Designing Topological Quantum Matter

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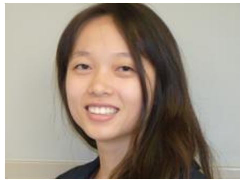
Collaborators

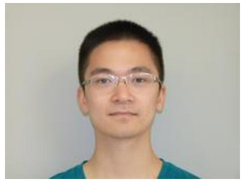
2020: PRL
2021: 2x PRB, 2x PRX Quantum
2022: 3x SciPost submissions
2023: arXiv

Dmitry Green

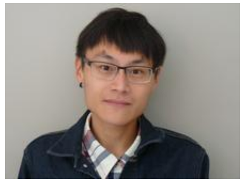
Zhi-Cheng Yang



Shiyu Zhou



Elliot Yu



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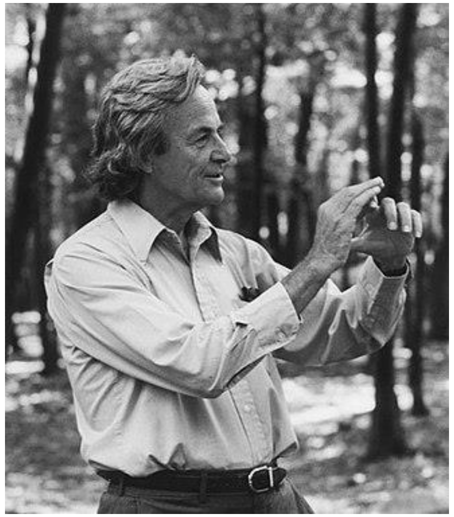
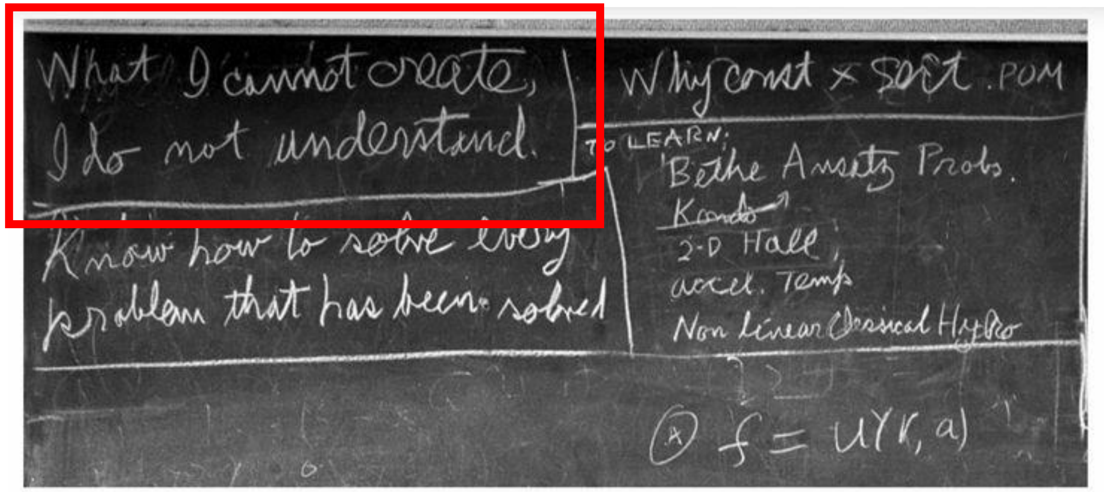
Motivation

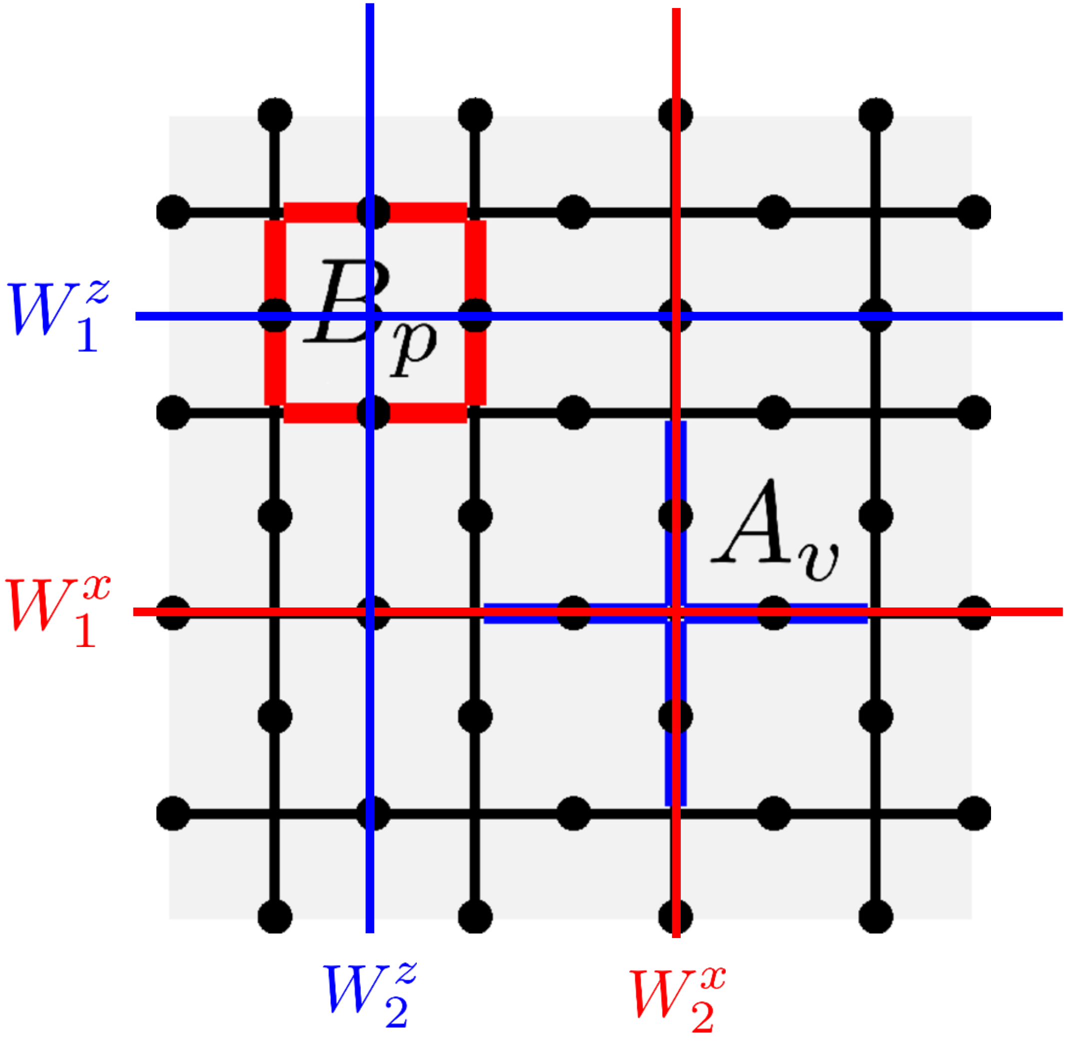
Build topological phases (e.g., toric code or gauge models) with ***physical*** interactions (2-spin interactions or Josephson couplings)





Example: $\mathbb{Z}_2$ Toric Code

A. Kitaev (1997)



$$A_v = \prod_{i \in v} \sigma_i^z \qquad B_p = \prod_{i \in p} \sigma_i^x$$

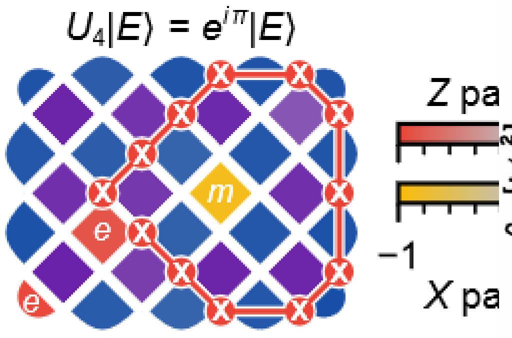
$$H = -\lambda_A \sum_s A_s - \lambda_B \sum_p B_p$$

constraints on the torus: $\prod_v A_v = \prod_p B_p = \mathbb{1}$

ground state degeneracy: 2^2

Motivation

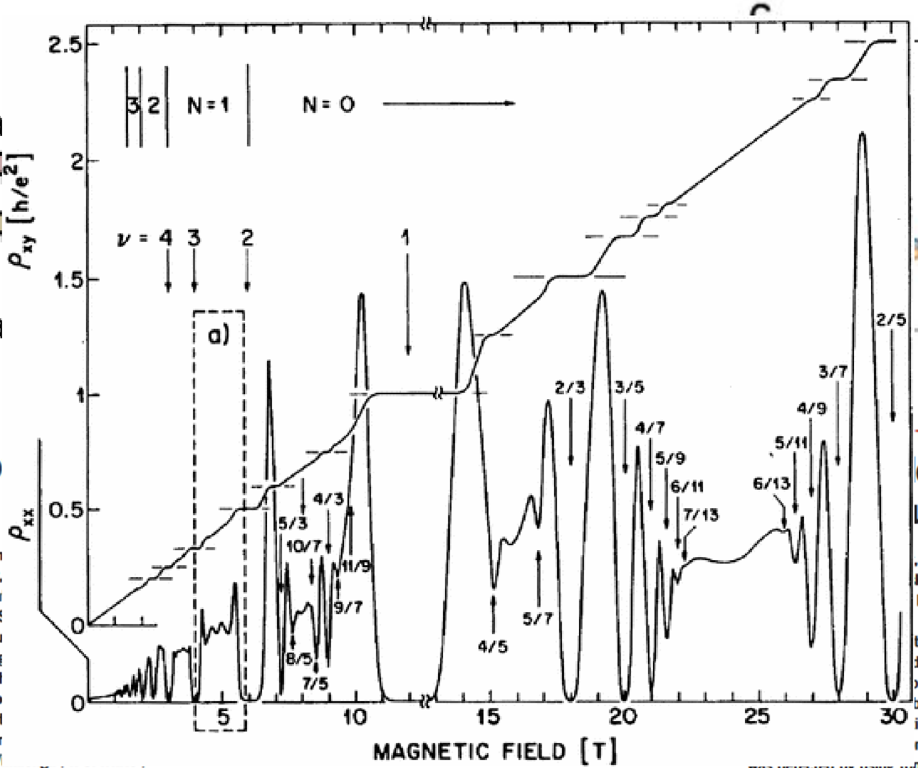
Build topological phases (e.g., toric code or gauge models) with ***physical*** interactions (2-spin interactions or Josephson couplings)



