Localization in homogeneous systems due to geometric frustration

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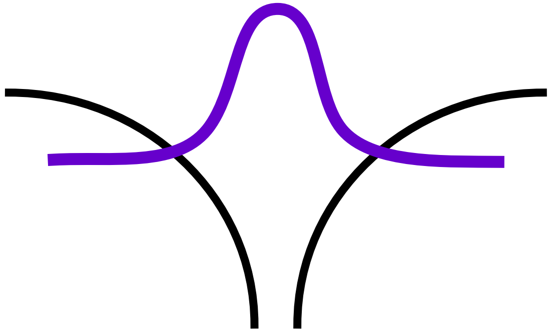
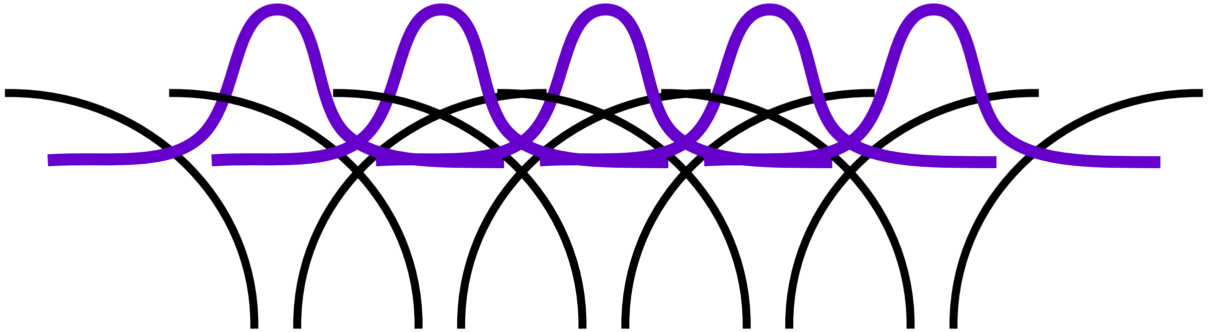
Localization – defeating kinetic energy

- Kinetic energy allows physical objects to move with various velocities
- Localization → defeating kinetic energy
- Strong trapping potential



- However, be aware of the uncertainty effect in quantum mechanics



→ metallic materials

Localizing quantum objects

1. Suppressing KE below the scale of other competing physics
   a. Thermal fluctuation
   b. Intra-atomic repulsion (realistic Mott insulators)
   c. Electron-lattice coupling (self-trapped polarons, other types of bosons too)
   d. Short-range binding (phase separation)

2. Damaging KE itself
   a. Disorder (Anderson insulators)
   b. Geometric frustration

Physical consequences
   a. Transport properties (electric conductivity, thermal conductivity, …)
   b. Information application? (entanglement entropy)
   c. Thermal properties (inability of thermalization)

Many-body effects on localization with damaged KE?

1. Disorder + interaction (many-body localization)

2. Geometric frustration + interaction ?

- Development of superfluidity and higher-order ordering

Yi-Zhuang You, Zhu Chen, Xiao-Qi Sun, and Hui Zhai. Phys. Rev. Lett. **109**, 265302 (2012).

- Quantum geometry

D Aleksi Julku, Georg M. Bruun, and Päivi Törmä. Phys. Rev. Lett. **127**, 170404 (2021).

- Generation of effective KE in both cases (and many others)

Q1: How robust is the geometric frustration-induced localization?

Q2: Can local density-density interaction defeat such localization?

Homogeneous quantum Bose metal

T. Hegg, J. Hou, and Wei Ku, PNAS **118**, 10.1073 (2021)

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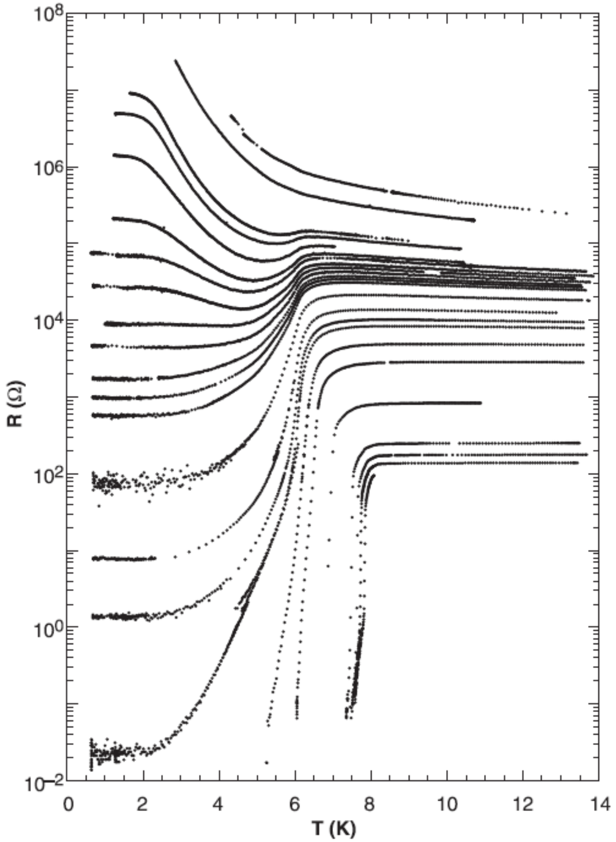
Quantum Bose metal? A long theoretical pursue

Ga films



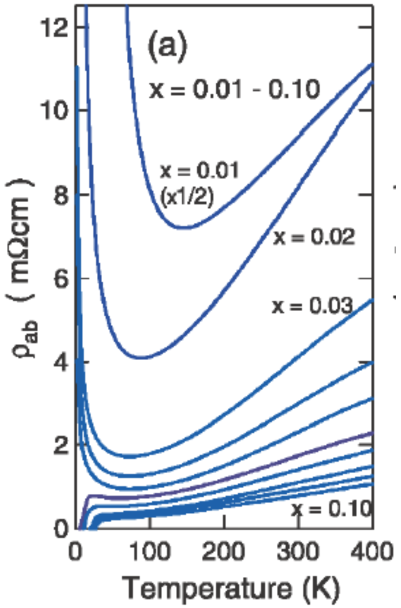
Philip Phillips et al., Science, vol. 302, 243-247 (2003)

Low T → finite resistivity

Sheet resistance→ $h/4e^2$→ charge 2e

$La_{2-x}Sr_xCuO_4$

Pseudogap range with low doping



PRL 93, 267001 (2004)

→ At low T, not consistent with $\sim e^{-\frac{\Delta}{T}}$

→ Not insulator Not superconductor

→ metallic

Quantum resistance → charge 2e boson

Nature 472, 458 (2011)

A stable Bose metal phase

Proposed theories

Superconducting grain models

Phys. Rev. B 64, 132502 (2001)

Ring exchange models

Phys. Rev. B 66, 054526 (2002)

“moatband models”

Phys Rev. Lett. 87, 037004 (2001)

…

…

Philip Phillips et al., Science, vol. 302, 243-247 (2003)

Inapplicable in a stable phase

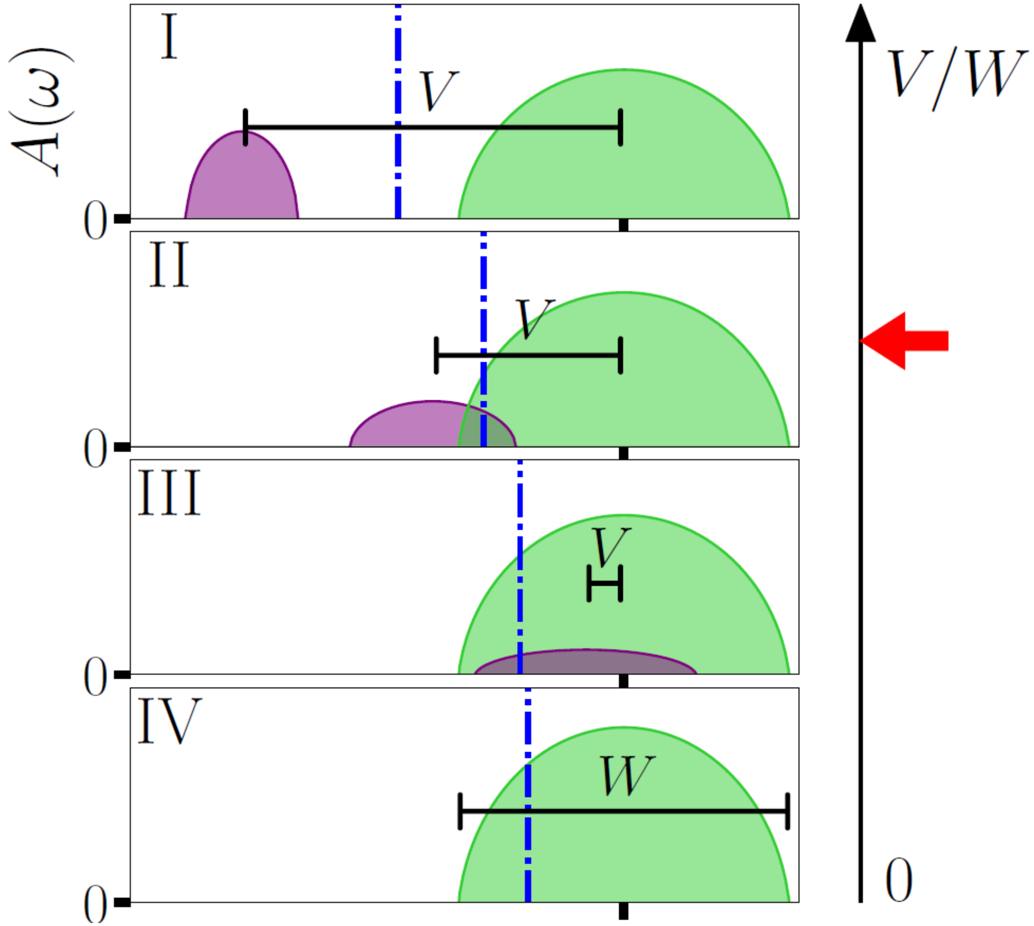
Emergent Bose liquid – schematic of reality

Under strong enough binding:

- Not too strong (phase separation or BEC limit)
- Not too weak (BCS SC)
- Carriers most likely as part of the pair
- Small chance to be "free"

→ Properties dominated by bosonic component

→ Fermionic one-body properties strongly modified by bosons, a sensitive probe to EBL



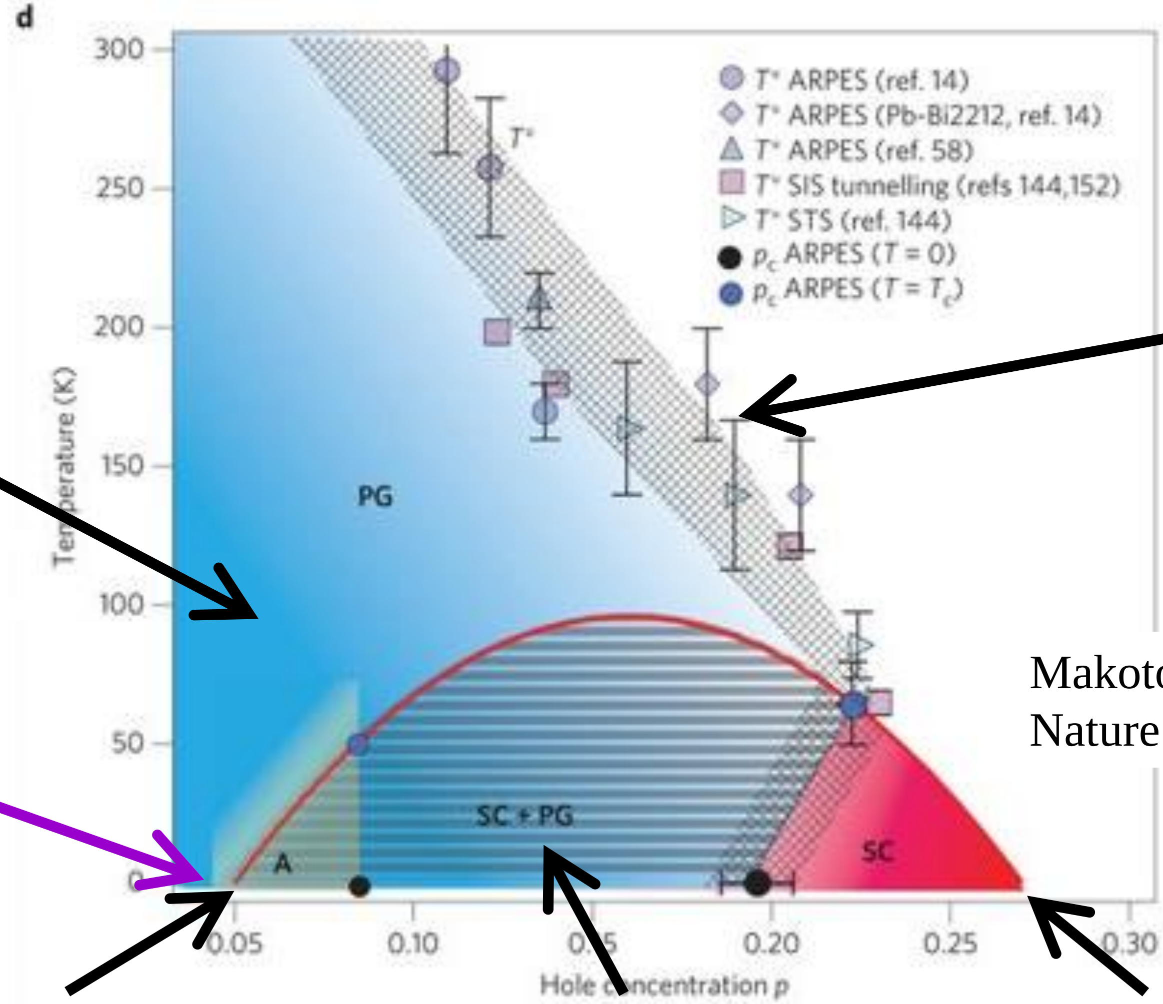
Y. Yildirim and Wei Ku, *Phys. Rev. X* **1**, 011011 (2011)

