Seeing topological charges by naked eyes

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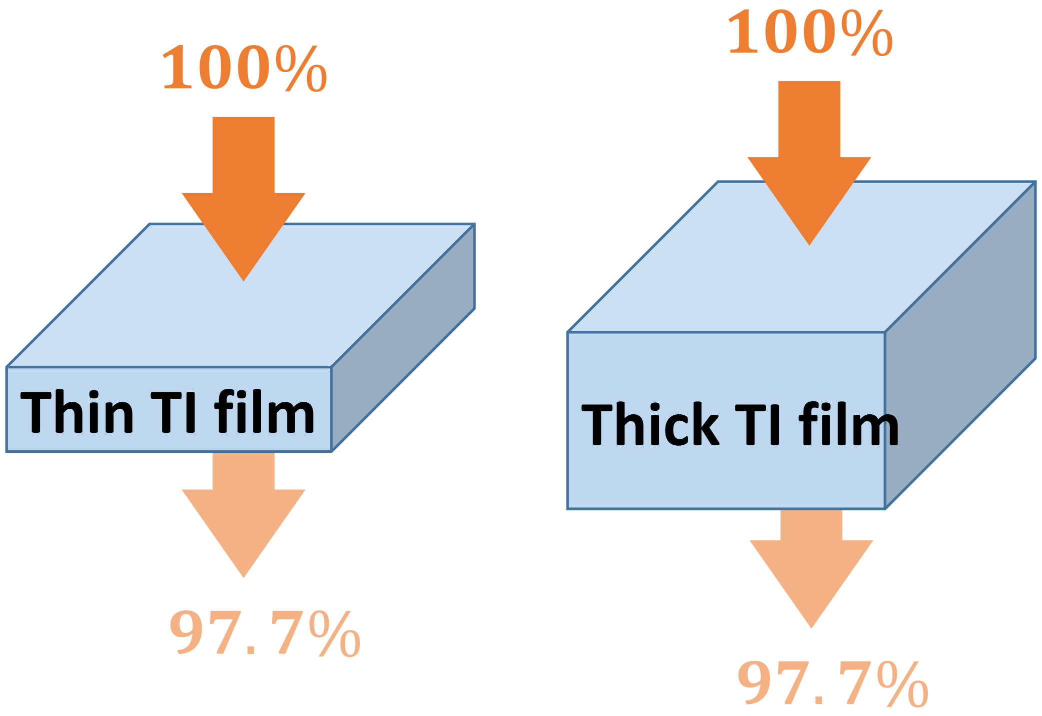


Outline

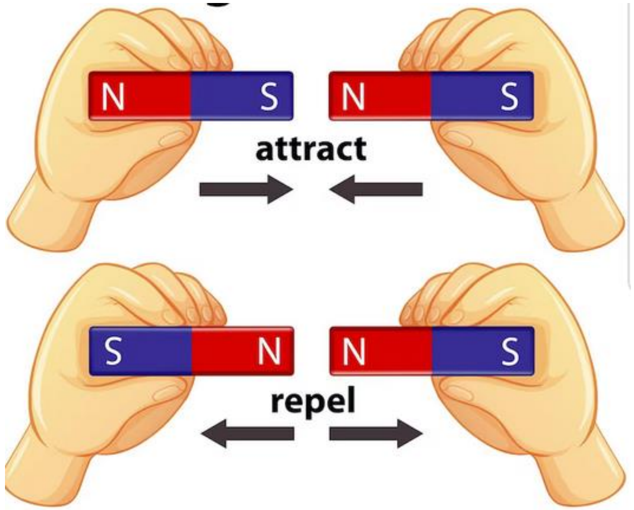
Seeing the topological charge of various materials by naked eyes through an infrared lens:

- ❑ Opacity of graphene $\pi\alpha \approx 2.3\%$ as a topological charge
- ❑ Topologically protected fine-structure constant
- ❑ 3D topological insulator thin films of any thickness have the same opacity $\pi\alpha \approx 2.3\%$
- ❑ Seeing topological surface states by naked eyes
- ❑ Darkness of 3D Weyl semimetals as a topological charge

Can topological order of materials be observed by human perception?

