School on Nonlinear Dynamics, Complex Networks, Information Theory and Machine Learning in Neuroscience, 22-26 May 2023

Nonlinear time series analysis

Cristina Masoller

Departamento de Fisica

Universitat Politecnica de Catalunya

Class 1: From dynamical systems to complex systems

Class 2: Univariate time series analysis

Class 3: Bivariate and multivariate analysis

UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH

Campus d'Excel·lència Internacional

ICTP SAIFR International Centre for Theoretical Physics South American Institute for Fundamental Research

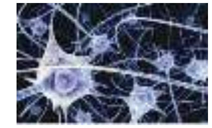
cristina.masoller@upc.edu @cristinamasoll1

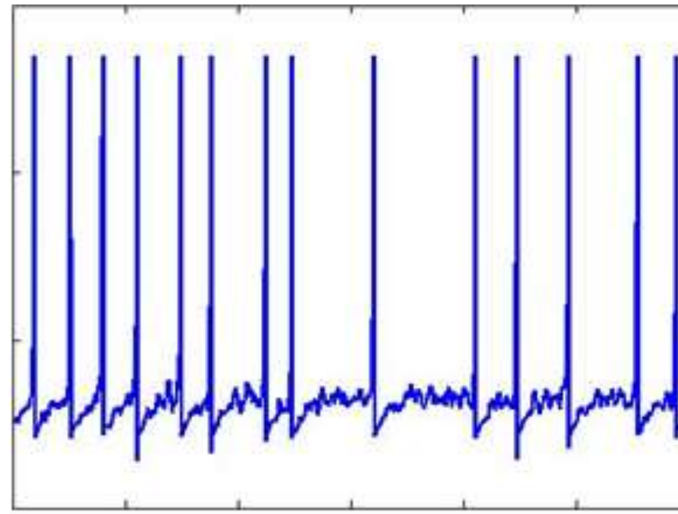
Outline

- Uncovering similarities between lasers and neurons
- Analysis of regime transitions: EEG signals and vegetation fields
- Network based analysis of retina fundus images

cristina.masoller@upc.edu @cristinamasoll1

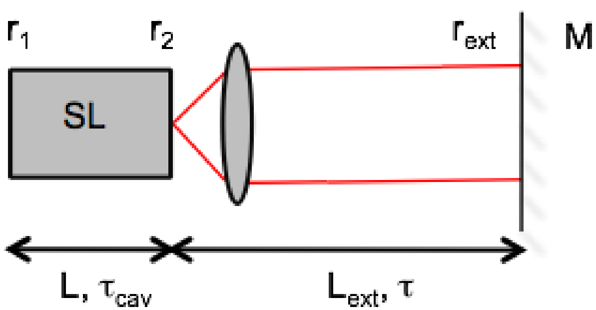
Data analysis methods allow to discover statistical similarities in very different systems





Time 10^{-9} s

Time 10^{-3} s



$$\varepsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y\,,$$

$$\frac{dy}{dt} = x + a + D\xi(t)\,.$$

(High-dimensional, Memory)

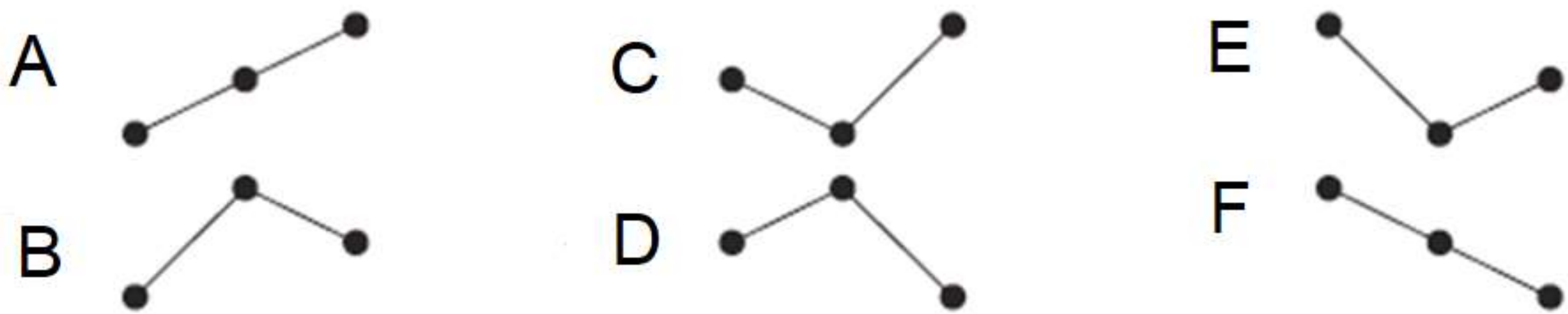
cristina.masoller@upc.edu @cristinamasoll1

Data analysis method: ordinal analysis

$$\{\ldots x_i,\ x_{i+1},\ x_{i+2},\ \ldots\}$$

Possible order relations among three numbers (e.g., 2, 5, 7)

{…2, 5, 7…} {…5, 2, 7…} {…7, 2, 5 …}

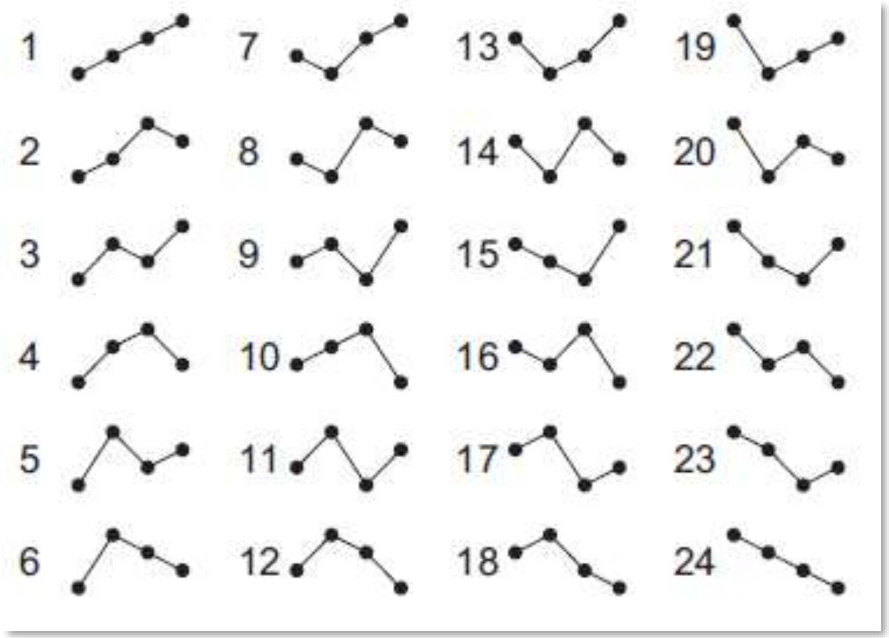


{…2, 7, 5…} {…5, 7, 2…} {…7, 5, 2…}

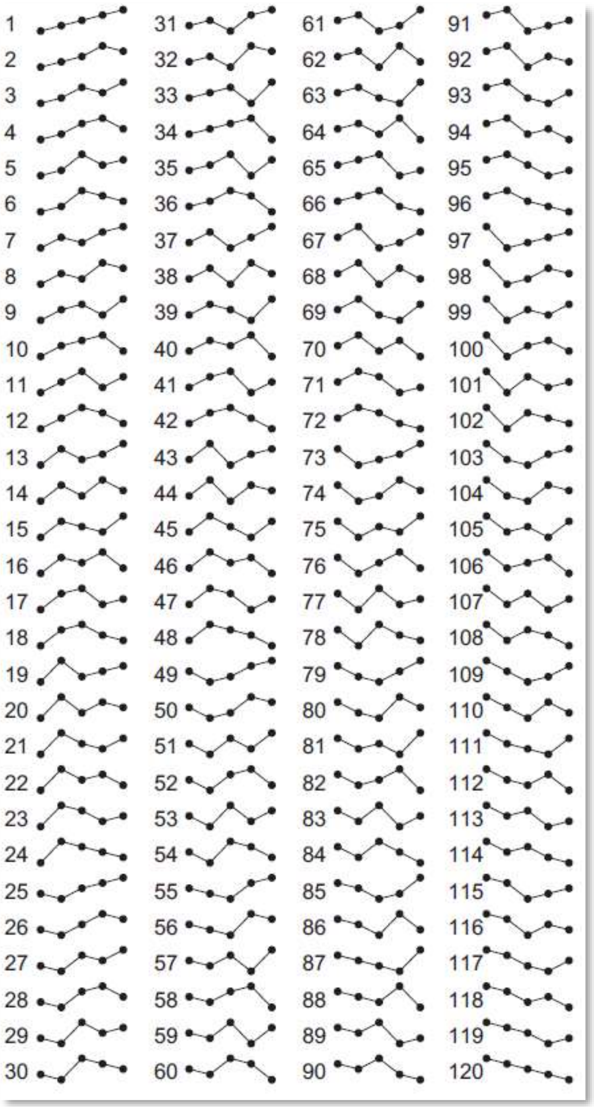
Bandt and Pompe: Phys. Rev. Lett. 2002

cristina.masoller@upc.edu @cristinamasoll1

The number of ordinal patterns increases as D!



A problem for short datasets.



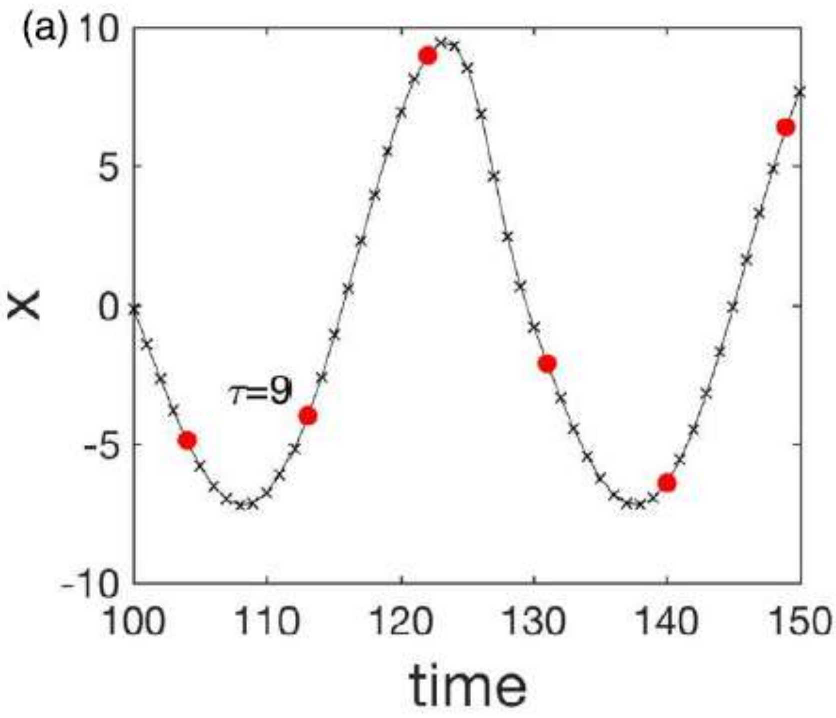
U. Parlitz et al. / Computers in Biology and Medicine 42 (2012) 319–327

cristina.masoller@upc.edu @cristinamasoll1

Ordinal patterns can be defined using a lag

Example: climatological time series (monthly sampled)

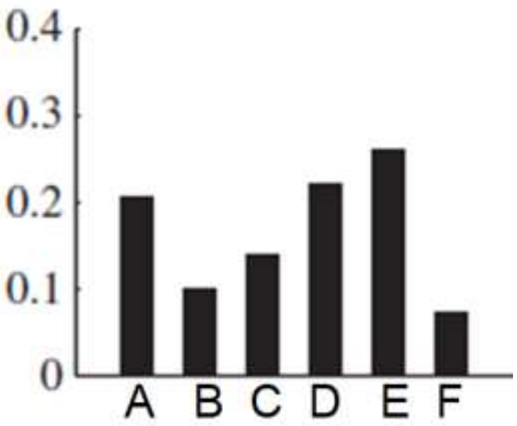
- Consecutive months:
  $[\ldots x_i(t), x_i(t+1), x_i(t+2)\ldots]$
- Consecutive years:
  $[\ldots x_i(t),\ldots x_i(t+12),\ldots x_i(t+24)\ldots]$



Y. Zou et al. Phys. Rep. 787, 1 (2019)

cristina.masoller@upc.edu @cristinamasoll1

From a time series, by counting the number of times the different patterns appear, we can calculate the set of "ordinal probabilities"

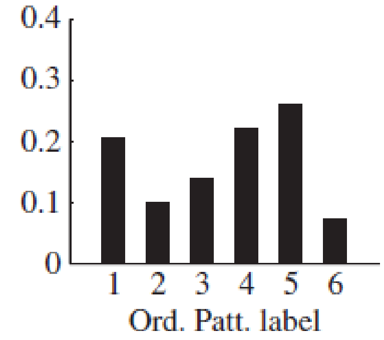


?

