Quench dynamics of the Kondo effect

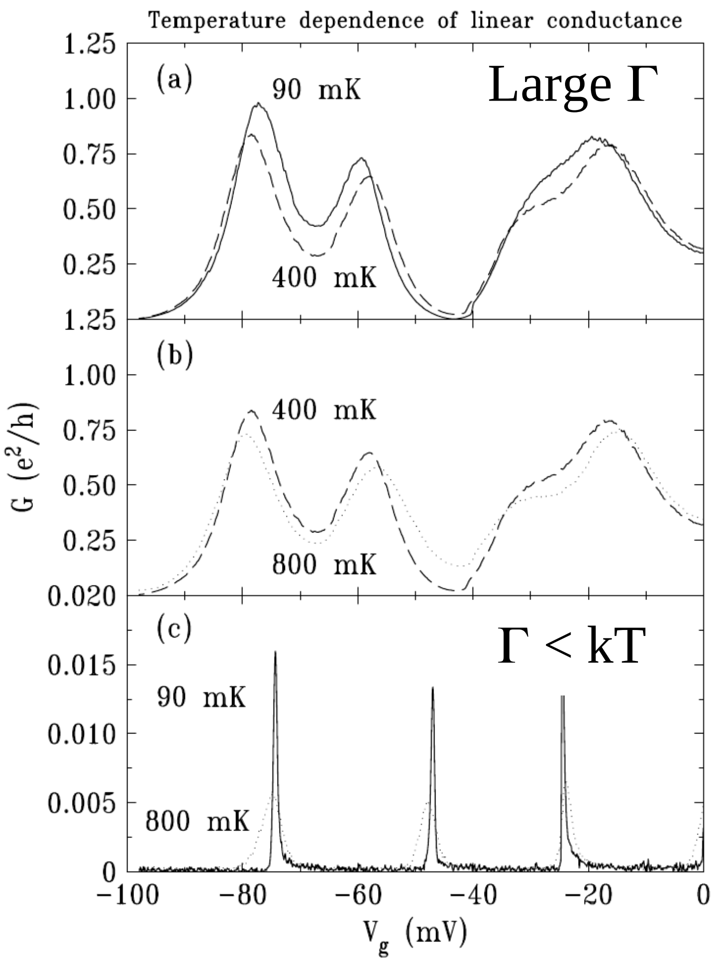
Maria Carolina O. Aguiar

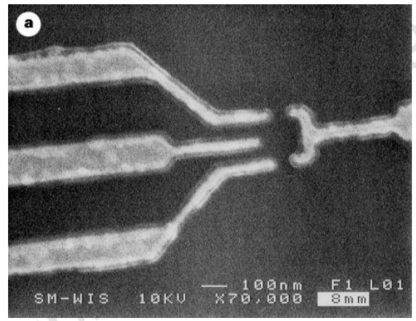
UFMG

Federal University of Minas Gerais, Brazil

aguiar@fisica.ufmg.br

✓ **First time I studied Kondo effect was for Eduardo's many body course in 1998 at UNICAMP**





Metallic gates on a GaAs/AlGaAs heterostructure containing a 2D electron gas

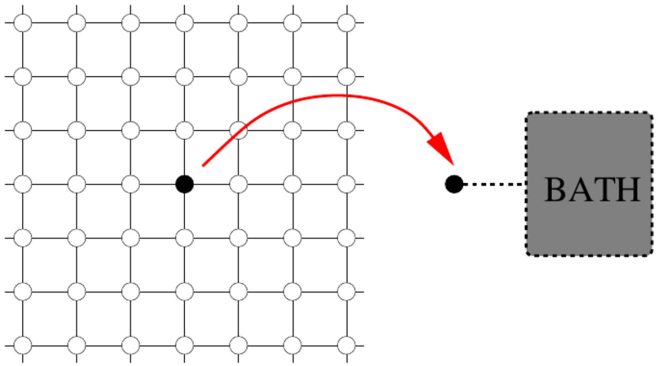
For odd *N*, an unpaired electron can form a singlet with conduction electrons through the Kondo effect, resulting in a larger DOS at the Fermi level and higher conductance

Energy Γ: coupling of electronic states on the artificial atom to those on the leads, resulting from tunnelling

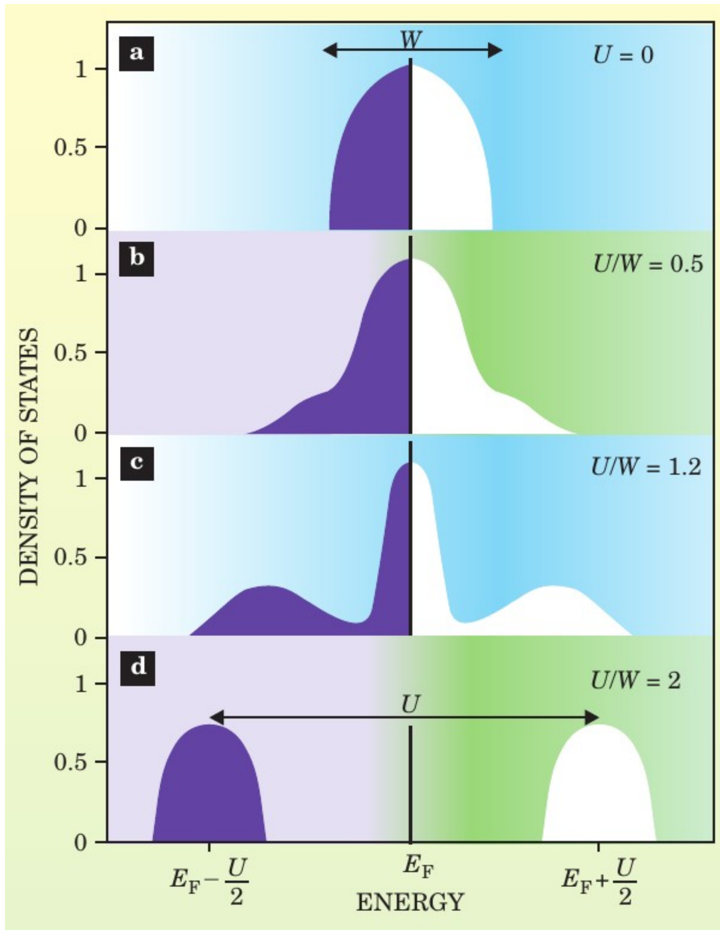
Kondo effect in a single-electron transistor, Goldhaber-Gordon *et al.*, Nature **391**, 156 (1998)

Impurity problem within DMFT

✓ **One year later, I started my PhD with Eduardo and started studying Dynamical Mean Field Theory**



Lattice problem $\Rightarrow$
single-impurity problem +
bath of conduction electrons



DMFT solution for the Hubbard model

Outline

✓ Introduction

- Quantum quench and experimental motivation

✓ Dynamics after connecting an impurity to interacting chains

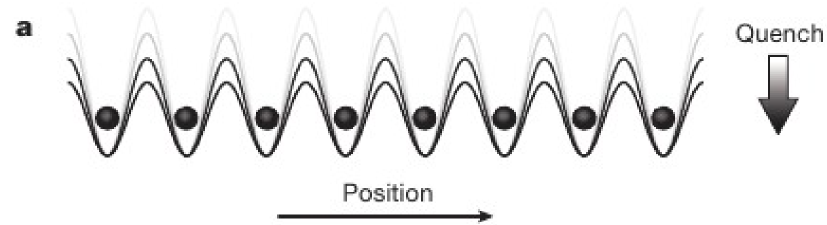
- **Magnetization at the impurity site decays faster if we increase the interaction *U* in the chains**, even though the spin velocity decreases
- For small *U*, we obtain an **analytical expression for the Kondo time $\tau_K(U)$**, which confirms that ***U* favors the formation of the Kondo cloud**

