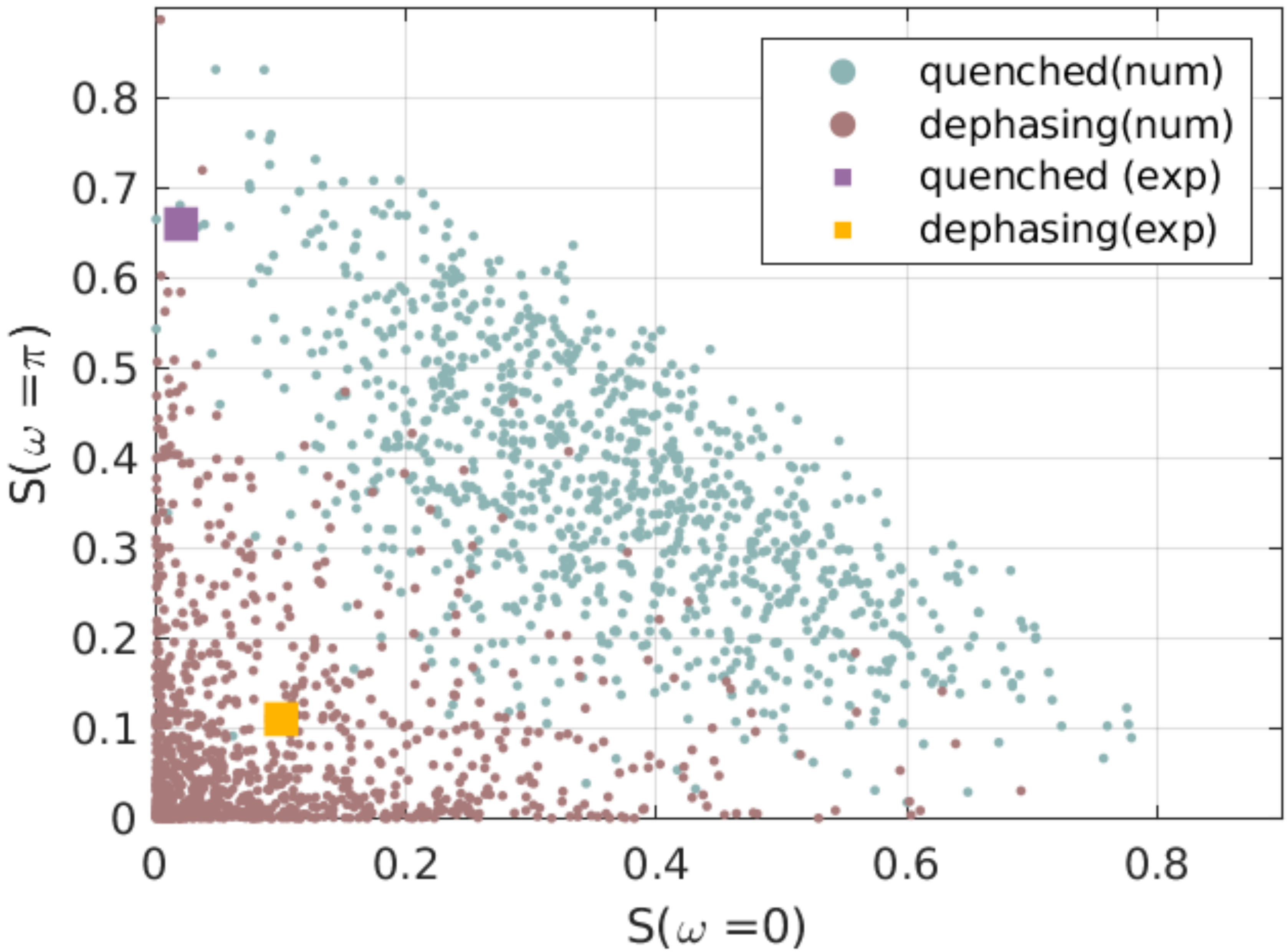
# Observation of Topological Criticality




Tobias Micklitz

CBPF
Centro Brasileiro de Pesquisas Físicas

# Motivation

disorder in *1d: strong Anderson localization*

single known exception: *topological critical point*

can we *observe* the *critical state*?

# Collaborators

theory

**Alexander Altland**
**Dmitry Bagrets**
**Kun Woo Kim**

experiment

**Sonja Barkhofen**
**Syamsundar De**
**Jan Sperling**
**Christine Silberhorn**

INSTITUT FÜR PHOTONISCHE QUANTENSYSTEME (PHOQS)

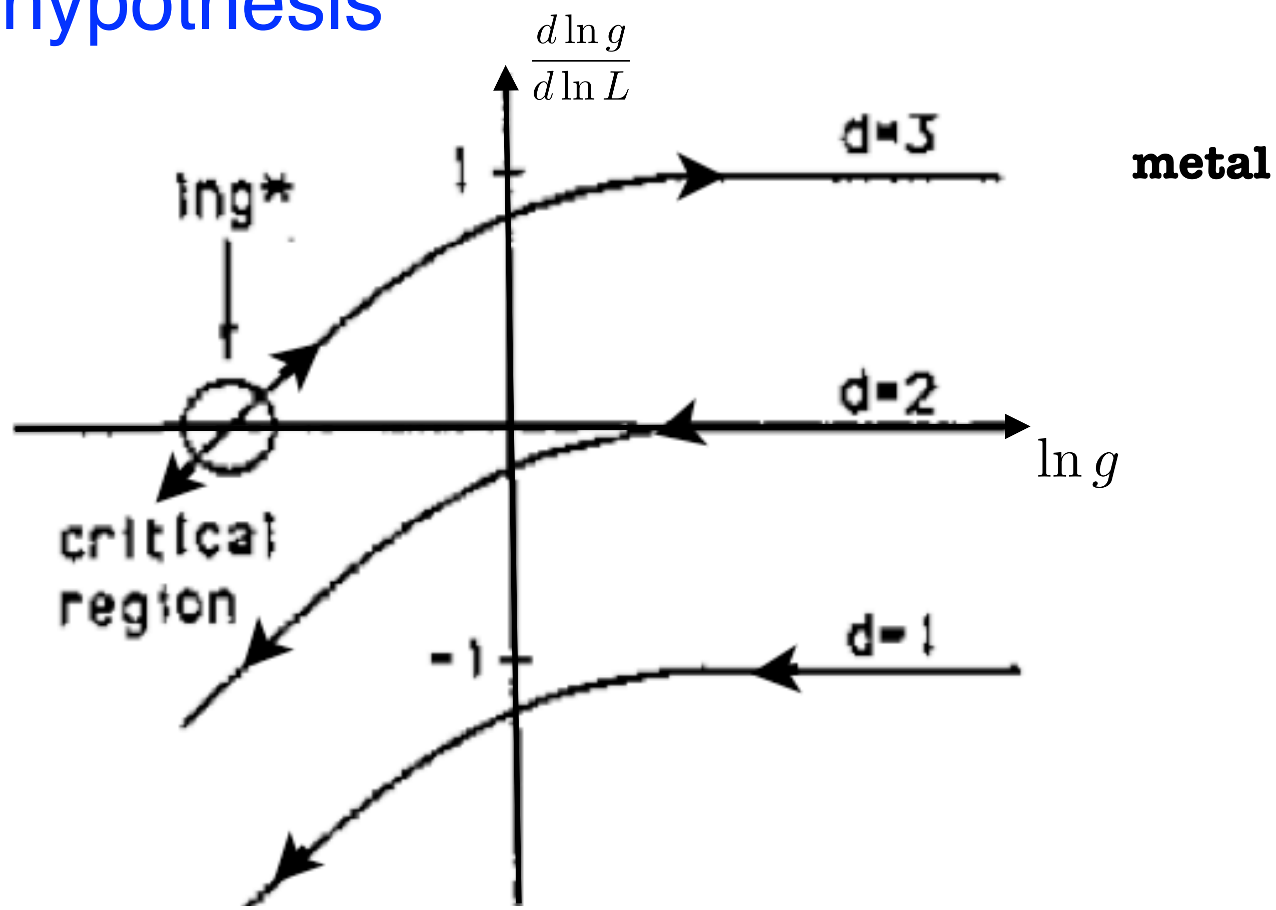
# Anderson localization in a nutshell

P.W. Anderson: “Absence of Diffusion in Certain Random Lattices“ (1958)

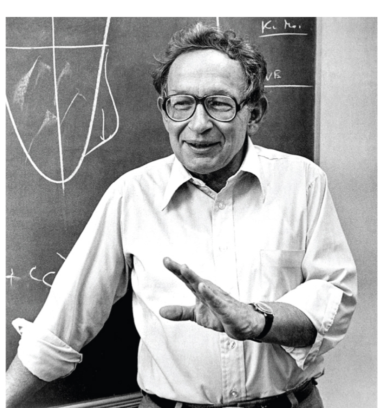
Abrahams, Anderson, Licciardello, Ramakrishnan: “Scaling theory of localization”(1979)

## Single-parameter scaling hypothesis

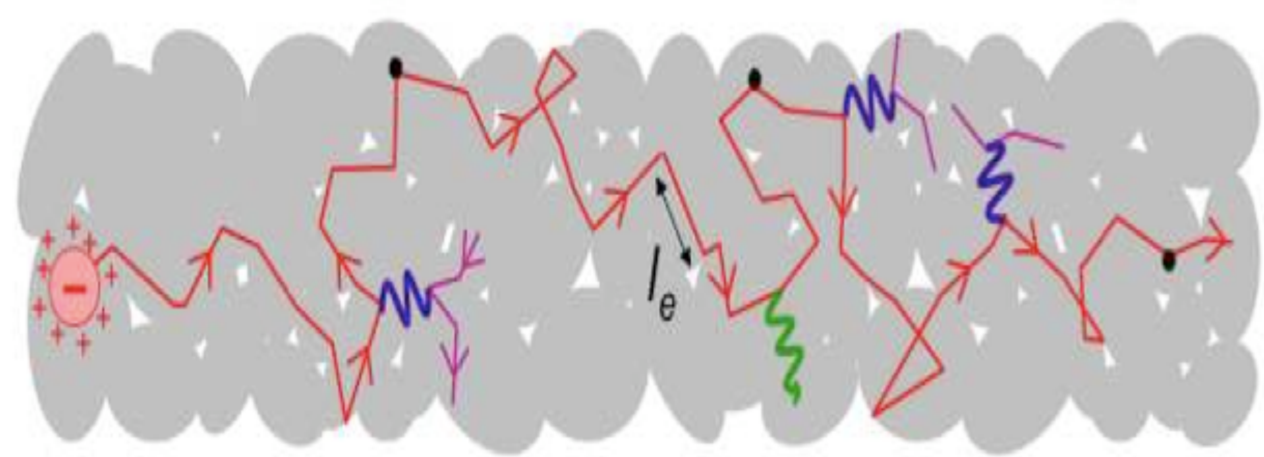
**assumption: conductance *g* is the single relevant parameter**




**“Anderson insulat**
absence of transpo



*“Very few believed [localization] at the time, and even fewer saw its importance; among those who failed to fully understand it at first was certainly its author…”* [Nobel lecture, 1977]




**real systems:** coupling to “bath”: decoherence, activation

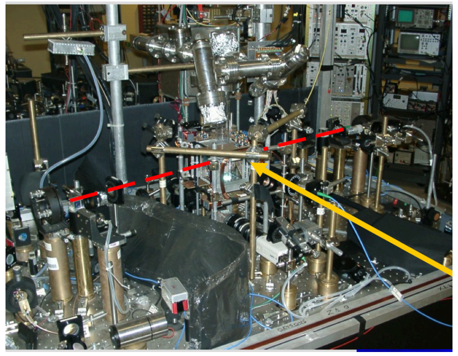
# Quantum Simulator of the *3d Disorder-Driven* Anderson Localization Transition