A. Analyze the probabilities (are differences statistically significant?)
B. Compute information theory measures (entropy, complexity)

cristina.masoller@upc.edu @cristinamasoll1

Are the *D*! ordinal patterns equally probable?



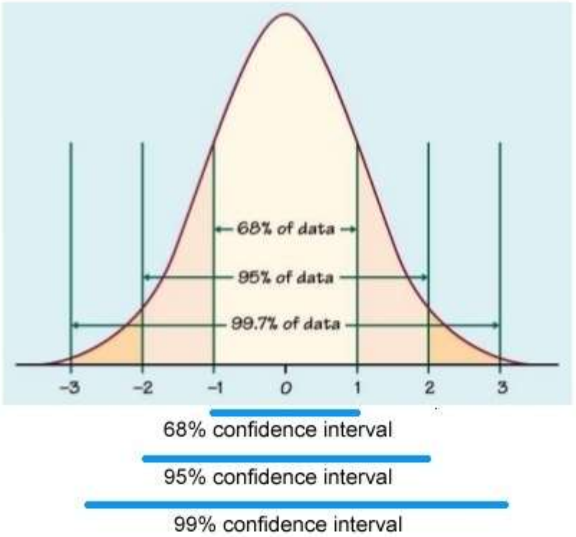
- **Null hypothesis**:

  $p_i = p = 1/D!$ for all $i = 1 \ldots D!$

- If at least one probability **is not** in the interval $p \pm 3\sigma$ with $\sigma = \sqrt{p(1-p)/N}$ and N the number of ordinal patterns:

  We **reject** the NH with 99.74% confidence level.

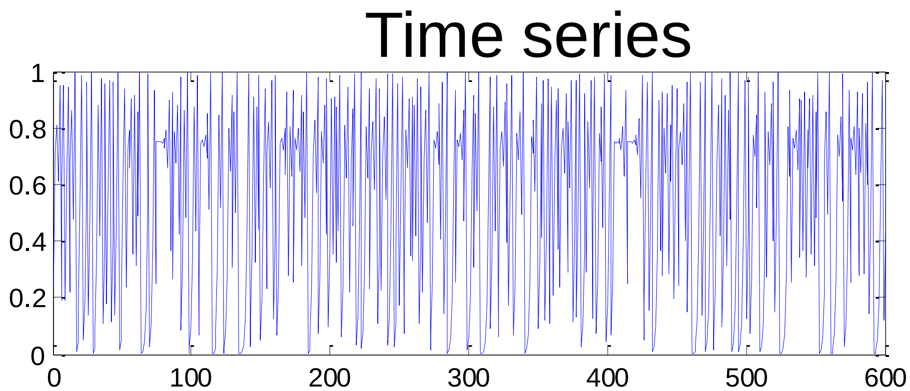
- Else, we **fail to reject** the NH with 99.74% confidence level.

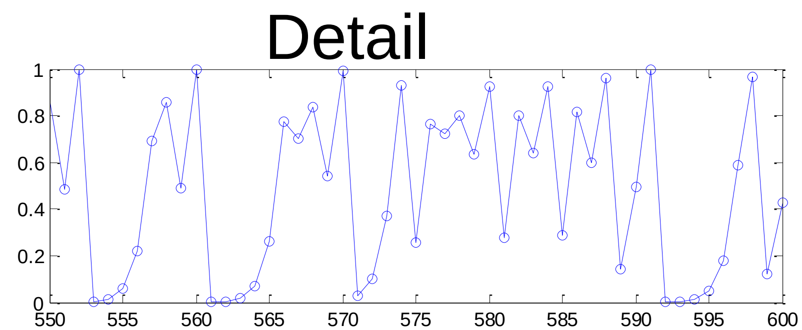


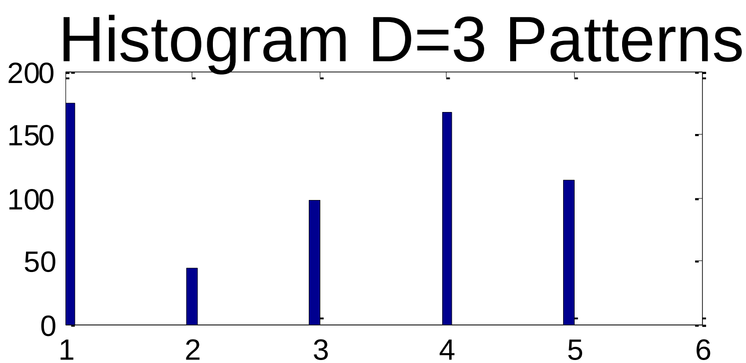
cristina.masoller@upc.edu @cristinamasoll1

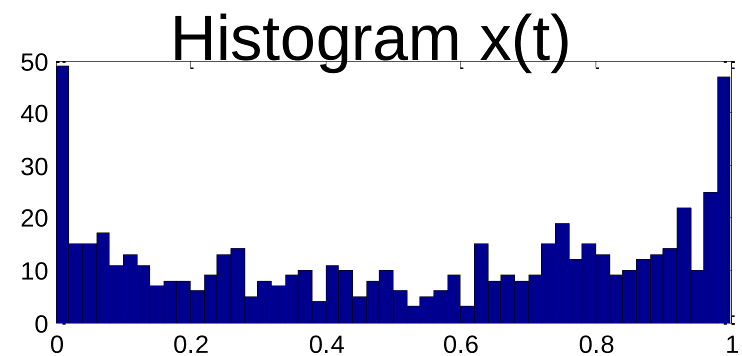
Example: chaotic time series generated with the Logistic map

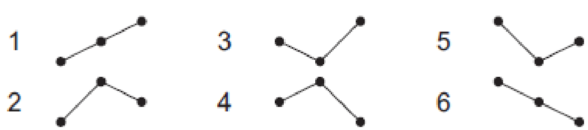
$$x(i+1) = r\, x(i)[1 - x(i)] \quad \text{r=3.99}$$









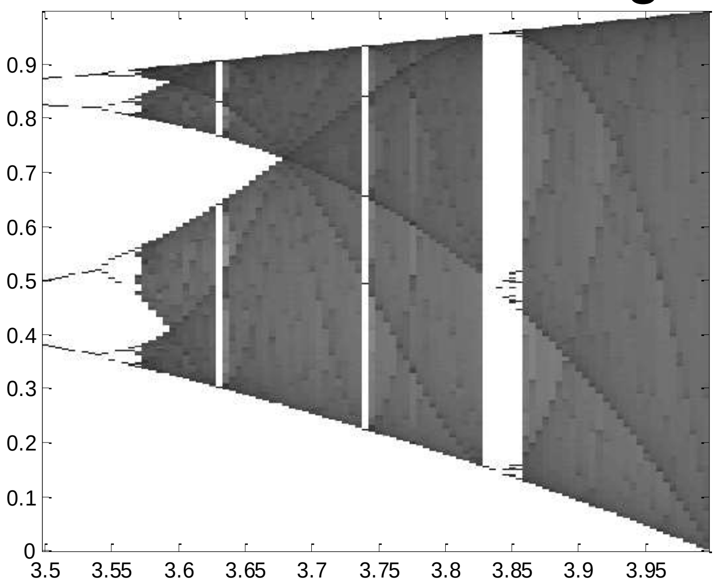


↑ **forbidden**

cristina.masoller@upc.edu @cristinamasoll1

“Normal” and “Ordinal” bifurcation diagrams of the Logistic map

Normal bifurcation diagram

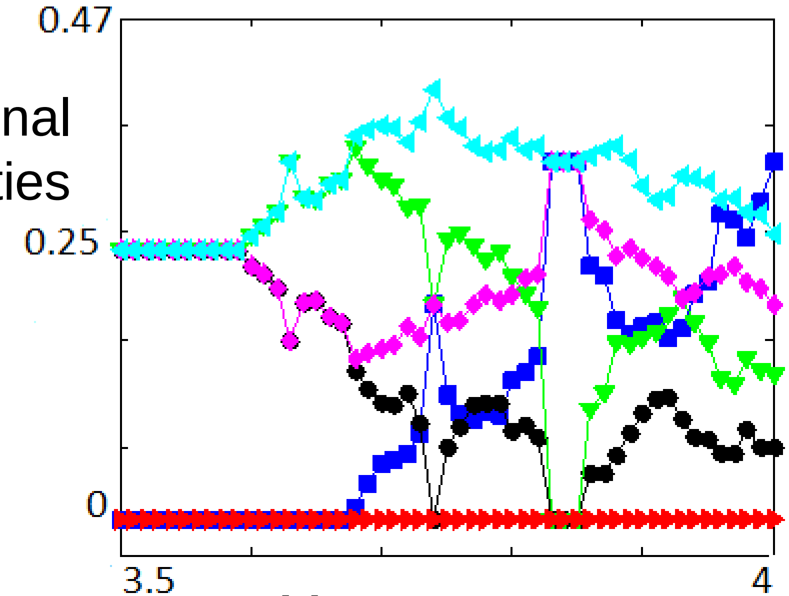


X_i

Map parameter

Ordinal diagram with D=3

Ordinal Probabilities



Map parameter, r

012 021 102 120 201 210

Pattern **210** is always forbidden; pattern **012** is more probable as r increases

cristina.masoller@upc.edu @cristinamasoll1

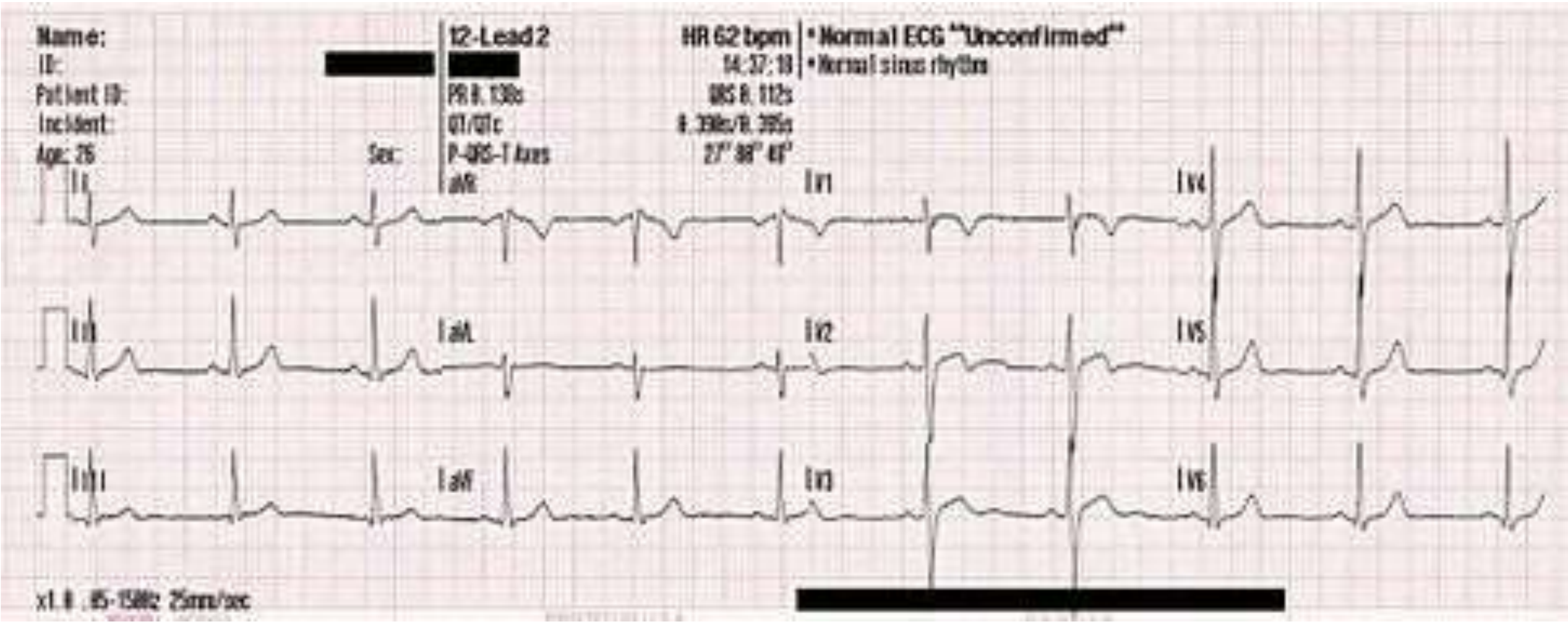
Ordinal analysis has been extensively used:

- To test if a model is good for the data
- To fit the parameters of a model
- To classify different types of data based on the probabilities of ordinal patterns

I. Leyva, J. M. Martinez, C. Masoller, O. A. Rosso, M. Zanin, "*20 Years of Ordinal Patterns: Perspectives and Challenges*", EPL 138, 31001 (2022).