Landau order parameters can be easily perceived in macroscopic scale

Magnetic force between bar magnets

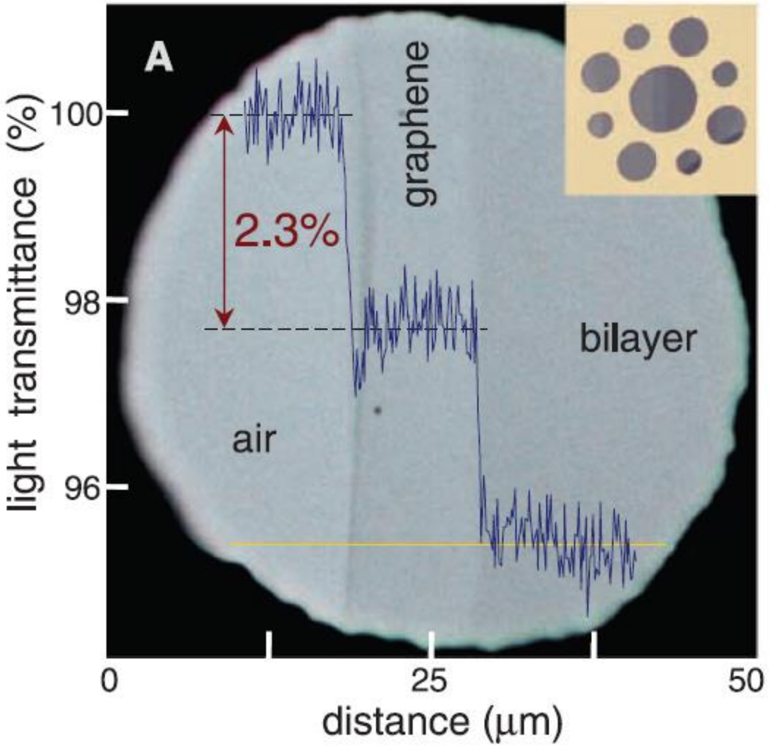


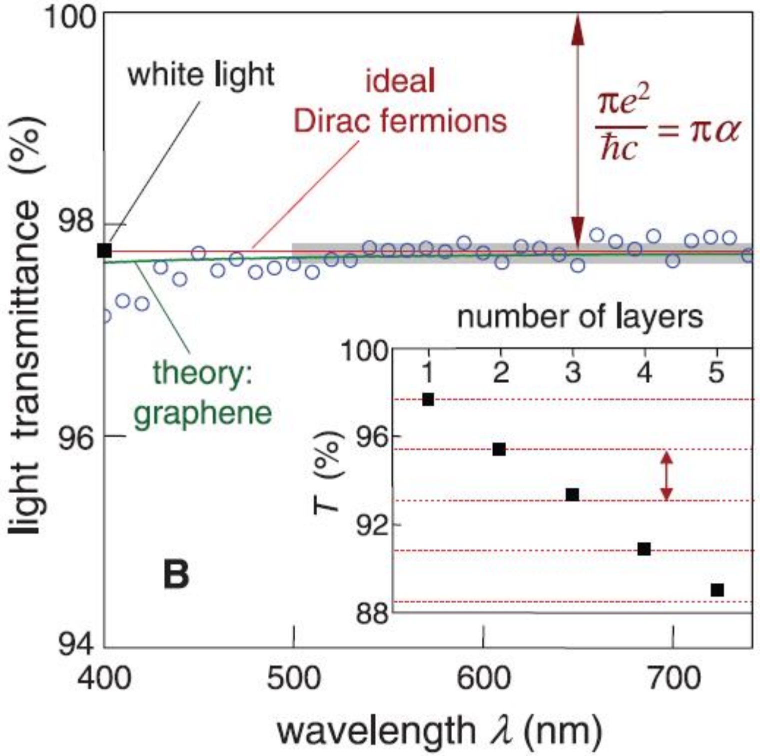
Magnetic levitation by superconductors

Can topological order of materials also be perceived in macroscopic scale?

Opacity of graphene

Opacity of graphene is known to be roughly given by $\pi\alpha \approx 2.3\%$ independent of the frequency and polarization of the light





Nair et al, Science 320, 1308 (2008)

Opacity = $\dfrac{\text{Absorption power}}{\text{Incident power}}$

Light with frequency ω and polarization μ

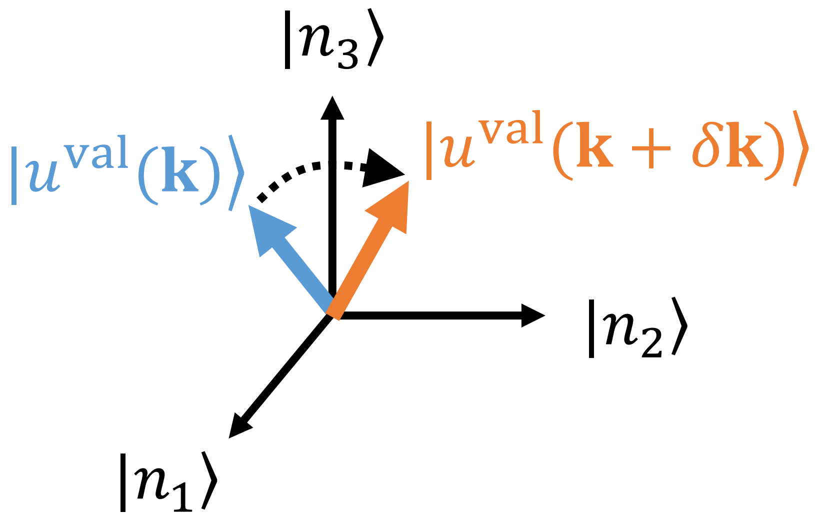
$$\mathcal{O}(\omega) = \frac{W_a^\mu(\omega)}{W_i} \qquad W_i = c\varepsilon_0 E_0^2/2$$

Absorption power is given by optical conductivity $W_a^\mu(\omega) = \langle j_\mu(\omega,t)E_\mu(\omega,t)\rangle_t = \frac{1}{2}\sigma_{\mu\mu}(\omega)E_0^2$