Featured in Physics | Editors' Suggestion

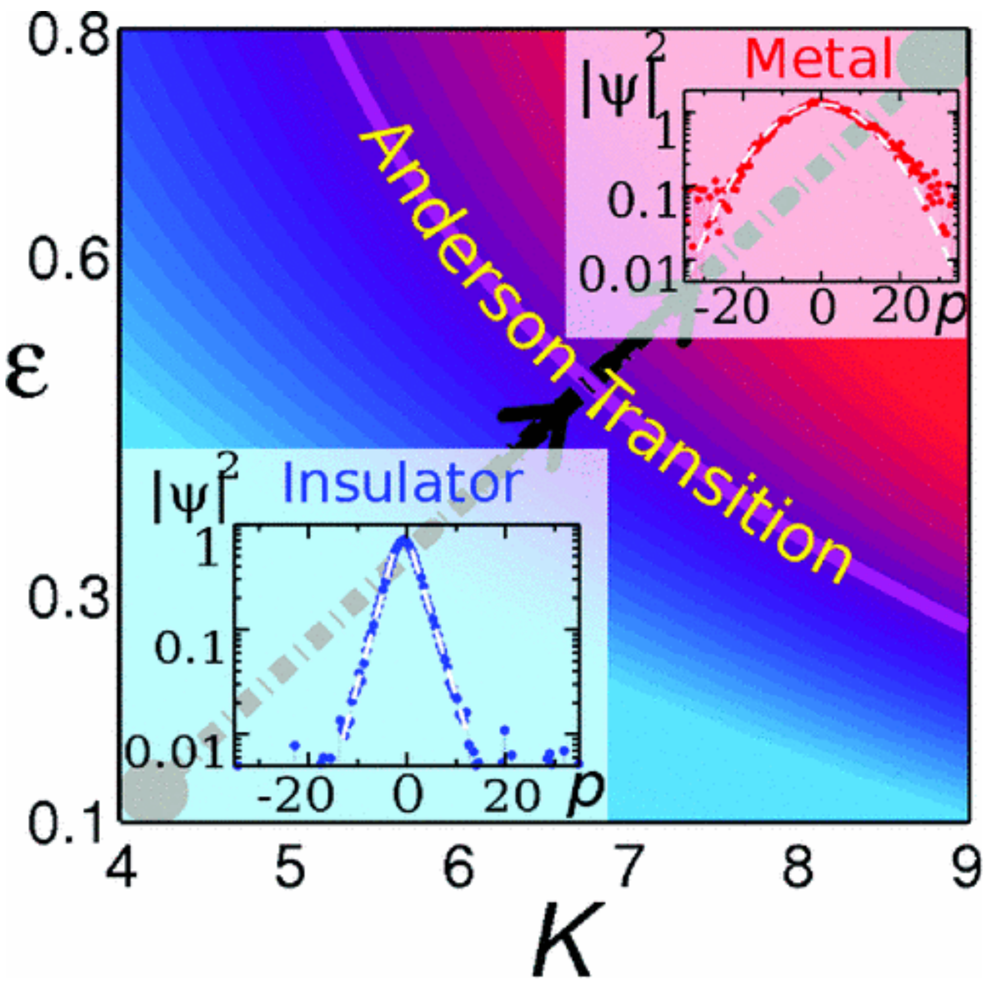
Experimental Observation of the Anderson Metal-Insulator Transition with Atomic Matter Waves

Julien Chabé, Gabriel Lemarié, Benoît Grémaud, Dominique Delande, Pascal Szriftgiser, and Jean Claude Garreau

Phys. Rev. Lett. **101**, 255702 – Published 15 December 2008

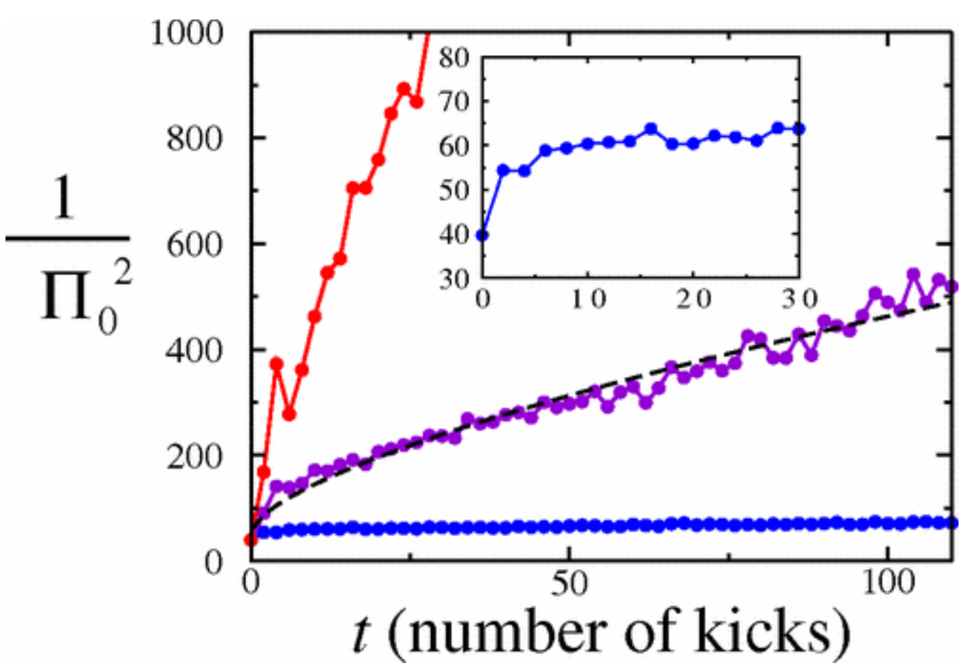


- controlled observation of a ***3d*** Anderson-insulator transition



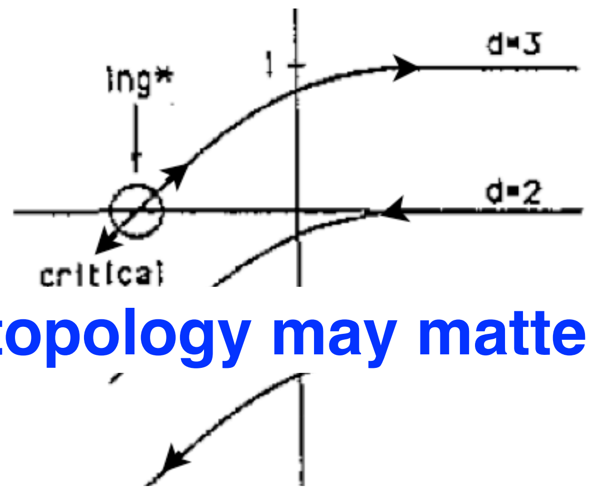

from ***1d*** **kicked rotor**

$$\hat{H} = \hbar^2 \partial_x^2 + K \cos(\hat{x}) \sum_n \delta(t-n)$$



# Topological Anderson Insulators

single-parameter scaling




**topology may matter!**

topological insulators

| Symmetry | | | | $d$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| AZ | $\Theta$ | $\Xi$ | $\Pi$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| A | 0 | 0 | 0 | 0 | $\mathbb{Z}$ | 0 | $\mathbb{Z}$ | 0 | $\mathbb{Z}$ | 0 | $\mathbb{Z}$ |
| AIII | 0 | 0 | 1 | $\mathbb{Z}$ | 0 | $\mathbb{Z}$ | 0 | $\mathbb{Z}$ | 0 | $\mathbb{Z}$ | 0 |
| AI | 1 | 0 | 0 | 0 | 0 | 0 | $\mathbb{Z}$ | 0 | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | $\mathbb{Z}$ |
| BDI | 1 | 1 | 1 | $\mathbb{Z}$ | 0 | 0 | 0 | $\mathbb{Z}$ | 0 | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ |
| D | 0 | 1 | 0 | $\mathbb{Z}_2$ | $\mathbb{Z}$ | 0 | 0 | 0 | $\mathbb{Z}$ | 0 | $\mathbb{Z}_2$ |
| DIII | $-1$ | 1 | 1 | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | $\mathbb{Z}$ | 0 | 0 | 0 | $\mathbb{Z}$ | 0 |
| AII | $-1$ | 0 | 0 | 0 | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | $\mathbb{Z}$ | 0 | 0 | 0 | $\mathbb{Z}$ |
| CII | $-1$ | $-1$ | 1 | $\mathbb{Z}$ | 0 | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | $\mathbb{Z}$ | 0 | 0 | 0 |
| C | 0 | $-1$ | 0 | 0 | $\mathbb{Z}$ | 0 | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | $\mathbb{Z}$ | 0 | 0 |
| CI | 1 | $-1$ | 1 | 0 | 0 | $\mathbb{Z}$ | 0 | $\mathbb{Z}_2$ | $\mathbb{Z}_2$ | $\mathbb{Z}$ | 0 |

## two-parameter scaling

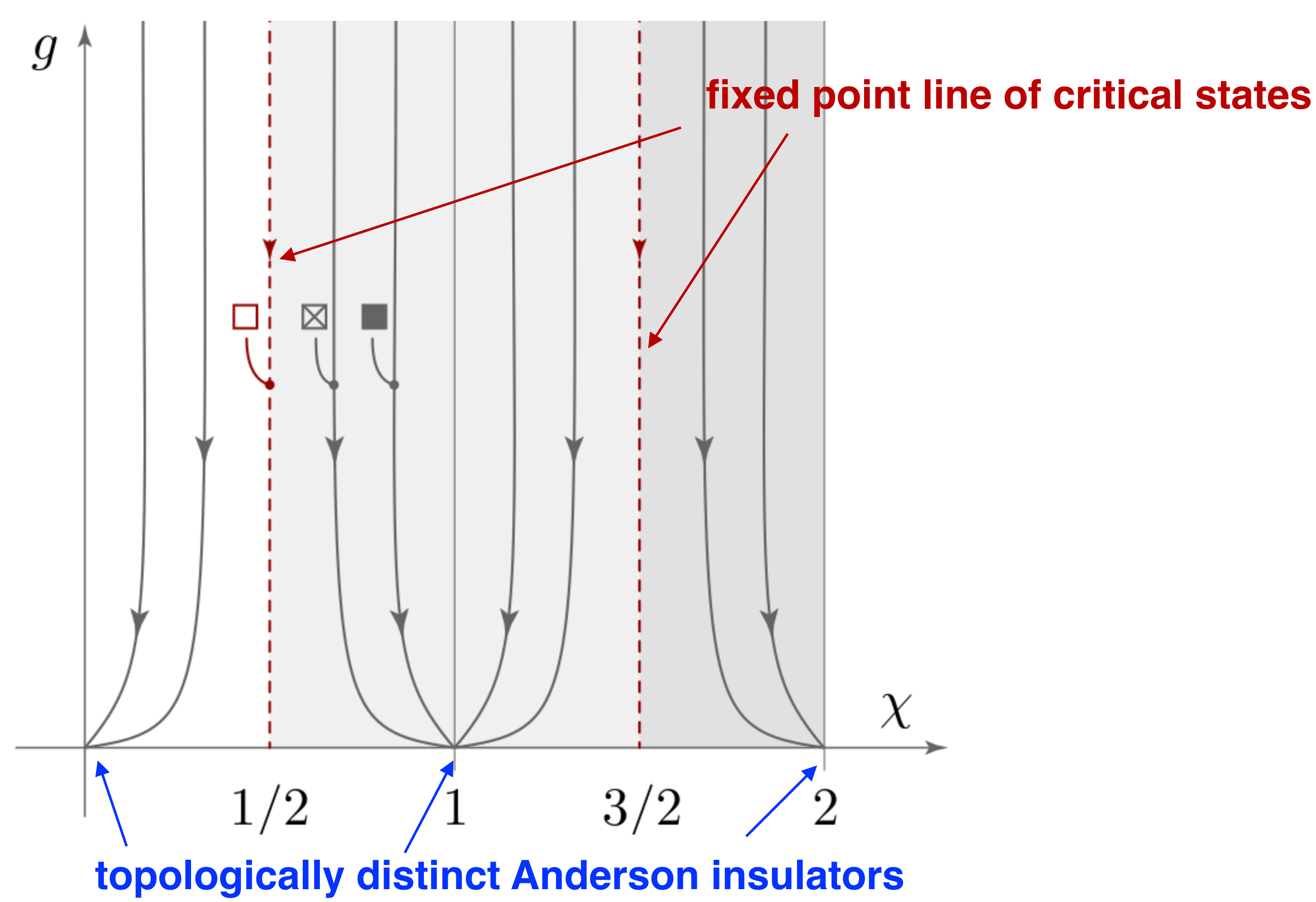
**two relevant parameters:**
**conductance *g*,**
**topological number $\chi$**

