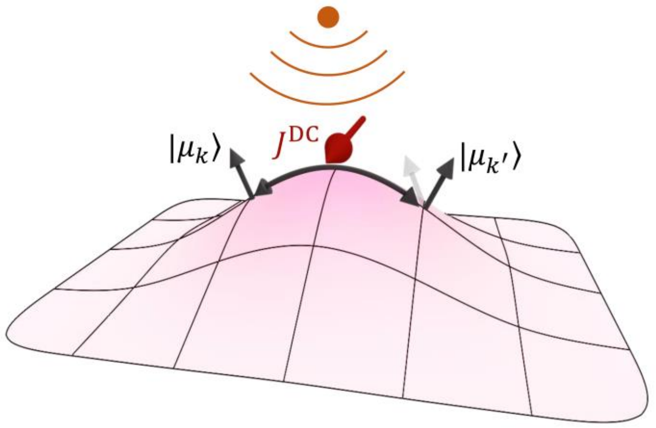
Workshop on Strong Electron Correlations in Quantum Materials: Inhomogeneities Frustration, and Topology

Nonlinear Hall effect induced by a quantum metric dipole in antiferromagnetic heterostructures

Thaís Victa Trevisan (trevisan@berkeley.edu)

Lawrence Berkeley National Laboratory and UC Berkeley



Gao, Liu, Qiu, Gosh, Trevisan et al., Science (2023) - DOI: 10.1126/science.adf1506

ICTP, São Paulo, Brazil – June 19th 2023

Professor Eduardo Miranda taught me:

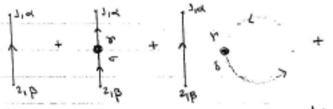
Classical Mechanics II

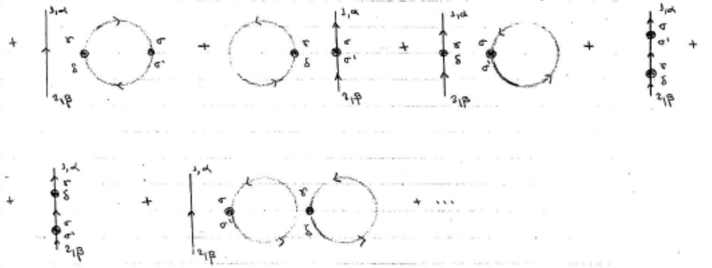
Quantum Theory of Many-Body Systems

Electrodynamics I

Quantum Theory of Many-Body Systems

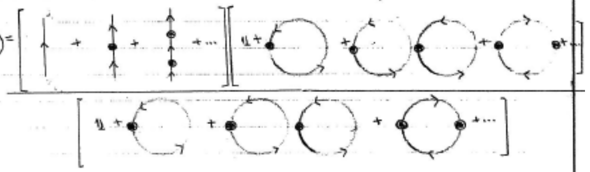
Portanto:

$\langle \Phi_0 | T[\tilde{U}(+\infty,-\infty)\hat{\psi}_{\alpha I}(\vec{x}_1,t_1)\hat{\psi}^{\dagger}_{\beta I}(\vec{x}_2,t_2)] | \Phi_0 \rangle =$ 

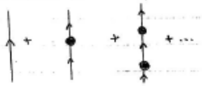


$\langle \Phi_0 | \tilde{U}(+\infty,-\infty) | \Phi_0 \rangle =$ 

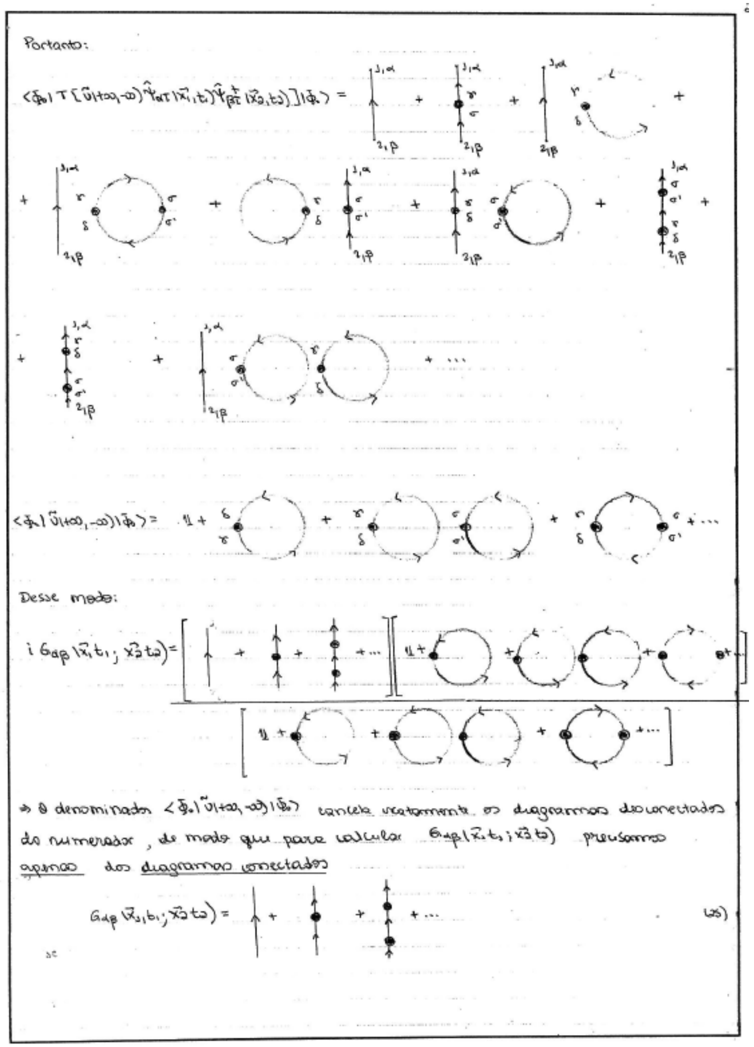
Desse modo:

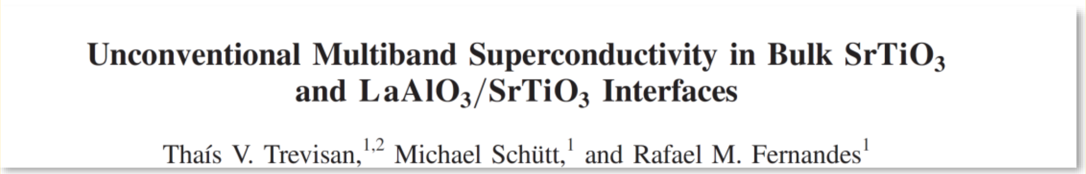
$i\, G_{\alpha\beta}(\vec{x}_1,t_1;\vec{x}_2 t_2) =$ 

⇒ O denominador $\langle \Phi_0 | \tilde{U}(+\infty,-\infty) | \Phi_0 \rangle$ cancela exatamente os diagramas desconectados do numerador, de modo que para calcular $G_{\alpha\beta}(\vec{x}_1 t_1;\vec{x}_2 t_2)$ precisamos apenas dos diagramas conectados

$G_{\alpha\beta}(\vec{x}_1,t_1;\vec{x}_2 t_2) =$  (25)

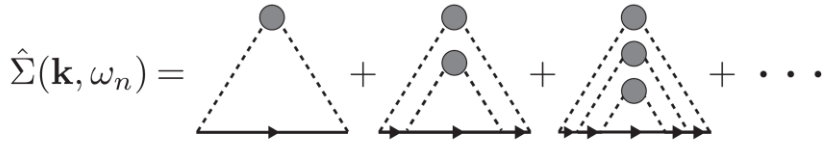
Quantum Theory of Many-Body Systems



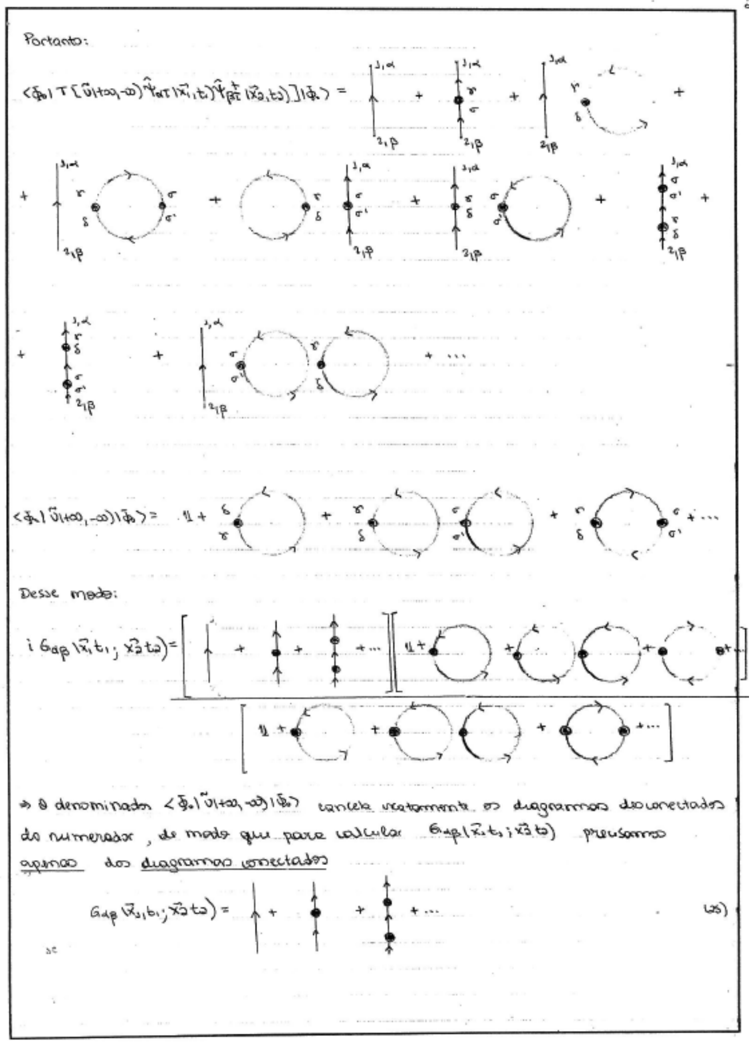


Unconventional Multiband Superconductivity in Bulk $SrTiO_3$ and $LaAlO_3/SrTiO_3$ Interfaces

Thaís V. Trevisan,[1,2] Michael Schütt,[1] and Rafael M. Fernandes[1]



Quantum Theory of Many-Body Systems



Portanto:

$\langle \Phi_0 | T[\hat{U}(+\infty,-\infty)\hat{\psi}_{\alpha}(\vec{x}_1,t_1)\hat{\psi}^{\dagger}_{\beta}(\vec{x}_2,t_2)]|\Phi_0\rangle = \ldots$

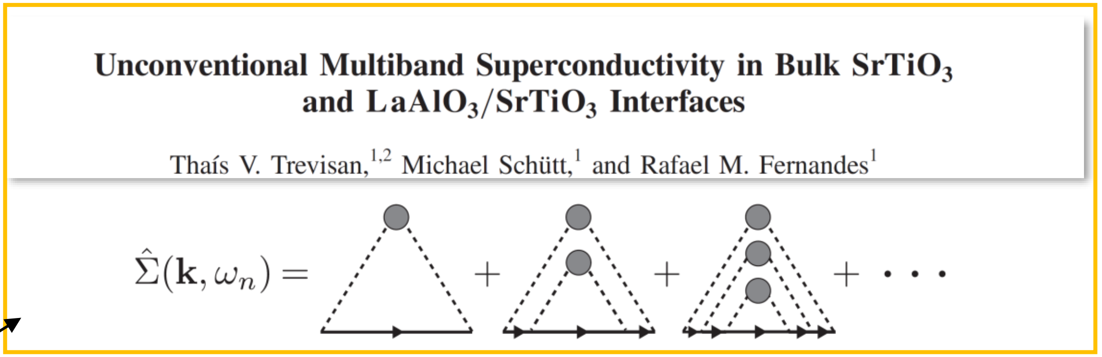
$\langle \Phi_0 | \hat{U}(+\infty,-\infty)|\Phi_0\rangle = 1 + \ldots$

Desse modo:

$i\,G_{\alpha\beta}(\vec{x}_1,t_1;\vec{x}_2,t_2) = \ldots$

⇒ o denominador $\langle \Phi_0 | \hat{U}(+\infty,-\infty)|\Phi_0\rangle$ cancela exatamente os diagramas desconectados do numerador, de modo que para calcular $G_{\alpha\beta}(\vec{x}_1,t_1;\vec{x}_2,t_2)$ precisamos apenas dos diagramas conectados

$G_{\alpha\beta}(\vec{x}_1,t_1;\vec{x}_2,t_2) = \ldots$ (25)



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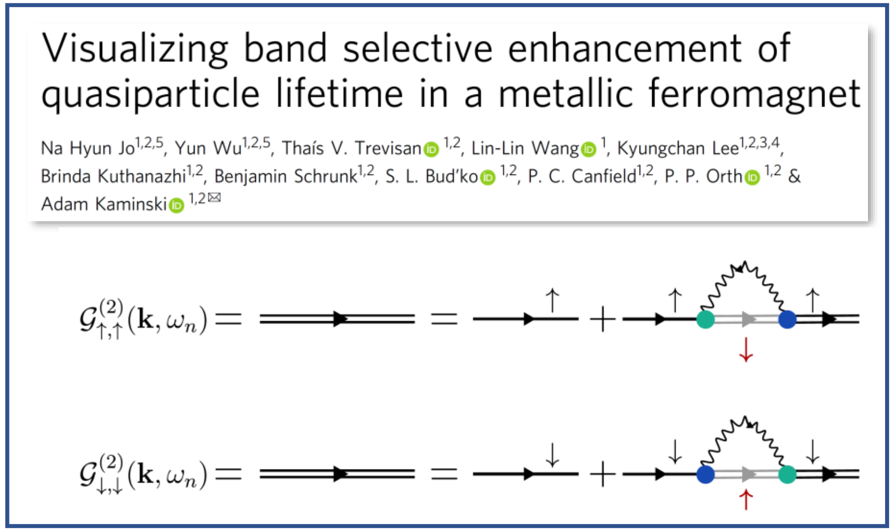
Thaís V. Trevisan,[1,2] Michael Schütt,[1] and Rafael M. Fernandes[1]

$\hat{\Sigma}(\mathbf{k},\omega_n) = \ldots + \ldots + \ldots + \cdots$



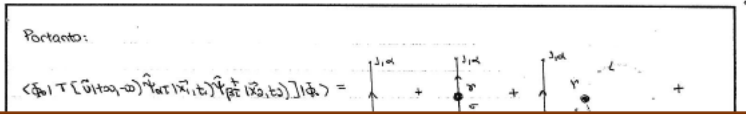
Visualizing band selective enhancement of quasiparticle lifetime in a metallic ferromagnet

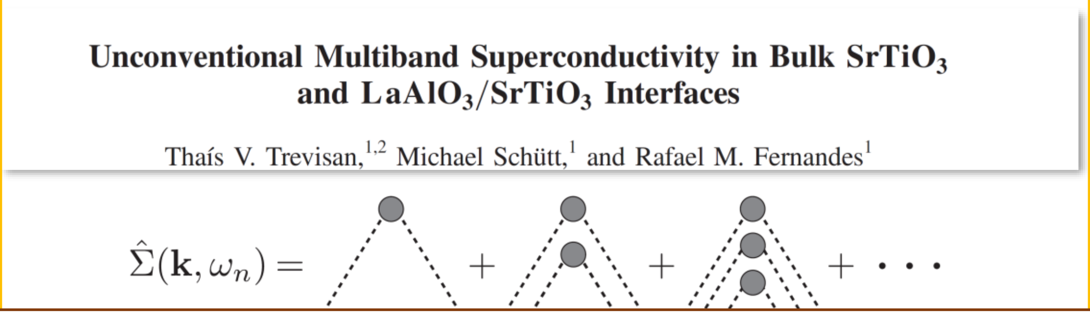
Na Hyun Jo[1,2,5], Yun Wu[1,2,5], Thaís V. Trevisan[1,2], Lin-Lin Wang[1], Kyungchan Lee[1,2,3,4], Brinda Kuthanazhi[1,2], Benjamin Schrunk[1,2], S. L. Bud'ko[1,2], P. C. Canfield[1,2], P. P. Orth[1,2] & Adam Kaminski[1,2]

$\mathcal{G}^{(2)}_{\uparrow,\uparrow}(\mathbf{k},\omega_n) = \ldots$

$\mathcal{G}^{(2)}_{\downarrow,\downarrow}(\mathbf{k},\omega_n) = \ldots$

Quantum Theory of Many-Body Systems

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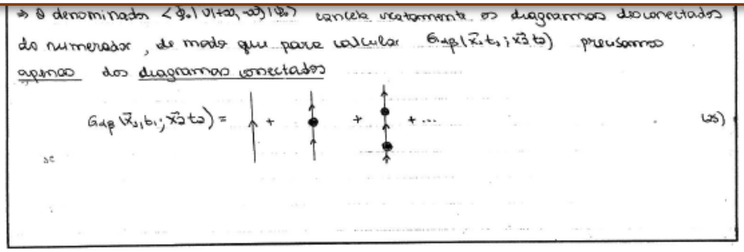
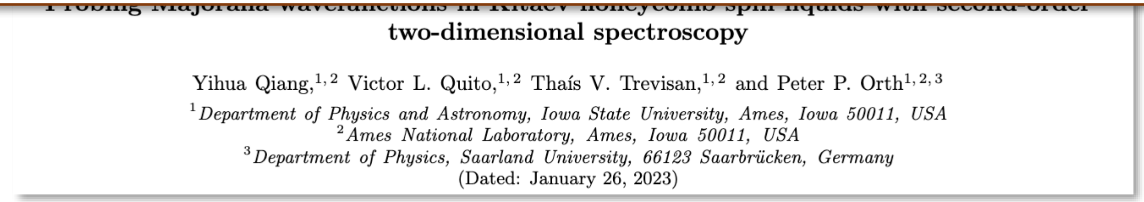
$$\hat{\Sigma}(\mathbf{k}, \omega_n) = \quad + \quad + \quad + \cdots$$

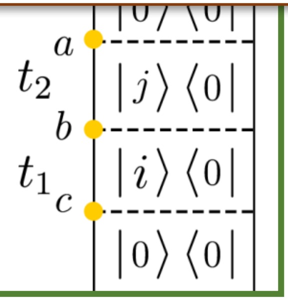
Thank you, Eduardo!




two-dimensional spectroscopy

Yihua Qiang,[1,2] Victor L. Quito,[1,2] Thaís V. Trevisan,[1,2] and Peter P. Orth[1,2,3]

[1] *Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA*
[2] *Ames National Laboratory, Ames, Iowa 50011, USA*
[3] *Department of Physics, Saarland University, 66123 Saarbrücken, Germany*
(Dated: January 26, 2023)



(Victor Quito talk on Wednesday)

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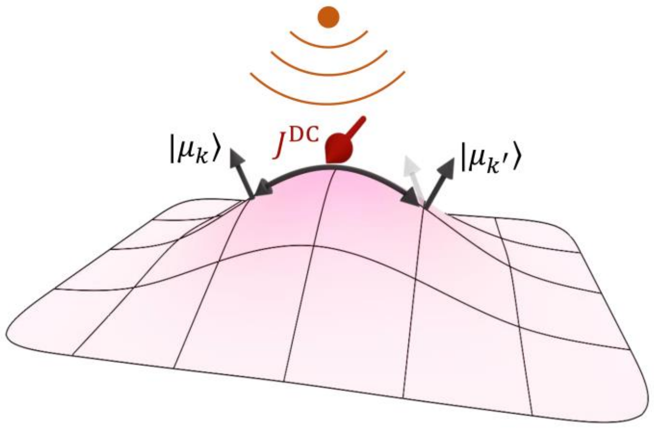