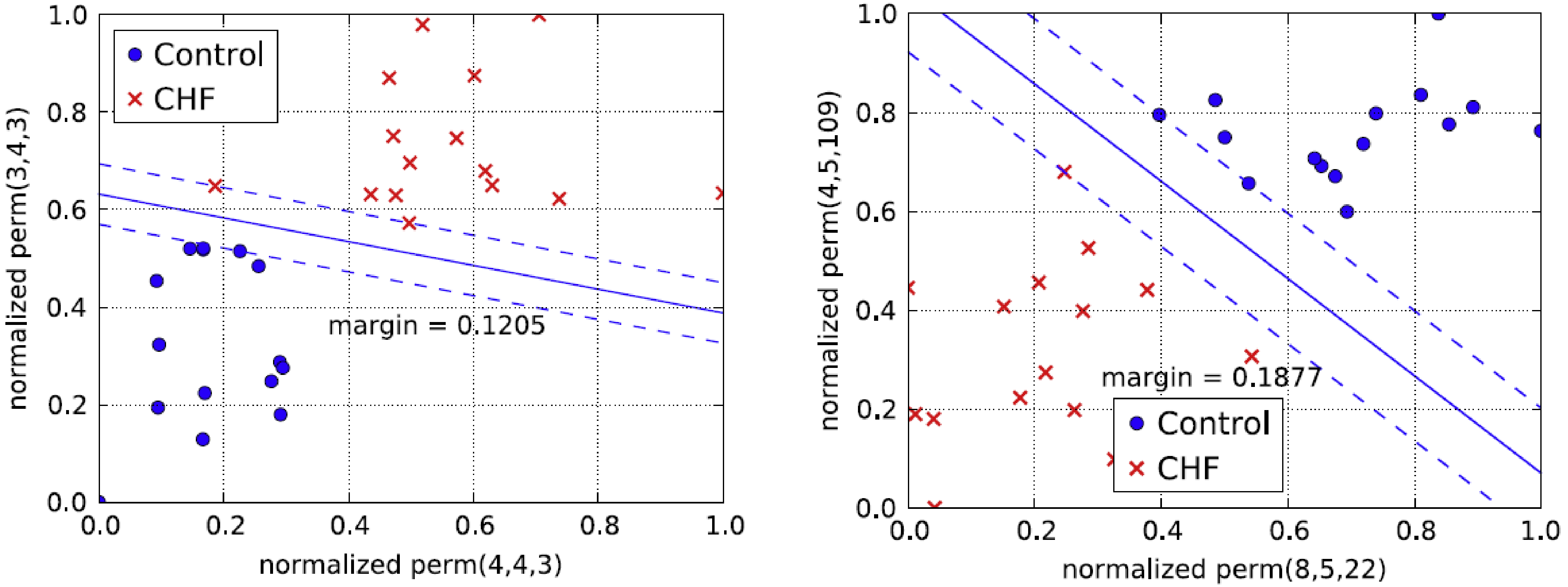
cristina.masoller@upc.edu @cristinamasoll1

ECG signals: analysis of time series of inter-beat intervals



cristina.masoller@upc.edu @cristinamasoll1

Classifying ECG signals according to ordinal probabilities

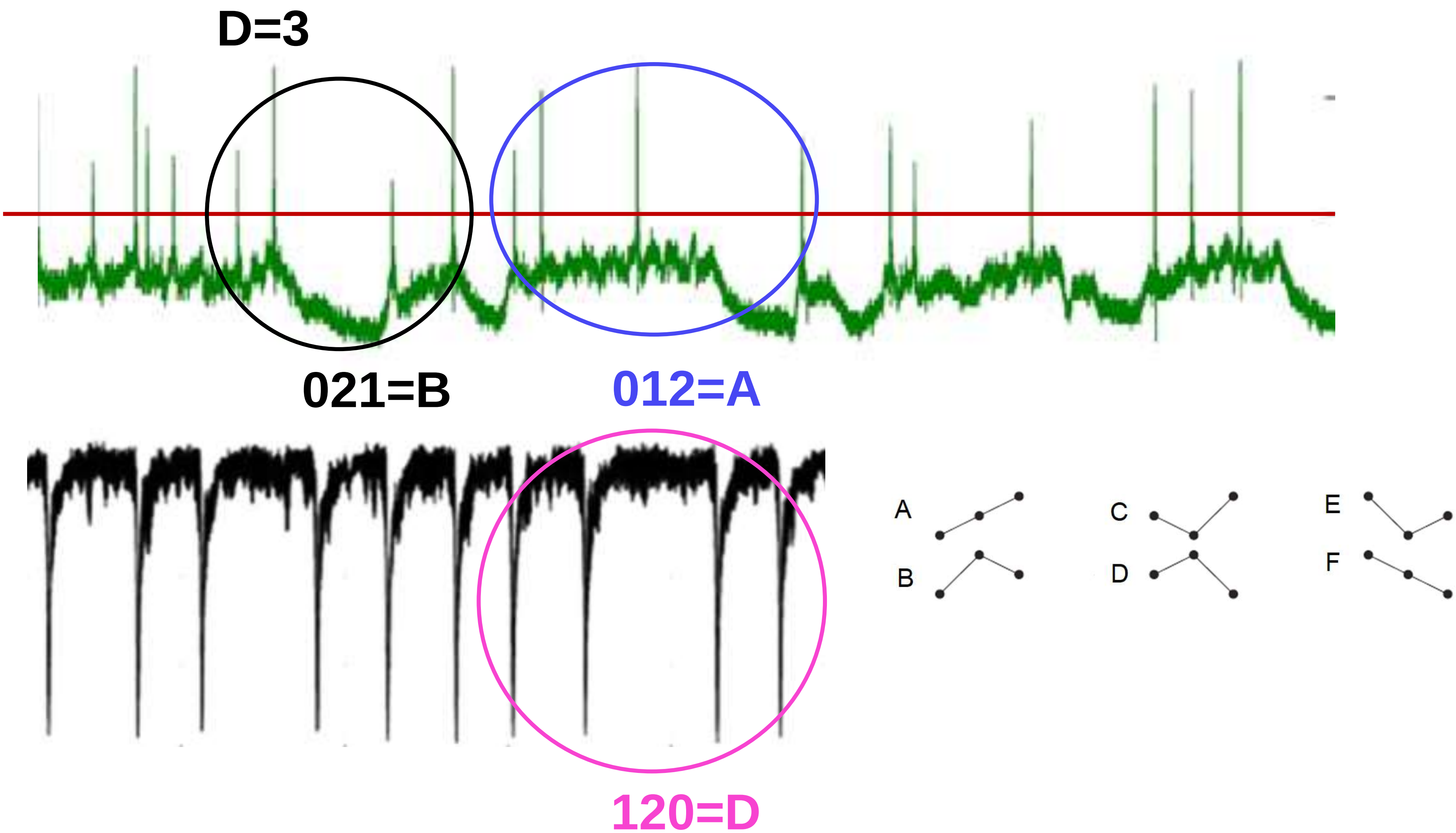


- Analysis of raw data (statistics of ordinal patterns is almost unaffected by a few extreme values)
- The probabilities are normalized with respect to the smallest and the largest value occurring in the data set.

U. Parlitz et al. Computers in Biology and Medicine 42, 319 (2012)

cristina.masoller@upc.edu @cristinamasoll1

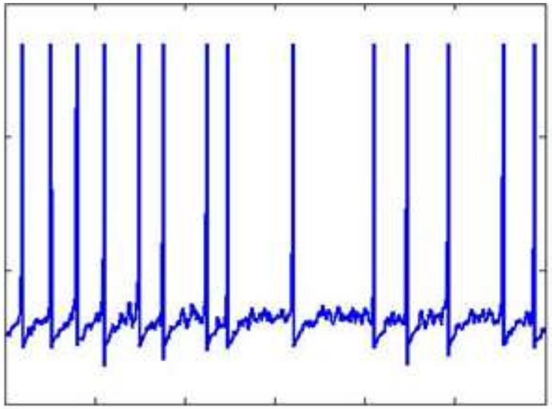
Sequence of inter-spike-intervals (ISIs) ⇒ sequence of ordinal patterns



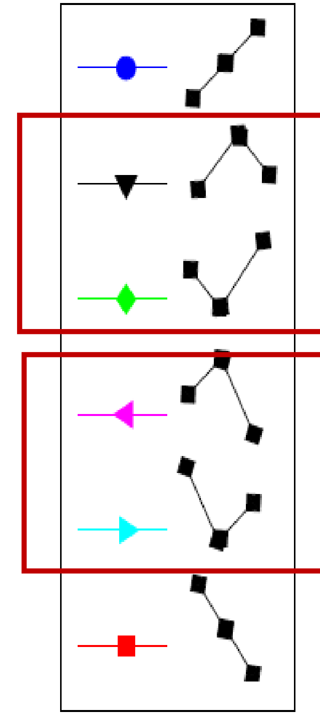
cristina.masoller@upc.edu @cristinamasoll1

The analysis of the ordinal probabilities uncovers similarities in ISI sequences

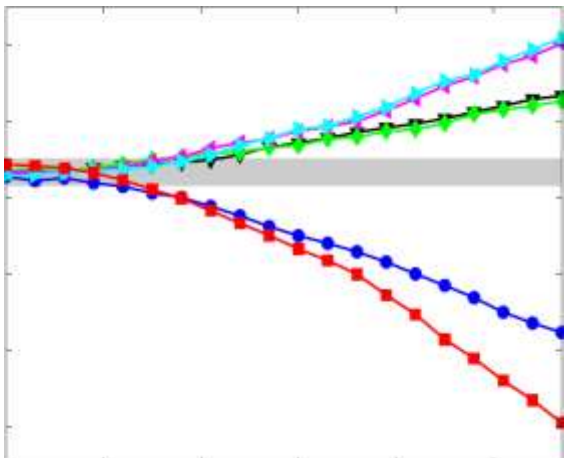
Neuron model



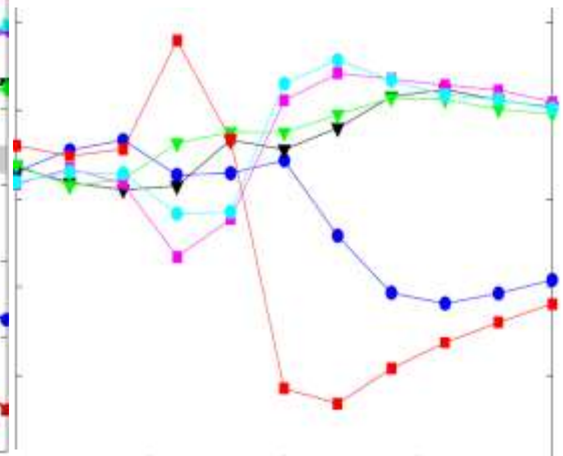
Diode laser



Ordinal probabilities



Forcing amplitude



Forcing amplitude

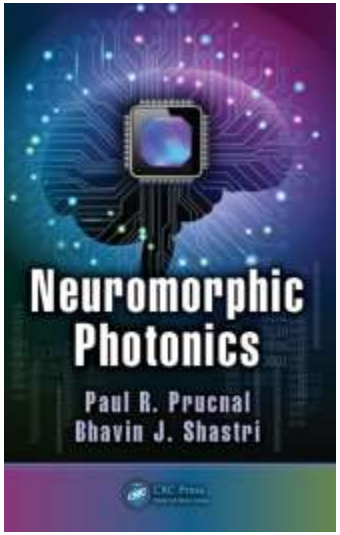
J. M. Aparicio-Reinoso et al PRE 94, 032218 (2016) A. Aragoneses et al, Sci. Rep. **4**, 4696 (2014)

cristina.masoller@upc.edu @cristinamasoll1

Uncovering similarities between neurons and lasers… Interesting but relevant?