**at quantum criticality:**

2 competing principles

Anderson localization due to disorder

⇔ change of topological number



*"why should we care?"*

a **quantum simulator** for **topological disordered systems?**

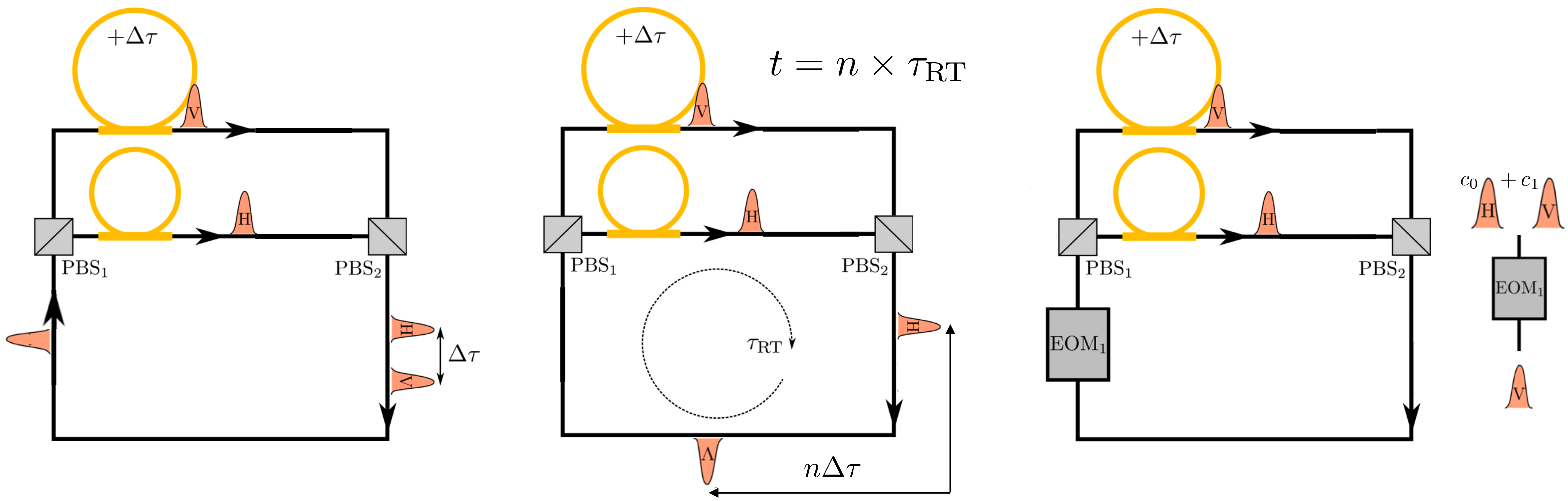
*"a simulator for topological quantum matter in presence of engineered randomness"*

**A Quantum Simulator of the *Topology-Driven* Anderson Localization Transition?**

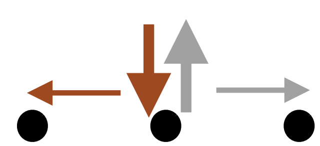
## challenges

- precision control over internal degree of freedom (`spin')
- observable?

# Optical Linear Networks




"chiral hopping on a *1d* lattice"



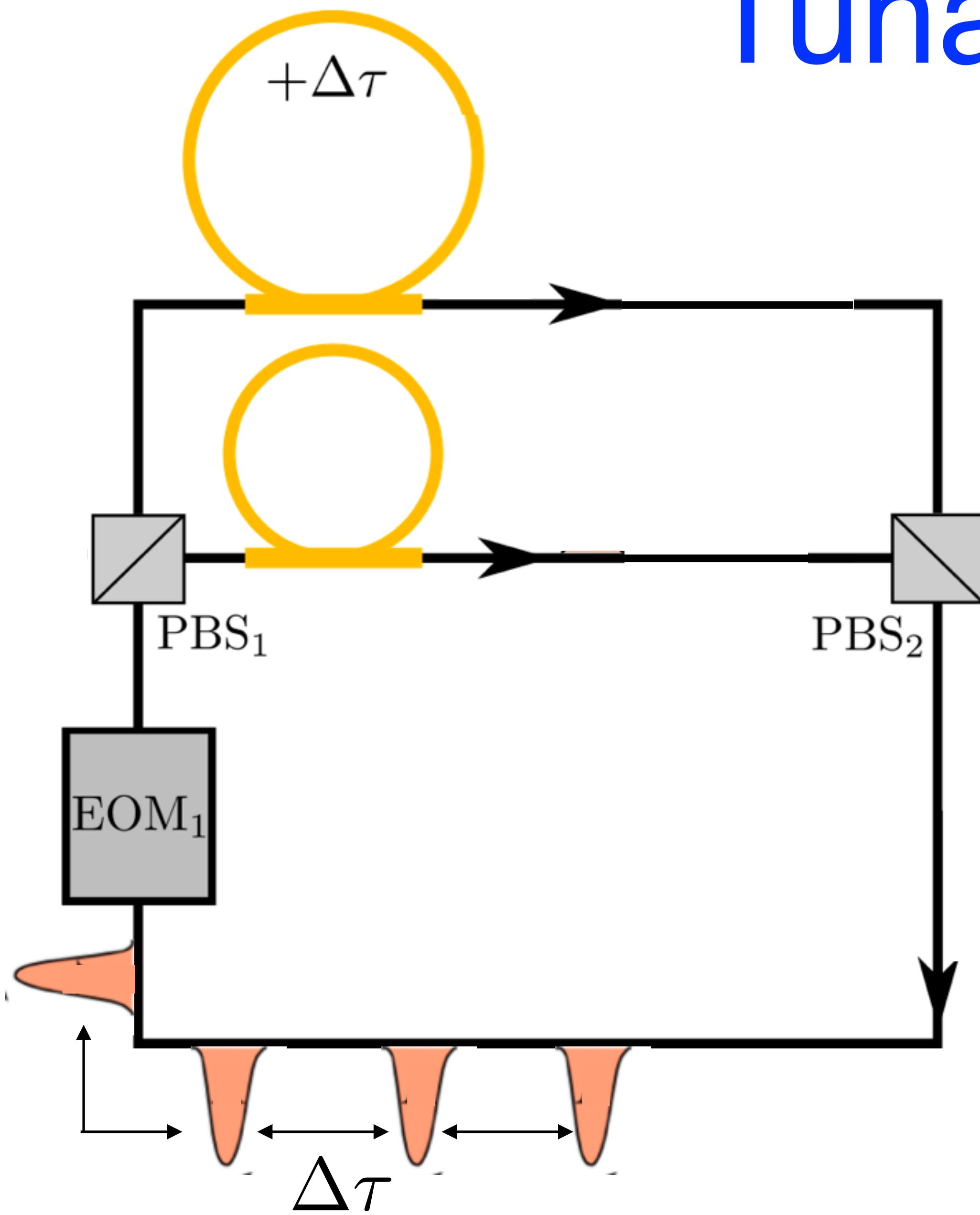
$\hat{T}$ ("step")

"spin-rotations"

↑ $\mapsto c_+$ ↑ + $c_-$ ↓ $\hat{R}$ ("coin")

"quantum walk of **spin-1/2 particle** with **tunable disorder**"

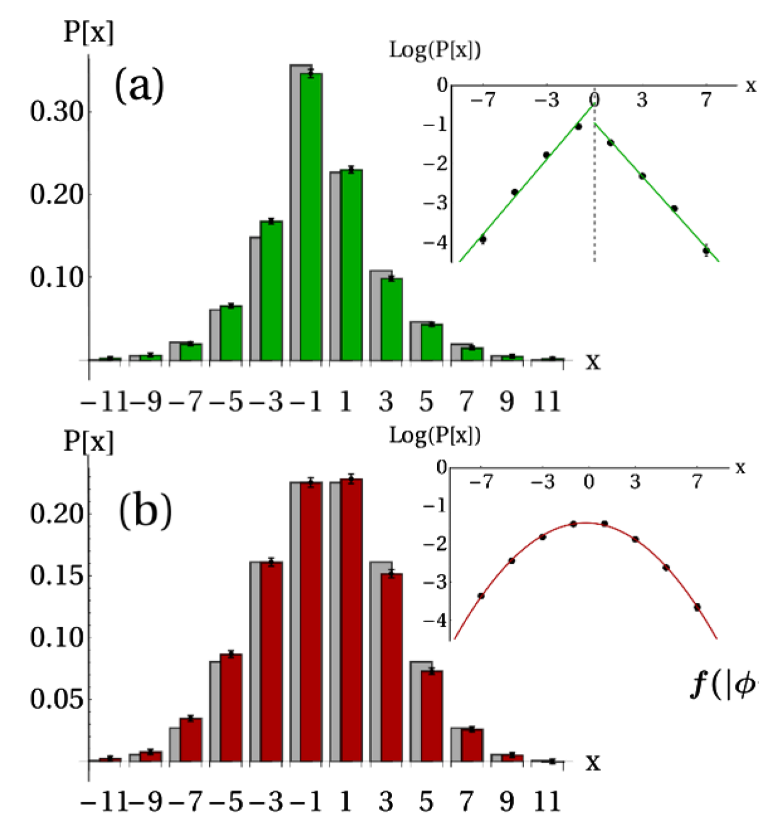
# Tunable Disorder

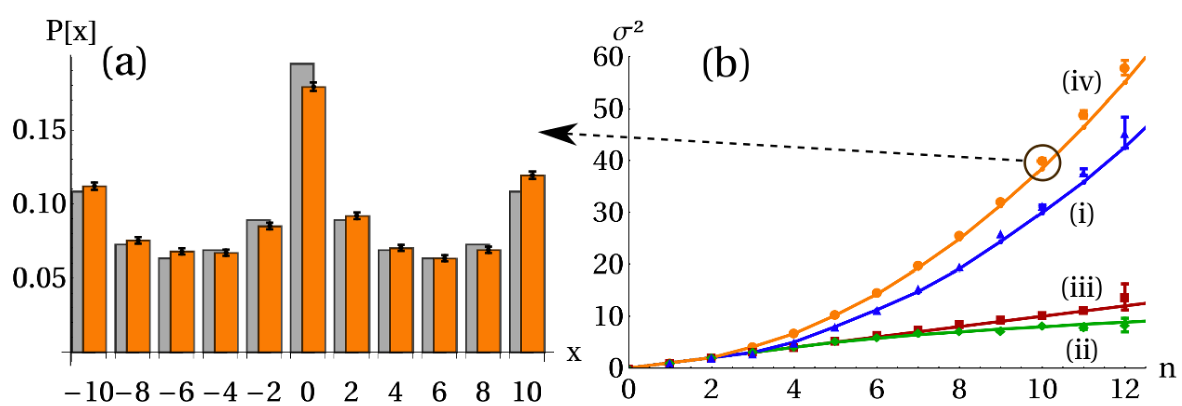



PRL 106, 180403 (2011) PHYSICAL REVIEW LETTERS week ending 6 MAY 2011

**Decoherence and Disorder in Quantum Walks: From Ballistic Spread to Localization**

A. Schreiber,[1,*] K. N. Cassemiro,[1] V. Potoček,[2] A. Gábris,[2,†] I. Jex,[2] and Ch. Silberhorn[1,3]
[1]*Max Planck Institute for the Science of Light, Günther-Scharowsky-Straße 1/Bau 24, 91058 Erlangen, Germany*
[2]*Department of Physics, FNSPE, Czech Technical University in Prague, Břehová 7, 115 19 Praha, Czech Republic*
[3]*University of Paderborn, Applied Physics, Warburger Straße 100, 33098 Paderborn, Germany*
(Received 13 January 2011; published 6 May 2011)



