Holographic baryonic matter without flavor branes

Carlos Hoyos

Universidad de Oviedo

Holography@25

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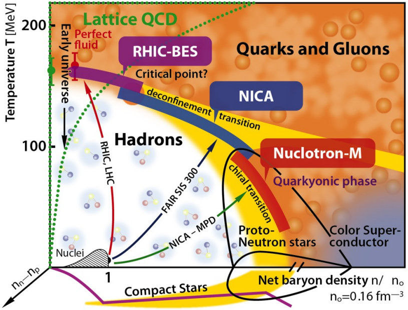
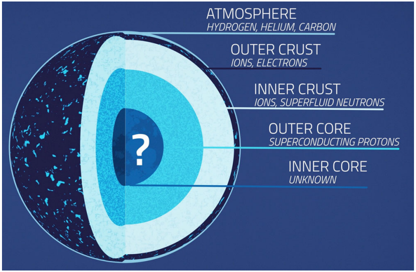
A. Faedo, C.H., J. Subils: 2212.04996, 2304.07257

Introduction

Motivation





- Large baryon density phases relevant for astrophisical observations and heavy ion experiments
- Outside the range of first principles methods suchs as lattice QCD and perturbative QCD.

Motivation

- Gauge/gravity duality can give us information about the large density regime in strongly coupled theories
- Top-down models: try to extract qualitative properties and general lessons
- Bottom-up models: try to fit available QCD data

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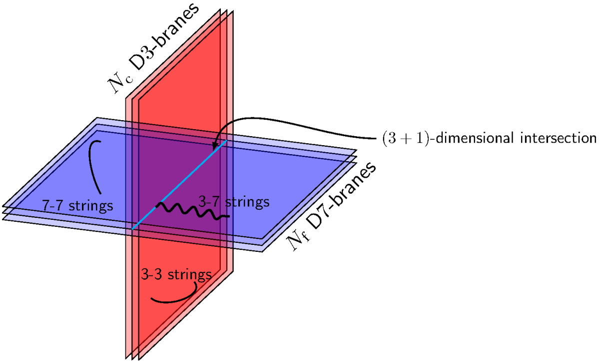
Baryon symmetry in holographic models

Baryon symmetry can be realized by adding additional branes [Karch, Katz '02]



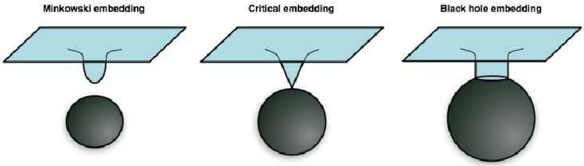
Abelian gauge field on flavor branes $\longleftrightarrow$ $U(1)_B$ current

Non-zero baryon density

- Baryon density = radial electric flux of dual gauge field

$$\langle J^{\mu}_{B} \rangle \simeq \sqrt{-g} F^{\mu r}$$

- No $U(1)_B$ charged fields on the flavor brane
- Deconfined phase: charge behind black hole horizon



Non-zero baryon density

- Confined phase: baryon = soliton

$$\mathcal{B} \sim \epsilon_{i_1 \cdots i_{N_c}} \underbrace{\psi^{i_1} \cdots \psi^{i_{N_c}}}_{\sim N_c \text{ fields}}$$

  - Witten-Sakai-Sugimoto: D4 wrapped on S^4

$$\int_{D4 \cap D8} \mathrm{tr}\,(F \wedge F) \neq 0$$

  Instanton on the D8 branes

- Multi-instanton solutions: non-homogeneous

  [Rho, Sin, Zahed '09; Kaplunovsky, Melnikov, Sonnenschein '12]

- Homogeneous: singular solutions

  [Bergman, Lifschytz, Lippert '07; Rozali, Shieh, Van Raamsdonk, Wu '07; Kim, Sin, Zahed '07]

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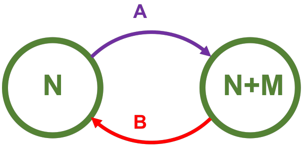
Other realizations

