An Exact Amorphous Chiral Spin Liquid

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ICTP – June 2023

arXiv:2208.08246

Imperial College London

NEUTRONS FOR SOCIETY

Imperial College PhD Students

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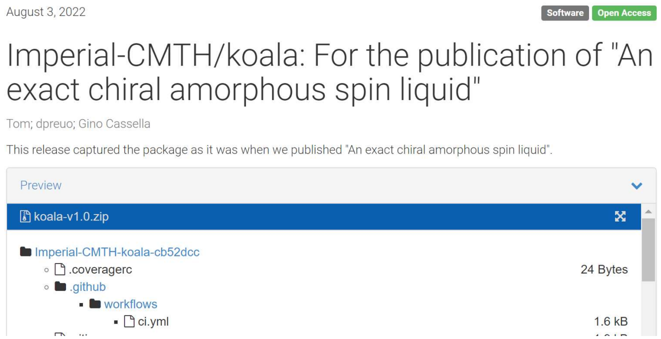
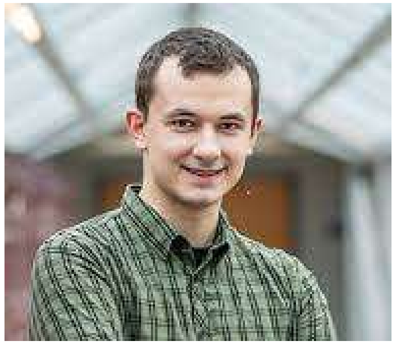
Creators of the open-source package KOALA (**K**itaev **O**n **A**morphous **La**ttices)



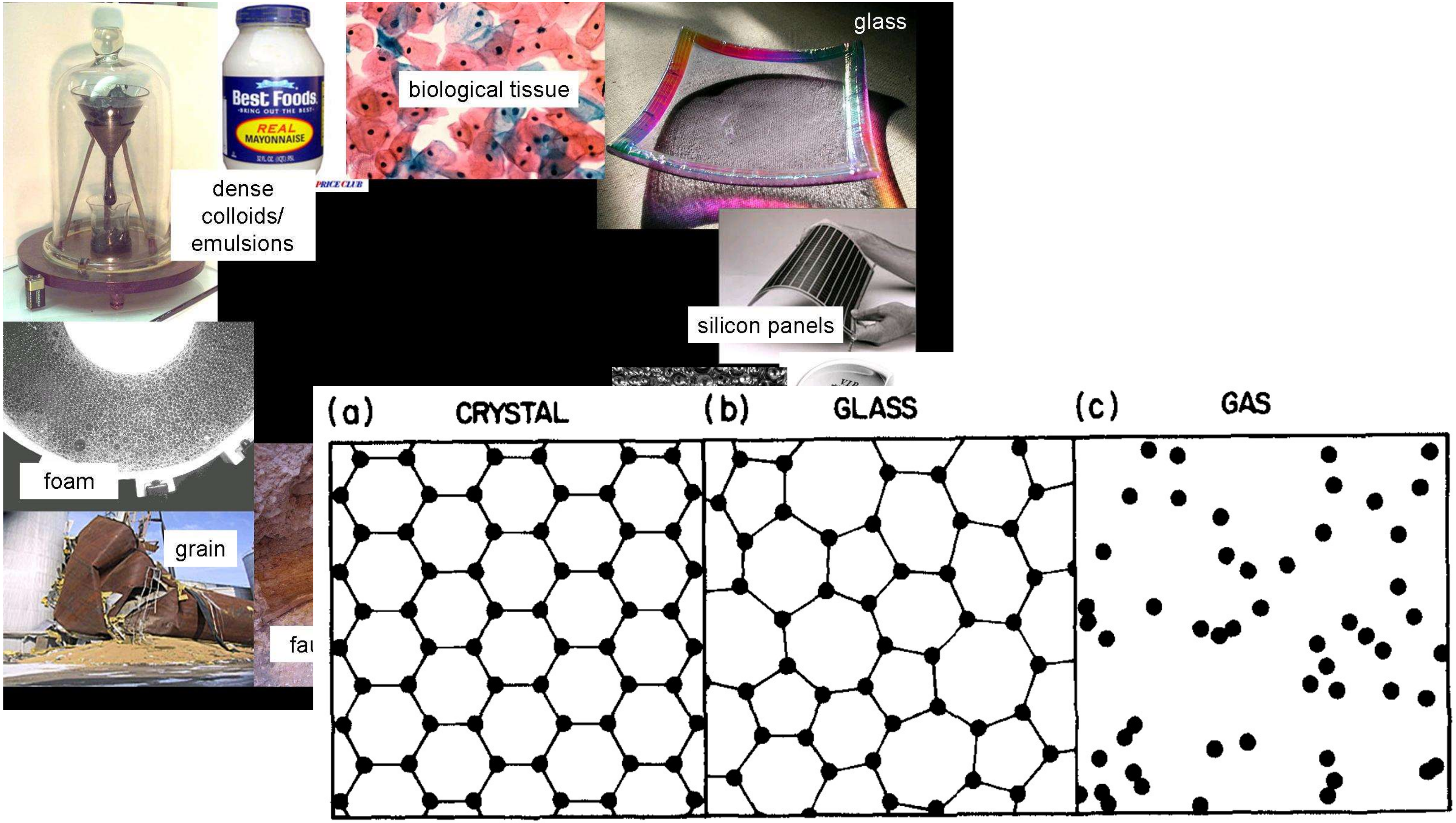
Johannes Knolle
Technische Universität München

Classical MC by:

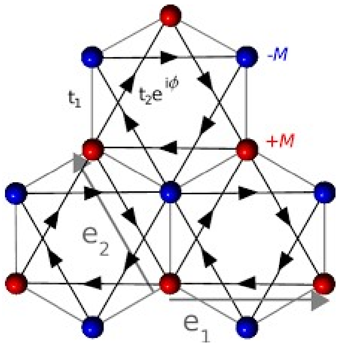


Andrey Zelenskiy
Dalhousie University

Amorphous Materials



Topological amorphous materials



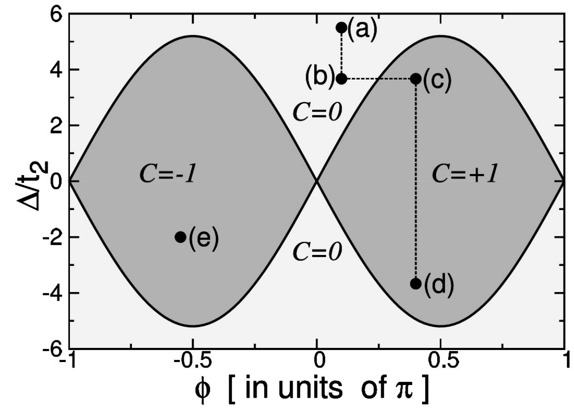
A Topological invariant:
the Chern number

$$C = -\frac{1}{\pi}\frac{(2\pi)^2}{A_c}\mathrm{Im}\ \mathrm{tr}_{\mathrm{cell}}\{PxQy\} = \frac{4\pi}{A_c}\mathrm{Im}\ \mathrm{tr}_{\mathrm{cell}}\{PxPy\},$$

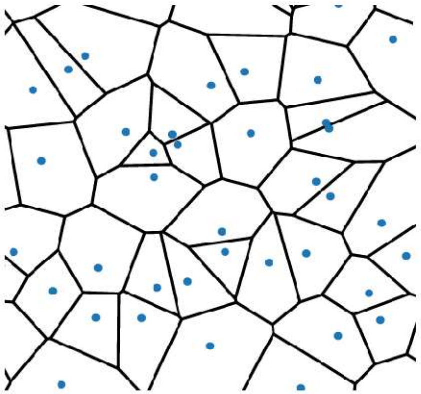
$$P = \frac{A_c}{(2\pi)^2}\sum_{n=1}^{N_c}\int_{\mathrm{BZ}} d\mathbf{k} |\psi_{n\mathbf{k}}\rangle\langle\psi_{n\mathbf{k}}|,$$

$$Q = \frac{A_c}{(2\pi)^2}\sum_{n'=N_c+1}^{\infty}\int_{\mathrm{BZ}} d\mathbf{k}' |\psi_{n'\mathbf{k}'}\rangle\langle\psi_{n'\mathbf{k}'}|.$$

Bianco and Resta,
PRB **84**, 241106(R)



Application: topological phases in amorphous semiconductors



Marsal, Varjas, and Grushin
PNAS **117**, 30260

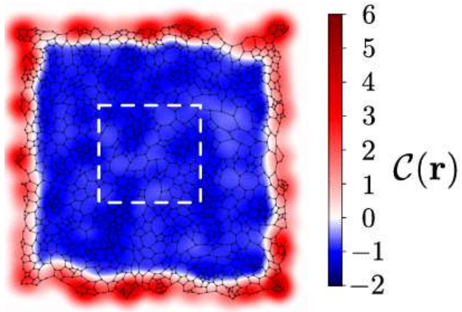
$$H_{\mathrm{WT}} = \sum_{i,j\neq j'}^{z} V_{jj'}^{(i)}|i,j\rangle\langle i,j'| + \sum_{i\neq i',j}^{z} W_{ii'}^{(j)}|i,j\rangle\langle i',j|.$$