# Simulator of a Chiral Quantum Walk

single time-step evolution

$$U(\theta) = R(\theta/2)\, T\, R(\theta/2)$$

$$T = \sum_q \left( |q+1, \uparrow\rangle\langle\uparrow, q| + |q-1, \downarrow\rangle\langle\downarrow, q| \right)$$

$$R(\theta) = \sum_q |q\rangle \begin{pmatrix} \cos\theta_q & i\sin\theta_q \\ i\sin\theta_q & \cos\theta_q \end{pmatrix} \langle q|$$

random angles: **“static disorder”**

[D. Bagrets, K. W. Kim, S. Barkhofen, S. De, J. Sperling, C. Silberhorn, A. Altland, TM, PRR (2021)]

**(chiral symmetry)**

$$\sigma_2 U \sigma_2 = U^\dagger$$

$$\sigma_2 |\pm\rangle = \pm|\pm\rangle$$

***L/R*-circular polarized photons**

| Symmetry | | | | $d$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| AZ | $\Theta$ | $\Xi$ | $\Pi$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| A | 0 | 0 | 0 | 0 | $\mathbb{Z}$ | 0 | $\mathbb{Z}$ | 0 | $\mathbb{Z}$ | 0 | $\mathbb{Z}$ |
| AIII | 0 | 0 | 1 | $\mathbb{Z}$ | 0 | $\mathbb{Z}$ | 0 | $\mathbb{Z}$ | 0 | $\mathbb{Z}$ | 0 |

# Effective Field Theory

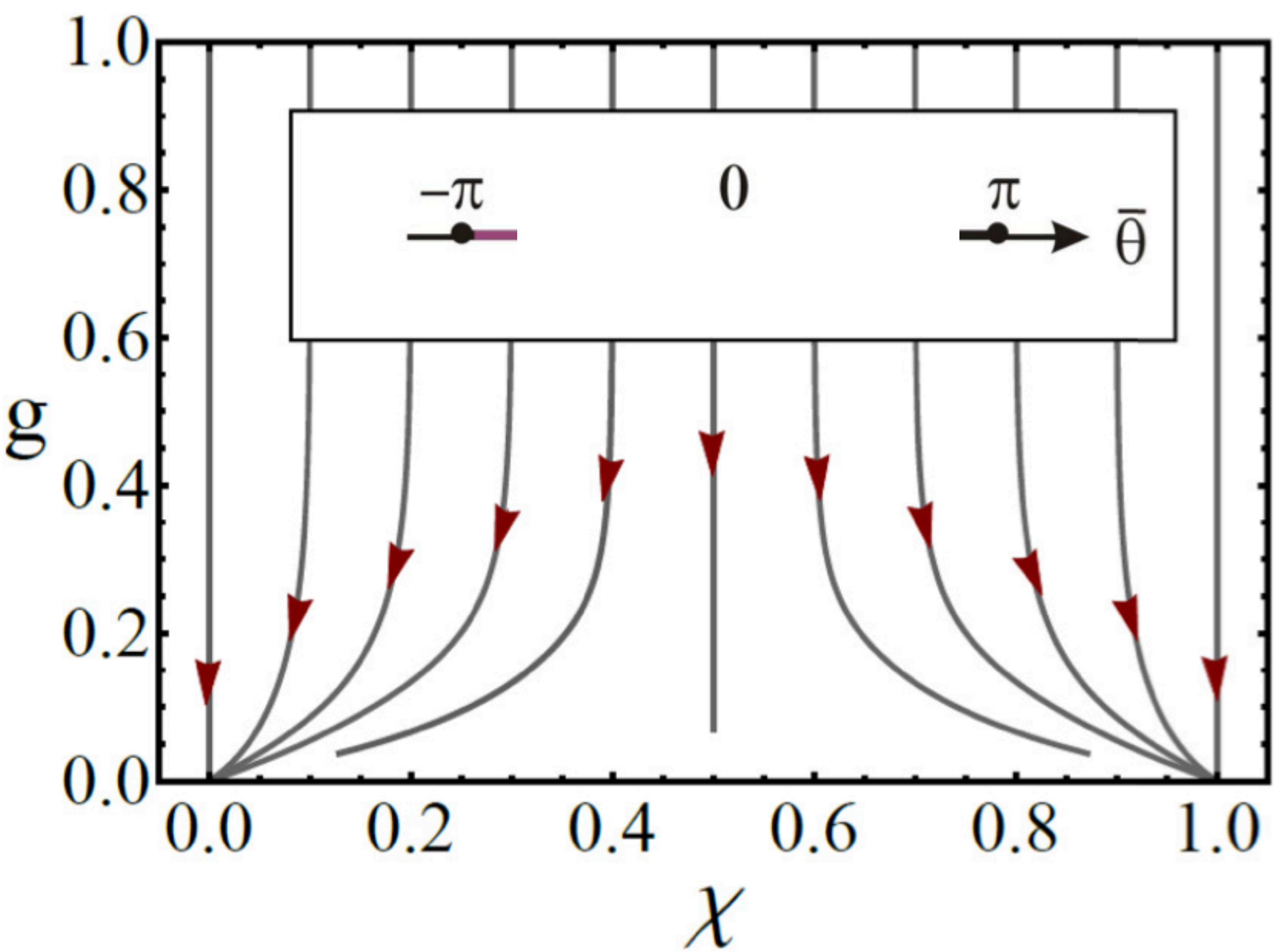
$$S_{\rm eff} = S_0 + S_{\rm top}$$

$$S_0 = \frac{1}{2}\int dx\, \mathrm{str}\left(\xi \partial_x Q^{-1}\partial_x Q + i\omega(Q+Q^{-1})\right)$$

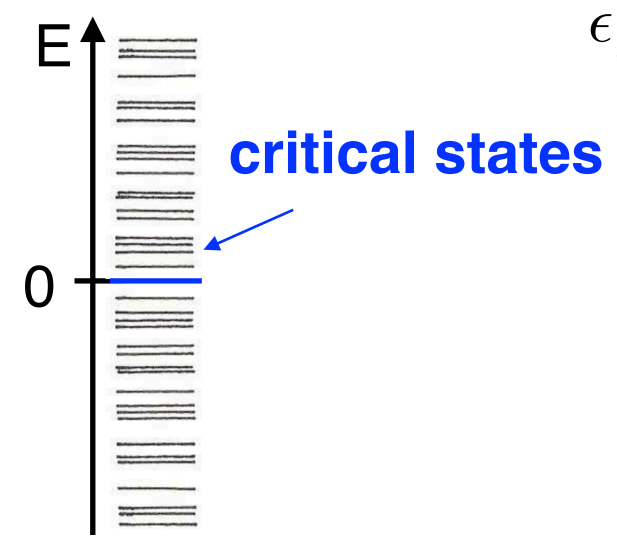
$$S_{\rm top} = \chi \int dx\, \mathrm{str}\left(Q^{-1}\partial_x Q\right)$$

**diffusive nonlinear sigma model with** topological term

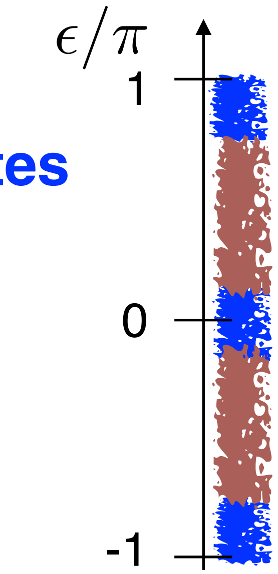
[D. Bagrets, K. W. Kim, S. Barkhofen, S. De, J. Sperling, C. Silberhorn, A. Altland, TM, PRR (2021)]




experiment typically probes states from **entire (quasi)energy spectrum**!



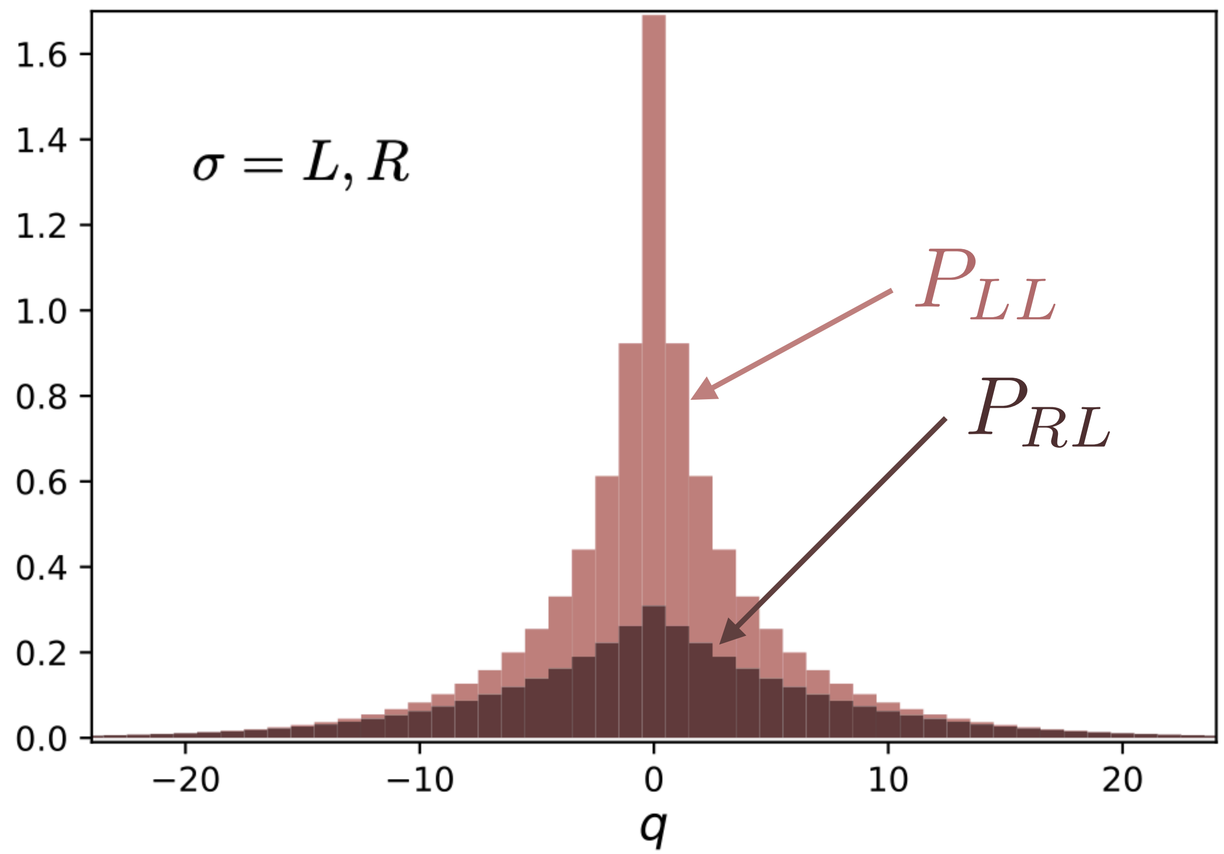

“mirror symmetric spectrum” (chiral symmetry!)