ICTP, São Paulo, Brazil – June 19th 2023

BERKELEY LAB

AMES LABORATORY

Collaborators

Multi-institute collaboration through the Center for the Advancement of Topological Semimetals

Theory collaborators:

- Peter P. Orth (Iowa State University)
- Liang Fu (MIT)
- Arun Bansil (Northeastern University)
- David C. Bell (Harvard)
- Bahadur Singh (Tata Institute of Fundamental Research)
- Tay-Rong Chang (National Cheng Kung University)

Experiment collaborators:

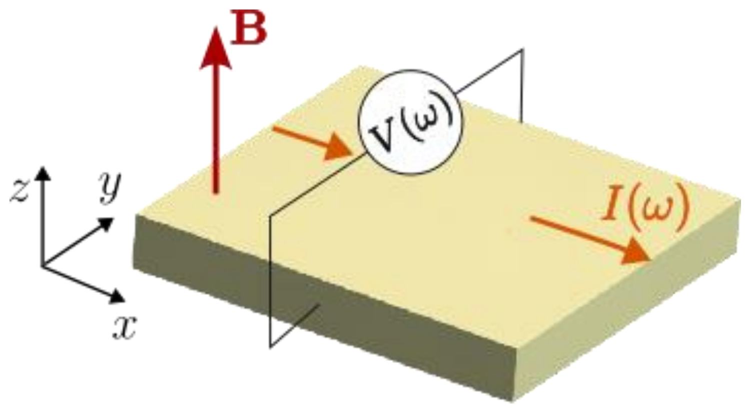
- Su-Yang Xu (Harvard)
- Qiong Ma (Boston College)
- Chunhui Rita Du (UC San Diego)
- Ni Ni (UCLA)
- Takashi Taniguchi (National Institute for Materials Science)

Outline

- **Introduction**
  - Generalities of the Hall effects
  - Quantum metric: what it is and where it appears
  - Theory of the intrinsic quantum metric anomalous Hall effect
- **The quantum metric Hall effect in $MnBi_2Te_4$ and BP heterostructure**
  - An ideal platform for the quantum metric Hall effect
  - Minimal model
- **Summary**

The Hall effect

Linear Hall effect:



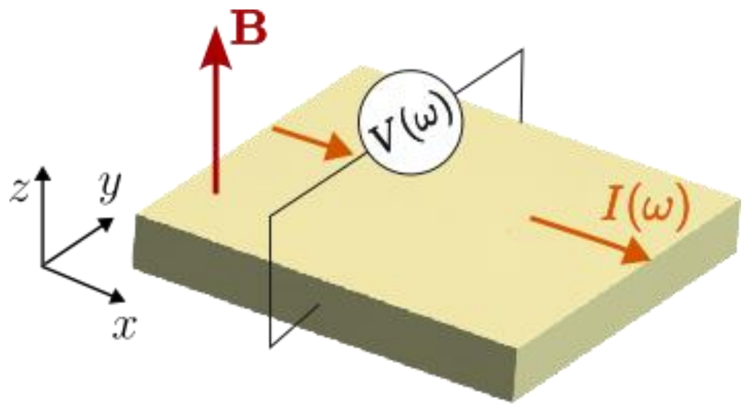
- External $\boldsymbol{B} \Rightarrow$ normal Hall

[E. Hall, Am. J. Math. (1879)]

[N. Nagaosa et al., Rev. Mod. Phys. 82 (2010)]

The Hall effect

Linear Hall effect:



$$J_x(\omega) = \sigma_{xy}(\omega) E_y(\omega)$$

Hall conductivity

[N. Nagaosa et al., Rev. Mod. Phys. 82 (2010)]

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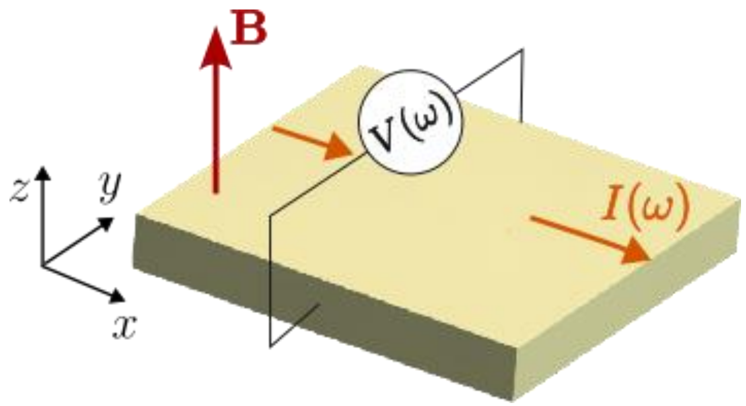

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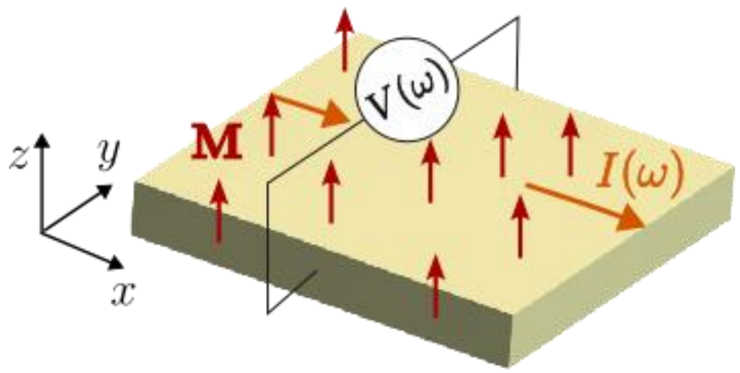
$$\rho_{xy} = \frac{1}{\sigma_{xy}} = \frac{1}{ne} B$$

carrier density

The Hall effect

Linear Hall effect:



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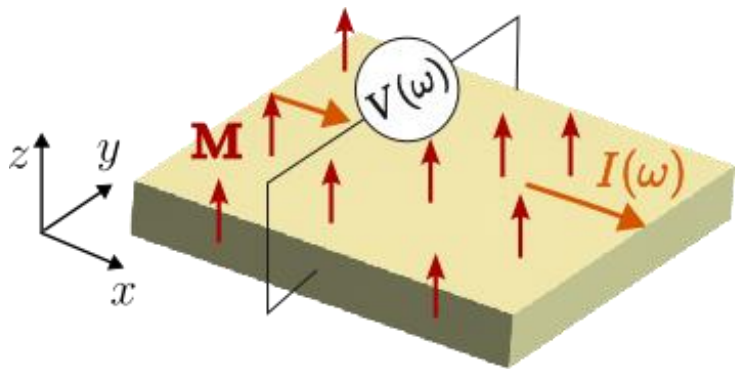

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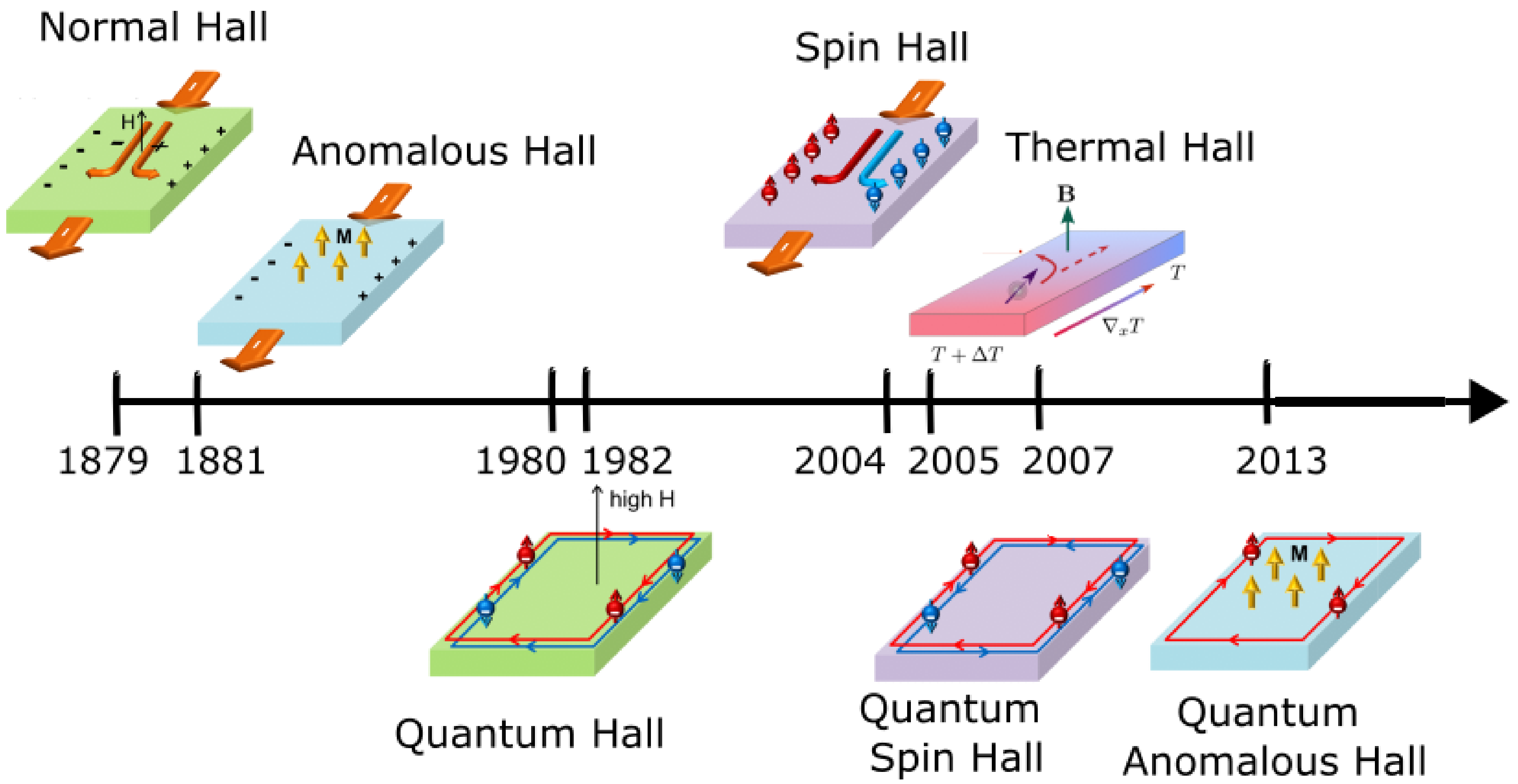
- Intrinsic $\boldsymbol{M}$ ⇒ anomalous Hall

[E. Hall, Philos. Mag. (1881)]

➡ Require break of time-reversal symmetry

➡ Different microscopic mechanism

The Hall effect family grows



1980: Integer Quantum Hall
1982: Fractional Quantum Hall

[Adapted from C. Chang et al. J. Phys. Cond. Matt. (2016), R. Samajdar et al. PRB (2019) and Z.Z. Du et al. Nat. Phys. (2021)]

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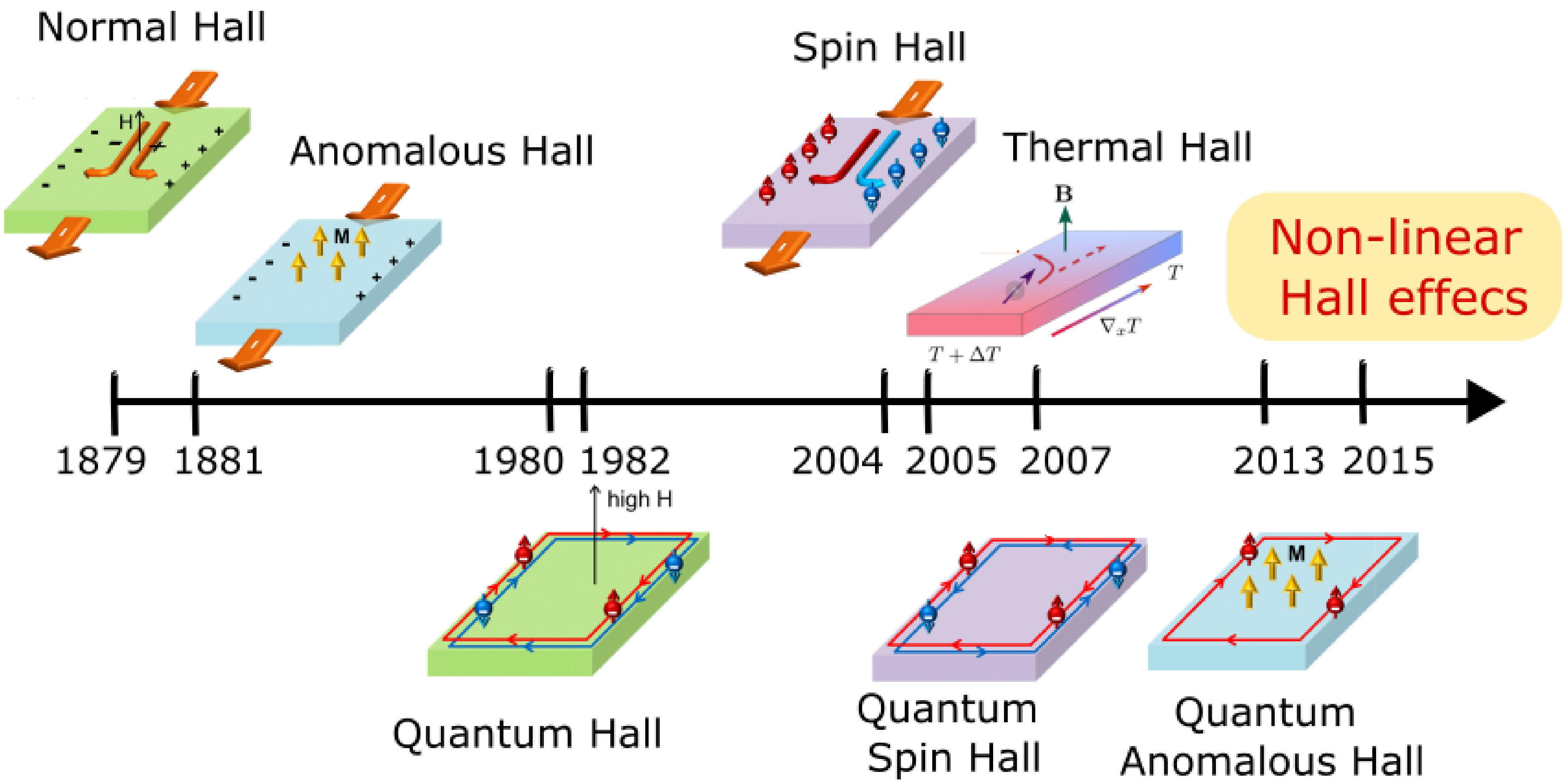

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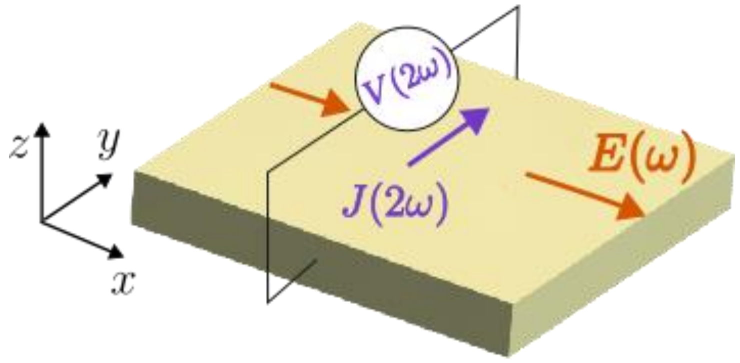
The non-linear anomalous Hall effects

- Hall current oscillates at a different frequency than the electric field

This talk: second-order anomalous Hall effect



$$J_y(\Sigma) = \sigma_{yxx}(\omega_1, \omega_2) E_x(\omega_1) E_x(\omega_2)$$

$\Sigma = \omega_1 + \omega_2$

antisymmetric in y and x

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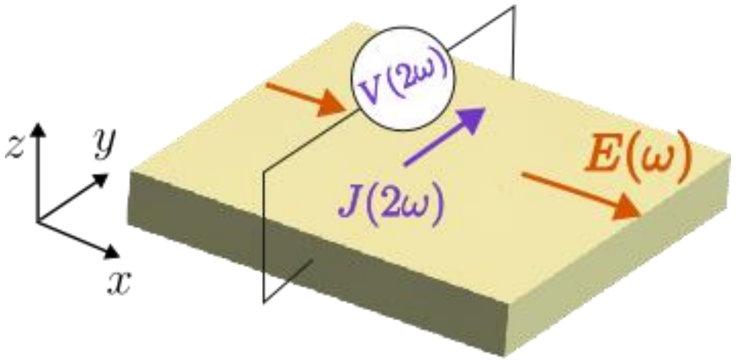

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antisymmetric in y and x

Two possibilities:

$a)\ \omega_1 = \omega_2 \Rightarrow \Sigma = 2\omega$

$b)\ \omega_2 = -\omega_1 \Rightarrow \Sigma = 0$

- Can happen even in the presence of time-reversal symmetry
- Require inversion symmetry breaking

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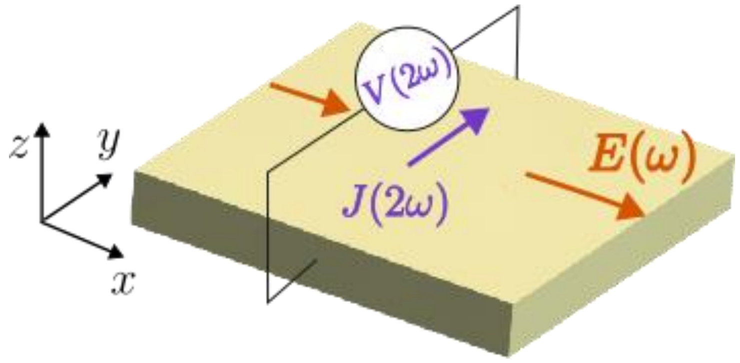

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- Can happen even in the presence of time-reversal symmetry
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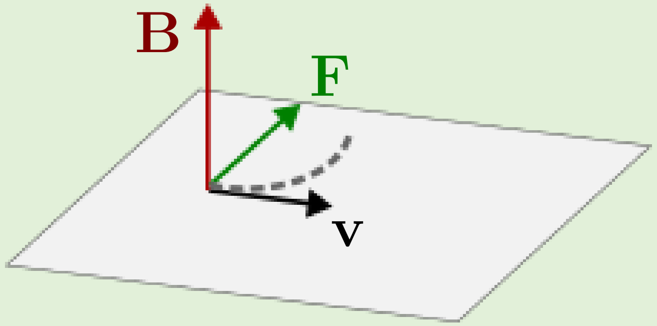
What is the microscopic mechanism?