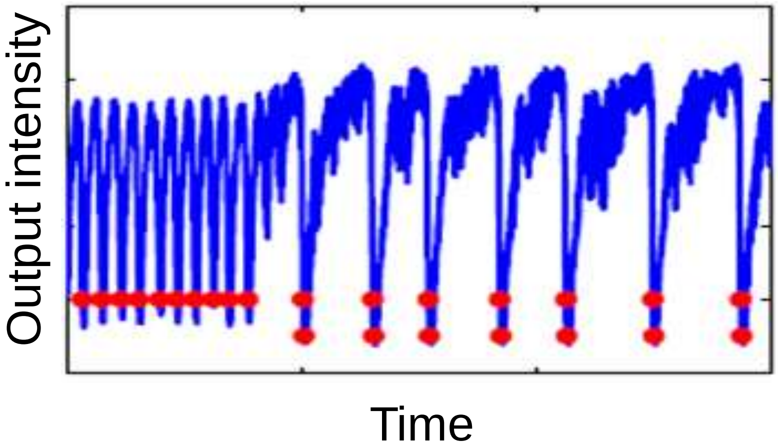
- Data centers, AI systems, HPC consume huge amounts of energy.
- Big concern in the context of climate change.
- The human brain processes huge amounts of information using only 19 Watts.
- Uncovering genuine similarities between neurons and lasers will allow to develop **photonic neurons**, able to process information as real neurons do, but
  - much faster,
  - with much less energy consumption.

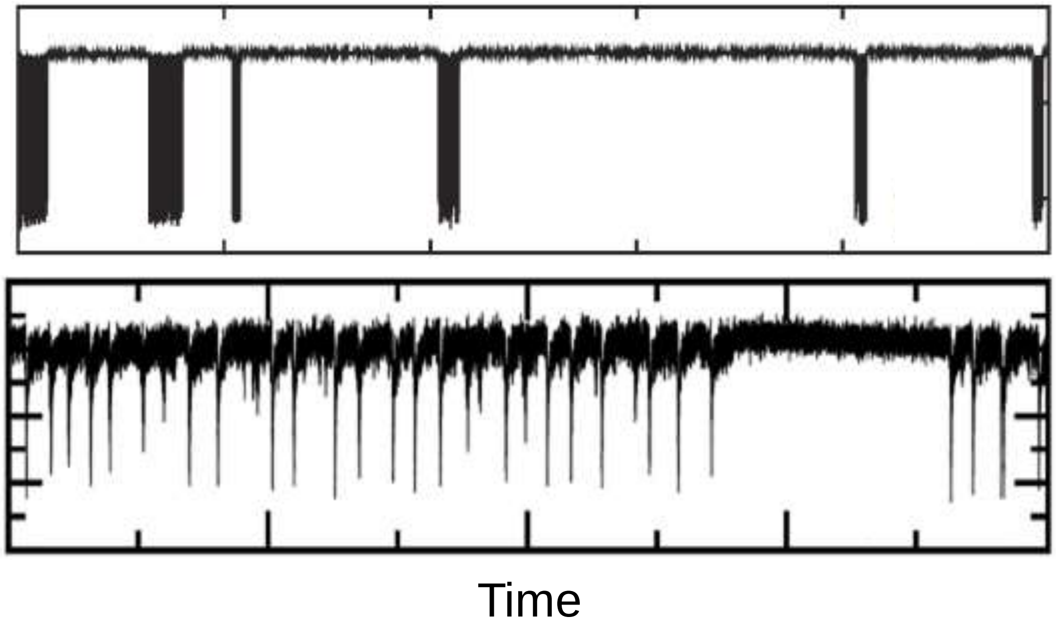
European Centre for Medium-Range Weather Forecasts, Reading, UK



cristina.masoller@upc.edu @cristinamasoll1

Time series recorded in our lab show excitability, tonic spikes, and bursting. Similar to real neurons?





A. Aragoneses, S. Perrone, T. Sorrentino, M. C. Torrent and C. Masoller, "*Unveiling the complex organization of recurrent patterns in spiking dynamical systems*", Sci. Rep. **4**, 4696 (2014).

C. Quintero-Quiroz, J. Tiana-Alsina, J. Roma, M. C. Torrent, and C. Masoller, "*Characterizing how complex optical signals emerge from noisy intensity fluctuations*", Sci. Rep. **6** 37510 (2016).

cristina.masoller@upc.edu @cristinamasoll1

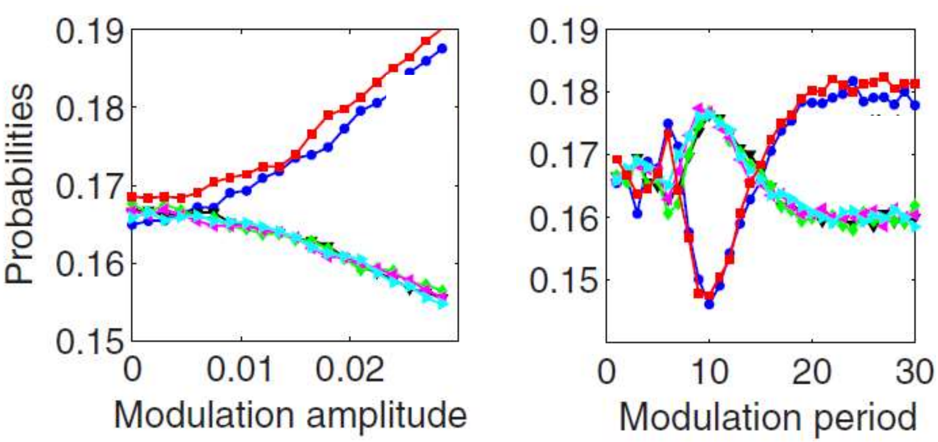
Main challenge

Understand how to mimic with lasers the way neurons encode and process information.

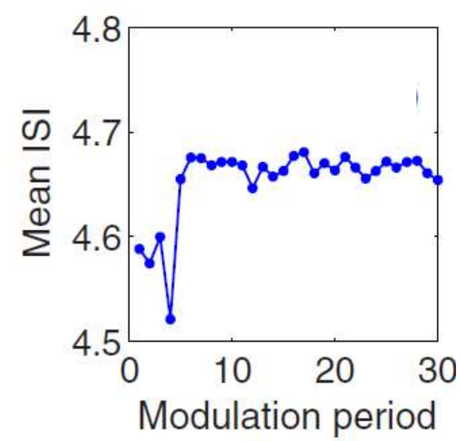
$$\epsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y,$$

$$\frac{dy}{dt} = x + a + \boxed{a_o \cos(2\pi t/T)} + D\xi(t),$$

Weak, subthreshold signal



Rate coding?



J. A. Reinoso, M. C. Torrent, and C. Masoller, "*Emergence of spike correlations in periodically forced excitable systems*", Phys. Rev. E. 94, 032218 (2016).

cristina.masoller@upc.edu @cristinamasoll1

Single-neuron vs ensemble encoding

- <u>Single-neuron encoding</u>: **slow** because long spike sequences are needed to estimate the ordinal probabilities.
- <u>Ensemble encoding</u>: can be **fast** because, from the ISI sequences of all the neurons, few spikes per neuron can be enough to accurately estimate the probabilities.

Weak, subthreshold signal

$$\epsilon \dot{u}_i = u_i - \frac{u_i^3}{3} - v_i + a_0 \cos(2\pi t/T) + \frac{\sigma}{k_i} \sum_j^N a_{ij}(u_j - u_i) + \sqrt{2D}\xi_i(t), \qquad i \neq j$$

$$\dot{v}_i = u_i + a.$$

$k_i = \sum_j a_{ij}$

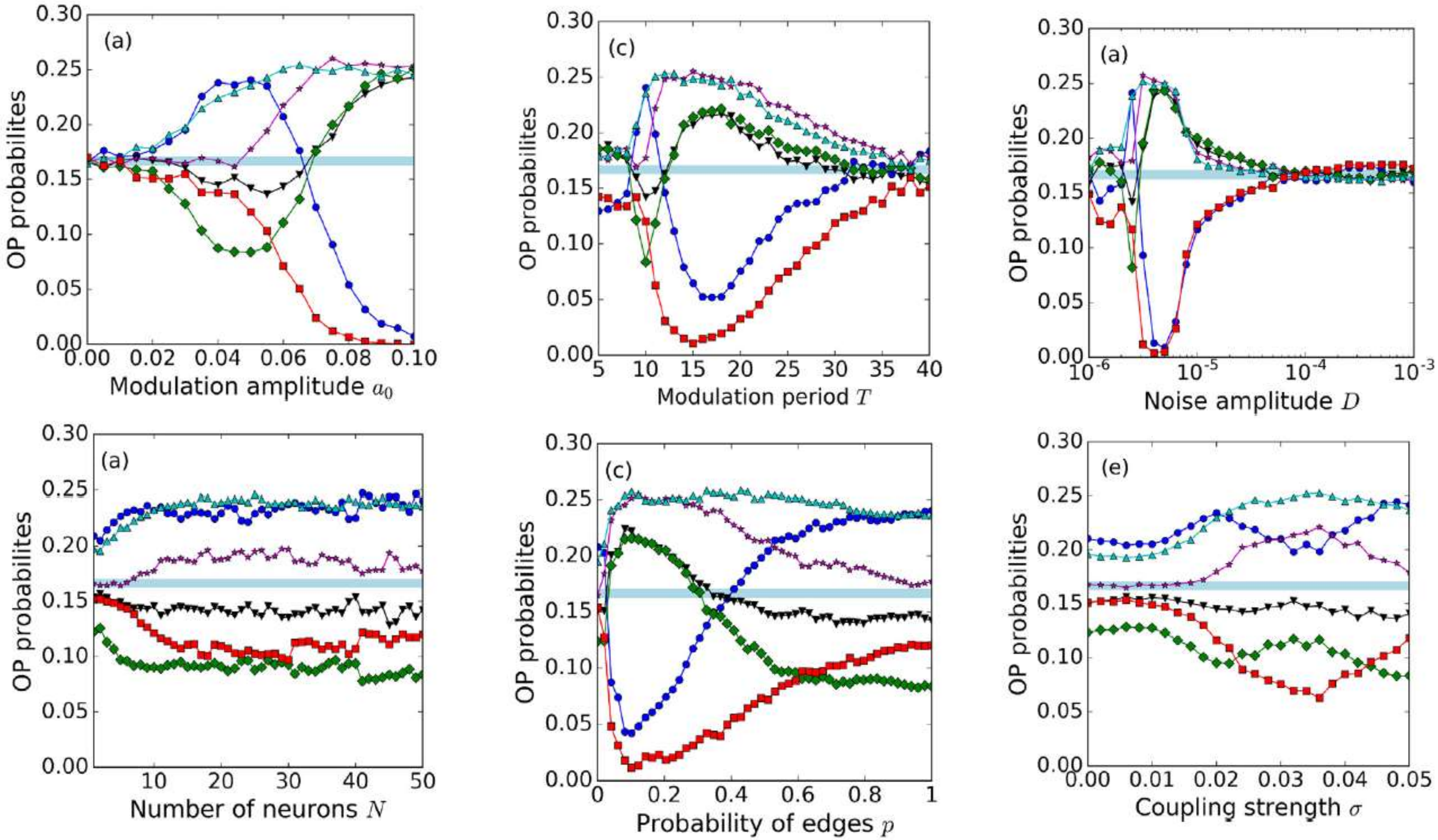
$a_{ij} = a_{ji} = 1$

$a_{ij} = a_{ji} = 0$

M. Masoliver and C. Masoller, "*Neuronal coupling benefits the encoding of weak periodic signals in symbolic spike patterns*", Commun. Nonlinear Sci. Numer. Simulat. 88, 105023 (2020).

