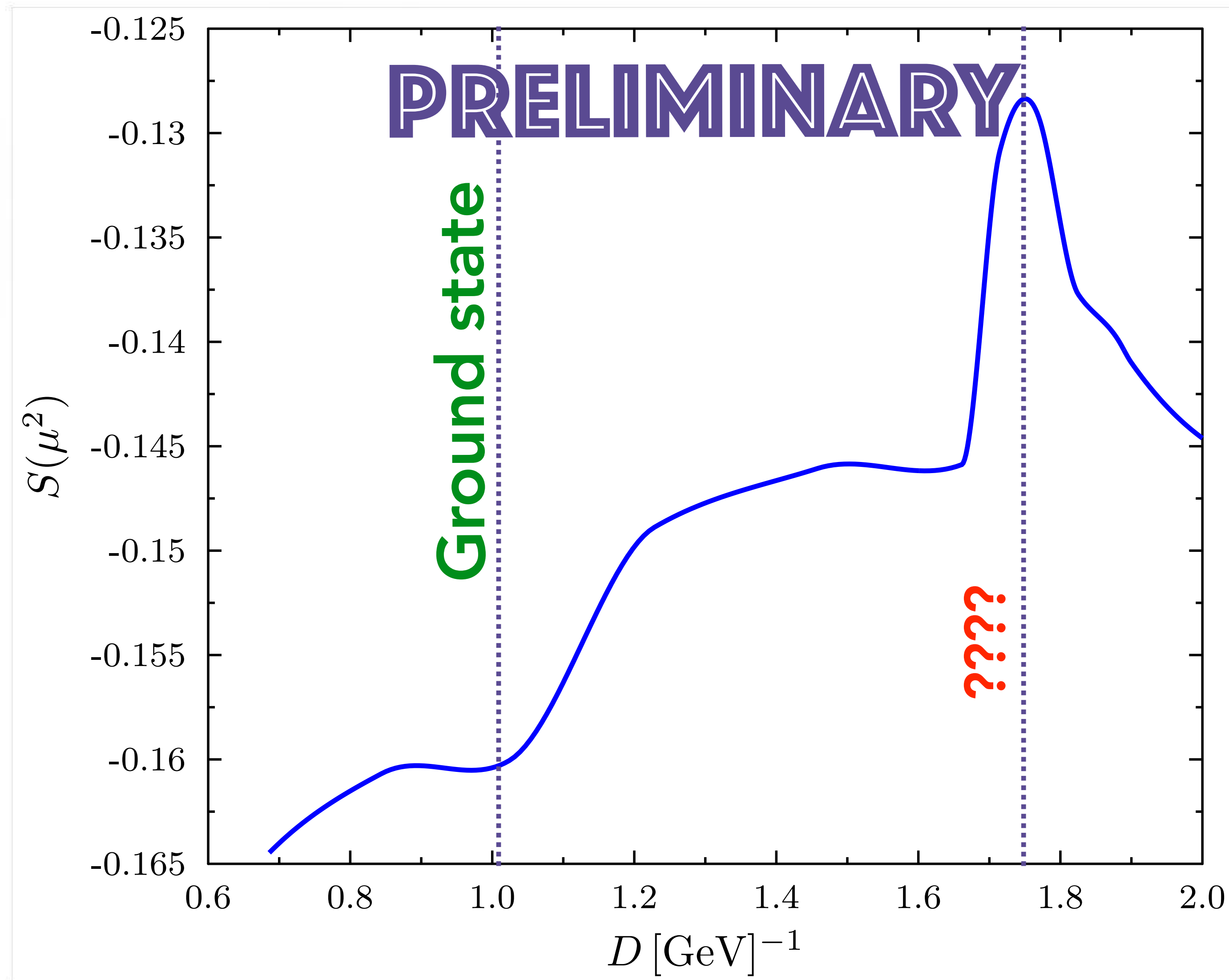
$$s(\mu^2) = -\int_0^1 \left\{ q_\pi^u(x,\mu^2)\log[q_\pi^u(x,\mu^2)] + q_\pi^d(x,\mu^2)\log[q_\pi^d(x,\mu^2)] \right\} dx$$

$$\frac{\mathcal{G}_f(q^2)}{q^2} = \mathcal{G}_f^{\rm IR}(q^2) + 4\pi\tilde{\alpha}_{\rm PT}(q^2)$$

QC-Model:

$$\mathcal{G}_f^{\rm IR}(q^2) = \frac{8\pi^2}{\omega_f^4} D_f\, e^{-q^2/\omega_f^2}$$

$$\omega = 0.5\,\mathrm{GeV}$$



Conclusions

- Good reproduction of the meson mass spectrum and their weak decay constants for pseudoscalar and vector channels .

- First predictions for **LCDAs** of vector D and D_s mesons.

- **LCDAs** of the J/Ψ and D_s^* mesons can readily be used in diffractive vector-meson production which are of interest to the experimental program of the **EIC**.

- We also are interested in study the entropy of the valence distribution of the valence quark distribution in the pion.

Thank you