Controlling the Stripe Order in a Diluted Frustrated Magnet

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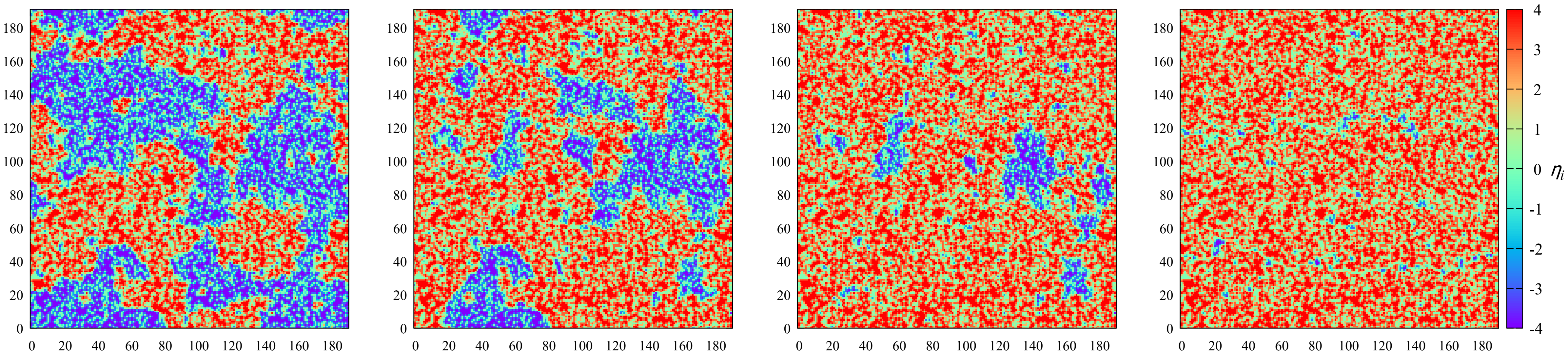
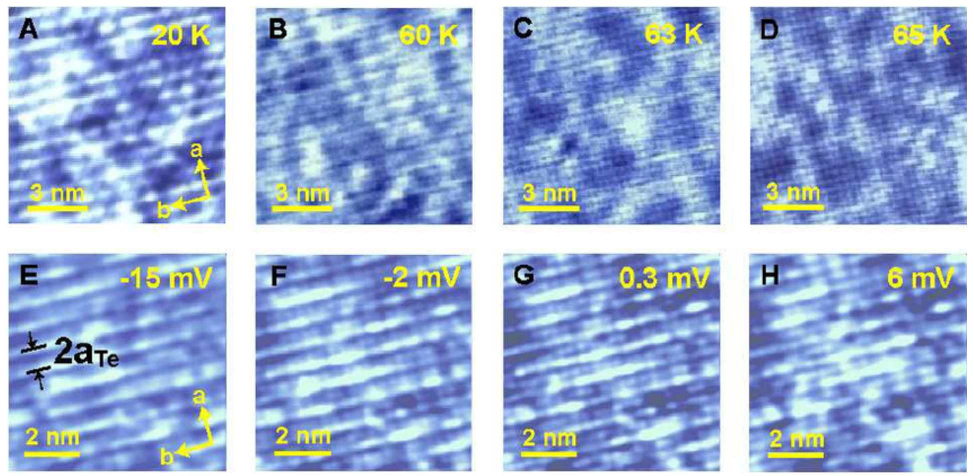
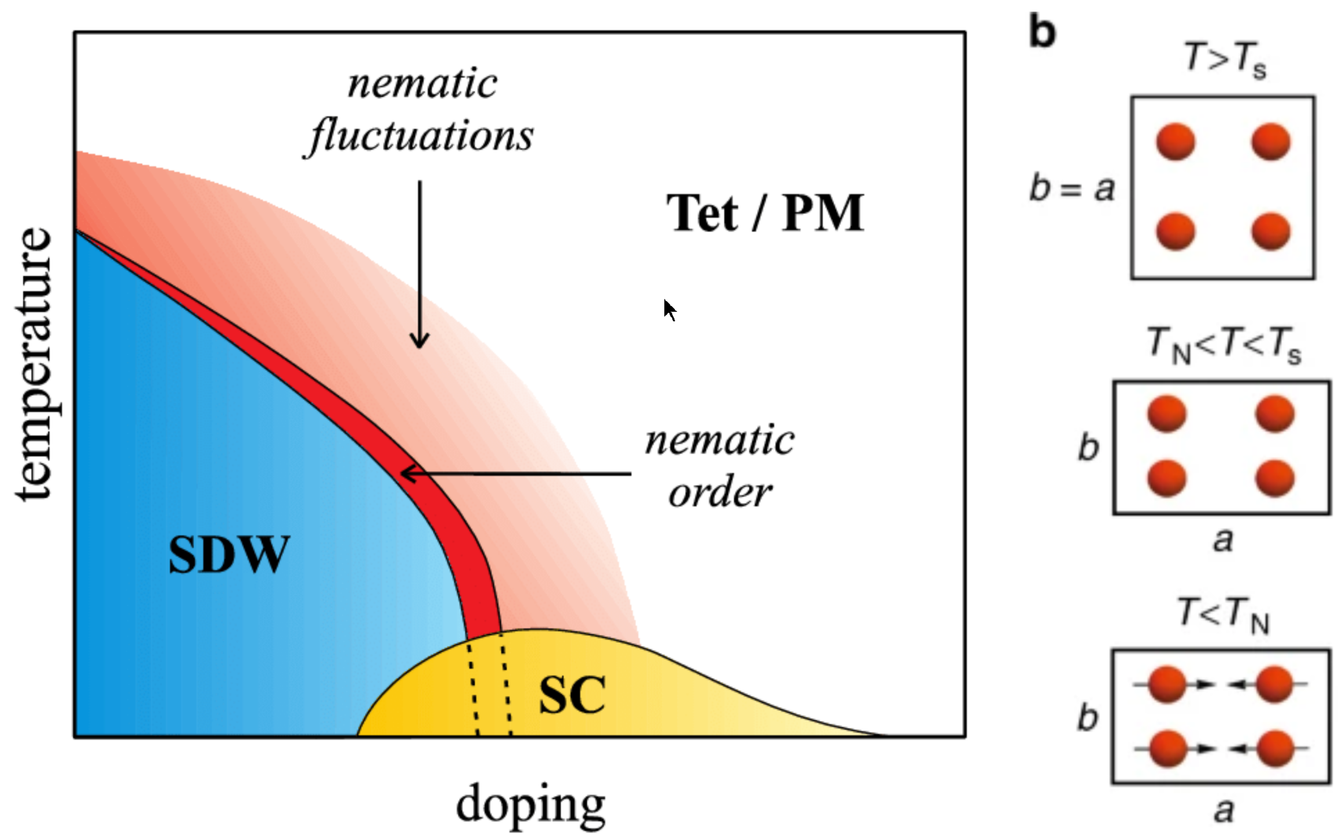


Motivation

Question: Effects of disorder on phases that break real-space symmetries?



charge "stripes" in cuprates, organic crystals, etc



nematic and spin-density wave orders in iron pnictides

Outline

- Symmetry-breaking and types of disorder
- Stripe order in the $J_1 - J_2$ Ising model
- Random fields from dilution
- Domain formation vs. stripe order
- Controlling the random-field mechanism

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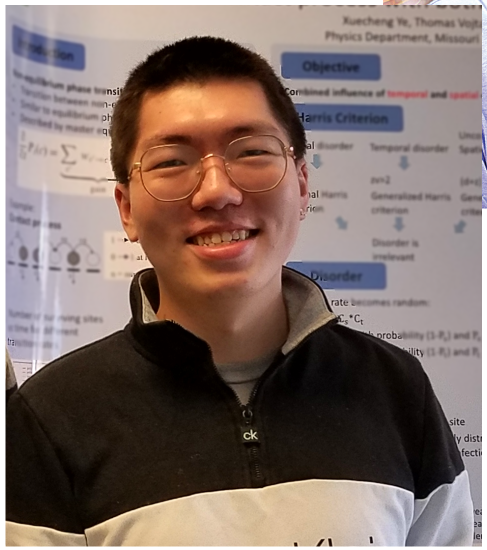


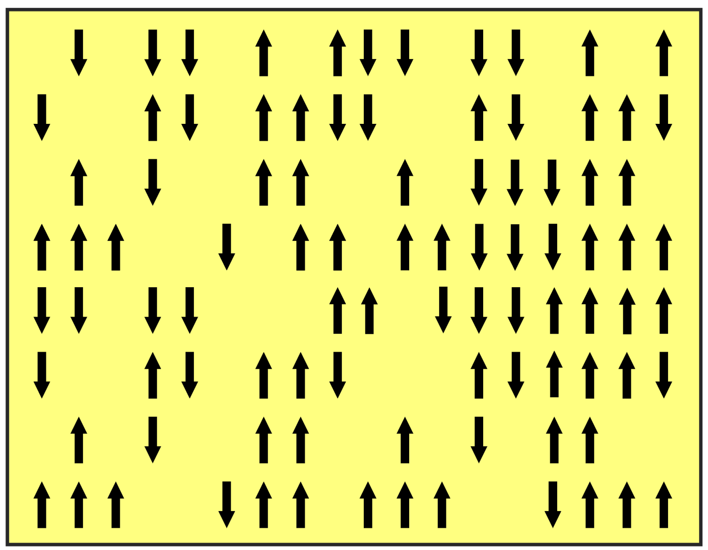
X.Y.