Floquet system

# Sinai Diffusion

$$P_{\sigma L}(t,q) = \langle |\langle q,\sigma|U^t|0,L\rangle|^2\rangle_\theta$$




"probability distribution of critical walker"

$$P^{\text{chiral}}_{\sigma'\sigma}(t,q) \propto \frac{1}{\ln^5 t}\sum_{n=1}^{\infty}(\sigma\sigma')^{(n+1)} n^2 e^{-n^2|q|/\xi_t}$$

$$P_{t,\sigma}(q) = F(q\xi_t^{-1}) \quad \xi_t \sim \ln^2 t$$

$$\langle |q| \rangle \sim \ln^2 t$$ (Sinai diffusion)


[L. Balents, MPA Fisher, PRB (1997)]
[A.Altland,D. Bagrets, A. Kamenev PRB (2015)]
[D. Bagrets, K. W. Kim, S. Barkhofen, S. De, J. Sperling, C. Silberhorn, A. Altland, TM, PRR (2021)]


## challenges

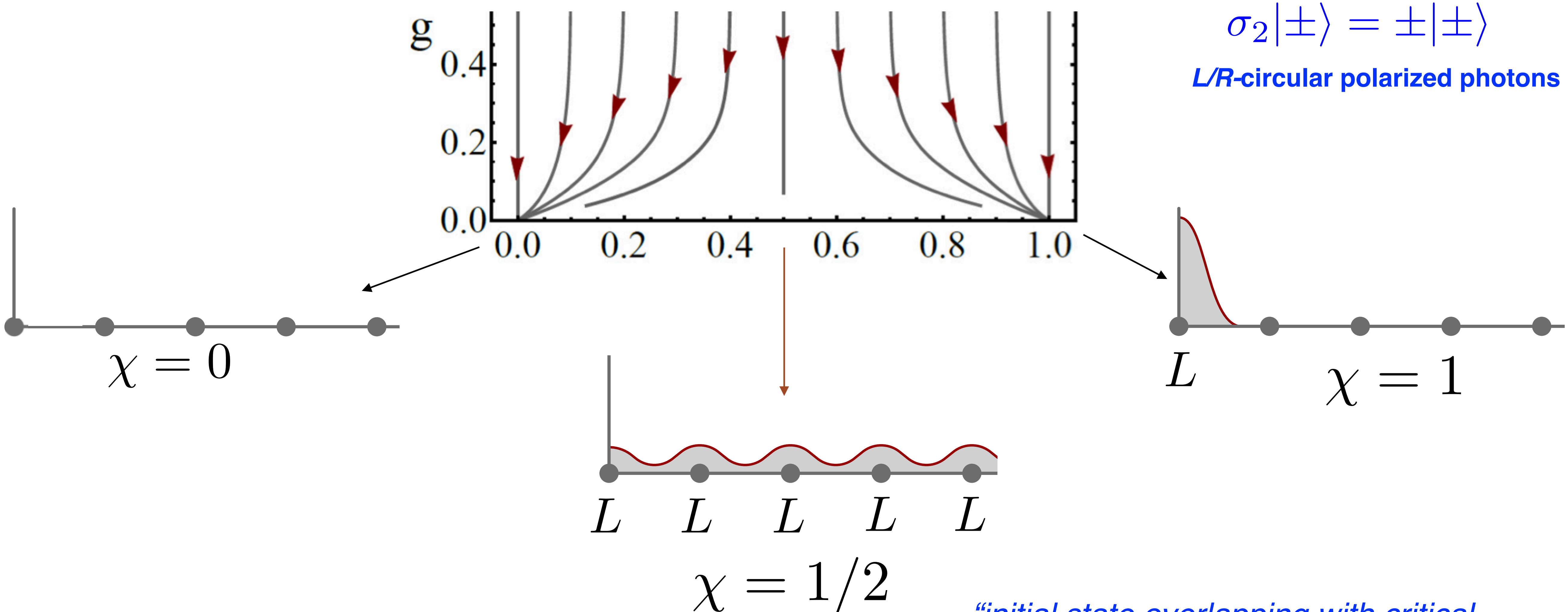
- precision control over an internal degree of freedom (`spin') ✓
- observable **?**

**Memory of initial polarization** notwithstanding random coin operators

# Circular Photon-Polarization

$\sigma_2|\pm\rangle = \pm|\pm\rangle$

***L/R*-circular polarized photons**




*"initial state overlapping with critical state preserves circular polarization"*

$$\Delta P(t) \equiv \sum_q \left(P_{LL}(t,q) - P_{RL}(t,q)\right)$$

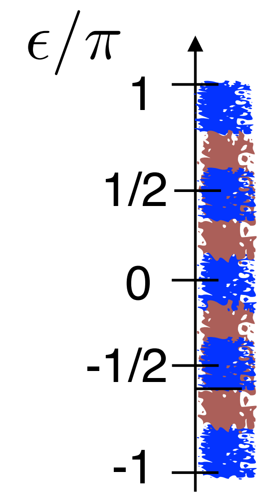
# Sub-Lattice Symmetry

$$U = \begin{pmatrix} & U_{eo} \\ U_{oe} & \end{pmatrix} \Longleftrightarrow [U,S]_+ = -U \quad S \equiv \sum_q |q\rangle(-1)^q\langle q|$$

"hopping between n.n. sites"

## 2nd chiral symmetry

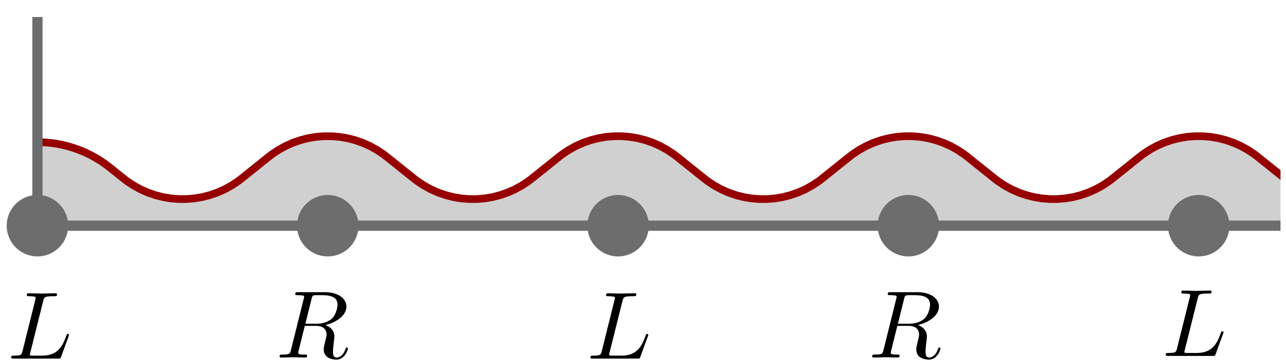
$$\sigma_2 S(iU)\sigma_2 S = (iU)^\dagger$$




eigenstates

$\{|2q,R\rangle, |2q-1,L\rangle\}$

$\{|2q,L\rangle, |2q-1,R\rangle\}$




$\chi = 1/2$

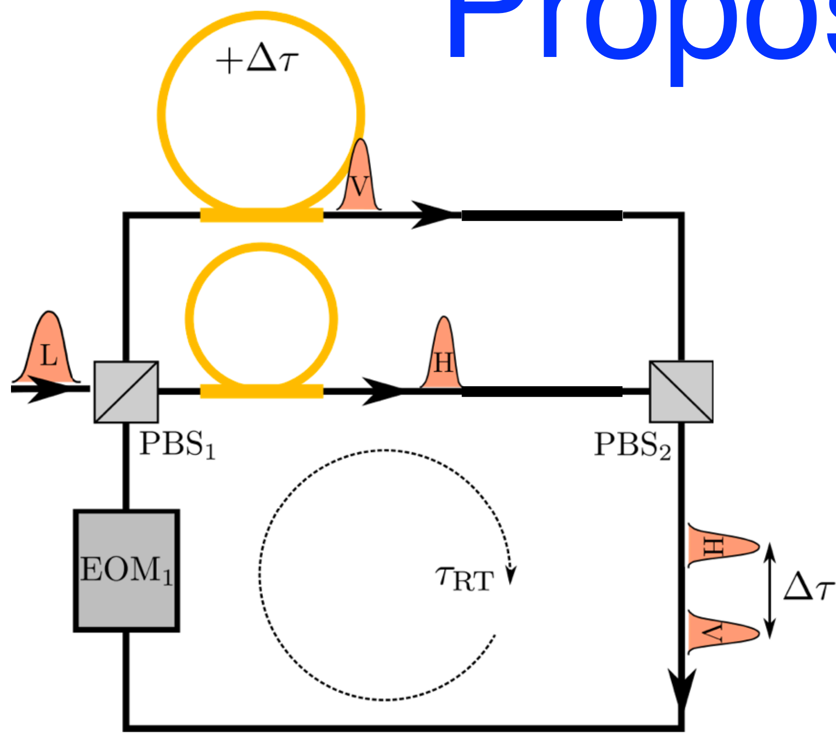
*"photons alternating circular polarization every site"*

"initial states overlapping with critical state preserve *"staggered"* circular polarization"

# Proposal

**initial state**:
*L*-circular polarized photon localized on single site

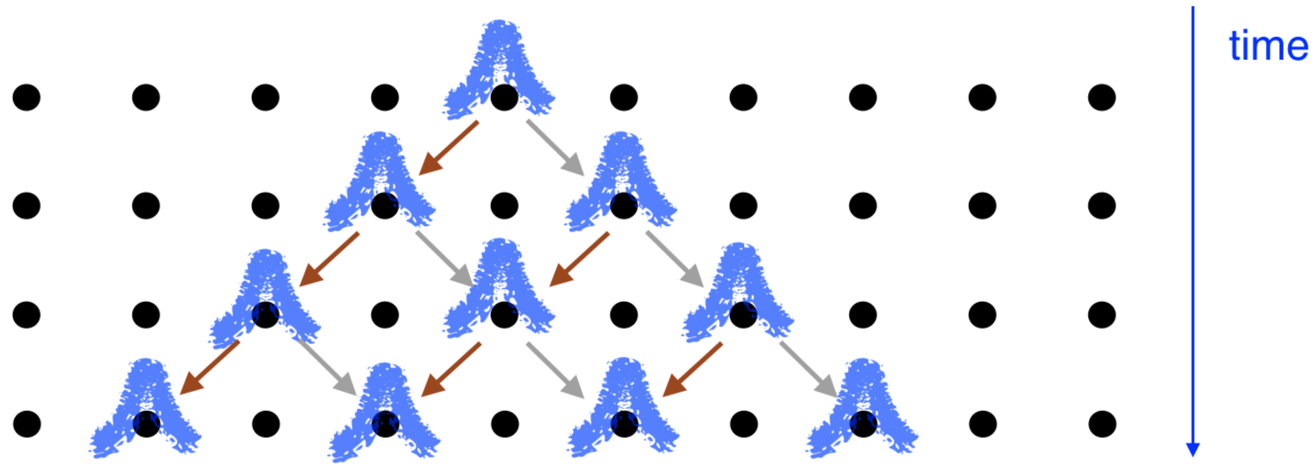
$$|\mathrm{in}\rangle = |0\rangle \otimes |L\rangle$$