Theory of the linear Hall effects

Normal Hall

Lorentz force $\mathbf{F} = -e\mathbf{v} \times \mathbf{B}$



[N. Nagaosa et al., Rev. Mod. Phys. 82 (2010)]

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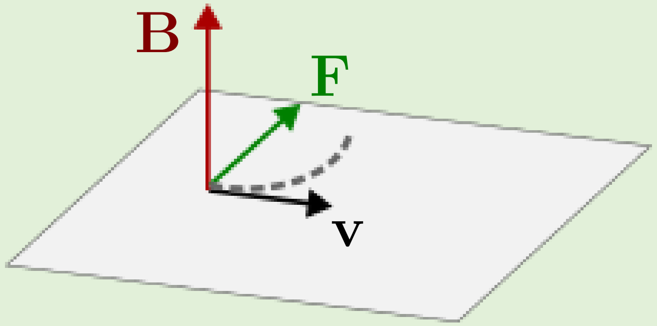
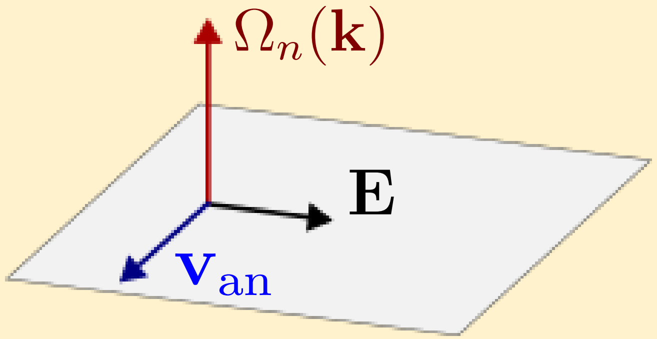


(Linear) anomalous Hall

Extrinsic: disorder → depend on electron lifetime τ

Intrinsic: anomalous velocity → independent of τ



Cannot be explained by band structure alone!

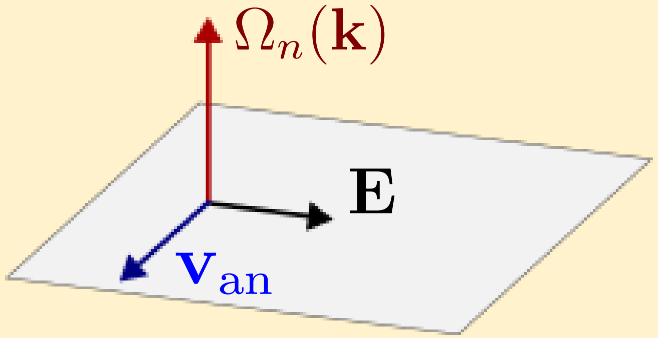
[N. Nagaosa et al., Rev. Mod. Phys. 82 (2010)]

Theory of the linear Hall effects

(Linear) anomalous Hall

Intrinsic: anomalous velocity → independent of τ

$$\mathbf{v}_{\mathrm{an}} = \frac{e}{\hbar}\mathbf{E} \times \Omega_n(\mathbf{k})$$



[N. Nagaosa et al., Rev. Mod. Phys. (2010), Jungwirth et al., Phys. Rev. Lett. (2002), Handane et al. PRL (2004)]

Theory of the linear Hall effects

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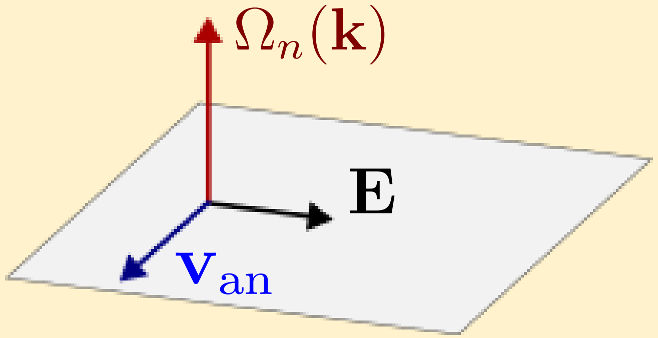
Berry curvature!



Electrons in solids:

$$\psi_{n,\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} \langle \mathbf{r} | u_{n,\mathbf{k}} \rangle$$

$$\boldsymbol{\Omega}_n(\mathbf{k}) = \boldsymbol{\nabla}_{\mathbf{k}} \times \langle u_{n,\mathbf{k}} | \boldsymbol{\nabla}_{\mathbf{k}} u_{n,\mathbf{k}} \rangle$$

[N. Nagaosa et al., Rev. Mod. Phys. (2010), Jungwirth et al., Phys. Rev. Lett. (2002), Handane et al. PRL (2004)]

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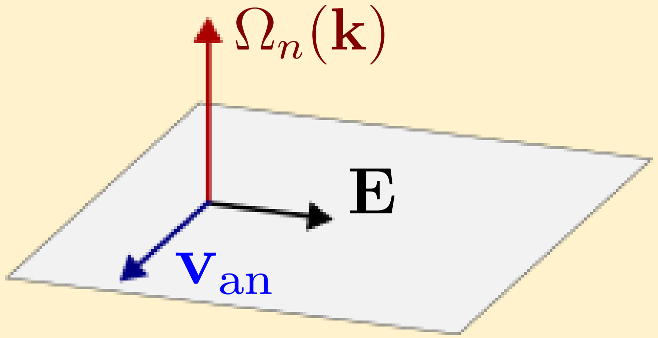

Electrons in solids:

cell periodic

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Theory of the linear Hall effects

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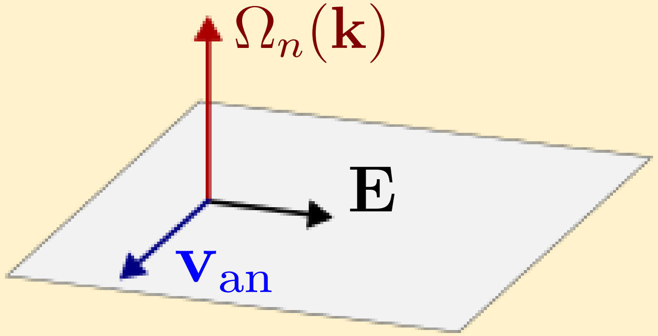
Berry curvature!



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$$\boldsymbol{\Omega}_n(\mathbf{k}) = \boldsymbol{\nabla}_{\mathbf{k}} \times \langle u_{n,\mathbf{k}} | \boldsymbol{\nabla}_{\mathbf{k}} u_{n,\mathbf{k}} \rangle$$

Electric current:

$$\mathbf{J} = -e \int_{\mathbf{k}} f_0(\mathbf{k}) \mathbf{v}_{\mathrm{an}} \quad \Longrightarrow \quad \boxed{\sigma_{yx} = \frac{e^2}{\hbar} \sum_{n \in \mathrm{occ}} \int \frac{d^d k}{(2\pi)^d} f_0(\mathbf{k}) \Omega_n^z(\mathbf{k})}$$

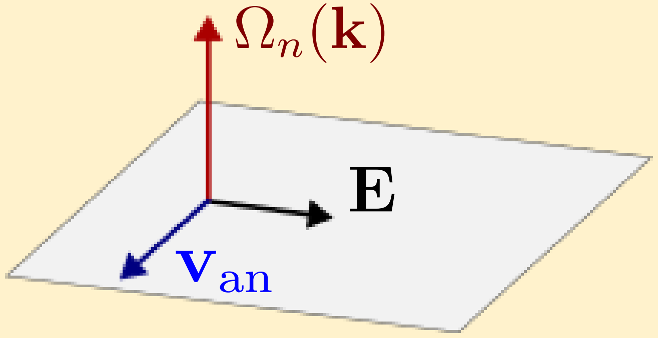
[N. Nagaosa et al., Rev. Mod. Phys. (2010), Jungwirth et al., Phys. Rev. Lett. (2002), Handane et al. PRL (2004)]

Theory of the linear Hall effects

(Linear) anomalous Hall

Intrinsic: anomalous velocity → independent of τ

$$\mathbf{v}_{\mathrm{an}} = \frac{e}{\hbar}\mathbf{E} \times \Omega_n(\mathbf{k})$$ Berry curvature!



Electrons in solids:

$$\psi_{n,\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} \langle \mathbf{r} | u_{n,\mathbf{k}} \rangle$$ cell periodic

$$\mathbf{\Omega}_n(\mathbf{k}) = \mathbf{\nabla}_{\mathbf{k}} \times \langle u_{n,\mathbf{k}} | \mathbf{\nabla}_{\mathbf{k}} u_{n,\mathbf{k}} \rangle$$

$$\boxed{\sigma_{yx} = \frac{e^2}{\hbar} \sum_{n \in \mathrm{occ}} \int \frac{d^d k}{(2\pi)^d} f_0(\mathbf{k}) \Omega_n^z(\mathbf{k})}$$

Requires $\mathcal{T}$ breaking!

Time reversal ($\mathcal{T}$):

$$\Omega_n(\mathbf{k}) \xrightarrow{\mathcal{T}} -\Omega_n(-\mathbf{k})$$

$$f_0(\mathbf{k}) \xrightarrow{\mathcal{T}} f_0(-\mathbf{k})$$

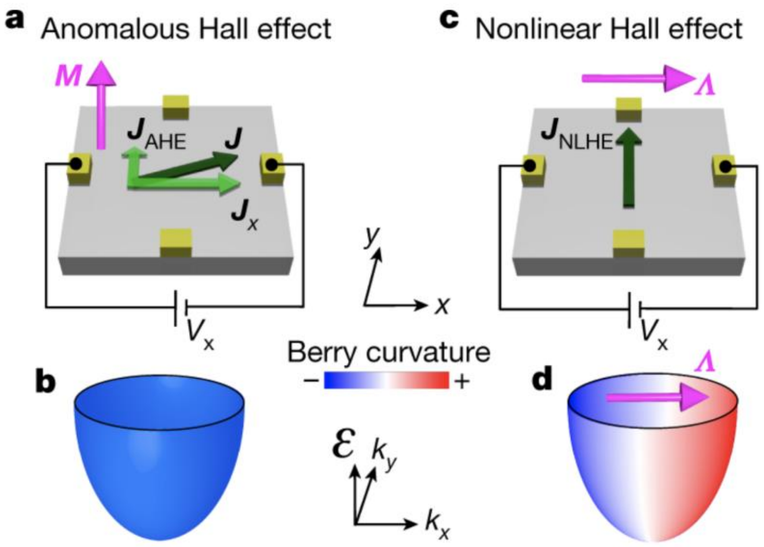
[N. Nagaosa et al., Rev. Mod. Phys. (2010), Jungwirth et al., Phys. Rev. Lett. (2002), Handane et al. PRL (2004)]

Non-linear Hall: beyond the Berry curvature

- Extrinsic contributions to the non-linear Hall effect also involves the Berry curvature

$$\chi_{\mathrm{abc}} = \varepsilon_{\mathrm{adc}} \frac{e^3 \tau}{2(1 + i\omega\tau)} \int_k \boxed{(\partial_b f_0)\Omega_d}.$$

Berry curvature dipole



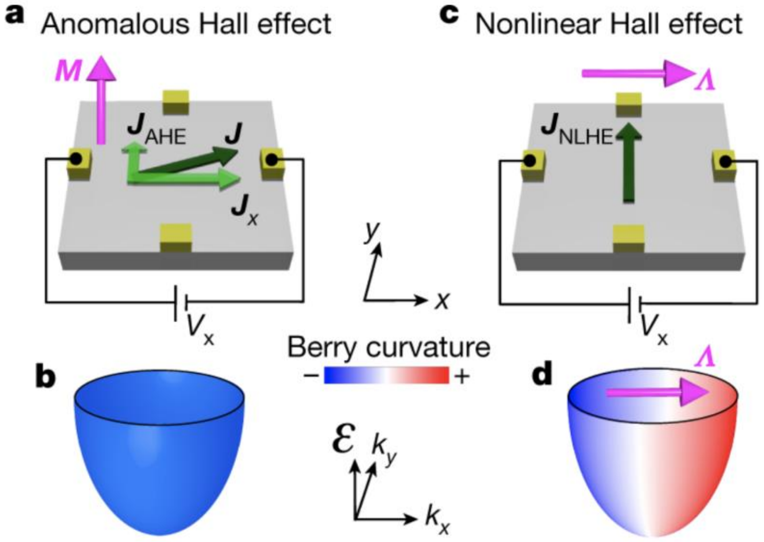
[Sodemann, Fu (2014; Ma *et al.* (2018); Kang *et al.* (2018)]

Non-linear Hall: beyond the Berry curvature

- Extrinsic contributions to the non-linear Hall effect also involves the Berry curvature

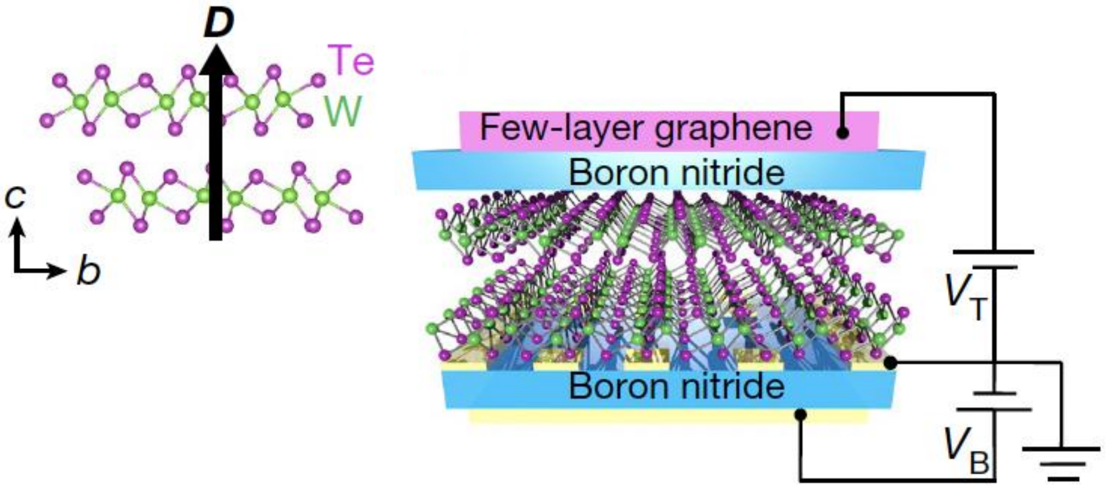
$$\chi_{\mathrm{abc}} = \varepsilon_{\mathrm{adc}} \frac{e^3\tau}{2(1+i\omega\tau)} \int_k \boxed{(\partial_b f_0)\Omega_d}.$$

Berry curvature dipole



Observed in bilayer WTe_2

From Ma *et al.* (2018):



[Sodemann, Fu (2014; Ma *et al.* (2018); Kang *et al.* (2018)]

- Intrinsic second-order Hall effect is generated by dipoles of the quantum metric

The quantum geometry of the electrons

- Quantum geometric tensor: geometric properties of the electron wave functions

$$\boxed{\mathcal{Q}_{\mu\nu}^{(n)}(\mathbf{k}) = \langle \partial_\mu u_n | \partial_\nu u_n \rangle - \langle \partial_\mu u_n | u_n \rangle \langle u_n | \partial_\nu u_n \rangle = -\sum_{m \neq n} \mathcal{A}_{mn}^{(\mu)}(\mathbf{k}) \mathcal{A}_{nm}^{(\nu)}(\mathbf{k})}$$

$$\mathcal{A}_{mn}^{(\mu)} = i \langle u_{m,\mathbf{k}} | \partial_\mu u_{n,\mathbf{k}} \rangle$$

Non-Abelian Berry connection

[J. P. Provost and G. Valle et al. Communications in Mathematical Physics (1980)]

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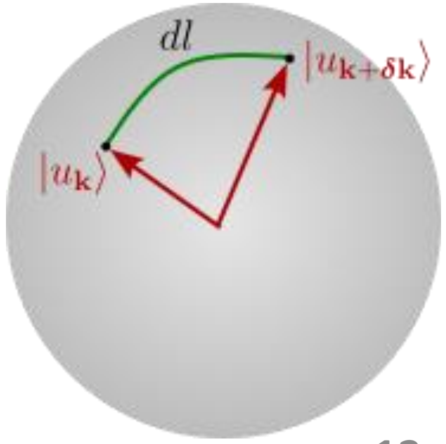
$$\boxed{\mathrm{Re}\mathcal{Q}_{\mu\nu}^{(n)}(\mathbf{k}) = g_{\mu\nu}(\mathbf{k})} \longleftarrow$$ Quantum metric

Distance between quantum states

$$dl^2 = || u_{\mathbf{k}+\boldsymbol{\delta}\mathbf{k}} \rangle - | u_{\mathbf{k}} \rangle |^2$$



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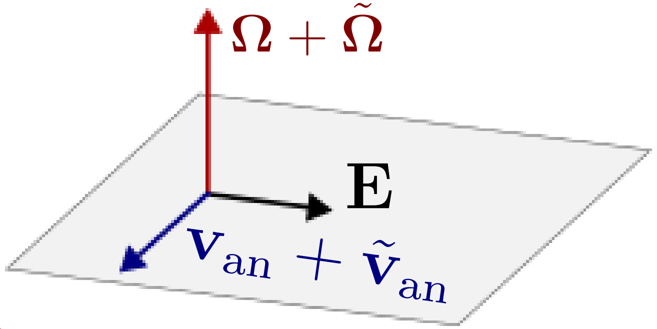
Theory of the intrinsic non-linear Hall effect: beyond Berry curvature

- Semiclassical description (second order in $\boldsymbol{E}$, with $\boldsymbol{B} = 0$) [Y. Gao et al. PRL 112 (2014)]

$$\begin{cases} \dot{\mathbf{r}} = \frac{1}{\hbar}\boldsymbol{\nabla}_{\mathbf{k}}\tilde{\varepsilon}_{n,\mathbf{k}} - \dot{\mathbf{k}} \times \Omega_n(\mathbf{k}) - \dot{\mathbf{k}} \times \left(\boldsymbol{\nabla}_{\mathbf{k}} \times \overset{\leftrightarrow}{G}_n \mathbf{E}\right) \\ \dot{\mathbf{k}} = -\frac{e}{\hbar}\mathbf{E} \end{cases}$$

field correction to the anomalous velocity ($\tilde{v}_{an}$)



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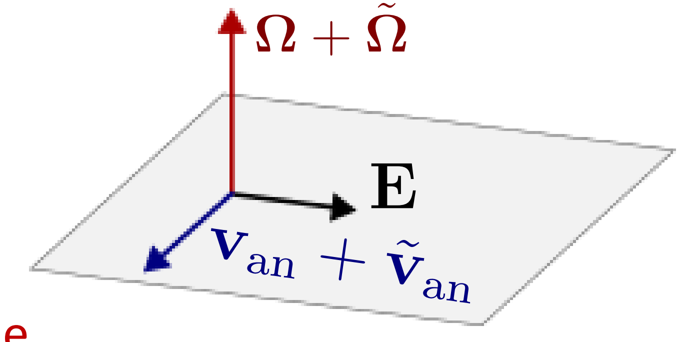

For $E\hat{x}$: $\tilde{v}_{\mathrm{an},y} = \dfrac{e}{\hbar}\left(\partial_x G_{yx} - \partial_y G_{xx}\right)$

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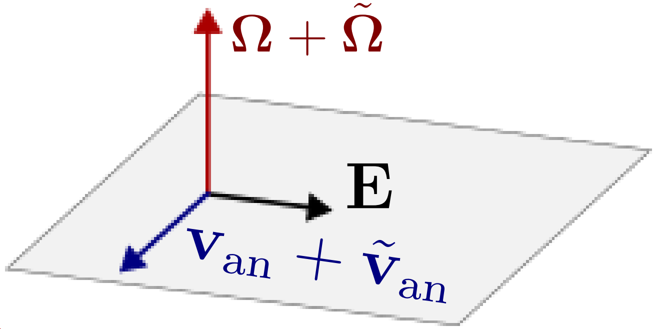