cristina.masoller@upc.edu @cristinamasoll1

Ensemble encoding of a weak sinusoidal signal in the frequencies of occurrence of ordinal patterns



M. Masoliver and C. Masoller, Commun. Nonlinear Sci. Numer. Simulat. 88, 105023 (2020).

cristina.masoller@upc.edu @cristinamasoll1

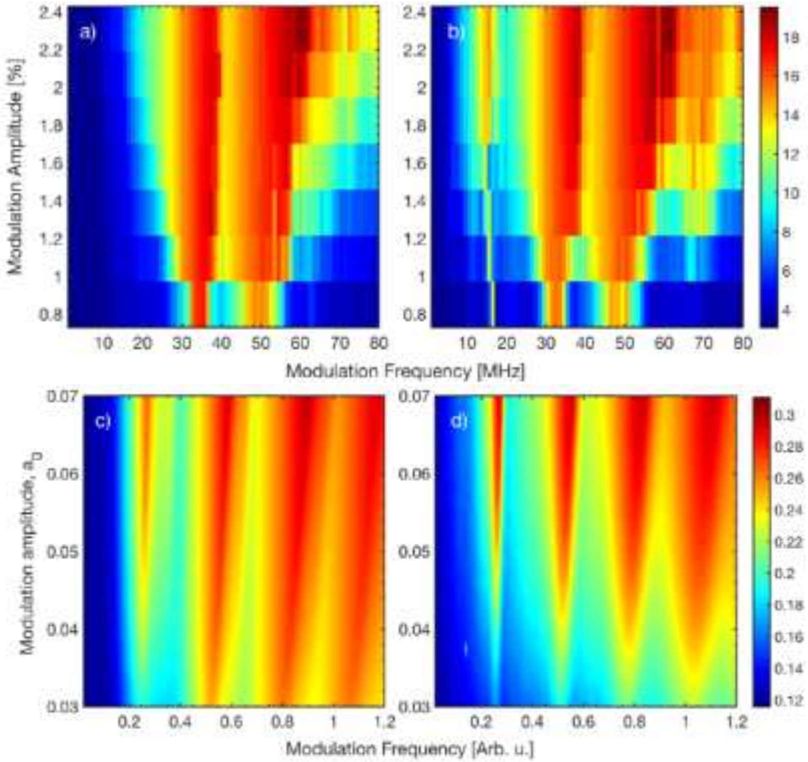
Laser-neuron comparison: encoding a weak periodic signal using spike rate code

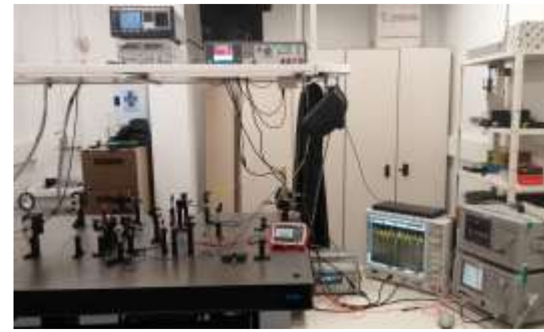
Spike rate in color code

Sinusoidal   Pulsed signal

Experiments modulating the laser current

Neuron model with the same input signal





$$\varepsilon \frac{dx}{dt} = x - \frac{x^3}{3} - y\,,$$

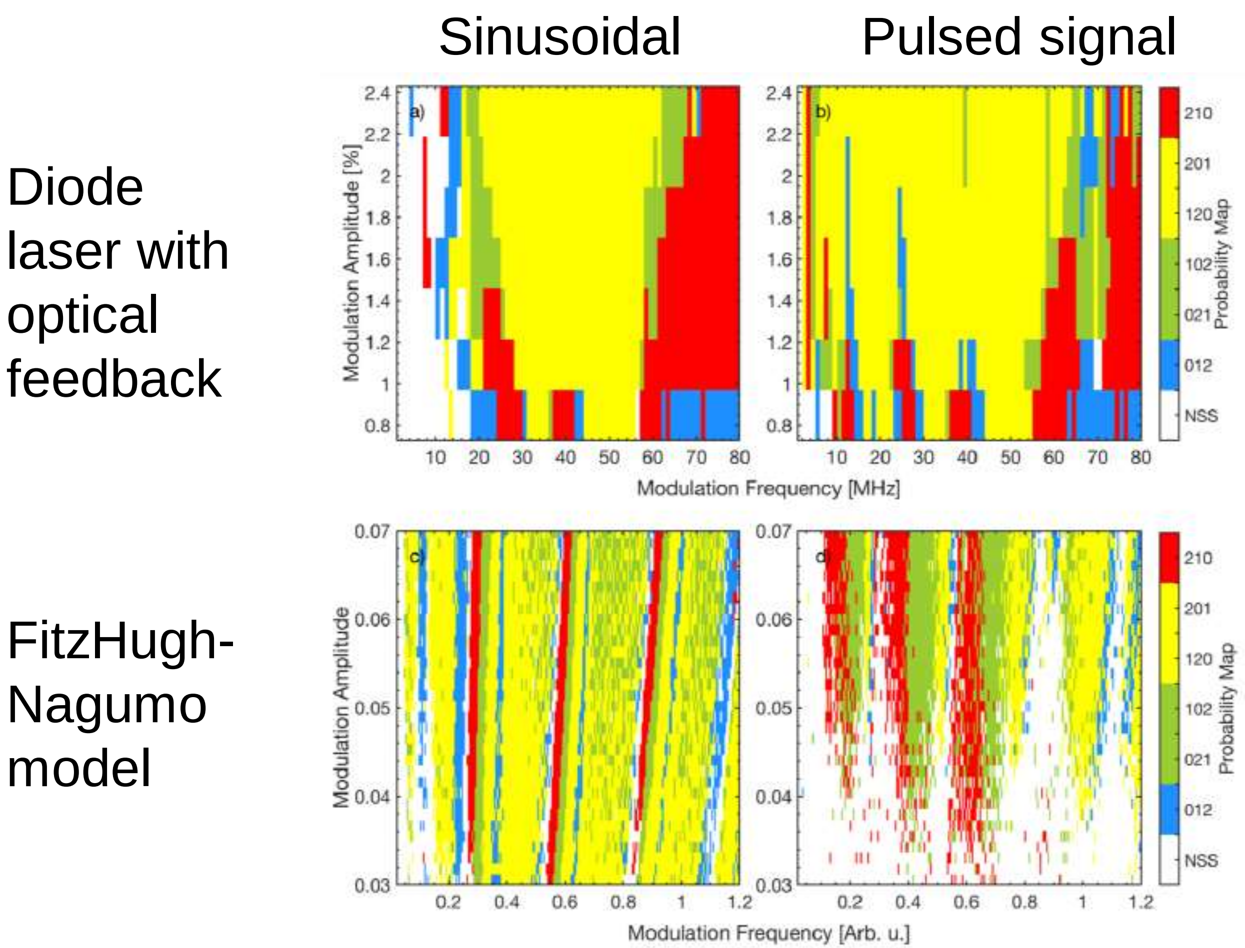
$$\frac{dy}{dt} = x + a + D\xi(t)\,.$$

J. Tiana-Alsina, C. Quintero-Quiroz and C. Masoller, "*Comparing the dynamics of periodically forced lasers and neurons*", New J. of Phys. 21, 103039 (2019) (2019).

J. Tiana-Alsina, C. Masoller, "*Time crystal dynamics in a weakly modulated stochastic time delayed system*", Sci. Rep. 12, 4914 (2022).

cristina.masoller@upc.edu @cristinamasoll1

How about the temporal code?
Ordinal analysis uncovers differences in spike timing.



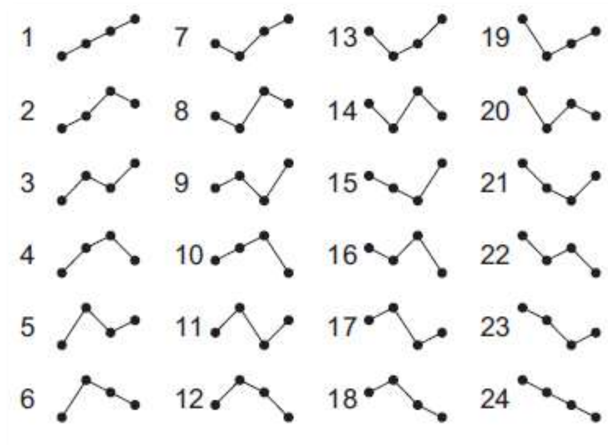
Most probable pattern in color code

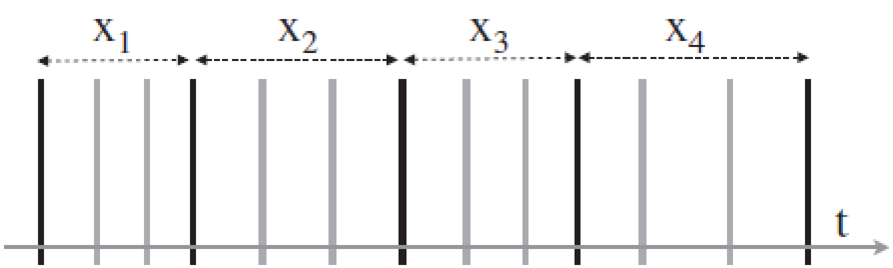
J. Tiana-Alsina, C. Quintero-Quiroz and C. Masoller, New J. of Phys. 21, 103039 (2019).

cristina.masoller@upc.edu @cristinamasoll1

Software

Python and Matlab codes for computing the ordinal pattern **index** are available here: [U. Parlitz et al. Computers in Biology and Medicine 42, 319 (2012)]()

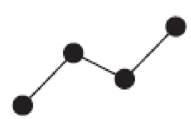




World length (wl): 4

Lag = 3 (skip 2 points)

Result:

indcs= 3 

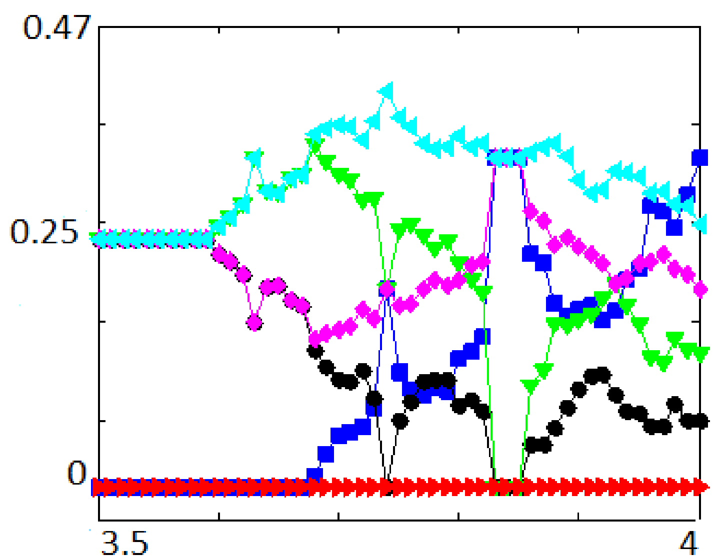
```
function indcs = perm_indices(ts, wl, lag) ;
m = length(ts)-(wl-1)*lag;
 indcs = zeros(m,1) ;
for i = 1:wl-1 ;
   st = ts(1+(i-1)*lag : m+(i-1)*lag) ;
   for j = i:wl-1 ;
      indcs = indcs+(st > ts(1+j*lag : m+j*lag));
   end
   indcs = indcs*(wl-i) ;
end
indcs=indcs + 1 ;
```


cristina.masoller@upc.edu @cristinamasoll1

Hands on activity: Ordinal analysis of the logistic map

$$\boxed{x(i+1) = r\, x(i)[1 - x(i)]}$$

1. Test the “ordinal pattern” program with some “hand made” examples (e.g., the sequence [5,2,7] returns index 3).
2. For the logistic map with r=3.99, calculate the probabilities of the 6 D=3 ordinal patterns and plot the distribution.
3. Calculate the ordinal bifurcation diagram with r in (3.5,4).



cristina.masoller@upc.edu @cristinamasoll1

Outline

- Uncovering similarities between lasers and neurons
- Analysis of regime transitions: EEG signals and vegetation fields
- Network based analysis of retina fundus images

cristina.masoller@upc.edu @cristinamasoll1

Analysis of eyes-closed eyes-open transition in EEG recordings of healthy subjects.