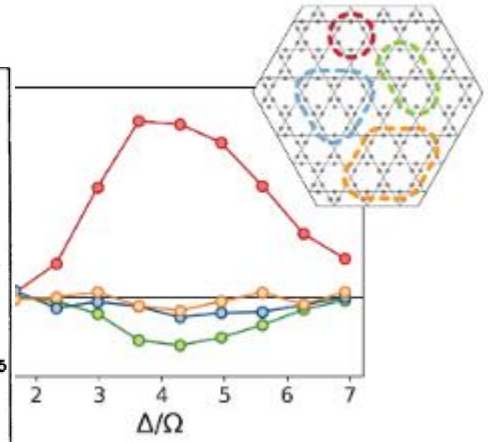
TOPOLOGICAL MATTER

Realizing topologically ordered states o quantum processor

K. J. Satzinger[1*], Y.-J Liu[2,3], A. Smith[2,4,5], C. Knapp[6,7†], M. Newman[1], C. Jon X. Mi[1], A. Dunsworth[1], C. Gidney[1], I. Aleiner[1], F. Arute[1], K. Arya[1], J. Atalaya[1], J. C. Bardin[1,8], R. Barends[1], J. Basso[1], A. Bengtsson[1], A. Bilmes[1], M. Brough D. A. Buell[1], B. Burkett[1], N. Bushnell[1], B. Chiaro[1], R. Collins[1], W. Courtney[1], D. Eppens[1], C. Erickson[1], L. Faoro[9], E. Farhi[1], A. G. Fowler[1], B. Foxen[1], M. Giu J. A. Gross[1], M. P. Harrigan[1], S. D. Harrington[1], J. Hilton[1], S. Hong[1], T. Huang L. B. Ioffe[1], S. V. Isakov[1], E. Jeffrey[1], Z. Jiang[1], D. Kafri[1], K. Kechedzhi[1], T. Kl P. V. Klimov[1], A. N. Korotkov[1,11], F. Kostritsa[1], D. Landhuis[1], P. Laptev[1], A. Lo O. Martin[1], J. R. McClean[1], M. McEwen[1,12], K. C. Miao[1], M. Mohseni[1], S. Mont J. Mutus[1], O. Naaman[1], M. Neeley[1], C. Neill[1], M. Y. Niu[1], T. E. O'Brien[1], A. Opr A. Petukhov[1], N. C. Rubin[1], D. Sank[1], V. Shvarts[1], D. Strain[1], M. Szalay[1], B. V Z. Yao[1], P. Yeh[1], J. Yoo[1], A. Zalcman[1], H. Neven[1], S. Boixo[1], A. Megrant[1], Y. Chen[1], J. Kelly[1], V. Smelyanskiy[1], A. Kitaev[1,6,7], M. Knap[2,3,13], F. Pollmann[2,3*], P. Roushan[1*]



quantum Hall effect



TER

logical spin liquids on a programmable ulator

, A. Keesling[1,2], S. Ebadi[1], T. T. Wang[1], D. Bluvstein[1], R. Verresen[1], ki[1], R. Samajdar[1], A. Omran[1,2], S. Sachdev[1,5], A. Vishwanath[1*], M. D. Lukin[1*]

tic phases of matter with topological order, have been a major focus in physics des. Such phases feature long-range quantum entanglement that can realize robust quantum computation. We used a 219-atom programmable be quantum spin liquid states. In our approach, arrays of atoms were placed on ice, and evolution under Rydberg blockade created frustrated quantum states nset of a quantum spin liquid phase of the paradigmatic toric code type was detected by using topological string operators that provide direct signatures of topological order and quantum correlations. Our observations enable the controlled experimental exploration of topological matter and protected quantum information processing.

Motivation

Build topological phases (e.g., toric code or $\mathbb{Z}_2$ gauge models) with ***physical*** interactions (2-spin interactions or Josephson couplings)

Our goal is to build static Hamiltonians hosting topological ground states!!!

Ground state is a quiet place

Motivation

Build topological phases (e.g., toric code or gauge models) with ***physical*** interactions (2-spin interactions or Josephson couplings)

Ioffe and Feigel'man, PRB 2002

Ioffe, Feigel'man, Ioselevich, Ivanov, Troyer, and Blatter, Nature 2002

Douçot, Feigel'man, and Ioffe, PRL 2003, PRB 2005

Gladchenko, Olaya, Dupont-Ferrier, Douçot, Ioffe, and Gershenson, Nat. Phys. 2009

Douçot and Ioffe, Rep. Prog. Phys. 2012

Jordan and Farhi, Sci. Adv. 2016

J. D. Biamonte, PRA 2008

Bravyi, DiVincenzo, Loss, and Terhal, PRL 2008

Leib, Zoller, and Lechner, Quant. Sci. and Tech. 2016

Subas and Jarzynski, PRA 2016

Chancellor, Zohren, and Warburton, Quant. Info. 2017

Gaps are perturbative: how can we try to increase these gaps?

Motivation

Build topological phases (e.g., toric code or $\mathbb{Z}_2$ gauge models) with ***physical*** interactions (2-spin interactions or Josephson couplings)

“The definition of insanity is doing the same thing over and over again and expecting different results.”

Albert Einstein often gets the credit for this saying, but you probably won’t be surprised to learn that he never actually said it. This misattributed quotation has been well documented: it appears to have originated around 1980 in literature published by Narcotics Anonymous (Becker; “Insanity”).

Becker, Michael. “Einstein Probably Didn’t Say That Famous Quote about Insanity.” *Becker’s Online Journal*, 13 Nov. 2012, www.news.hypercrit.net/2012/11/13/einstein-on-insanity.

https://style.mla.org/five-commonly-misattributed-quotations/

Design **exact** gauge symmetries

Combinatorial gauge symmetry

EXACT

NOT emergent

Combinatorial gauge symmetry

What is it?

Chamon, Green, and Yang, Phys. Rev. Lett. (2020)

What symmetries preserve commutation relations for *n* spins?

Compare with the case of *n* fermions or bosons

$$\psi_i \to \tilde{\psi}_i = \sum_j U_{ij}\, \psi_j$$

$$\{\psi_i\, , \psi_j^\dagger\} = \{\tilde{\psi}_i\, , \tilde{\psi}_j^\dagger\}$$

$$\phi_i \to \tilde{\phi}_i = \sum_j U_{ij}\, \phi_j$$

$$[\phi_i\, , \phi_j^\dagger] = [\tilde{\phi}_i\, , \tilde{\phi}_j^\dagger]$$

What symmetries preserve commutation relations for *n* spins?

Eg.: 2 fermions

$$\tilde{\psi}_1 = \frac{1}{\sqrt{2}}\left(\psi_1 + \psi_2\right)$$

$$\tilde{\psi}_2 = \frac{1}{\sqrt{2}}\left(\psi_1 - \psi_2\right)$$

All anti-commutation relations are preserved

What symmetries preserve commutation relations for *n* spins?

Now say we try this (please don't) for spins

$$\tilde{\sigma}_1^a = \frac{1}{\sqrt{2}}\left(\sigma_1^a + \sigma_2^a\right)$$

$$\tilde{\sigma}_2^a = \frac{1}{\sqrt{2}}\left(\sigma_1^a - \sigma_2^a\right)$$

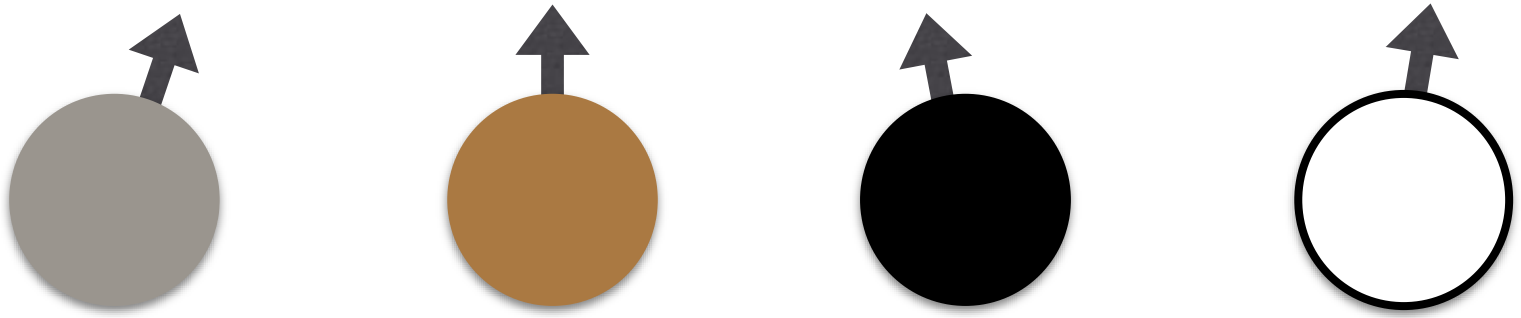
commutation and anti-commutation relations are messed up

Which transformations are allowed?

$$\sigma_i^a \rightarrow U\,\sigma_i^a\,U^\dagger \qquad U \in \mathrm{SU}(2^n)$$

$$\sigma_i^a \rightarrow \sum_j \sum_b R_{ij}^{ab}\,\sigma_j^b + \sum_{jk} \sum_{bc} \Lambda_{i,j\neq k}^{abc}\,\sigma_j^b\,\sigma_k^c + \dots$$

$$U \in \mathrm{SU}(2) \otimes \mathrm{SU}(2) \otimes \cdots \otimes \mathrm{SU}(2)$$



Which transformations are allowed?

$$\sigma_i^a \to U\, \sigma_i^a\, U^\dagger \qquad U \in \mathrm{SU}(2^n)$$

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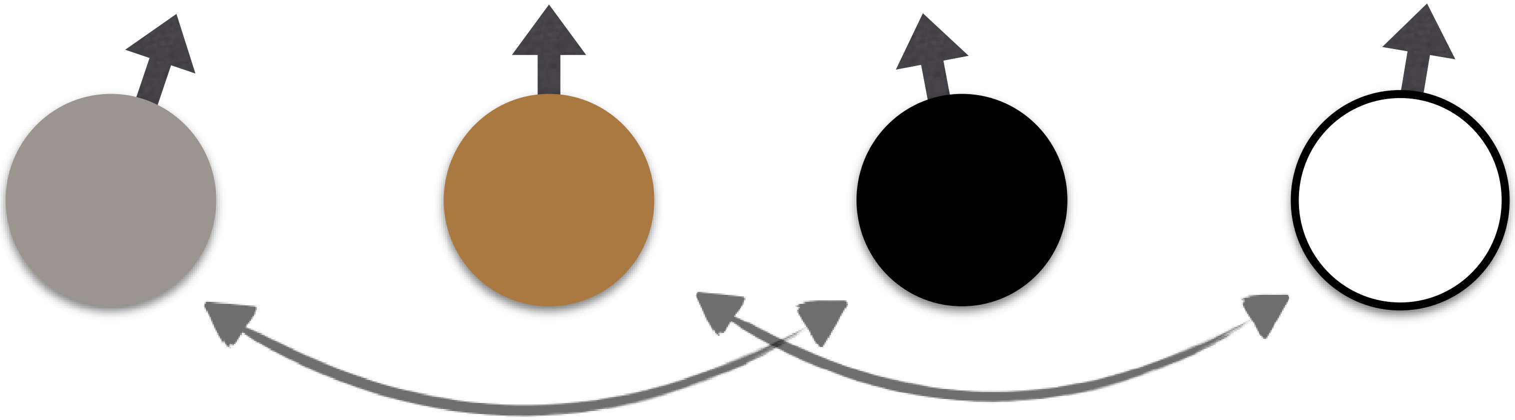
$$U \in \mathrm{SU}(2) \otimes \mathrm{SU}(2) \otimes \cdots \otimes \mathrm{SU}(2)$$