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← Berry curvature polarizability

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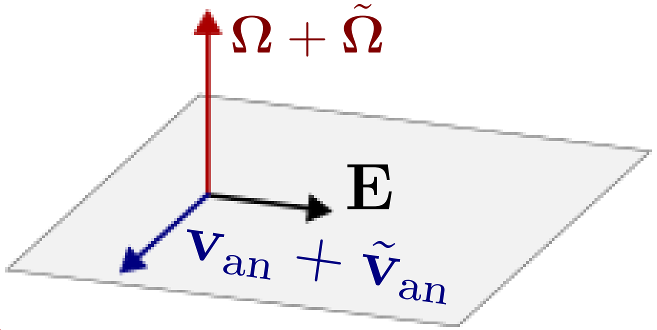

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Theory of the intrinsic non-linear Hall effect

- From the field correction to the anomalous velocity:

$$\sigma_{yxx} = 2e^3 \sum_n \int \frac{d^d k}{(2\pi)^d} \frac{v_y^n g_{xx}^{(n)}(\mathbf{k}) - v_x^n g_{yx}^{(n)}(\mathbf{k})}{\varepsilon_{n,\mathbf{k}} - \varepsilon_{\bar{n},\mathbf{k}}} \delta(\varepsilon_{n,\mathbf{k}} - \varepsilon_F) + \mathrm{AIC}$$

Antisymmetric:

$\sigma_{yxx} = -\sigma_{xyx}$

[Y. Gao et al. PRL 112 (2014), C. Wang et al. PRL 127 (2021)
Gao, Liu, Qiu, Gosh, Trevisan et al., Science (2023)]

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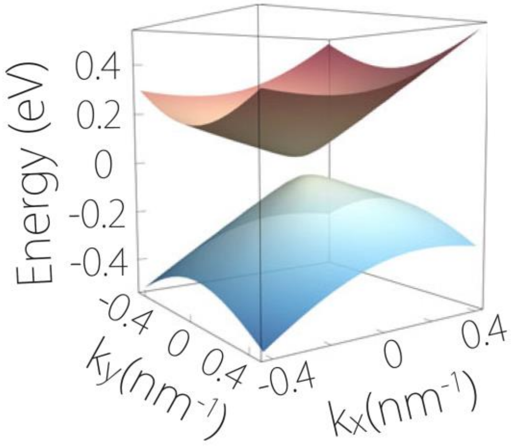
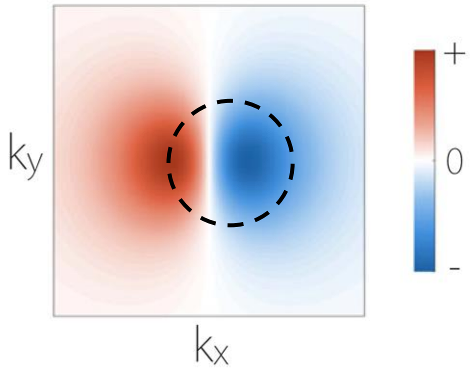
Quantum metric dipole: $D_{yxx}(\mathbf{k})$



$D_{yxx}(\mathbf{k})$



[Adapted from Liu et al. PRL 127 (2021)]

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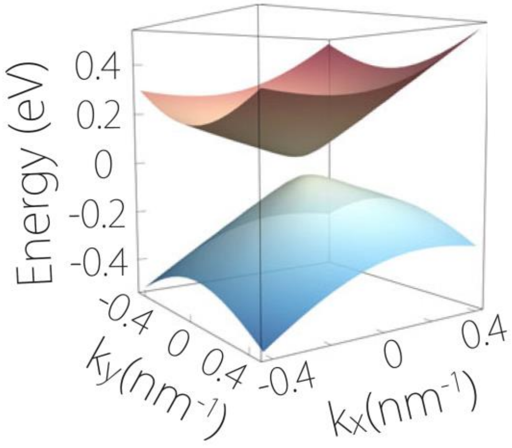
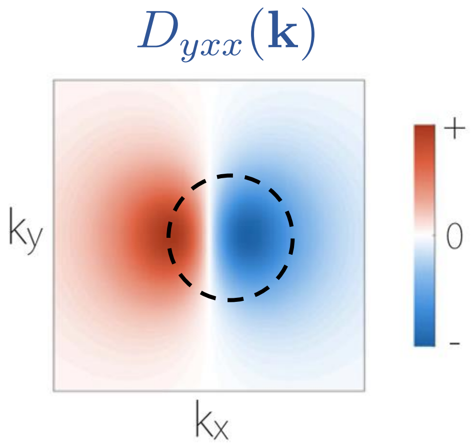
Additional interband contributions





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[Y. Gao et al. PRL 112 (2014), C. Wang et al. PRL 127 (2021)
Gao, Liu, Qiu, Gosh, Trevisan et al., Science (2023)]

Symmetry considerations

- What about the original anomalous velocity contribution?

$$\dot{\mathbf{r}} = \frac{1}{\hbar}\boldsymbol{\nabla}_{\mathbf{k}}\tilde{\varepsilon}_{n,\mathbf{k}} - \dot{\mathbf{k}} \times \Omega_n(\mathbf{k}) - \dot{\mathbf{k}} \times \left(\boldsymbol{\nabla}_{\mathbf{k}} \times \overset{\leftrightarrow}{G}_n \mathbf{E}\right)$$

- Quantum metric Hall effect dominates in $\mathcal{PT}$ symmetric materials

Time-reversal ($\mathcal{T}$): $\Omega_n(\mathbf{k}) \xrightarrow{\mathcal{T}} -\Omega_n(-\mathbf{k})$

Inversion ($\mathcal{P}$): $\Omega_n(\mathbf{k}) \xrightarrow{\mathcal{P}} \Omega_n(-\mathbf{k})$

$$\Rightarrow \quad \Omega_n(\mathbf{k}) \overset{\mathcal{PT}}{=} 0$$

[Y. Gao et al. PRL 112 (2014), C. Wang et al. PRL 127 (2021)]

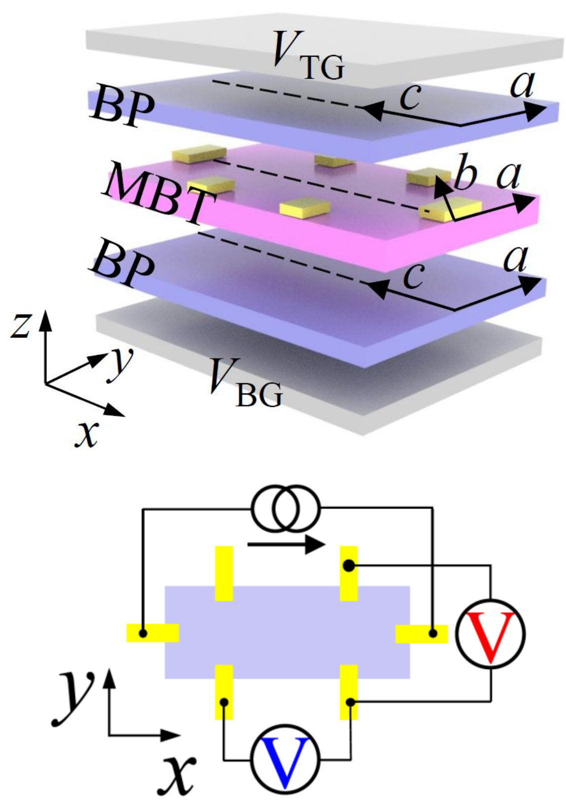
First observation in a heterostructure of $MnBi_2Te_4$ and BP

Experimentally measured anomalous Hall effect

Suyang Xu
Harvard

- Material: heterostructure composed by $MnBi_2Te_4$ (MBT) and black phosphorus (BP)

SETUP:



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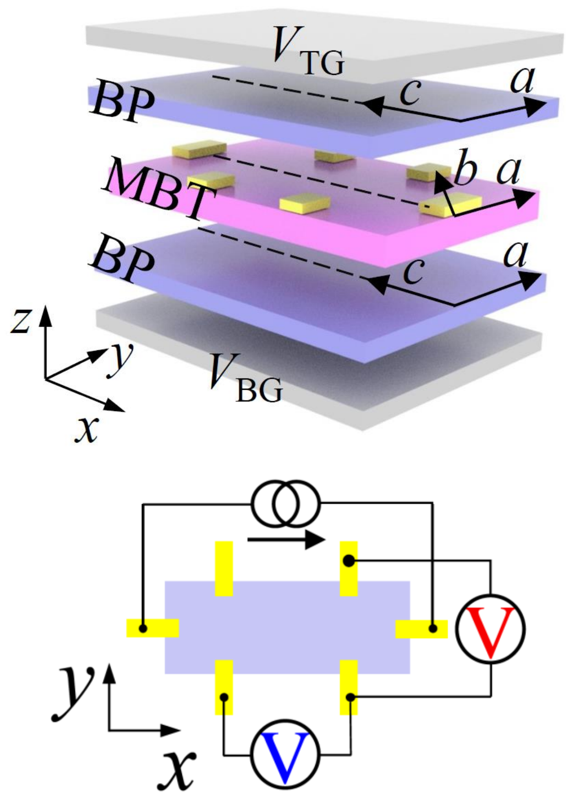
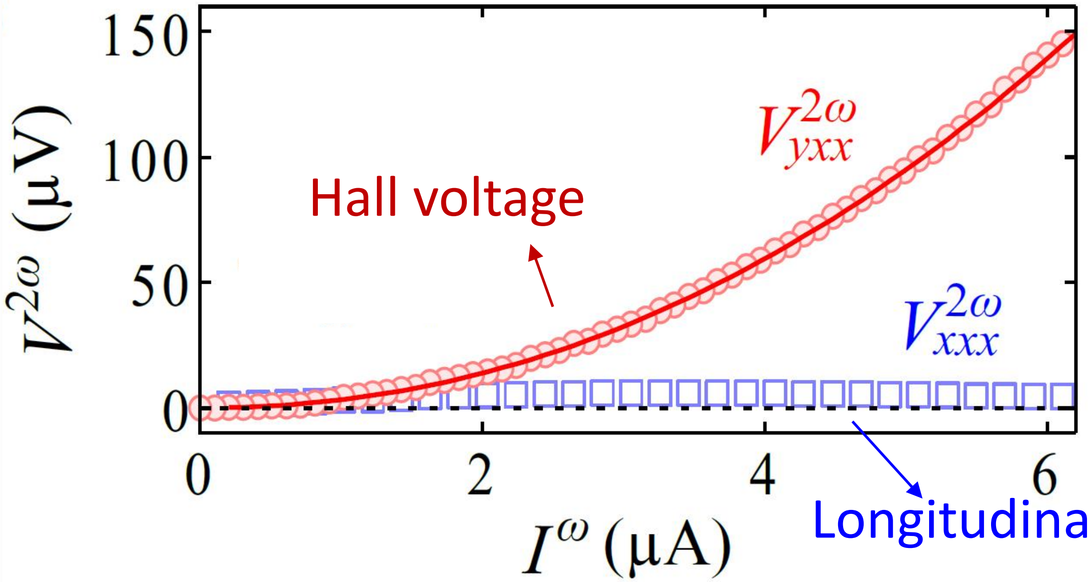




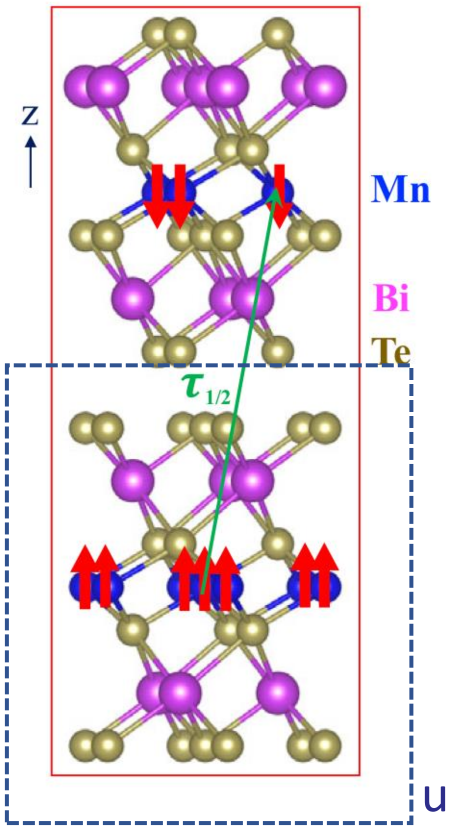
Longitudinal voltage

⟶ Dominance of the Hall response

[Gao, Liu, Qiu, Gosh, Trevisan et al., Science (2023)]

Properties of $MnBi_2Te_4$

CRYSTAL STRUCTURE



unit cell:
Septuple layer (SL)

References

- Crystal space group: $R\bar{3}m$ $\Rightarrow$ $\left\{ \begin{array}{l} C_{3z} ☑ \\ \mathcal{P} ☑ \text{ (spatial inversion)} \end{array} \right.$

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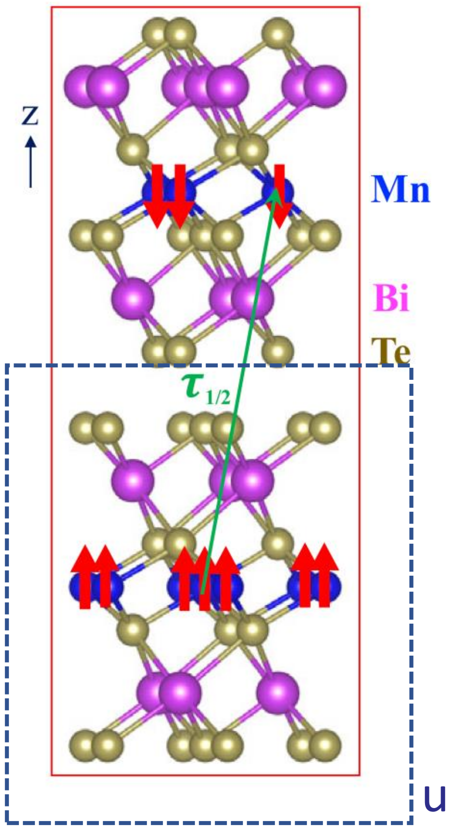

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- Mn moments ferromagnetically in each SL below $\sim 25\mathrm{K}$
- Ground state is AFM

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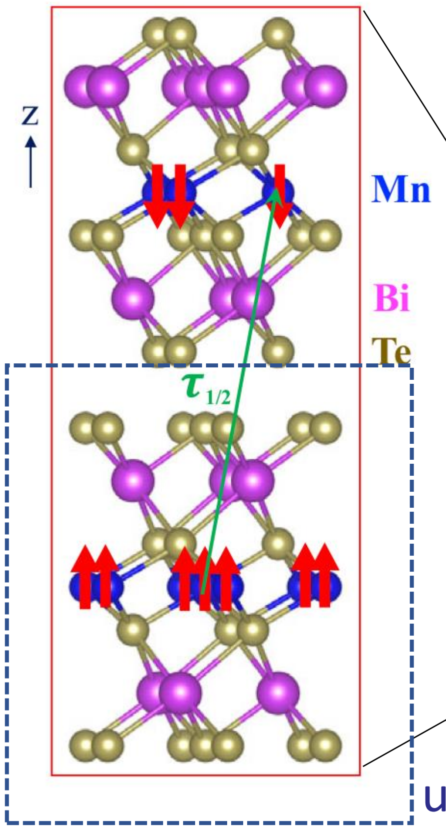

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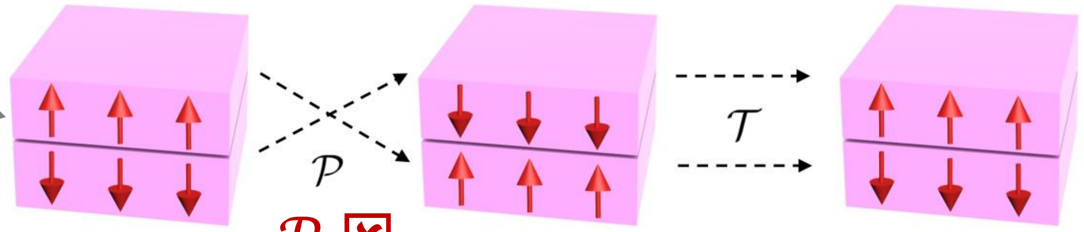
- Ground state is AFM
- SLs coupling is van der Waals type
- Even layer structures:

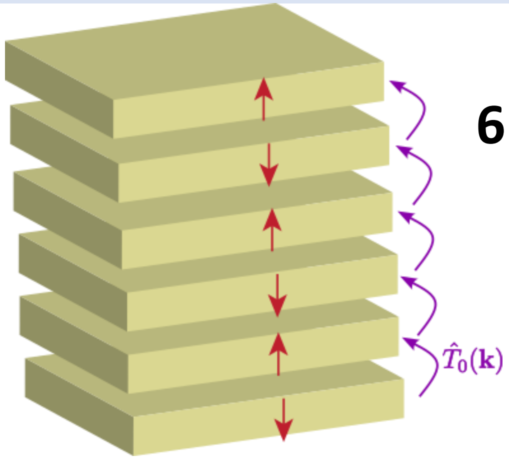


$\mathcal{P}$ ☒
$\mathcal{T}$ ☒

$\mathcal{PT}$ ☑

$\mathcal{PT}$ symmetric AFM⇒ $\boldsymbol{\Omega}(\boldsymbol{k}) = 0$

Fermi surface and quantum metric dipole in the 6SL MBT



6 SL

- Low energy physics dominated by p_z orbitals of Bi and Te atoms

Each SL: k.p model around the center of BZ

[B. Lian et al. PRL 124 (2020)]

$$\hat{h}_\varsigma(\mathbf{k}) = \hat{h}_N(\mathbf{k}) + (-1)^\varsigma \gamma_{af} \hat{h}_{\mathrm{M}}(\mathbf{k})$$

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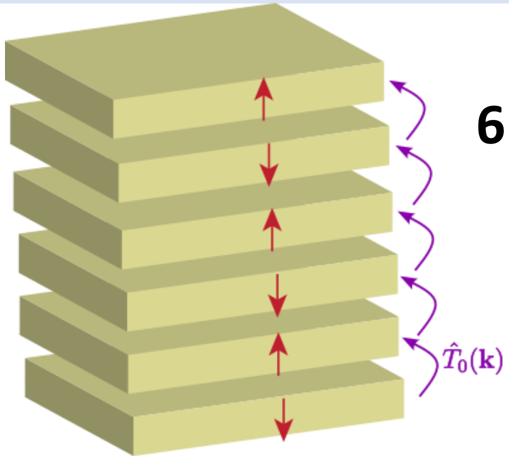

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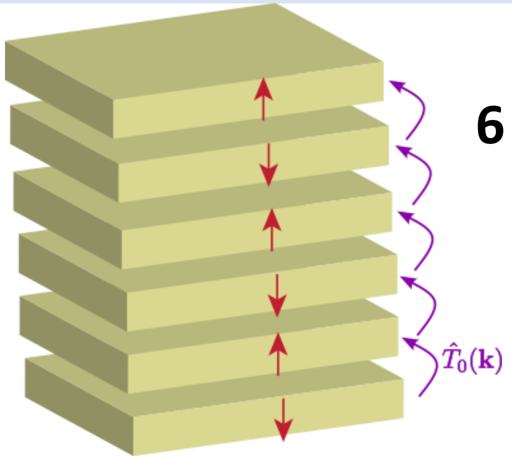