Unconventional superconductivity & its exotic normal state

non-FL
pseudogap

non-SF
quantum state
Bose metal

Fixed *H* & parameters
since 2011



bad metal
strange metal
weak insulating

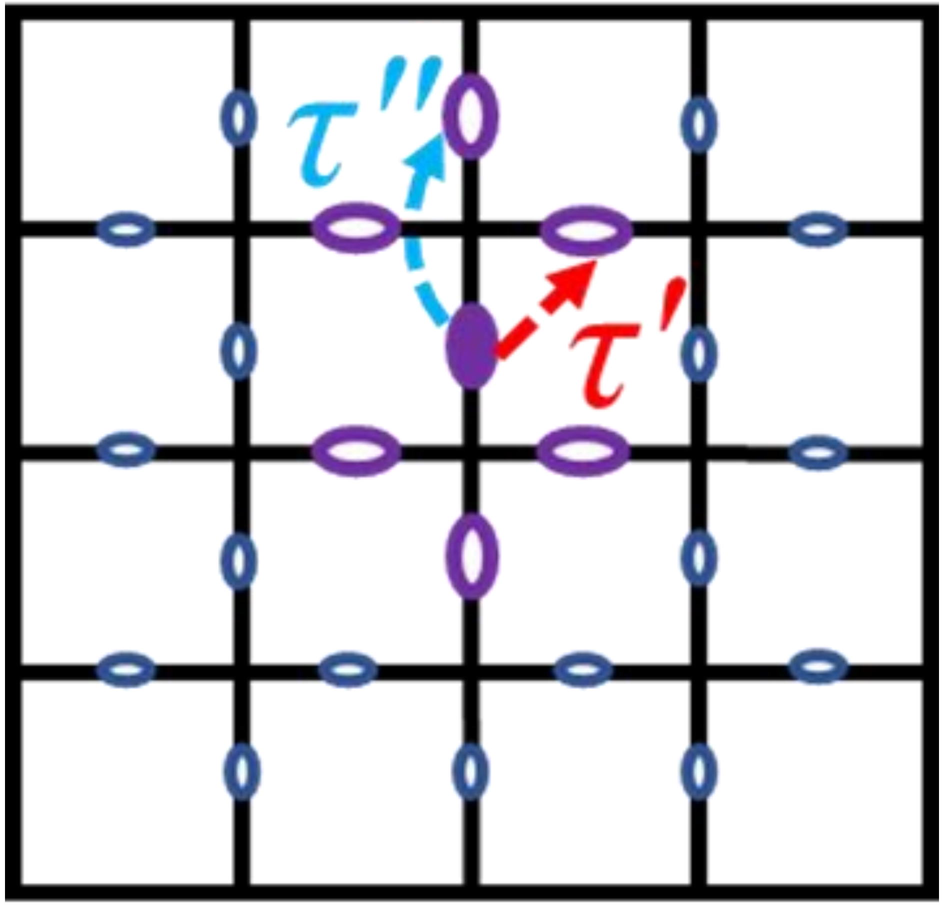
Makoto Hashimoto *et al*,
Nature Physics **10**, 483–495 (2014)

p-competition
m → ∞

d-wave SF &
2nd kind of SC gap

jamming
m → ∞

Geometric frustration of checkerboard lattice



$$H = \tau' \sum_{\langle i,i' \rangle} a_i^{+} a_{i'} + \tau'' \sum_{\langle\langle i,i' \rangle\rangle} a_i^{+} a_{i'}$$

- τ' and τ'' compete if both positive

Y. Yildirim and Wei Ku, *Phys. Rev. X* **1**, 011011 (2011)

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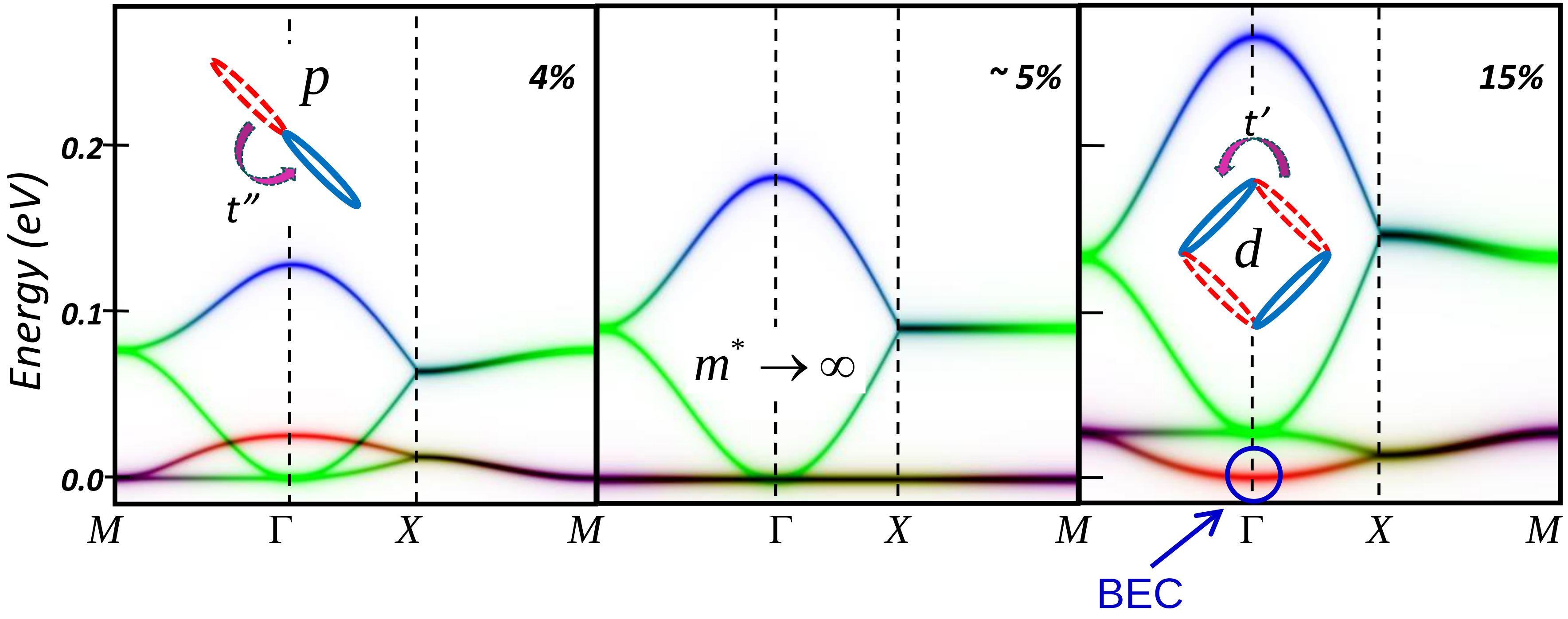
Non-SF Bosonic quantum state of the cuprates?
A Bose metal ?



- $\delta < 5\%$: infinite degeneracy → incoherent p-wave

Y. Yildirim and Wei Ku, PRB **92**, 180501(R) (2015)

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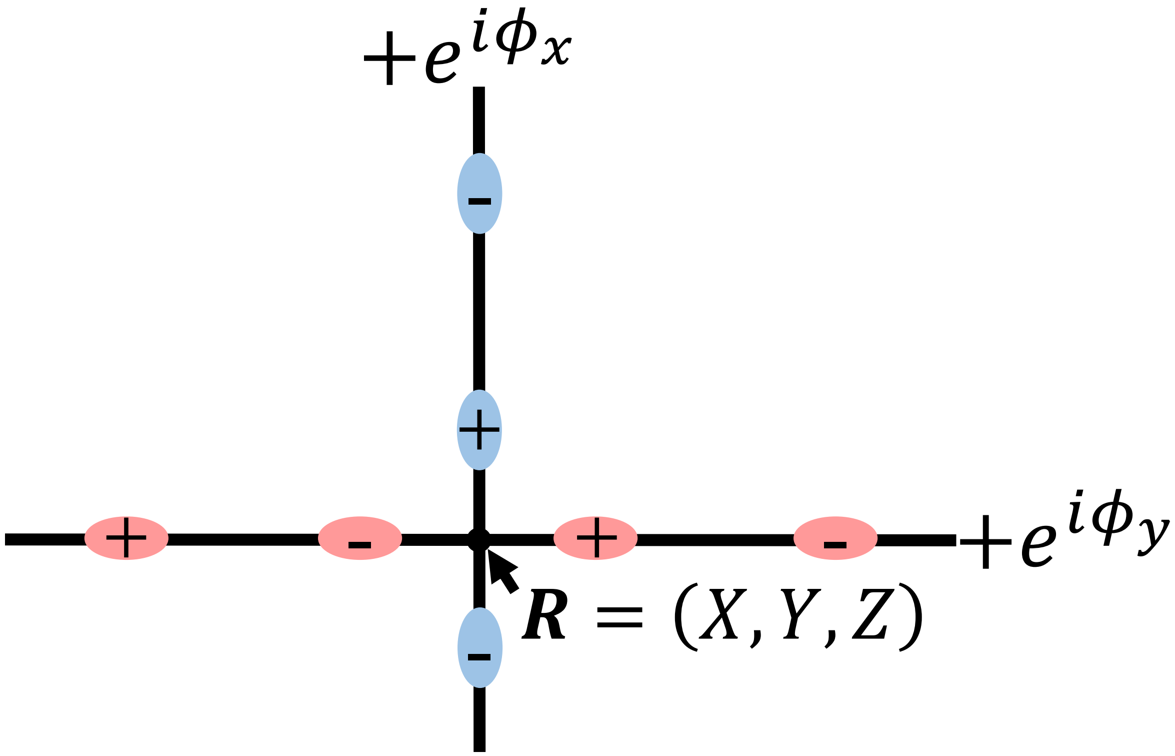
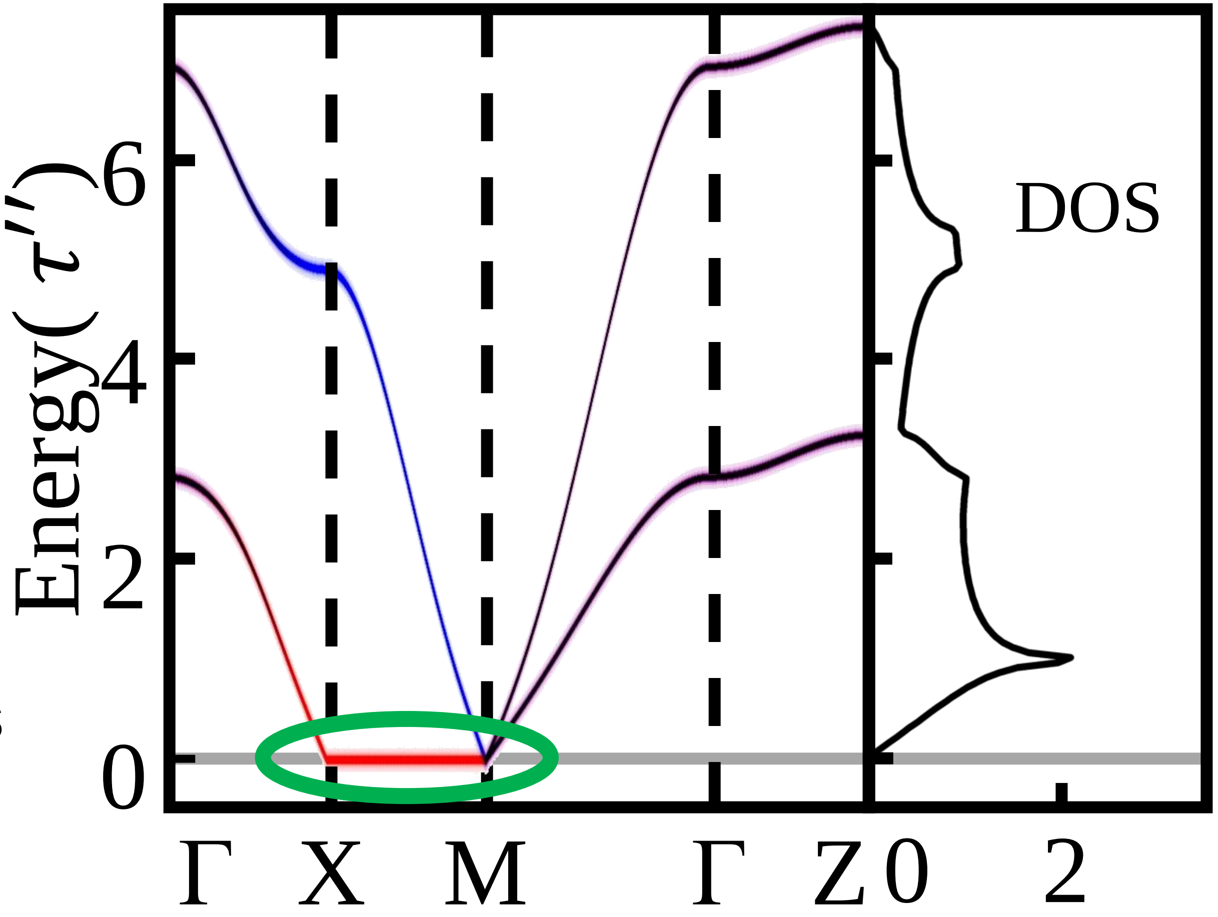
1D localization via geometric frustration