Single-spin rotations
+ permutations

Monomial transformations

Eg.: 4 spins

$$\begin{pmatrix} \vec{\sigma}_1 \\ \vec{\sigma}_2 \\ \vec{\sigma}_3 \\ \vec{\sigma}_4 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & g_1 & 0 \\ 0 & 0 & 0 & g_2 \\ g_3 & 0 & 0 & 0 \\ 0 & g_4 & 0 & 0 \end{pmatrix} \begin{pmatrix} \vec{\sigma}_1 \\ \vec{\sigma}_2 \\ \vec{\sigma}_3 \\ \vec{\sigma}_4 \end{pmatrix}$$

$$g_i \in SO(3)$$

Spin commutation relations are all preserved

Monomial transformations

$G \; P$ ⋉ semi-direct or product of two groups

$$\begin{pmatrix} \vec{\sigma}_1 \\ \vec{\sigma}_2 \\ \vec{\sigma}_3 \\ \vec{\sigma}_4 \end{pmatrix} \to \begin{pmatrix} 0 & 0 & g_1 & 0 \\ 0 & 0 & 0 & g_2 \\ g_3 & 0 & 0 & 0 \\ 0 & g_4 & 0 & 0 \end{pmatrix} \begin{pmatrix} \vec{\sigma}_1 \\ \vec{\sigma}_2 \\ \vec{\sigma}_3 \\ \vec{\sigma}_4 \end{pmatrix}$$

G rotations

P permutations

Monomial transformations

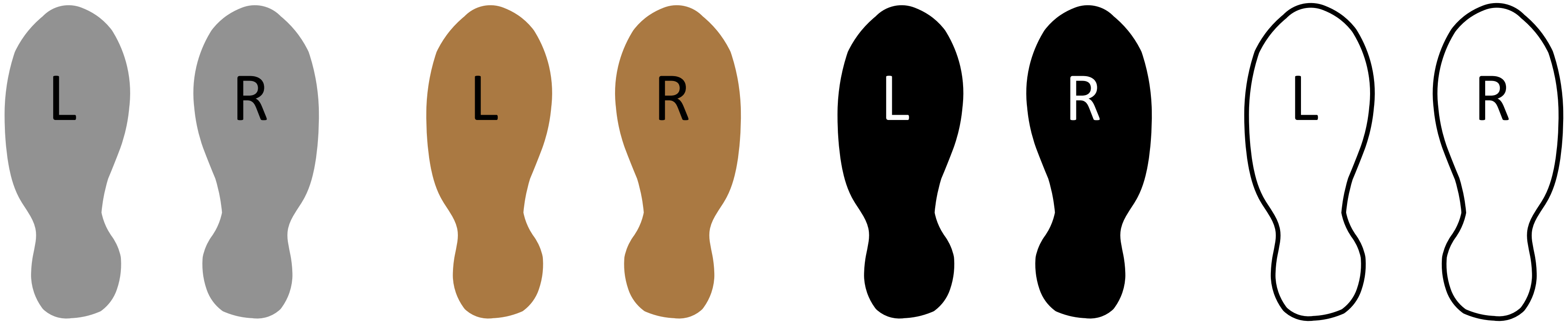
$$G \; P$$

$\ltimes$ semi-direct or product of two groups

$$\begin{pmatrix} \vec{\sigma}_1 \\ \vec{\sigma}_2 \\ \vec{\sigma}_3 \\ \vec{\sigma}_4 \end{pmatrix} \to \begin{pmatrix} 0 & 0 & g_1 & 0 \\ 0 & 0 & 0 & g_2 \\ g_3 & 0 & 0 & 0 \\ 0 & g_4 & 0 & 0 \end{pmatrix} \begin{pmatrix} \vec{\sigma}_1 \\ \vec{\sigma}_2 \\ \vec{\sigma}_3 \\ \vec{\sigma}_4 \end{pmatrix}$$

G rotations

P permutations



Monomial transformations

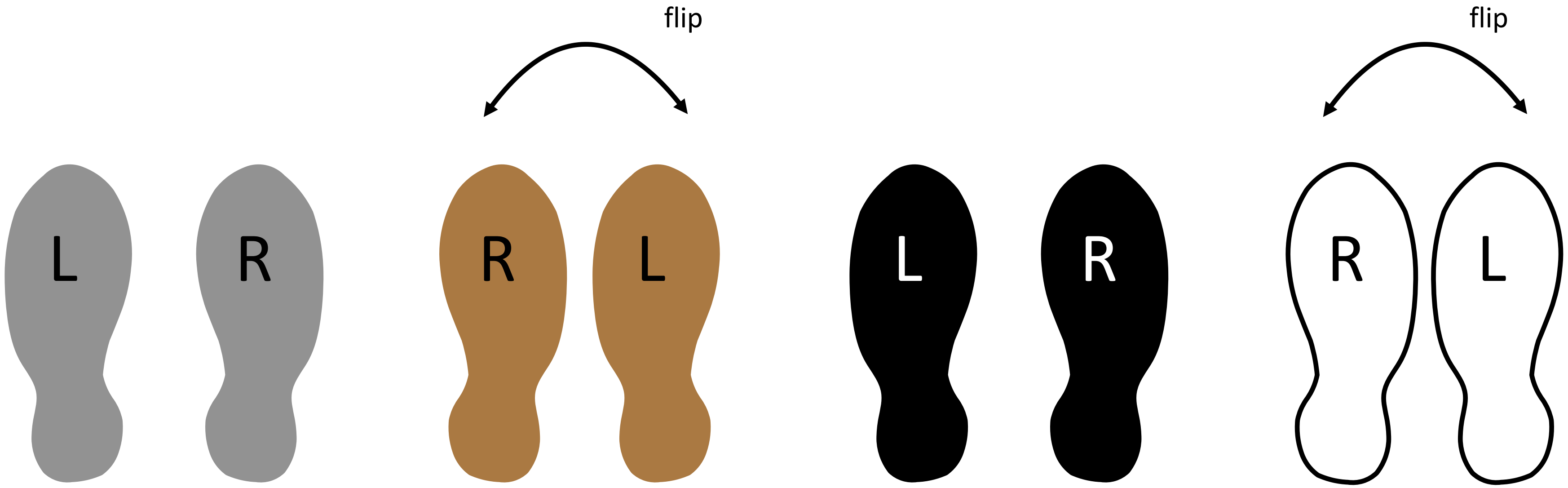
$G\ P$

$\ltimes$ semi-direct or product of two groups

$$\begin{pmatrix} \vec{\sigma}_1 \\ \vec{\sigma}_2 \\ \vec{\sigma}_3 \\ \vec{\sigma}_4 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & g_1 & 0 \\ 0 & 0 & 0 & g_2 \\ g_3 & 0 & 0 & 0 \\ 0 & g_4 & 0 & 0 \end{pmatrix} \begin{pmatrix} \vec{\sigma}_1 \\ \vec{\sigma}_2 \\ \vec{\sigma}_3 \\ \vec{\sigma}_4 \end{pmatrix}$$

G rotations

P permutations



Monomial transformations

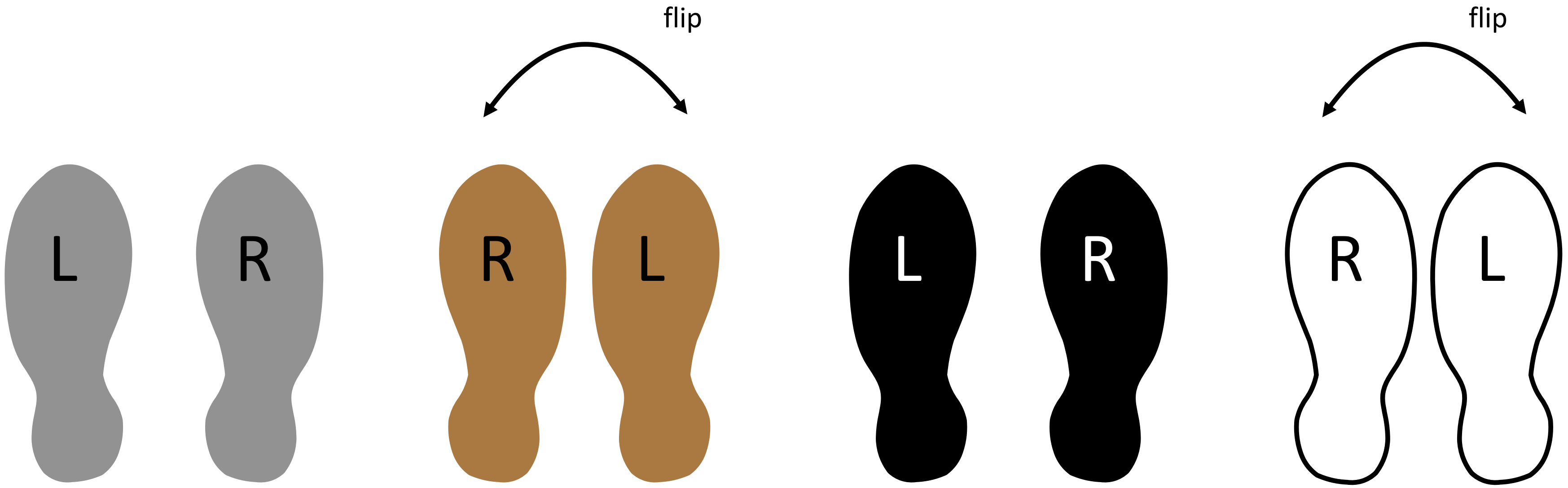
$G \ltimes P$

$\ltimes$ semi-direct or product of two groups

$$\begin{pmatrix} \vec{\sigma}_1 \\ \vec{\sigma}_2 \\ \vec{\sigma}_3 \\ \vec{\sigma}_4 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & g_1 & 0 \\ 0 & 0 & 0 & g_2 \\ g_3 & 0 & 0 & 0 \\ 0 & g_4 & 0 & 0 \end{pmatrix} \begin{pmatrix} \vec{\sigma}_1 \\ \vec{\sigma}_2 \\ \vec{\sigma}_3 \\ \vec{\sigma}_4 \end{pmatrix}$$

G rotations

P permutations



Monomial transformations

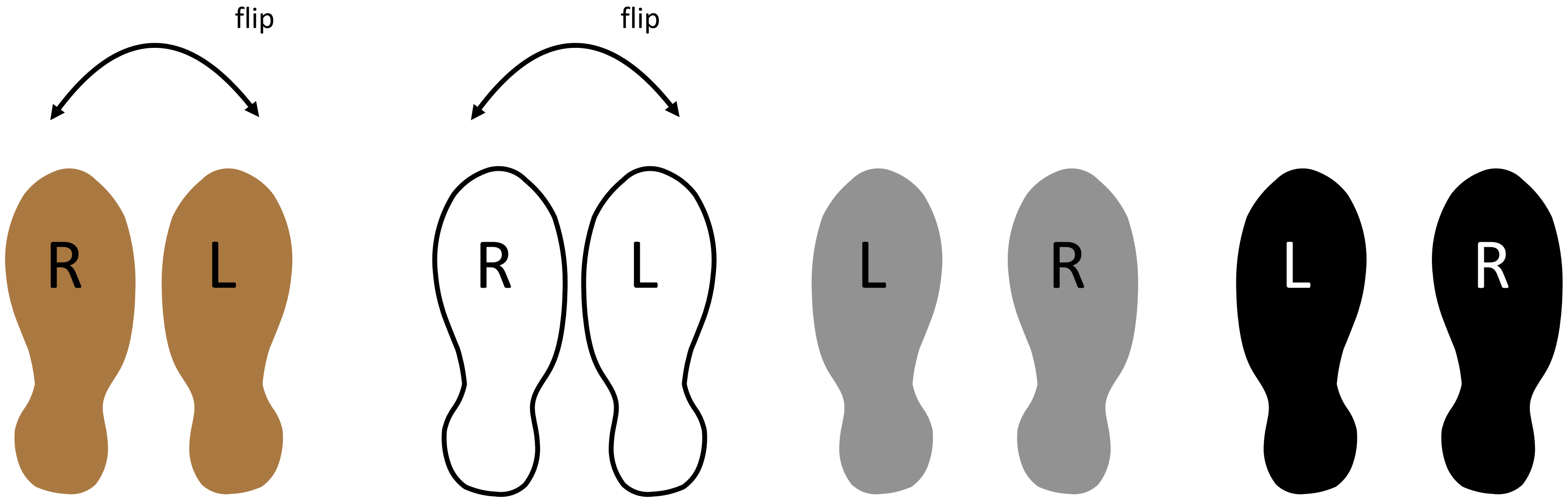
$$G \; P$$

$\ltimes$ semi-direct or product of two groups

$$\begin{pmatrix} \vec{\sigma}_1 \\ \vec{\sigma}_2 \\ \vec{\sigma}_3 \\ \vec{\sigma}_4 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & g_1 & 0 \\ 0 & 0 & 0 & g_2 \\ g_3 & 0 & 0 & 0 \\ 0 & g_4 & 0 & 0 \end{pmatrix} \begin{pmatrix} \vec{\sigma}_1 \\ \vec{\sigma}_2 \\ \vec{\sigma}_3 \\ \vec{\sigma}_4 \end{pmatrix}$$