Linking optical conductivity to quantum metric

The N_--particle valence band state of semiconductors

$$|u^{\rm val}(\mathbf{k})\rangle = \epsilon^{n_1 n_2 \ldots n_{N-}} |n_1\rangle|n_2\rangle\ldots|n_{N_-}\rangle/\sqrt{N_-!}$$



Provost and Vallee, Comm. Math. Phys. 76, 289 (1980)

Quantum metric on the BZ manifold

$$|\langle u^{\rm val}(\mathbf{k})|u^{\rm val}(\mathbf{k}+\delta\mathbf{k})\rangle| = 1 - \frac{1}{2} g_{\mu\nu}(\mathbf{k})\delta k^\mu \delta k^\nu$$

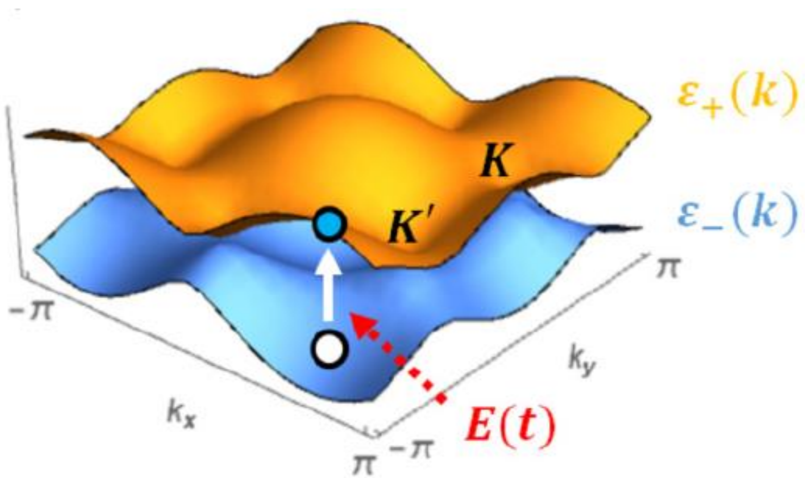
$$g_{\mu\mu}(\mathbf{k}) = \langle \partial_\mu u^{\rm val}|\partial_\mu u^{\rm val}\rangle - \langle \partial_\mu u^{\rm val}|u^{\rm val}\rangle\langle u^{\rm val}|\partial_\mu u^{\rm val}\rangle$$
$$= \sum_{nm} \langle \partial_\mu n|m\rangle\langle m|\partial_\mu n\rangle.$$

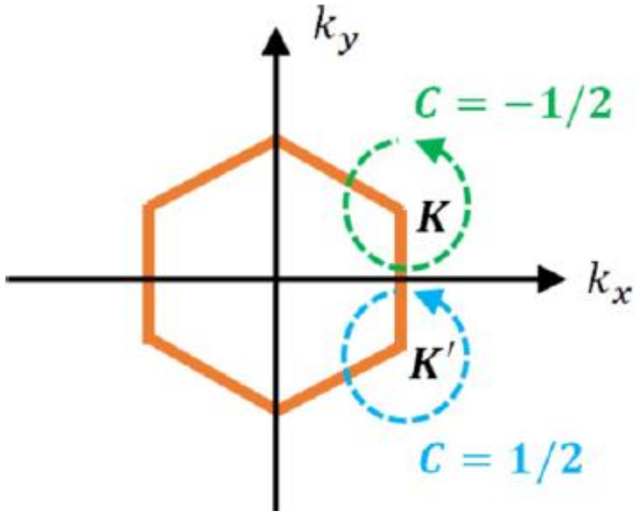
Given dipole energy $E \cdot \hat{\mu} = E i \partial_\mu$, optical conductivity at momentum k is related to $g_{\mu\mu}$

$$\sigma_{\mu\mu}(\mathbf{k},\omega) = \frac{\pi e^2 \hbar\omega}{V_D} \sum_{nm} \underbrace{\langle \partial_\mu n|m\rangle\langle m|\partial_\mu n\rangle}_{g_{\mu\mu}} \left[f(\varepsilon_n^{\mathbf{k}}) - f(\varepsilon_m^{\mathbf{k}})\right] \delta\left(\omega + \frac{\varepsilon_n^{\mathbf{k}}}{\hbar} - \frac{\varepsilon_m^{\mathbf{k}}}{\hbar}\right)$$

Linking quantum metric to topological charge

Band structure and topological charge of graphene $H_0^{\mathbf{K},\mathbf{K}'}(\mathbf{k}) = v_F\left(\pm k_y\sigma_x - k_x\sigma_y\right)$





$$\oint \frac{d\phi}{2\pi}\langle n^{\mathbf{K}\uparrow}|i\partial_\phi|n^{\mathbf{K}\uparrow}\rangle$$
$$= -\oint \frac{d\phi}{2\pi}\langle n^{\mathbf{K}'\uparrow}|i\partial_\phi|n^{\mathbf{K}'\uparrow}\rangle$$
$$= -1/2 \equiv -\mathcal{C}$$