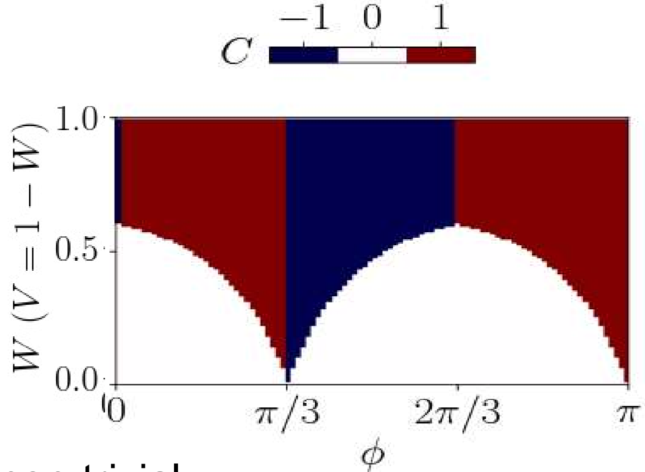
$$V \rightarrow Ve^{i\phi}$$

Real space Chern marker:

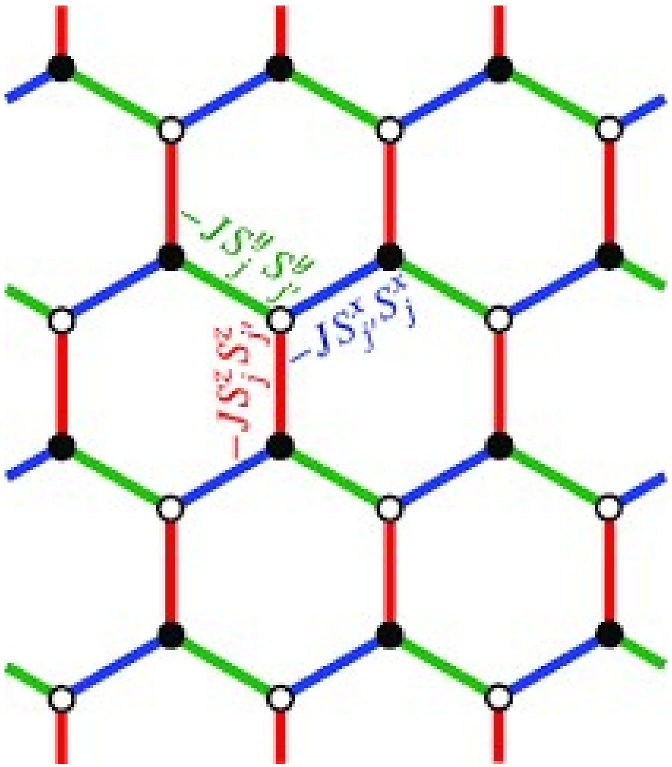
$$\mathcal{C}(\mathbf{r}) = 2\pi\mathrm{Im}\,\langle\mathbf{r}|\left[\hat{Q}\hat{x},\hat{P}\hat{y}\right]|\mathbf{r}\rangle$$

$$C = \frac{1}{A_{\mathrm{sys}}}\mathrm{Tr}\mathcal{C}(\mathbf{r})$$



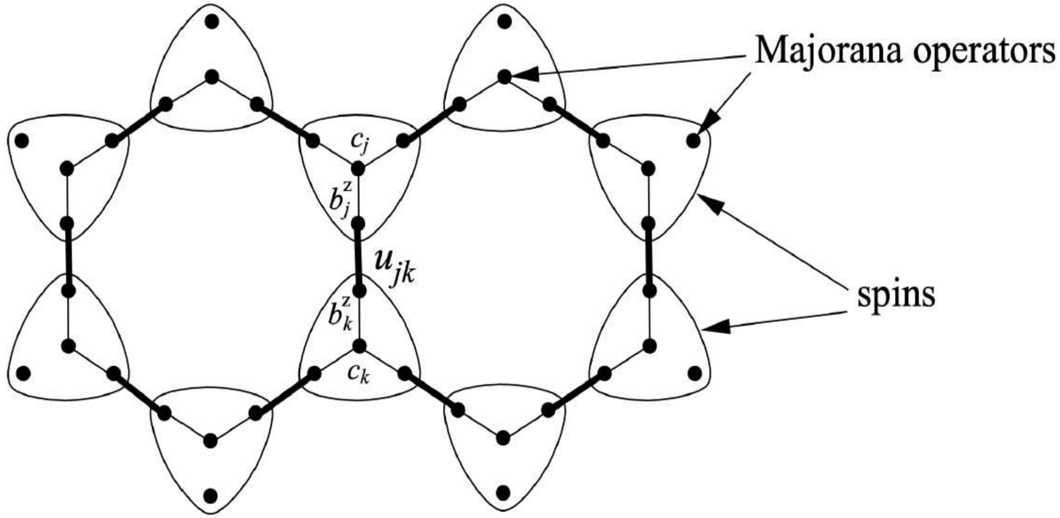


What about topologically non-trivial **strongly correlated phases**?



$$\mathcal{H} = -\sum_{\langle j,k\rangle_\alpha} J^\alpha \sigma_j^\alpha \sigma_k^\alpha,$$

$$W_p = \prod_{\langle jk\rangle_\gamma \in p} \sigma_j^\gamma \sigma_k^\gamma$$



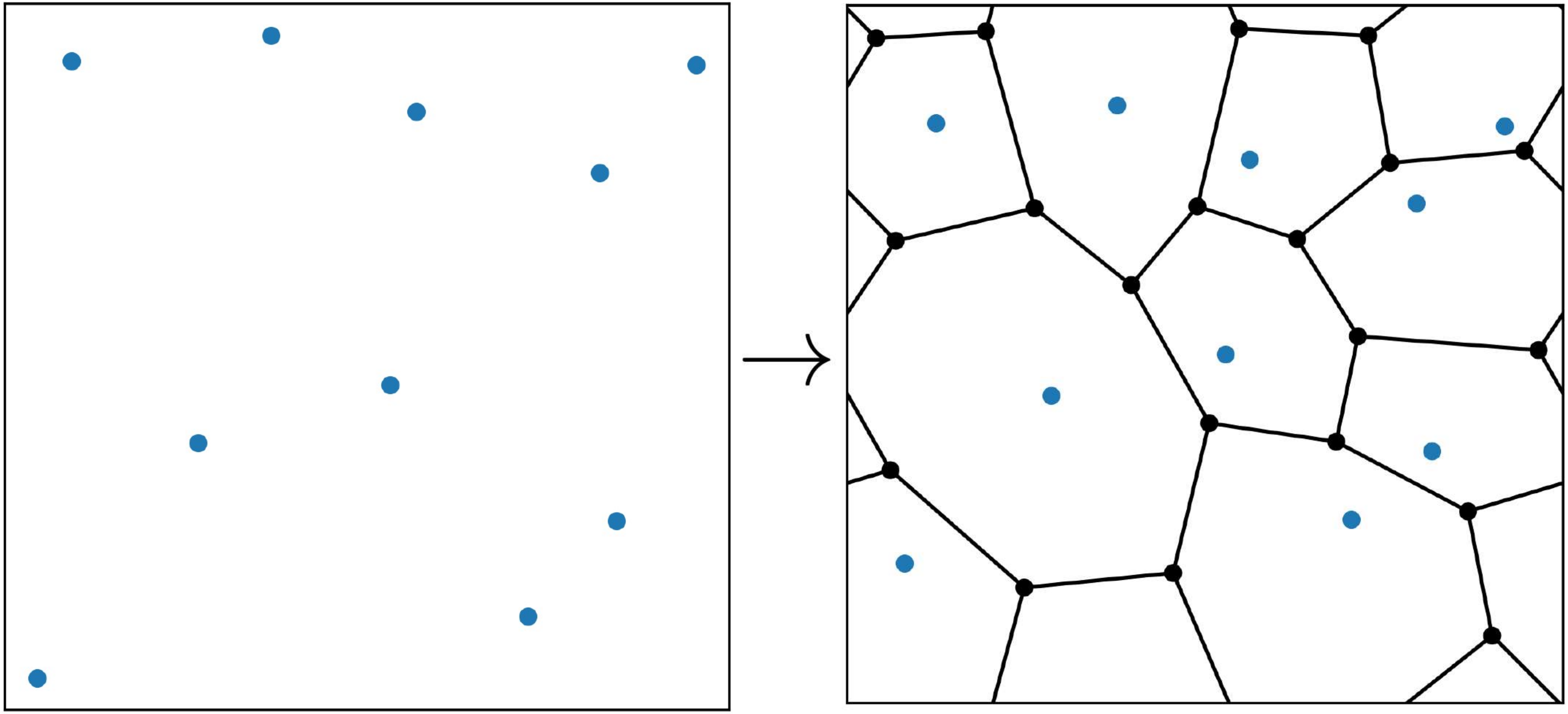
$$\sigma_i^\alpha = i b_i^\alpha c_i \qquad \hat{u}_{jk} = i b_j^\alpha b_k^\alpha$$