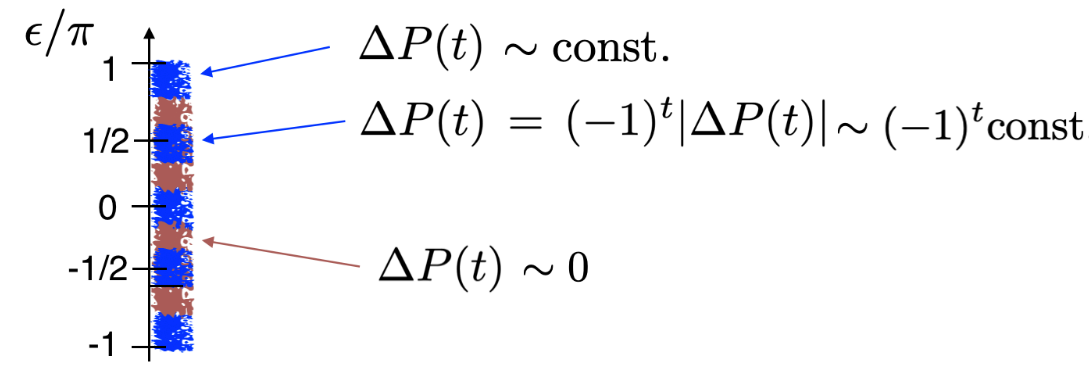



measure **circular photon polarization** after *t* time steps

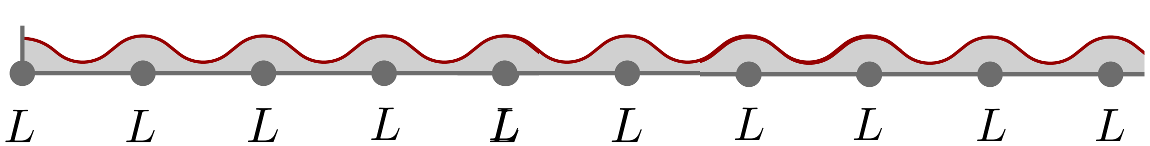
$$\Delta P(t) \equiv \sum_q \left(P_{LL}(t,q) - P_{RL}(t,q)\right)$$

**@topological critical point**

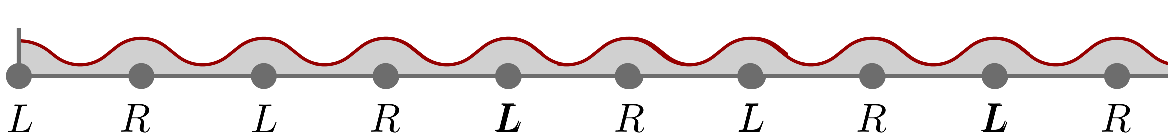



$\epsilon \simeq 0, \pi$




$\epsilon \simeq -\pi/2, \pi/2$




$$\Delta P(\omega)$$

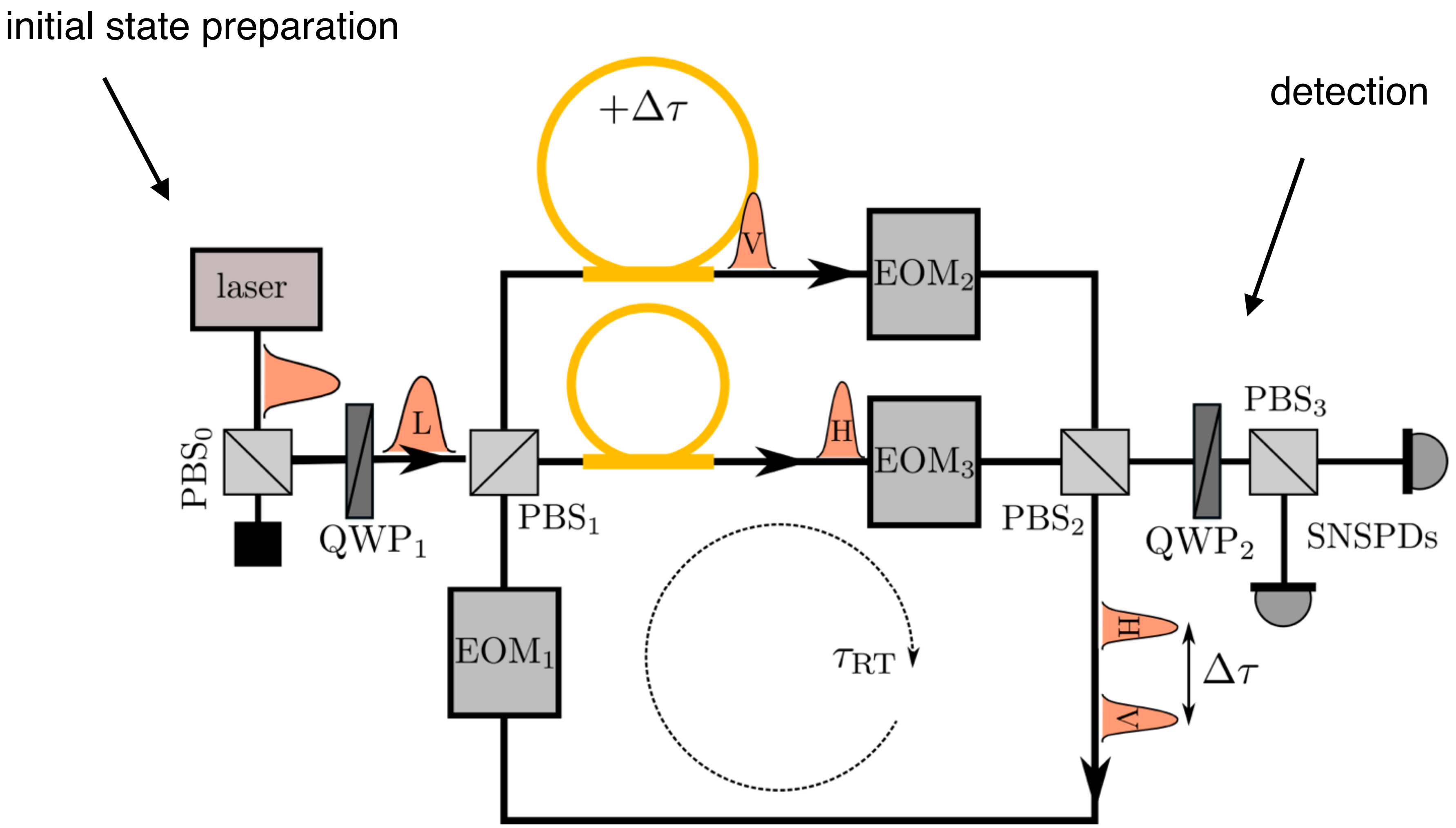
peaks at $\omega = 0, \pi$

# Experiment

UNIVERSITÄT PADERBORN
INSTITUT FÜR PHOTONISCHE QUANTENSYSTEME (PHOQS)

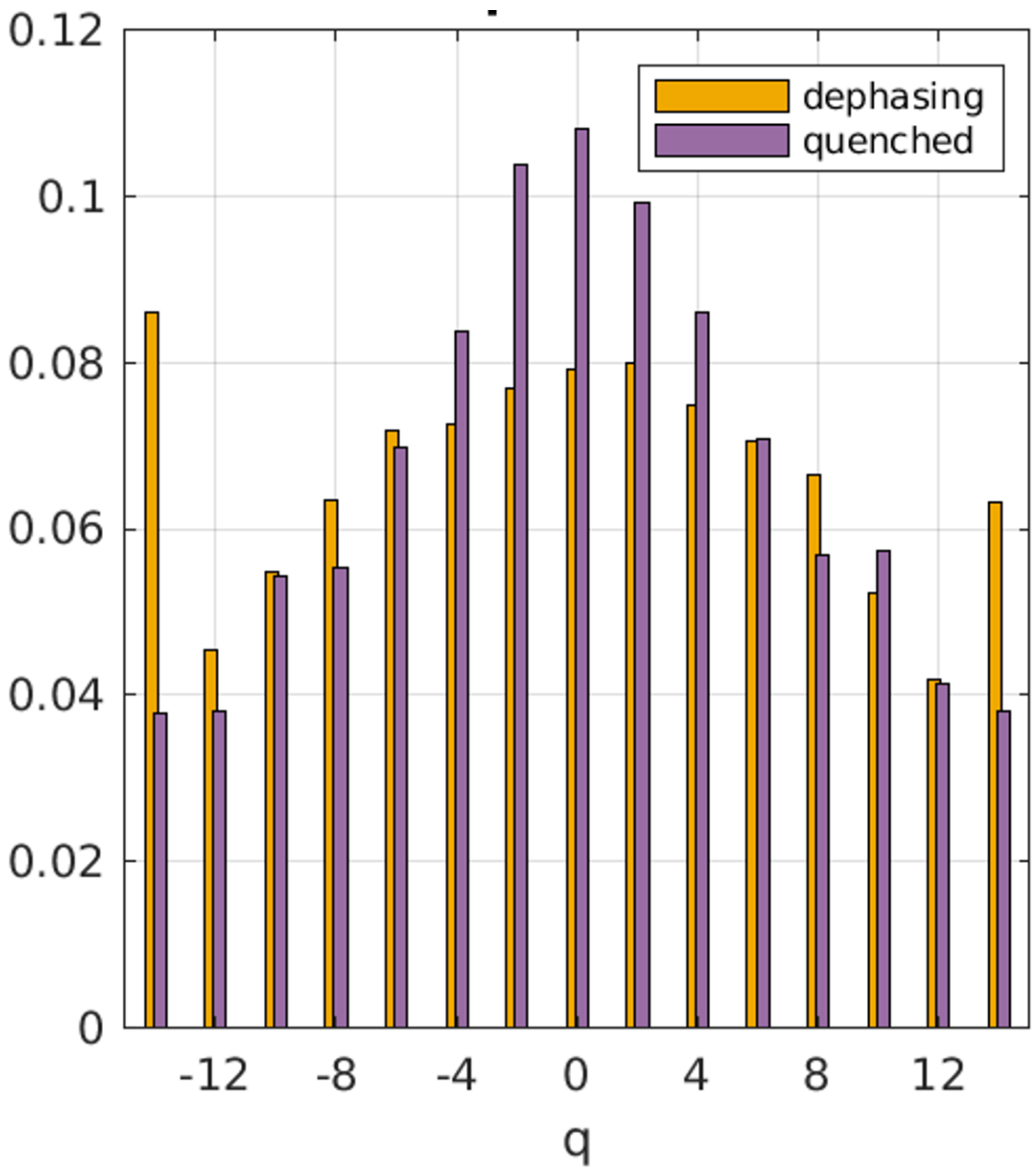
Sonja Barkhofen
Syamsundar De
Jan Sperling
Christine Silberhorn

[S. Barkhofen, *et al.*, arXiv:2301.05428]

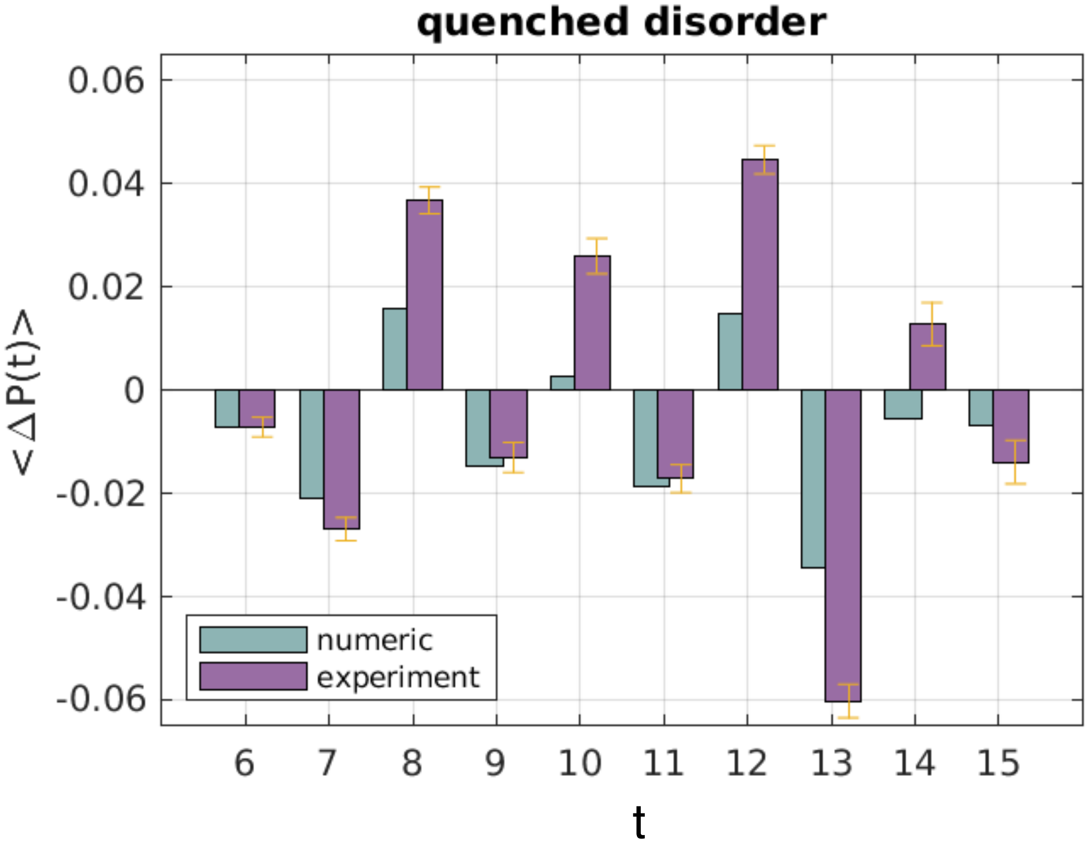



- binary disorder $\theta_q \in \{\pm\pi/8\}$ (500 realizations)
- simulating static and dynamic (dephasing) disorder
- *t=15* time steps