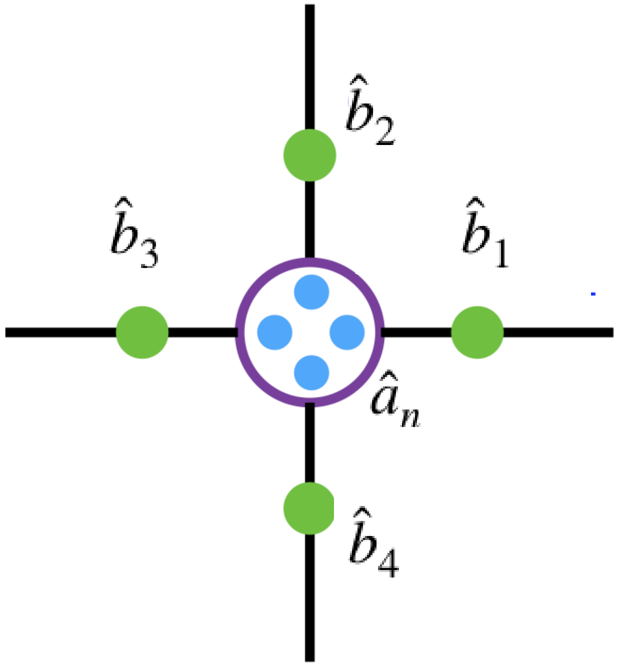
G rotations

P permutations

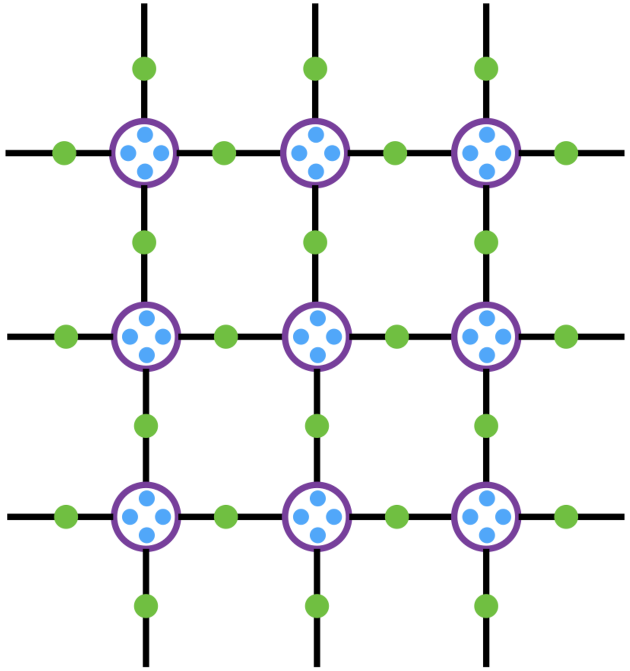


Combinatorial gauge symmetry on lattice

- “Matter” fields $\hat{a}_n$ at each vertex ($i = 1\ldots4$)
- “Gauge” fields $\hat{b}_i$ shared by vertices ($n = 1\ldots4$)



star s



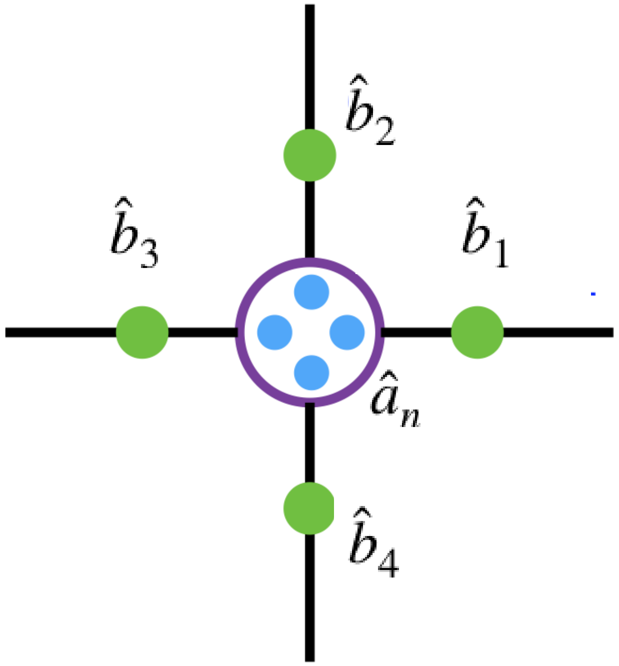
Combinatorial gauge symmetry on lattice

- “Matter” fields $\hat{a}_n$ at each vertex ($i = 1\ldots4$)
- “Gauge” fields $\hat{b}_i$ shared by vertices ($n = 1\ldots4$)

e.g.

$$\hat{a}_m = \mu^z_m$$

$$\hat{b}_i = \sigma^z_i$$



star s

Operators transform as:

$$\hat{a}_n \to \sum_m \hat{a}_m \, (L^{-1})_{mn} \quad \text{and} \quad \hat{b}_i \to \sum_j R_{ij} \, \hat{b}_j$$

$\hat{a}$ $\hat{b}$, can be spins, phases operators, etc.

L, R are monomial matrices

Combinatorial gauge symmetry on lattice

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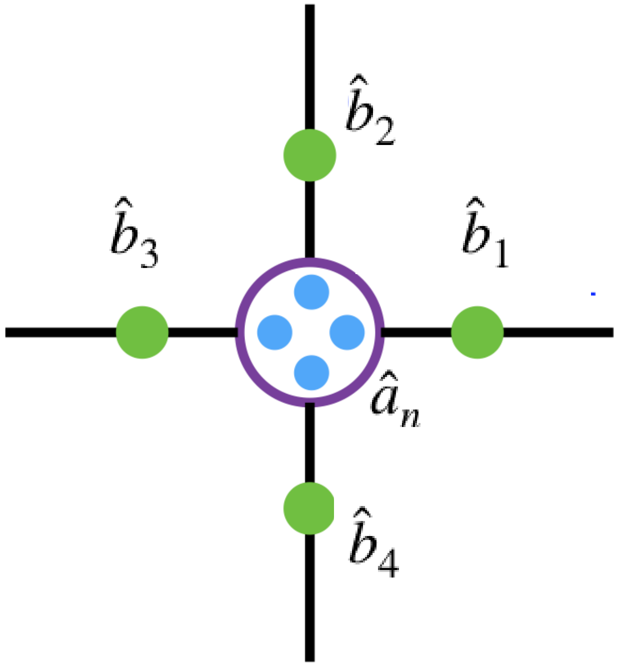


star s

Hamiltonian

$$H_J = -J \sum_s \sum_{n,i \in s} W_{ni} \left(\hat{a}_n^\dagger \, \hat{b}_i + \hat{b}_i^\dagger \, \hat{a}_n \right)$$

$$W = \begin{pmatrix} -1 & +1 & +1 & +1 \\ +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & +1 \\ +1 & +1 & +1 & -1 \end{pmatrix}$$

4x4 Hadamard

Symmetry (Automorphism)

$$L^{-1} \, W \, R = W$$

Mathematically: Hadamard automorphism

$$L^{-1}\, W R = W$$

Hadamard automorphism

$$\begin{pmatrix} 0 & +1 & 0 & 0 \\ +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \qquad \begin{pmatrix} -1 & +1 & +1 & +1 \\ +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & +1 \\ +1 & +1 & +1 & -1 \end{pmatrix} \qquad \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \end{pmatrix}$$

Monomial matrix

Monomial matrix

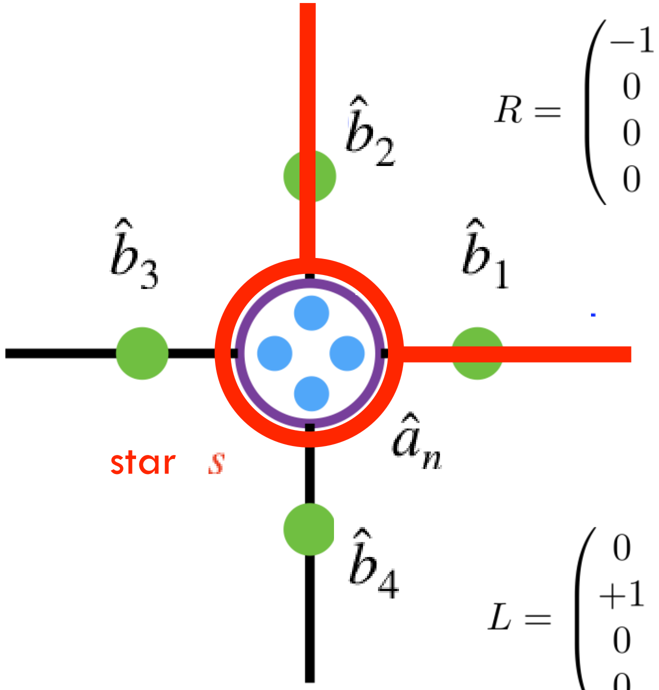
$$\hat{a}_n = \sum_{m=1}^{4} \hat{a}_m \,(L^{-1})_{m,n}$$

$$\hat{b}_i = \sum_{j=1}^{4} R_{i,j}\, \hat{b}_j$$

$$R \Rightarrow L \qquad L = W\ R\, W^{-1}$$

Combinatorial gauge symmetry on lattice

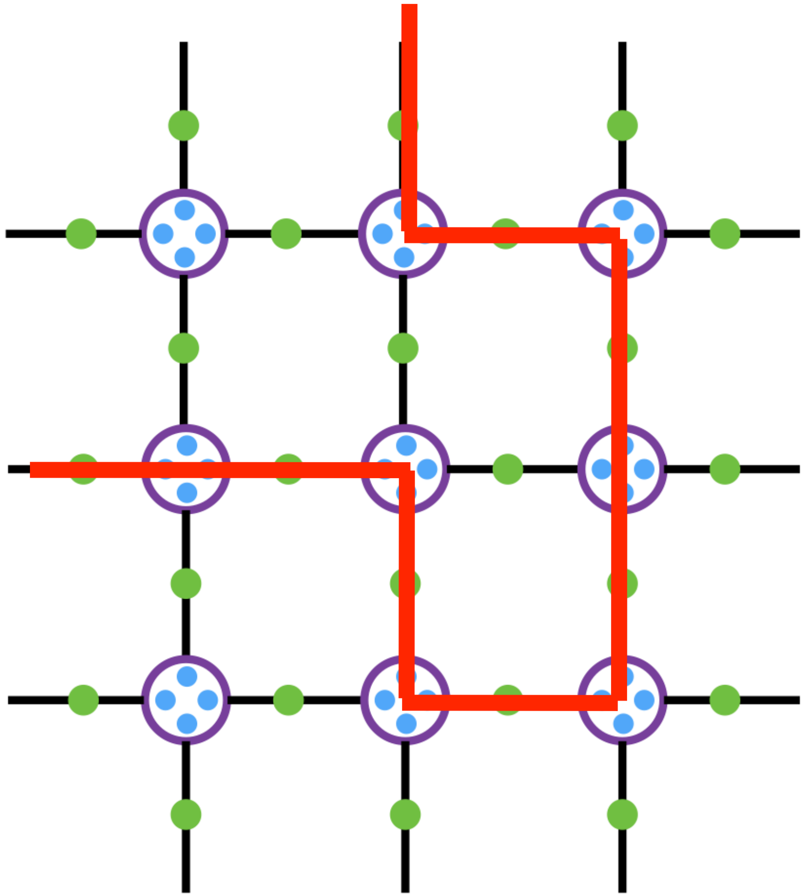
- “Matter” fields $\hat{a}_n$ at each vertex ($i = 1\ldots4$)
- “Gauge” fields $\hat{b}_i$ shared by vertices ($n = 1\ldots4$)