Disorder effects on long-range ordered phases

- **coupling** of order parameter to impurities and defects governed by **order parameter symmetries**

Random-T_c disorder

- does **not** break **order-parameter symmetries**
- **no** change in character of the bulk phases
- **spatial variation** of coupling strength
- example: diluted ferromagnet

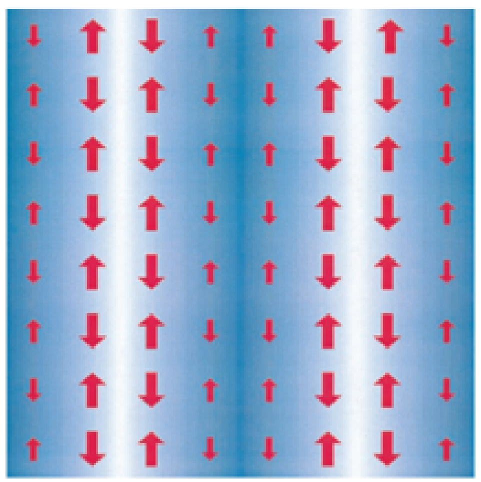


Random-field disorder

- locally **breaks order-parameter symmetries**
- **competes** with formation of long-range order

How can random fields arise?

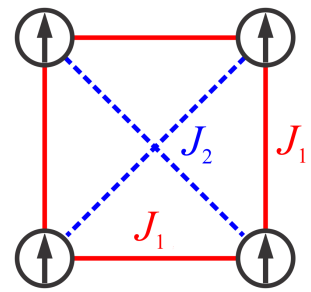
- if OP breaks real space symmetry, impurities expected to prefer particular OP direction
- impurities expected to create random-field disorder
- example: charge or spin density wave phase



Two-dimensional J_1–J_2 Ising model

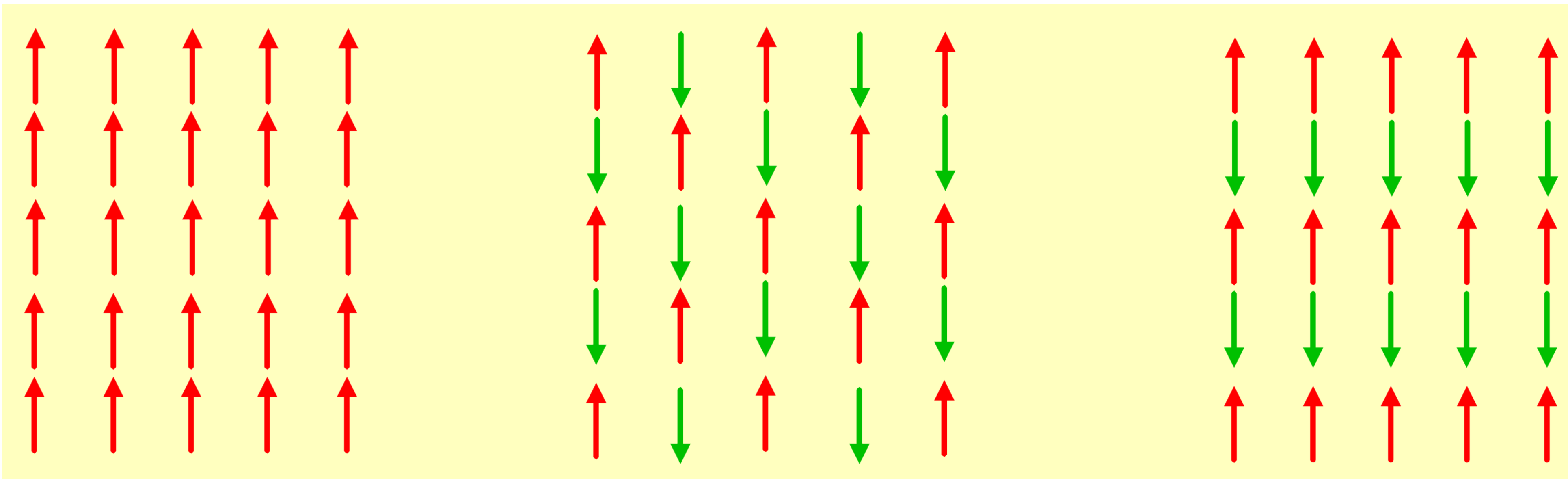
Hamiltonian:

$$H = -J_1 \sum_{\langle ij \rangle} S_i S_j - J_2 \sum_{\langle\langle ij \rangle\rangle} S_i S_j$$



negative $J_2 \Rightarrow$ frustration

Ground state phases:

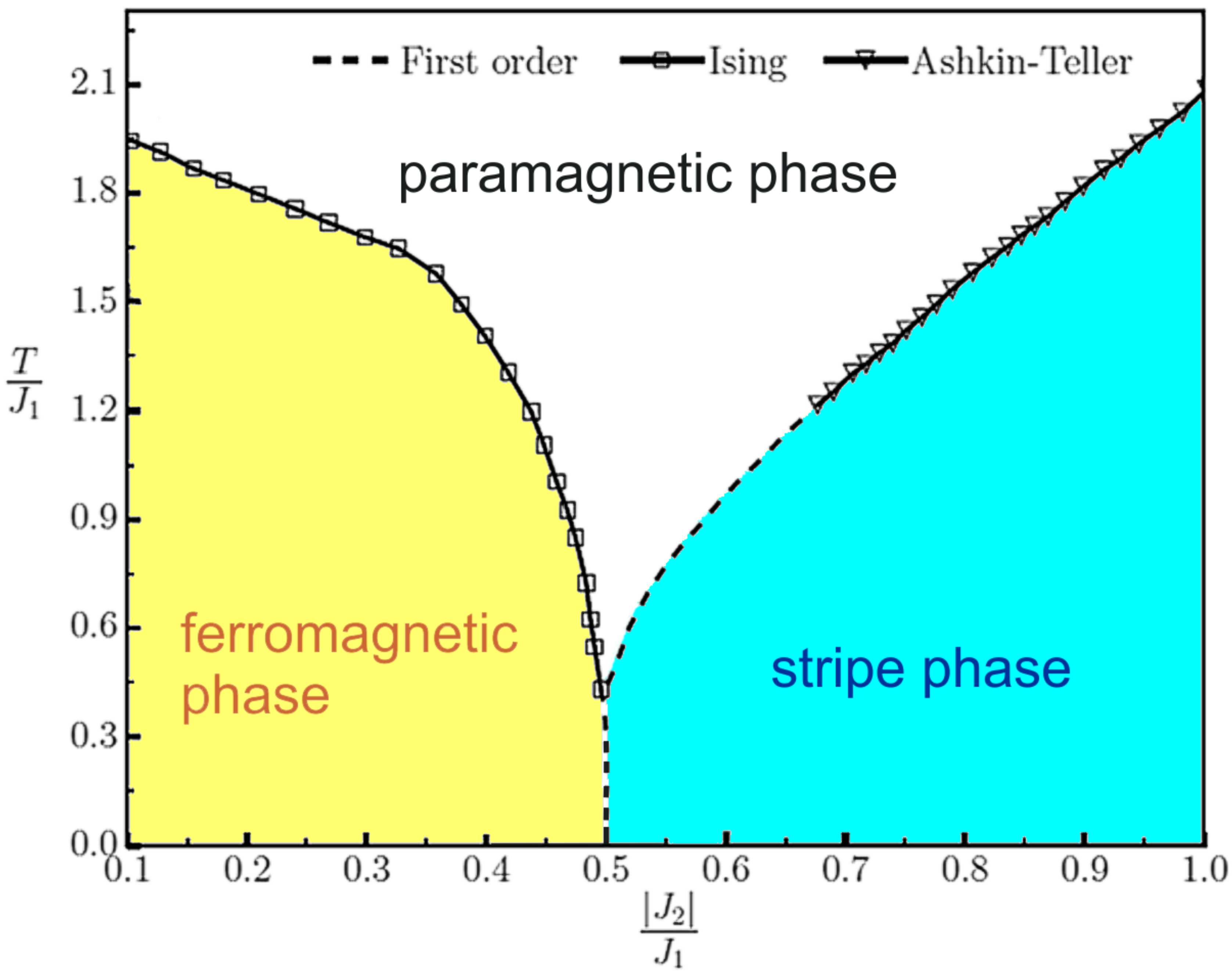


$J_1 > 0,\ J_2/J_1 > -0.5$,
ferromagnetic order,
breaks Ising up-down
symmetry

$J_1 < 0,\ J_2/|J_1| > -0.5$,
antiferromagnetic order,
breaks Ising up-down
symmetry, doubles unit cell

