A Cloaked Griffiths Phase in a Low dimensional superconductor

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Talk given at the Workshop on Strong Correlations at ICTP-SAIFR

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arXiv:2206.10215

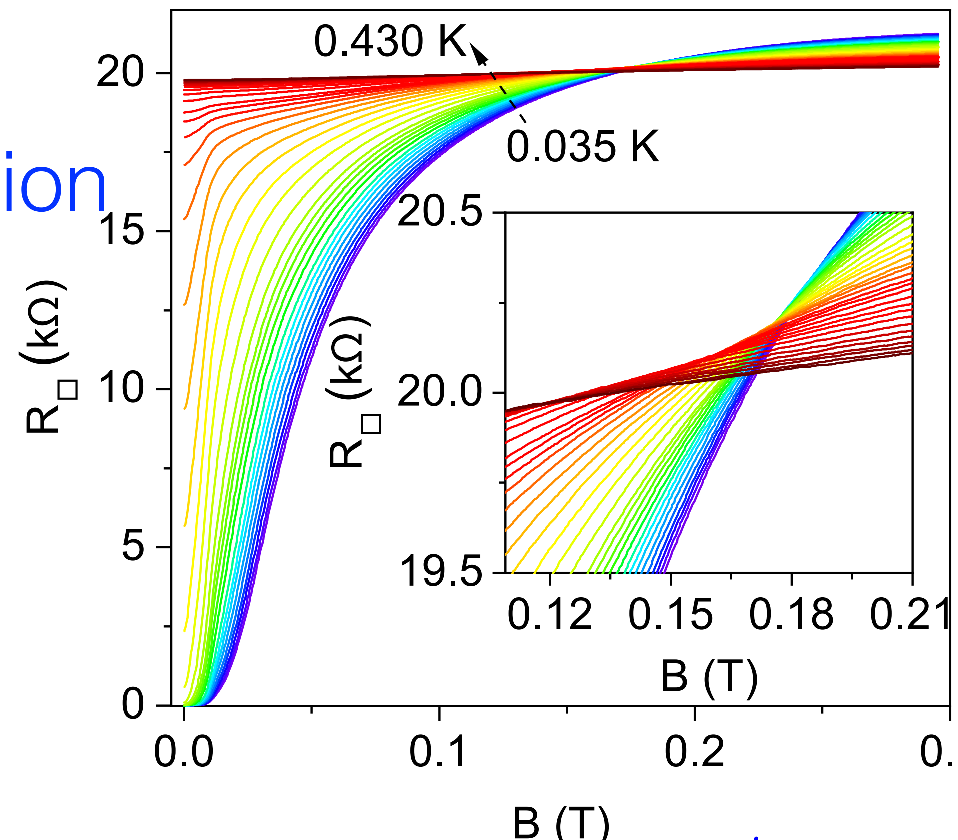
Superconductor-Normal State Transition

Superconducting Transition as a function

Of an applied magnetic field

Example of T=0 QPT

Tuning parameter magnetic field B



$\mathrm{LaScO_3/SrTiO_3}$

arXiv:2206.10215

Effective Field Theory

$$S = \sum_{q,\omega_n} \varphi(\mathbf{q},\omega_\mathbf{n})[\delta_\mathbf{0} + \xi_\mathbf{0}^\mathbf{2}\mathbf{q}^\mathbf{2} + \gamma|\omega_\mathbf{n}|]\varphi(-\mathbf{q},-\omega_\mathbf{n})$$
$$+\frac{u}{2N}\int \mathrm{d}^\mathrm{d}x\mathrm{d}\tau\varphi^4(x.\tau).$$

Hertz (1976), Millis (1993), Belitz and Kirkpatrick (1996)

$$\delta_0 = \frac{\mathrm{B}-\mathrm{B_c}}{\mathrm{B_c}} \quad \rightarrow$$ Non-thermal Tuning parameter

$|\omega_n| \rightarrow$ Damping from the underlying fermions.

Equivalent to a $1/|\tau-\tau'|^2$ interaction in imaginary time

Impact of disorder

$\delta_0 \to \delta_0(x) \to$ Random mass disorder $\to$

Runaway flow

Belitz and Kirkpatrick (1996)
R Narayanan, Vojta, Belitz and Kirkpatrick (1999)

The problem is tackled by the SDRG

Introduced by Ma, Dasgupta, Hu (1979), and further developed by D. Fisher (1992, 1995)

Application to problem at hand: Hoyos, Kotabage, Vojta (2007)

Vojta, Kotabage, Hoyos (2009)

Discretized Large-N theory

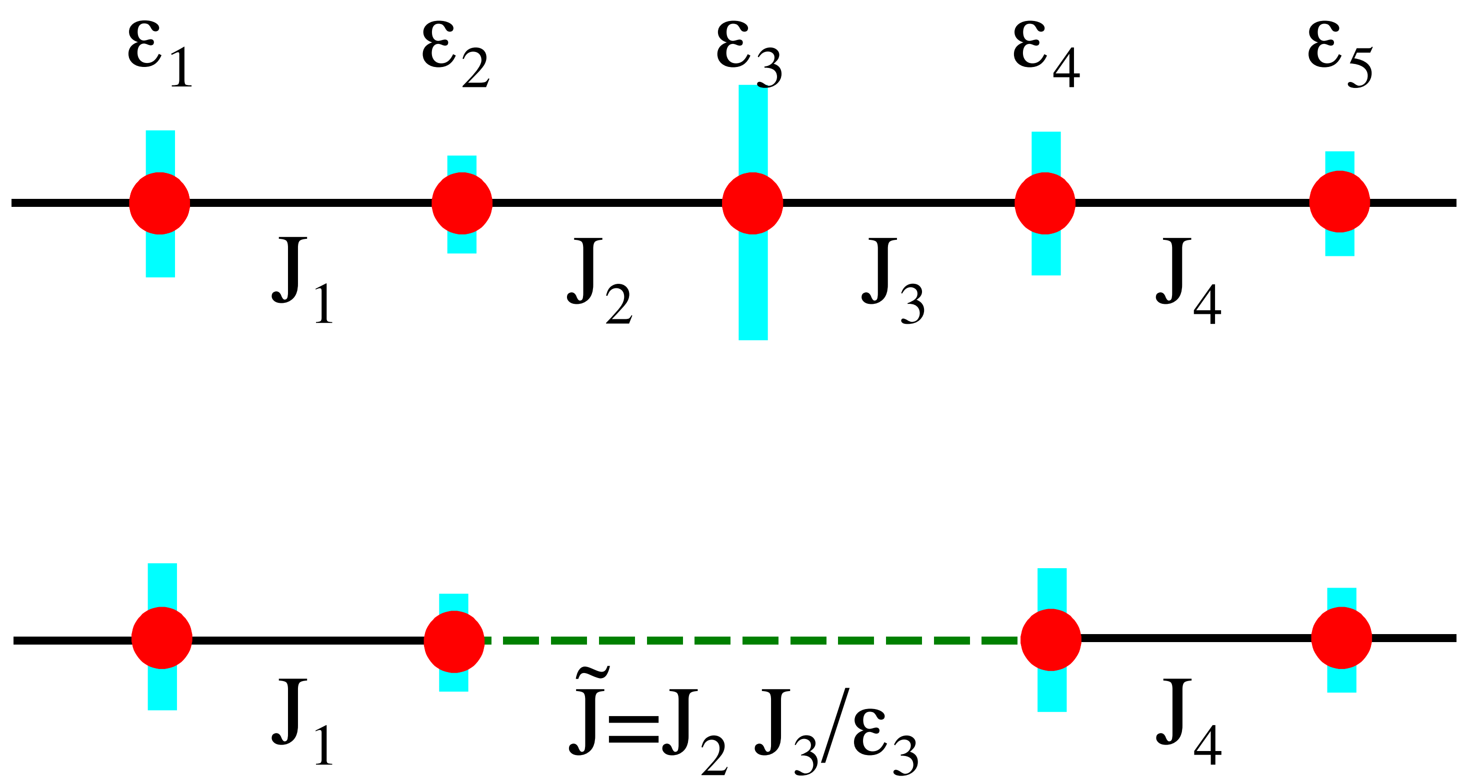
$$S = T\sum_{i,\omega_n}(\epsilon_i + \gamma_i|\omega_n|^{2/z})|\phi_i(\omega_n)|^2 - \sum_{i,\omega_\mathbf{n}} J_i\phi_i(-\omega_\mathbf{n})\phi_{i+1}(\omega_\mathbf{n})$$