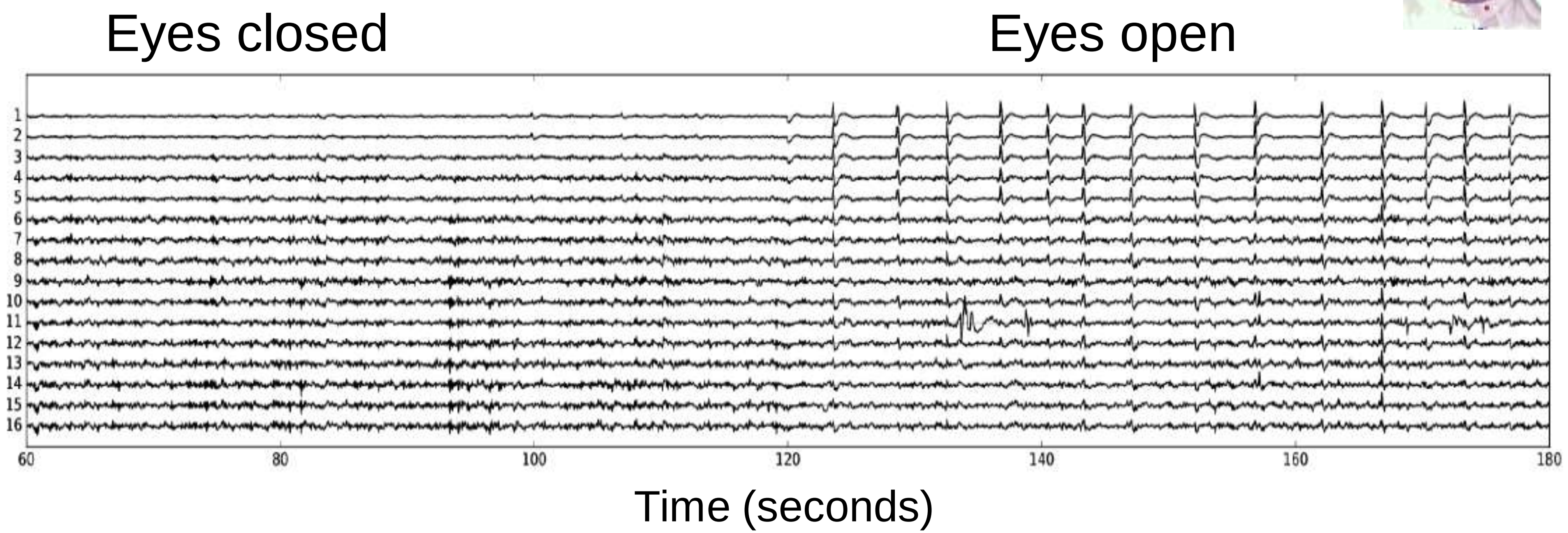


TABLE I. Description of the datasets used.

	DTS1	DTS2
Sampling rate (Hz)	256	160
Time task (seg)	120	60
Total points	30 720	9600
Number of electrodes	16	64
Number of subjects	71	109

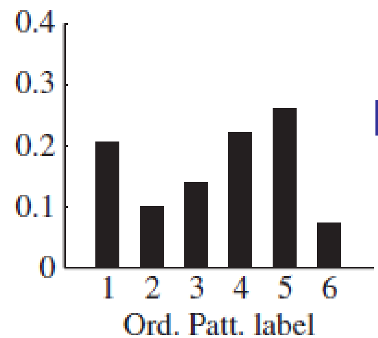
DTS1: Britbrain (Zaragoza)
DTS2: Physionet

cristina.masoller@upc.edu @cristinamasoll1

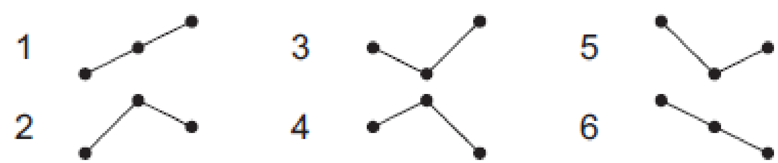
Permutation entropy: Shannon's entropy computed from ordinal probabilities

Time series

Ordinal probabilities



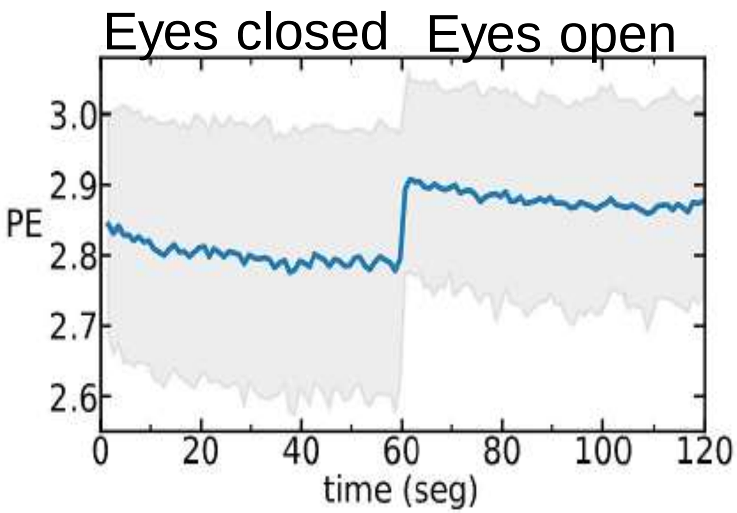
$$\sum_{i=1}^{N} p_i = 1$$

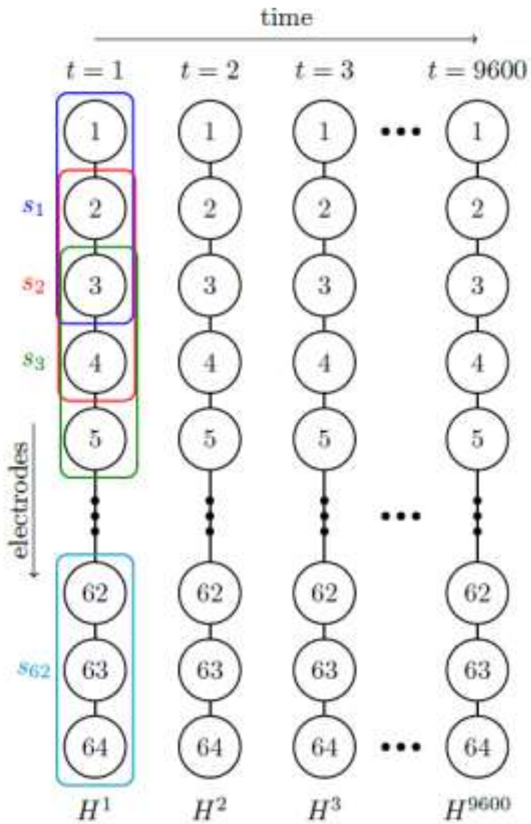


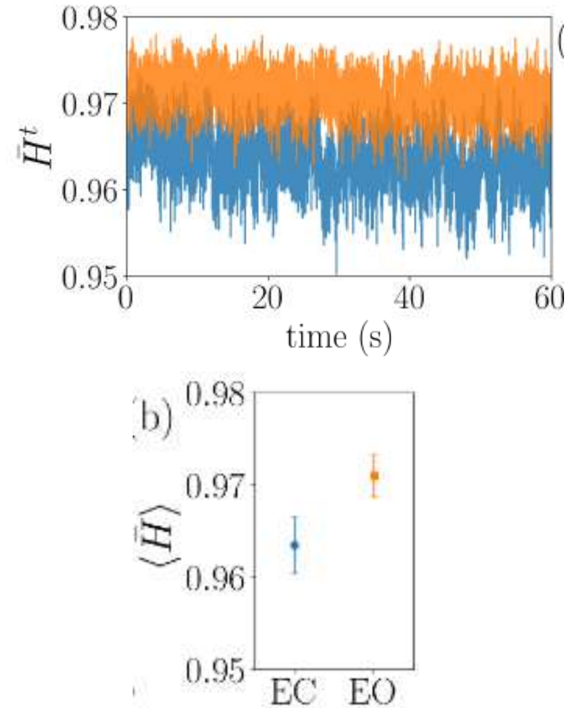
Permutation entropy

$$H = -\sum_{i=1}^{N} p_i \ln p_i$$

PE partially distinguishes EC-EO states



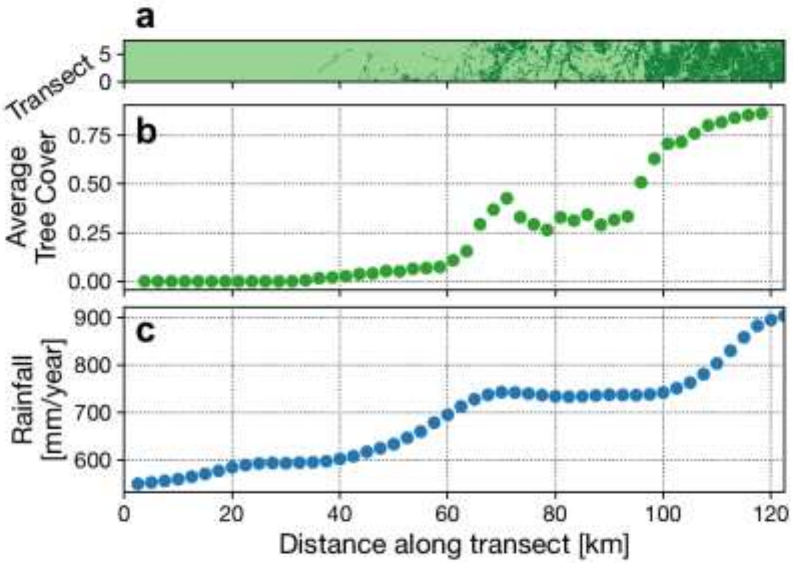


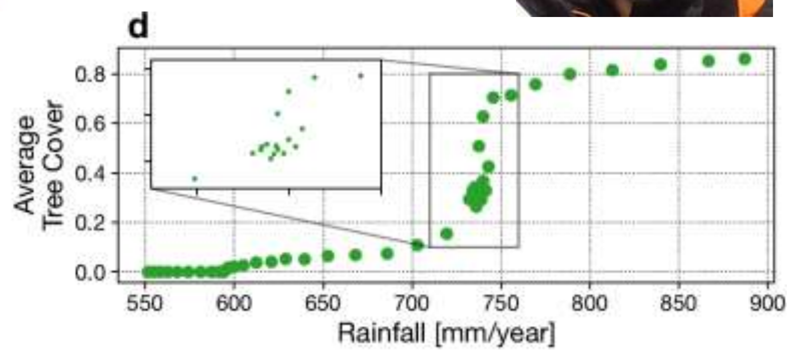
C. Quintero-Quiroz, L. Montesano, A. J. Pons, M. C. Torrent, J. García-Ojalvo, C. Masoller, "*Differentiating resting brain states using ordinal symbolic analysis*", Chaos 28, 106307 (2018).