Quantum quench

$$H_i\,|\Psi\rangle_i = E_i\,|\Psi\rangle_i$$

$$H_i \Rightarrow H_f$$

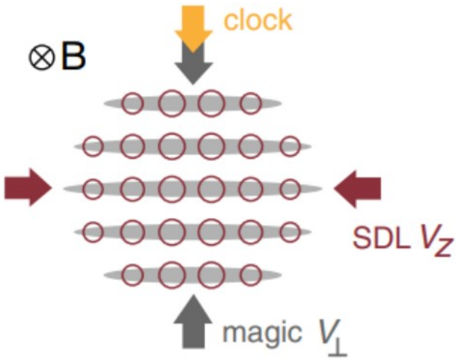
$$|\Psi(t)\rangle = e^{-iH_f t}\,|\Psi\rangle_i$$



Quench in Rb optical lattice
Cheneau et al., Nature **481**, 484 (2012)

Quench in Yb optical lattice

Experimental realization of interaction between localized magnetic moments and delocalized states



Quasi-1D tubes

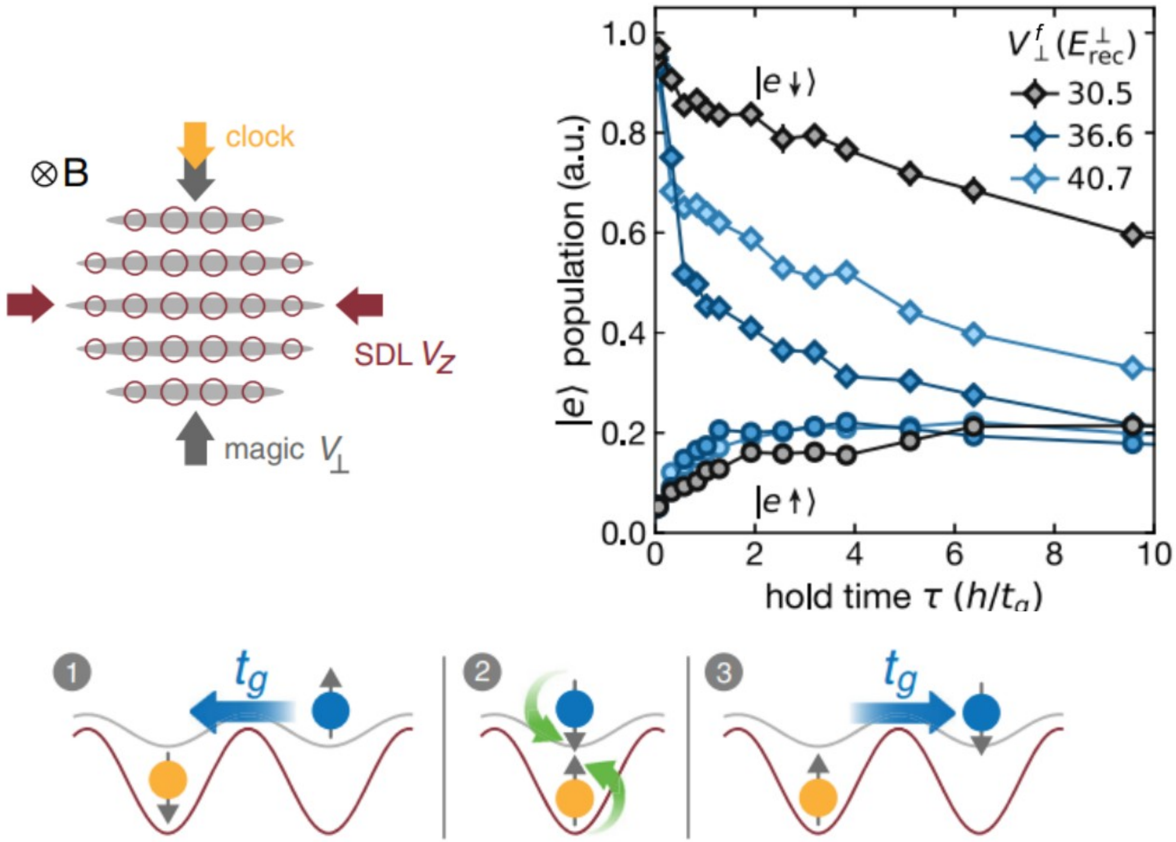
Gray: shallow potential for **mobile atoms |g>**

Red: lattice wells for **localized atoms |e>**

Initial state: particles with ↓ spins are localized at the wells (magnetic field of 20G)

Riegger *et al.*, Phys. Rev. Lett. **120**, 143601 (2018)

Experimental realization of interaction between localized magnetic moments and delocalized states



Temporal evolution of ↑ and ↓ spin populations, showing spin relaxation of localized ↓ spin particles (initial state), after the following quench:

B = 20 G → 1G

V_z = 8 → 5.7

(units of E_{rec}^z/h = 2.57 kHz)

$V_\perp$ = 45 → $V_\perp^f$

(units of $E_{rec}^\perp/h$ = 2.00 kHz)

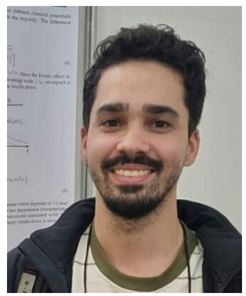
Spin-flip process between **localized magnetic moments (yellow)** and **delocalized particles (blue)**

Riegger *et al.*, Phys. Rev. Lett. **120**, 143601 (2018)

Quench dynamics of the Kondo effect

Spin relaxation: Phys. Rev. B **103**, 125152 (2021)

Transport across the impurity: Phys. Rev. B **107**, 075110 (2023)

Helena Bragança (1, 2)

Moallison F. Cavalcante (1)

Rodrigo G. Pereira (3)

M. Carolina O. Aguiar (1)

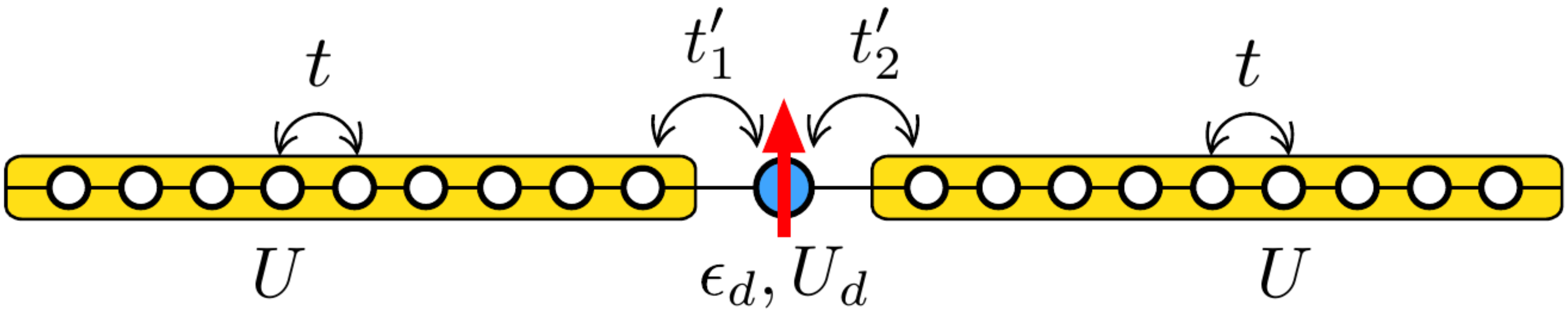
UFMG (1)