- Protected by local point-group symmetries
- # of symmetries ~ system size !
- Still respected even with local interaction

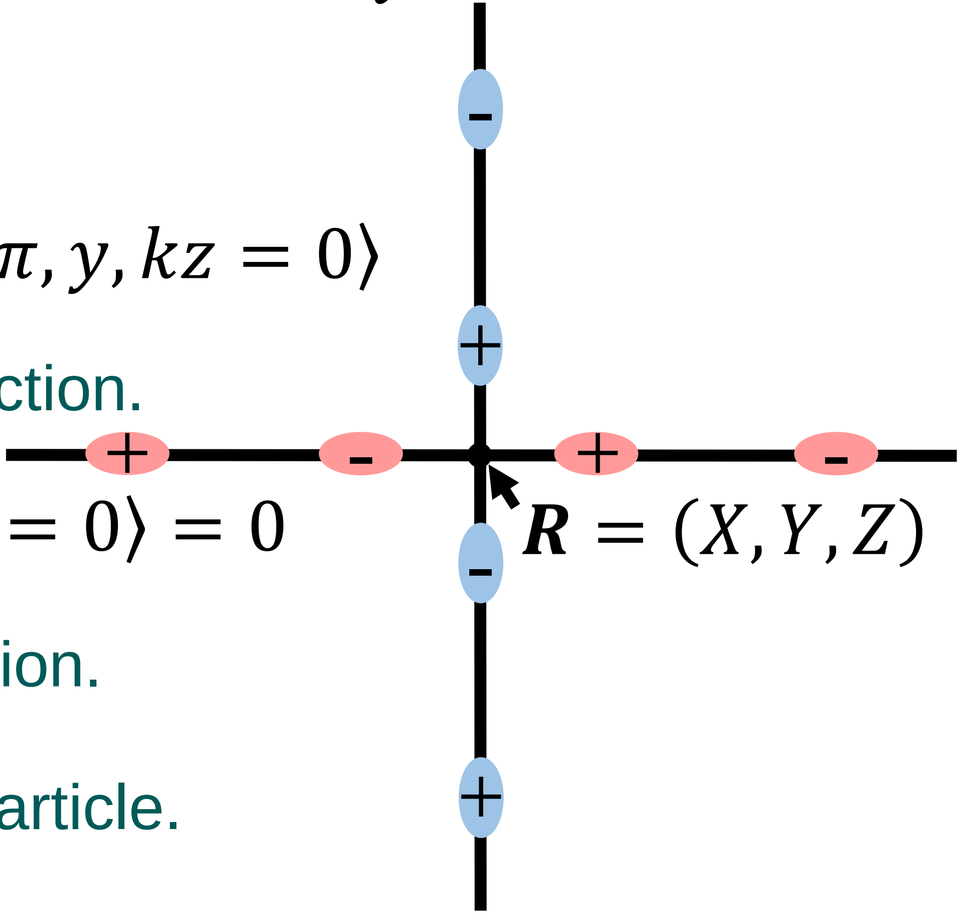


T. Hegg, J. Hou, and Wei Ku, PNAS **118**, 10.1073 (2021)

Localized one-body 2D state in a 3D system

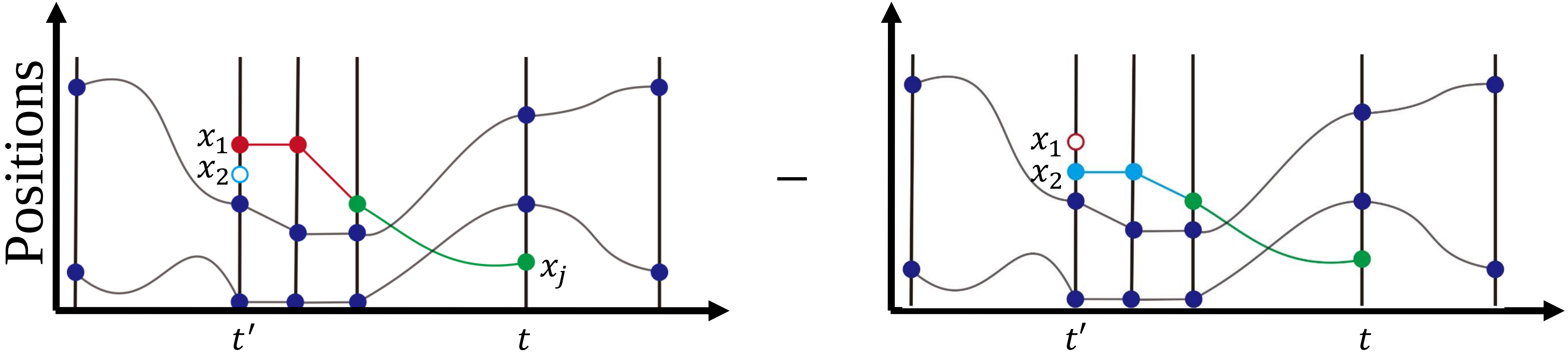
$$H = T + U = \tau' \sum_{\langle i,i' \rangle} a_i^+ a_{i'} + \tau'' \sum_{\langle\langle i,i' \rangle\rangle} a_i^+ a_{i'} + U \sum_i a_i^+ a_i^+ a_i a_i$$

- Only T moves particles.
  - But, $T\,|kx = \pi, y, kz = 0\rangle = -2\tau''\,|kx = \pi, y, kz = 0\rangle$
  - $|kx = \pi, y, kz = 0\rangle$ does not move in y direction.
- Similarly, the current operator $J_y\,|kx = \pi, y, kz = 0\rangle = 0$
  - $|kx = \pi, y, kz = 0\rangle$ does not flow in y direction.
- Interaction $U \sum_i a_i^+ a_i^+ a_i a_i$ does not move the particle.



Path integral demonstration of the Green's function

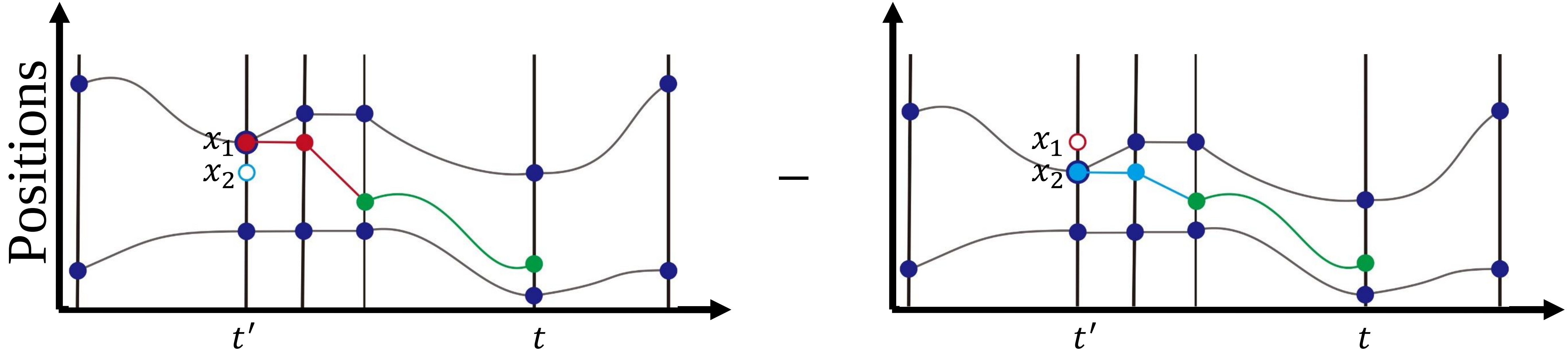
- $|m\rangle = |kx = \pi, y, kz = 0\rangle = (|1\rangle - |2\rangle + |3\rangle - |4\rangle + |5\rangle - |6\rangle + \cdots)/\sqrt{M}$
- $\sqrt{M}\, G(j, m; t, t') = G(j, 1; t, t') - G(j, 2; t, t') + G(j, 3; t, t') - \cdots$



- contributions always cancel ← geometric frustration

Path integral demonstration of the Green's function

- $|m\rangle = (|1\rangle - |2\rangle + |3\rangle - |4\rangle + |5\rangle - |6\rangle)/\sqrt{M}$
- $\sqrt{M}\, G(j,m;t,t') = G(j,1;t,t') - G(j,2;t,t') + G(j,3;t,t') - \cdots$



- contributions always cancel

  (as long as T is not seriously challenged by Pauli exclusion or U)
- $G(j,m;t,t') = 0$ for all j outside the plane.

1D localized many-body eigenstates

In diagonal representation,

$$H = \tau' \sum_{\langle i,i'\rangle} a_i^+ a_{i'} + \tau'' \sum_{\langle\langle i,i'\rangle\rangle} a_i^+ a_{i'} + U \sum_i a_i^+ a_i^+ a_i a_i$$ is simply

$$= \sum_i \epsilon_i b_i^+ b_i + \sum_{ij} \epsilon_{ij} b_i^+ b_j^+ b_j b_i + \sum_{ijk} \epsilon_{ijk} b_i^+ b_j^+ b_k^+ b_k b_j b_i + \cdots$$ (N-body term)

with eigenstates $|n_1, n_2, \dots\rangle = \Pi_i (b_i^+)^{n_i}|0\rangle$ $\quad$ ($\sum_i n_i = N$), where

$$b_i^+ = U^+ a_i^+ U$$

$$= \sum_j v_i^j a_j^+ + \sum_{jkk'} v_i^{jkk'} a_j^+ a_k^+ a_{k'} + \sum_{jklk'l'} v_i^{jklk'l'} a_j^+ a_k^+ a_l^+ a_{l'} a_{k'} + \cdots$$

gives the many-body wave function of each particle.

(U^+ is found by demanding the diagonal representation.)

1D localized many-body eigenstates

Since U^+ is found by demanding the diagonal representation, it must respect all symmetries of H: $[U, S] = 0$ for all S that satisfies $[H, S] = 0$.