Competing local energies

ϵ_i $\to$ favouring the disordered phase

$\mathrm{J_i}$ $\to$ favouring the ordered phase

Site Decimation



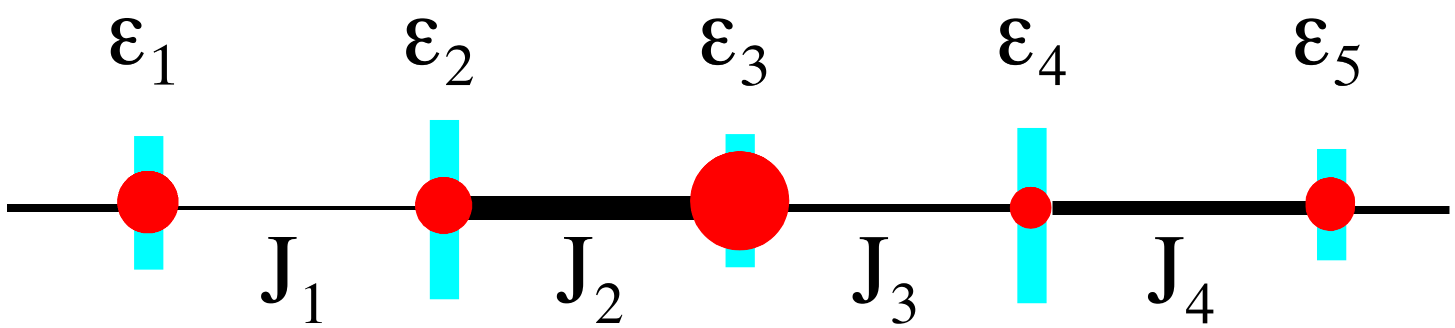
If the largest energy scale is $\epsilon_3 \gg J_2, J_3$:

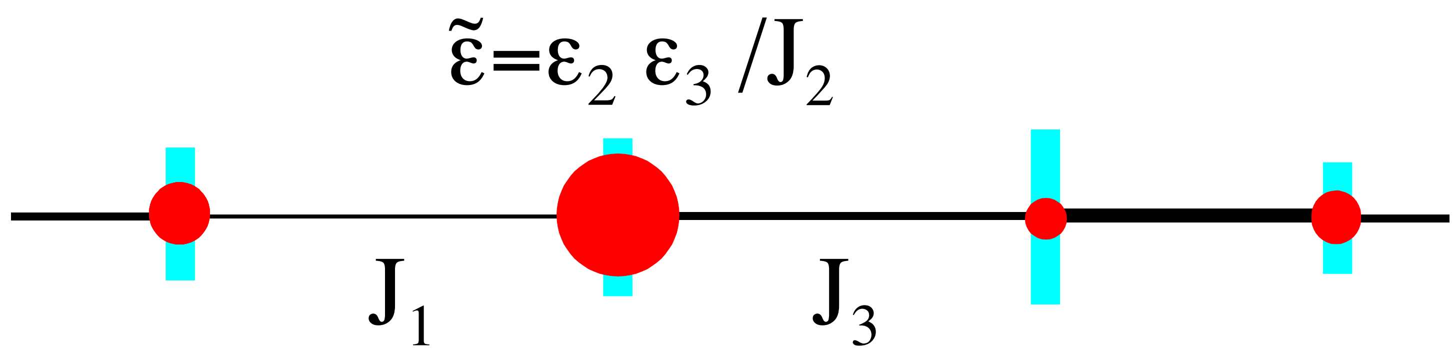
Site 3 is removed from the system

New weaker coupling mediates between sites 1 and 4

$$\tilde{J} = \frac{J_2 J_3}{\epsilon_3}$$

Bond Decimation





The greatest energy scales is $J_2 \gg \epsilon_2, \epsilon_3$:

Fuse the rotors together with a renormalized gap on the composite rotor $\tilde{\epsilon} = \dfrac{\epsilon_2 \epsilon_3}{J_2}$

Results from SDRG

$$S = T\sum_{i,\omega_n}(\epsilon_i + \gamma_i|\omega_n|^{2/z})|\phi_i(\omega_n)|^2 - \sum_{i,\omega_\mathbf{n}} J_i\phi_i(-\omega_\mathbf{n})\phi_{i+1}(\omega_\mathbf{n})$$

$z = 2$ (Ohmic damping)

Recursion relation: $\tilde{J} = \frac{J_2 J_3}{\epsilon_3}$ and $\tilde{\epsilon} = \frac{\epsilon_2\epsilon_3}{J_2}$ exactly equivalent

Random Transverse Field Ising Model Fisher (1994), (1995)

A special transition called Infinite Disorder Transition

Critical behavior lies in same universality class

Supports activated scaling $\ln\xi_\tau \sim \xi^{1/2}$ unlike conventional $\xi_\tau \sim \xi^z$

Hoyos, Kotabage, Vojta (2007) Vojta, Kotabage, Hoyos (2009)

$1 < z < 2$ Super-Ohmic damping

Disordered KT like behaviour

Vojta, Hoyos, Mohan, Narayanan (2010)

Results from SDRG (contd.)