UnB (2)

INTERNATIONAL INSTITUTE OF PHYSICS
Federal University of Rio Grande do Norte (3)

Our work: hybridization quench

We study the **out-of-equilibrium dynamics after a local quench that connects one impurity to interacting Hubbard chains**



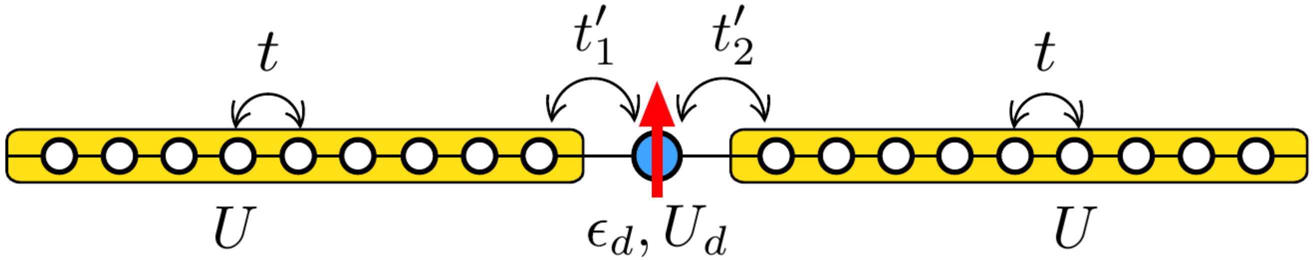
Initial state: $\boldsymbol{\tau < 0}$ $\quad |\Psi_0\rangle = |GS\rangle_L \otimes |\uparrow\rangle \otimes |GS\rangle_R$

After the quench: $\boldsymbol{\tau > 0}$

We use time-dependent density matrix renormalization group (t-DMRG), as well as bosonization within Tomonaga-Luttinger liquid theory

Our interest is in the **Kondo effect**, specially the **role played by electronic interactions in the chains**

Impurity coupled to two interacting chains



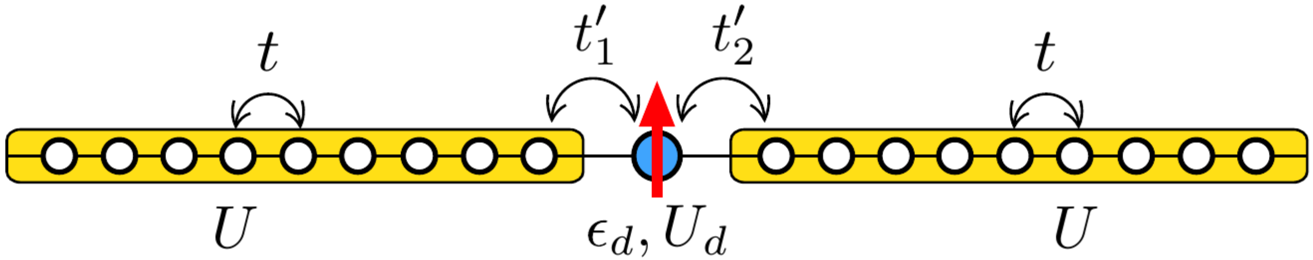
$L_1 = 23$, $L_2 = 24$, $t = 1$, $t'_{1,2} = 0.5$, ¼ filling

Hamiltonian after the quench: impurity coupled to interacting chains

$$H(\tau) = H_{\text{leads}} + H_{\text{imp}} + \Theta(\tau) H_{\text{hyb}}$$

$$H_{\text{leads}} = \sum_{\ell=1}^{2} H_\ell \qquad H_{1/2} = -t \sum_{i\sigma} (c^\dagger_{i\sigma} c_{i+1\sigma} + \text{h.c.}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Impurity coupled to two interacting chains



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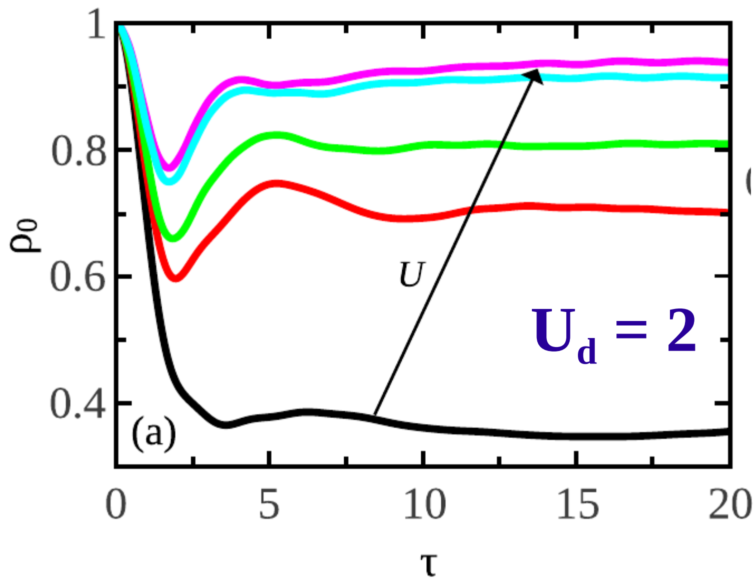
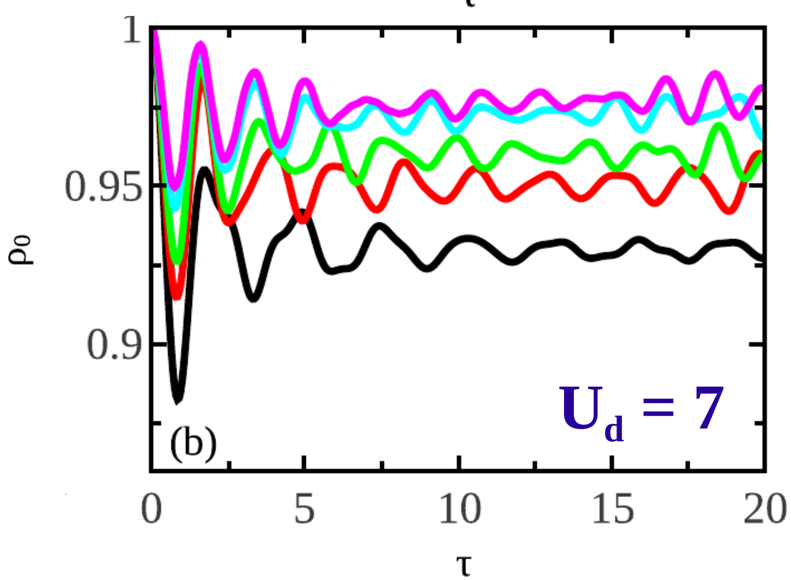
$$H_{\text{imp}} = \epsilon_d n_0 + U_d n_{0\uparrow} n_{0\downarrow} \qquad n_0 = n_{0\uparrow} + n_{0\downarrow} \qquad \epsilon_d = -U_d/2$$

$$H_{\text{hyb}} = -t'_1 \sum_\sigma c^\dagger_{-1\sigma} c_{0\sigma} - t'_2 \sum_\sigma c^\dagger_{0\sigma} c_{1\sigma} + \text{H.c.}$$