The odd parity of a_m^+, $P_x a_m^+ P_x = -a_m^+$ would gives

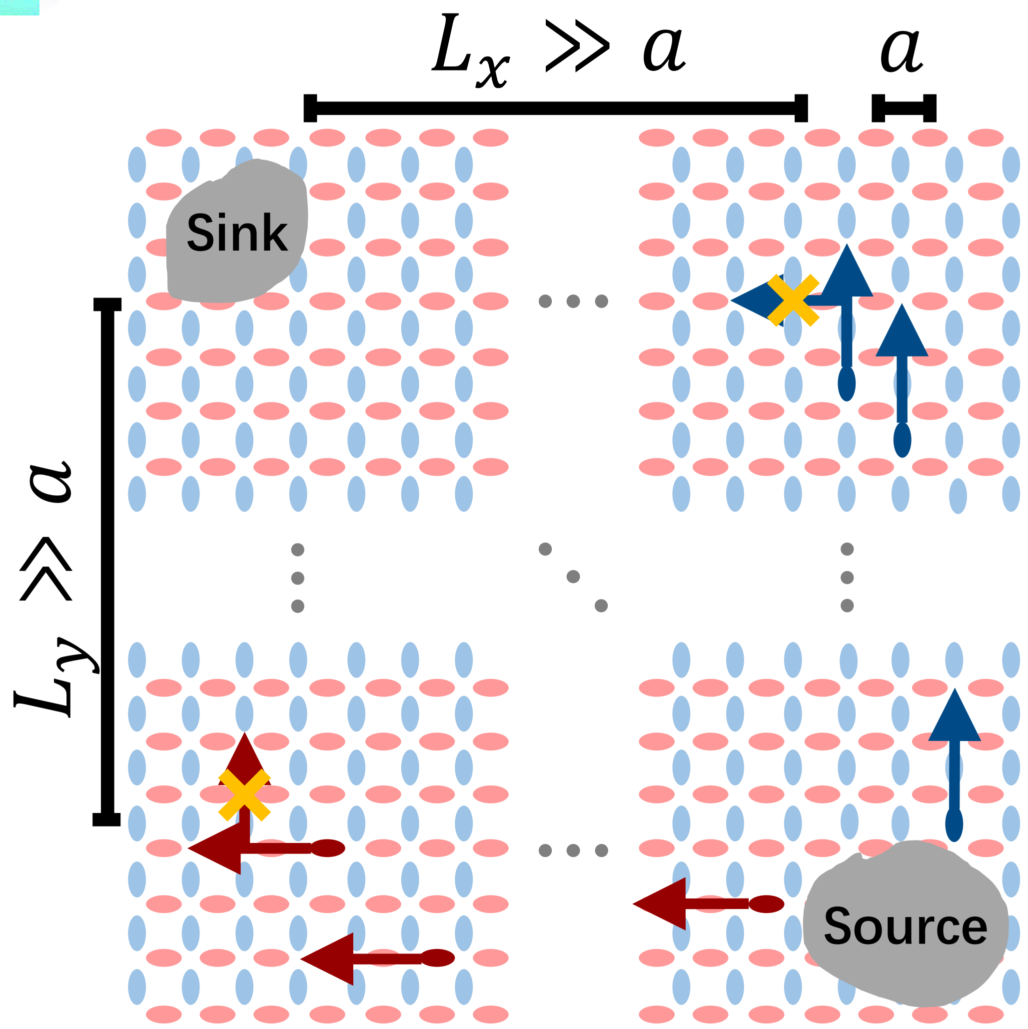
$b_m^+ = U^+ a_m^+ U$ odd parity as well, while a_j^+ is obviously even.

So, $\langle a_j b_m^+ \rangle = -\langle a_j b_m^+ \rangle = 0$

b_m^+ cannot move out even one step due to local point symmetry.

All other destination went through the first j and suffer from the same interference.

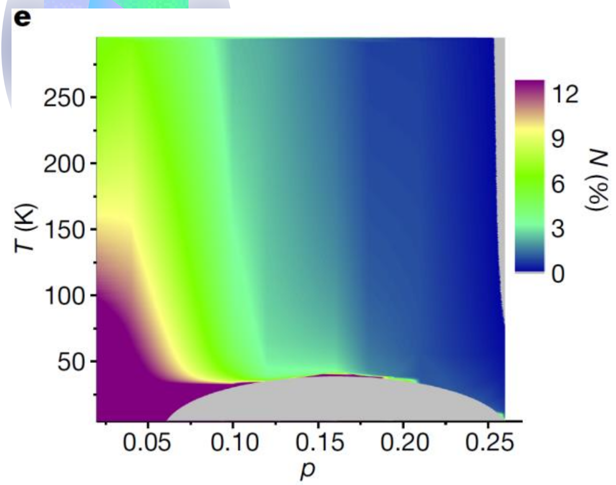
Lack of in-plane superfluidity → quantum Bose metal !

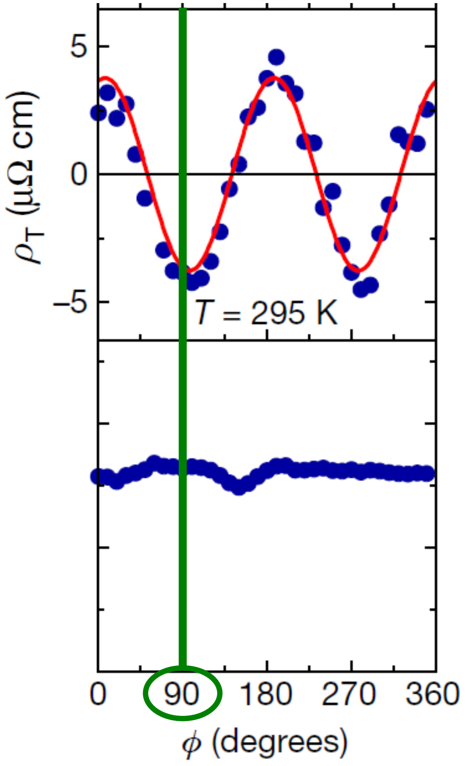


First theoretical uniform Bose metal

T. Hegg, J. Hou, and Wei Ku, PNAS **118**, 10.1073 (2021)

Exp. verification





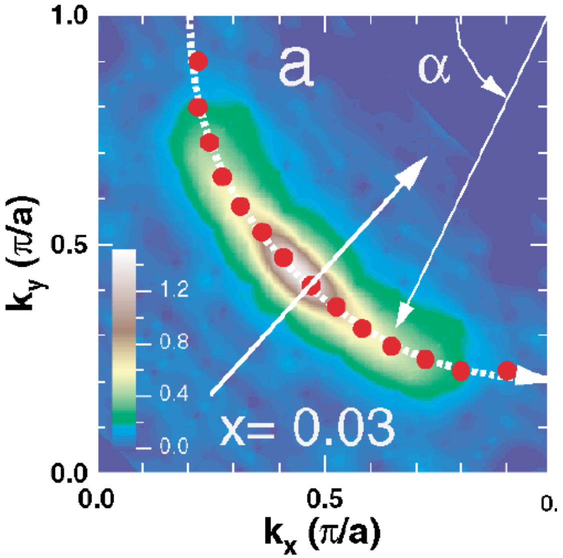
J. Wu et al., *Nature*, **547**, 432-435 (2017)

Bose metal has 4-fold anisotropy

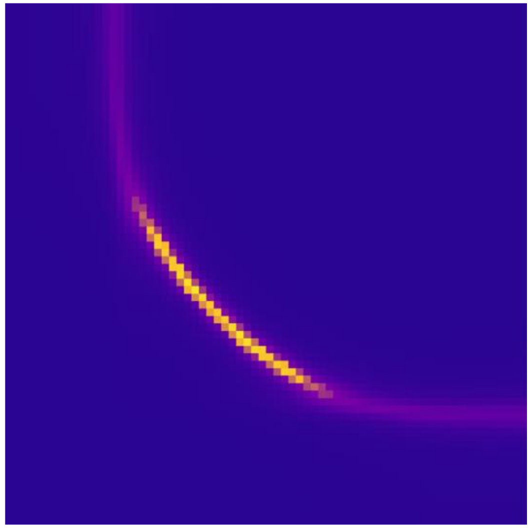
Lower along a slab, as in observations

Fermi arcs suggest the opposite?!

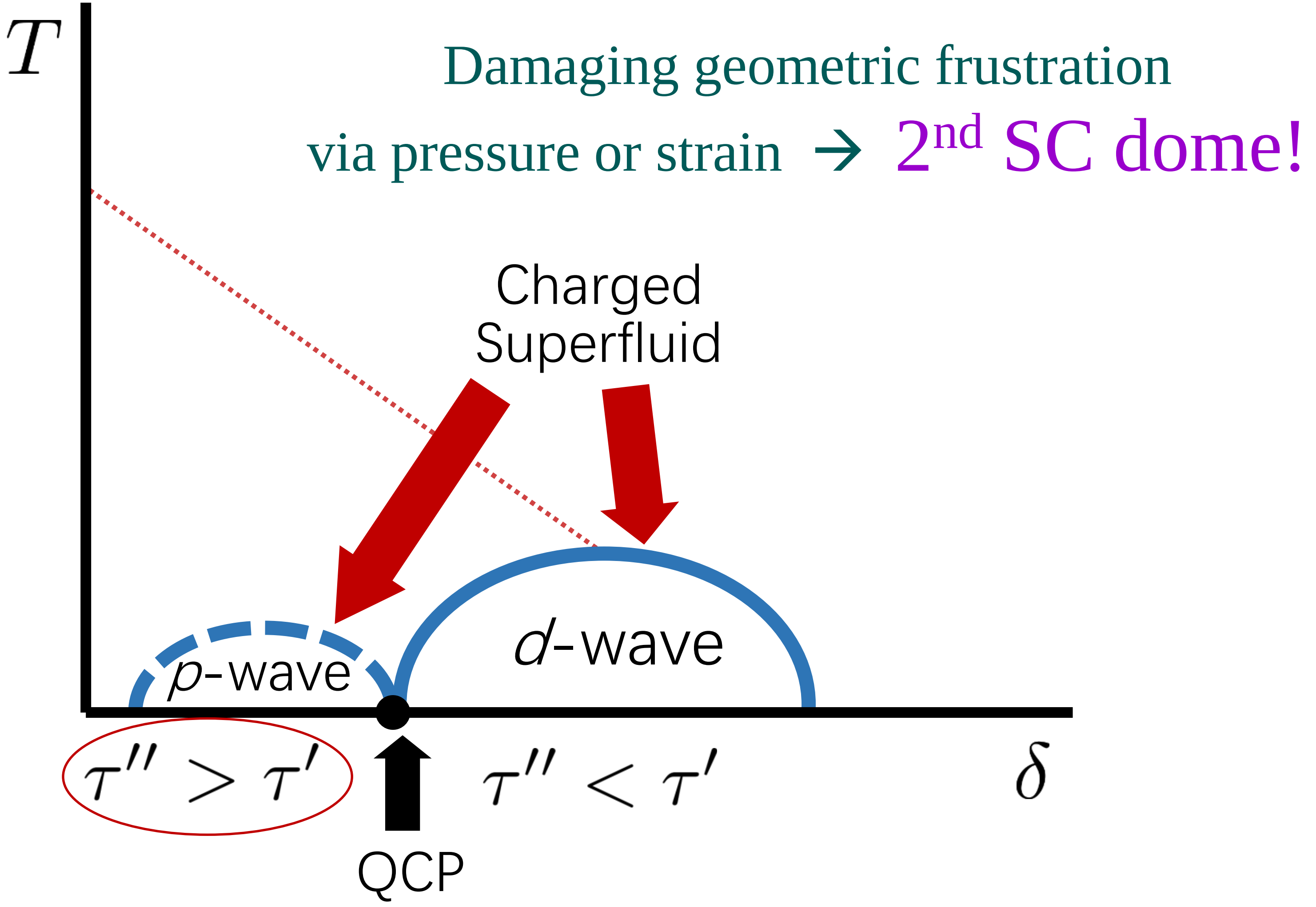
Fermionic scattering off of Bose metal accounts for both!!



T. Yoshida et al., *PRL*, **91**, 027001 (2003)



Prediction: a 2nd SC dome upon obstruction of frustration



T. Hegg, J. Hou, and Wei Ku, PNAS **118**, 10.1073 (2021)

Probing quantum Bose metal with electrons: inescapable non-Fermi liquid scattering rate and pseudogap formation

X Yue, A Hegg, X Li, and Wei Ku,
New J. Phys. 25, 053007 (2023)

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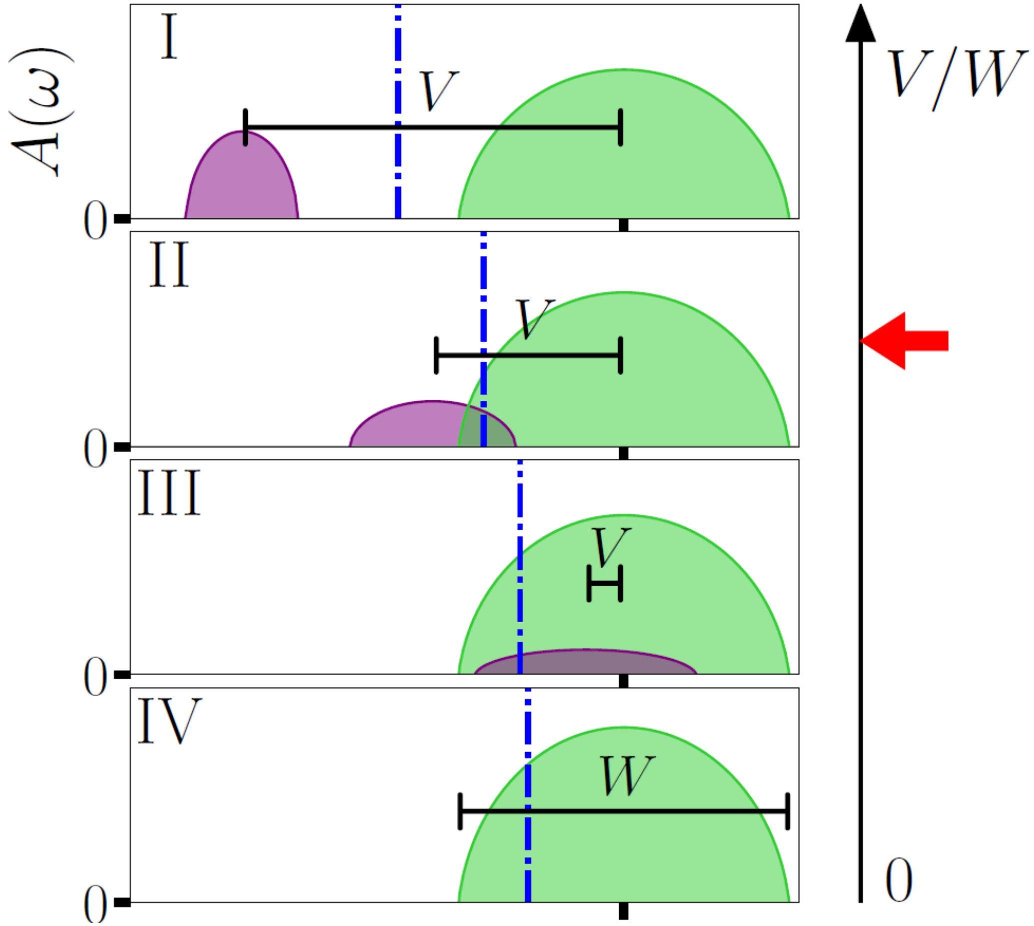
Probing the “probe” in emergent Bose liquid