Off critical solution: $\langle \ln \epsilon \rangle_{\rm dis} - \langle \ln \mathrm{J} \rangle_{\rm dis} = \delta_0$

$\delta_0 \rightarrow$ Distance to criticality

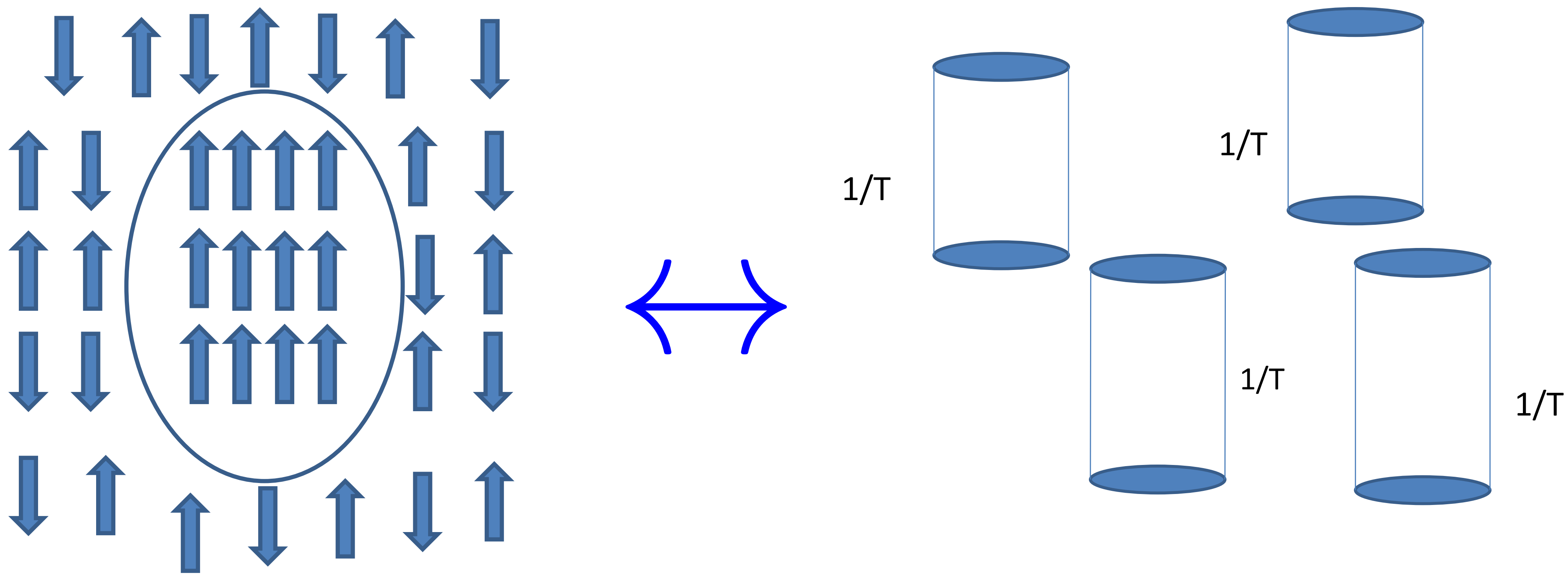
Off criticality one can show that $\xi_\tau \sim \xi^{\frac{1}{2\delta_0}}$

Implies a non-universal dynamical exponent that varies with tuning parameter

Smoking gun is the non-universal dynamical exponent z

Look for a dynamical exponent that diverges as we approach B_c

Quantum Griffiths Effects



- Finite spatially
- Infinite extent in time

- Endowed with very slow dynamics due to infinite extent in imaginary time.
- Slow dynamics responsible for anomalous effects in thermodynamic observables.
- In particular: Look at the lower critical dimension of Rare Regions

Vojta Review (2013)

Resistivity Measurements in 2-DEGS

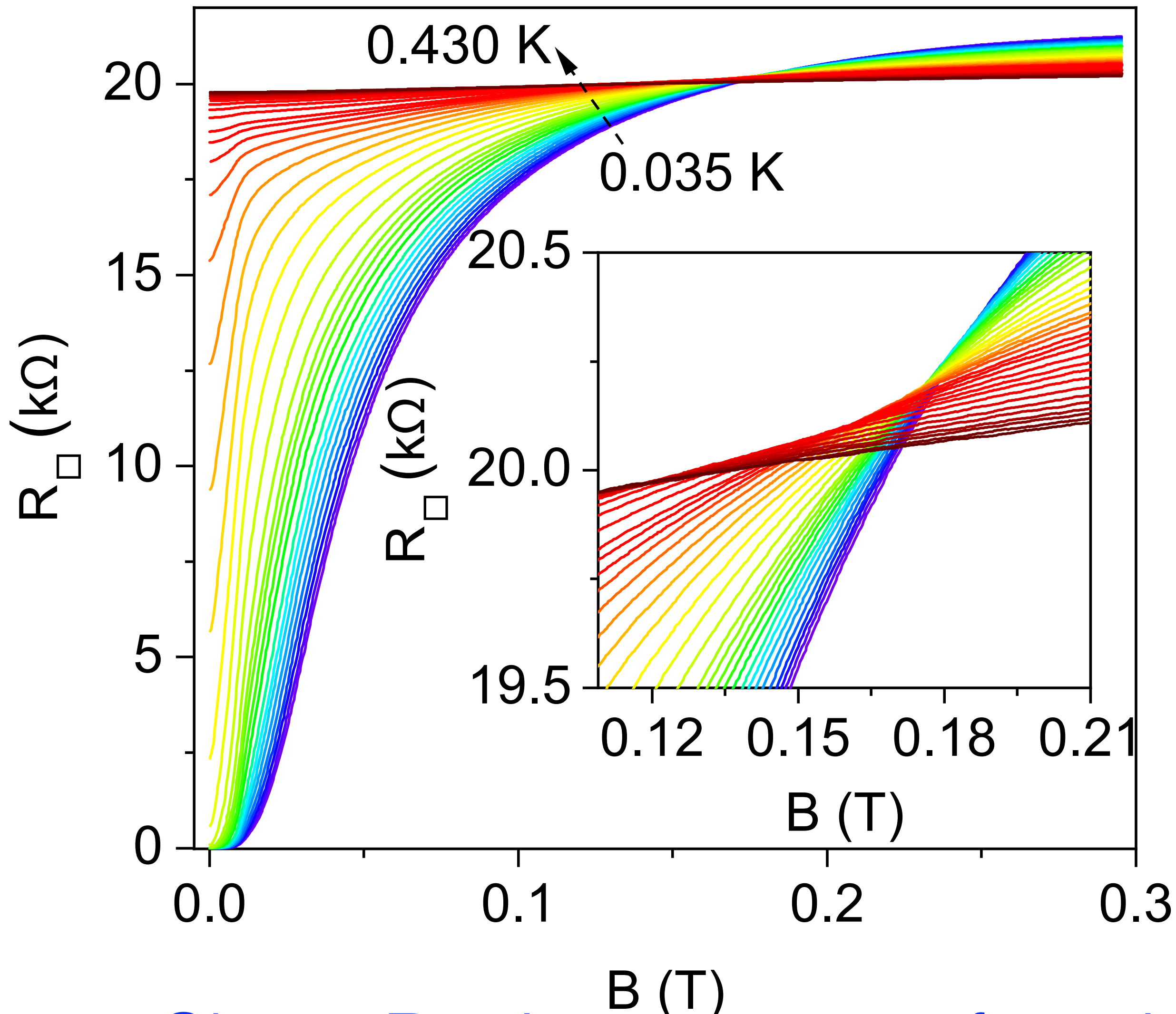
$LaScO_3/SrTiO_3$

Heterostructure

- Superconductor-Insulator transition as a function of magnetic field

- Drift in the crossing point due to leading irrelevant operator.