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Inter-SL hopping: $T_0(\boldsymbol{k})$

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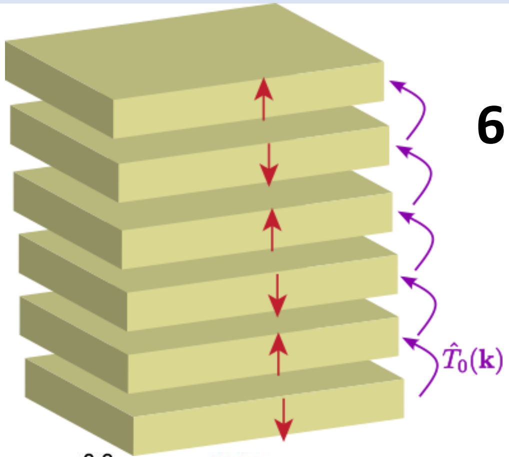

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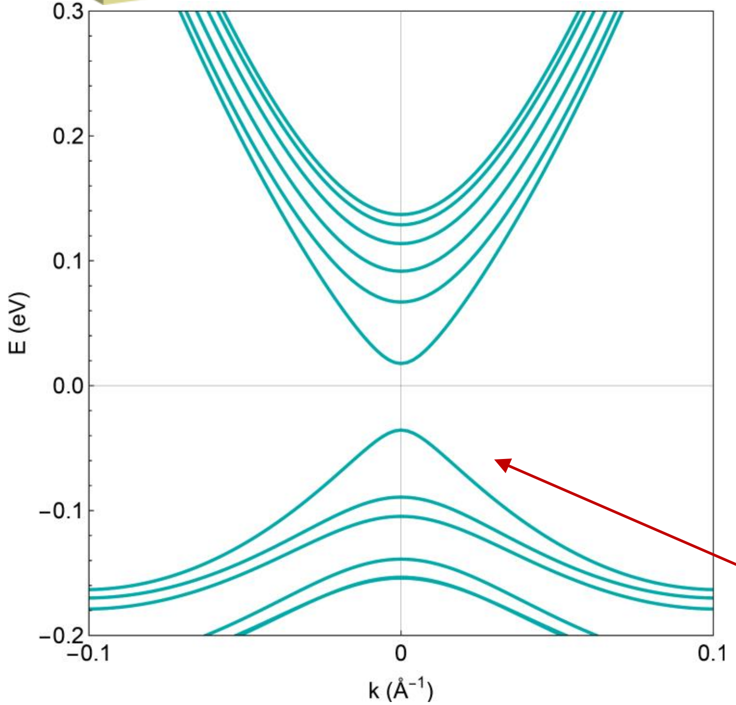

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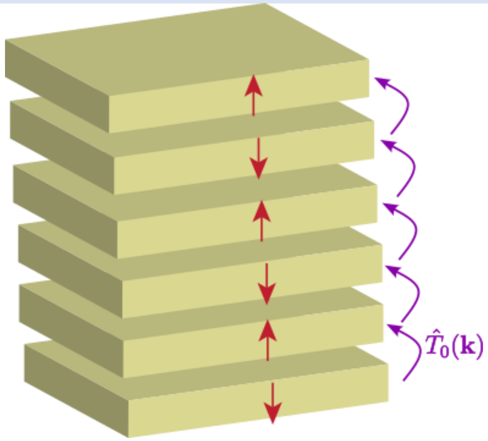
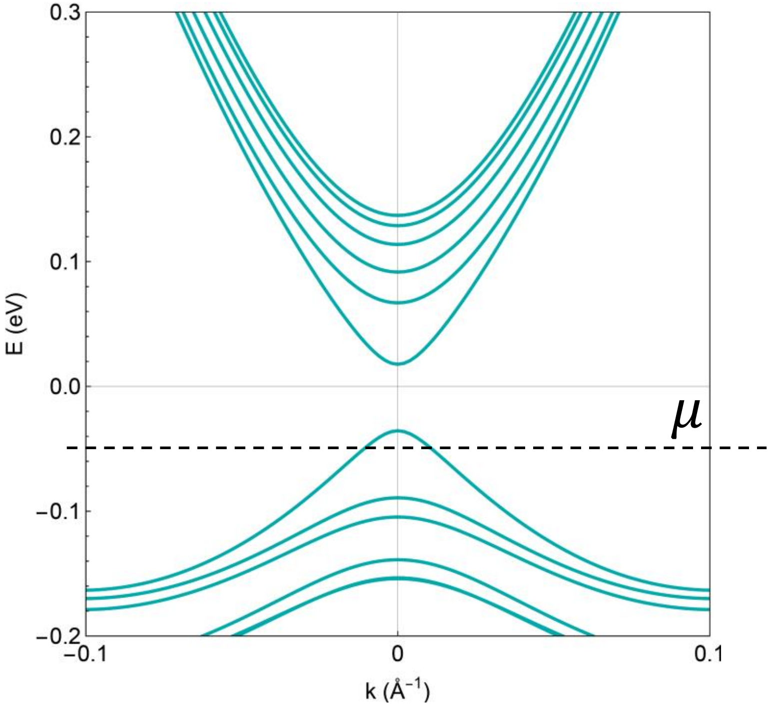
Large quantum metric!

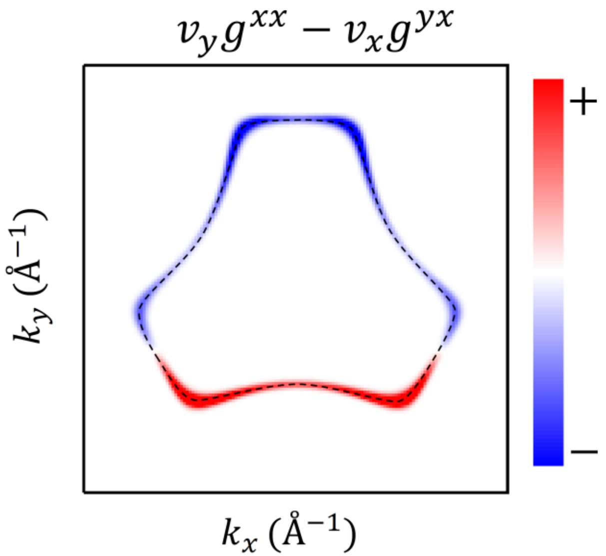
Fermi surface and quantum metric dipole in the 6SL MBT





For $\mu = -50$ meV :

Quantum metric dipole:



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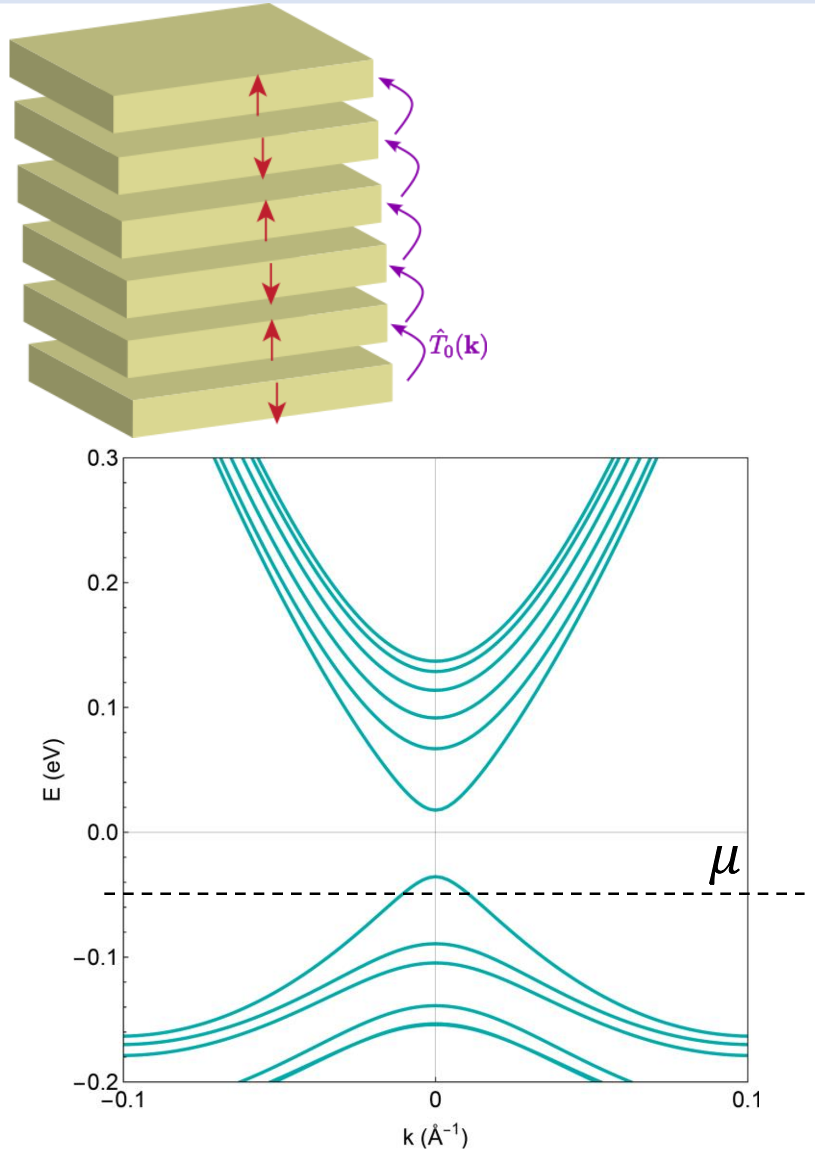

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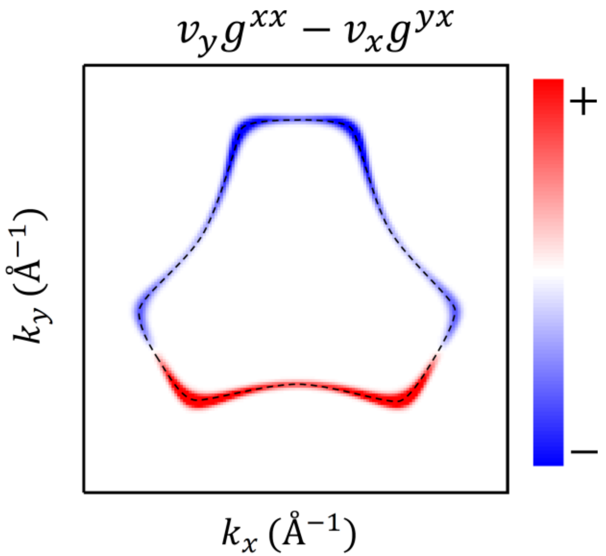
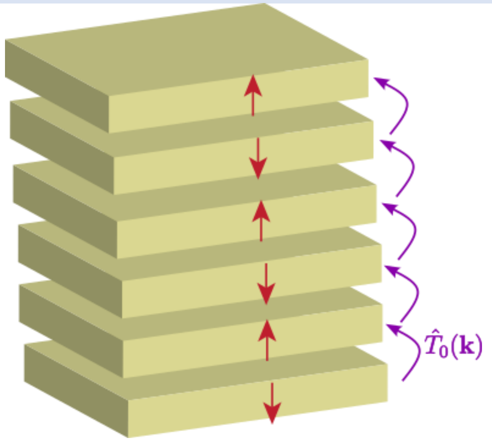


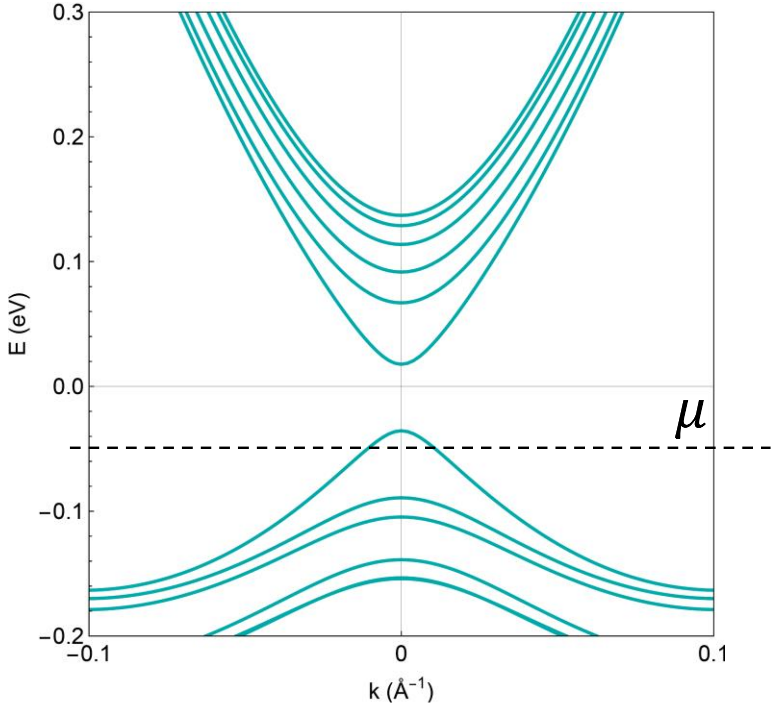
C_{3z}

$$\int_{\mathrm{FS}} d\mathbf{k}\, [v_y g_{xx} - v_x g_{yx}] = 0$$

No quantum metric Hall effect

Fermi surface and quantum metric dipole in the 6SL MBT





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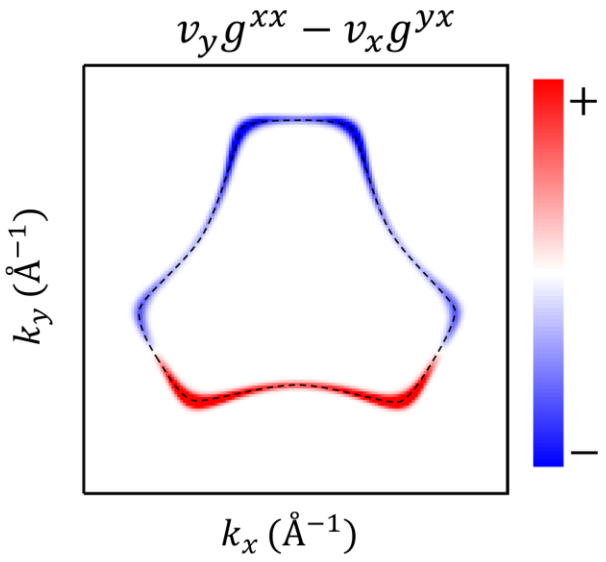


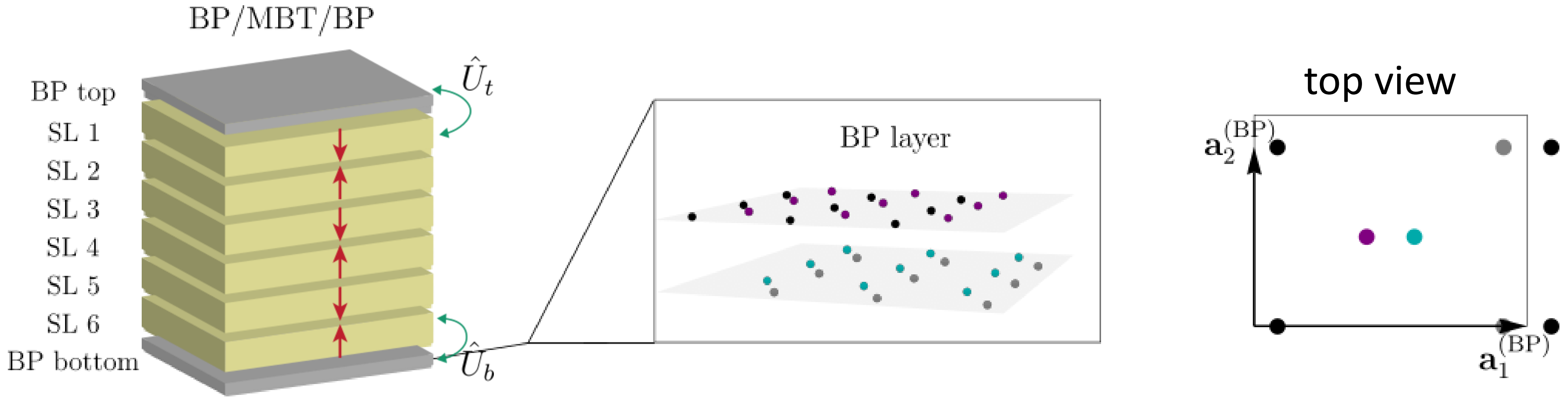
C_{3z}

$$\int_{\mathrm{FS}} d\mathbf{k}\,[v_y g_{xx} - v_x g_{yx}] = 0$$

No quantum metric Hall effect

We need to break C_{3z} symmetry and keep $\mathcal{PT}$…
How?