Baryon symmetries in quiver theories: e.g. Klebanov-Strassler



- Confining $\mathcal{N} = 1$ theory in $3 + 1$ dimensions
- Baryon symmetry: $A \to e^{i\alpha} A, \quad B \to e^{-i\alpha} B$
- Duality cascade $N = nM$
  $U(N) \times U(N+M) \to U(N-M) \times U(N) \to \cdots \to U(M) \times U(2M) \to U(M)$

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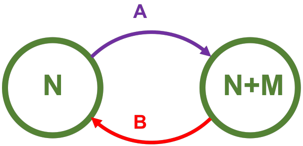

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- Baryon current dual to components of supergravity fields
- There are no fields carrying baryon charge: baryons $=$ wrapped D-branes

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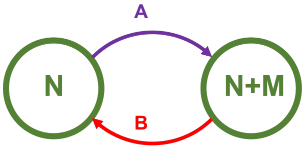

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- Baryon current dual to components of supergravity fields
- There are no fields carrying baryon charge: baryons $=$ wrapped D-branes

Almost the same as flavor branes... except baryon gauge field mixes with other vector fields

Holographic dual of a confining theory

Supergravity solutions

Type IIA SUGRA solutions (D2 branes on a cone with G_2 holonomy) [Cvetic, Gibbons, Lu, Pope '01]

$$\mathrm{d}s^2_{\rm st} = h^{-\frac{1}{2}}\left(-\mathsf{b}\ \mathrm{d}t^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2\right) + h^{\frac{1}{2}}\left(\frac{\mathrm{d}r^2}{\mathsf{b}} + e^{2f}\mathrm{d}\Omega_4^2 + e^{2g}\left[\left(E^1\right)^2 + \left(E^2\right)^2\right]\right)$$

Internal space $\sim \mathbb{CP}^3$ (more generally nearly-Kähler 6d space)

$$N = N_{D2} \sim \int_{\mathbb{CP}^3} F_6, \quad M = N_{D4} \sim \int_{\mathbb{CP}^2} F_4, \quad k = N_{D6} \sim \int_{\mathbb{CP}^1} F_2$$

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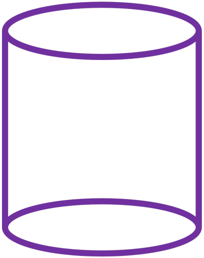
- Black brane: $\mathsf{b}(r_h) = 0$
- Regular: $e^{2g(r_0)} = 0$
- M-theory uplift: $\left\{ \right.$
  - $k \neq 0$: $\mathbb{R}^4 \times S^4$ topology
  - $k = 0$: $\mathbb{R}^3 \times S^1 \times S^4$ topology

Supergravity solutions

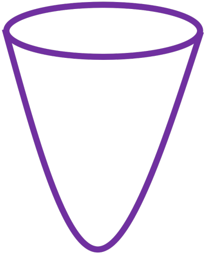
M2-brane on M-theory circle $\longrightarrow$ fundamental string

$k = 0$



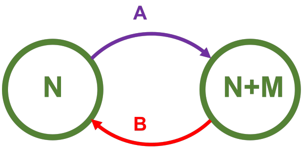
Confinement

$k \neq 0$



Not confining

Dual field theory



- $\mathcal{N} = 1$ supersymmetry in $2 + 1$ dimensions
- Gauge group $U(N) \times U(N + M)$, (anti)bifundamentals A_i, B_i, $i = 1, 2$

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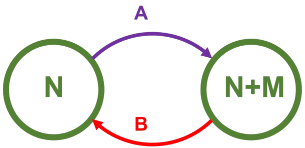

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- Duality cascade $N = nM$
  $U(N)\times U(N+M) \to U(N-M)\times U(N) \to \cdots \to U(M)\times U(2M) \to U(M)$
  Similar to ABJ [Aharony, Hashimoto, Hirano, Ouyang '09; Hashimoto, Hirano, Ouyang '10]

Dual field theory

$$S_{CS} = \frac{k}{4\pi}\int (\mathrm{tr}\ \omega_3(A_1) - \mathrm{tr}\ \omega_3(A_2))$$

$$\omega_3(A_i) = A_i \wedge dA_i + \frac{2}{3} A_i \wedge A_i \wedge A_i$$

- Chern-Simons level k $\left\{ \begin{array}{l} \bullet\ k \neq 0\text{: mass gap w/o confinement} \\ \bullet\ k = 0\text{: mass gap \& confinement} \end{array} \right.$