PRB 99, 054515 (2019).



Sheet Resistance as a function of Magnetic Field

Strategy use the scaling ansatz to extract νz

$$R_{\square} = R_c f\left[(B - B_c)\left(\frac{T_0}{T}\right)^{1/\nu z}\right].$$

$\nu z \rightarrow$ Universal $\rightarrow$ conventional

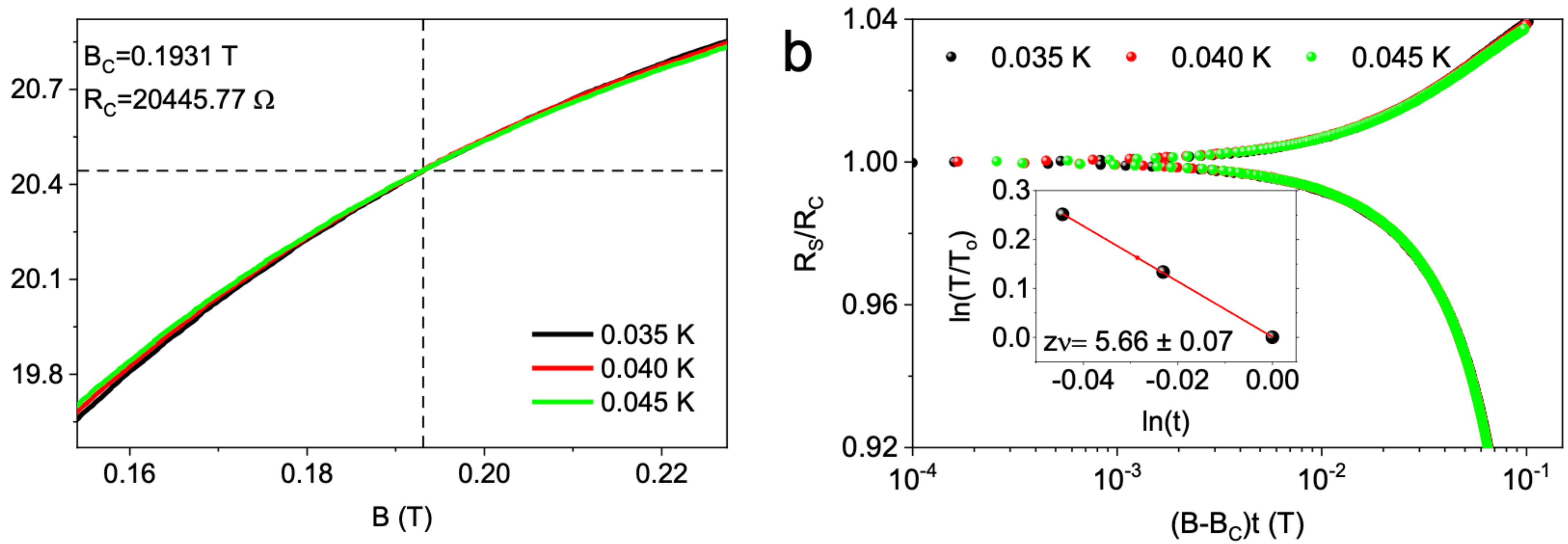
$\nu z \rightarrow$ Varies with B $\rightarrow$ Griffiths Behavior

Extracting an effective dynamical exponent

Use the power-law scaling ansatz

$$R_{\square} = R_c f \left[(B - B_c) \left(\frac{T_0}{T} \right)^{1/\nu z} \right] .$$

Extract and effective νz as a function of T and B



Strategy:

For each set of resistance isotherms find B_C and R_C

Perform a data collapse on these isotherms and obtain effective $(\nu z)_{\mathrm{eff}}(T)$

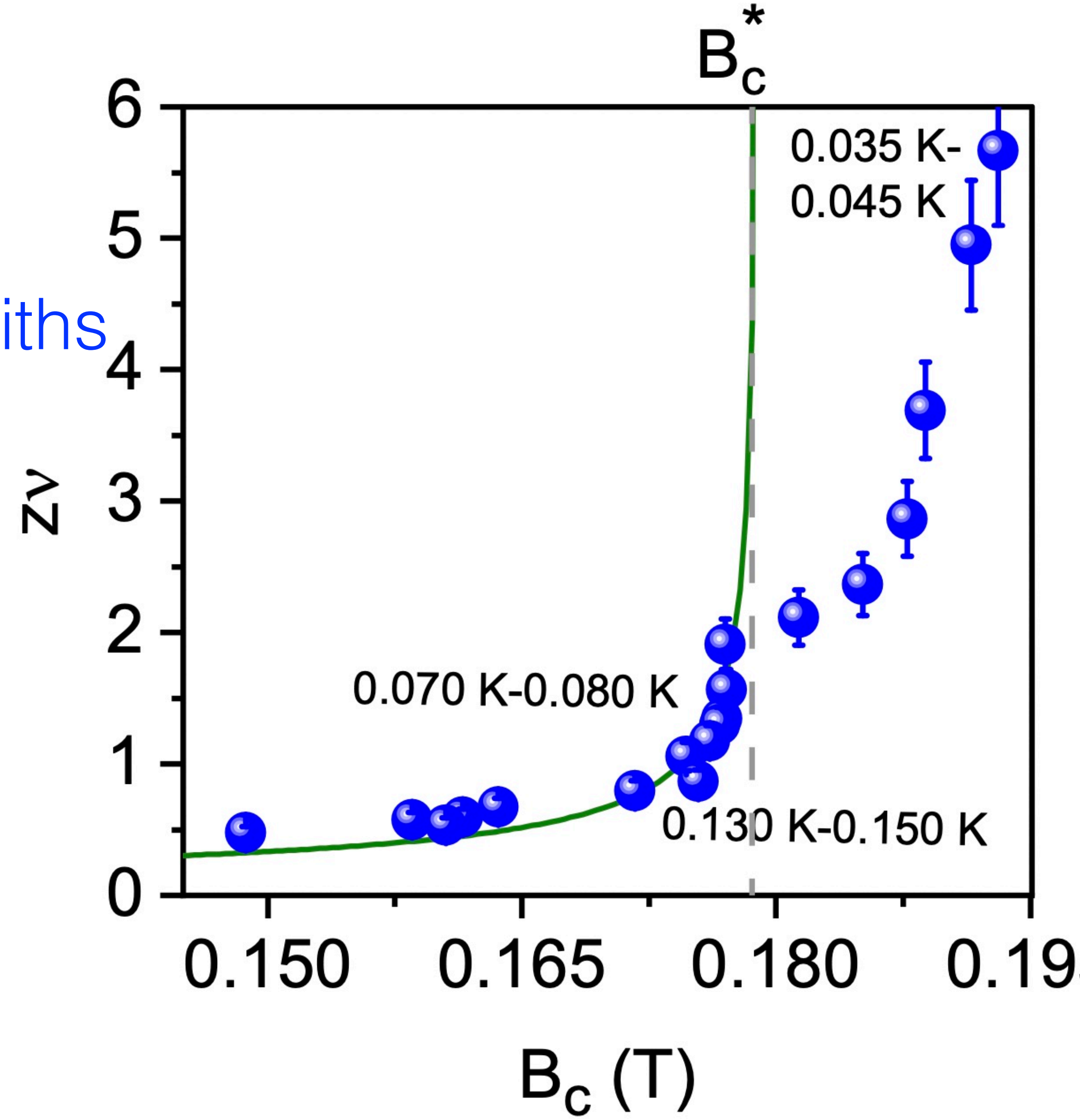
Smoking Gun: The first divergence

Extract $\nu z(B_c)$

Note the first divergence in line with

$z\nu \sim ({B_c}^* - B_c)^{-\nu\psi}$ IRFP and the Griffiths phase

However, the kink like feature at lower temperature is not consistent with



puts constraint on functional form of

$$\left(\frac{1}{\nu z}\right)_{\text{eff}} = \frac{1}{\nu\psi}\frac{1}{\ln(T_0/T)}$$