Under strong enough binding:

- Not too strong (phase separation or BEC limit)
- Not too weak (BCS SC)
- Carriers most likely as part of the pair
- Small chance to be “free”

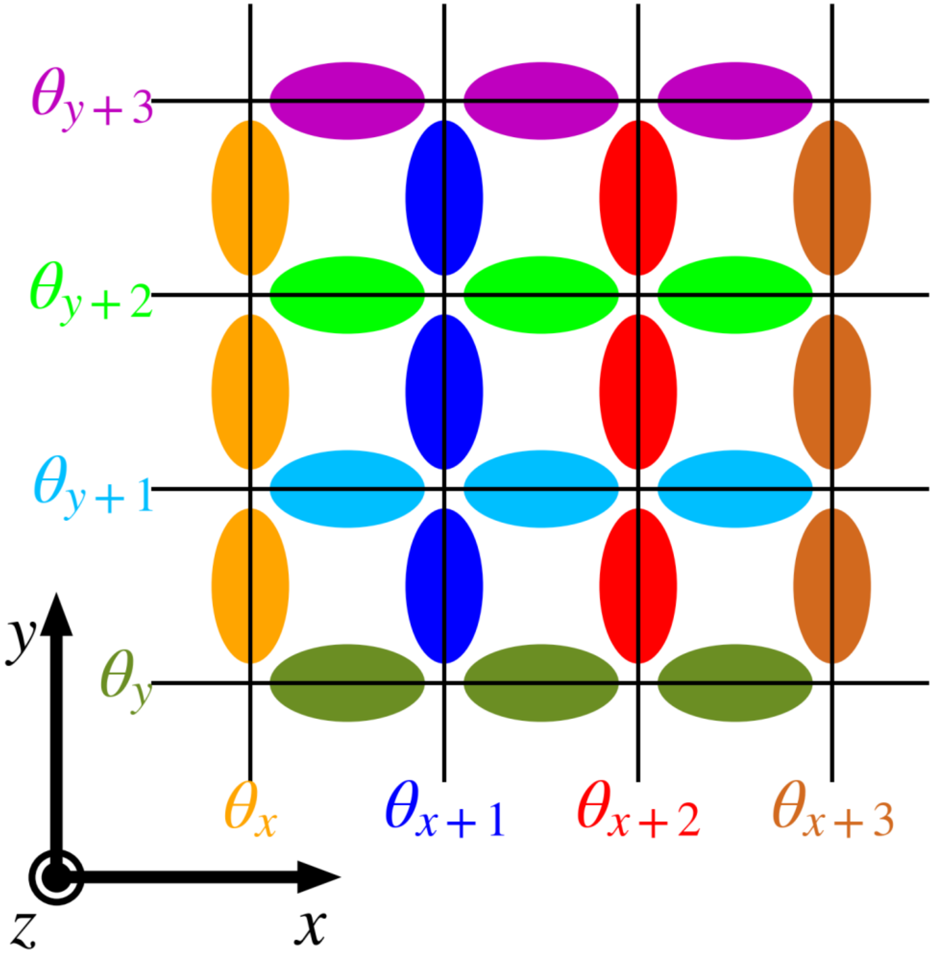
  → Properties dominated by bosonic component

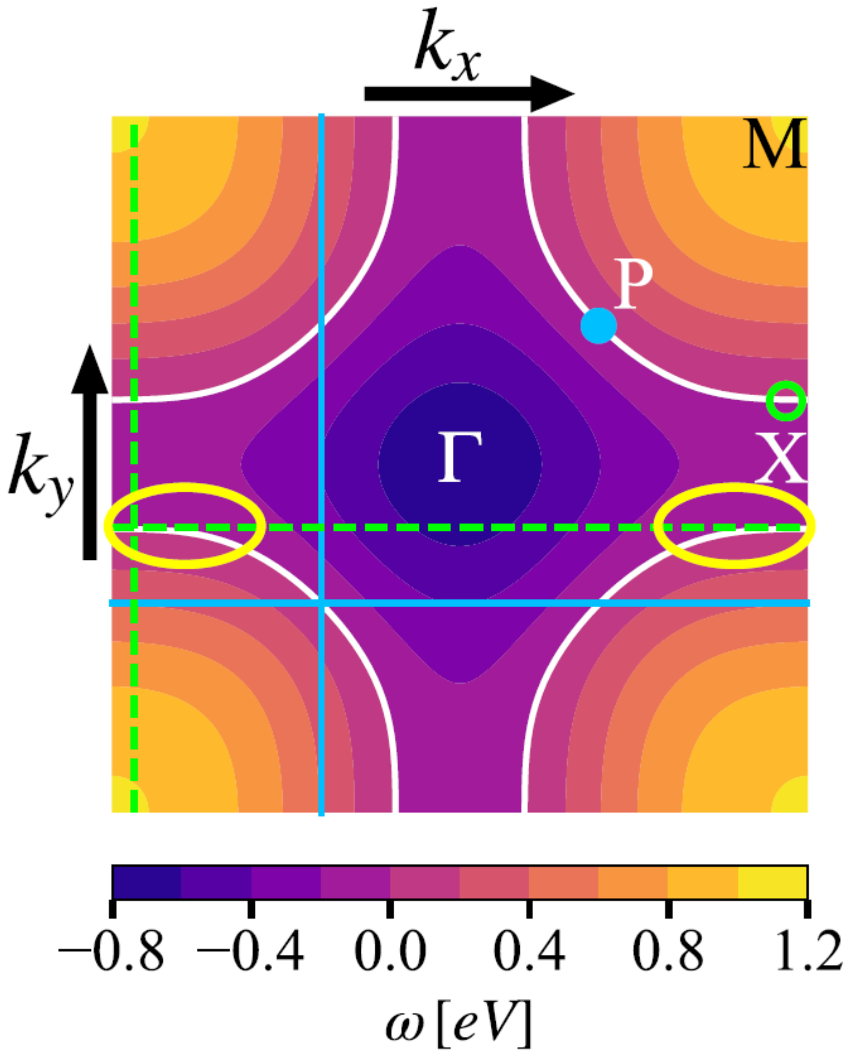
  → Fermionic one-body properties strongly modified by bosons, a sensitive probe to EBL



Y. Yildirim and Wei Ku, *Phys. Rev. X* **1**, 011011 (2011)

An unusual bosonic field with incomplete coherence

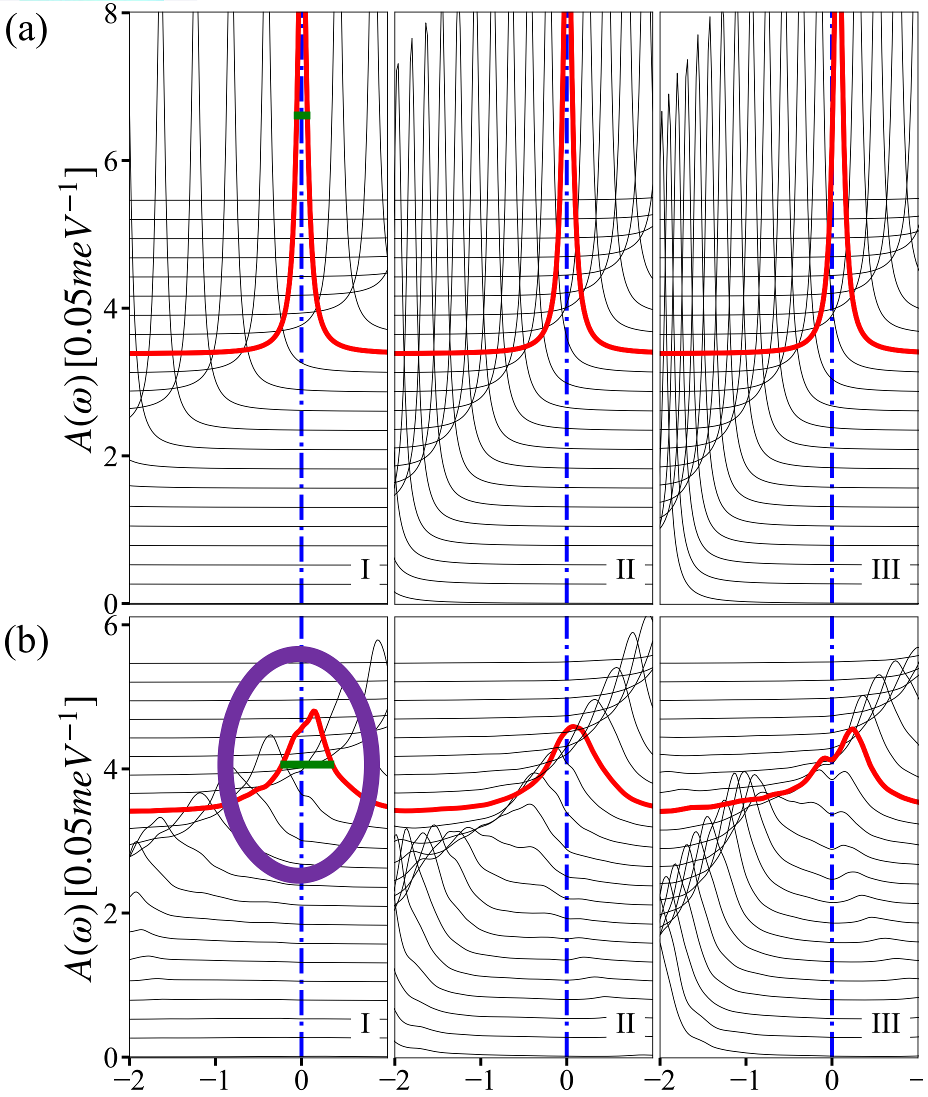




- Partially incoherent scatting ! ← key to the pseudogap & non-FL

X Yue, A Hegg, X Li, Wei Ku, New J. Phys. **25**, 053007 (2023)

Non-Fermi liquid scattering rate



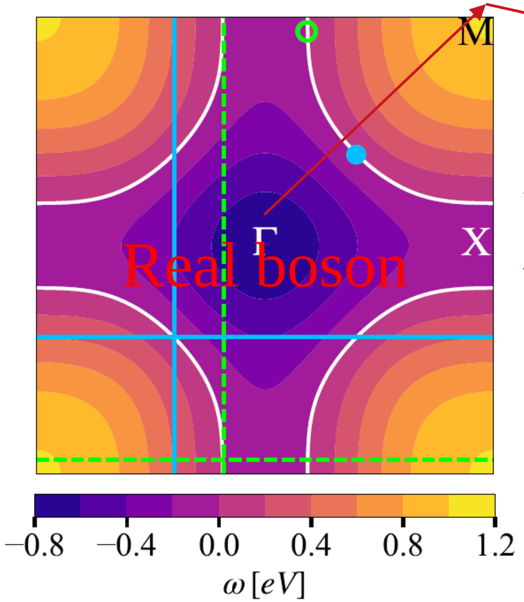
Zero frequency + zero temperature
+ No impurity

↓

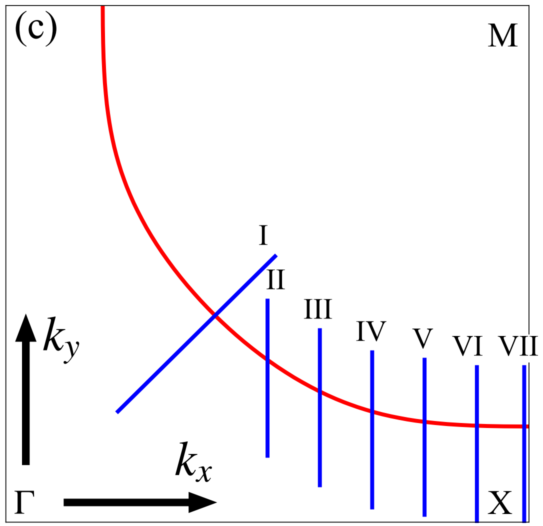
Non-zero scattering

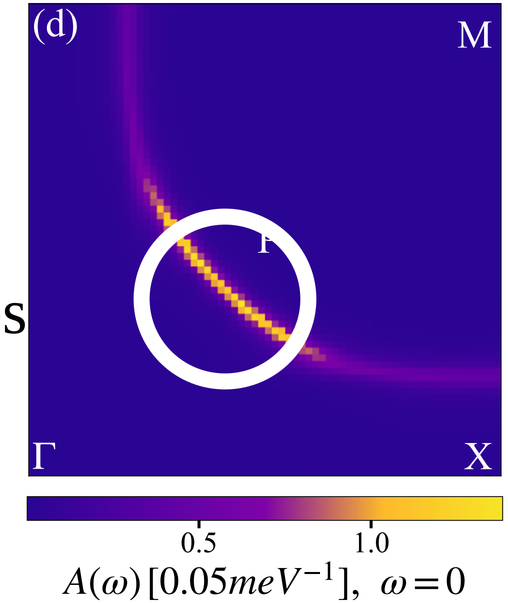
↓

Non-Fermi liquid scattering at
zeroth order!