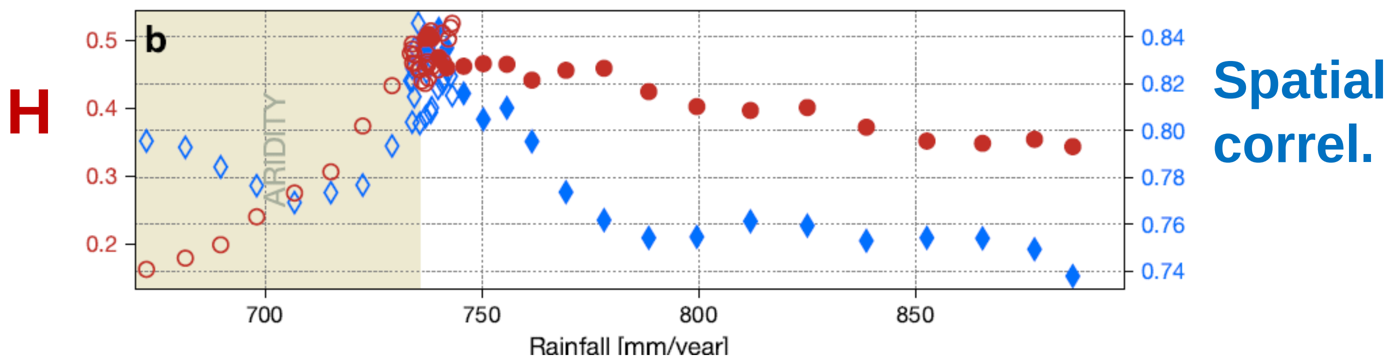
B. R. R. Boaretto, R. C. Budzinski, K. L. Rossi, C. Masoller, E. E. N. Macau, "*Spatial permutation entropy distinguishes resting brain states*", Chaos, Solitons & Fractals 171, 113453 (2023).

cristina.masoller@upc.edu @cristinamasoll1

The spatial permutation entropy can be an early indicator of vegetation transitions







G. Tirabassi and C. Masoller, "*Entropy-based early detection of critical transitions in spatial vegetation fields*", PNAS 120, e2215667120 (2022).

cristina.masoller@upc.edu @cristinamasoll1

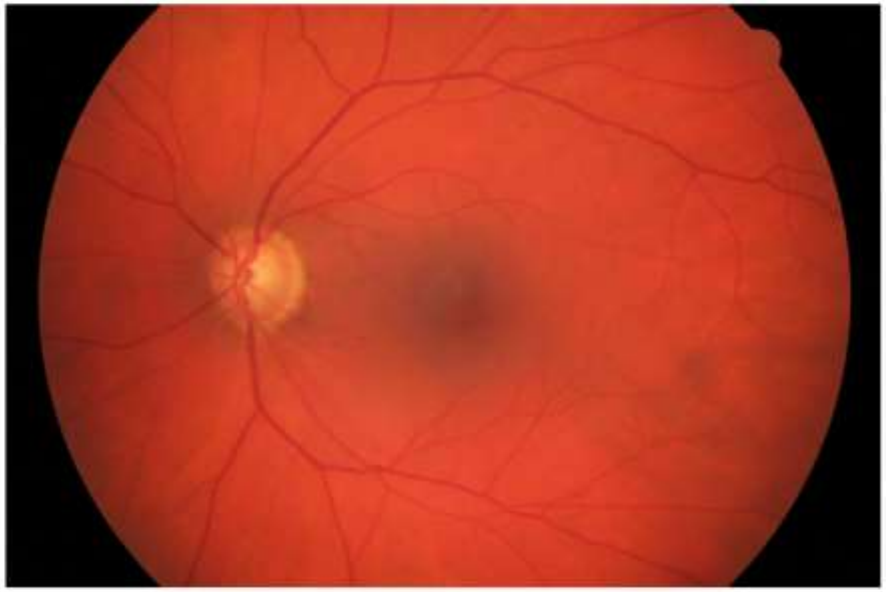
Outline

- Uncovering similarities between lasers and neurons
- Analysis of regime transitions: EEG signals and vegetation fields
- Network based analysis of retina fundus images

cristina.masoller@upc.edu @cristinamasoll1

Network-based analysis of retina fundus images

- For the diagnosis of eye diseases & follow up of treatments.
- Biometric identity identification.
- The retina is a window to the brain.
- Opportunity to detect other diseases: alterations in retina network may reflect alterations in other arterial systems.



BE-OPTICAL

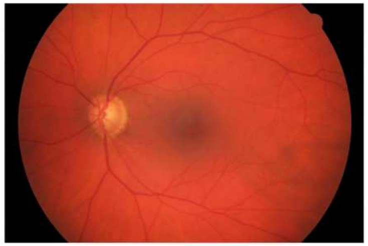
Advanced Biomedical Optical Imaging and Data Analysis

H2020-675512

cristina.masoller@upc.edu @cristinamasoll1

Data and image analysis steps

- 45 high resolution images (3504 × 2336 pixels)
  15 healthy subjects
  15 glaucoma
  15 diabetic retinopathy
- For every subject we had:
  – fundus photography
  – manual segmentation done by an expert ophthalmologist.



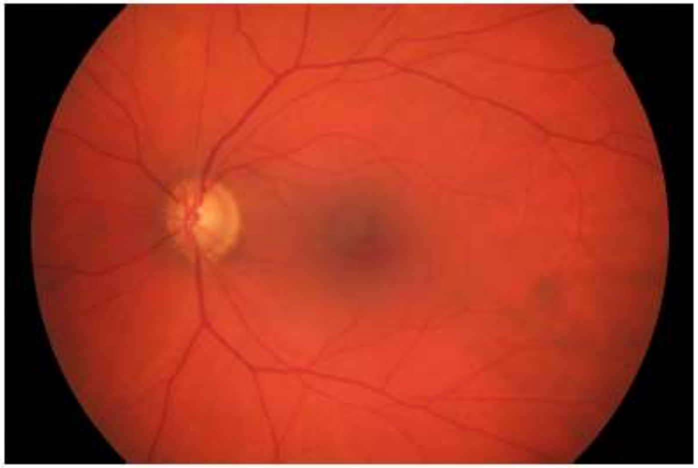
Steps:

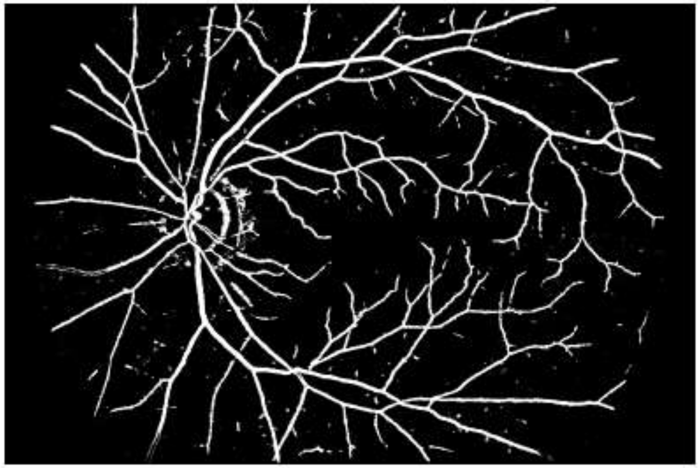
1. Pre-process and un-supervisely, segment the images.
2. Extract network.
3. Compare networks obtained from different images.
4. Classify the images.

https://www5.cs.fau.de/research/data/fundus-images/

cristina.masoller@upc.edu @cristinamasoll1

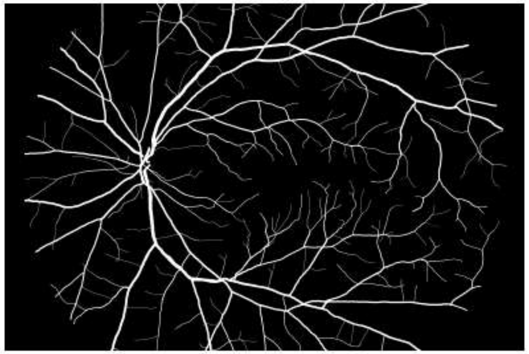
Step 1: Pre-process and segmentation





We adapted an *unsupervised* algorithm, originally developed for segmenting images of **cultured neural networks**.

Manual segmentation

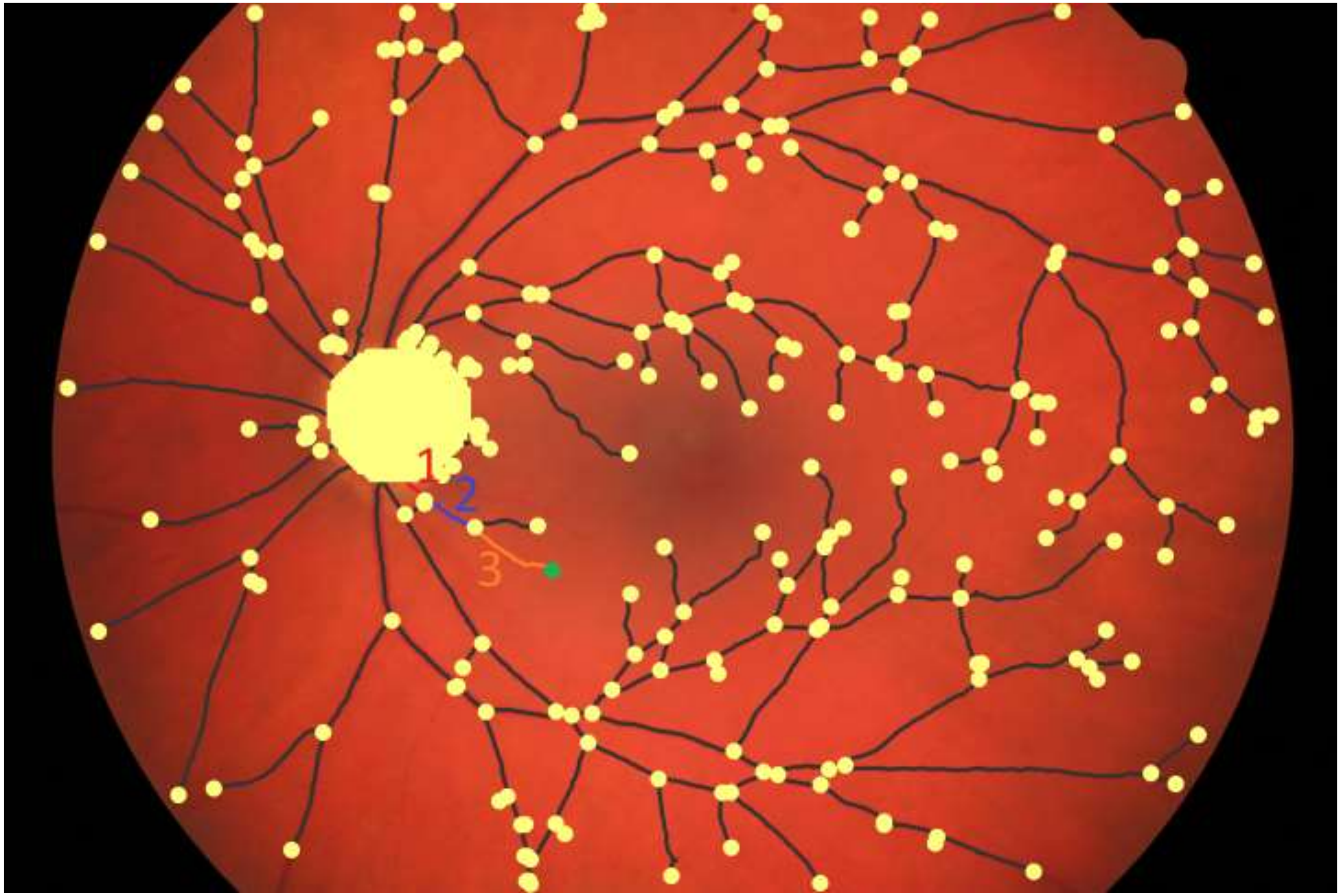


D. Santos-Sierra, I. Sendiña-Nadal, I. Leyva et al. Cytometry Part A. 87, 513 (2015).

P. Amil, F. Reyes-Manzano, L. Guzmán-Vargas, I. Sendiña-Nadal, C. Masoller, "*Network-based features for retinal fundus vessel structure analysis*", PLoS ONE 14, e0220132 (2019).

cristina.masoller@upc.edu **@cristinamasoll1**

Step 2: extract the network (identification of the optical nerve, nodes and links and assign weights to the links).