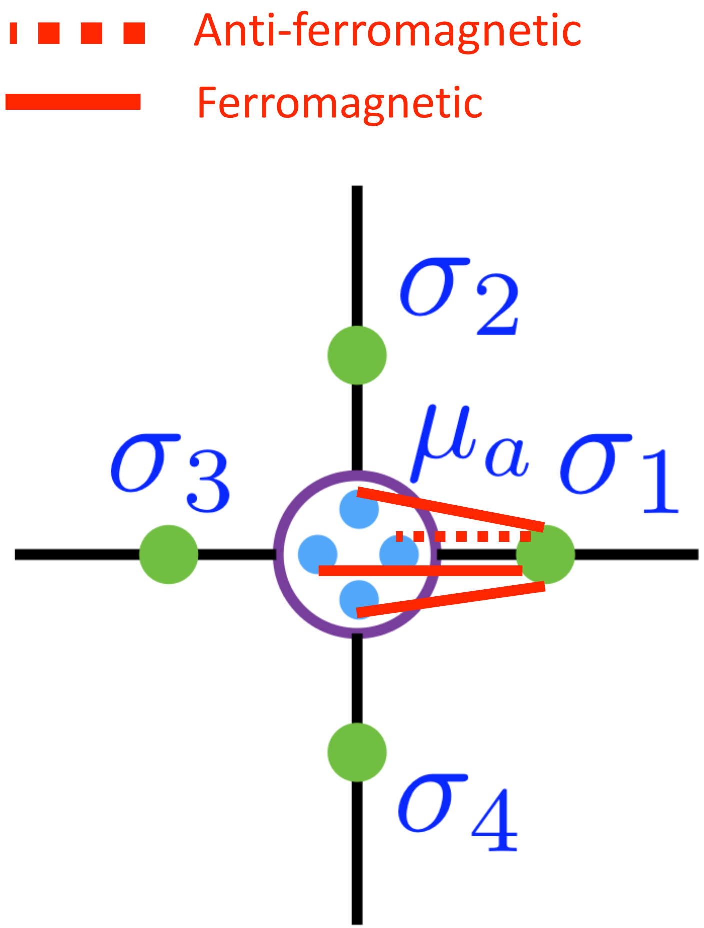
$$R = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \end{pmatrix}$$

Local $\mathbb{Z}_2$

$$L = \begin{pmatrix} 0 & +1 & 0 & 0 \\ +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$



E.g.: Spin model with gauge-matter spin-spin interaction



$$H_0 = -J \sum_{a=1}^{4} \sum_{i=1}^{4} W_{ai}\, \sigma_i^z \mu_a^z$$

$$W = \begin{pmatrix} -1 & +1 & +1 & +1 \\ +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & +1 \\ +1 & +1 & +1 & -1 \end{pmatrix}$$

4 x 4 Hadamard matrix

Invariance for *all* $J, \Gamma, \tilde{\Gamma}$

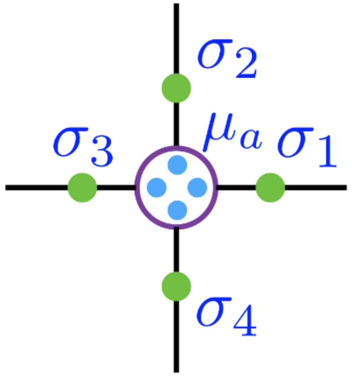
$$H = -J \sum_{s} \sum_{a,i \in s} W_{ai}\, \mu_a^z \sigma_i^z \; - \Gamma \sum_{a} \mu_a^x \; - \tilde{\Gamma} \sum_{i} \sigma_i^x$$

Transverse fields: invariant under spin flips and permutations

Monomial transformations preserve spin algebra

Warning: would lead to small gaps (sanity check!)

Simple limit: single star $\Gamma \gg J$



μ in effective field of σ

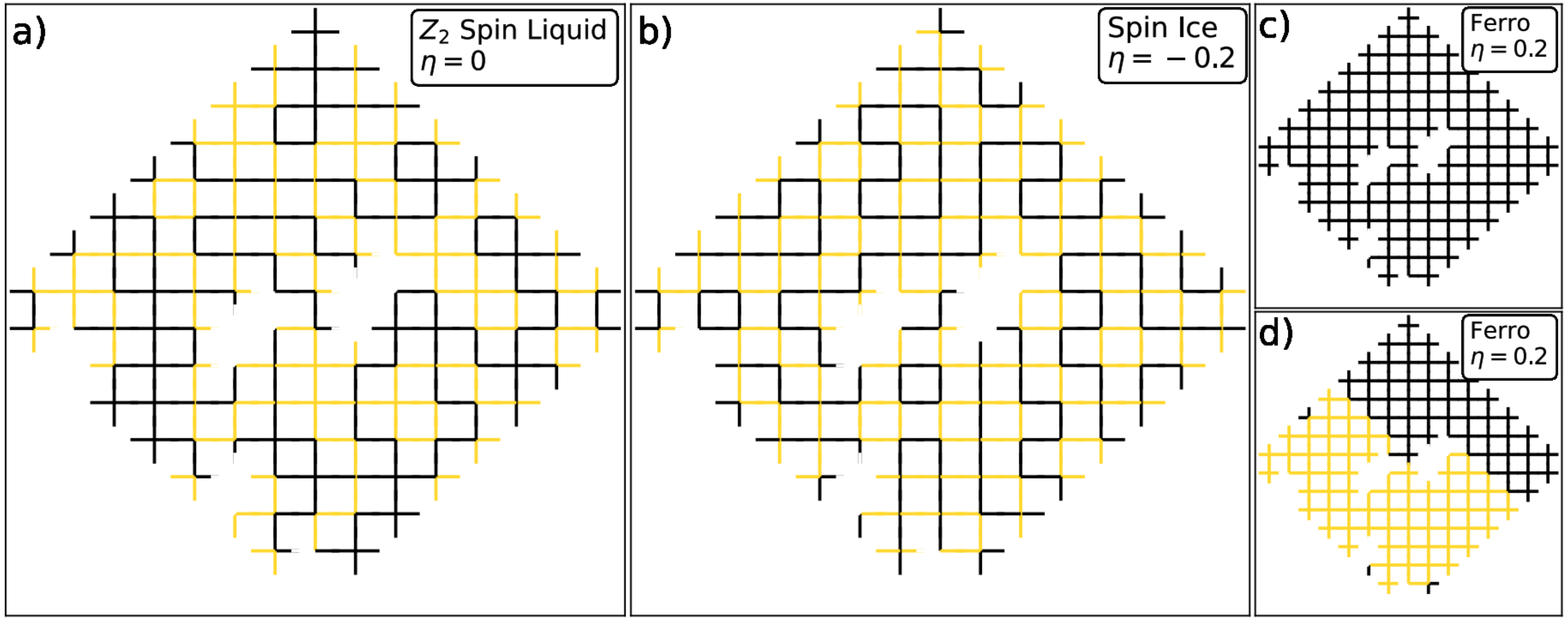
$$H = -J \sum_{a=1}^{4} \left(\sum_{i=1}^{4} W_{ai} \sigma_i^z \right) \mu_a^z - \Gamma \sum_{a=1}^{4} \mu_a^x$$

$B_z \, \mu^z$ $\qquad$ $B_x \, \mu^x$

$$E \sim -\sum \sqrt{(W\sigma^z)^2 + \Gamma^2} \sim \boxed{\text{const} - \sigma_1^z \sigma_2^z \sigma_3^z \sigma_4^z}$$

Realization in D-Wave DW-2000Q for spins (classical limit only)

First* experimental 8-vertex model (classical $\mathbb{Z}_2$ spin liquid)

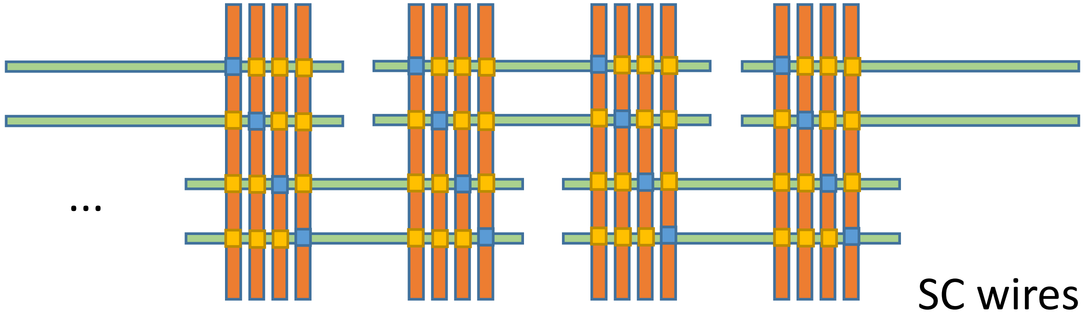


* As far as we know

Zhou, Green, Dahl, Chamon, Phys. Rev. B (2021)