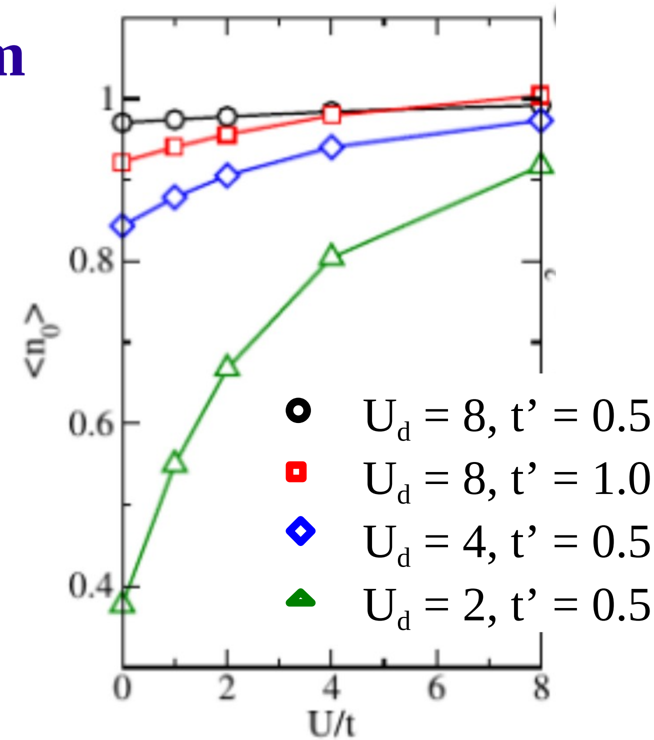
Occupation at the impurity as a function of time





U = 0, 2, 4, 8, 10

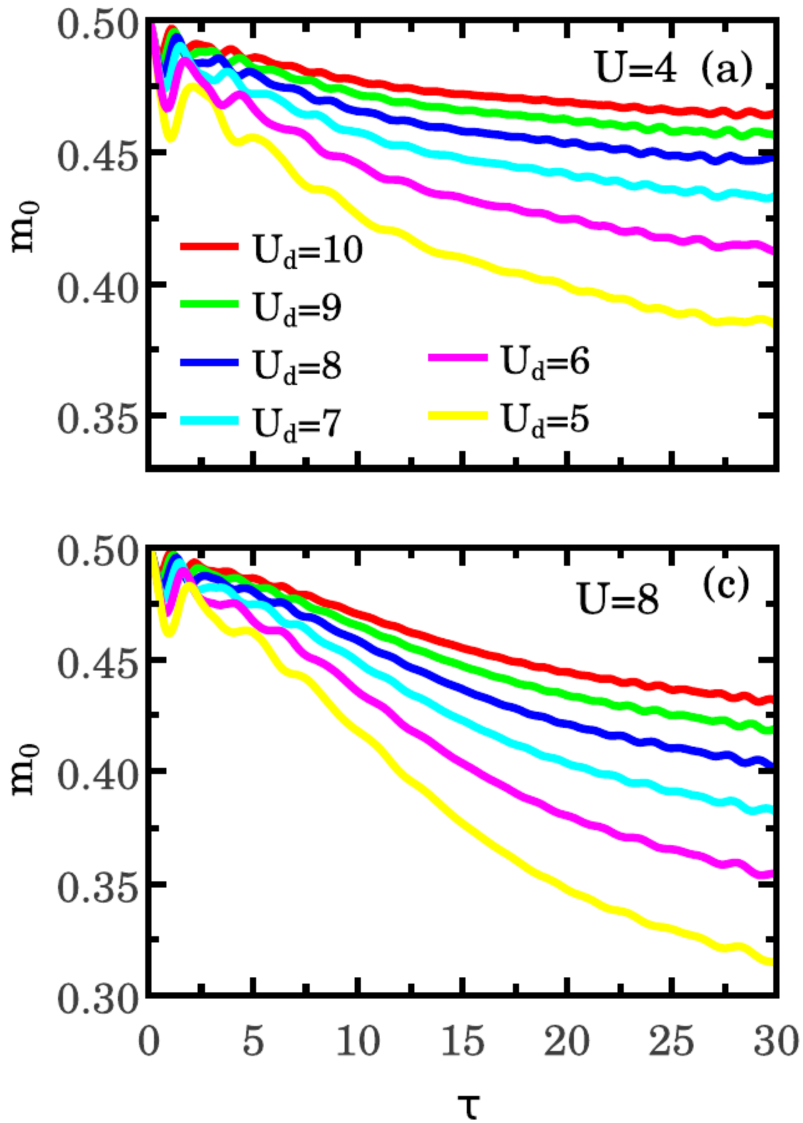
Equilibrium results



Costamagna, Gazza, Torio, and Riera, Phys. Rev. B 74, 195103 (2006)

Occupation at the impurity approaches the equilibrium values; deviation from single occupancy decreases as we increase both U and U_d

Sz at the impurity as a function of time

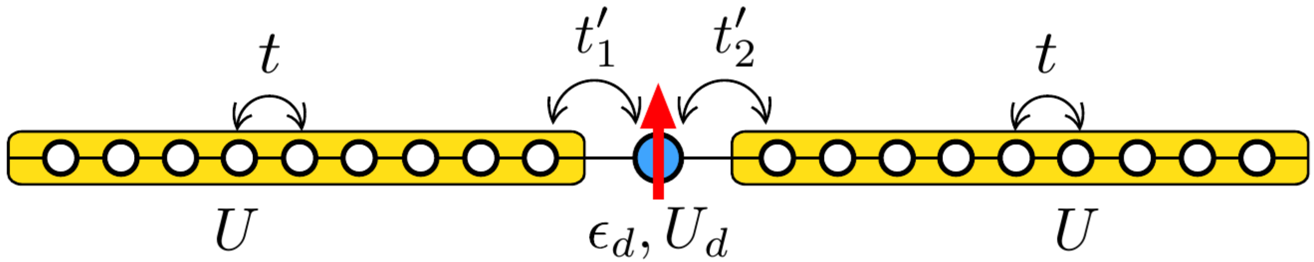


Single wire

- **Sz at the impurity** (m_0) **approaches the equilibrium value**, which is zero due to time reversal symmetry
- Total magnetization of the system is conserved: m_0 can only decay because it is transported away from the impurity
- For fixed U, **magnetization decays faster with decreasing U_d**; for fixed U_d, it decays **faster if U increases**

Next: we use a field theory approach to better understand the dependence of m_0 with U and U_d...