# $t$=15 steps



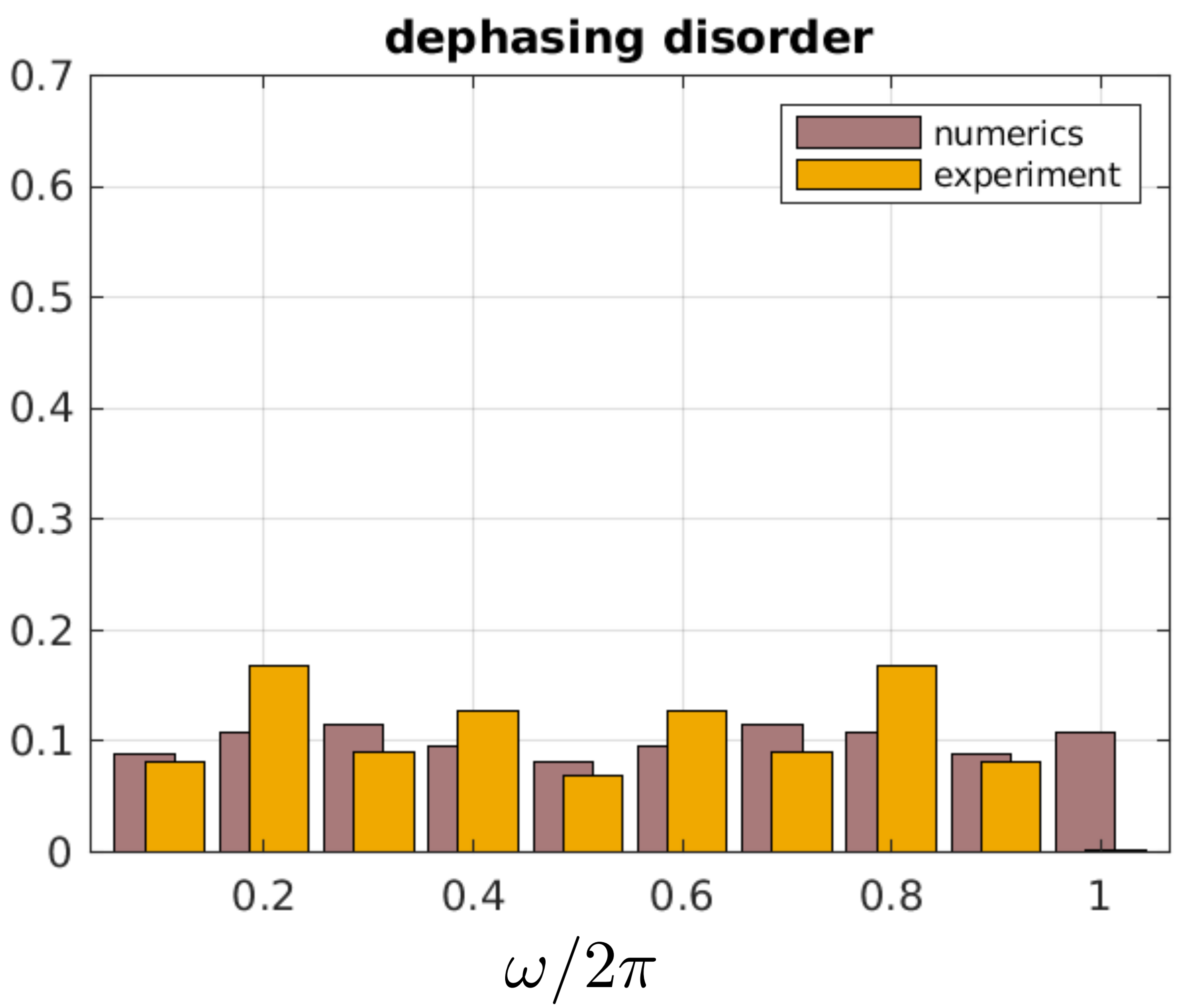

$\sum_q \left(P_{LL}(t,q) + P_{RL}(t,q)\right)$



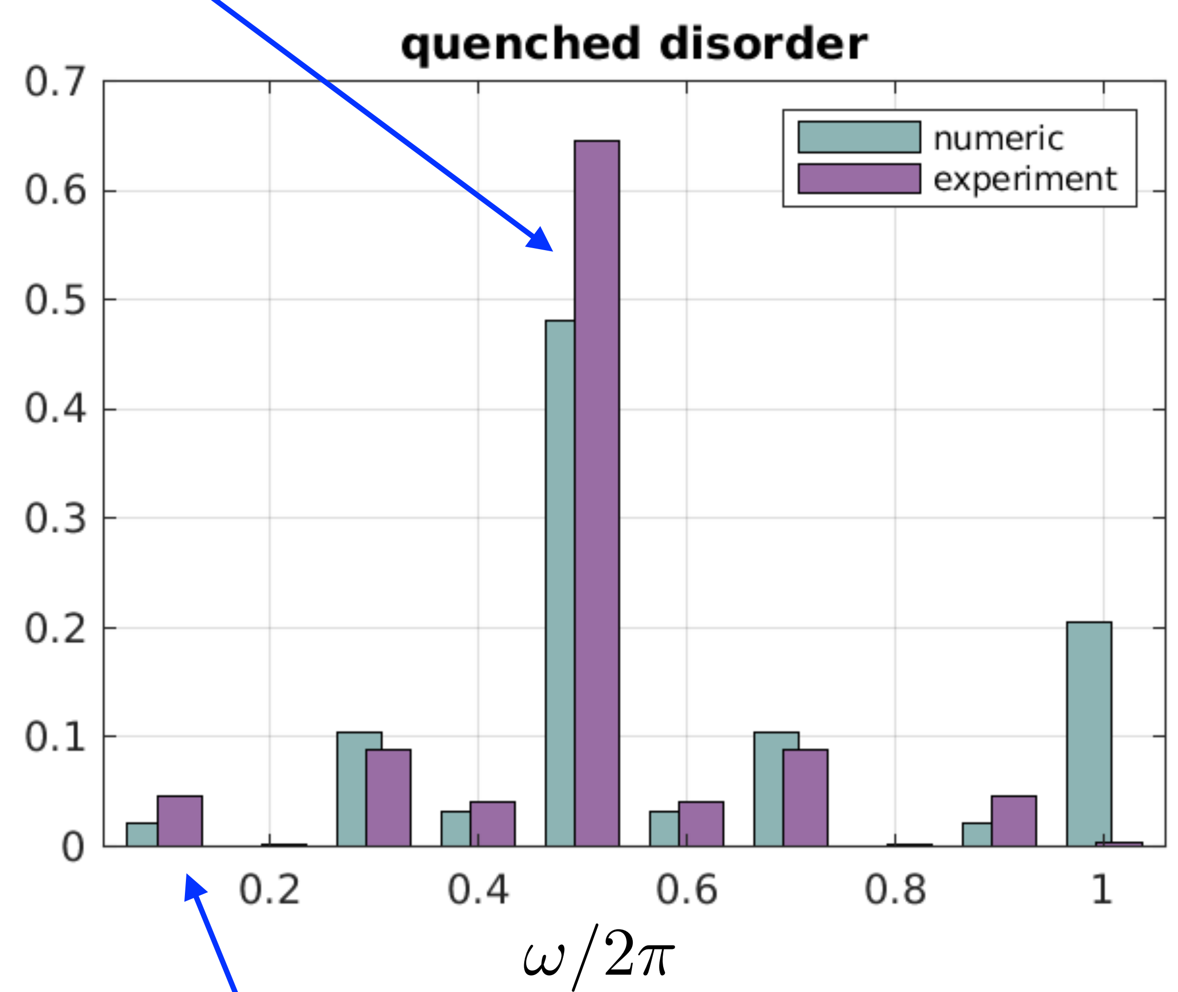

$$\Delta P(t) \equiv \sum_q \left(P_{LL}(t,q) - P_{RL}(t,q)\right)$$

# Power spectrum

$$\Delta P(\omega)$$




“critical state”




**peak at w=0?**

# Statistical Detuning

$t \sim \mathcal{O}(10)$ *sites visited* $\theta_q \in \{\pm\pi/8\}$ *500* realizations

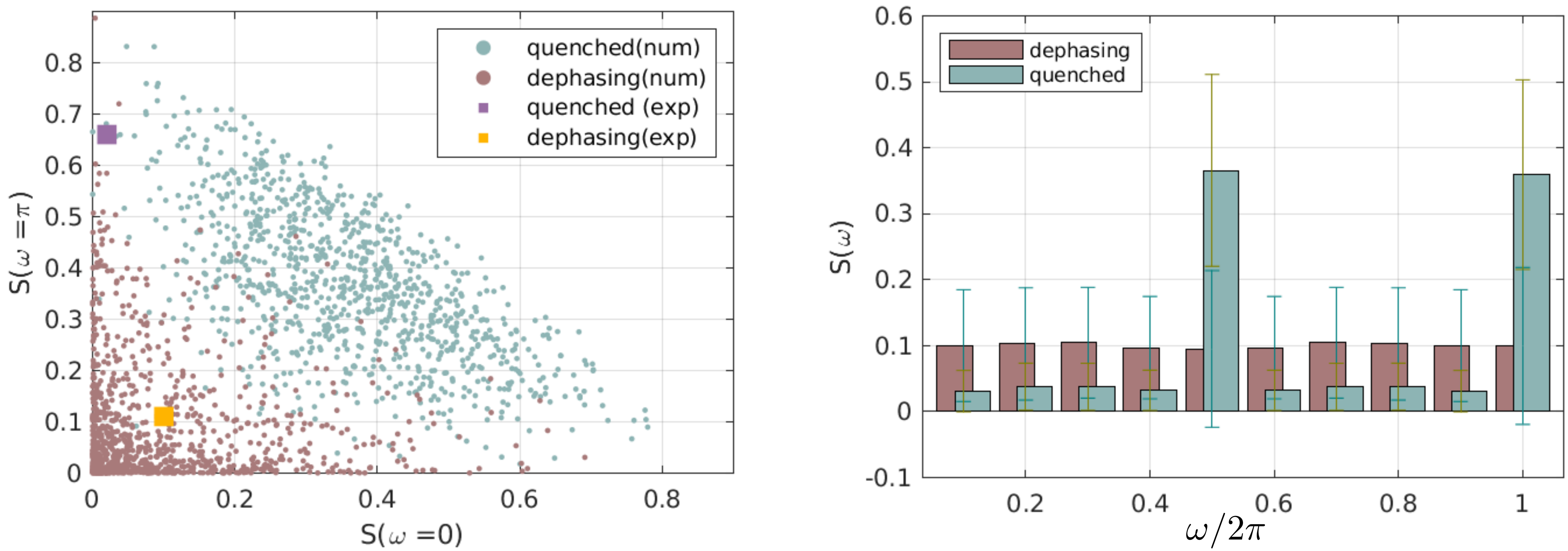
$$\frac{1}{t}\sum_q \theta_q \sim 1/\sqrt{t}$$

detuning of critical states $\sigma_2$

$$\frac{1}{t}\sum_q (-1)^q \theta_q \sim 1/\sqrt{t}$$

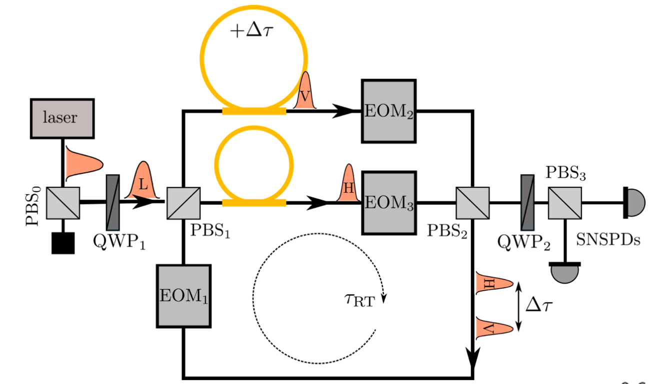
detuning of critical states $\sigma_2 S$