[Faedo, Mateos, Pravos, Subils '17; Elander, Faedo, Mateos, Subils '20]

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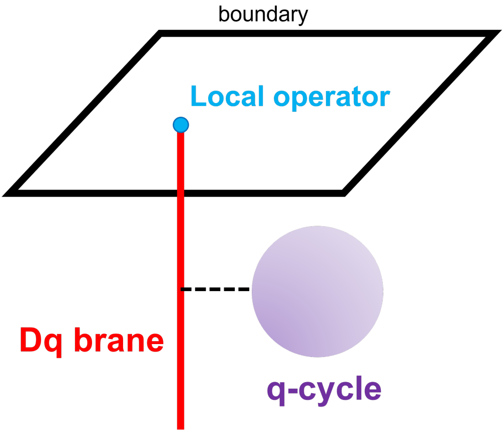
[Faedo, Mateos, Pravos, Subils '17; Elander, Faedo, Mateos, Subils '20]

- We will restrict to confining theories $k = 0$

Scale of mass gap (for $k = 0$)

$$\Lambda_{\text{QCD}} = \lambda \left(\frac{M}{N} \right)^3$$

Local operators as wrapped D-branes



Local operators as wrapped D-branes

In ABJM/ABJ:

- **Monopoles** $\mathcal{M}_{m_1,m_2}$

$$m_a = \frac{1}{2\pi}\int_{S^2} \mathrm{tr}\,(F_a)\,,\;\; a = 1,2$$

$$\boxed{\mathcal{M}_{1,1} \leftrightarrow D0 \qquad\qquad \mathcal{M}_{1,-1} \leftrightarrow D2}$$

$$\star_3 J_{\mathcal{M}} = \mathrm{tr}\,F_1 + \mathrm{tr}\,F_2$$

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- **(Di)baryons**

$$\mathcal{D}^{\beta_{N+1}\cdots\beta_{N+M}} = \epsilon^{\alpha_1\cdots\alpha_{N+M}}\epsilon_{\beta_1\cdots\beta_N} A_{\alpha_1}^{\;\beta_1}\cdots A_{\alpha_{N+M}}^{\;\beta_{N+M}}$$

$$\mathcal{B} = \epsilon_{\beta_1\cdots\beta_M\cdots\beta_{N-M+1}\cdots\beta_N} D^{\beta_1\cdots\beta_M}\cdots D^{\beta_{N-M+1}\cdots\beta_N}$$

$$\boxed{\mathcal{D} \leftrightarrow D4 \qquad\qquad \mathcal{B} \leftrightarrow D6}$$

$$J_{\mathcal{B}\,\mu} = iA^\dagger \overleftrightarrow{\partial}_\mu A - iB^\dagger \overleftrightarrow{\partial}_\mu B$$

U vs SU

[Bergman, Tachikawa, Zafrir '20]

Gauge group	Symmetry	Local operators	Dual branes
$U(N) \times U(N+M)$	$U(1)_{\mathcal{M}}$	Monopoles	D0, D2
$SU(N) \times SU(N+M)$	$U(1)_{\mathcal{B}}$	(Di)baryons	D4, D6

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Brane	Electric coupling	Magnetic coupling
D0	C_1	C_7
D2	C_3	C_5
D4	C_5	C_3
D6	C_7	C_1

Dirichlet b.c. $C_{p+1} \leftrightarrow$ Neumann b.c. C_{7-p}
Dp brane can end on the boundary $\leftrightarrow$ D$(6-p)$ cannot end on the boundary

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$SU(N) \times SU(N+M)$	C_5, C_7	C_1, C_3

Non-zero density

Vector fields

type IIA SUGRA 10 dimensions $\rightarrow$ SUGRA 4 dimensions

$$
\begin{aligned}
B_2 &= b_2 \\
C_1 &= a_1 \\
C_3 &= \tilde{a}_1 \wedge X_2 + \hat{a}_1 \wedge J_2
\end{aligned}
$$