Effective model in the Kondo regime



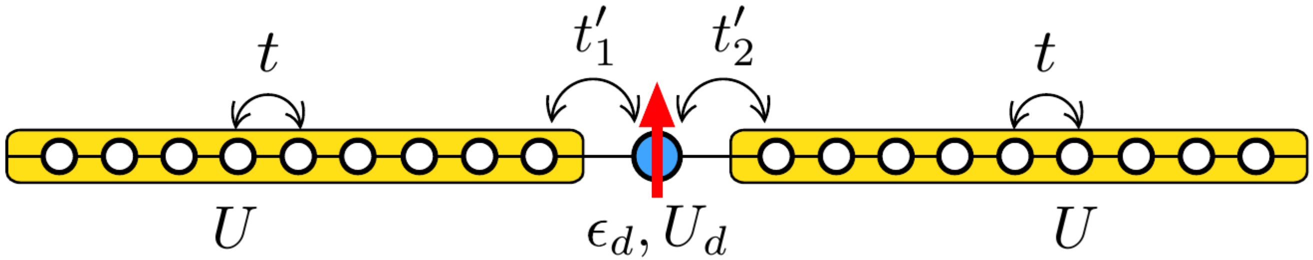
$$t'_{1,2} \ll -\epsilon_d, U_d \qquad\qquad H_{\text{eff}}(\tau) = H_{\text{leads}} + \Theta(\tau) H_K$$

$$t'_1 = t'_2 \qquad\qquad H_K = J_K \mathbf{S}_0 \cdot \underbrace{(\kappa_1 c^\dagger_{-1} + \kappa_2 c^\dagger_1)\frac{\boldsymbol{\sigma}}{2}(\kappa_1 c_{-1} + \kappa_2 c_1)}_{\text{Effective spin involving 1st site in each wire}}$$

We use **Luttinger model** to describe the disconnected **interacting leads** and then apply **bosonization**; Kondo term is also written in bosonized form. Next we consider an **expansion in powers of the Kondo coupling $\lambda_K \propto J_K$**

Sz at the impurity as a function of time

We calculate the magnetization at the impurity site



$$m_0(\tau) = \langle \Psi_I(\tau) | S^z_{0,I}(\tau) | \Psi_I(\tau) \rangle$$

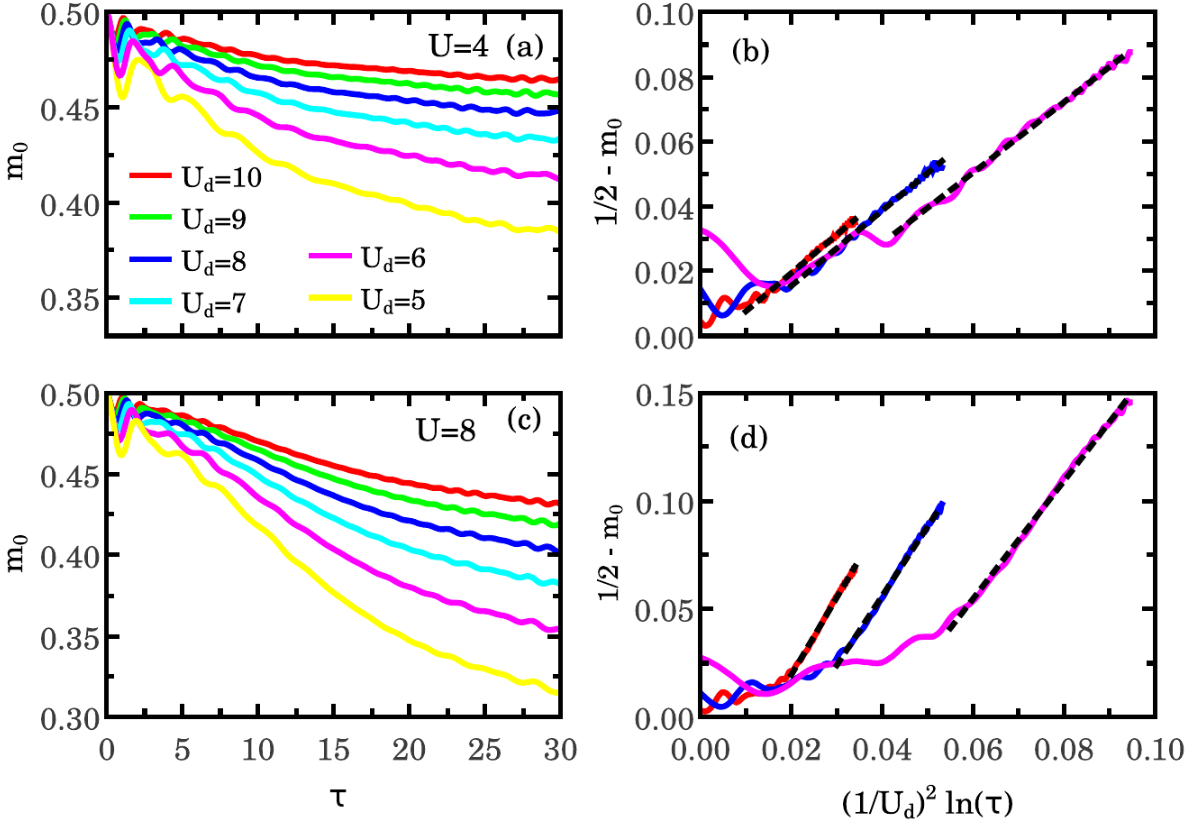
$$\Lambda^{-1} \lesssim \tau \ll \tau_K$$

$$\tau_K \sim \Lambda^{-1} e^{\pi v_F / \lambda_K}$$

For $\tau \sim \tau_K$ higher-order corrections begin to renormalize the effective Kondo coupling

$\Lambda \sim v_F/\alpha$ is an ultraviolet cutoff and α is a short-distance cutoff