How to get large (non-perturbative) gaps, back to the program

SC wire array

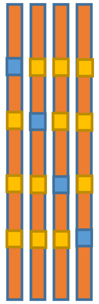


π junction

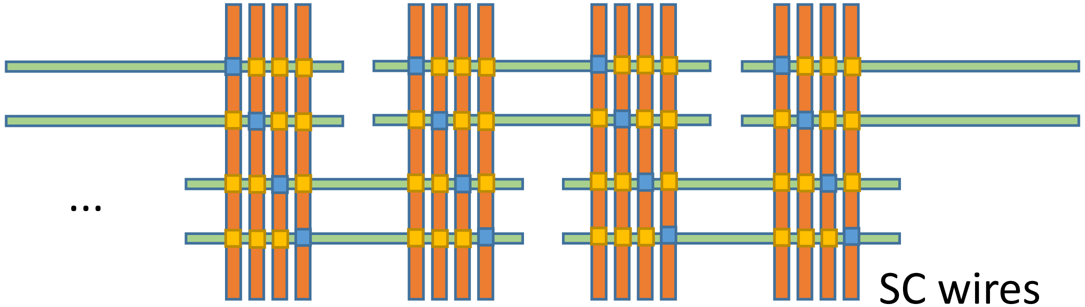
regular junction

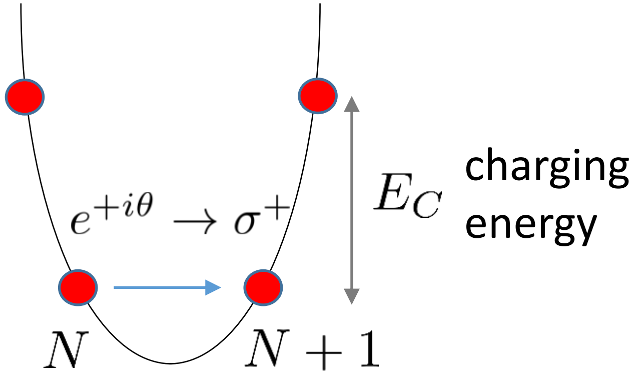
$$H_J = -J \sum_{ia} W_{ia}\, e^{i\phi_i}\, e^{-i\theta_a} + H.c.$$

$$W = \begin{pmatrix} - & + & + & + \\ + & - & + & + \\ + & + & - & + \\ + & + & + & - \end{pmatrix}$$



SC wire array





π junction

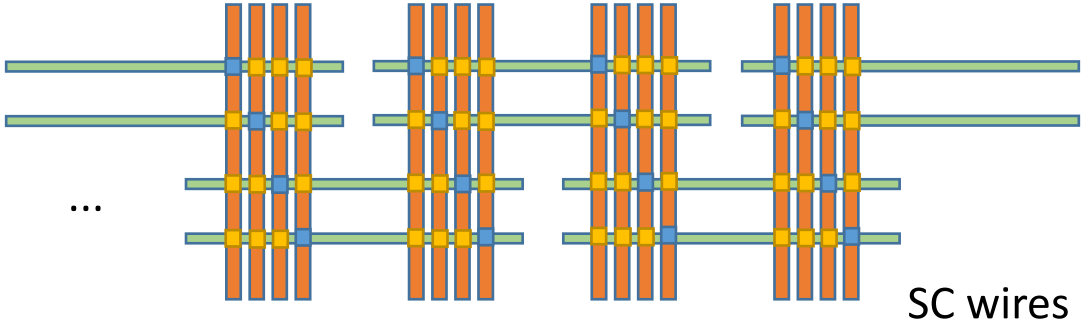
regular junction

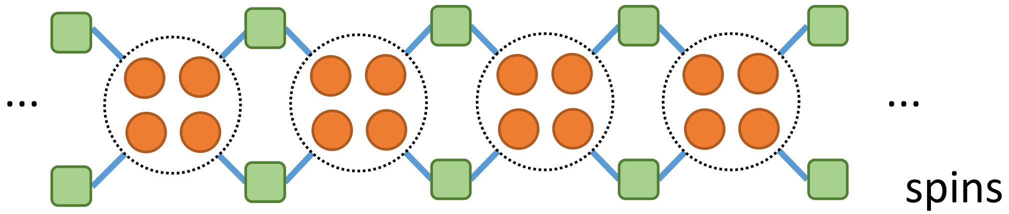
Small capacitance limit (charge degenerate point)

$$H_J = -J \sum_{ia} W_{ia}\, \mu_i^+ \, \sigma_a^- + H.c.$$

WXY model

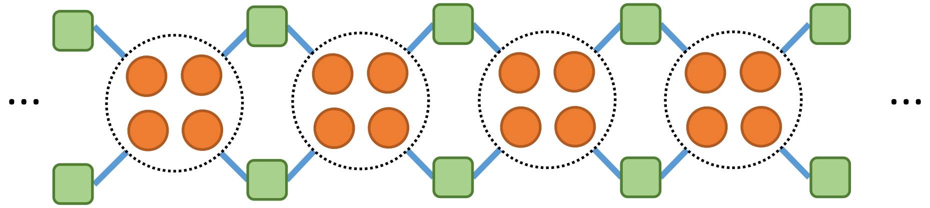
WXY model



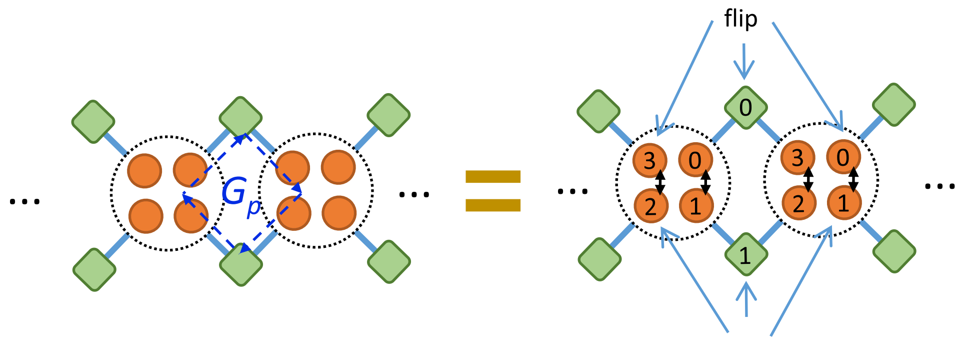


$$H_J = -J \sum_{ia} W_{ia}\, \mu_i^+\, \sigma_a^- + H.c.$$

WXY model symmetries



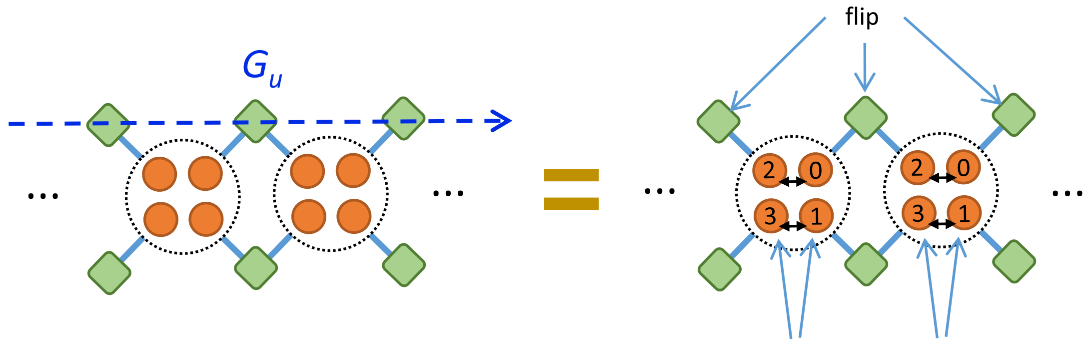
Local symmetries



plaquette operators

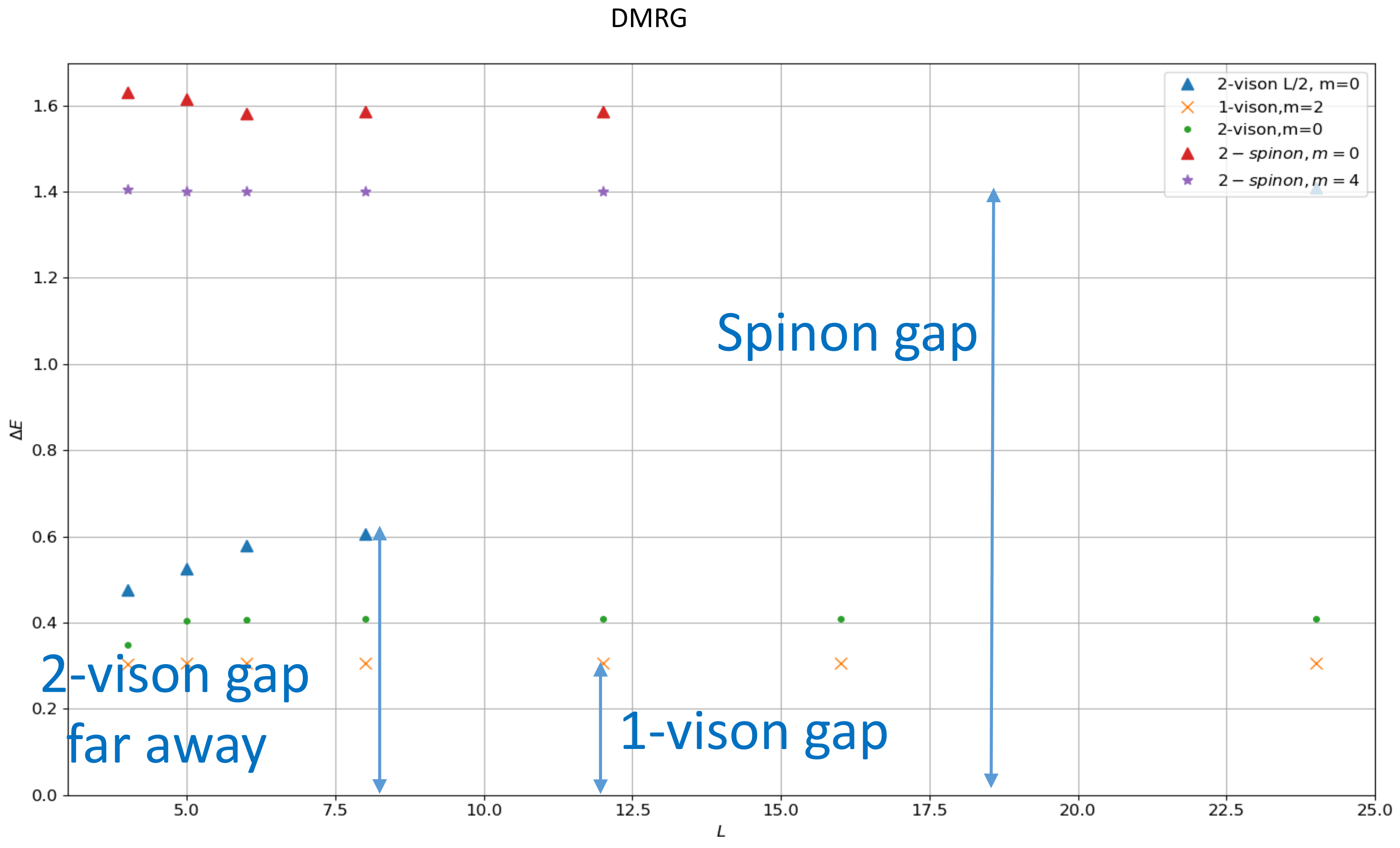
$$[H, G_p] = 0$$

Nonlocal symmetries

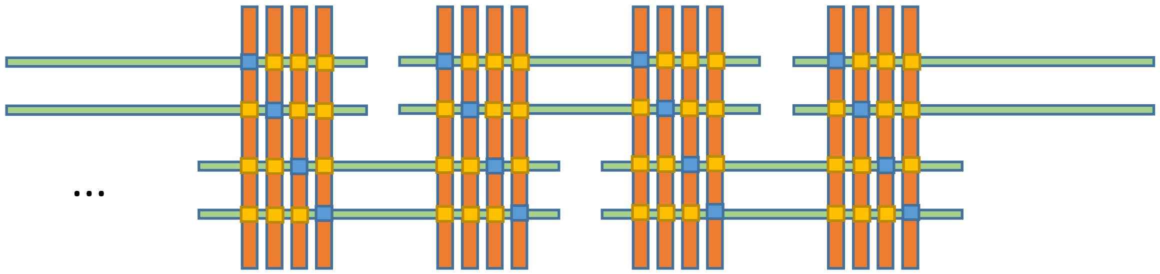


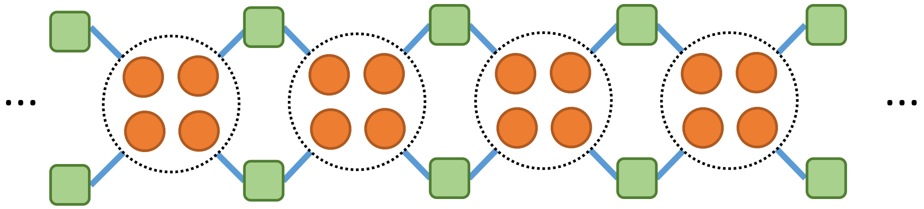
$$[H, G_u] = 0$$

WXY ladder spectrum (preliminary data)