$J_2/|J_1| < -0.5$
stripe order
breaks Ising up-down sym.
and C_4 lattice rotation sym.

J_1–J_2 Ising model phase diagram ($J_1 > 0,\ J_2 < 0$)



Site-diluted J_1–J_2 Ising model

Hamiltonian:

$$H = -J_1 \sum_{\langle ij \rangle} \epsilon_i \epsilon_j S_i S_j - J_2 \sum_{\langle\langle ij \rangle\rangle} \epsilon_i \epsilon_j S_i S_j$$

dilution: quenched random variables ϵ_i

$$\epsilon_i = \begin{cases} 0 & \text{with probability } p \\ 1 & \text{with probability } 1-p \end{cases}$$

Question: Effects of random site dilution on the ferromagnetic and stripe phases?

Ferromagnetic phase

- order parameter breaks Ising symmetry
- order parameter **does not break** real-space symmetries
- impurities produce **random-T_c disorder**
- ferromagnetic phase **survives** in presence of impurities

Stripe phase

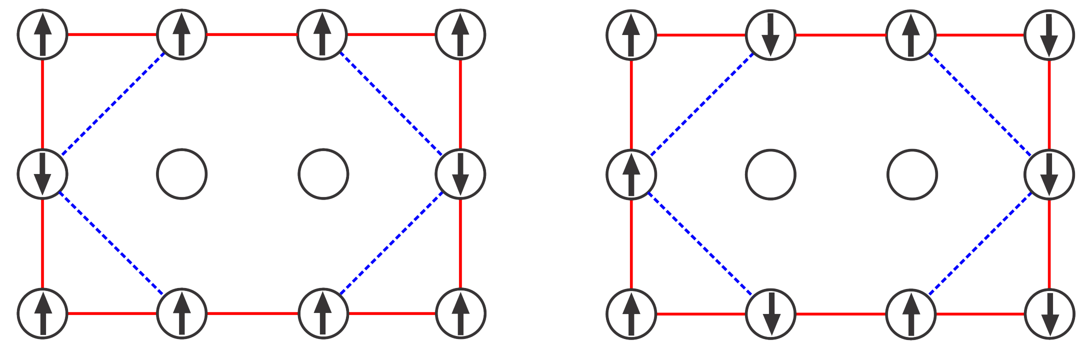
- order parameter **breaks** both Ising symmetry and C_4 **lattice rotation symmetry**
- stronger coupling between impurities and order parameter expected

Objective:

Study stability of stripe phase against spinless impurities

Random fields from site dilution

Effect of impurities on stripe phase:



- single vacancy does not break symmetry between stripe directions
- vacancy pair on horizontal nearest-neighbor sites prefers horizontal stripes by $2J_1$

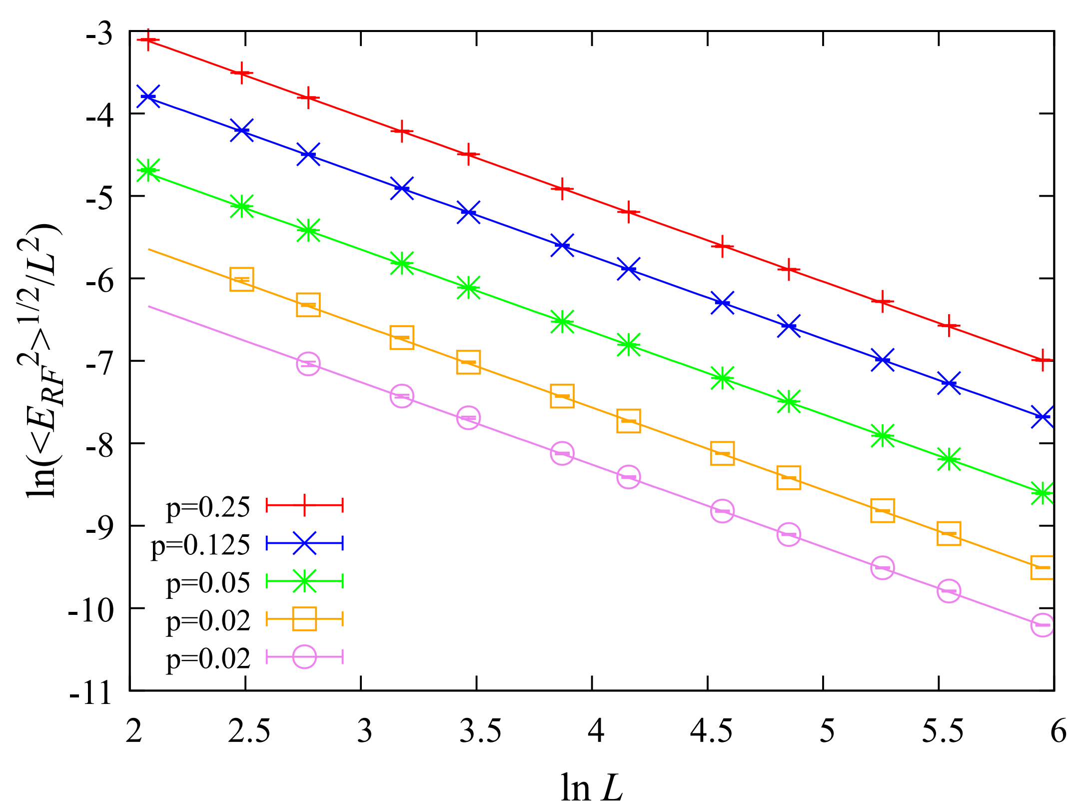
$\Rightarrow$ impurity pairs **locally break** C_4 lattice rotation symmetry

$\Rightarrow$ impurity pairs create **random-field** disorder for **nematic** order parameter

Random field strength:

- estimated from number of vacancy pairs via **central limit theorem**

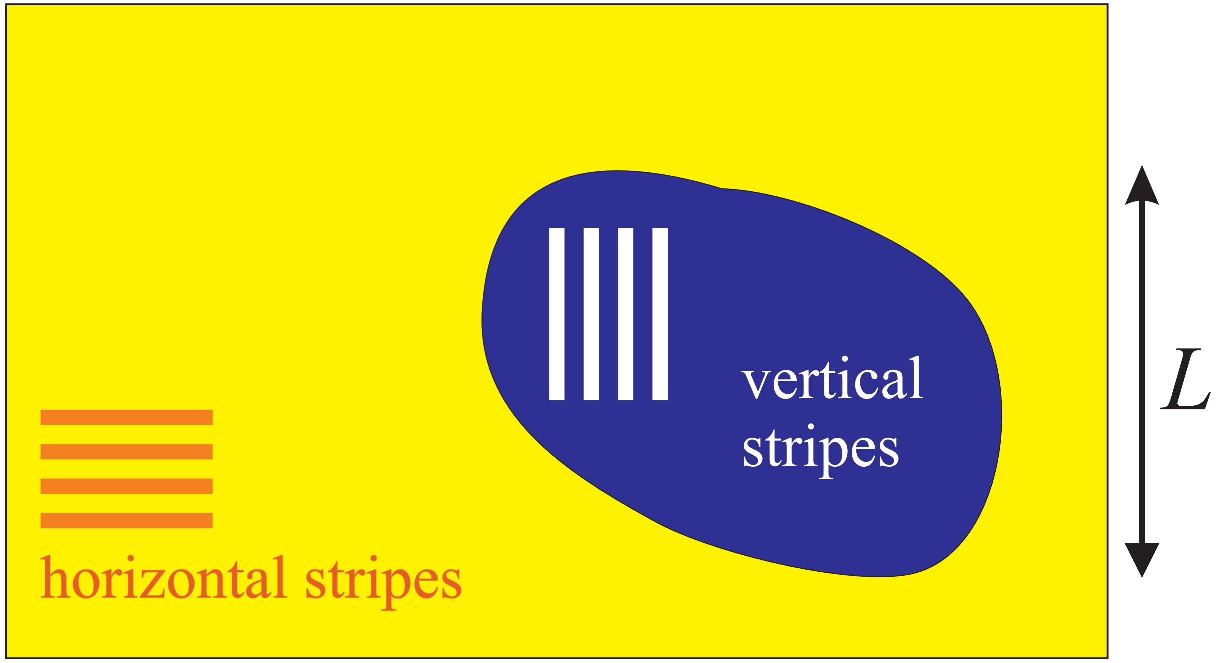
$$\langle E_{RF}^2(L)\rangle = 2L^2 p^2 J_1^2 = h_{\rm eff}^2 L^2$$