$$\downarrow$$

$$\mathcal{H} = \frac{i}{2} \sum_{j,k} [J^\alpha u_{jk}] \, c_j c_k$$

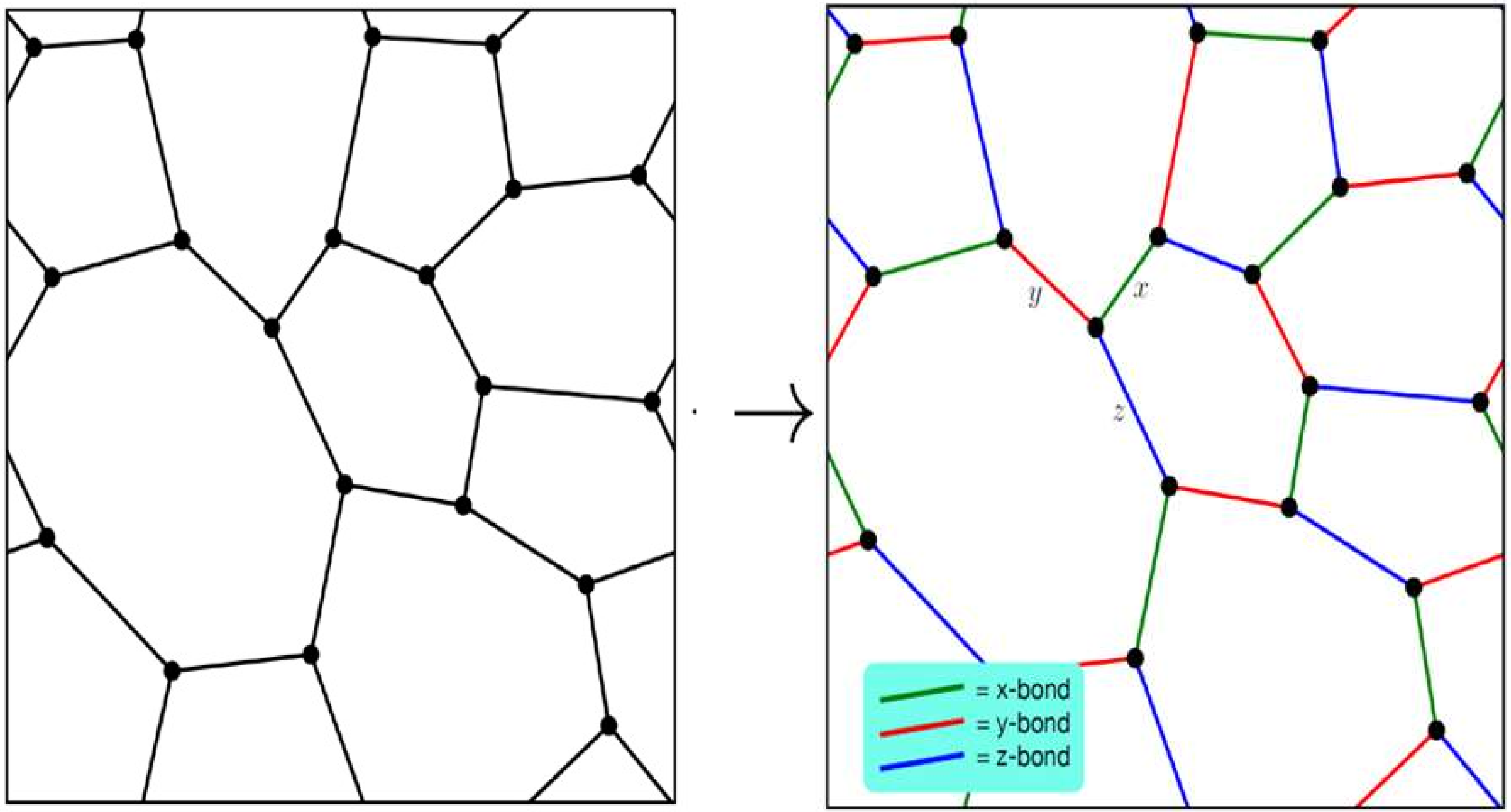
A. Kitaev,
Ann. Phys. 321, 2 (2006)

Voronoi Tessellation



arXiv:2208.08246

Bond-Coloring Problem



arXiv:2208.08246

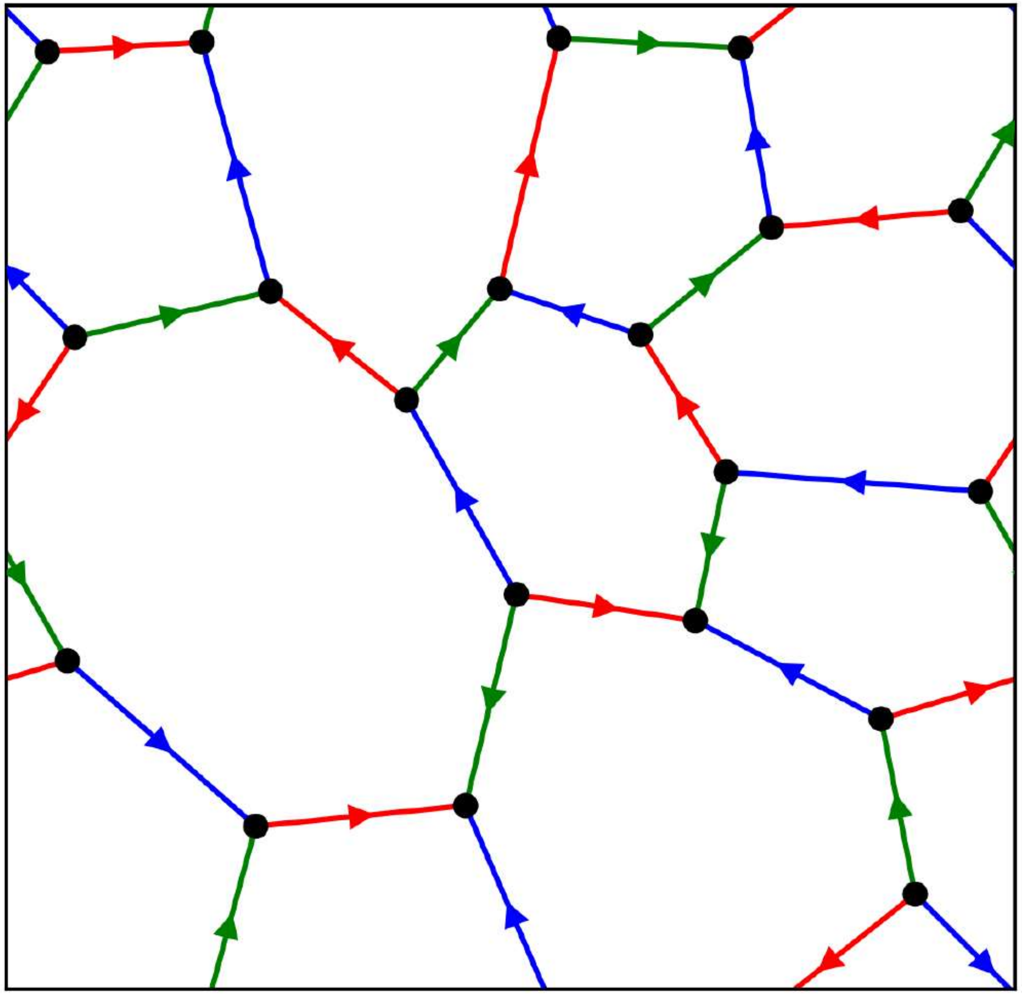
$$\mathcal{H} = \frac{i}{2}\sum_{j,k}\left[J^{\alpha}u_{jk}\right]c_j c_k$$

$$\hat{u}_{jk} = i b_j^{\alpha} b_k^{\alpha}$$

$$u_{jk} = \pm 1$$

$$u_{jk} = -u_{kj}$$

arXiv:2208.08246



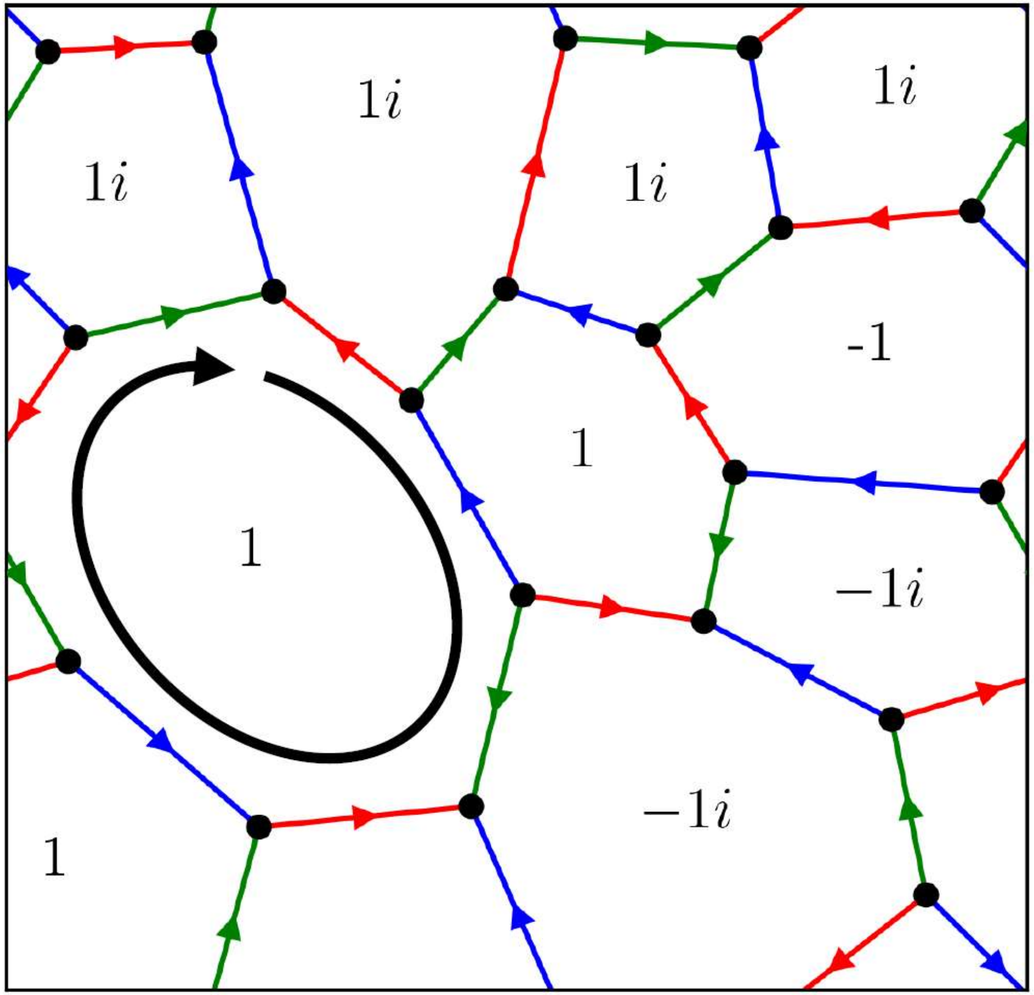
$$W_p = \prod_{\langle jk \rangle_\gamma \in p} \sigma_j^\gamma \sigma_k^\gamma$$