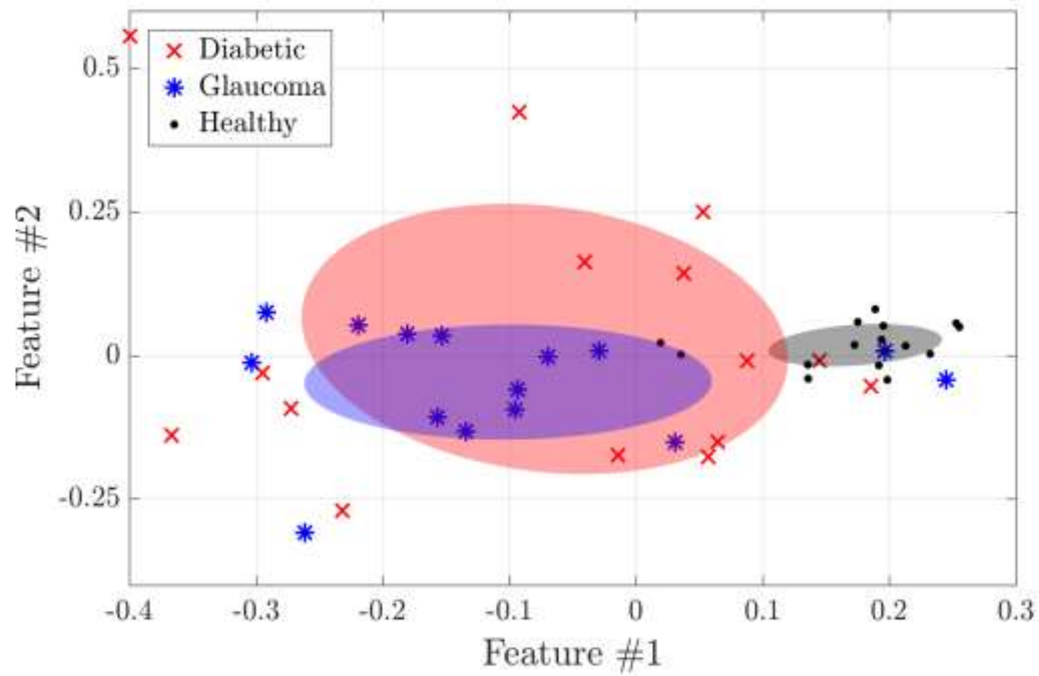


cristina.masoller@upc.edu @cristinamasoll1

Steps 3 and 4: Compare the networks extracted from different images and classify the images.

- $\{p_{i,j}\}$: distances between probability distributions that characterize the networks obtained from images i and j.
- We used nonlinear dimensionality reduction (*Isomap*) to reduce the set of 45x45 $\{p_{i,j}\}$ values to only two features.

Distance distribution to the central node in the *manual* segmentation

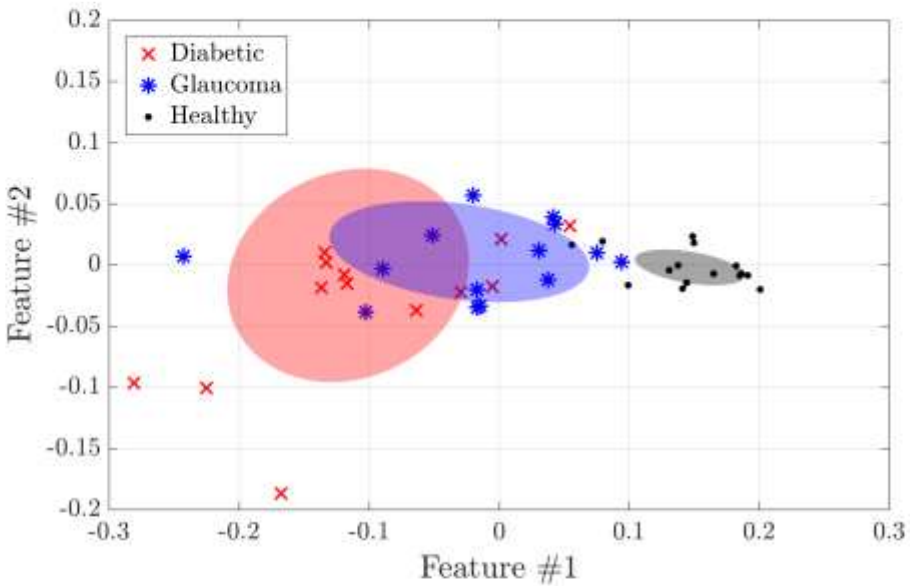


P. Amil et al, Network-based features for retinal fundus vessel structure analysis, PLoS ONE 14 e0220132 (2019).

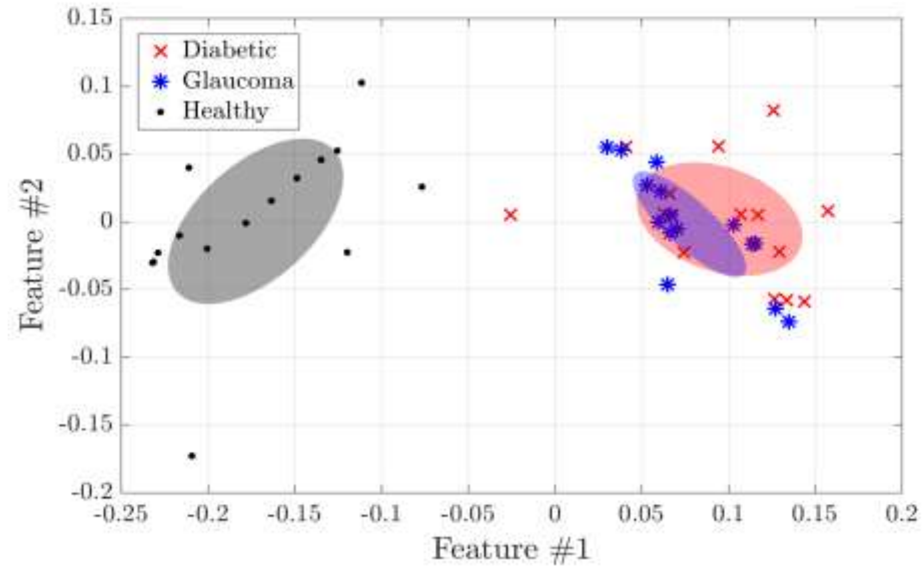
cristina.masoller@upc.edu @cristinamasoll1

Performance of network features in the *manual* segmentation

Distribution of weights along the shortest path to central node



Distribution of weighted degrees

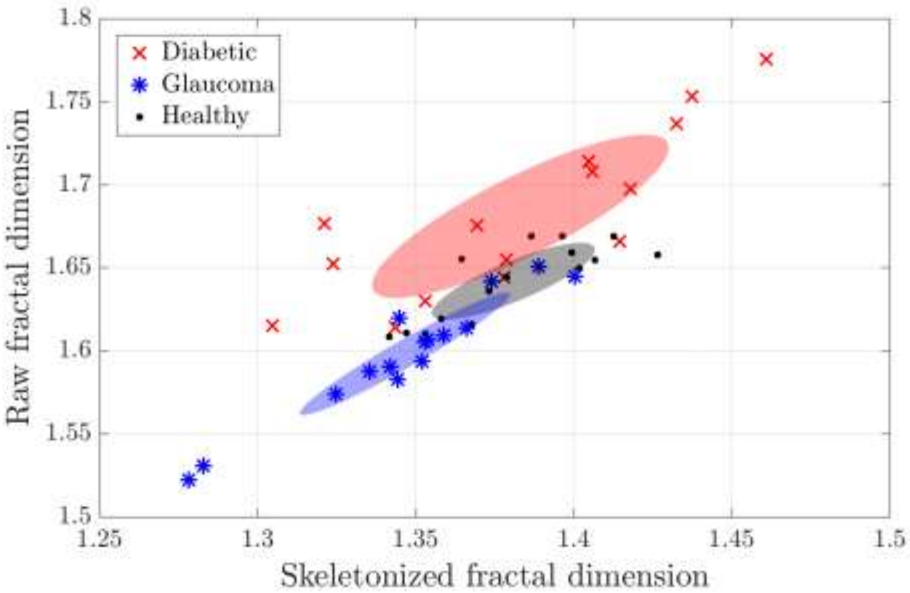


P. Amil et al, Network-based features for retinal fundus vessel structure analysis, PLoS ONE 14 e0220132 (2019).

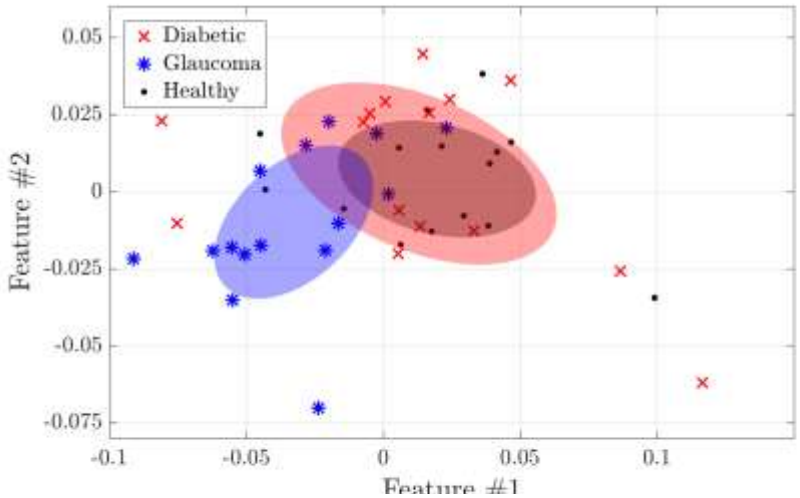
cristina.masoller@upc.edu @cristinamasoll1

In the automated segmentation

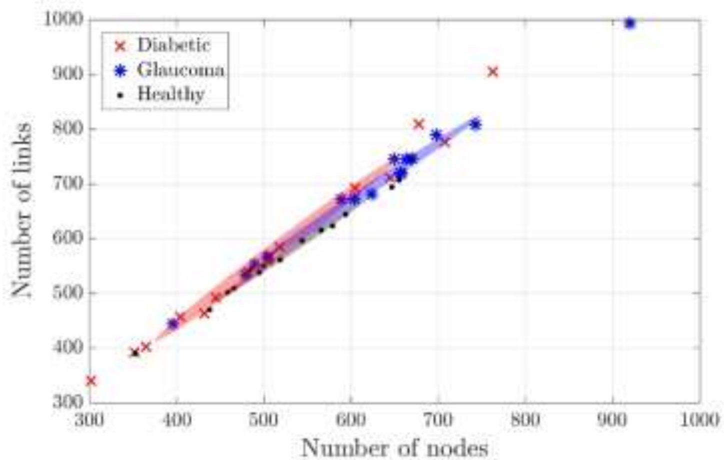
Fractal dimension $D = \lim_{\varepsilon \to 0} \frac{\log(N(\varepsilon))}{\log(1/\varepsilon)}$



Mean weight distribution along the shortest path to central node



Simple network features do not differentiate



P. Amil et al, Network-based features for retinal fundus vessel structure analysis, PLoS ONE 14 e0220132 (2019).

cristina.masoller@upc.edu @cristinamasoll1

Take home messages

- Data analysis techniques allow us to uncover similarities in very different systems.
- Different methods provide *complementary* information, and can be adapted to study very different systems.
- “Surrogate” tests are needed to determine if the numerical values are statistically significant.
- Holger Kantz: “*Every data set bears its own difficulties: data analysis is never routine*”.

cristina.masoller@upc.edu @cristinamasoll1

References

- A. Aragoneses et. al, "*Unveiling the complex organization of recurrent patterns in spiking dynamical systems*", Sci. Rep. 4, 4696 (2014).
- J. A. Reinoso et. al, "*Emergence of spike correlations in periodically forced excitable systems*", Phys. Rev. E. 94, 032218 (2016).
- J. Tiana-Alsina et. al, "*Comparing the dynamics of periodically forced lasers and neurons*", New J. of Phys. 21, 103039 (2019).
- M. Masoliver and C. Masoller, "*Neuronal coupling benefits the encoding of weak periodic signals in symbolic spike patterns*", Commun. Nonlinear Sci. Numer. Simulat. 88, 105023 (2020).

- C. Quintero-Quiroz et. al, "*Differentiating resting brain states using ordinal symbolic analysis*", Chaos 28, 106307 (2018).
- G. Tirabassi and C. Masoller, "*Entropy-based early detection of critical transitions in spatial vegetation fields*", PNAS 120, e2215667120 (2022).
- B. R. R. Boaretto, R. C. Budzinski, K. L. Rossi, C. Masoller, E. E. N. Macau, "*Spatial permutation entropy distinguishes resting brain states*", Chaos, Solitons & Fractals 171, 113453 (2023).

- P. Amil et. al, "*Network-based features for retinal fundus vessel structure analysis*", PLoS ONE 14, e0220132 (2019).

cristina.masoller@upc.edu @cristinamasoll1