Linear dependence
to the next order
without singularities





X Yue, A Hegg, X Li, Wei Ku, New J. Phys. **25**, 053007 (2023)

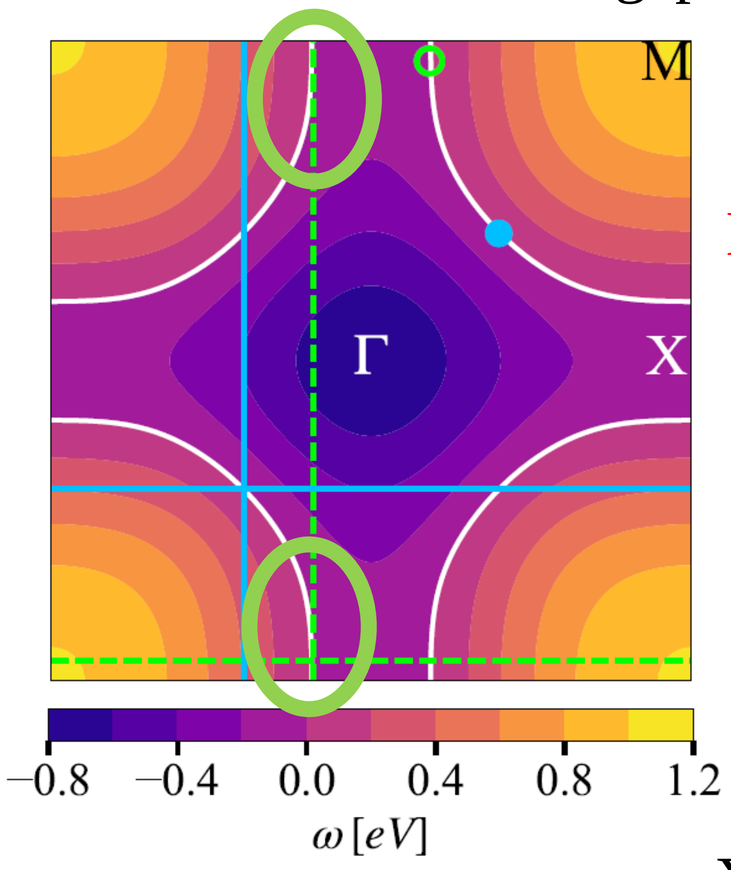
Pseudogap formation

(a)

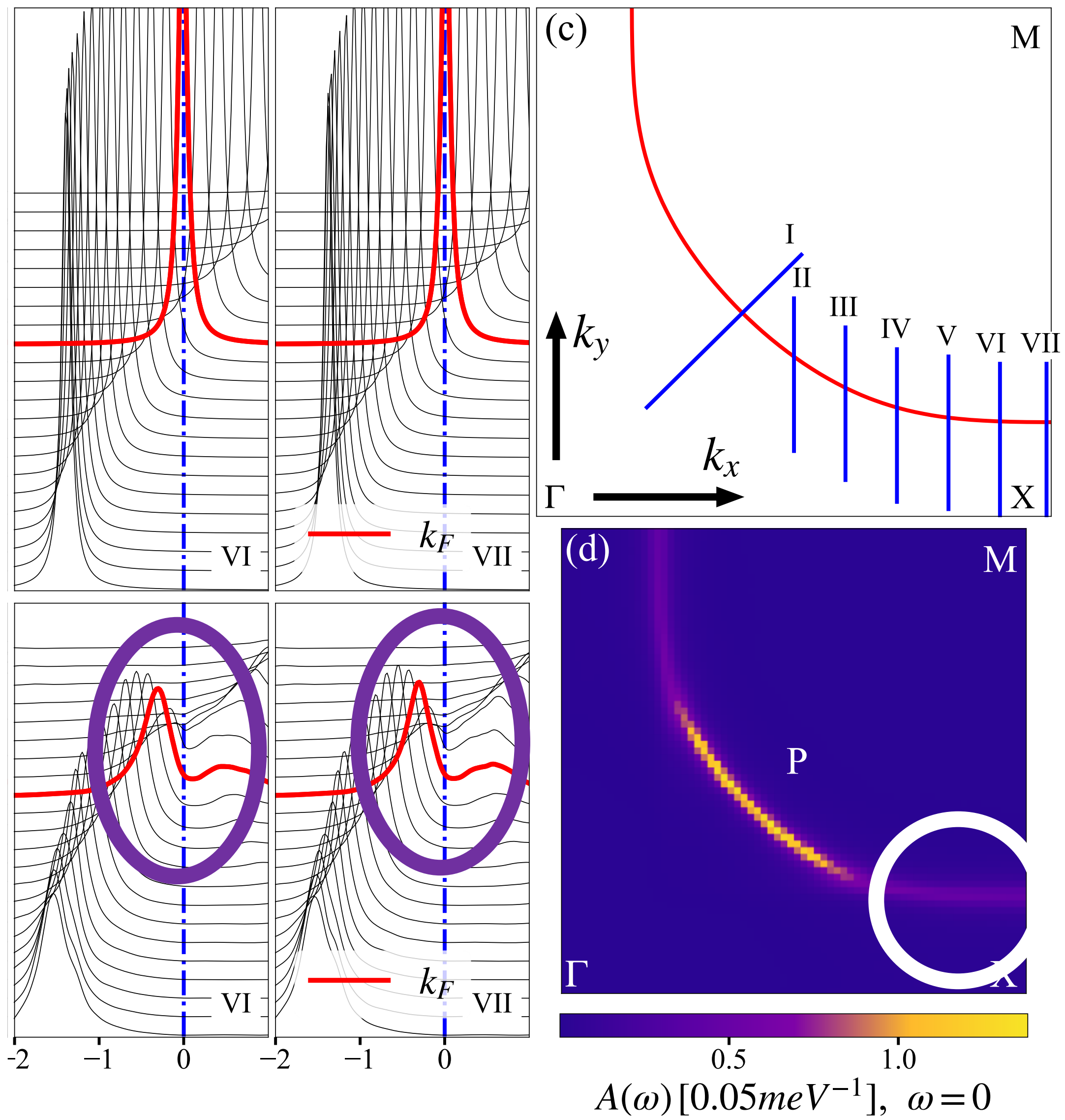
Gap-like but finite spectral weight

Pseudogap!

(b)

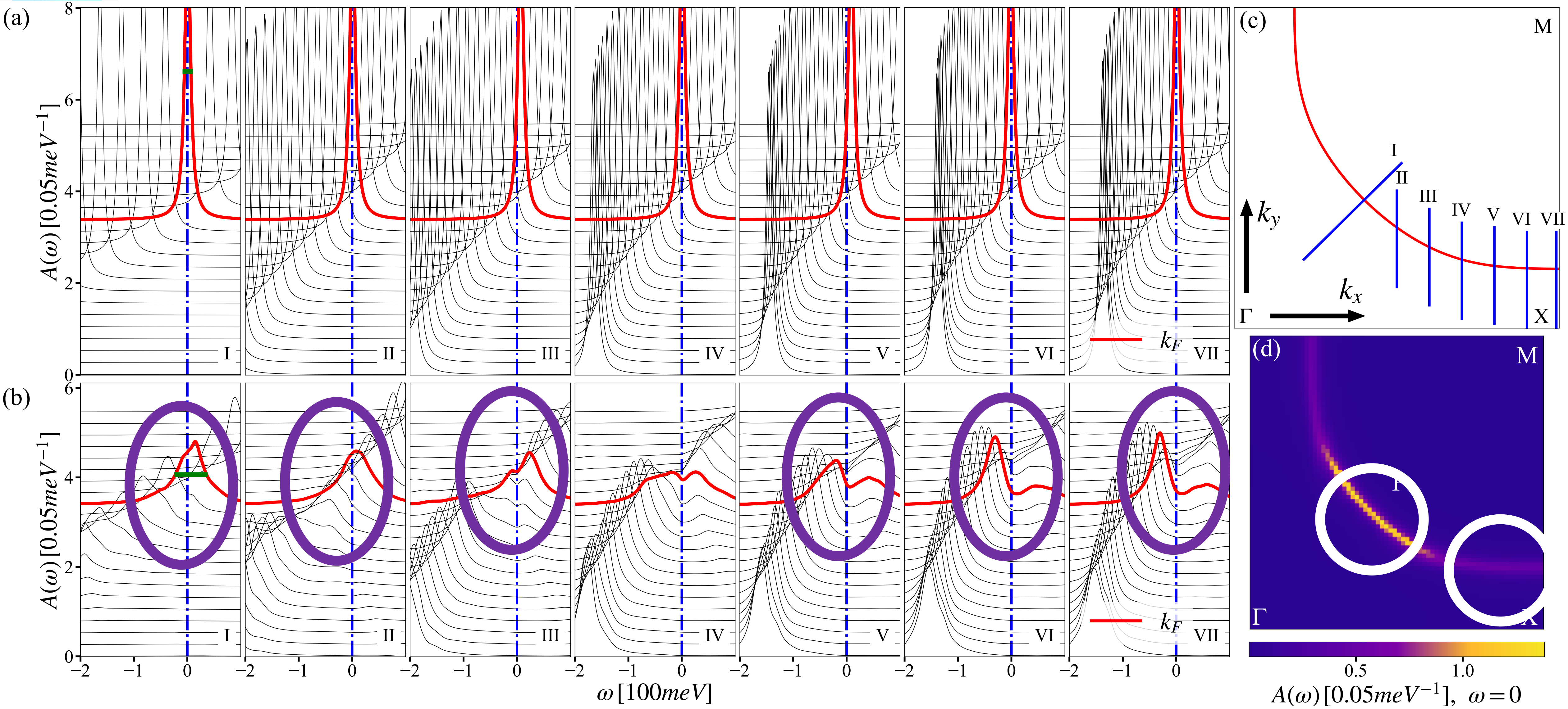


No need for quantum critical fluctuation or near phase boundary



X Yue, A Hegg, X Li, Wei Ku, New J. Phys. **25**, 053007 (2023)