$$W_p \to \phi_p = \prod_{(j,k) \in \partial p} -iu_{jk}$$

even plaquette: ϕ_p real

odd plaquette: ϕ_p imaginary

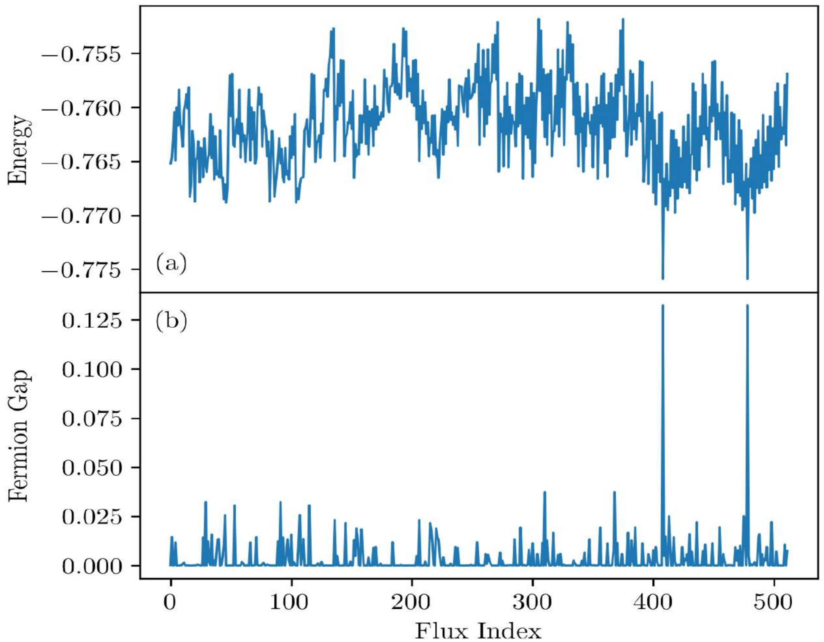
arXiv:2208.08246

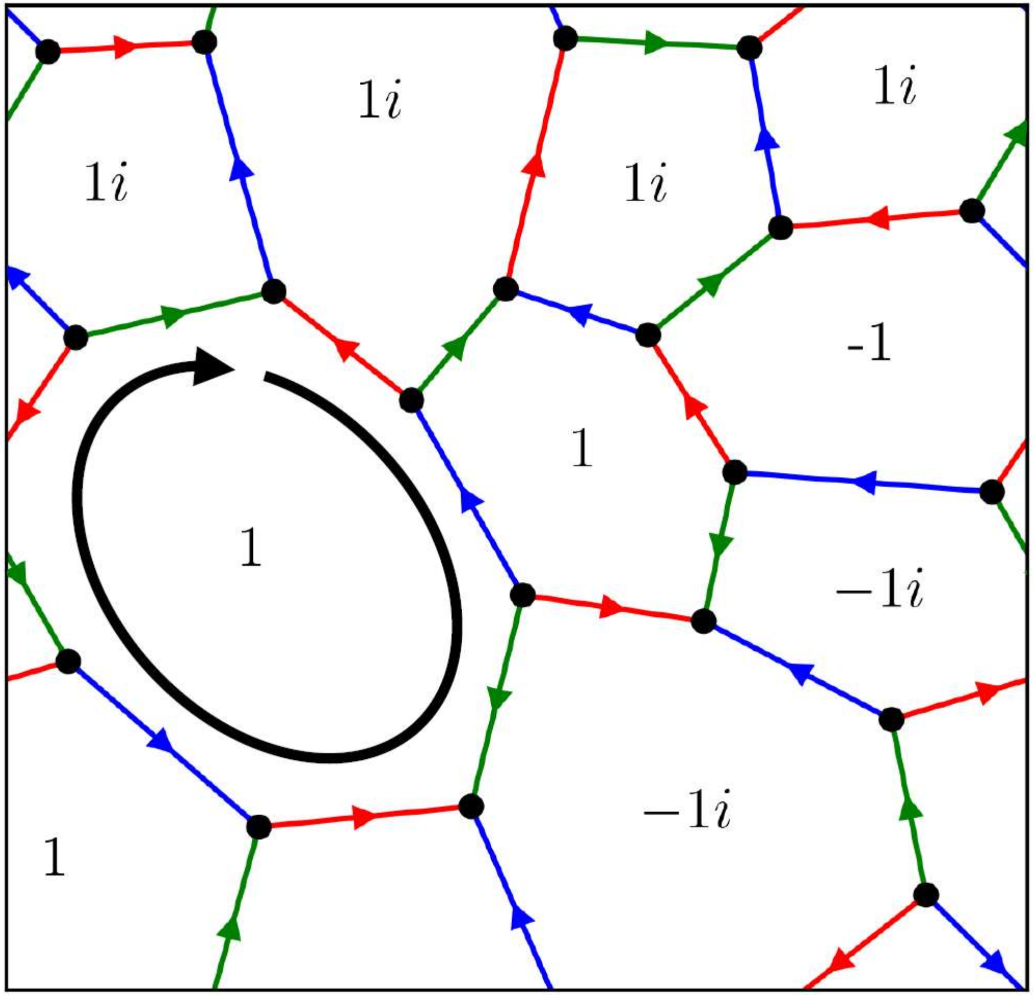


$\#\text{Flux sectors} \sim 2^{n_p - 1}$

10 plaquettes = 512 flux sectors — half a minute
16 plaquettes = 32,000 — half an hour
25 plaquettes = 16,000,000 — two weeks

Example of 10-plaquette exact diagonalization





arXiv:2208.08246

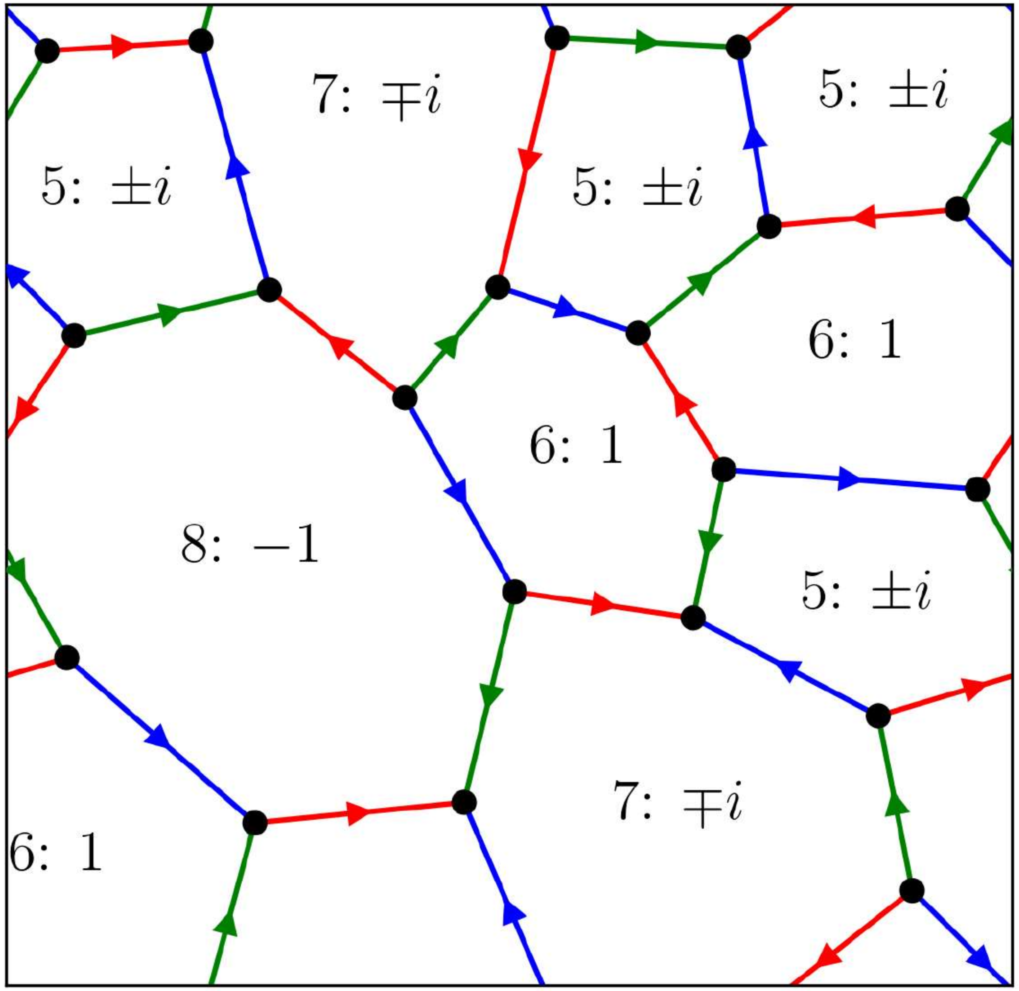
“Lieb’s Theorem” for Amorphous Lattices

- Use a 16-plaquette amorphous realization as the unit cell.
- Diagonalize the periodic lattice for every ~32000 flux sectors
- Find the flux sector ground state

Ground state flux-sector depends only on the number of sides of the plaquettes:

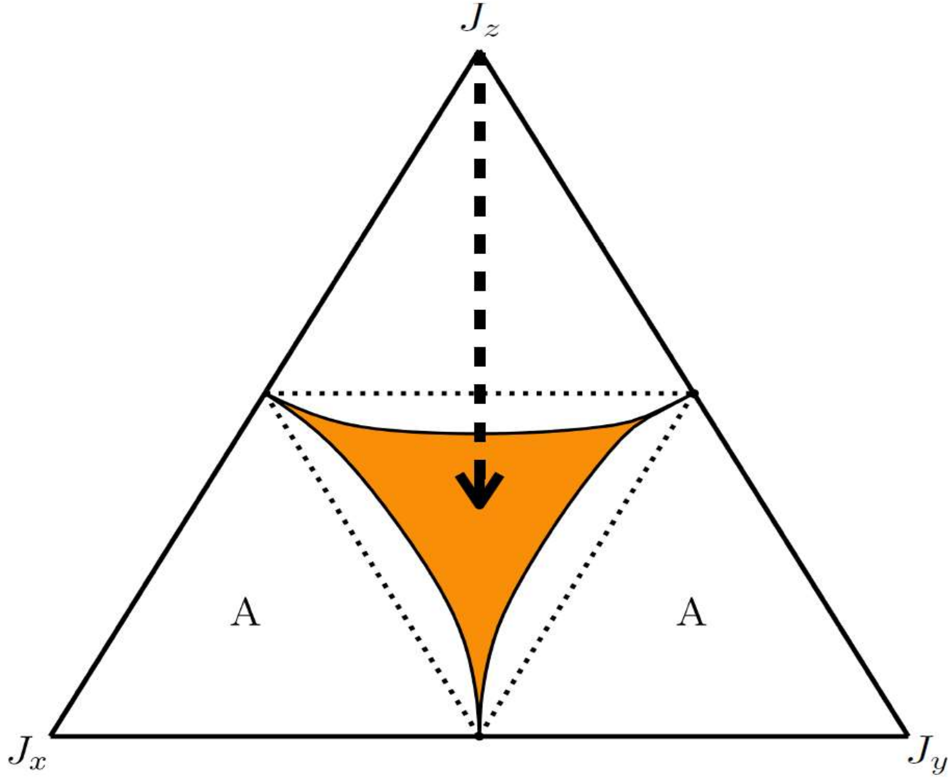
$$\phi_{\text{g.s.}} = -(\pm i)^{n_{\text{sides}}}$$

arXiv:2208.08246

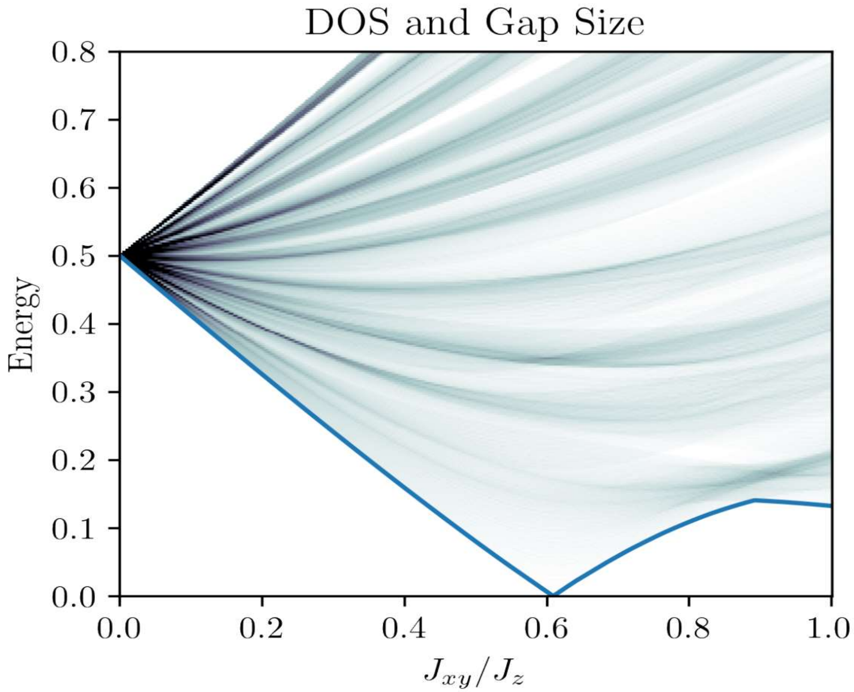


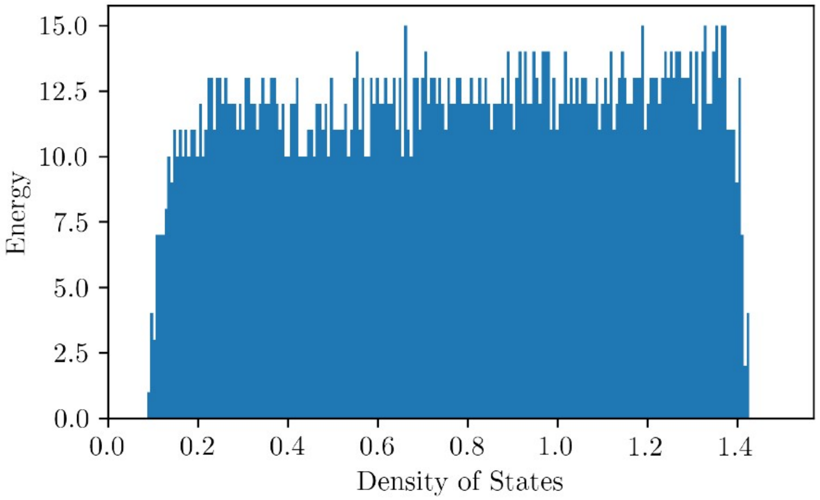
$$\mathcal{H} = -\sum_{\langle j,k\rangle_\alpha} J^\alpha \sigma_j^\alpha \sigma_k^\alpha,$$