Azimuthal quantum metric turns out to be equal to the topological charge

$$g_{\phi\phi}^{\gamma} = |\langle m^\gamma|i\partial_\phi|n^\gamma\rangle|^2 = |\langle n^\gamma|i\partial_\phi|n^\gamma\rangle|^2 = \mathcal{C}^2 = \frac{1}{4}$$

Metric-curvature correspondence

von Gersdorff and Chen, PRB 104, 195133 (2021)

Azimuthal quantum metric converted back to Cartesian coordinate

$$g_{\mu\mu}^{\gamma} = \frac{\sin^2\phi}{k^2} g_{\phi\phi}^{\gamma} = \frac{\sin^2\phi}{k^2}\mathcal{C}^2$$

Linking opacity of graphene to topological charge

Optical conductivity is real space is given by momentum integration

$$\sigma_{\mu\mu}(\omega) = V_D \int \frac{d^D\mathbf{k}}{(2\pi\hbar)^D}\,\sigma_{\mu\mu}(\mathbf{k},\omega) = \frac{\pi e^2}{\hbar^{D-1}}\,\omega \int \frac{d^D\mathbf{k}}{(2\pi)^D}\, g^{\gamma}_{\mu\mu}\left[f(\varepsilon^{\mathbf{k}}_n) - f(\varepsilon^{\mathbf{k}}_m)\right]\delta\left(\omega + \frac{\varepsilon^{\mathbf{k}}_n}{\hbar} - \frac{\varepsilon^{\mathbf{k}}_m}{\hbar}\right)$$

$$g^{\gamma}_{\mu\mu} = \frac{\sin^2\phi}{k^2}\, g^{\gamma}_{\phi\phi} = \frac{\sin^2\phi}{k^2}\,\mathcal{C}^2$$

Optical conductivity is proportional to topological charge

$$\sigma_{\mu\mu}(\omega) = \frac{e^2}{\hbar}\mathcal{C}^2\left[f\left(-\frac{\hbar\omega}{2}\right) - f\left(\frac{\hbar\omega}{2}\right)\right]$$

Topological charge is hiding
In the optical conductivity

As a result, the opacity is given by $\pi\alpha \times$ topological charge

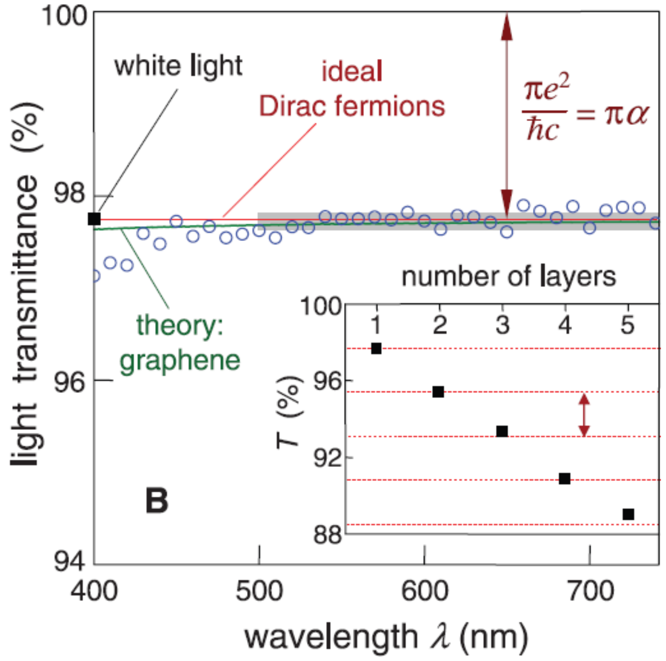
$$\mathcal{O}(\omega) = \frac{W^{\mu}_{a}(\omega)}{W_i} = \pi\alpha \times 4\mathcal{C}^2\left[f\left(-\frac{\hbar\omega}{2}\right) - f\left(\frac{\hbar\omega}{2}\right)\right]$$

de Sousa, Cruz, and Chen, arXiv:2303.14549

Seeing topological charge of graphene by naked eyes

Zero temperature limit of the opacity is independent of frequency and polarization of light

$$\lim_{T\to 0} \mathcal{O}(\omega) = \pi\alpha \times 4\mathcal{C}^2 = \pi\alpha \approx 2.3\%$$



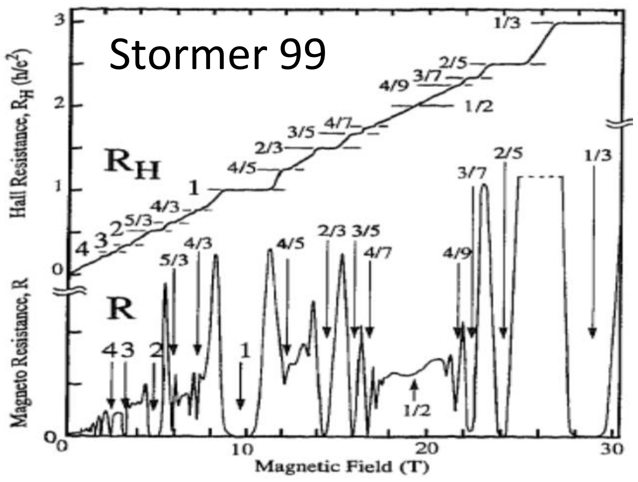
This implies:

de Sousa, Cruz, and Chen, arXiv:2303.14549

(1) One can literally see the topological charge by naked eyes

(2) All 2D Dirac semimetals have the same opacity at infrared

(3) The fine-structure constant is topologically protected

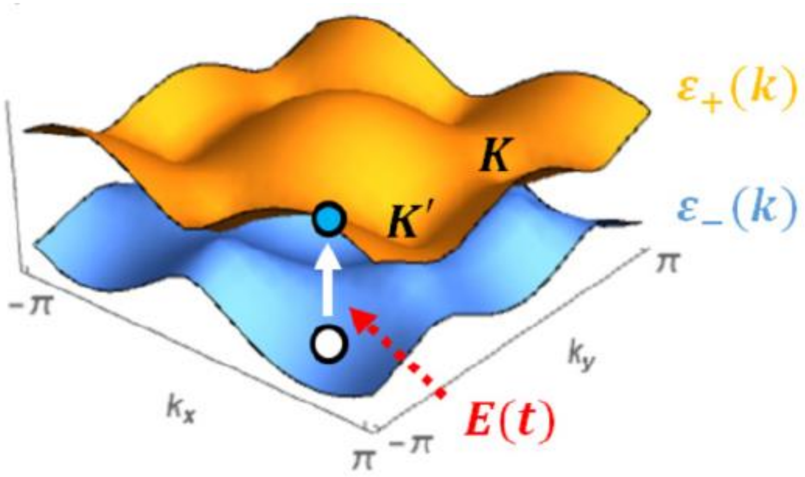


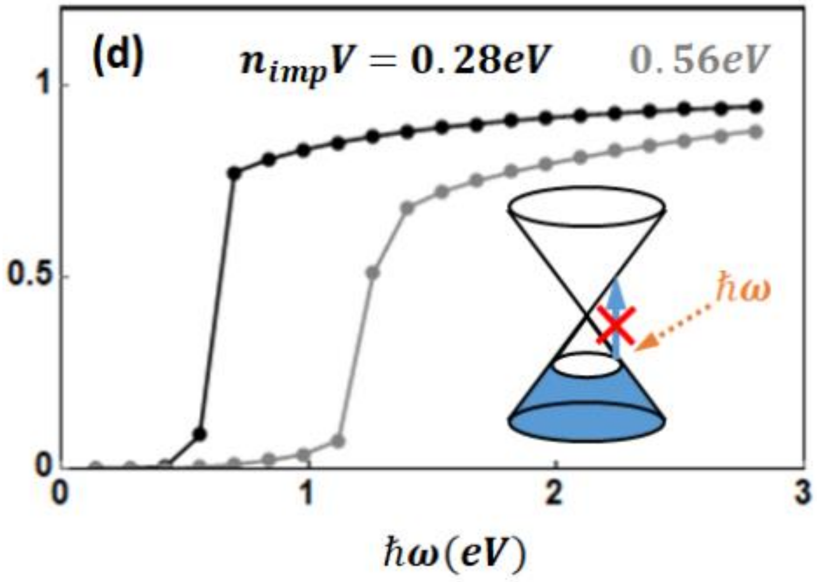
The only other topologically protected constant is the von Klitzing constant h/e^2 in QHE. Our paper thus plays the same role as the TKNN paper that links the quantized Hall conductance to a topological invariant.

Thouless, Kohmoto, Nightingale, and den Nijs, PRL 49, 405 (1982).

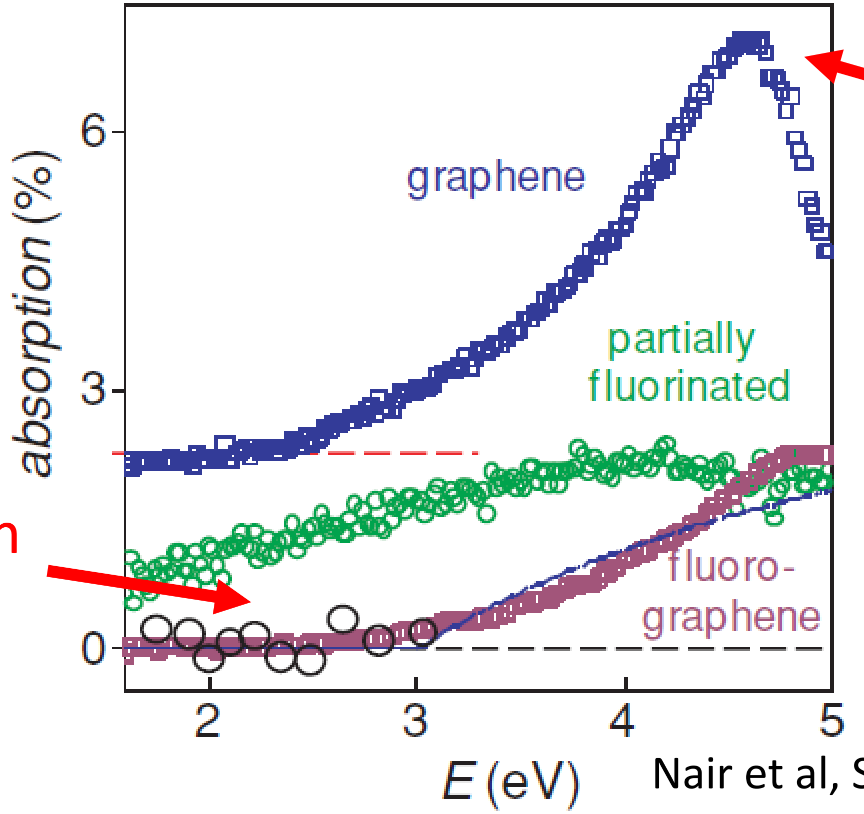
However, the reality is more complicated…