- a_1 massless $\rightarrow$ dual to conserved $U(1)$ current
- $\tilde{a}_1 + \hat{a}_1$ Stueckelberg mass
- $\tilde{a}_1 - \hat{a}_1$ massive through topological term

$$
\sim \int (\tilde{a}_1 - \hat{a}_1) \wedge db_2
$$

Vector fields

- Note

$$A_{D0} \sim C_1 \sim a_1, \qquad\qquad A_{D2} \sim \int_{\mathbb{C}\mathrm{P}^1} C_3 \sim \tilde{a}_1 - \hat{a}_1$$

- Dirichlet b.c. for C_1, $C_3 \to U(N) \times U(N+M)$ theory

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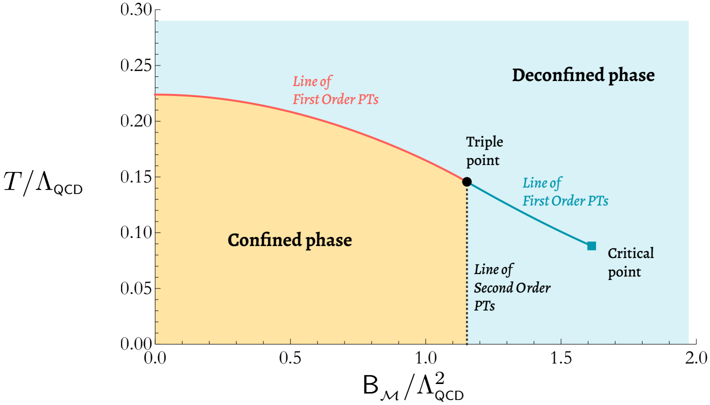
- $U(1)_{\mathcal{M}}$ symmetry: a_1 dual to monopole current $a_1 = g_s \ell_s \frac{M^2}{N} \mathsf{A}_\mu dx^\mu$
- Ansatz for non-zero density and magnetic field:

$$a_1 = a_t(r)dt + g_s \ell_s \frac{M^2}{N} \frac{\mathsf{B}_{\mathcal{M}}}{2}(x_1 dx_2 - x_2 dx_1)$$

Polyakov confinement



- Phase diagram for $\mu_{\mathcal{M}} = 0$
- Charge $Q_{\mathcal{M}} = 0$ in confined phase
- Magnetization $\text{M}_{\mathcal{M}} \neq 0$ in all phases

Particle-vortex duality in $2+1$ dimensions

Conserved current in a superfluid:

$$J_\mu = \partial_\mu \theta, \qquad \partial_\mu J^\mu = 0$$

Dual gauge field a_μ

$$J^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda} f_{\nu\lambda} = \epsilon^{\mu\nu\lambda}\partial_\nu a_\lambda$$

Charges (particles) $\longleftrightarrow$ magnetic flux (vortices)

$$\rho = J^0 = \epsilon^{ij}\partial_i a_j = b$$

From U to SU

- We should impose Dirichlet b.c. for

$$A_{D4} \sim \int_{\mathbb{C}\mathrm{P}^2} C_5 \sim \hat{A}_1, \qquad A_{D6} \sim \int_{\mathbb{C}\mathrm{P}^3} C_7 \sim A_1$$

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- A_1 is massless $\to$ dual to $U(1)_{\mathcal{B}}$ baryon current

$$S_{WZ}^{D6} = -T_{D6} V(\mathbb{C}\mathrm{P}^3) \int A_1 \quad \Rightarrow \quad A_1 = \frac{3}{2\pi^3}(2\pi\ell_s)^6 g_s \ell_s \frac{N^2}{M} \mathsf{A}_{\mathcal{B}}$$

- $\hat{A}_1$ is massive

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- $\hat{A}_1$ is massive
- Change in b.c. $\leftrightarrow$ particle-vortex duality

[Witten '03; Seiberg, Senthil, Wang, Witten '16]

$$S_{\text{PV dual}} = \frac{1}{2\kappa_4^2} \int \mathrm{d}a_1 \wedge \mathrm{d}A_1 = \frac{NM}{2\pi} \int \mathrm{d}\mathsf{A}_{\mathcal{M}} \wedge \mathrm{d}\mathsf{A}_{\mathcal{B}}$$