$$W_p = \prod \sigma_j^\alpha \sigma_k^\alpha$$

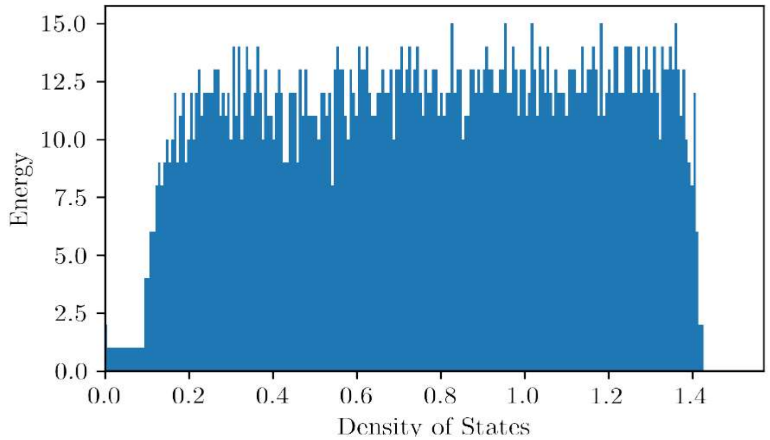


arXiv:2208.08246



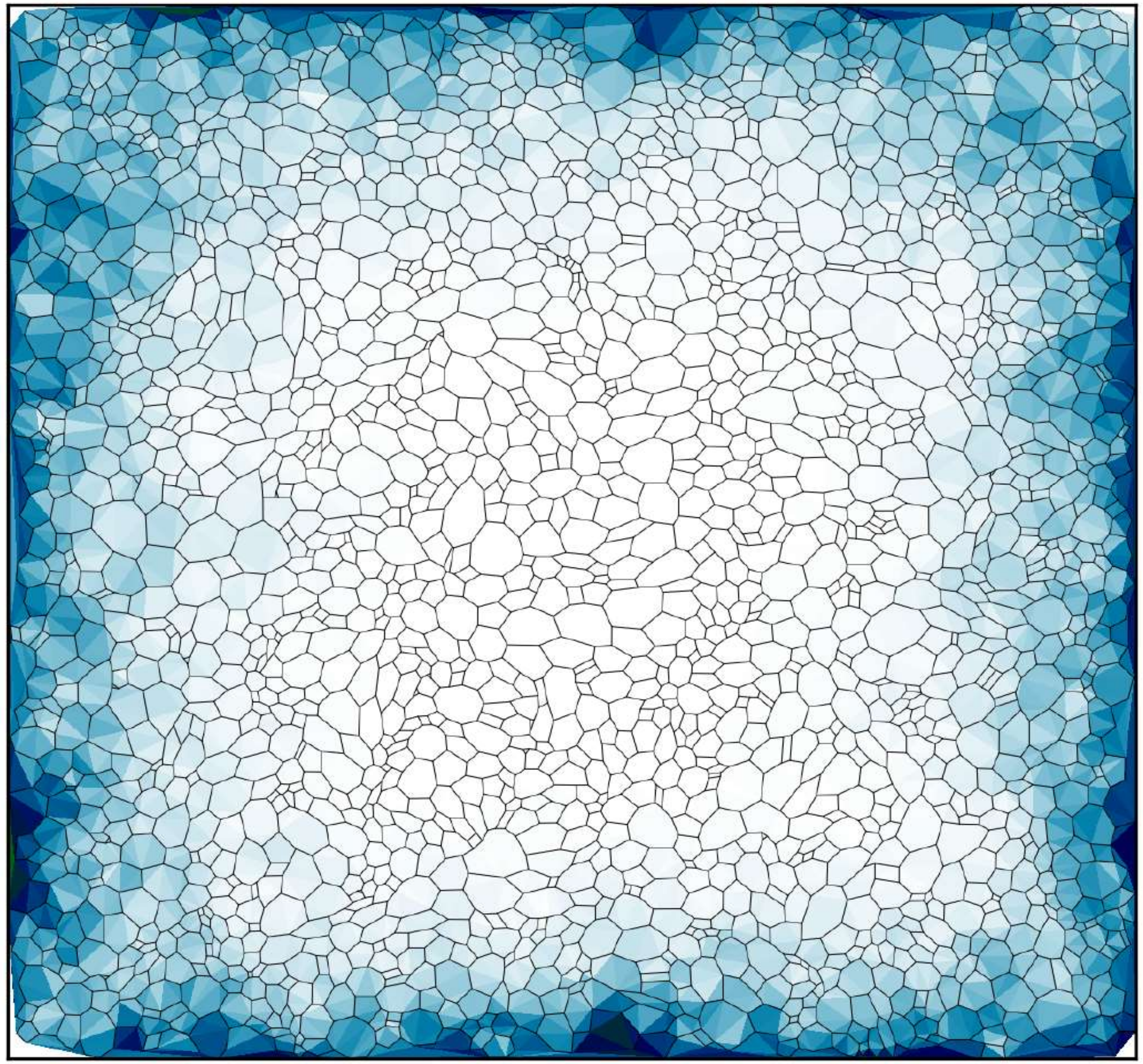


↓ Open boundary conditions

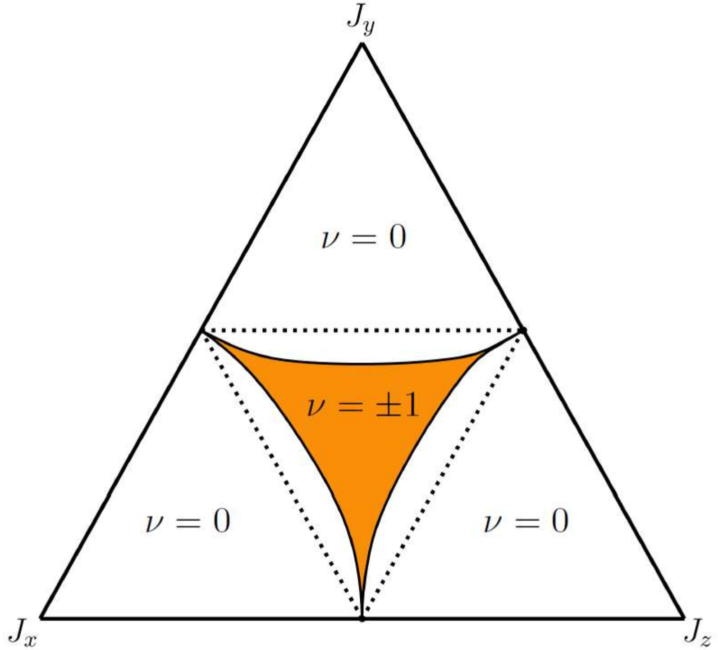


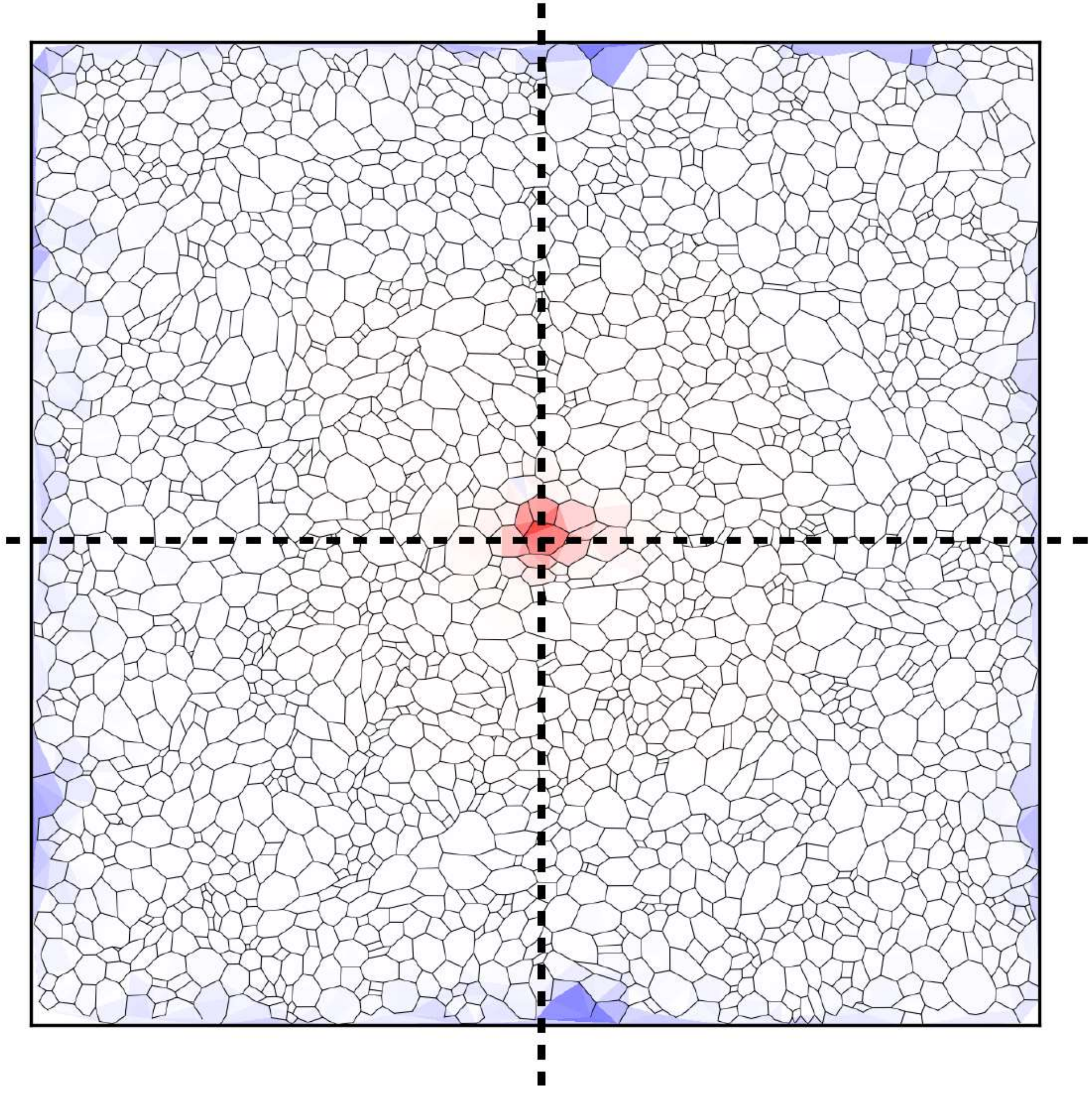
arXiv:2208.08246

$| \langle \mathbf{r} | \psi \rangle |$ for edge mode



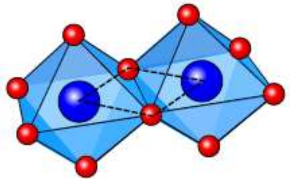
$$\nu(\mathbf{R}) = 4\pi \mathrm{Im}\ \mathrm{Tr}_{\mathrm{Bulk}} \left(P\theta_{R_x} P\theta_{R_y} P \right)$$

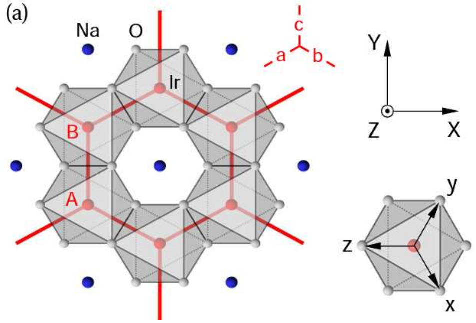




Possible implementations:

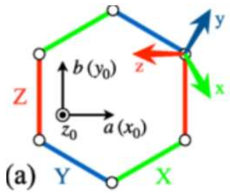
Layered honeycomb Mott insulators:



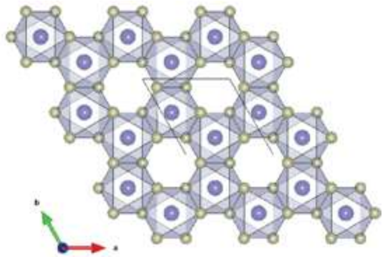


Nearest-neighbor exchanges allowed by symmetry is a four parameter model

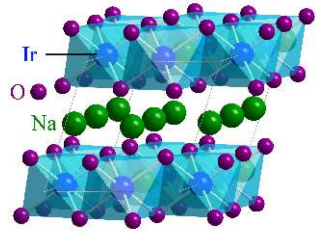
$$H_{JK\Gamma\Gamma'} = J\sum_{\langle ij\rangle} \mathbf{S}_i\cdot\mathbf{S}_j + K\sum_{\langle ij\rangle_\gamma} S_i^\gamma S_j^\gamma + \Gamma\sum_{\langle ij\rangle_\gamma}\left(S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha\right) + \Gamma'\sum_{\langle ij\rangle_\gamma}\left(S_i^\gamma S_j^\alpha + S_i^\alpha S_j^\gamma + S_i^\gamma S_j^\beta + S_i^\beta S_j^\gamma\right)$$



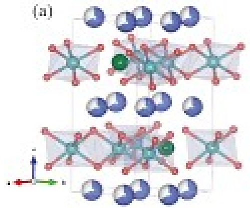
1_ Turn crystalline Kitaev materials into glass