Imry-Ma criterion: long-range order vs. domain formation

Imry and Ma:

compare energy gain from forming domain that takes advantage of disorder with energy cost of domain wall



random field energy gain:

$$E_{\rm RF} \sim h_{\rm eff}\, L^{d/2}$$

domain wall energy:

$$E_{\rm DW} \sim \begin{cases} J_2\, L^{d-1} & \textbf{(discrete)} \\ J_2\, L^{d-2} & \text{(continuous)} \end{cases}$$

• if $E_{\rm RF} > E_{\rm DW}$ for $L \to \infty$: domain formation favorable

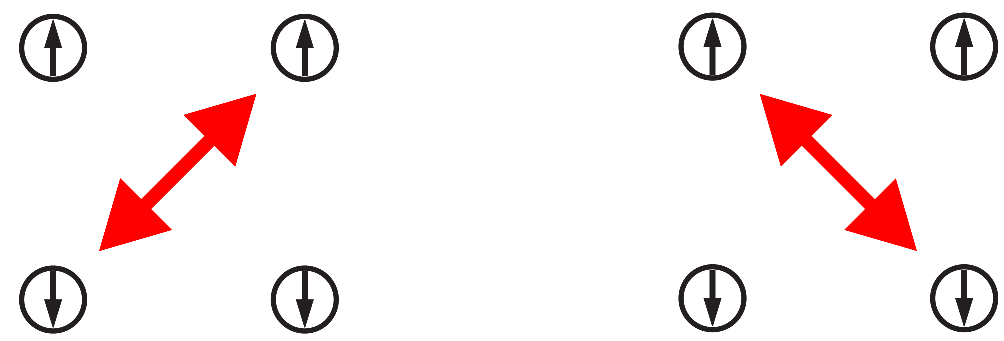
$\Rightarrow$ long-range stripe order **destroyed** by domain formation in $d \leq 2$

• minimum domain size (in 2D): $L_0 = A \exp(c J_2^2 / h_{\rm eff}^2)$

Computer simulations

Monte Carlo algorithms:

- single spin flips (Metropolis) and corner exchange moves
- system sizes up to 192^2
- site dilutions $p = 1/8$ and $1/4$
- 3,000 to 100,000 disorder configurations
- up to 2×10^6 equilibration and measurement sweeps

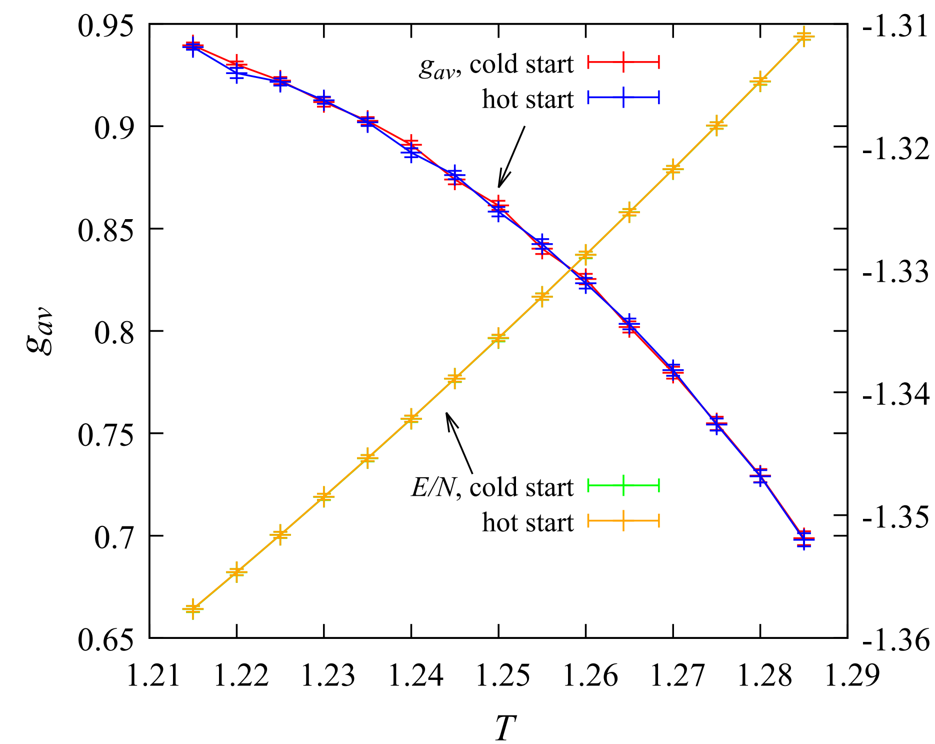


Observables:

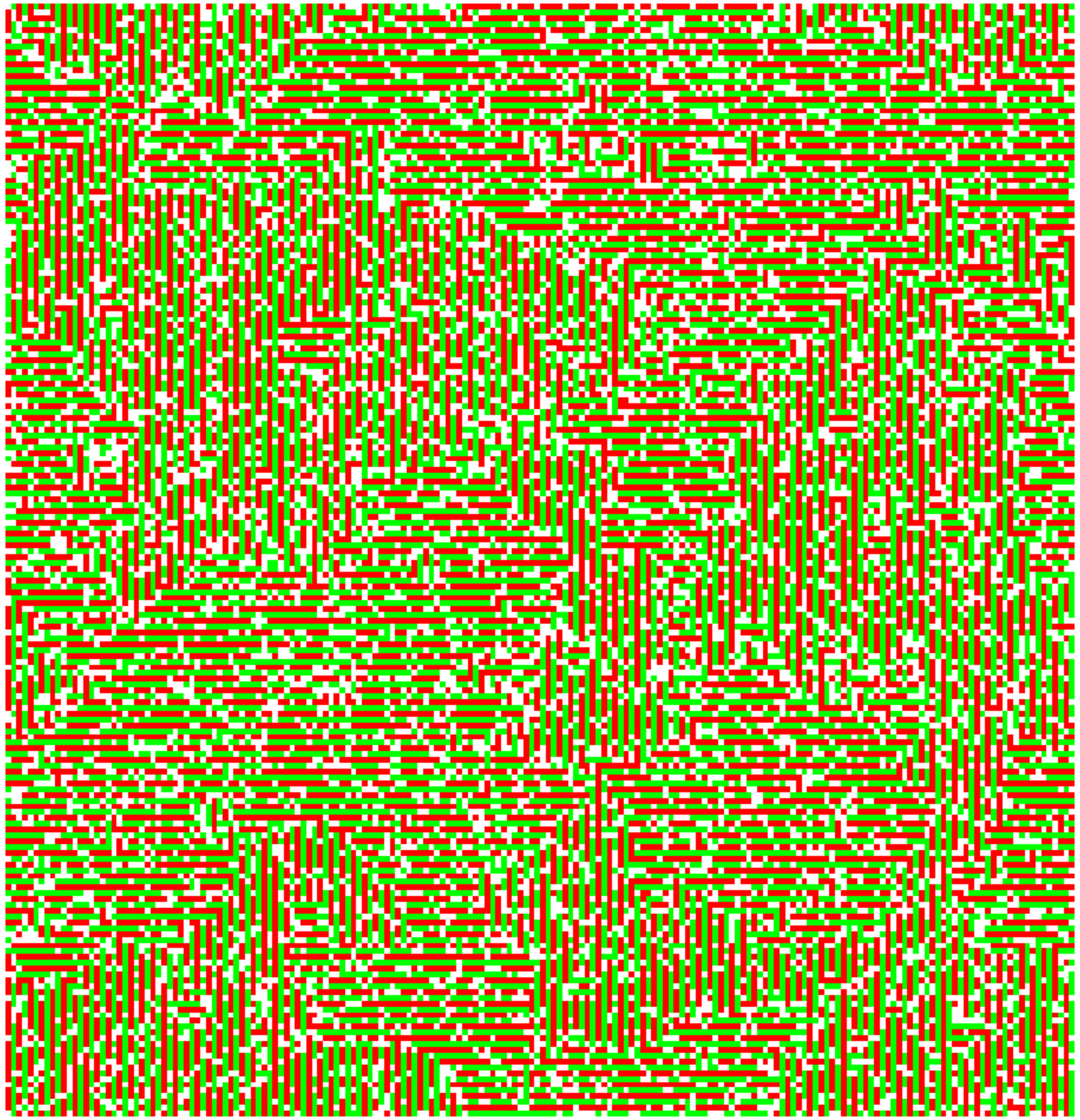
- two-component **stripe order parameter** $\psi = (\psi_h, \psi_v)$