Total derivative: eoms do not change

PV dual thermodynamics

- Free energy

$$G_{\mathcal{B}} = G_{\mathcal{M}} + \mu_{\mathcal{M}} Q_{\mathcal{M}} - \mu_{\mathcal{B}} Q_{\mathcal{B}}$$

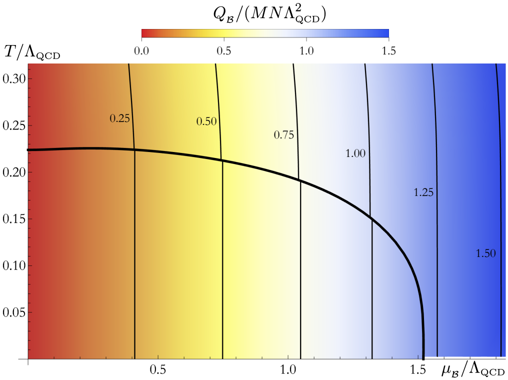
- Baryon chemical potentials and charge

$$\mu_{\mathcal{B}} = -\frac{2\pi}{NM}\frac{\partial G_{\mathcal{M}}}{\partial \mathsf{B}_{\mathcal{M}}} = \frac{2\pi}{NM}\mathsf{M}_{\mathcal{M}}, \qquad Q_{\mathcal{B}} = -\frac{NM}{2\pi}\mathsf{B}_{\mathcal{M}}$$

- Baryon magnetic field and magnetization

$$\mu_{\mathcal{M}} = \frac{2\pi}{NM}\frac{\partial G_{\mathcal{B}}}{\partial \mathsf{B}_{\mathcal{B}}} = -\frac{2\pi}{NM}\mathsf{M}_{\mathcal{B}}, \qquad Q_{\mathcal{M}} = \frac{NM}{2\pi}\mathsf{B}_{\mathcal{B}}$$

Phase diagram



- Phase diagram for $B_{\mathcal{B}} = 0$
- Charge $Q_{\mathcal{B}} \neq 0$ in confined phase

Phase diagram

- Confined phase: baryon superfluid

$$Q_{\mathcal{M}} = 0 \longleftrightarrow \; B_{\mathcal{B}} = 0$$

Arbitrary magnetization: Meissner effect

Phase diagram

- Confined phase: baryon superfluid

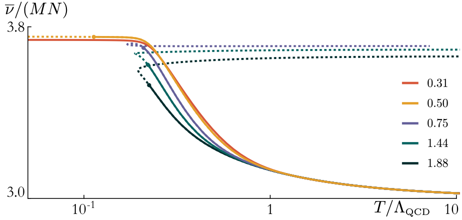
$$Q_{\mathcal{M}} = 0 \longleftrightarrow B_{\mathcal{B}} = 0$$

Arbitrary magnetization: Meissner effect

- Deconfined phase: baryon ferromagnet $G_{\mathcal{B}} \supset -\frac{T}{V_2}\int \mathrm{d}^3x\, \mathsf{M}_{\mathcal{B}}(T,\mu_{\mathcal{B}})\, \mathsf{B}_{\mathcal{B}}$

[Jensen, Kaminski, Kovtun, Meyer, Ritz, Yarom '11; Banerjee, Bhattacharya, Bhattacharyya, Jain, Minwalla, Sharma '12; Jensen, Kaminski, Kovtun, Meyer, Ritz, Yarom '12; Haehl, Rangamani '13]

$$\mathsf{M}_{\mathcal{B}} = -\mu_{\mathcal{B}}\, \frac{\bar{\nu}}{2\pi}$$



Outlook

Outlook

First example of a fully supergravity solution dual to a confining state with homogeneous baryon density

Possible future directions:

- Goldstone boson in baryon superfluid phase
- Phase diagram of theories with Chern-Simons terms $k \neq 0$
- Baryon density in four-dimensional theories: Klebanov-Strassler (deconfined solutions [Herzog, Klebanov, Pufu, Tesileanu '09])