WXY ladder spectrum (preliminary data)





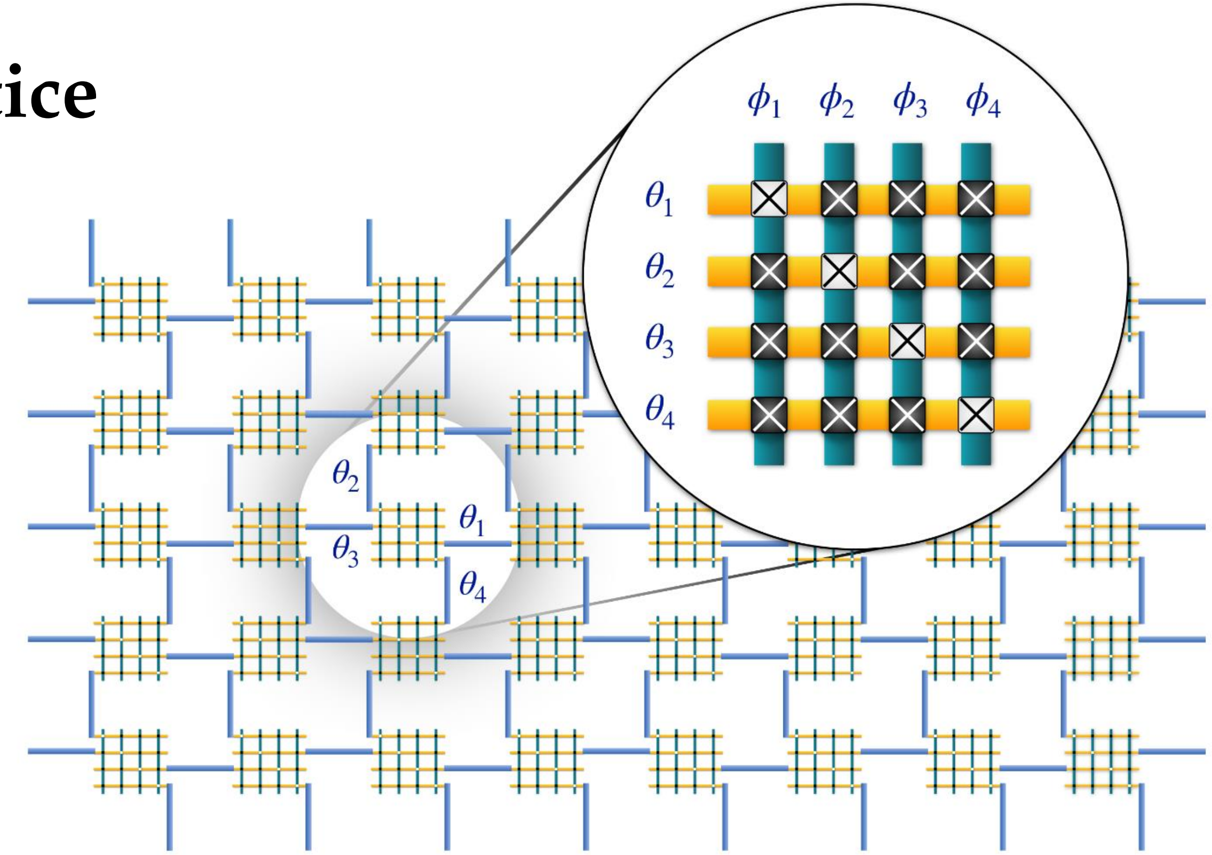
vison and spinon gaps

$$\Delta_v \sim 0.3\,J$$

$$\Delta_s \sim 1.4\,J$$

2D version

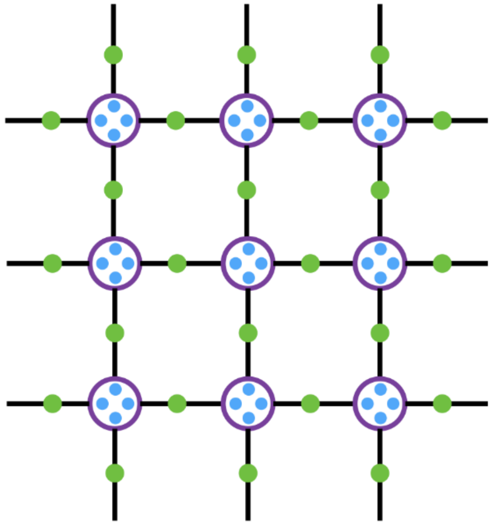
Lattice

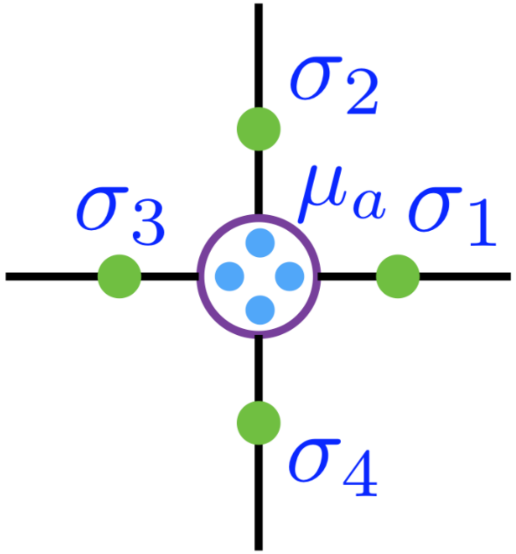


Chamon, Green, and Kerman, PRX Quantum 2021

WXY model

$$H_J = -J \sum_s \sum_{ia\in s} W_{ia}\, \mu_i^+\, \sigma_a^- + H.c.$$





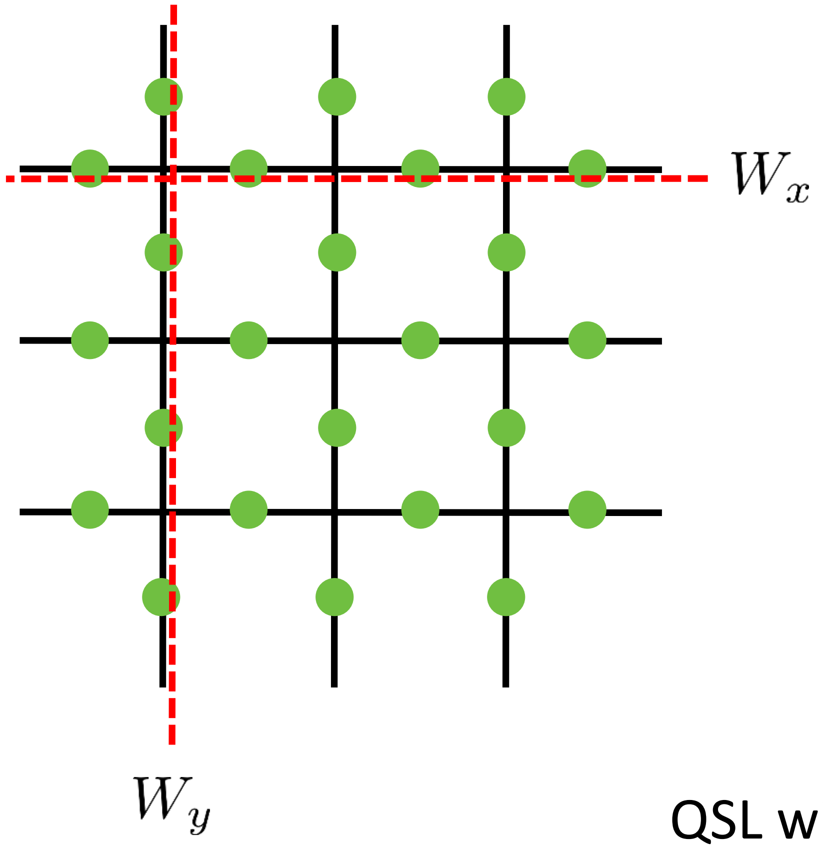
QSL with gap of order J ?

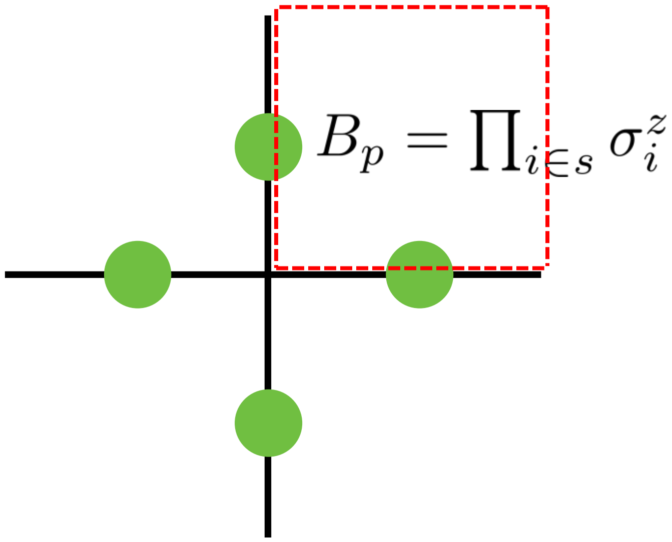
U(1) toric code – YES!

UV/IR mixing, strange topological degeneracies,
Hilbert space fragmentation, possibly non-Abelian

U(1) symmetry-enhanced toric

$$H_J = -J \sum_s \mathcal{A}_s - \lambda \sum_p B_p$$





$$\mathcal{A}_s = \sigma_1^+ \, \sigma_2^+ \, \sigma_3^- \, \sigma_4^-$$
$$+\, 5 \text{ terms}$$

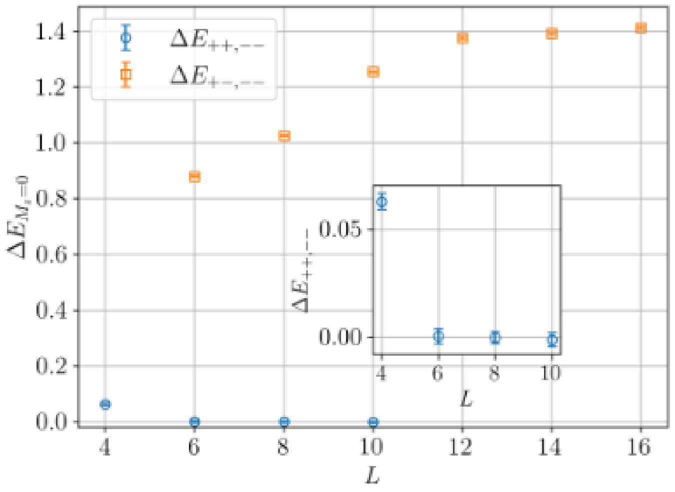
QSL with gap of order J !