Black phosphorus (BP) promotes the needed C_{3z} breaking



- Low energy physics: tight-biding dominated by p_z orbitals of P atoms

[Ezawa et al. NJP (2014)
Rudenko et al. PRB (2015)]

[Gao, Liu, Qiu, Gosh, Trevisan et al., Science (2023)]

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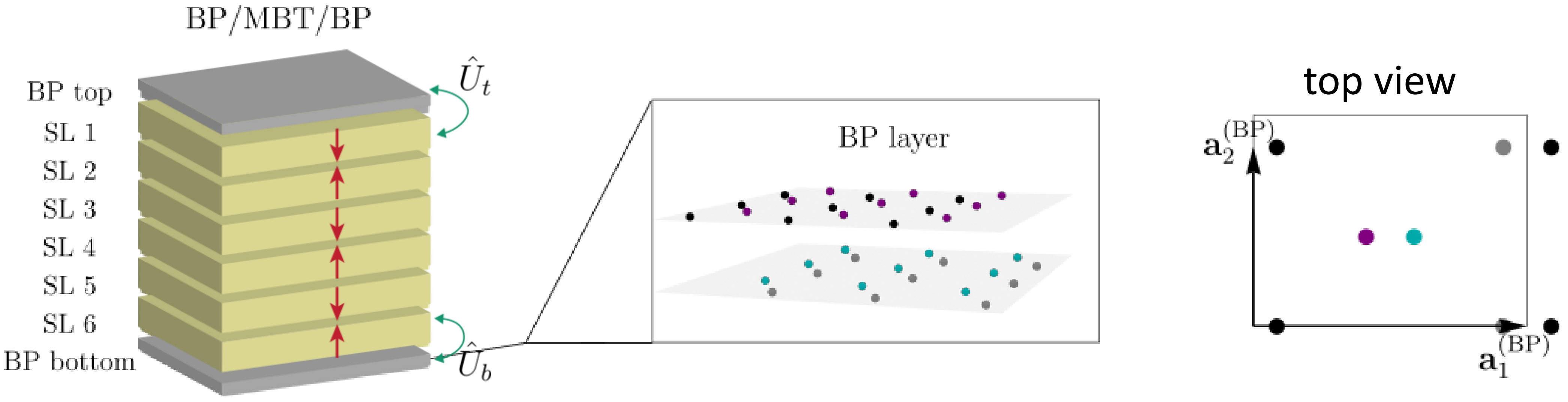


- Low energy physics: tight-biding dominated by p_z orbitals of P atoms [Ezawa et al. NJP (2014) Rudenko et al. PRB (2015)]
- BP tetragonal lattice breaks C_{3z}

Sources of C_{3z} breaking:

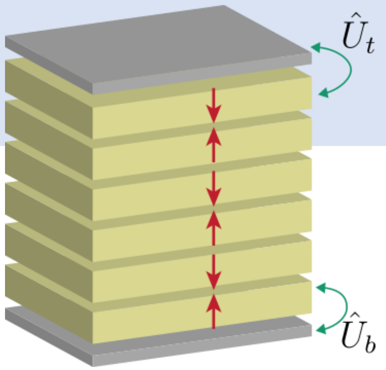
a) Hybridization of BP and MBT bands

Next-neighbor interlayer hoping: $\widehat{U}_b$ and $\widehat{U}_t$

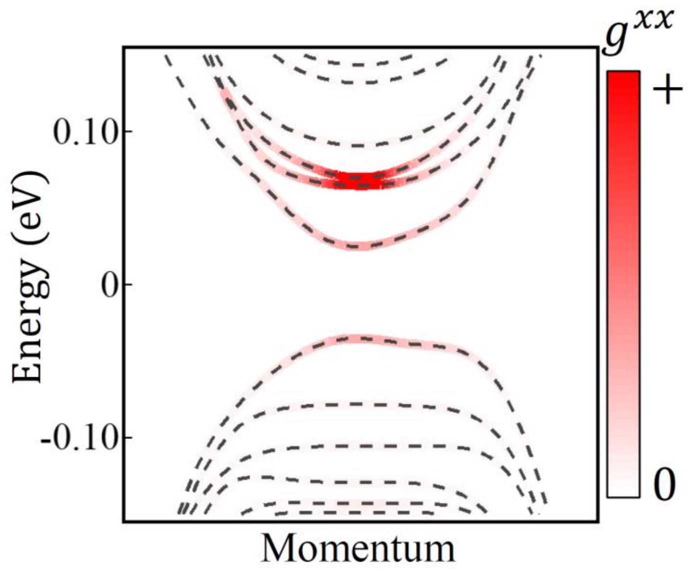
b) Lattice mismatch ⇒ strain

[Gao, Liu, Qiu, Gosh, Trevisan et al., Science (2023)]

The BP/MBT/BP heterostructure: results from theoretical modeling

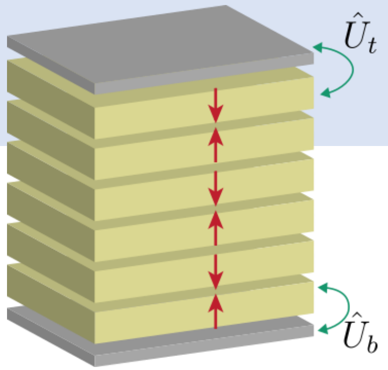


Band structure:



Large quantum metric at small band gaps

The BP/MBT/BP heterostructure: results from theoretical modeling



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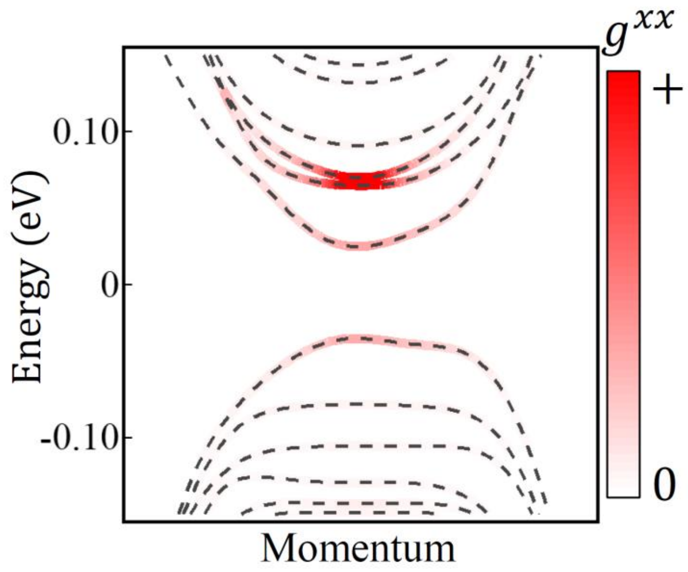
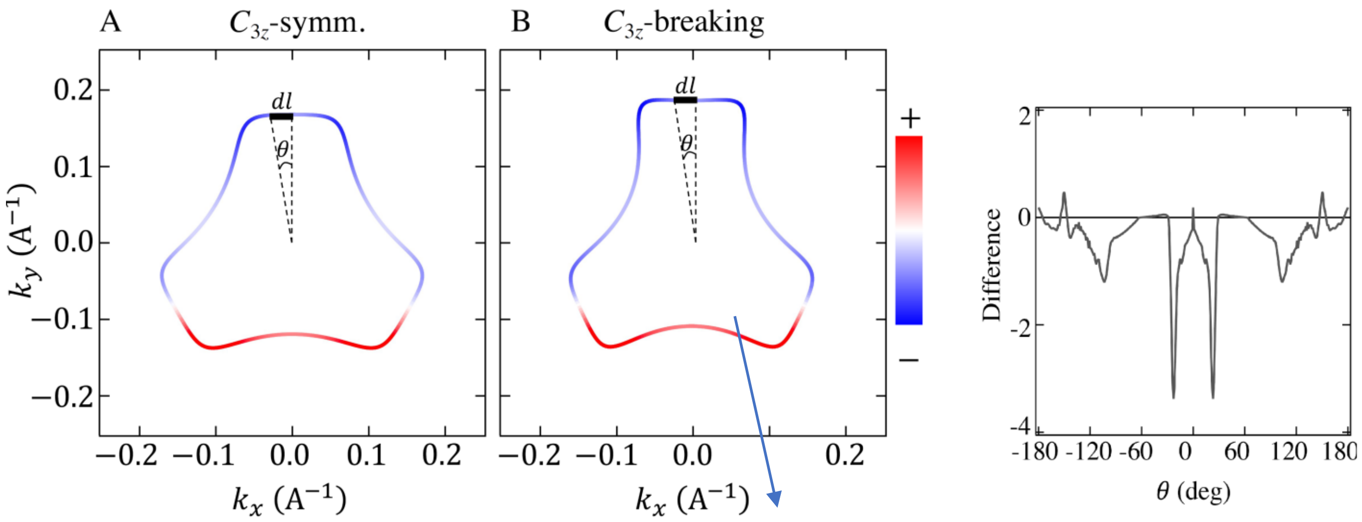


Large quantum metric at small band gaps

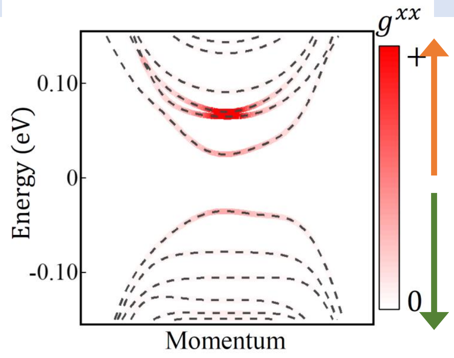
Quantum metric dipole:

For $\mu = -50$ meV:

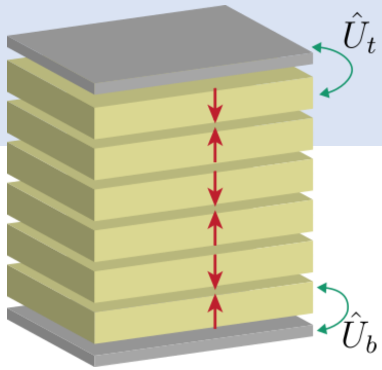


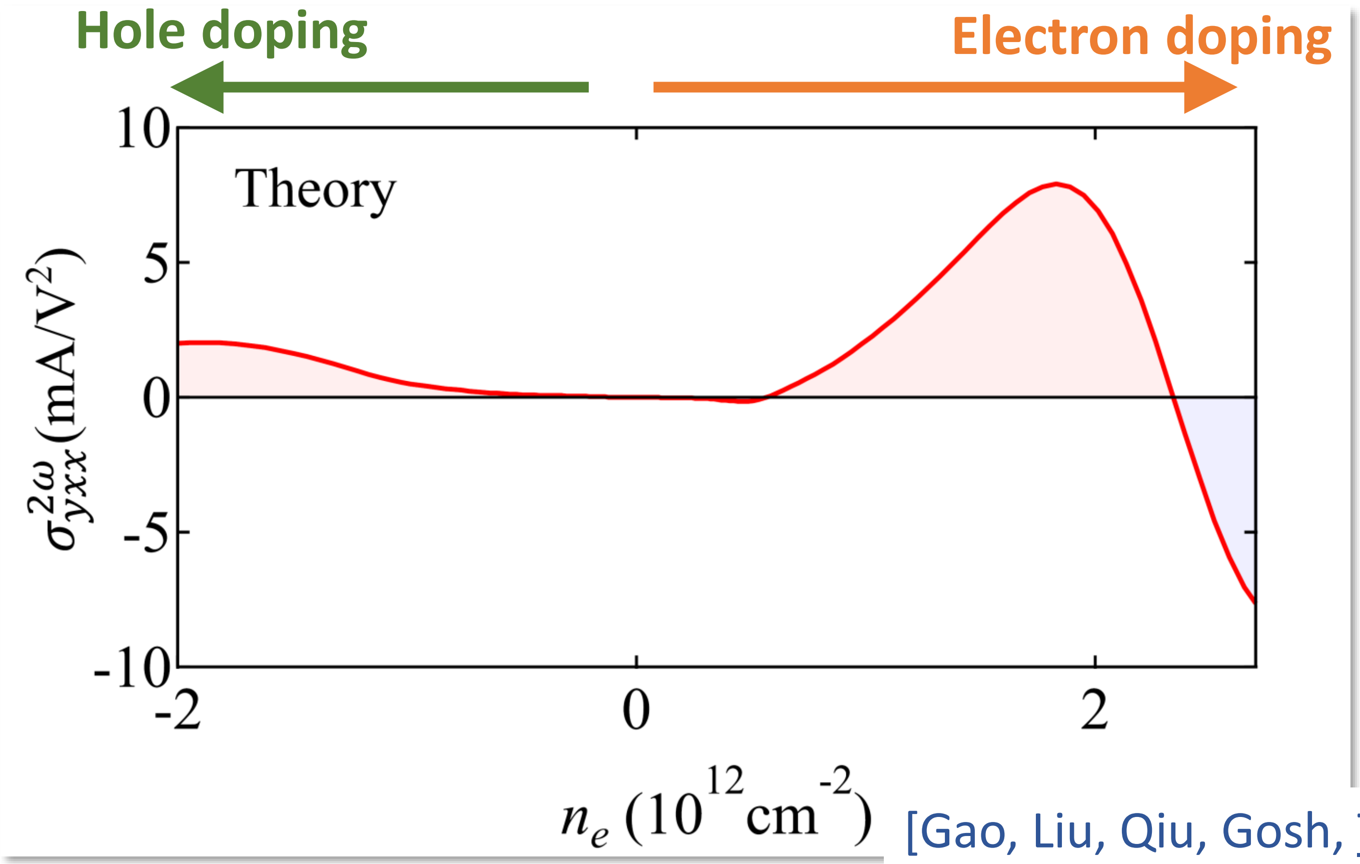
$$\int_{\mathrm{FS}} d\mathbf{k}\,[v_y g_{xx} - v_x g_{yx}] \neq 0$$

Energy dependence of the non-linear anomalous Hall effect



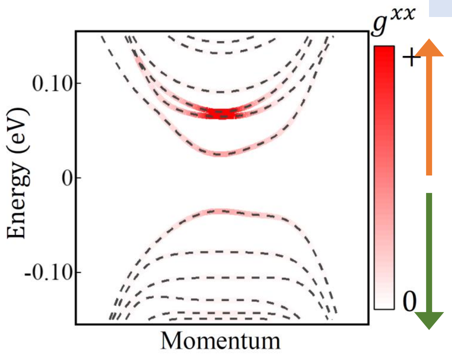
$$\sigma_{yxx} = 2e^3 \sum_n \int \frac{d^d k}{(2\pi)^d} \frac{v_y^n g_{xx}^{(n)}(\mathbf{k}) - v_x^n g_{yx}^{(n)}(\mathbf{k})}{\varepsilon_{n,\mathbf{k}} - \varepsilon_{\bar{n},\mathbf{k}}} \delta(\varepsilon_{n,\mathbf{k}} - \varepsilon_F) + \mathrm{AIC}$$

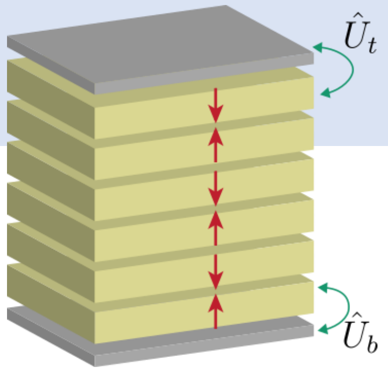




[Gao, Liu, Qiu, Gosh, Trevisan et al., Science (2023)]

Energy dependence of the non-linear anomalous Hall effect





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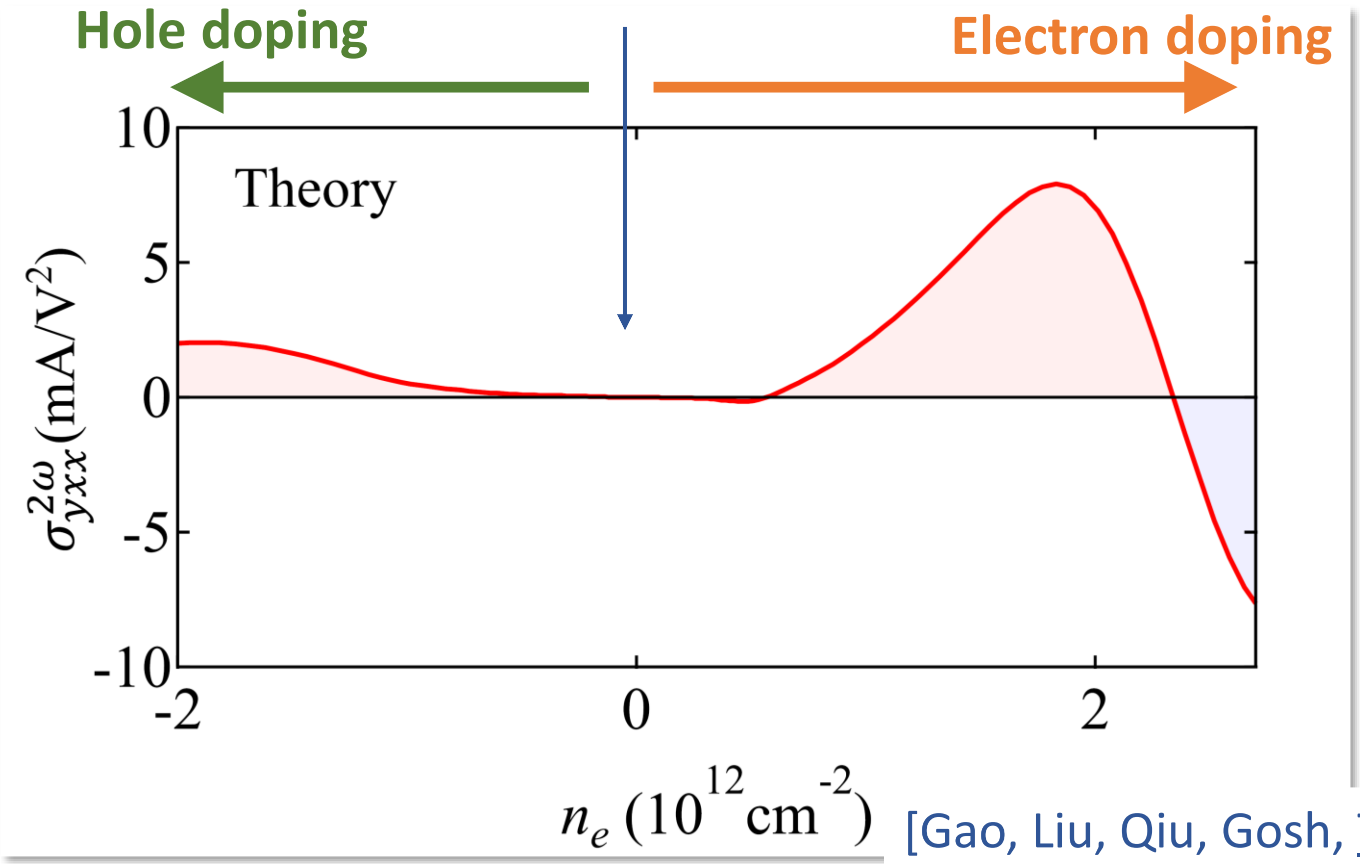
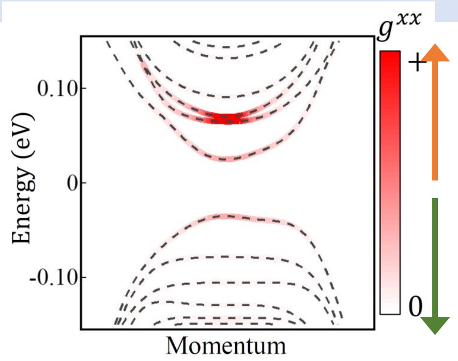
1) Zero at charge neutrality

⇒Fermi surface property

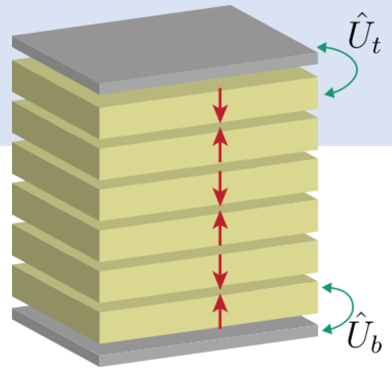


[Gao, Liu, Qiu, Gosh, Trevisan et al., Science (2023)]

Energy dependence of the non-linear anomalous Hall effect



$$\sigma_{yxx} = 2e^3 \sum_n \int \frac{d^d k}{(2\pi)^d} \frac{v_y^n g_{xx}^{(n)}(\mathbf{k}) - v_x^n g_{yx}^{(n)}(\mathbf{k})}{\varepsilon_{n,\mathbf{k}} - \varepsilon_{\bar{n},\mathbf{k}}} \delta(\varepsilon_{n,\mathbf{k}} - \varepsilon_F) + \mathrm{AIC}$$