$$\psi_h = \frac{1}{N}\sum_i (-1)^{y_i}\epsilon_i S_i \ , \quad \psi_v = \frac{1}{N}\sum_i (-1)^{x_i}\epsilon_i S_i$$

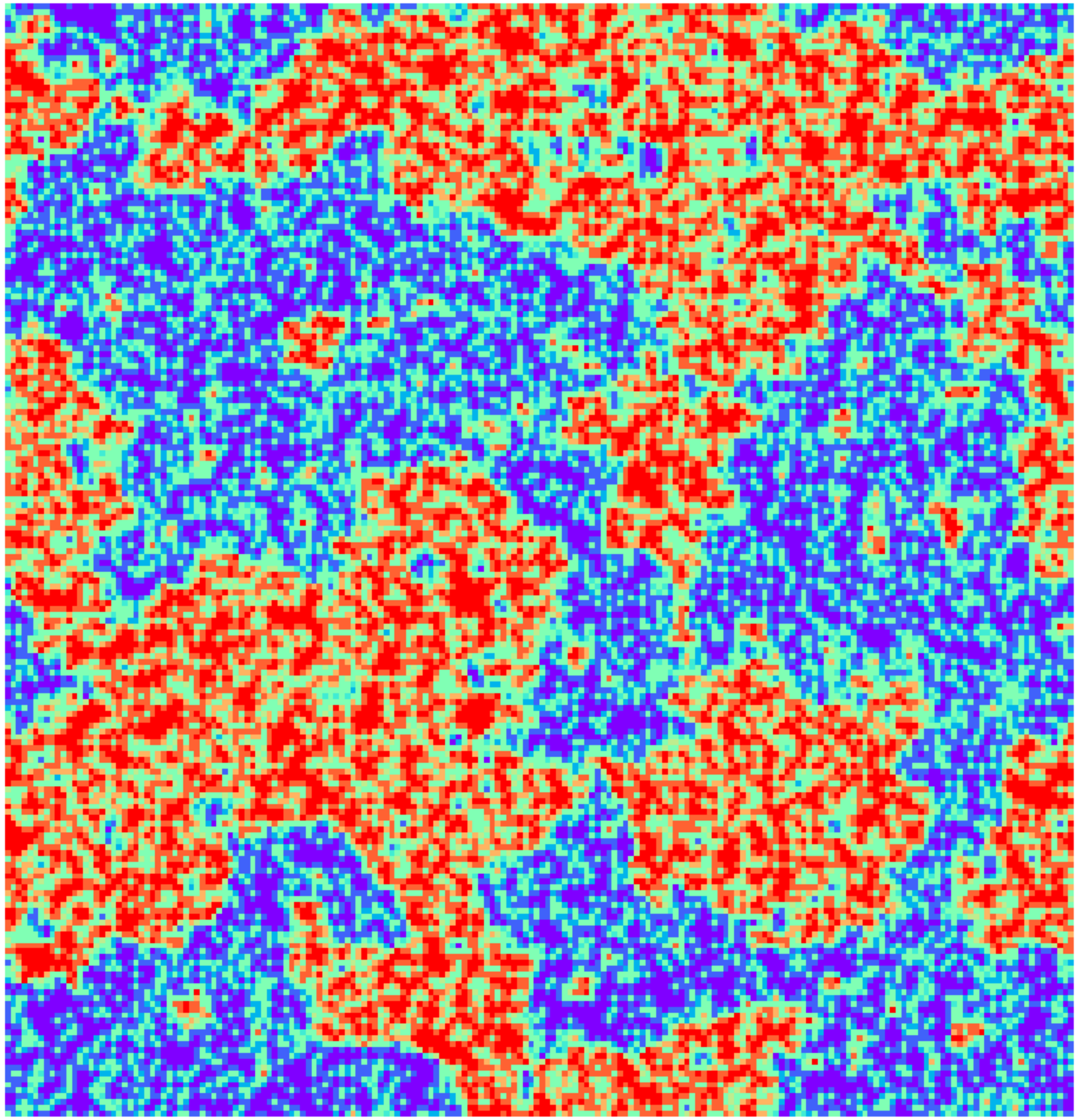
- Binder cumulant $\quad g_{\mathrm{av}} = \left[2 - \frac{\langle|\psi|^4\rangle}{\langle|\psi|^2\rangle^2}\right]_{\mathrm{dis}}$