Sz at the impurity as a function of time



Single wire

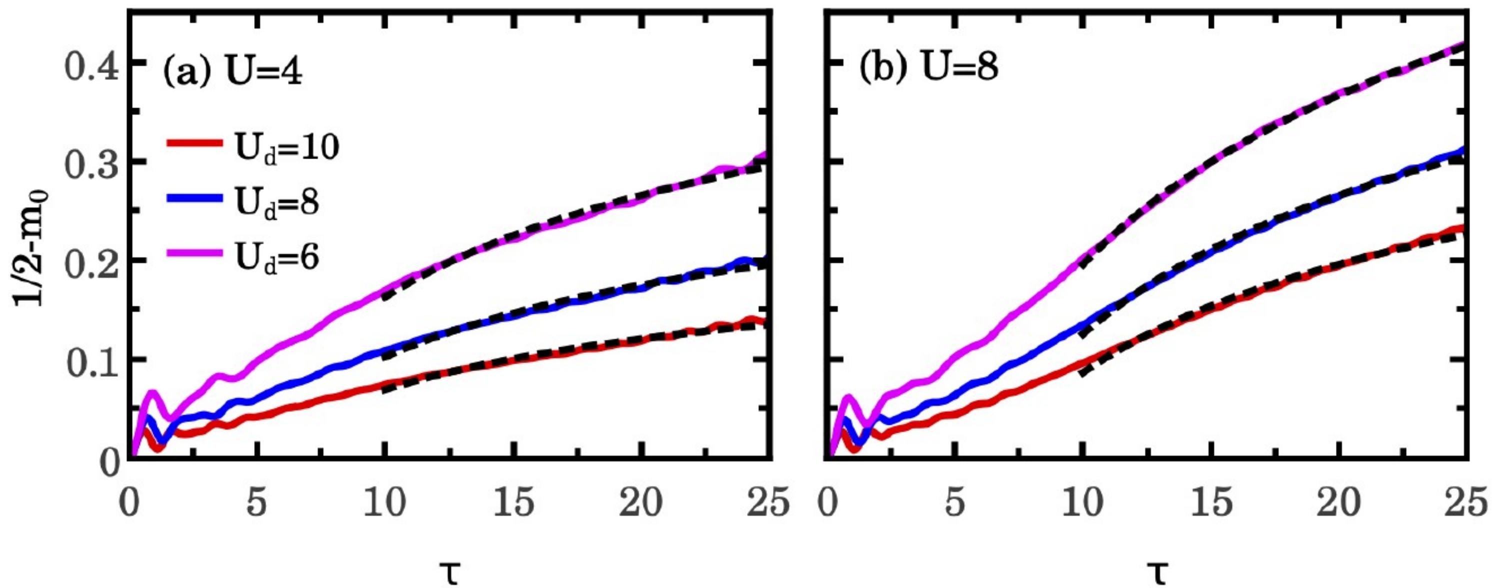
$$\Lambda^{-1} \ll \tau \ll \tau_K$$

$$\Delta m(0,\tau) \approx \frac{A}{U_d^2} \ln \tau + \mathrm{c}$$

$$A = 32(t_2')^4/(\pi\alpha\Lambda)^2$$

- For times $\tau \geq 10$, **variation in magnetization scales logarithmically with time**; different slopes show that prefactor of the logarithmic time dependence increases with U
- For fixed U, **magnetization decays faster for smaller U_d, for which the Kondo coupling is stronger** ($\lambda_K \propto 1/U_d$)

Sz at the impurity as a function of time



Two wires

K_c: Luttinger parameter

$$\Lambda^{-1} \ll \tau \ll \tau_K \qquad \Delta m(0,\tau) \approx \frac{2A}{U_d^2} \ln \tau - B\tau^{1-K_c^{-1}} + C$$

- Within the limited time range available numerically we are **not able** to unambiguously **distinguish between a pure logarithmic dependence**, as expected for the noninteracting case ($K_c \to 1$), **and the combination of a logarithm and a power law** with exponent $1 - K_c^{-1}$

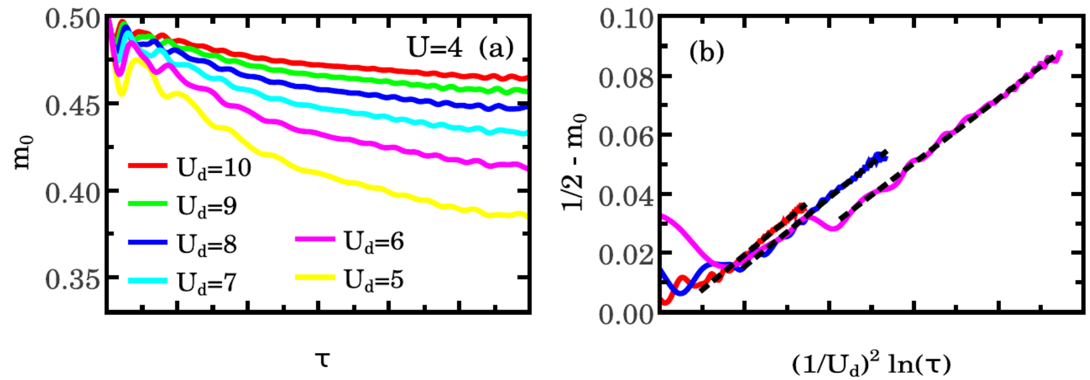
Magnetization and spin current – small τ

One wire

$\tau \ll \tau_K$

$$m_0(\tau) = \frac{1}{2} - \frac{1}{2}{\lambda_K^{(0)}}^2 \ln \Lambda\tau \left[1 + \lambda_K^{(0)} \ln \Lambda\tau\right] + \mathcal{O}(\lambda_K^4)$$

Here we go **beyond our previous results**, which were obtained up to 2nd order in the Kondo coupling



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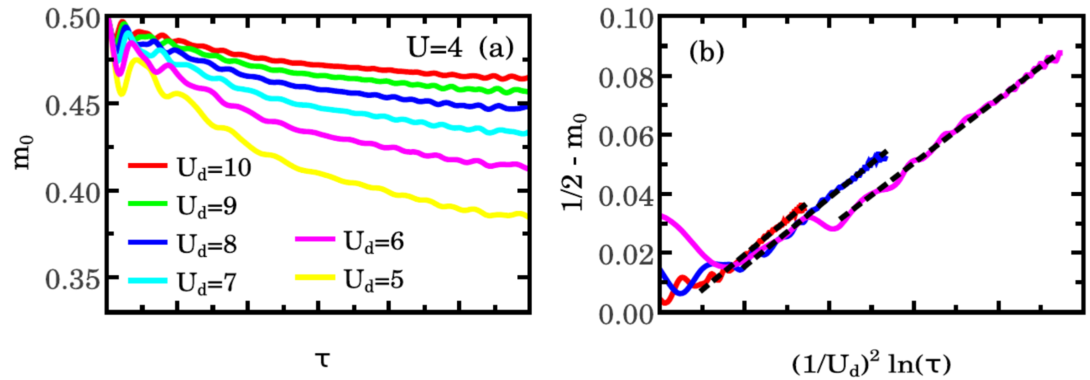
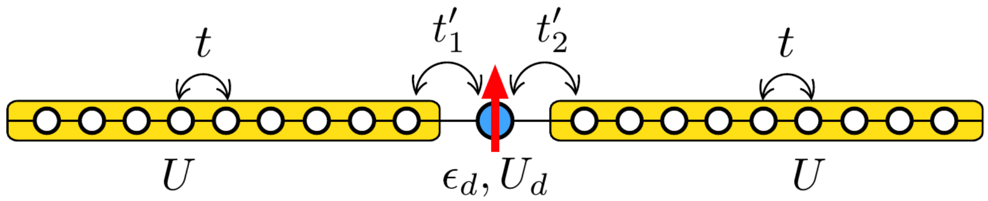


$$\langle j_s^z \rangle + \frac{d}{d\tau} m_0(\tau) = 0 \qquad j_s^z = \frac{d}{d\tau} S^z \qquad \langle j_s^z \rangle = \frac{1}{2\tau} \frac{1}{\ln^2\left(\tau_K/\tau\right)}$$

The impurity magnetization is not a function of τ_K/τ but its derivative – the spin current - is

Charge current across the impurity

To describe **transport** we turn on **chemical potentials** in each wire



$$H \to H + \sum_{\ell=1,2} \mu_\ell N_\ell$$

N_l is the number operator in wire l;
$e\mu = \mu_2 - \mu_1$ plays the role of a gate voltage

$$\hat{j}(\tau) = e\frac{d}{d\tau}(N_1 - N_2)$$

Current through the impurity

As before, we consider **perturbation theory in the Kondo coupling λ_K**

Up to λ_K^2 we recover the result for the current throug a non-magnetic impurity $\Rightarrow$ **to describe the Kondo effect we need to go beyond λ_K^2**