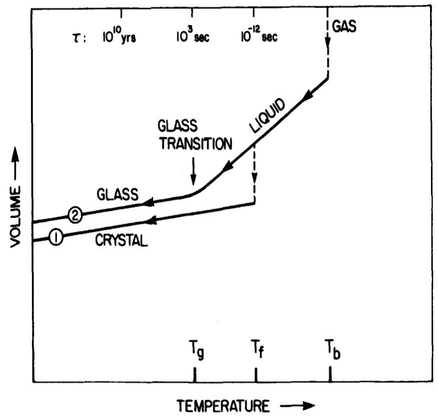
α-$RuCl_3$



Na_2IrO_3

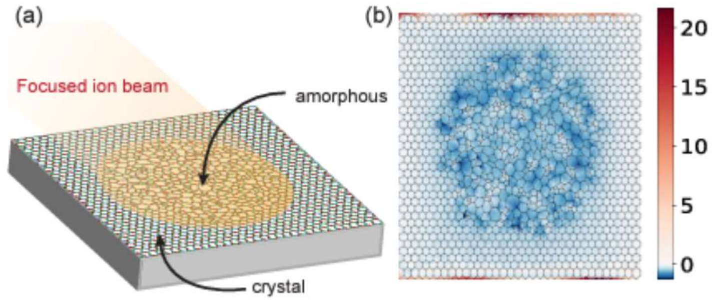


$Na_2Co_2TeO_6$ or $Na_2Co_2SbO_6$



Zallen, The Physics of Amorphous Solids

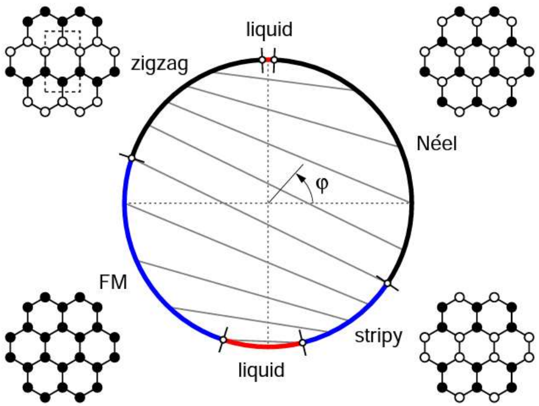
2_ Focused Ion Beam
(Grushin and Repellin, PRL 130 186702)



Preliminary Results of the Kitaev-Heisenberg Model

Results on the honeycomb lattice:

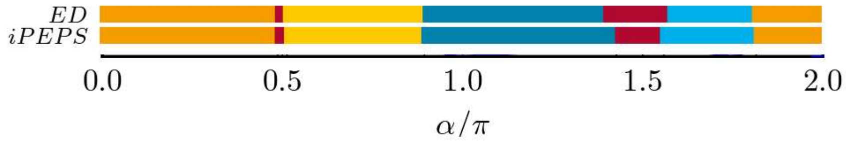
$$H = 2A\sin\phi \sum_{\langle ij\rangle_\gamma} S_i^\gamma S_j^\gamma + A\cos\phi \sum_{\langle ij\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$



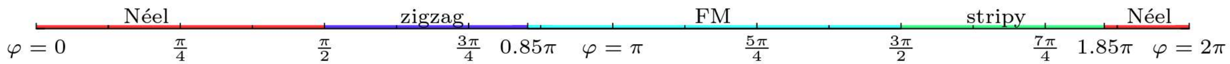
Phase diagram of 24-site cluster ED

Chaloupka, Jackelli, Khaliullin, PRL **110**, 097204

DMRG phase diagram (Gohlke et al., PRL 119, 157203):



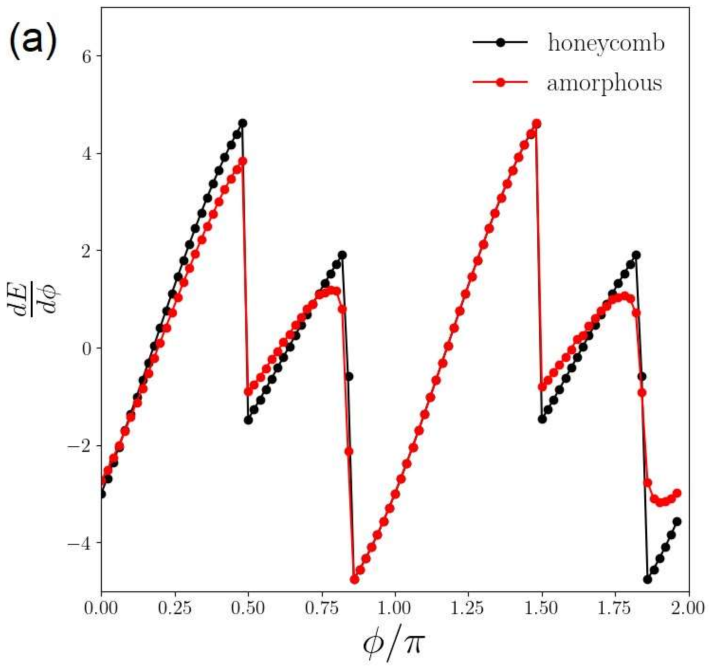
Classical MC (Janssen, Andrade and Vojta, PRL 117, 277202):



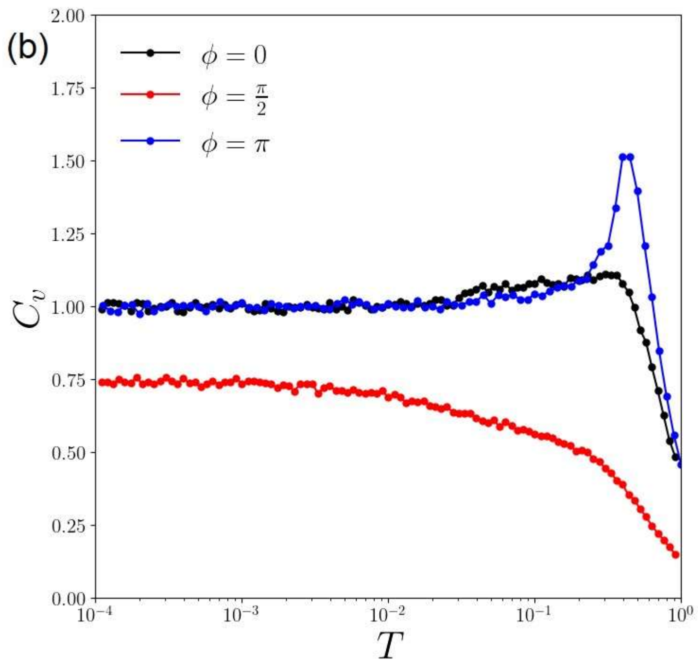
Preliminary Results of the Kitaev-Heisenberg Model

$$H = 2A\sin\phi \sum_{\langle ij \rangle_\gamma} S_i^\gamma S_j^\gamma + A\cos\phi \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

Phase transitions on the same points as the honeycomb lattice:



Specific Heat of the pure Kitaev and Heisenberg models:

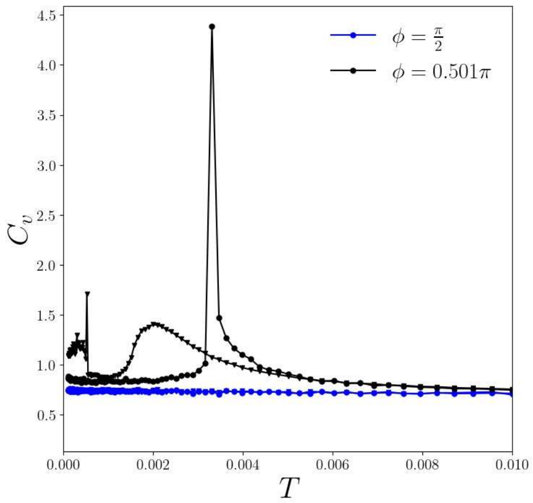


- Exponentially large number of zero-modes implies that $C_v < 1$ for T=0 (Baskaran, Sen, Shankar, PRB **78** 115116)
- Absence of perfect overlap in the two Heisenberg models.

A. Zelenszkiy, **WMHN** et al., in preparation

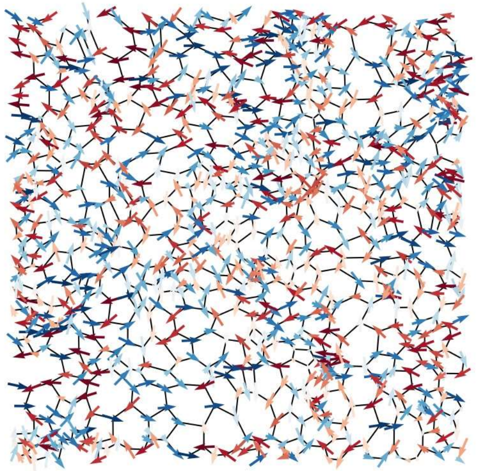
Preliminary Results of the Kitaev-Heisenberg Model

Pure Kitaev model
+ small FM Heisenberg:

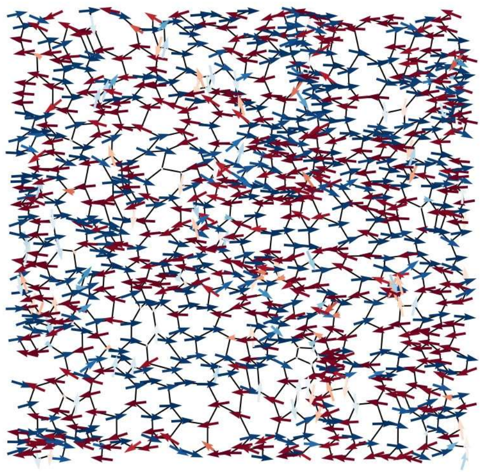


Re-entrant spin glass physics?