Domain formation

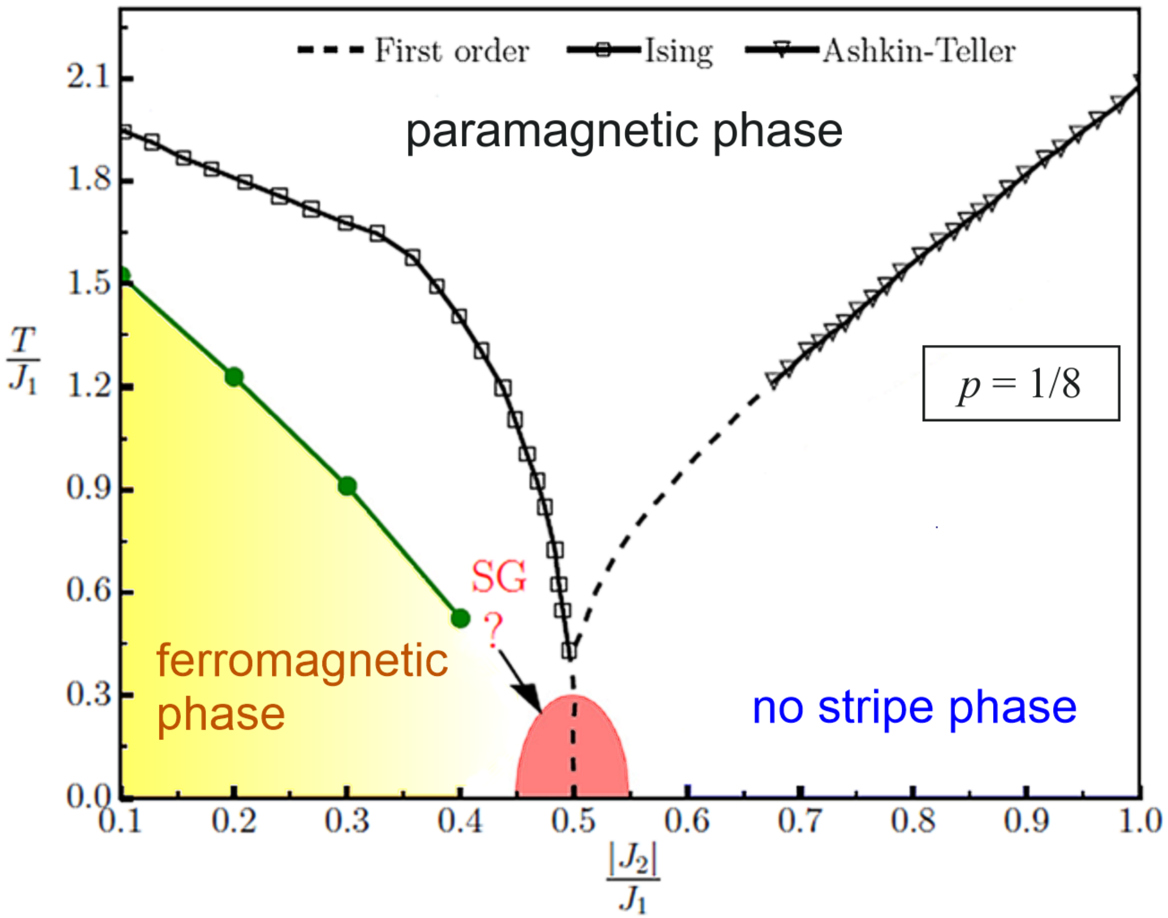


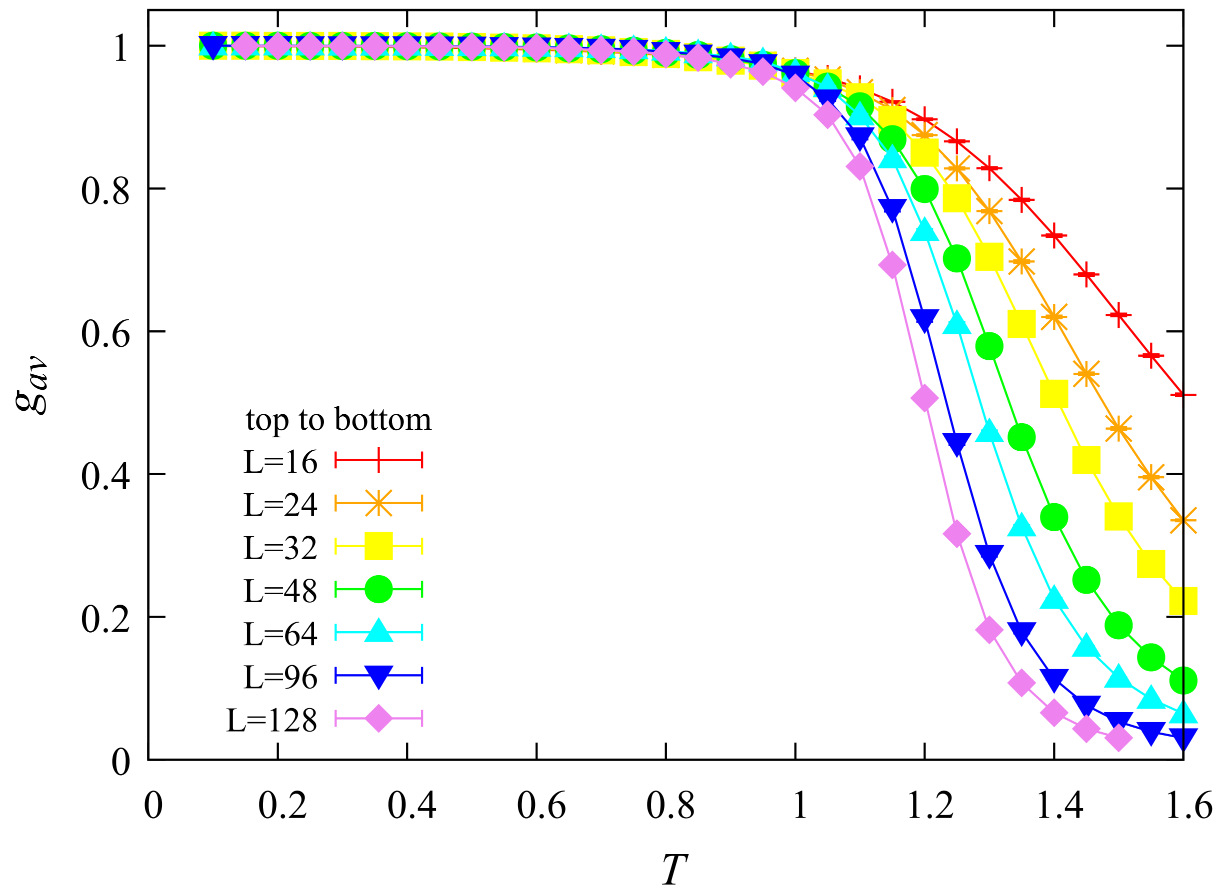
Snapshot of spin configuration
$T = 0.1$ $L = 192$, $p = 1/4$, $J_1 = -J_2 = 1$



Local nematic order parameter $\eta_i = \sum_j' \pm\epsilon_i\epsilon_j S_i S_j$
distinguishing **horizontal** and **vertical** stripes

Destruction of the stripe phase





- ferromagnetic phase **survives** with reduced T_c (impurities create **random-T_c disorder**)
- stripe phase **destroyed** by **random-field** induced domain formation

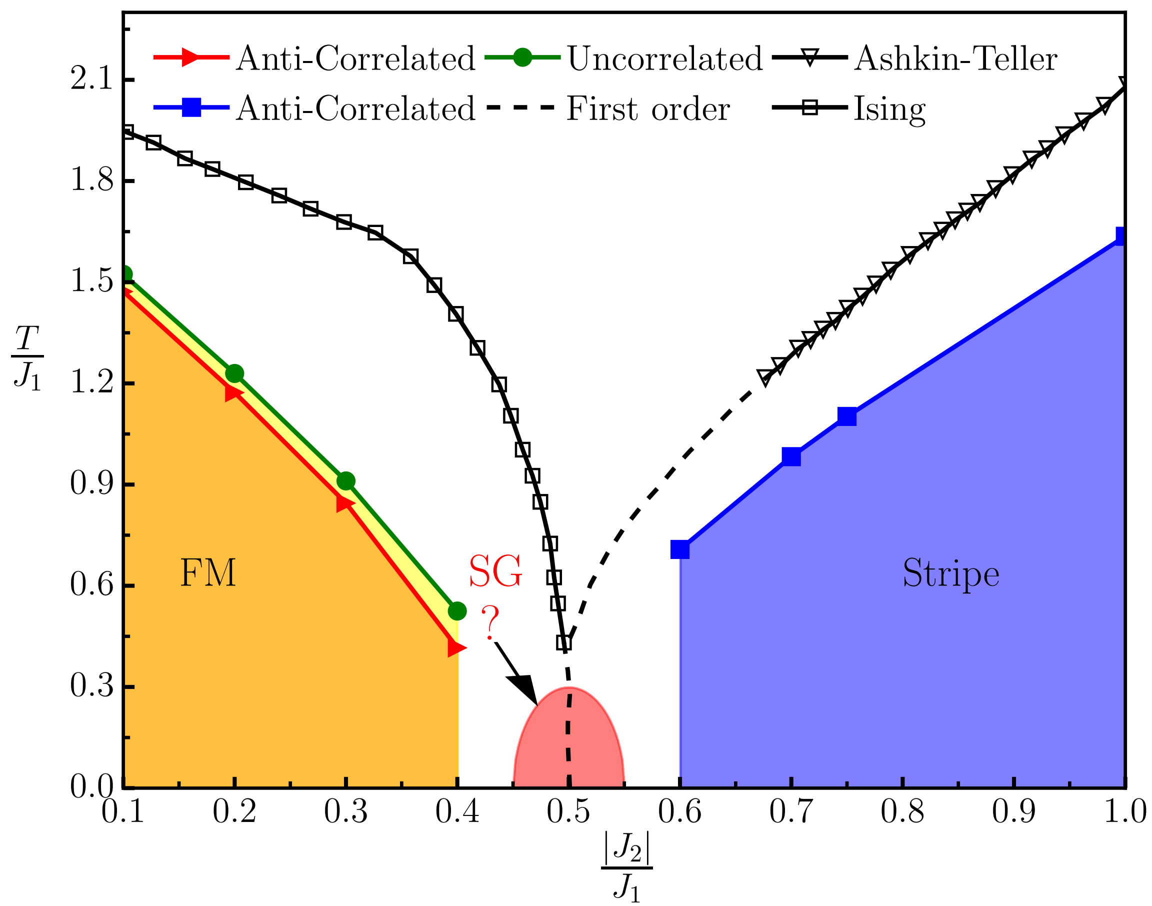
- stripe Binder cumulant vs. temperature, $p = 1/4$, $J_1 = -J_2 = 1$