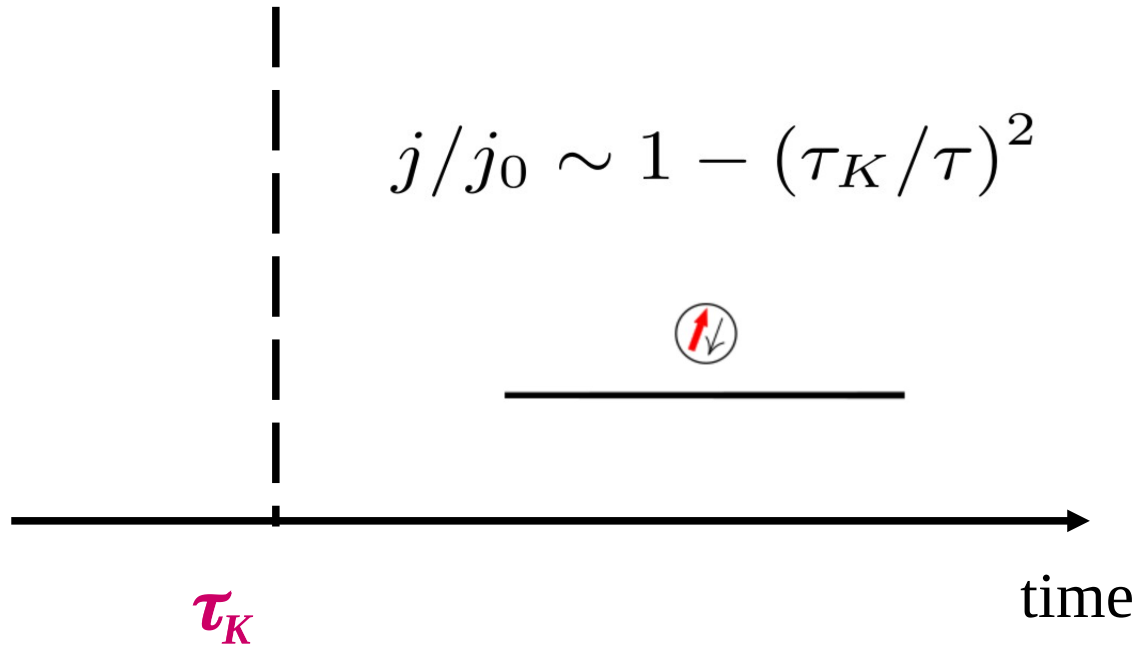
Time dependent transport

U = 0

$$\Lambda^{-1} \ll \tau$$

$$j/j_0 \sim \frac{1}{\ln^2\left(\tau_K/\tau\right)}$$

$$j/j_0 \sim 1 - \left(\tau_K/\tau\right)^2$$



For $\tau \sim \tau_K$, the problem is non-perturbative; since Kondo physics corresponds to a crossover (not a phase transition), **there should be a smooth function *j*(τ_K/τ) connecting the two results above**

U = 0

$$\Lambda^{-1} \ll \tau$$

$$j/j_0 \sim 1 - (\tau_K/\tau)^2$$

$$j/j_0 \sim \frac{1}{\ln^2(\tau_K/\tau)}$$

time

τ_K, τ_K*

$$j/j_0 \sim \frac{1}{(\Lambda\tau)^{K_c^{-1}-1}} \times$$

$$\left\{1 + 2\lambda_K^{(0)}\left[\ln(\Lambda\tau) + \beta(K_c)\right]\right\}$$

$$j/j_0 \sim \frac{1}{\tau^{K_c^{-1}-1}}$$

U > 0

Time dependent transport

Perturbation theory in the Kondo coupling **breaks down at τ_K and τ^* for U = 0 and small U**, respectively

$$\tau_K \sim \Lambda^{-1} \exp \lambda_K^{(0)^{-1}}$$

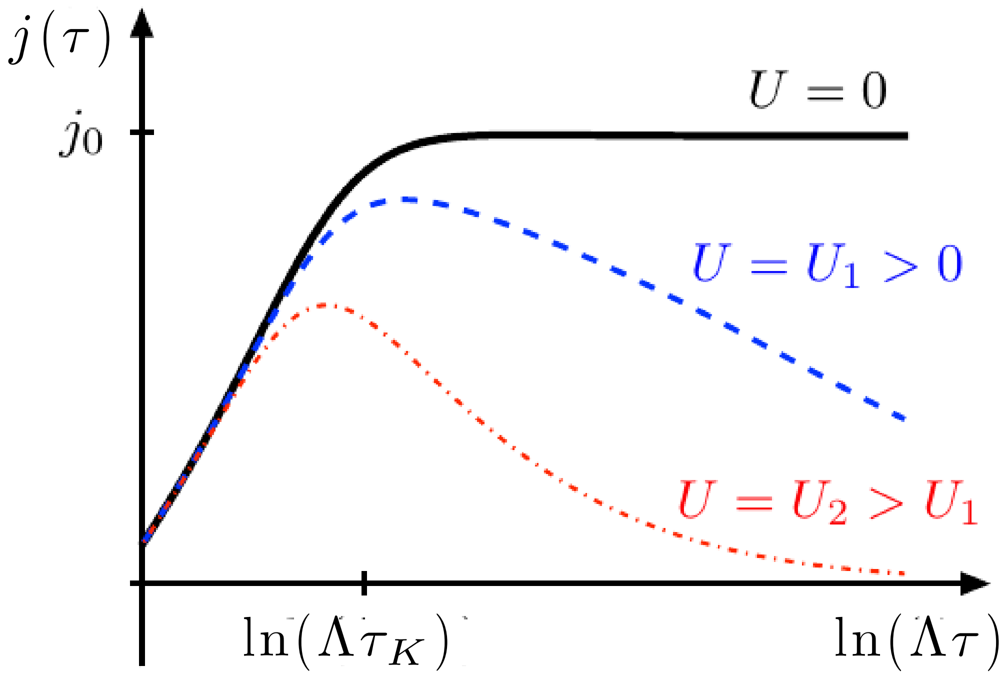
$$\tau_K^* \sim \tau_K \left[1 - \frac{\pi U}{12 v_F}\right]$$

$$K_c \sim 1$$

$$K_c \approx 1 - U/\pi v_F$$

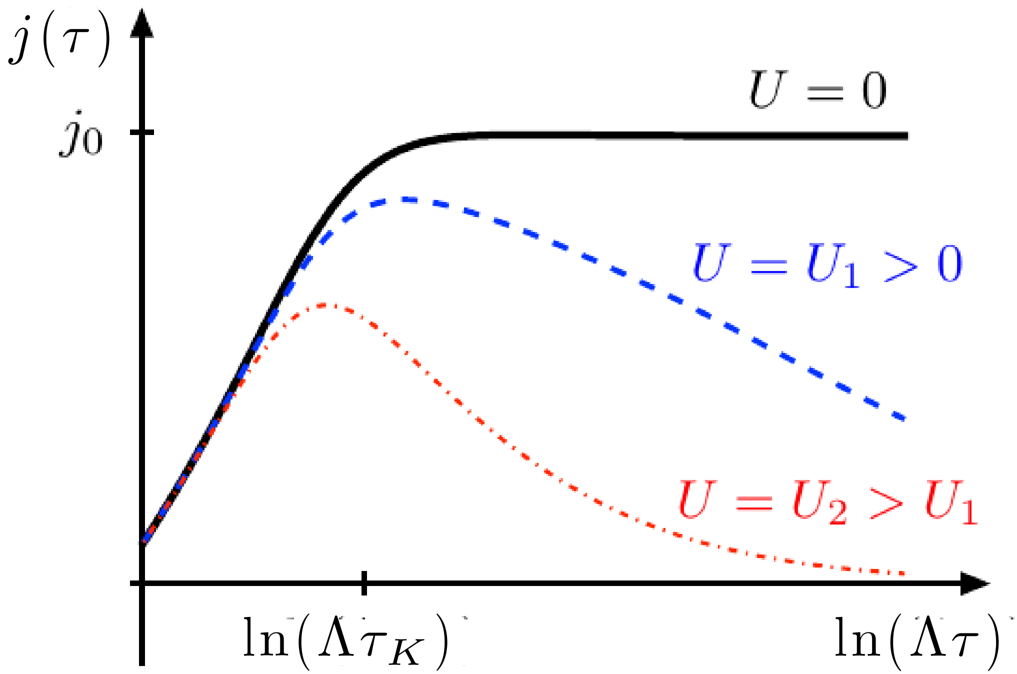
As we know, interactions in the wires destroy the universality of the Kondo problem; thus **the current is not a function of τ_K^*/τ**