in line with IRFP predictions.

* However for T<0.07 K the data does not fit with the IRFP scenario

For $T > 0.07$ Similar results: Lewellyn (2019), Xing (2015)

Cloaking of Griffiths phase

- In the range: $0.07\ K < T < 0.17$

$$\left(\frac{1}{\nu z}\right)_{\rm eff} = \frac{1}{\nu\psi}\frac{1}{\ln(T_0/T)}$$

- In the range $T < 0.07K$

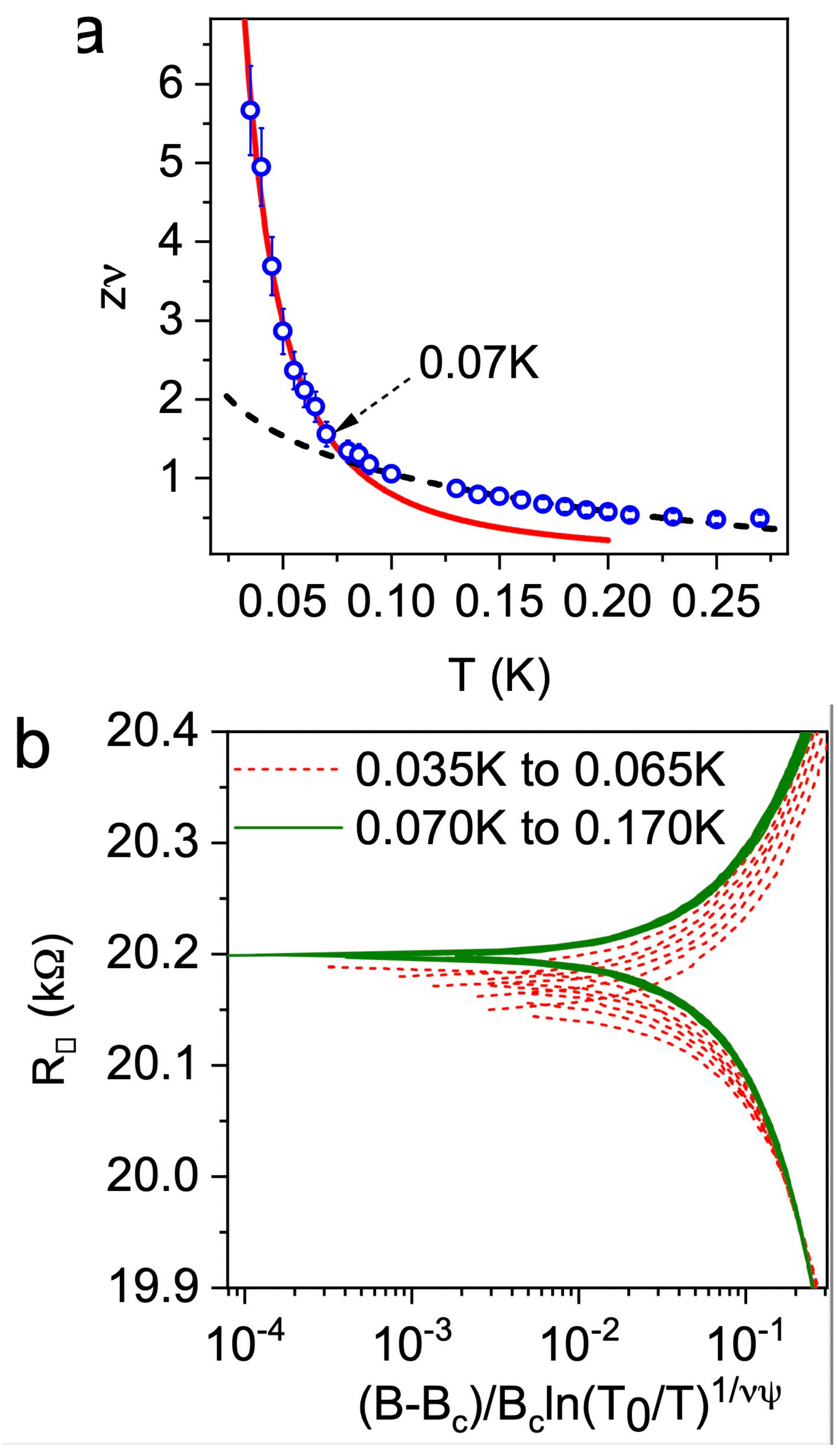
$z\nu(T) \sim T^{-1.9}$ $\Rightarrow$

Incompatible with the notion of Quantum Critical Point.