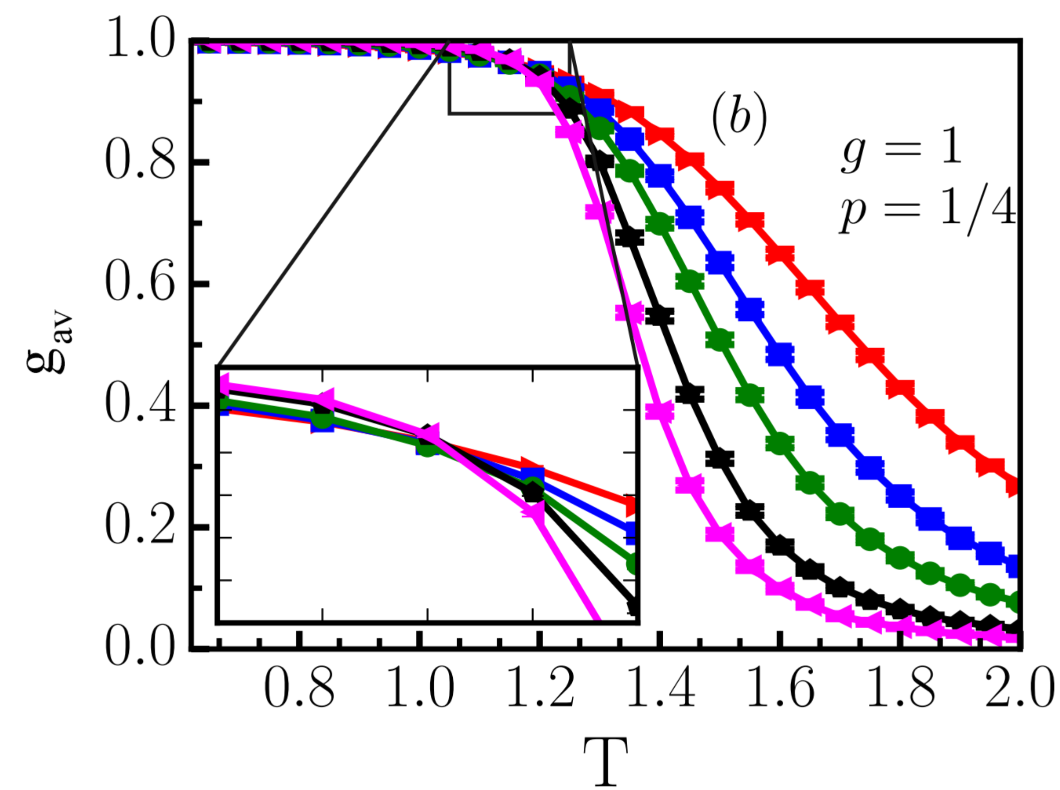
⇒ curves do not cross

⇒ **no phase transition** into stripe phase

Controlling the random-field mechanism I: impurity correlations

- random fields produced by vacancy pairs on nearest neighbor sites
- random fields **suppressed** if impurities are anti-correlated (repulse each other)



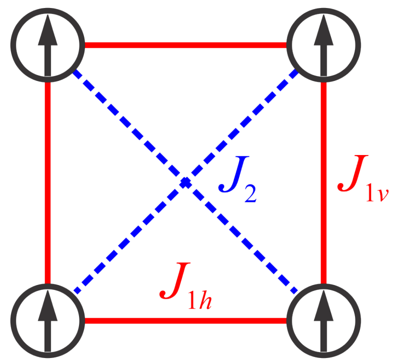


- perfect anticorrelations (no vacancy pairs): Binder cumulant curves cross

⇒ stripe phase is **restored**

⇒ T_c reduced w.r.t. clean case (vacancies act as **random-T_c disorder**)

Controlling the random-field mechanism II: global symmetry breaking



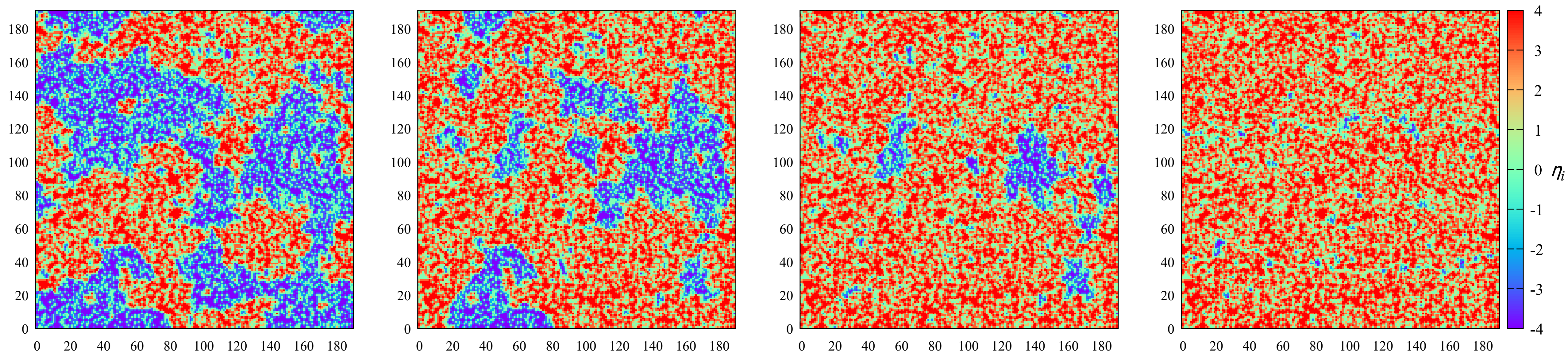
$J_{1h} = J_1 + \Delta J$
$J_{1v} = J_1 - \Delta J$

- symmetry between two lattice directions **weakly broken** (e.g., by external strain)

$$H = -J_{1h} \sum_{\langle ij \rangle_h} \epsilon_i \epsilon_j S_i S_j - J_{1v} \sum_{\langle ij \rangle_v} \epsilon_i \epsilon_j S_i S_j - J_2 \sum_{\langle\langle ij \rangle\rangle} \epsilon_i \epsilon_j S_i S_j$$