Fermi arc & nodal-antinodal dichotomy



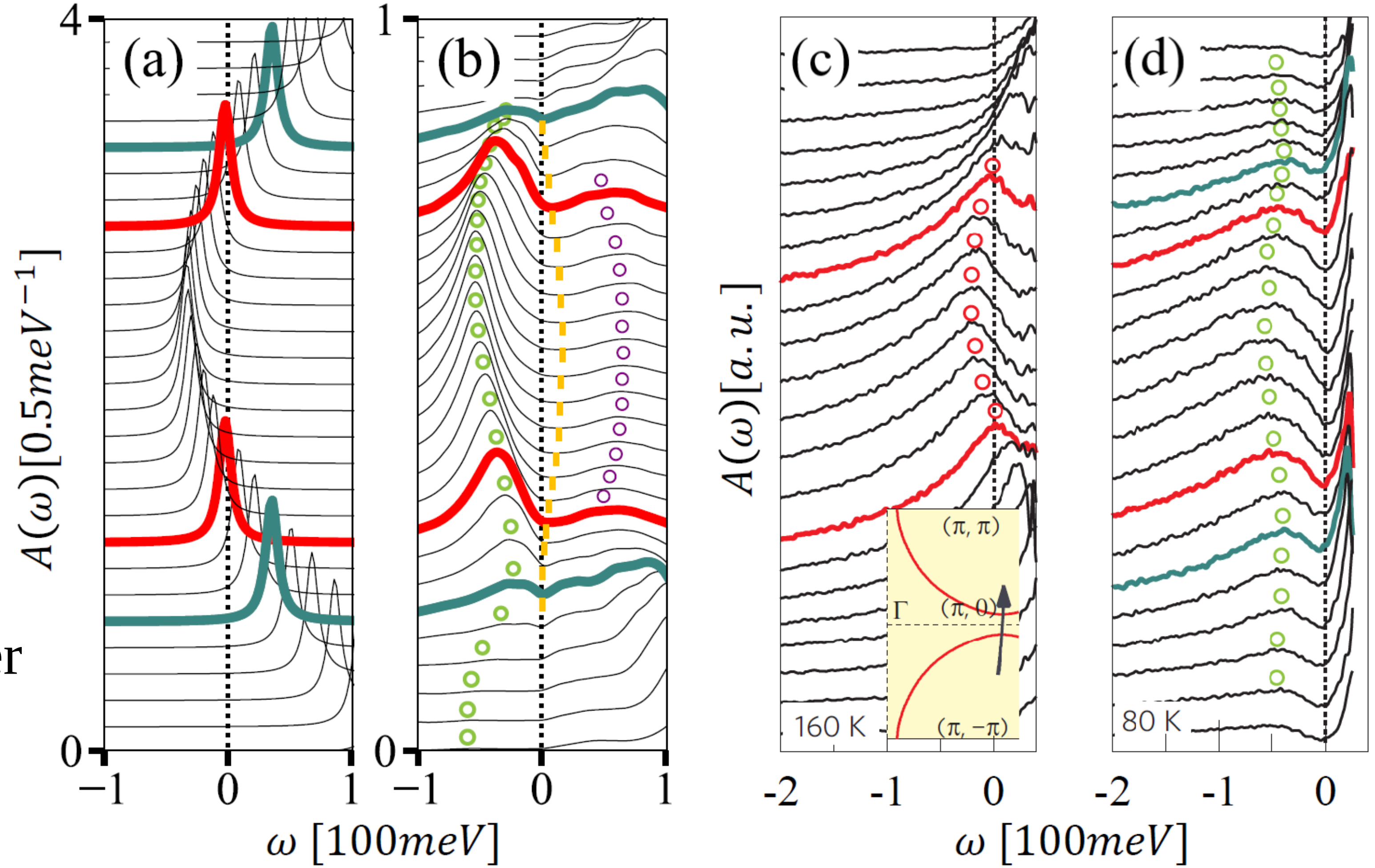
X Yue, A Hegg, X Li, Wei Ku, New J. Phys. **25**, 053007 (2023)

k_F offset of pseudogap

Shadow band

Fermi vector
≠
“Bending back” momentum

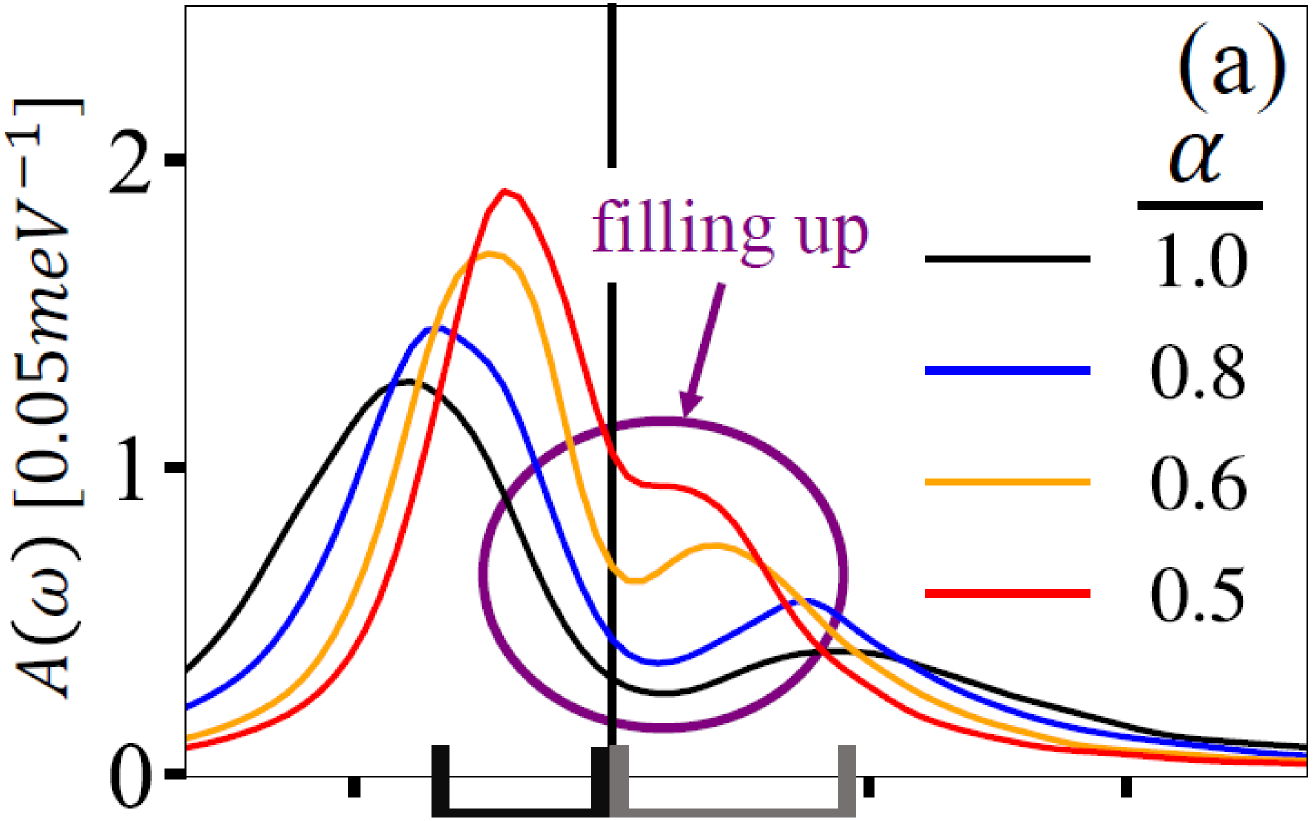
Back bending band and band center develop dispersion



Hashimoto M, et al. Nature Physics, 2010, 6(6): 414-418.

X Yue, A Hegg, X Li, Wei Ku, New J. Phys. **25**, 053007 (2023)

Filling of pseudogap against temperature



Gap scale

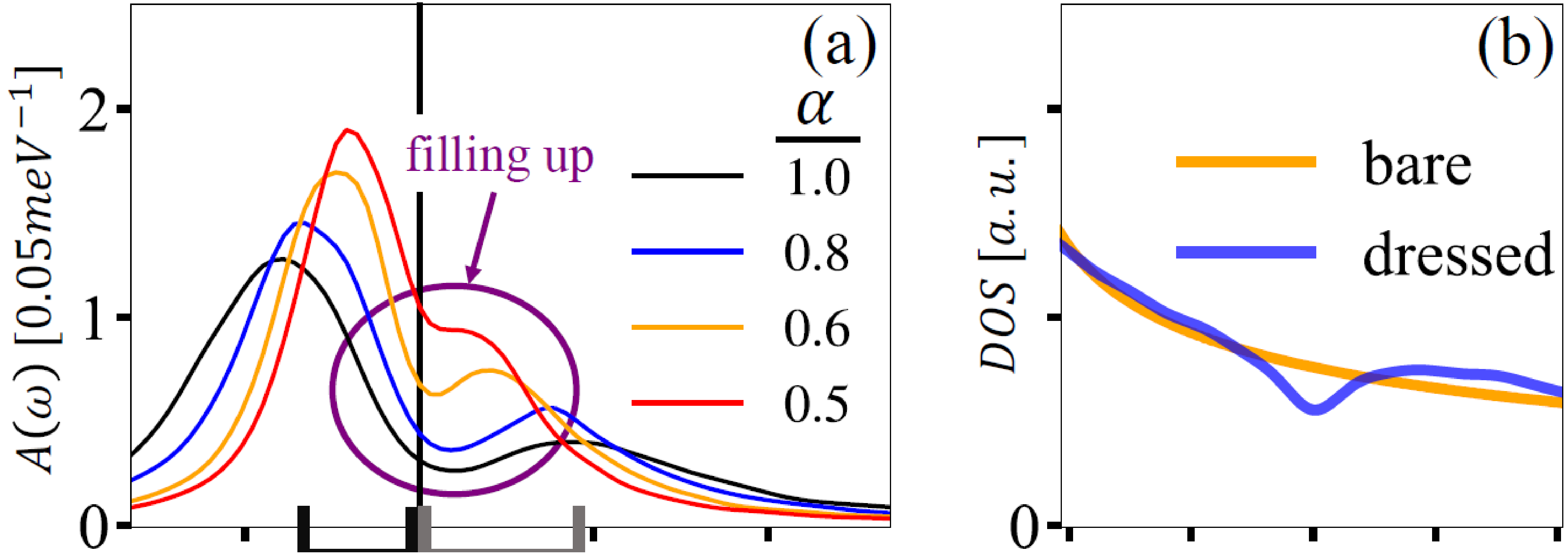
Effective coupling strength

Spectral weight scale

DOS of the coupled lines
+
Effective coupling strength

X Yue, A Hegg, X Li, Wei Ku, New J. Phys. **25**, 053007 (2023)

Pseudogap edge asymmetry vs symmetry



X Yue, A Hegg, X Li, Wei Ku, New J. Phys. **25**, 053007 (2023)

Many-body localization in homogeneous systems via geometric frustration

X Zhang, X Li, Z Fan, and Wei Ku, in preparation

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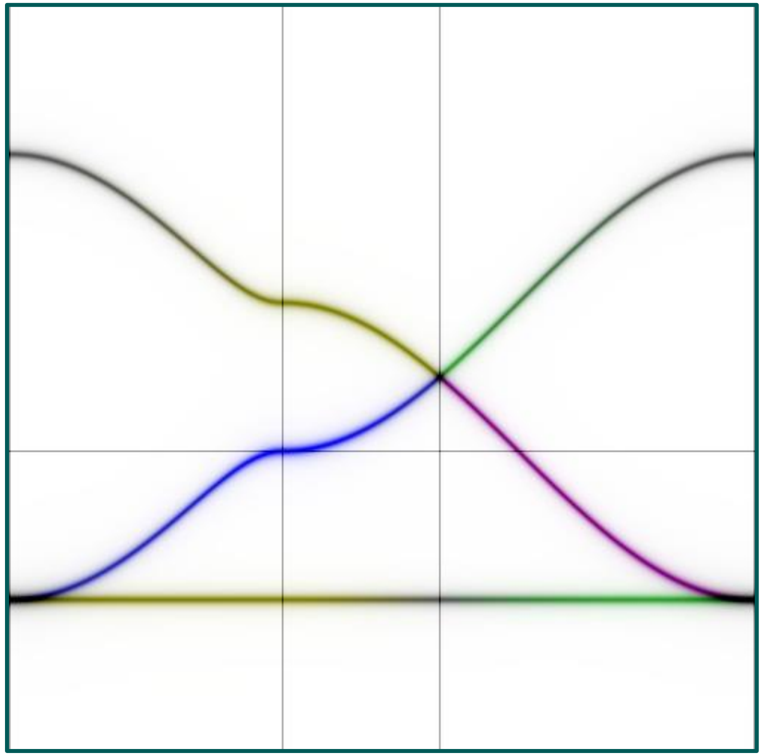
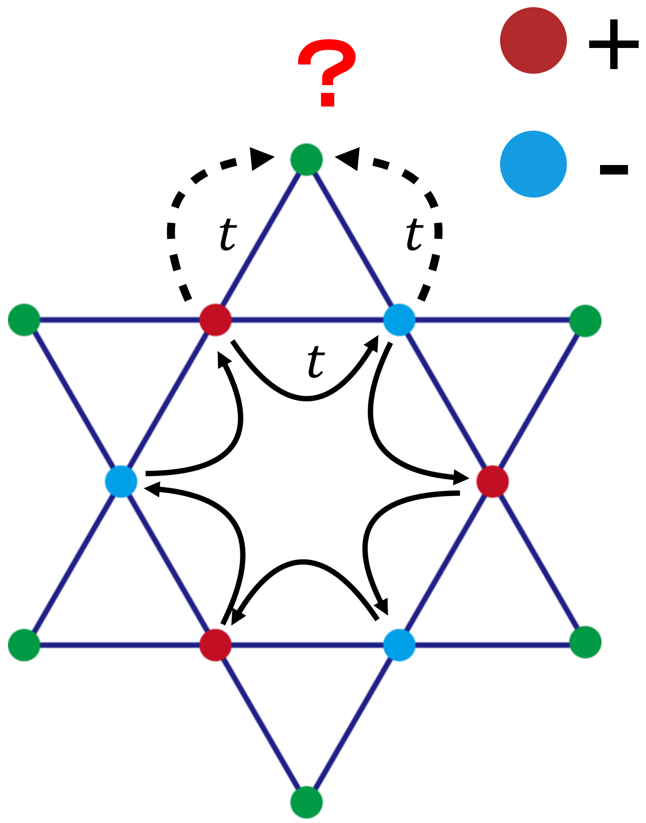
Geometric frustration of Kagome lattice

$$H = T + U = t \sum_{\langle i,i' \rangle} a_i^{+} a_{i'} + U \sum_{i} a_i^{+} a_i^{+} a_i a_i$$