- Scaling collapse with activated scaling: Remember $\ln\xi_\tau \sim \xi^{1/2}$

$$R_\square = R_c f\left[\frac{(B-B_c)}{B_c}\ln(T_0/T)^{1/\nu\psi}\right]$$

Data collapse break down for $T < 0.07$ K

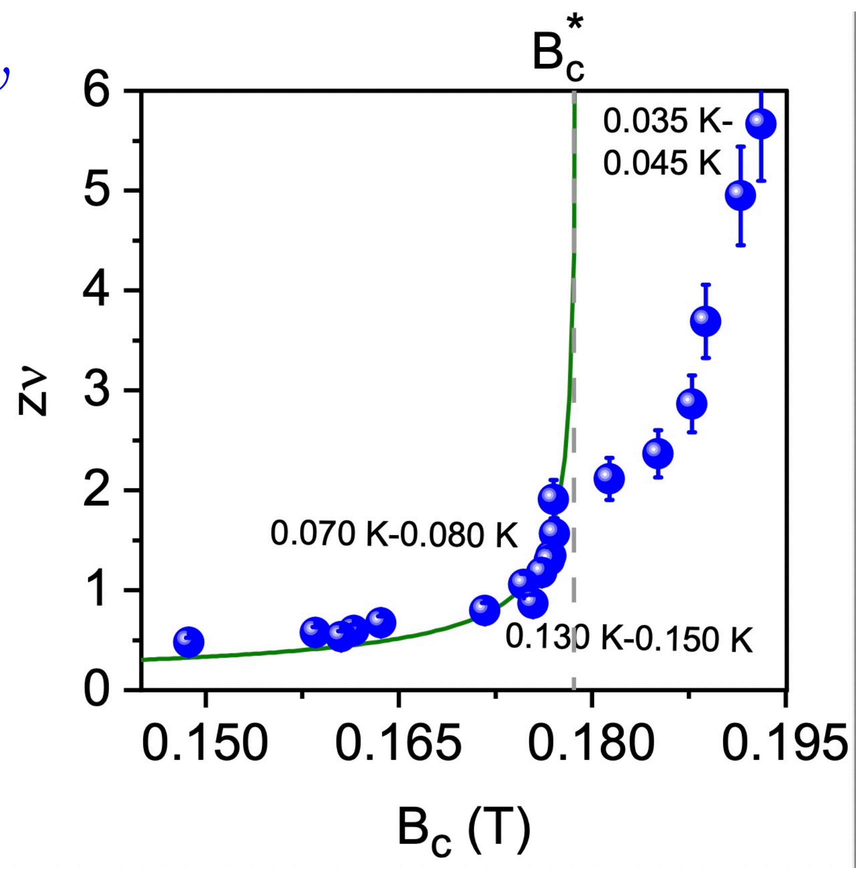


Summarizing

For T>0.07 K, the data is consistent with IRFP

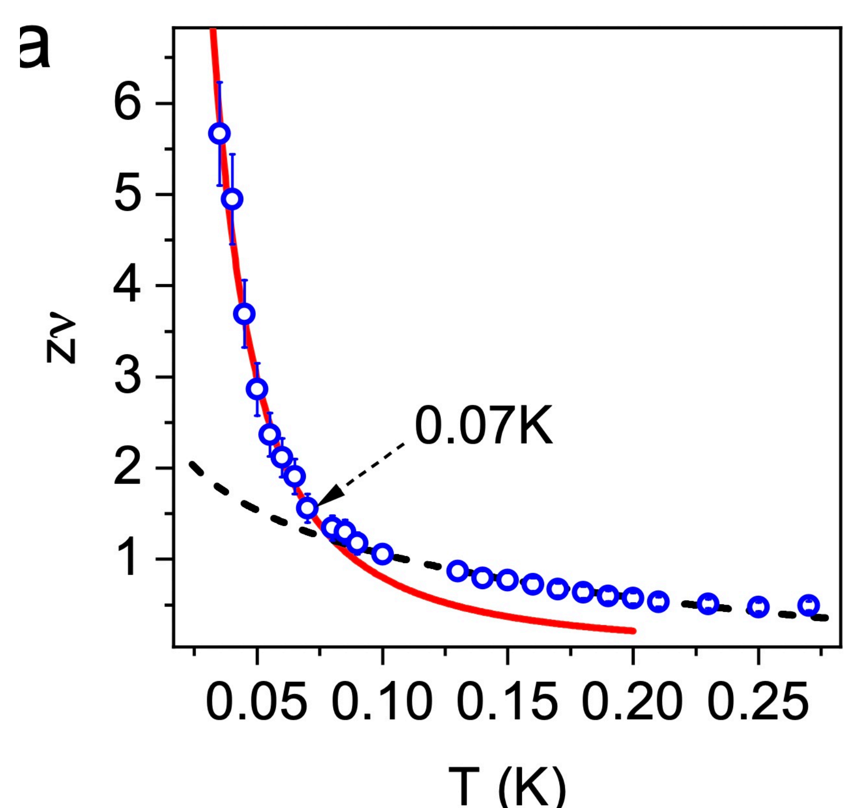
with attendant Griffiths phase: $z\nu \sim (B_c{}^* - B_c)^{-\nu\psi}$

and $$\left(\frac{1}{\nu z}\right)_{\text{eff}} = \frac{1}{\nu\psi}\frac{1}{\ln(T_0/T)}$$



For T<0.07 K, the data is inconsistent with IRFP $z\nu(T) \sim T^{-1.9}$

Conclusion: The Infinite Randomness fixed point is destabilised below $T < 0.07$

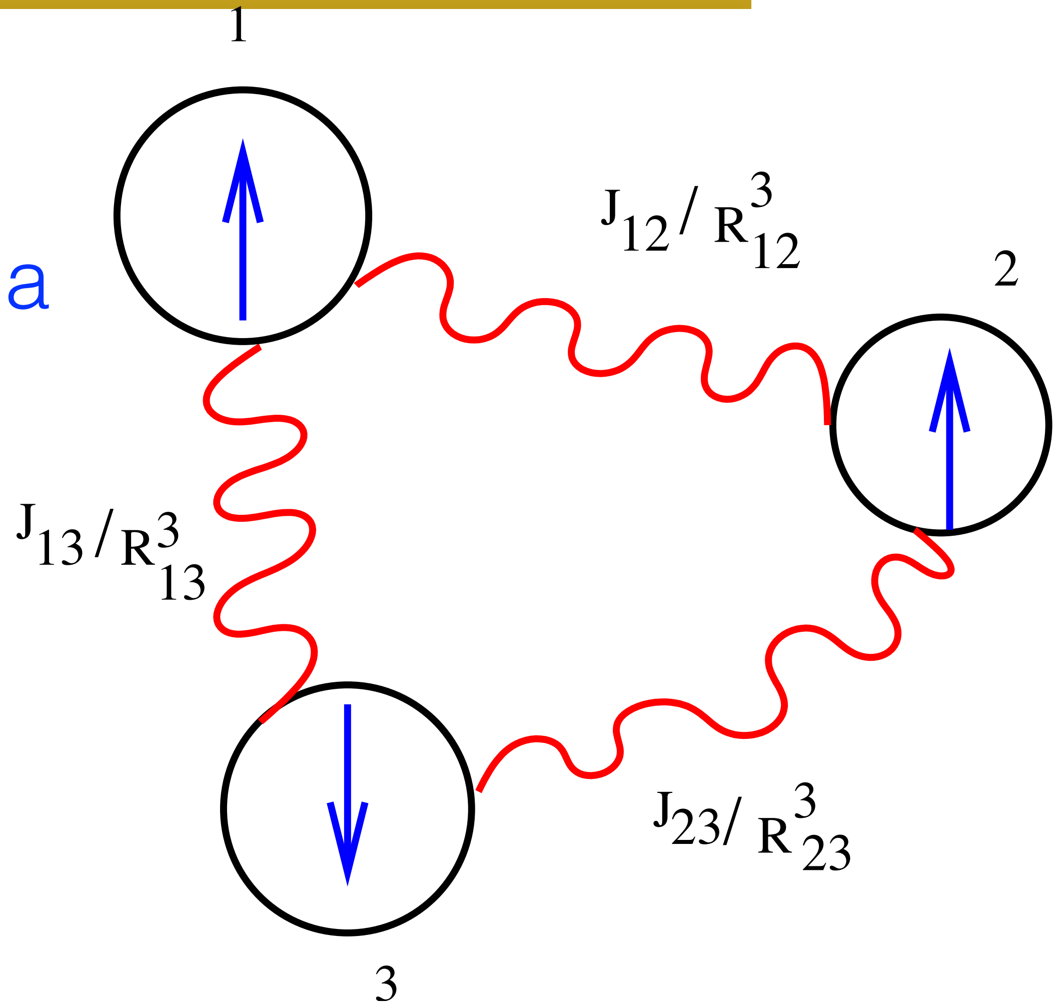


Can we come up with a physical mechanism that destabilises IRFP

Towards a plausible explanation

Dobrasaljveic and Miranda (2005)