- nearest-neighbor interaction depends on direction

$\Rightarrow$ horizontal stripes **preferred** for $\Delta J > 0$

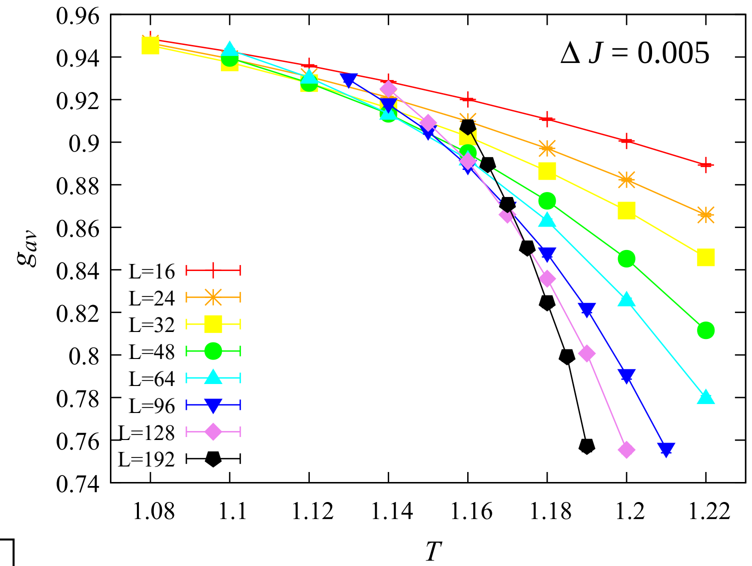


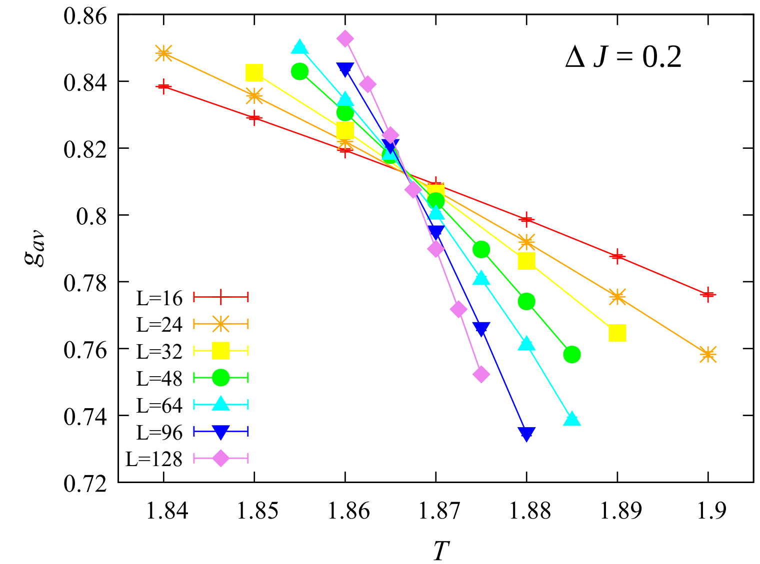
$\Delta J = 0$ 0.002 0.01 0.05

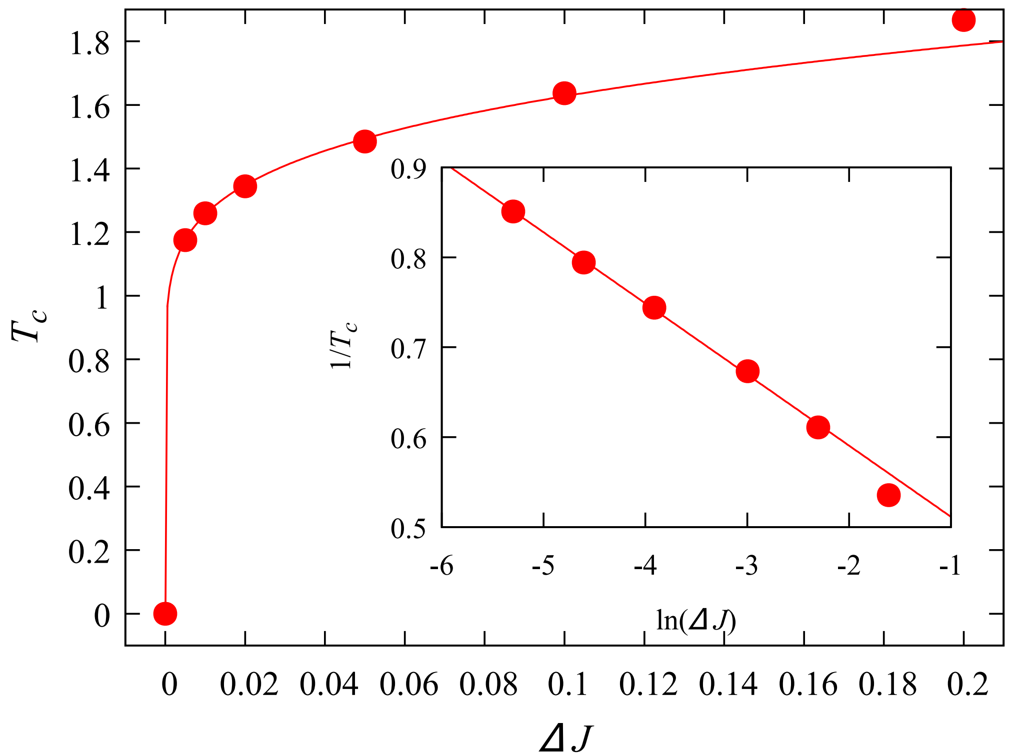
Stripe transition for $\Delta J > 0$

- Binder cumulant curves cross for **any nonzero ΔJ**

$\Rightarrow$ stripe phase is **restored**







- very small ΔJ sufficient to produce sizable T_c
- **logarithmic dependence:** $T_c \sim 1/|\ln(\Delta J)|$

percolation theory

- domain structure resembles percolation problem
- distance from percolation criticality $\sim \Delta J$
- Ising model on percolating lattice: $T_c \sim 1/|\ln(x - x_c)|$

Critical behavior

Symmetry expectation:

- for $\Delta J > 0$, order parameter has **Ising** symmetry
- impurities create **random-T_c** disorder

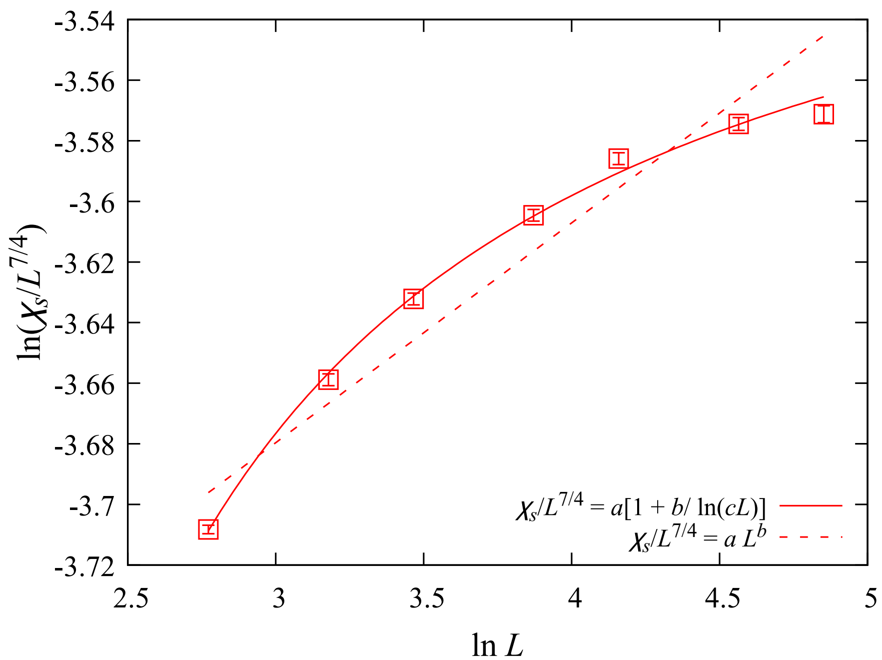
$\Rightarrow$ expect disordered 2D Ising universality class

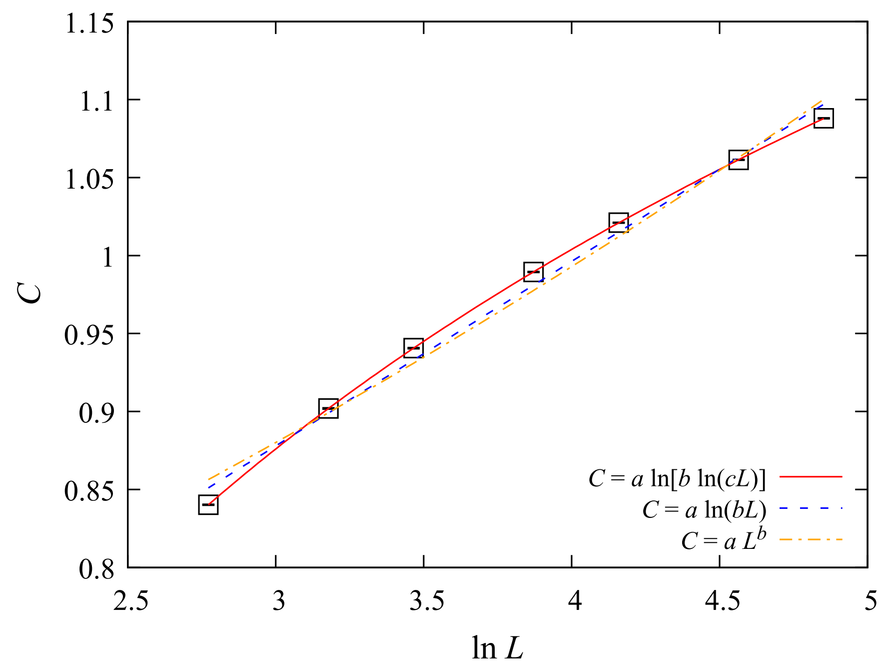
Harris criterion:

- disorder marginal w.r.t. Harris criterion $d\nu > 2$

RG prediction:

- critical behavior controlled by **clean Ising fixed point**
- disorder produces **universal logarithmic corrections**
- stripe susceptibility $\chi_s \sim L^{\gamma/\nu} [1 + O(1/\ln L)]$ with $\gamma/\nu = 7/4$
- specific heat: $C \sim \ln \ln L$





Conclusions

- when order parameter breaks a **real-space** symmetry
  $\Rightarrow$ impurities generically create both **random-T_c** disorder and **random-field** disorder
- random fields **destroy** the long-range ordered phase in dimensions $d \leq 2$ by domain formation
- long-range order can be restored by weak **global symmetry breaking** ("strain engineering")
- in J_1–J_2 Ising model, strength of random-fields can be tuned by repulsion between impurities

Manipulating the random fields provides a novel way of controlling the phase diagram.

S.S. Kunwar, A. Sen, T. Vojta and R. Narayanan, *Tuning a random field mechanism in a frustrated magnet*, Phys. Rev. B **98**, 024206 (2018), arXiv:1803.05597

X. Ye, R. Narayanan and T. Vojta, *Stripe order, impurities, and symmetry breaking in a diluted frustrated magnet*, Phys. Rev. B. **105**, 024201 (2022), arXiv:2111.00101