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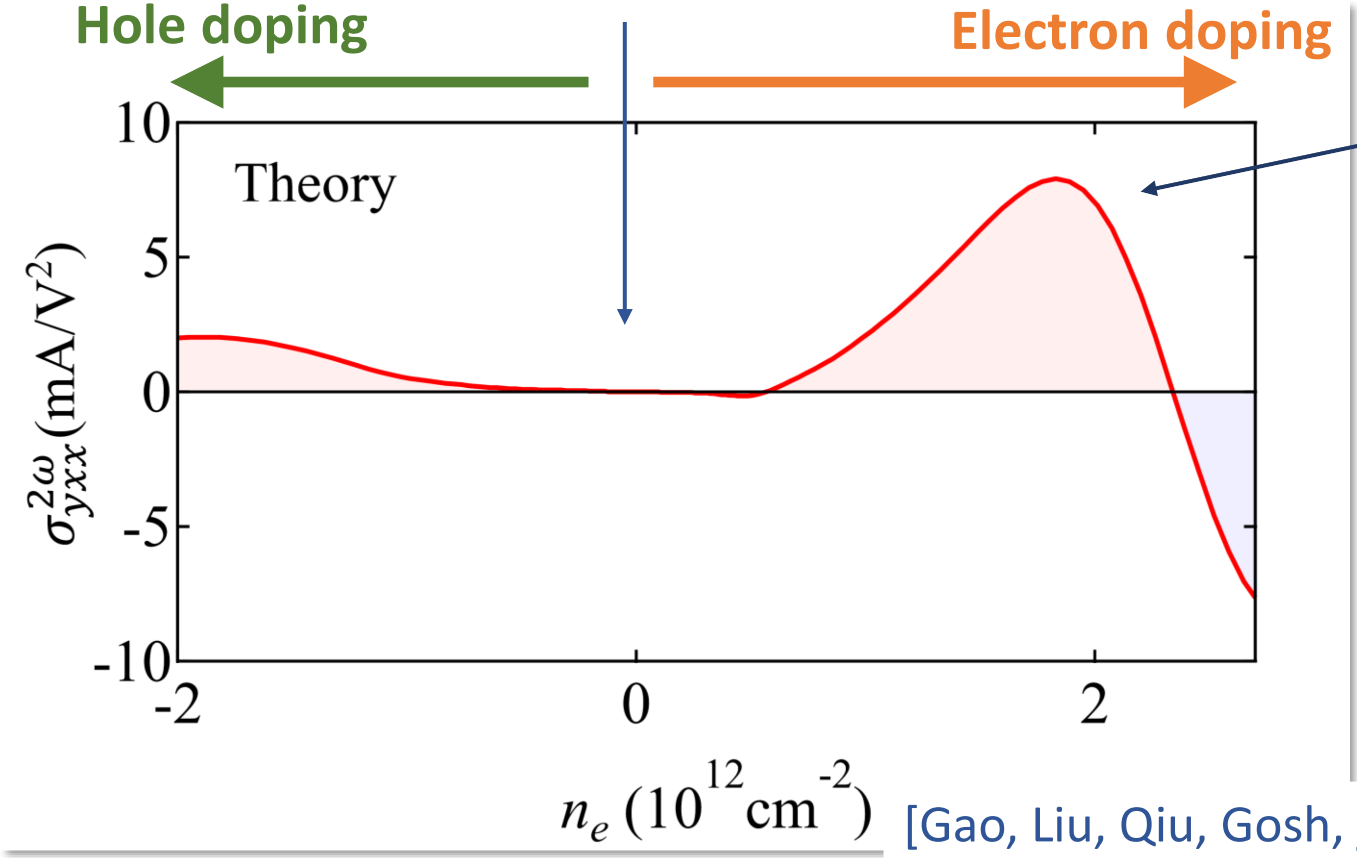
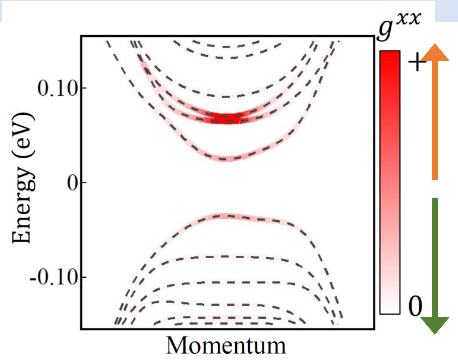


2) Larger for electron doping

⇒Larger quantum metric

[Gao, Liu, Qiu, Gosh, Trevisan et al., Science (2023)]

Energy dependence of the non-linear anomalous Hall effect



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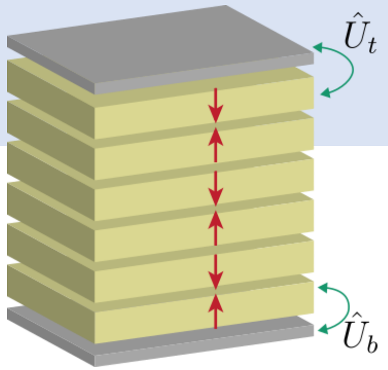

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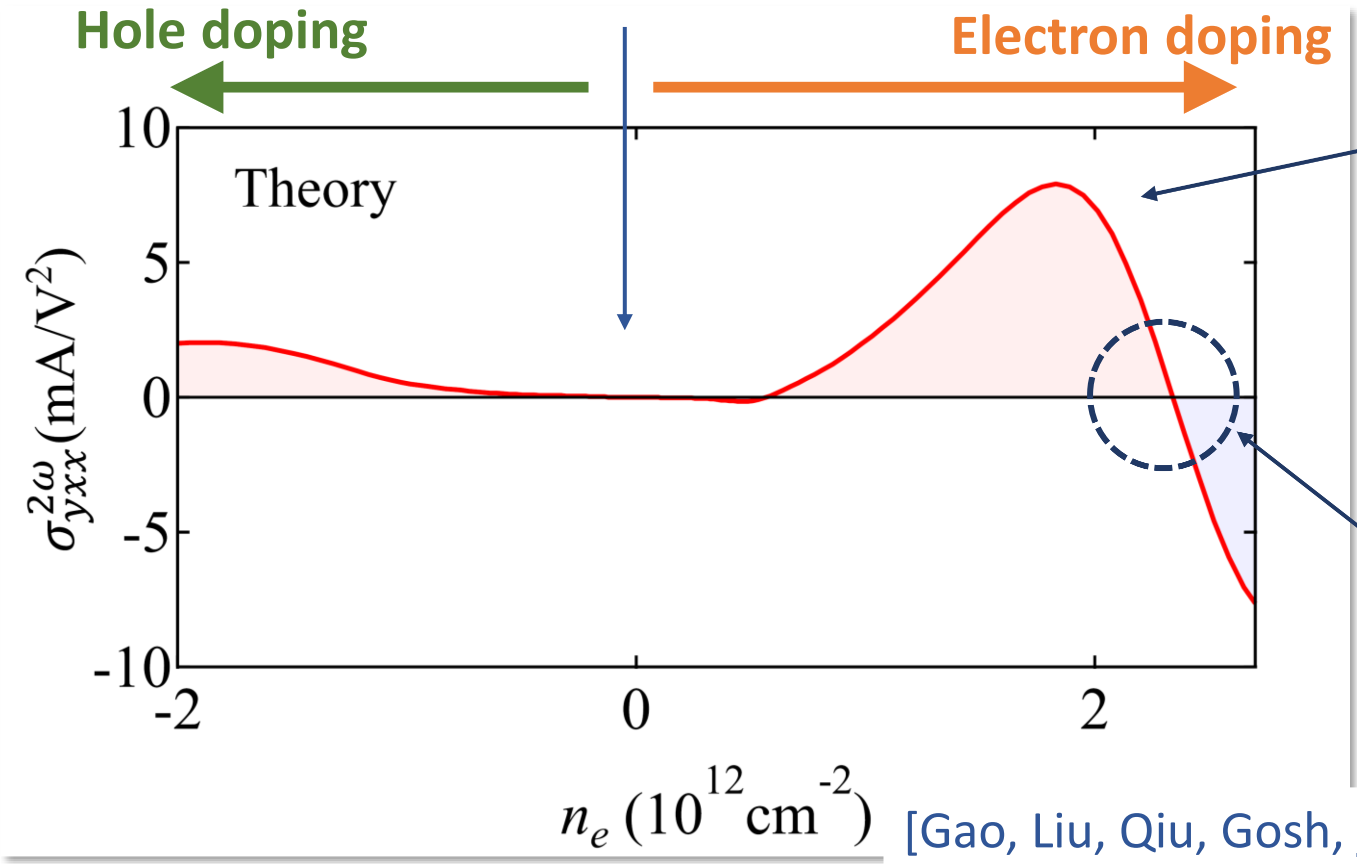


2) Larger for electron doping

⇒Larger quantum metric

3) Changes sign

⇒ hybridization between BP and MBT bands

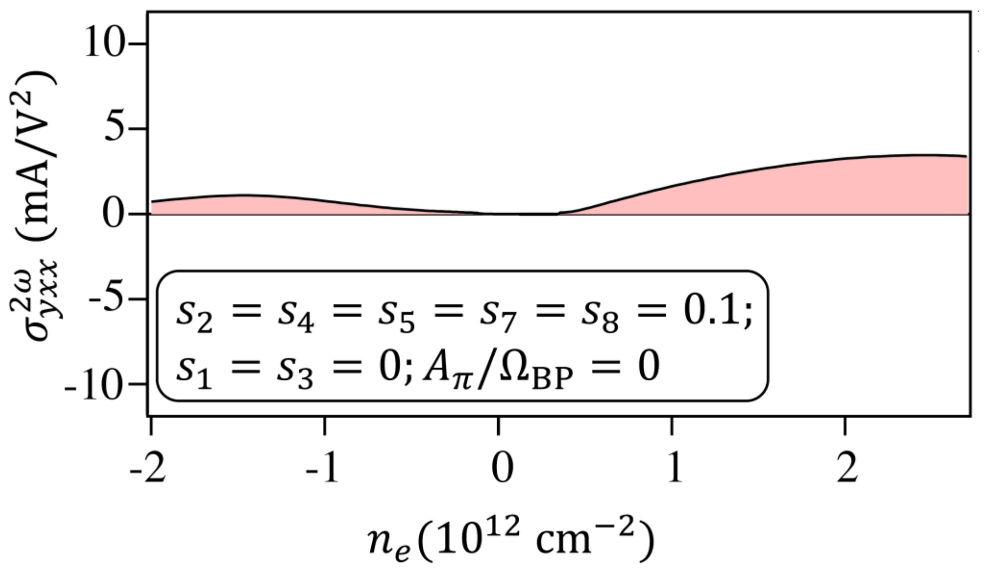
[Gao, Liu, Qiu, Gosh, Trevisan et al., Science (2023)]

Strain alone cannot capture the features of the observed Hall signal

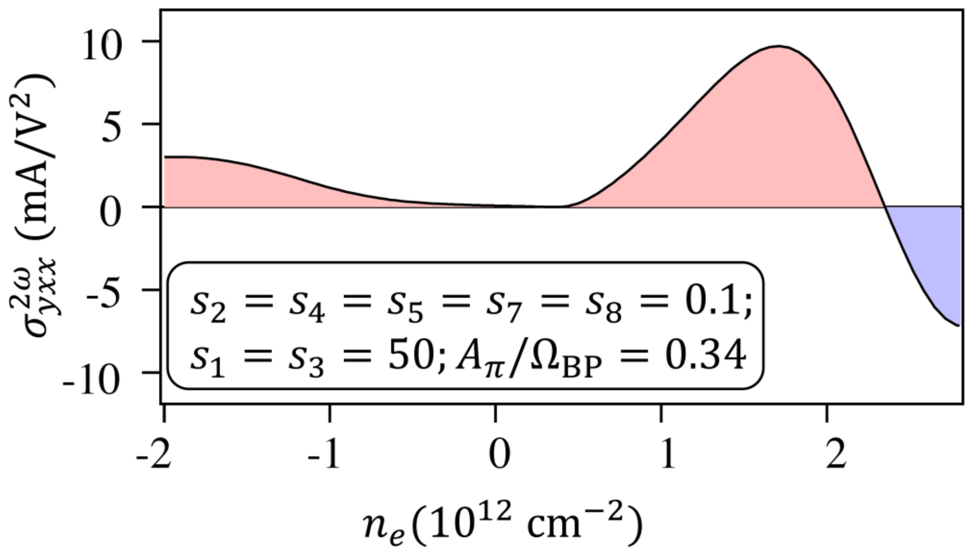
- Large mismatch between MBT and BP lattice induces strain
- New terms in the MBT minimal model

$$\hat{h}(\mathbf{k}) = \hat{h}_N(\mathbf{k}) + (-1)^{\zeta}\gamma_{\mathrm{af}}\hat{h}_{\mathrm{AFM}}(\mathbf{k}) + \gamma_s \hat{h}_s(\mathbf{k})$$

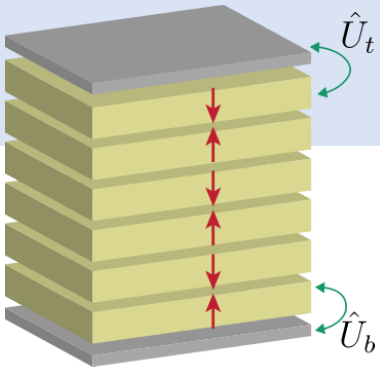
Strain only

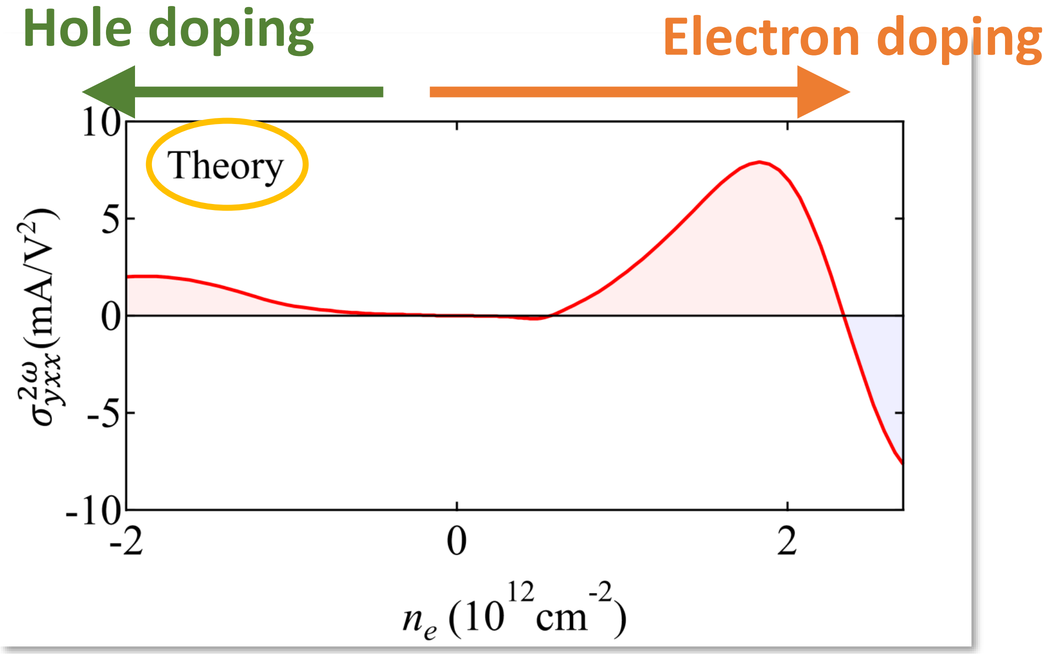


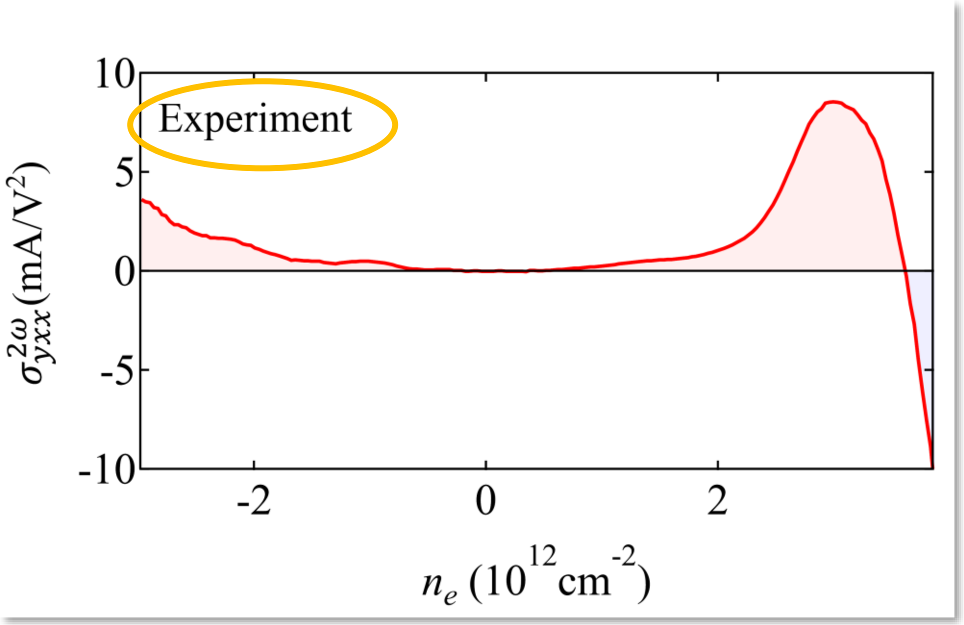
Strain and MBT-BP hybridization



Energy dependence of the non-linear anomalous Hall effect

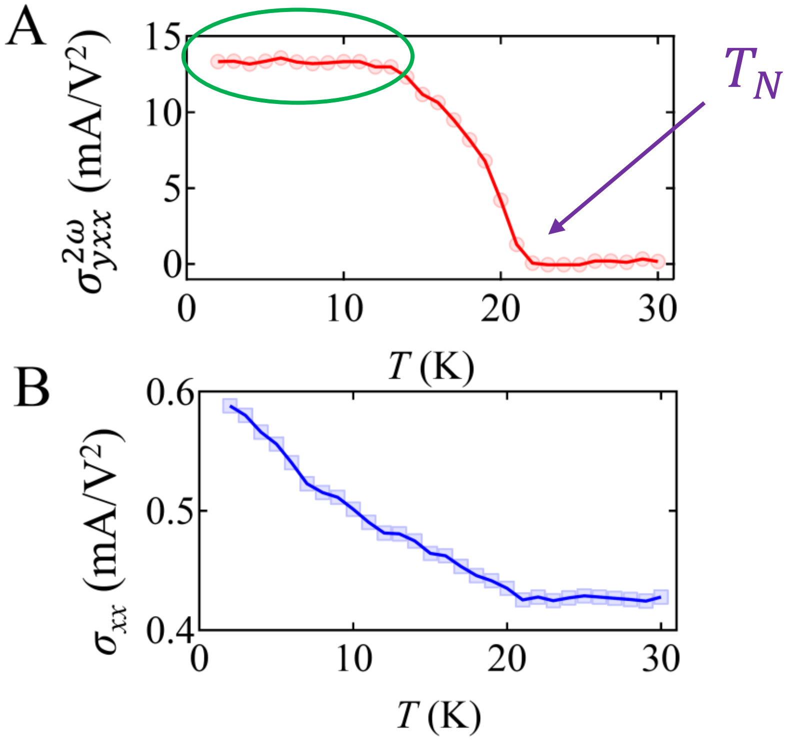






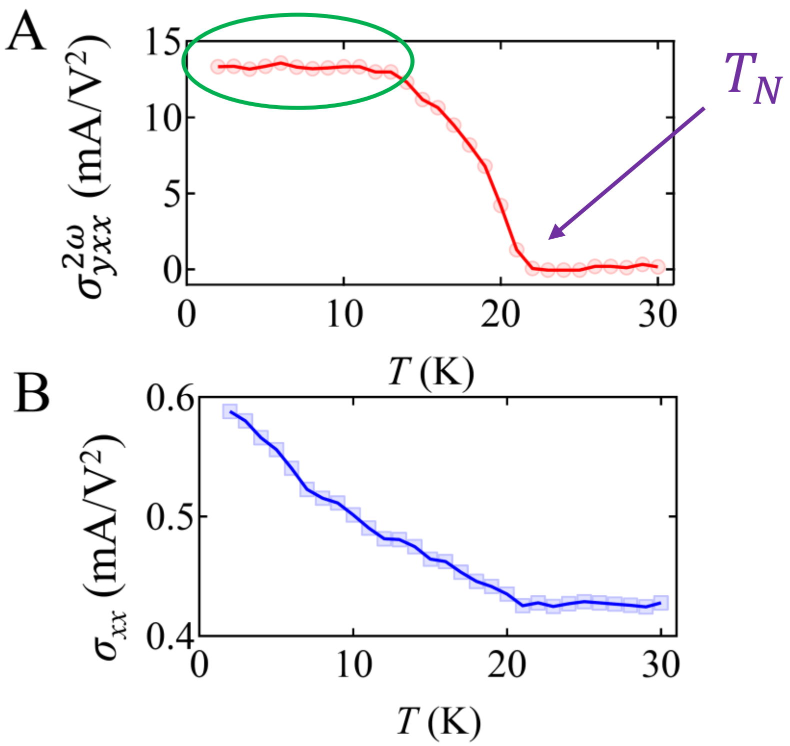
[Gao, Liu, Qiu, Gosh, Trevisan et al., Science (2023)]