Consider the rare-regions interacting via random long range interaction



$$S = S_{\rm RR} + S_{\rm RKKY}$$

$$S_{\rm RR} = \sum_{\omega_n} \phi_i(\omega_n)\left(\epsilon_i + |\omega_n|\right)\phi_i(-\omega_n) + O(\phi^4)$$

$$S_{\rm RKKY} = \frac{J_{ij}}{R^d_{ij}} \int d\tau \phi_i(\tau)\phi_j(\tau)$$

$J_{ij} \rightarrow$ random: Integrate out using Replicas

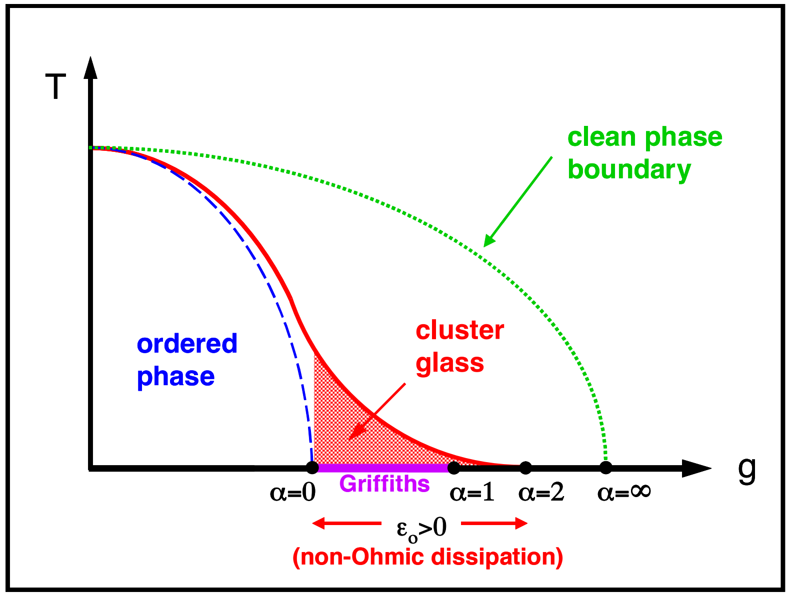
$$S_{\rm RKKY} \rightarrow \quad -\frac{J^2}{R^{2d}} \sum_{\alpha,\beta} \int d\tau d\tau' \phi_i^\alpha(\tau)\phi_j^\alpha(\tau)\phi_i^\beta(\tau')\phi_j^\beta(\tau')$$

$\Rightarrow$ HS decoupling+ saddle point approximation

$\Rightarrow Q^{\alpha\beta} \Rightarrow$ Evaluate in a self consistent manner

The General Case:

- $S_{\rm G} = \int d\omega \phi_i(\omega) \{\chi(\omega)\}_{\rm dis} \phi_i(\omega)$
- $\{\chi(\omega)\}_{\rm dis} \sim \int d\epsilon P(\epsilon) [\epsilon + |\omega|]^{-1}$
- Griffiths Phase $\Rightarrow P(\epsilon) \sim \epsilon^{\alpha-1}$
  Later !!
- Damping: $|\omega| \rightarrow |\omega|^{\alpha-1}$
  Reminder: $\alpha = d/z$

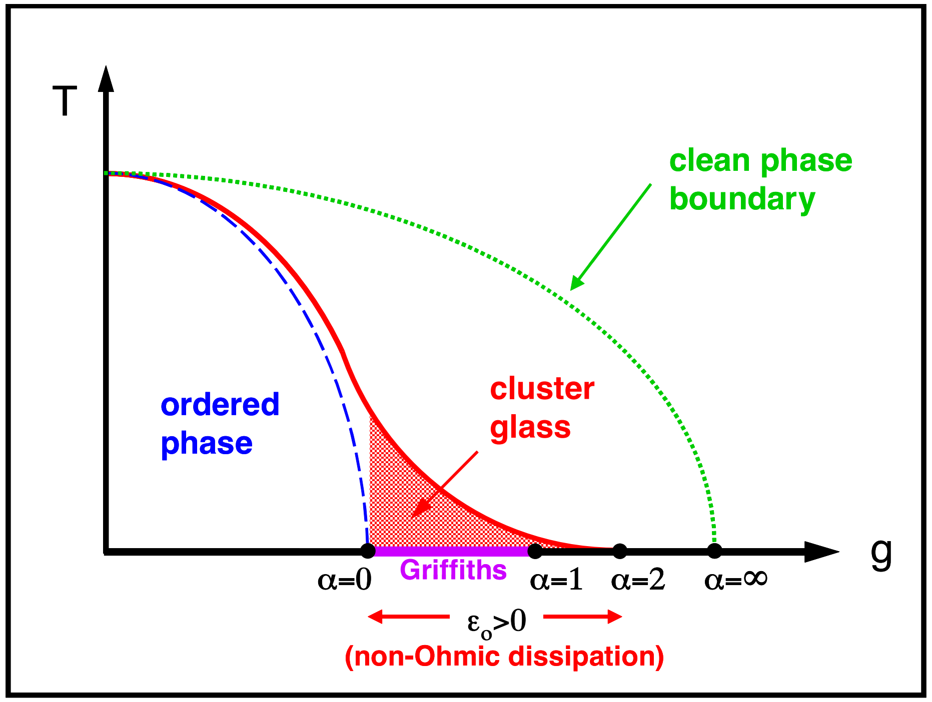


- The model maps onto an effective model 1-d model with slower than $1/r^2$ interaction
- $\Rightarrow$ System above the lower-critical dimension even in the Heisenberg case
- $\Rightarrow$Leads to the freezing of large droplets thus pre-empting the Griffiths phase.

- Ref: V. Dobrosavljevic and E. Miranda (2005).