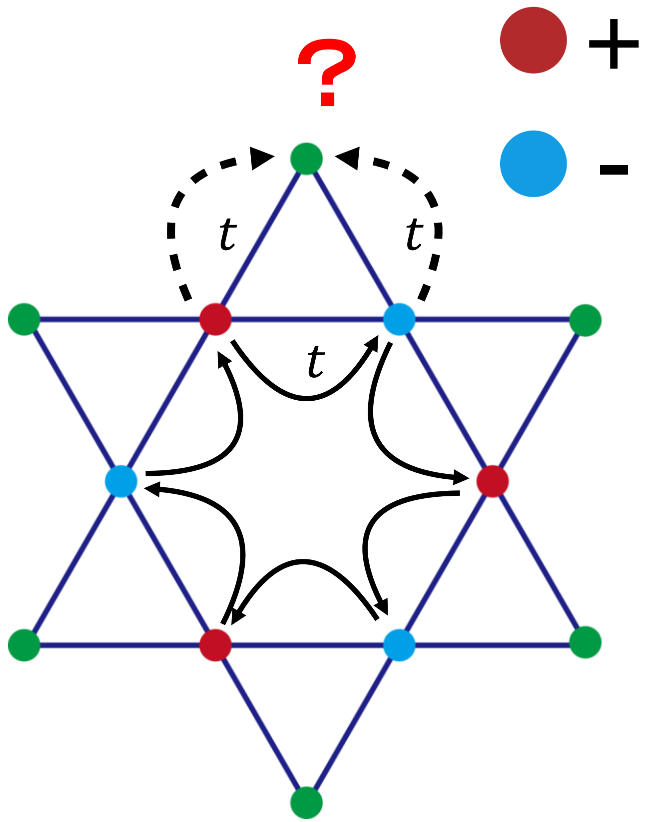
$$|R, m = 3\rangle \equiv$$



Localized one-body state

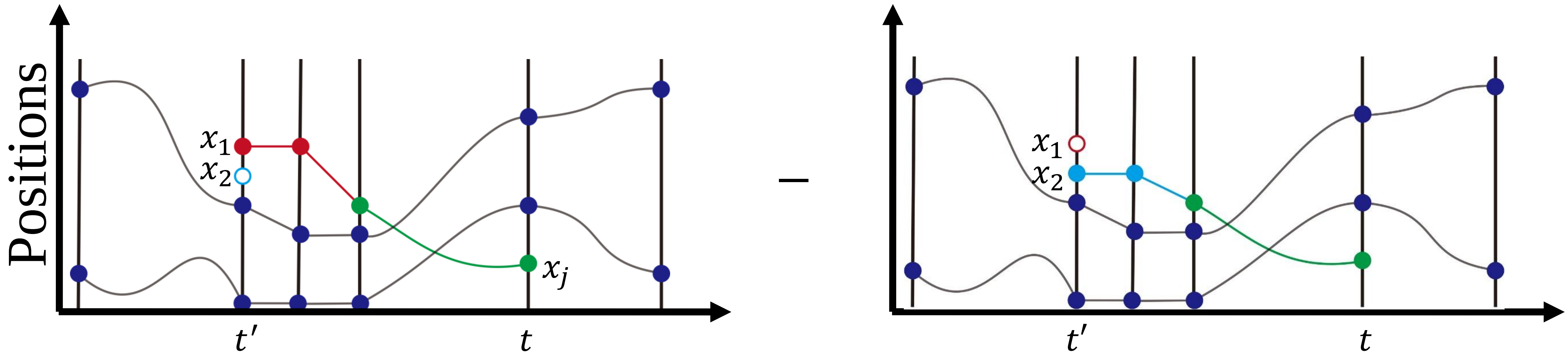
$$H = T + U = t \sum_{\langle i,i' \rangle} a_i^\dagger a_{i'} + U \sum_i a_i^\dagger a_i^\dagger a_i a_i$$

- Only T moves particles.
  - But, $T\,|R, m = 3\rangle = -2t\,|R, m = 3\rangle$
  - $|R, m = 3\rangle$ does not move.
- Similarly, the current operator $\mathbf{J}\,|R, m = 3\rangle = \mathbf{0}$
  - $|R, m = 3\rangle$ does not flow.
- Interaction $U \sum_i a_i^\dagger a_i^\dagger a_i a_i$ does not move the particle.



Path integral demonstration of the Green's function

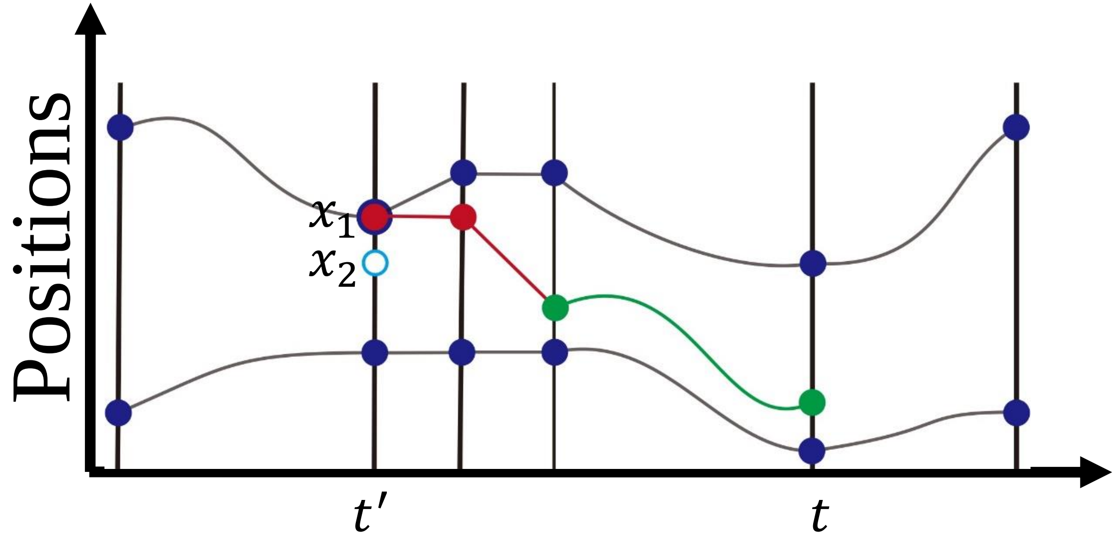
- $|R, m = 3\rangle = (|1\rangle - |2\rangle + |3\rangle - |4\rangle + |5\rangle - |6\rangle)/\sqrt{6}$
- $\sqrt{6}\, G(j, m = 3; t, t') = G(j, 1; t, t') - G(j, 2; t, t') + G(j, 3; t, t') - \cdots$



- contributions always cancel

Path integral demonstration of the Green's function

- $|R, m=3\rangle = (|1\rangle - |2\rangle + |3\rangle - |4\rangle + |5\rangle - |6\rangle)/\sqrt{6}$
- $\sqrt{6}\, G(j, m=3; t, t') = G(j, 1; t, t') - G(j, 2; t, t') + G(j, 3; t, t') - \cdots$



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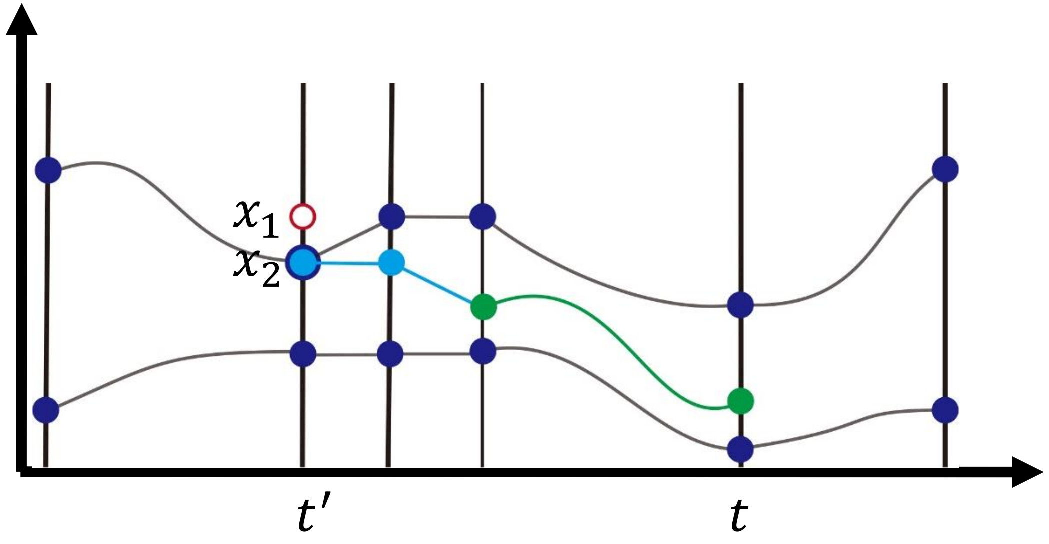


- contributions always cancel
- $G(j, m=3; t, t') = 0$

Localized many-body eigenstates

In diagonal representation,

$H = t\sum_{\langle i,i'\rangle} a_i^+ a_{i'} + U\sum_i a_i^+ a_i^+ a_i a_i$ is simply

$$= \sum_i \epsilon_i b_i^+ b_i + \sum_{ij} \epsilon_{ij} b_i^+ b_j^+ b_j b_i + \sum_{ijk} \epsilon_{ijk} b_i^+ b_j^+ b_k^+ b_k b_j b_i + \cdots$$ (N-body term)

with eigenstates $|n_1, n_2, \ldots\rangle = \Pi_i (b_i^+)^{n_i}|0\rangle$ $\quad (\Sigma_i\, n_i = N)$, where

$$b_i^+ = U^+ a_i^+ U$$

$$= \sum_j v_i^j a_j^+ + \sum_{jkk'} v_i^{jkk'} a_j^+ a_k^+ a_{k'} + \sum_{jklk'l'} v_i^{jklk'l'} a_j^+ a_k^+ a_l^+ a_{l'} a_{k'} + \cdots$$

gives the many-body wave function of each particle.

(U^+ is found by demanding the diagonal representation.)

Localized many-body eigenstates

Since U^+ is found by demanding the diagonal representation, it must respect all symmetries of H: $[U, S] = 0$ for all S that satisfies $[H, S] = 0$.

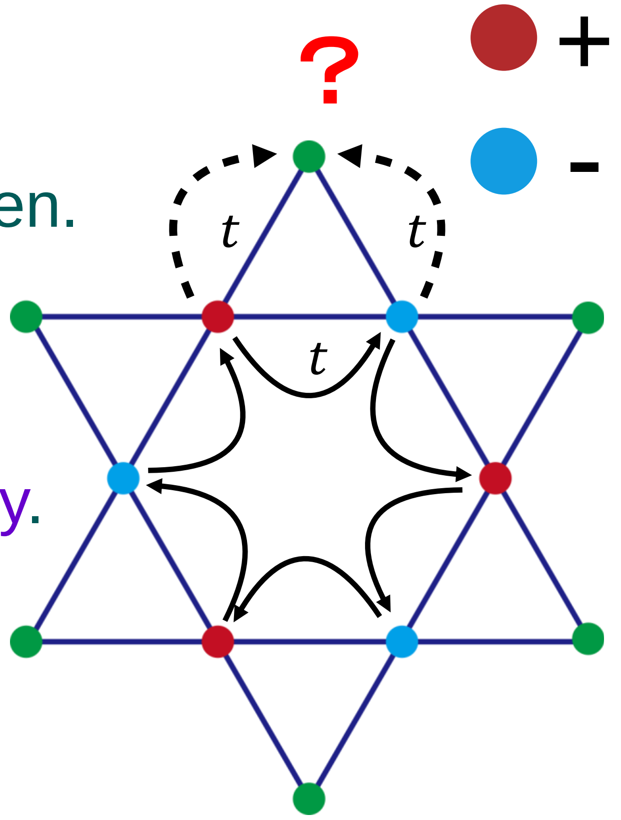
The odd parity of $a^+_{m=3}$, $P_x a^+_{m=3} P_x = -\, a^+_{m=3}$ would gives

$b^+_{m=3} = U^+ a^+_{m=3} U$ odd parity as well, while a^+_j is obviously even.

So, $\langle a_j b^+_{m=3} \rangle = -\langle a_j b^+_{m=3} \rangle = 0$

$b^+_{m=3}$ cannot move out even one step due to local point symmetry.

All other destination went through the first j and suffer from the same interference.



Geometric frustration induced many-body localization

- The one-body Green’s function for the localized subspace remains localized.
- Flat band survives moderate interaction.
- No phase stiffness possible for superfluidity
- $U(1)$ symmetry cannot be broken unless geometric frustration is broken first

(by breaking spatial symmetry, for example.)

- Many-body localization realized
- In many way similar to the “many-body scar”
- Implications on transport, entanglement entropy, and thermalization.

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