The band structure of graphene is not perfectly linear in the visible light range





Suppression by disorder



Van Hove singularity

Nair et al, Small 6, 2877 (2010)

The suppression of opacity at low frequency in fluorinated graphene can be explained by shift of chemical potential caused by impurities, which blocks optical absorption.

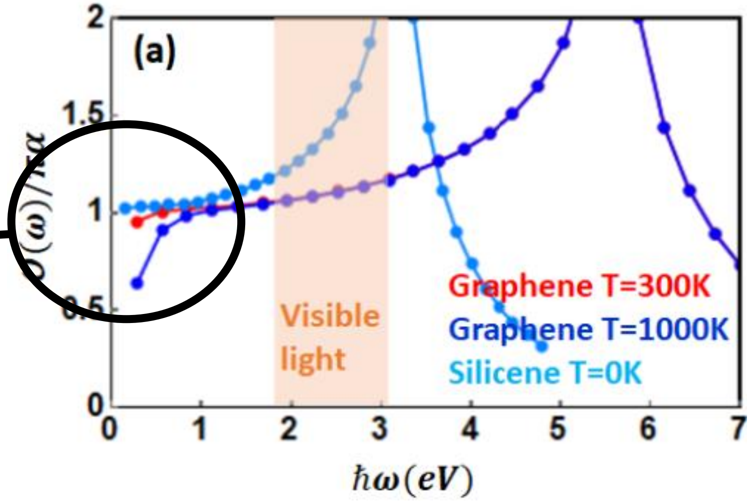
Extracting fine-structure constant accurately?

Given all these complications, can fine-structure constant be extracted accurately from opacity?

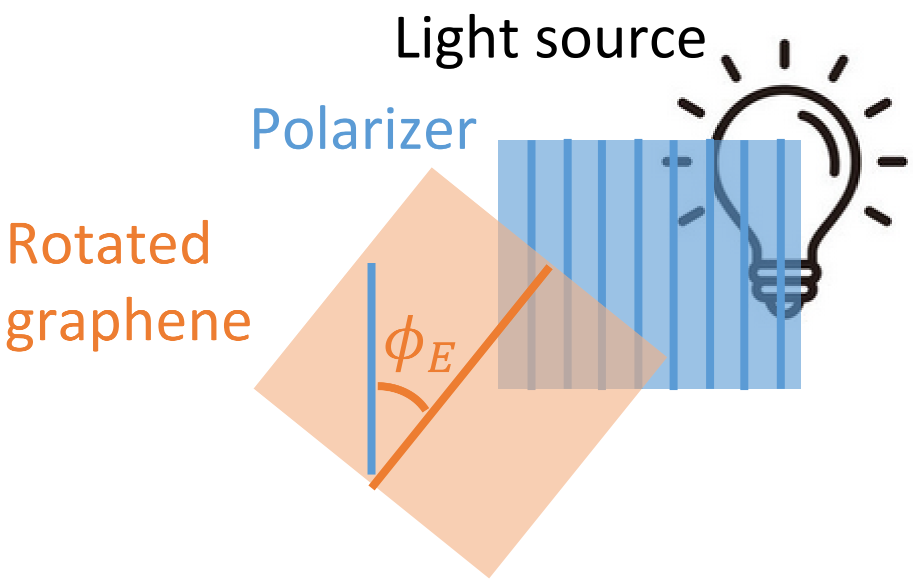
Fine-structure constant may be extracted accurately by fitting the quadratic frequency dependence in any 2D Dirac semimetals