Ruling out other sources of second-order AHE



1) Finite only below T_N $\Rightarrow$ Hall signal comes from the spin texture

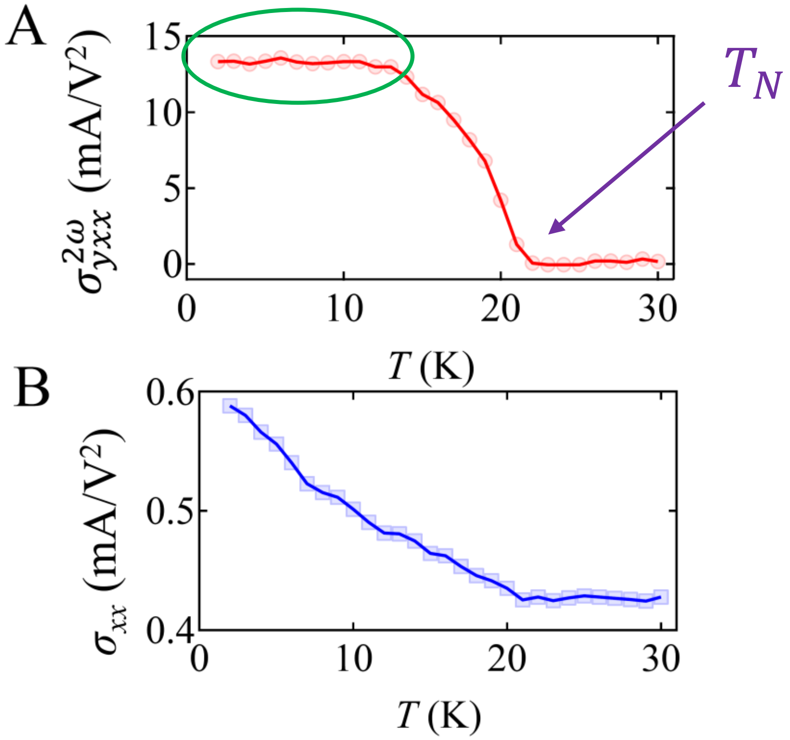
Ruling out other sources of second-order AHE

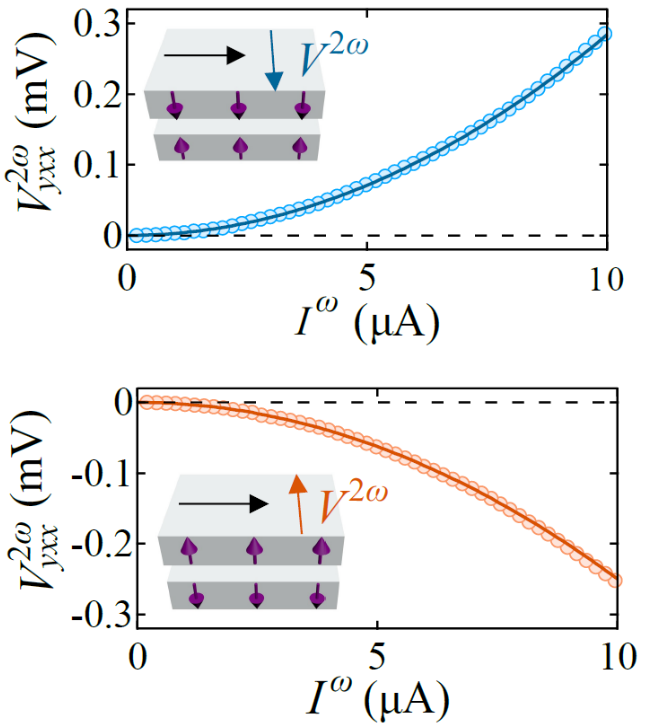


1) Finite only below T_N $\Rightarrow$ Hall signal comes from the spin texture

2) Independent of σ_{xx} $\Rightarrow$ intrinsic effect

Ruling out other sources of second-order AHE



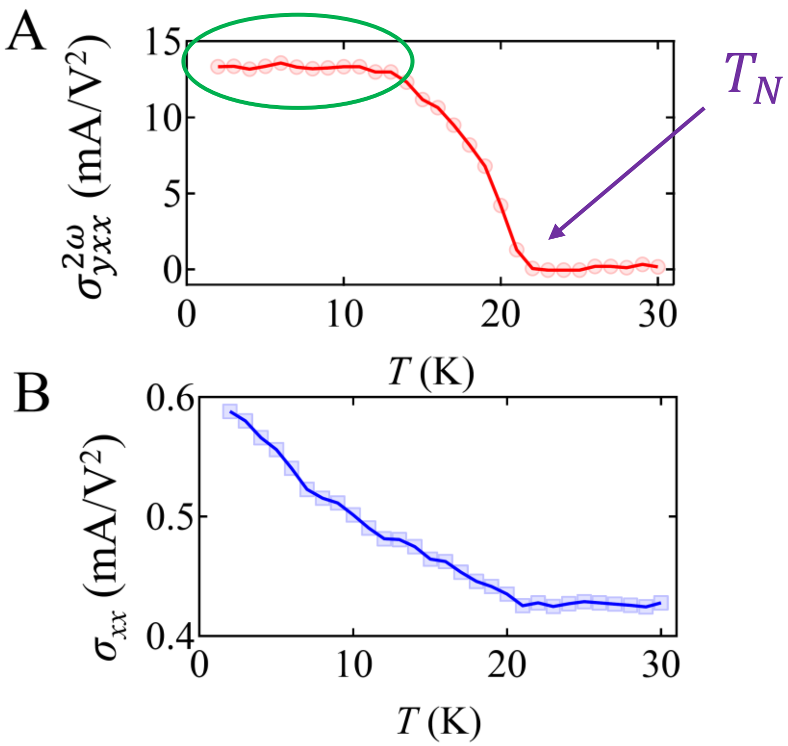


1) Finite only below T_N ⇒ Hall signal comes from the spin texture

2) Independent of σ_{xx} ⇒ intrinsic effect

3)Hall signal changes sign under time-reversal

Ruling out other sources of second-order AHE



1) Finite only below T_N $\Rightarrow$ Hall signal comes from the spin texture

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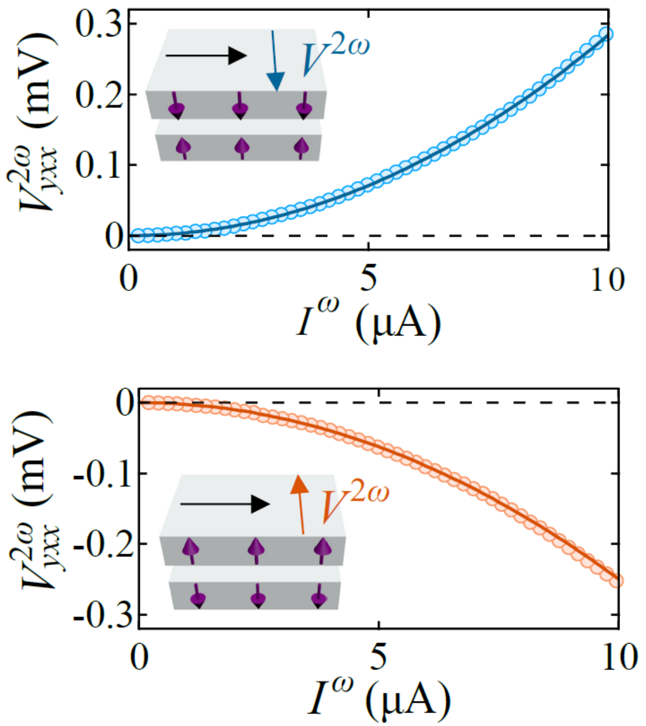
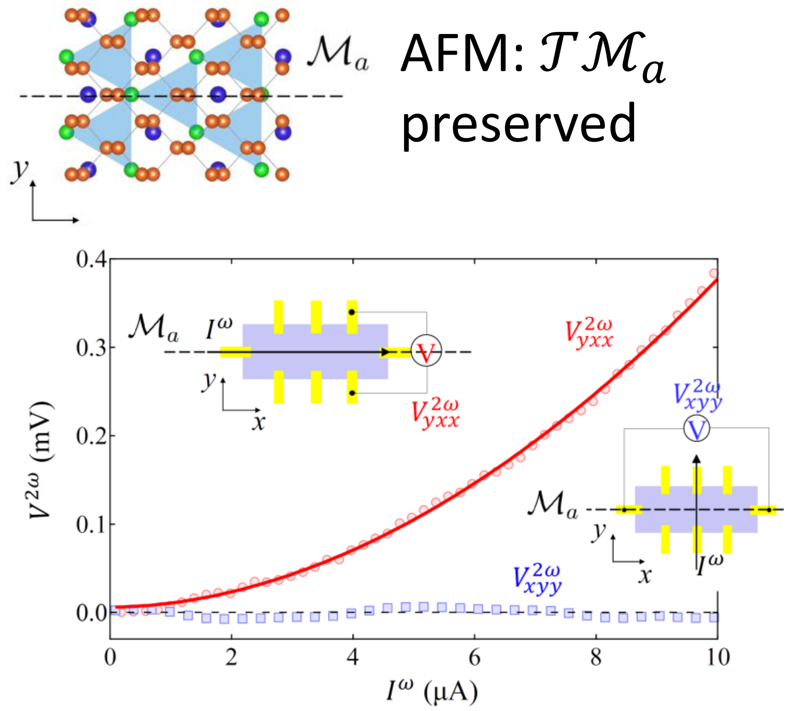
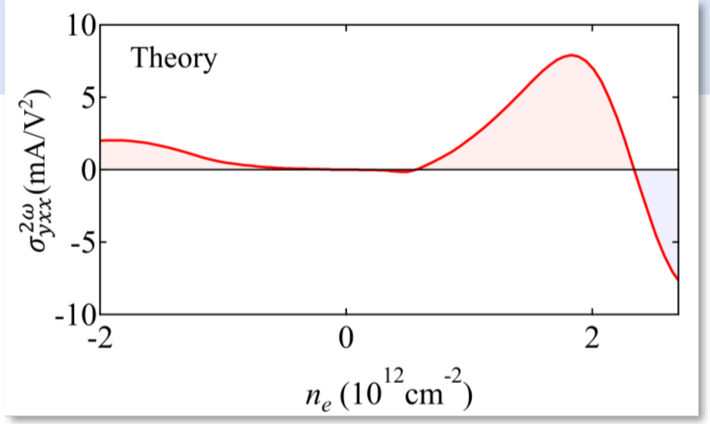


3)Hall signal changes sign under time-reversal



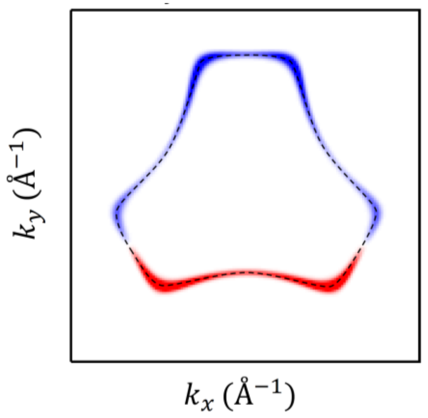
4) Symmetry constraint rule out Berry curvature dipole

Summary



- Second-order anomalous Hall effect
  - Extrinsic: disorder, Berry curvature multipoles
  - Intrinsic: dipoles of quantum metric

- Intrinsic component dominates in PT-symmetric materials with large quantum metric
  - BP/MBT/BP is an ideal platform
  - Main features captured by a minimal model
  - Hybridization between MBT and BP bands is essential to account for sign change of the Hall signal



- Future direction: what about multipoles of quantum metric?

[Gao, Liu, Qiu, Gosh, Trevisan et al., Science (2023)]

Thank you!

Extra Slides

Frequency dependence of the quantum metric anomalous Hall conductivity

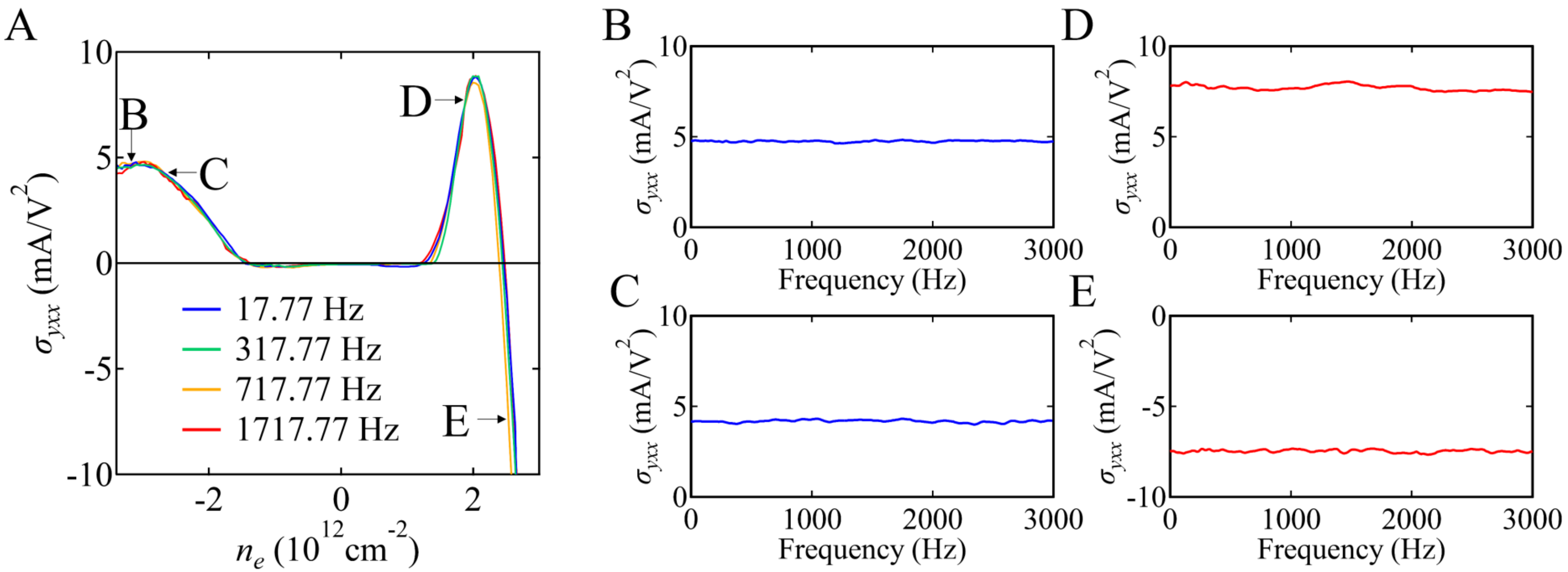


Fig. S23. Measured frequency dependence of the nonlinear Hall effect. (A) $\sigma^{2\omega}_{yxx} - n_e$ at discrete frequency ω values. (B-E) $\sigma^{2\omega}_{yxx} - \omega$ at four charge density n_e values (noted by the black arrows in panel A).

Dominant contribution of the quantum metric term

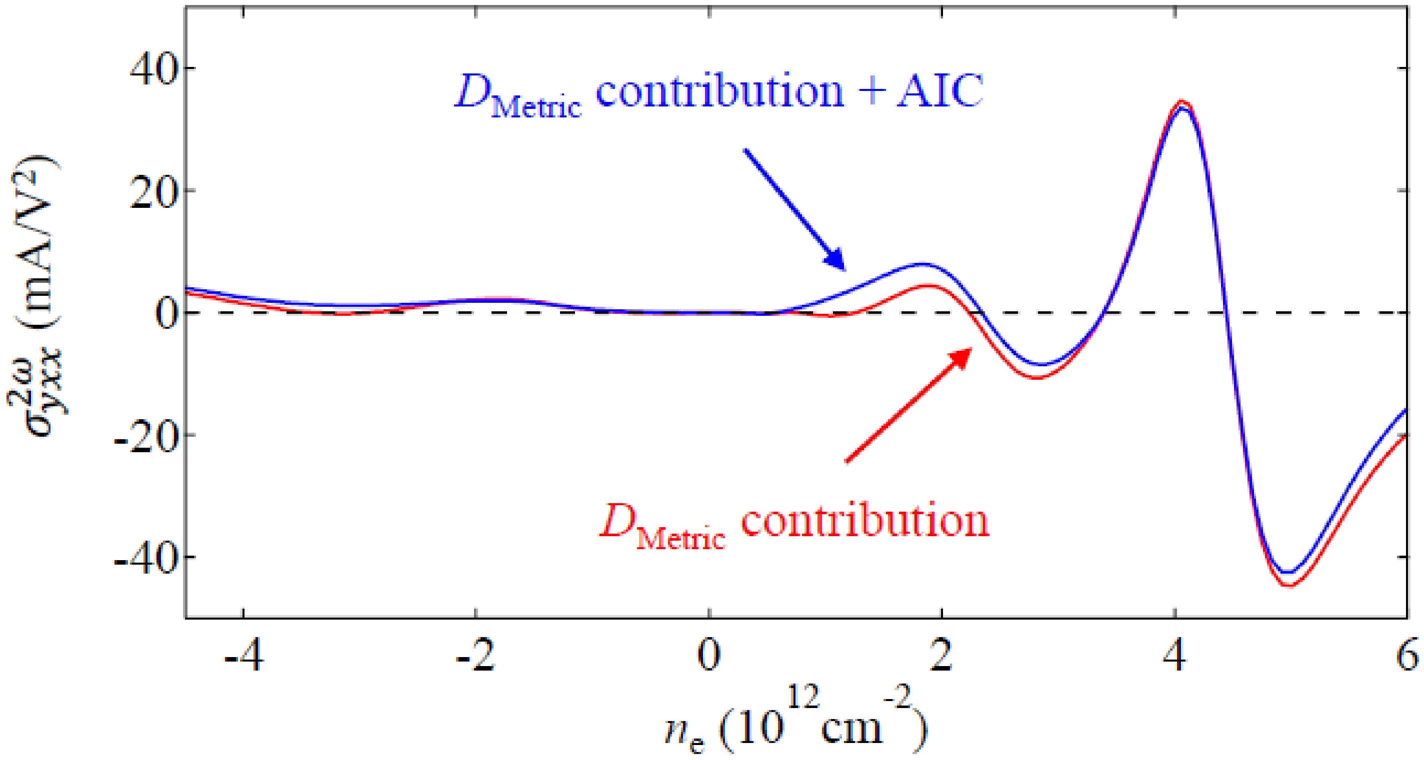


Fig. S42. Quantum metric dipole dominated nonlinear Hall conductivity. Two-band model (quantum metric dipole D_{Metric} contribution) and multiband model (D_{Metric}+ additional inter band contributions AIC) calculated nonlinear Hall conductivity as a function of carrier density. The nonlinear Hall signals are dominated by the quantum metric dipole contribution.