Destabilizing the Griffiths Phase

Interaction between the droplets
provides another source of dissipation



$$S_{\rm G} = \int d\omega \phi_i(\omega)[\chi(\omega)]_{\rm dis}\phi_i(\omega)$$

$$[\chi(\omega)]_{\rm dis} = \int d\epsilon \frac{\rho(\epsilon)}{\epsilon + |\omega_n|}$$

In the Griffiths phase $\rho(\epsilon) = \epsilon^{d/z-1}$

From Dobrasaljveic and Miranda

Performing the integration: $|\omega| \to |\omega|^{d/z-1}$

Rare-regions map to 1-d model with slower than $1/|\tau|^2$

Interaction $\Longrightarrow$ Each rare region can independently order

$\Longrightarrow$ Cut-off of Griffiths Phase $\Longrightarrow$ Smearing of IRFP

Cloaking and smearing

Interplay of long-ranged interactions

and disorder ⇒

Phase locked rare-regions

Each Josephson Junction undergoes transition

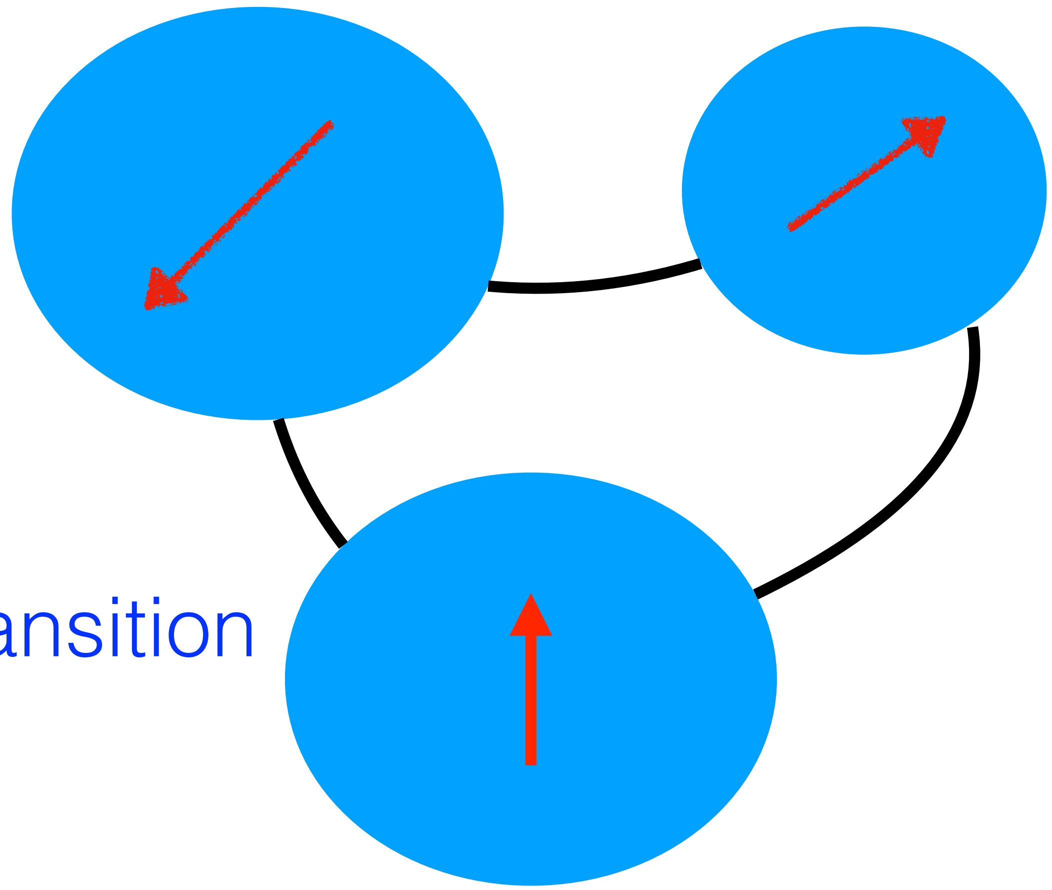
by itself ⇒ Smearing

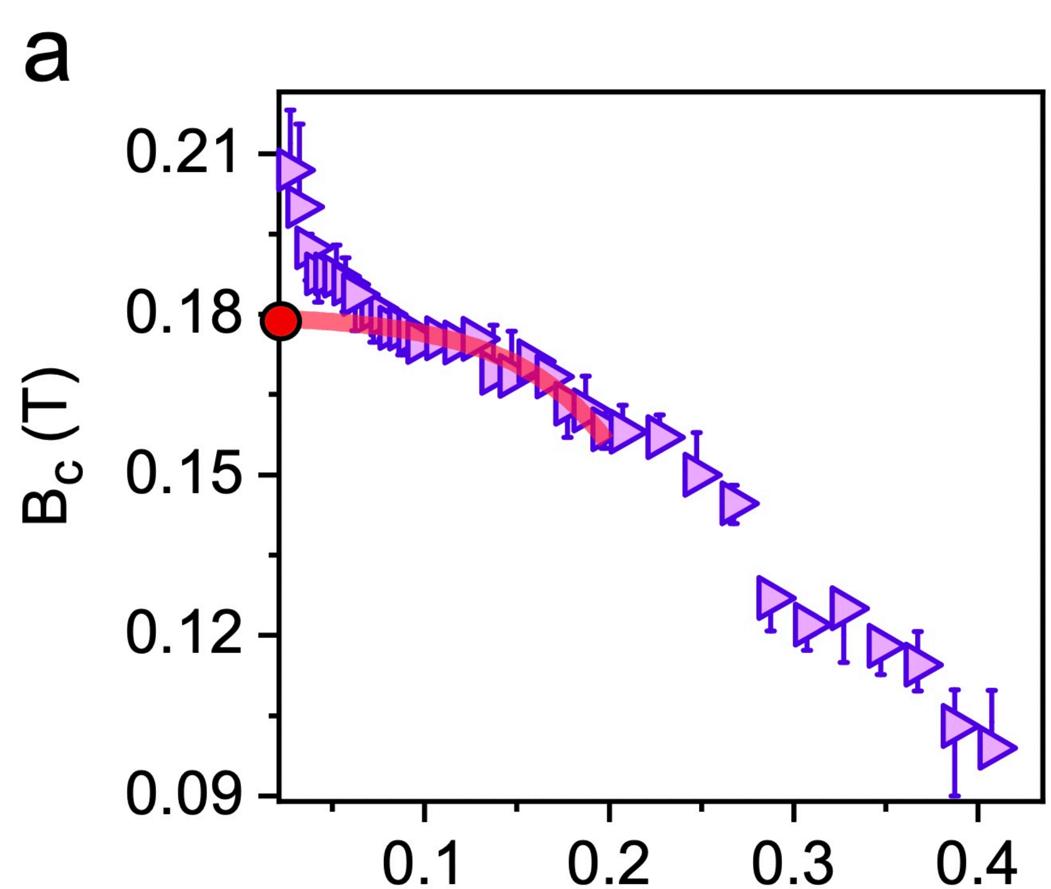
⇒ Tail to the phase diagram

Each rare-region orders by itself

Strange metal due to correlated

hopping of Cooper pairs





Open Questions and Conclusions

- Griffiths phase destroyed at low enough temperatures
- What is the nature of the ground-state
- How generic is this mechanism
- Is there an SDRG one can do to capture this phase

What we have not talked about:

- Spin-resolved disorder and MIT

S Kunwar, Madhuparna Karmakar, R. Narayanan (Unpublished).

- Higgs localization in disordered systems

Vishnu P. K., Martin Puschmann, R. Narayanan and T. Vojta

- Emergent U(1) phases in clock-model and disorder

Vishnu P. K., Gaurav Khairnar, T. Vojta and R. Narayanan

- Disorder Stabilized Breached Pair Phase

Madhuparna Karmakar, Subhojit Roy, Shantanu Mukherjee and R. Narayanan

- Disorder induced Bose-Fermi cross-overs

Madhuparna Karmakar,and R. Narayanan