**1000 simulations** (ensembles of 500 realizations)



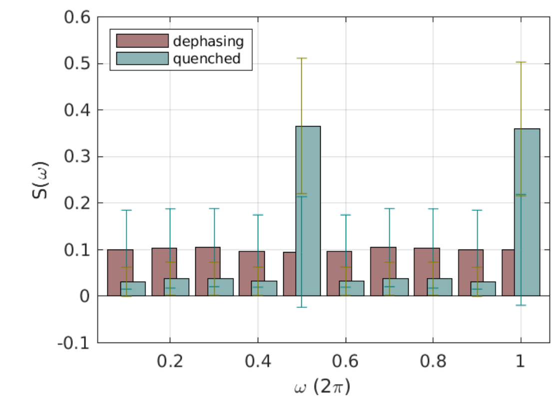
# Conclusions

- quantum simulator for topological criticality
- realizes chiral quantum walk with tunable disorder

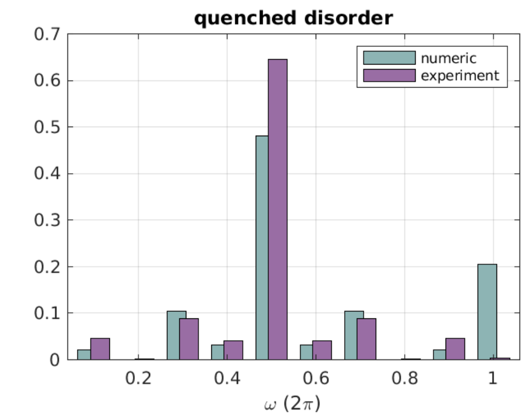



- prediction: peaks in power spectrum of dynamic spin polarization

  (interplay of chiral and sublattice symmetry)




- one of the peaks observed in experiment
- restricted number of time steps, ensemble: fluctuations matter!




- future: increase number of time-steps!   Study controlled detuning…


*"a simulator for topological quantum matter in presence of engineered randomness"*

[K. W. Kim, D. Bagrets, TM, A. Altland, PRX (2023)]

# Quantum Simulator of Topological Metals

simulation of higher dimensional dynamics via time-dependent protocols using incommensurate frequencies

PHYSICAL REVIEW X **13,** 011003 (2023)

**Floquet Simulators for Topological Surface States in Isolation**

Kun Woo Kim,[1,2,*] Dmitry Bagrets,[1] Tobias Micklitz,[3] and Alexander Altland[1]

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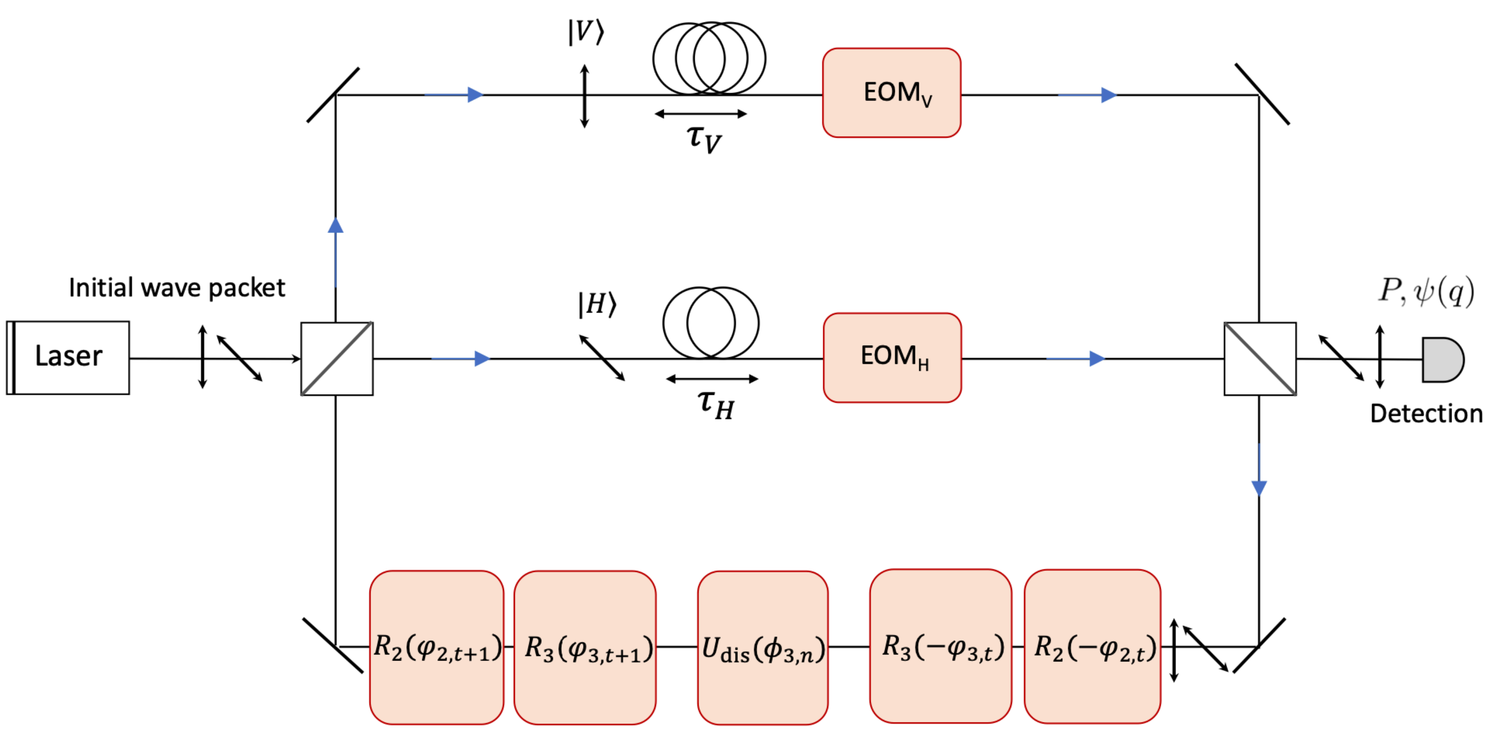
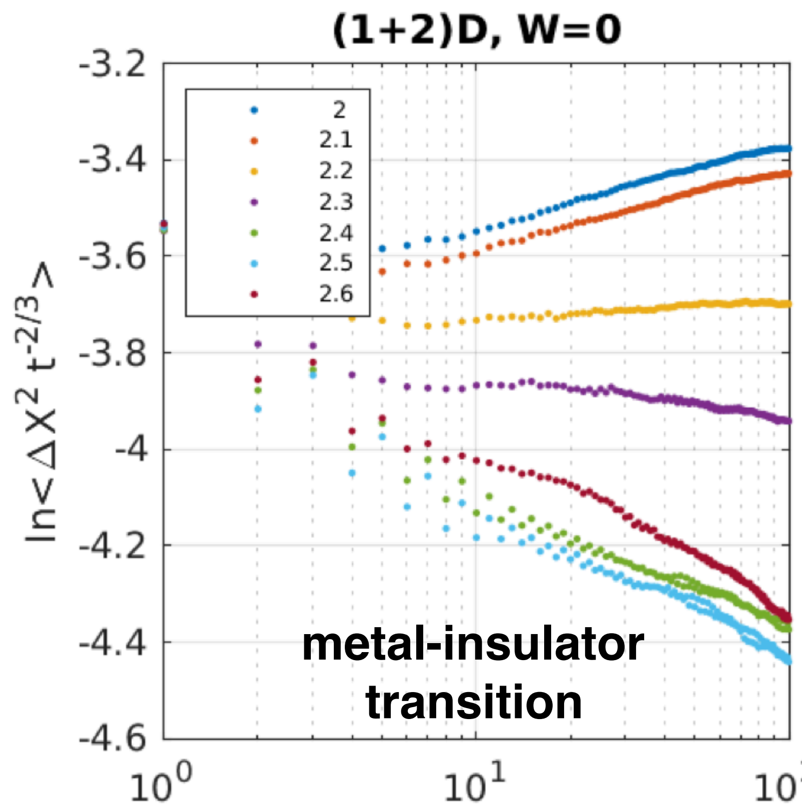
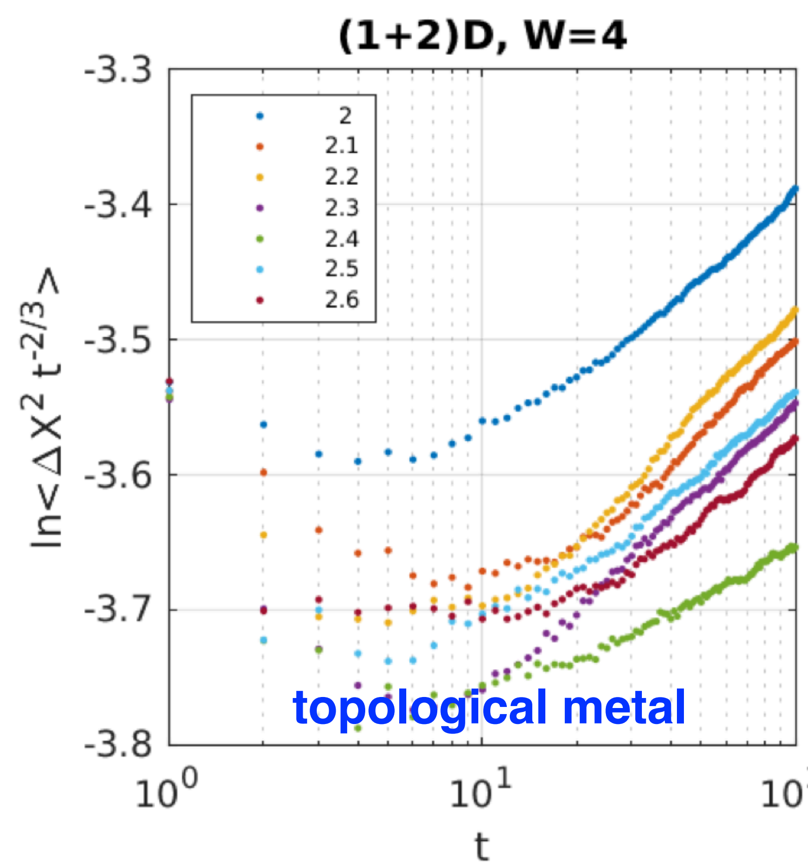
[2]*Department of Physics, Chung-Ang University, 06974 Seoul, Republic of Korea*

[3]*Centro Brasileiro de Pesquisas Físicas, Rua Xavier Sigaud 150, 22290-180, Rio de Janeiro, Brazil*

**Simulator of surfaces states of *4d* Quantum-Hall insulator**




**experimental realization…?**