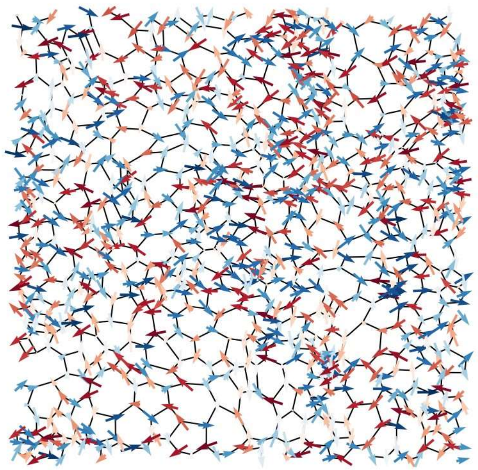
Spin Glass (?) ($T=10^{-4}$)



Nematic ($T=10^{-3}$)



Paramagnet ($T=10^{-2}$)



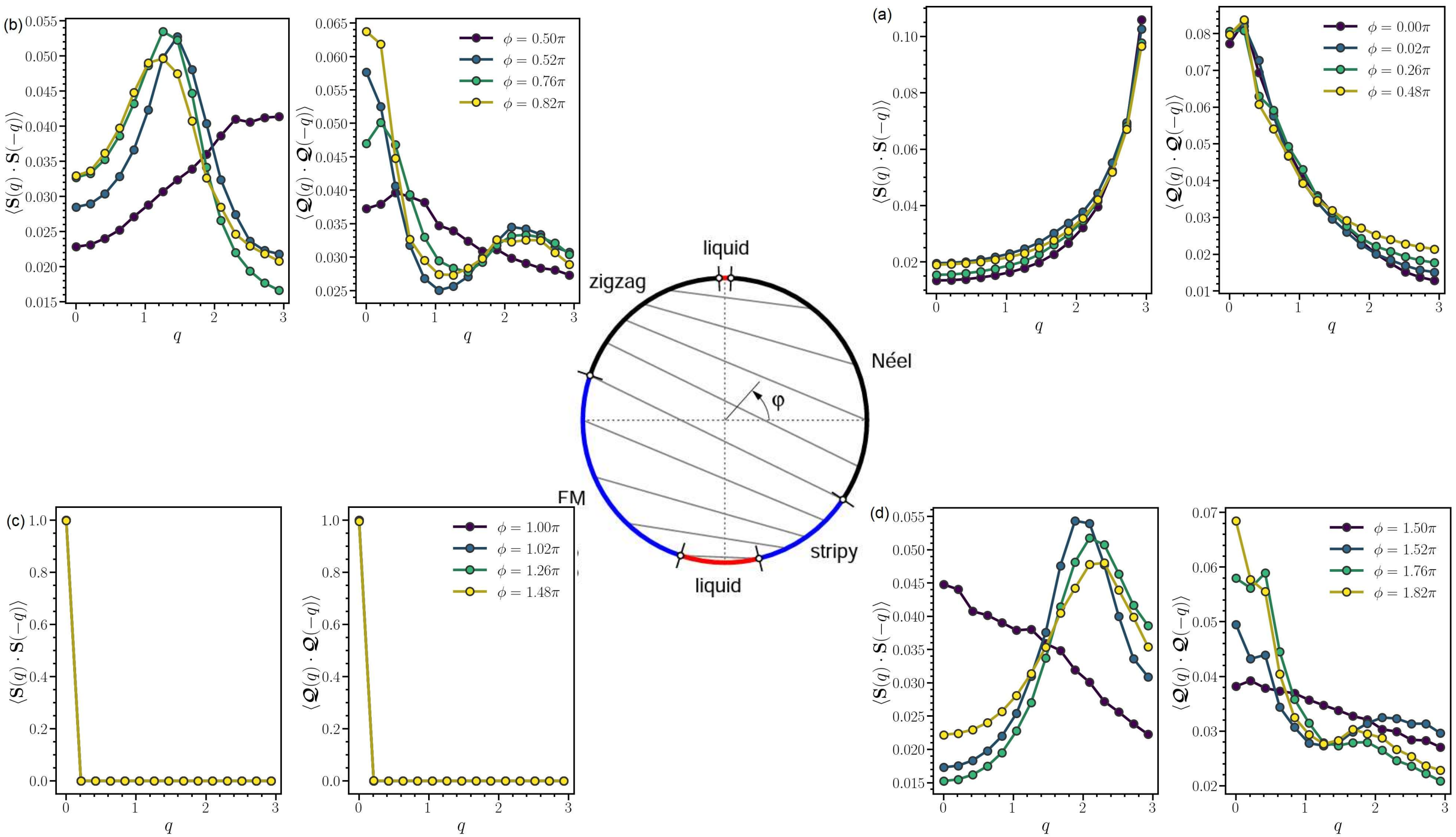
A. Zelenszkiy, **WMHN** et al., in preparation

Preliminary Results of the Kitaev-Heisenberg Model

Spin and Nematic Static Correlation Functions

$$S^{\alpha\beta}(q) = \int d\Omega \sum_{ij} \left\langle S_i^{\alpha} S_j^{\beta} \right\rangle e^{i\mathbf{q}\cdot(\mathbf{r}_i - \mathbf{r}_j)}$$
$$= \left\langle S_q^{\alpha} S_{-q}^{\beta} \right\rangle$$

$$\mathcal{Q}_i^{\alpha\beta} = \frac{3}{2} S_i^{\alpha} S_i^{\beta} - \frac{1}{2}\delta^{\alpha\beta}$$



A. Zelenszkiy, **WMHN** et al., in preparation

Outlook

1_ Spin Glass?

PHYSICAL REVIEW B **90**, 205112 (2014)

Magnetism in spin models for depleted honeycomb-lattice iridates: Spin-glass order towards percolation

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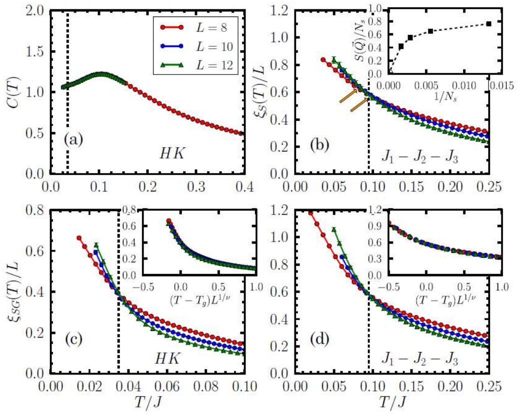
(Received 24 September 2013; revised manuscript received 16 October 2014; published 7 November 2014)



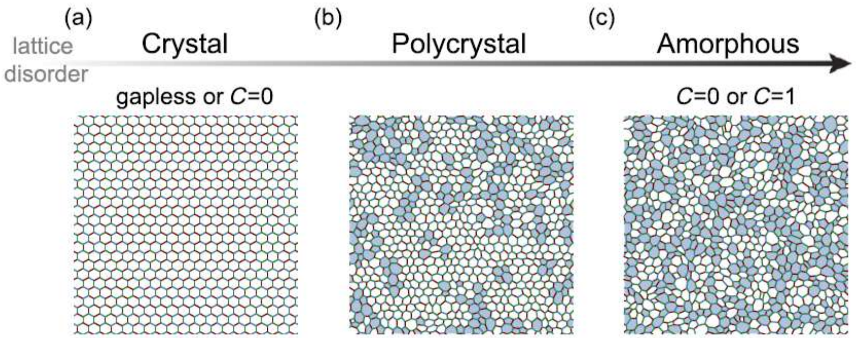
On depleted honeycomb, **interlayer coupling is required to observe SG**. Are we observing:

- A disordered ground state that becomes SG if we include interlayer interactions?

or

- A glassy phase emerging from the glass structure?

2_ Relationship between orders in amorphous and regular lattices

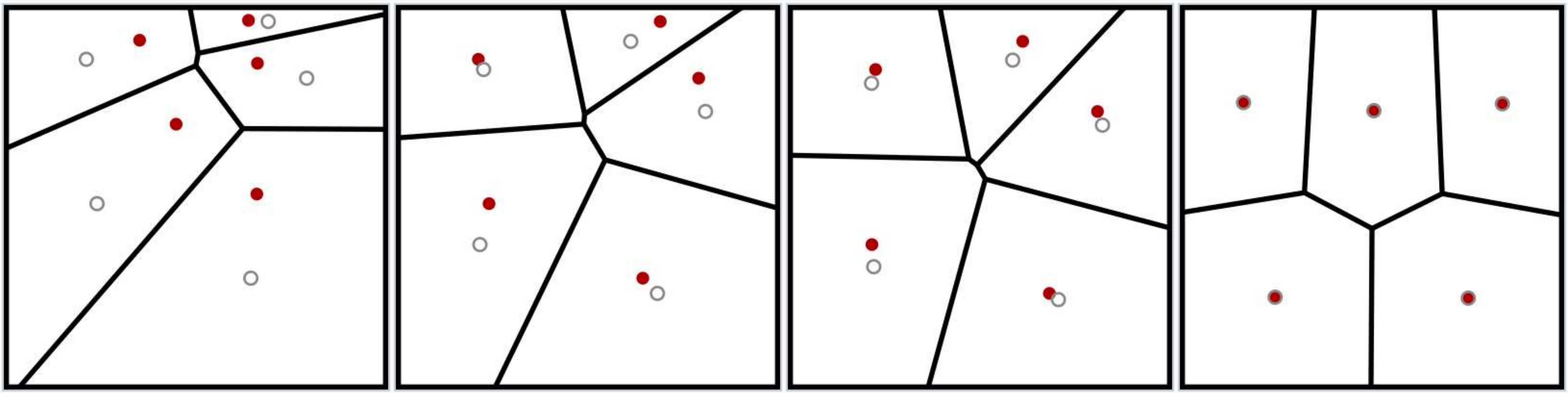


Grushin and Repellin, PRL **130**, 186702

- Is there a phase transition between the ordered crystalline and amorphous phases, or are they adiabatically connected?
- How to include quantum fluctuations to these orders?

Thanks!

Voronoi Iteration



Iteration 1 | Iteration 2 | Iteration 3 | Iteration 15