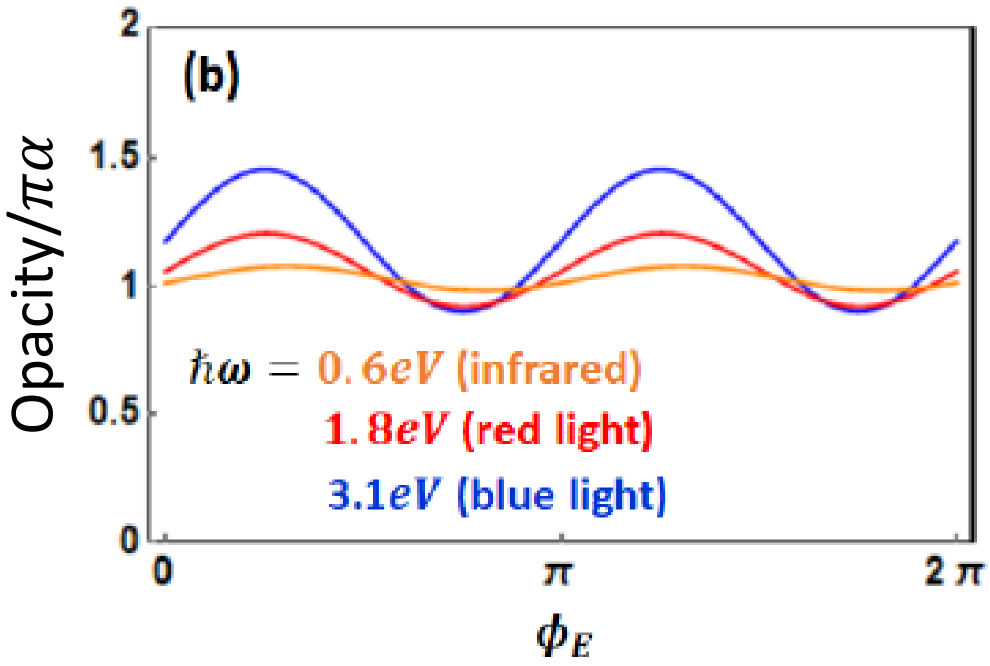
$$\lim_{\{T,\omega\}\to 0} \mathcal{O}(\omega) = \alpha + \beta\omega^2$$

Stauber, Peres, and Geim, PRB 78, 085432 (2008)



Seeing optical Hall conductance by rotating graphene





Graphene rotated by angle ϕ_E wrt polarization will have opacity

$$\mathcal{O}_E(\omega, \phi_E) = \cos^2\phi_E \mathcal{O}_{xx}(\omega) + 2\sin\phi_E\cos\phi_E \mathcal{O}_{xy}(\omega) + \sin^2\phi_E \mathcal{O}_{yy}(\omega)$$

The variation $\mathcal{O}_{xy}(\omega)/\pi\alpha \sim 20\%$ is roughly given by the optical Hall conductance, which is potentially visible by naked eyes

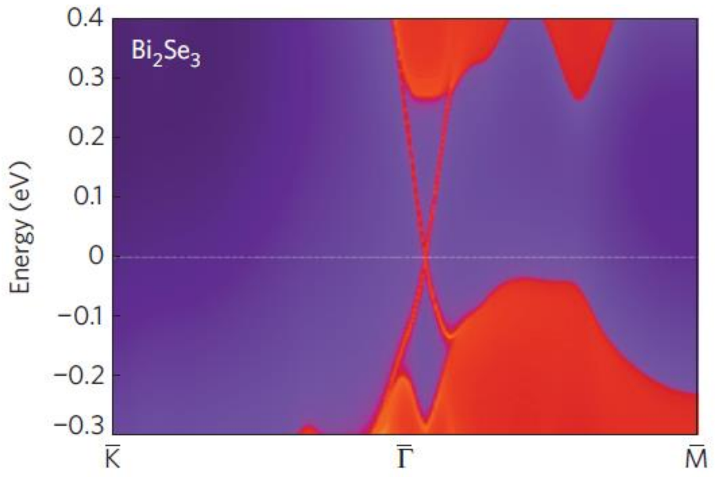
de Sousa, Cruz, and Chen, arXiv:2303.14549

All 3D topological insulator thin films have the same opacity $\pi\alpha$

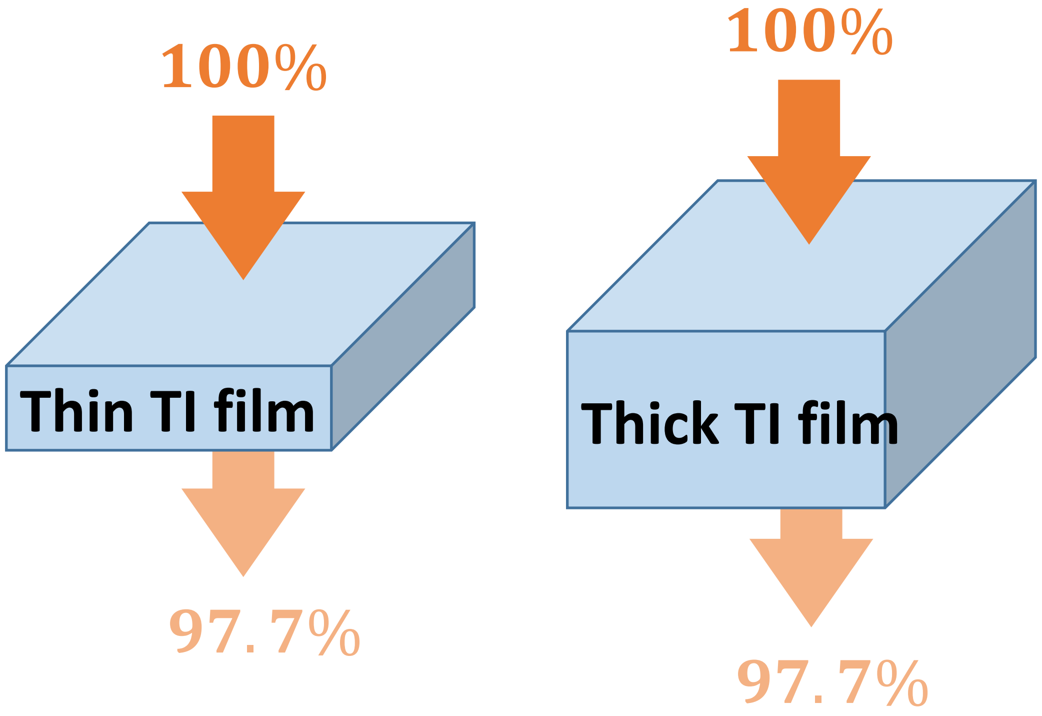
Surface states of 3D Tis are also described by the same Hamiltonian as graphene