Formation of the Kondo singlet cloud is favored by finite U in the wires, in accordance with our numerical results



Small U

$$\tau_K^* \sim \tau_K \left[1 - \frac{\pi U}{12 v_F}\right]$$



Small U

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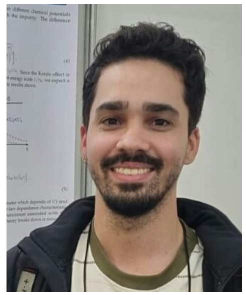
Poster session

Wednesday: Raman Response of the Charge Density Wave in Cuprate Superconductors

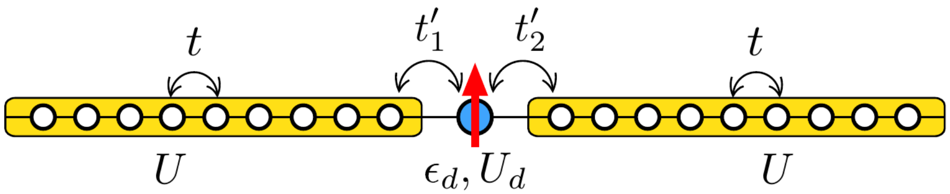
Thursday: Quench dynamics of the Kondo effect: transport across an impurity coupled to interacting wires



Moallison F. Cavalcante

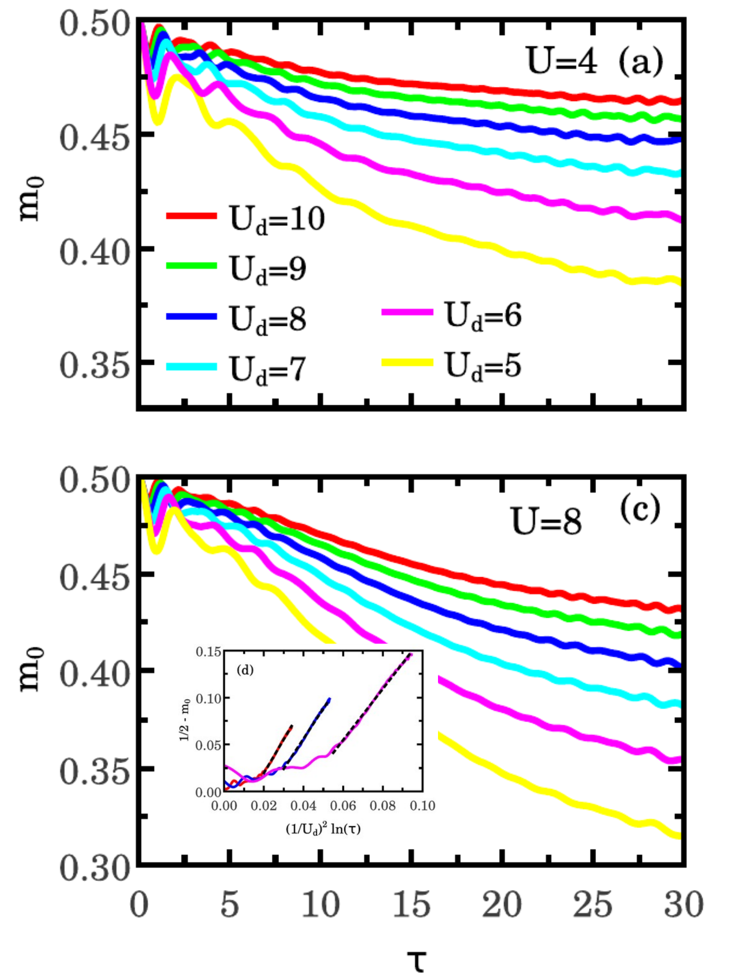
Conclusions

✓ **Dynamics after connecting an impurity to interacting chains**



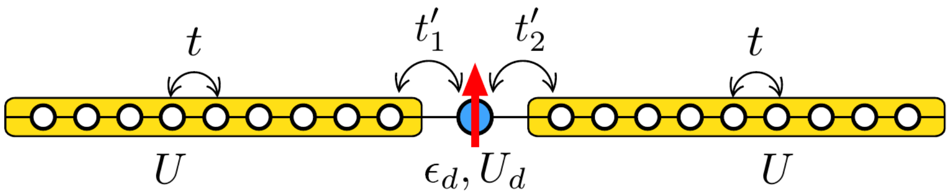
For fixed U_d, **magnetization at the impurity decays faster with increasing *U***; for fixed U, it decays **faster if U_d decreases**, in accordance with our bosonization results

Phys. Rev. B **103**, 125152 (2021)



Acknowledgement

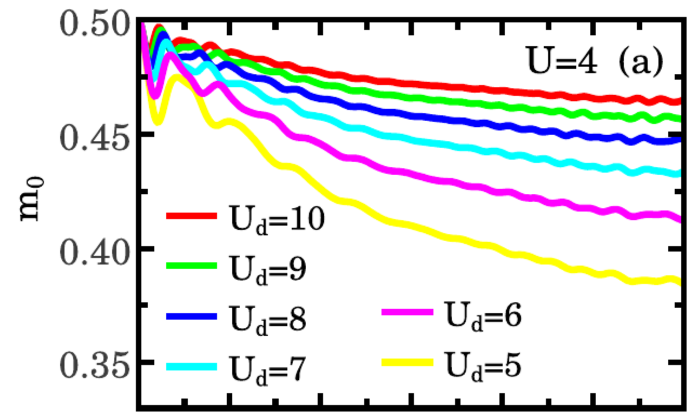
✓ **Dynamics after connecting an impurity to interacting chains**



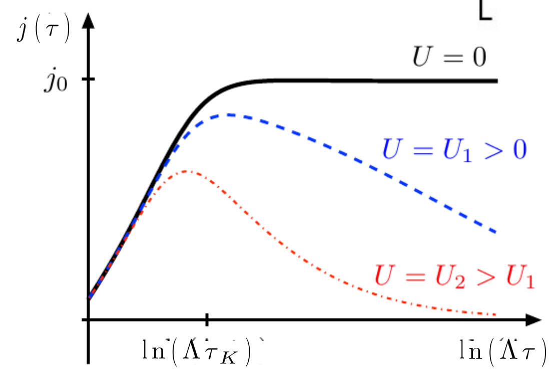
For fixed U_d, **magnetization at the impurity decays faster with increasing U**; for fixed U, it decays **faster if U_d decreases**, in accordance with our bosonization results

For small U, we obtain an analytical expression for the **Kondo time $\tau_K(U)$**, which indicates that **U favors the formation of the Kondo cloud**

Phys. Rev. B **103**, 125152 (2021)



$$\tau_K^* \sim \tau_K \left[1 - \frac{\pi U}{12 v_F}\right]$$



Phys. Rev. B **107**, 075110 (2023)

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CAPES

FAPEMIG Fundação de Amparo à Pesquisa do Estado de Minas Gerais

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