Wu, Khudorozhkov, Delfino, Green, and Chamon, arXiv Feb. 2013

U(1) symmetry-enhanced toric

0° compactification

$W_x W_y$

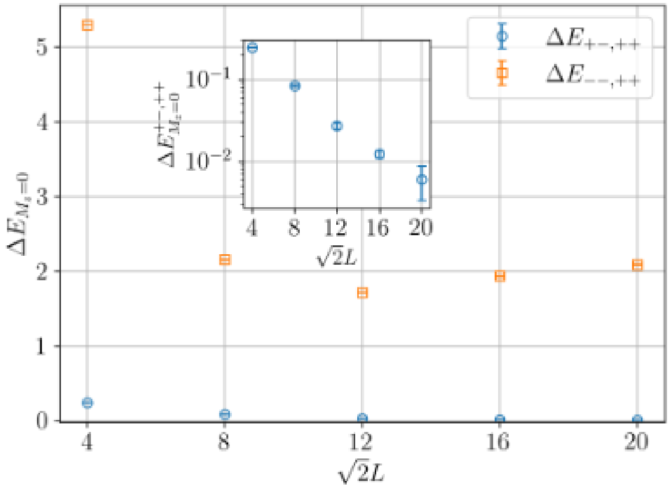


+- , -+

2-fold GSD

++ , --

45° compactification

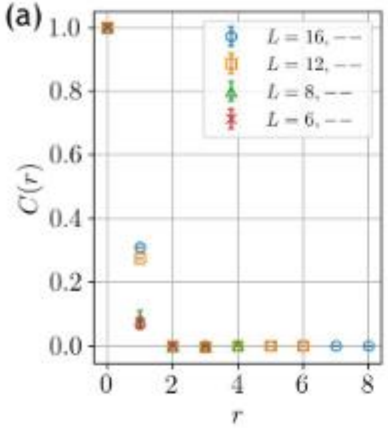


3-fold GSD

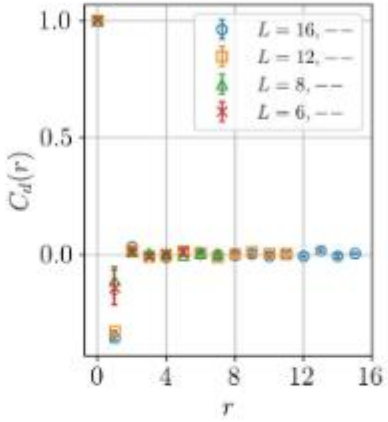
--

++, +- , -+

[1,0] direction



[1,1] direction



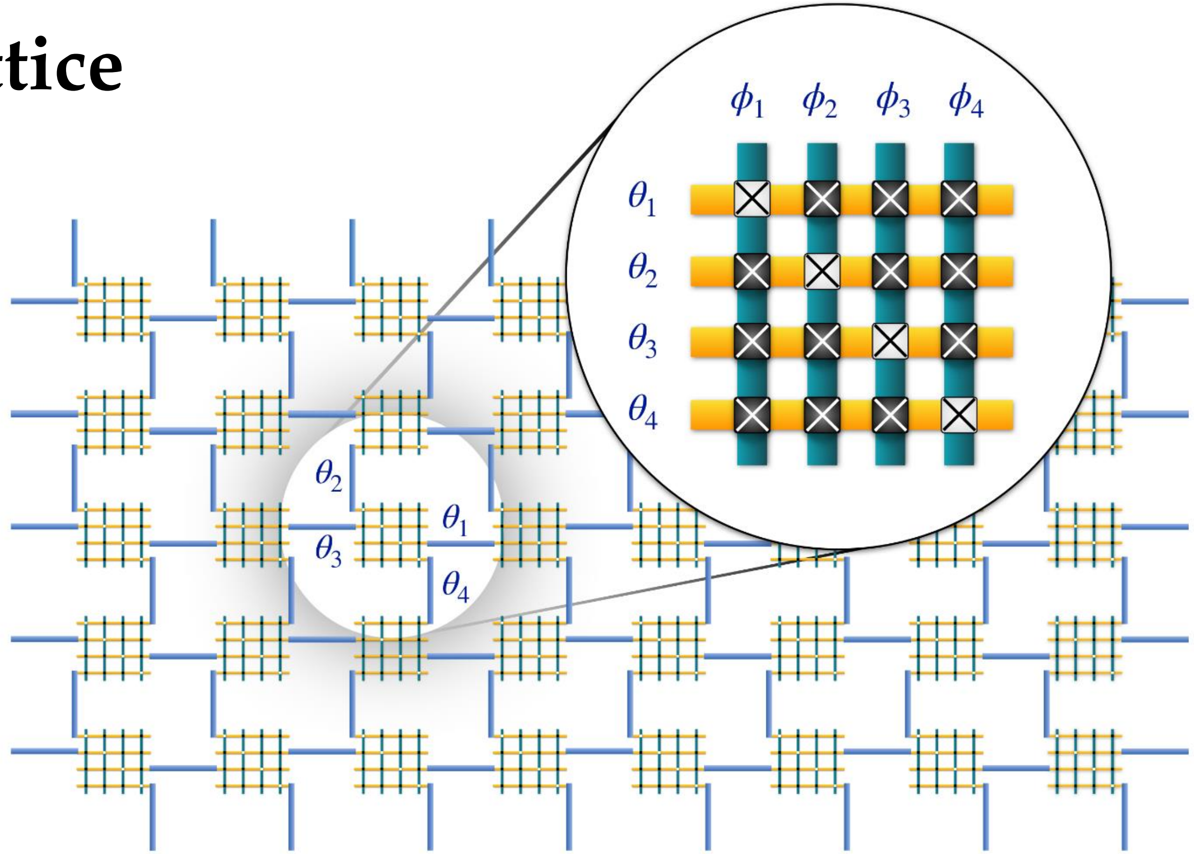
Gapped spin-liquid

Spin-spin correlation

UV/IR mixing, strange topological degeneracies,
Hilbert space fragmentation, possibly non-Abelian

Motivated by the SC wire array!

Lattice



Abelian combinatorial gauge symmetry

Generalized framework for all Abelian groups and lattice connectivities

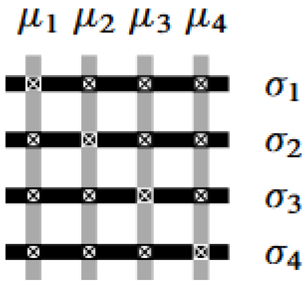
arXiv:2212.03880

Yu, Goldstein, Green, Ruckenstein, and Chamon

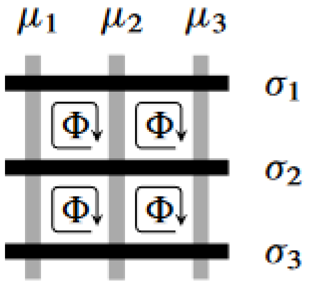
W matrices translate into "waffle" arrays

$\mathbb{Z}_2$ topological state on a square lattice

$$\begin{pmatrix} - & + & + & + \\ + & - & + & + \\ + & + & - & + \\ + & + & + & - \end{pmatrix}$$



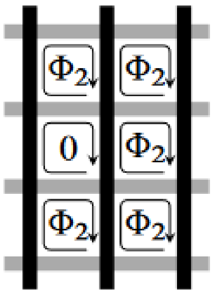
$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \overline{\omega} \\ 1 & \overline{\omega} & \omega \end{pmatrix}$$



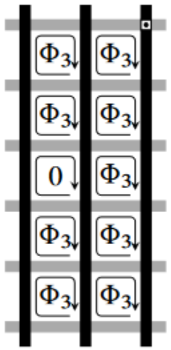
$\mathbb{Z}_3$ topological state on a honeycomb lattice

$\mathbb{Z}_2$ topological state on a honeycomb lattice

$$\begin{pmatrix} + & + & + \\ - & + & - \\ + & - & - \\ - & - & + \end{pmatrix} \Longleftrightarrow$$

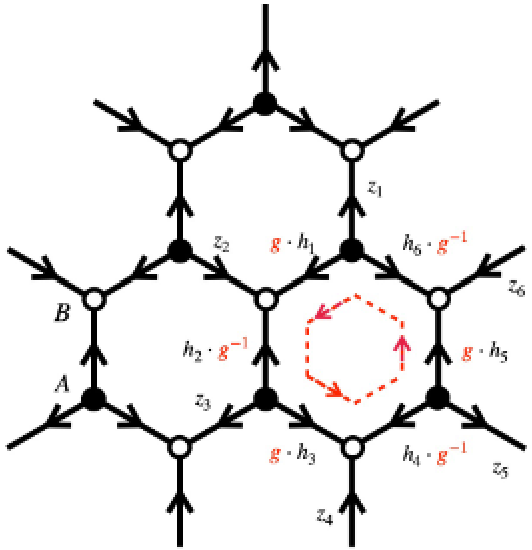


$$\begin{pmatrix} 1 & 1 & \omega \\ 1 & \omega & 1 \\ 1 & \overline{\omega} & \overline{\omega} \\ 1 & \overline{\omega} & 1 \\ 1 & 1 & \overline{\omega} \\ 1 & \omega & \omega \end{pmatrix} \Longleftrightarrow$$



Non-Abelian combinatorial gauge symmetry

arXiv:2209.14333
Green and Chamon



Quaternion group

$$
\begin{aligned}
v(+1) &= [++++] & v(-1) &= [----] \\
v(+i) &= [+-+-] & v(-i) &= [-+-+] \\
v(+j) &= [++--] & v(-j) &= [--++] \\
v(+k) &= [-++-] & v(-k) &= [+--+]
\end{aligned}
$$

$$
W = \frac{1}{4}\begin{bmatrix} v(f_1) & v(h_1) & v((f_1 h_1)^{-1}) \\ v(f_2) & v(h_2) & v((f_2 h_2)^{-1}) \\ \vdots & \vdots & \vdots \\ v(f_{64}) & v(h_{64}) & v((f_{64} h_{64})^{-1}) \end{bmatrix}
$$

64×12 matrix

lots of SC wires and junctions!

General (discrete) non-Abelian groups:
Yu, Green and Chamon, in preparation

Summary

- Framework for constructing systems with **exact** (not emergent) local **Abelian** and **non-Abelian gauge symmetries** using **physical** interactions

- Proposed a 2-leg ladder SC wire array with **non-perturbative spinon/vison gap**

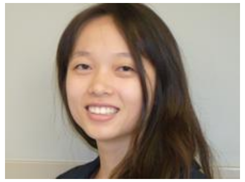
- Presented a U(1)-symmetry enhanced toric code with unusual topological features

Collaborators

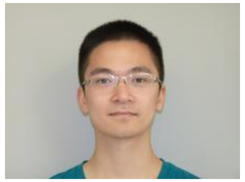
2020: PRL
2021: 2x PRB, 2x PRX Quantum
2022: 3x SciPost submissions
2023: arXiv

Dmitry Green

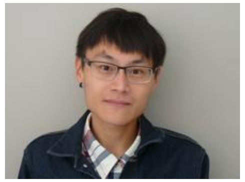
Zhi-Cheng Yang



Shiyu Zhou



Elliot Yu



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Obrigado!