$$H_0^{\mathbf{K},\mathbf{K}'}(\mathbf{k}) = v_F\left(\pm k_y\sigma_x - k_x\sigma_y\right)$$

But the linearity is only up to infrared region. Thus we predict That all 3D TI thin films of any thickness has the same opacity $\pi\alpha \approx 2.3\%$ in the infrared region



Zhang et al, Nat Phys 2009



This also implies that one can literally see the topological surface states by naked eyes though an infrared lens!

Optical absorption power of 3D Weyl semimetals as a topological charge

Hamiltonian of 3D Weyl semimetals

$$H^{\gamma}(\mathbf{k}) = \pm\left(vk_x\sigma_x + vk_y\sigma_y + vk_z\sigma_z\right) = \pm\mathbf{d}\cdot\boldsymbol{\sigma}$$

Topological charge of Weyl points

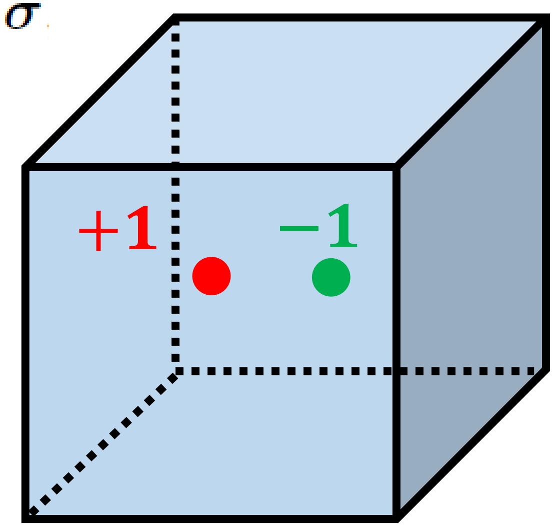
$$\mathcal{C} = \pm\frac{1}{4\pi}\int d\phi\int d\theta\frac{1}{d^3}\varepsilon^{ijk}d_i\partial_\theta d_j\partial_\phi d_k = \pm 1$$

Metric-curvature correspondence

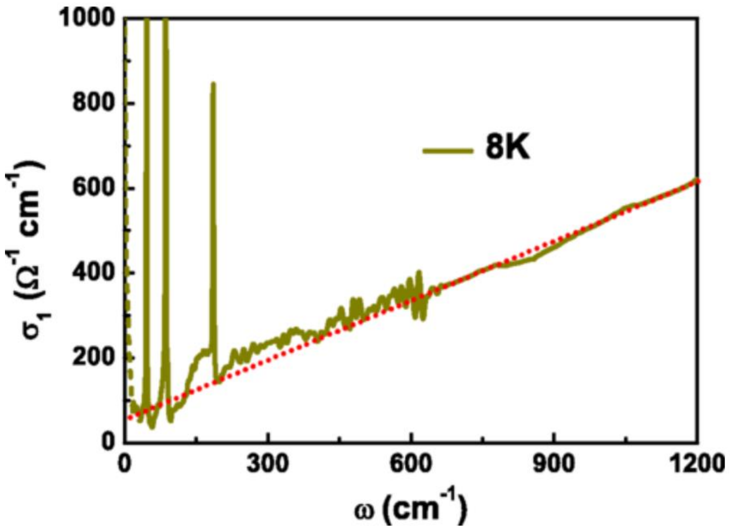
$$\sqrt{\det g^{\gamma}} = \frac{1}{4}\left|\,\varepsilon^{ijk}d_i\partial_\theta d_j\partial_\phi d_k/d^3\,\right|$$

Infrared optical conductivity is linear in frequency and topological charge

$$\sum_{\mu=x,y,z}\sigma_{\mu\mu}(\omega) = \frac{N_W e^2\omega|\mathcal{C}|}{8\pi\hbar v}$$



Chen et al, PRB 92, 075107 (2015)



The material looks darker under higher frequency light is a topological phenomenon, can be seen by naked eyes through infrared lens

Summary

- Opacity of graphene $\pi\alpha \approx 2.3\%$ as a topological charge
- Topologically protected fine-structure constant
- 3D topological insulator thin films of any thickness have the same opacity $\pi\alpha \approx 2.3\%$
- Seeing topological surface states by naked eyes
- Darkness of 3D Weyl semimetals as a topological charge