Holography in & beyond the AdS paradigm

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Diego M. Hofman (Amsterdam U.)
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★★★

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Wilson loops and integrability in Chern-Simons-matter theories

Diego H. Correa

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Based on:

2304.01924 [D.C., V. Giraldo-Rivera, M.Lagares]

Workshop Holography@25 - Sao Paulo, Brasil

An important paper in the history of AdS/CFT:

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Strings in flat space and pp waves from $\mathcal{N}=4$ super Yang Mills

David Berenstein, Juan Maldacena and Horatiu Nastase

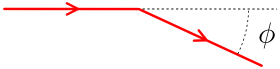
- First AdS/CFT verification for non-protected observables

1-loop anomalous dim of single trace operators	$\equiv$	energy of closed strings in a pp-wave background

Possible in BMN limit: $\frac{\lambda}{L^2} \ll 1$ For length of trace L large

My talk today

Cusp anomalous dimension in
$\mathcal{N}=6$ super Chern-Simons-matter (ABJM)



$$\langle W_{\text{cusp}} \rangle = e^{-\Gamma_{\text{cusp}}(\phi)\log\left(\frac{\Lambda_{\text{IR}}}{\Lambda_{\text{UV}}}\right)}$$

$$\Gamma_{\text{cusp}}(\phi) = -\lambda\left(\frac{1}{\cos\frac{\phi}{2}}-1\right) - \lambda^2\left(\frac{1}{\cos\frac{\phi}{2}}-1\right)\log\left(\cos\frac{\phi}{2}\right)^2 + \mathcal{O}(\lambda^3)$$

(Griguolo,Marmiroli,Martelloni,Seminara 12)

- **We proposed a TBA system to compute $\Gamma_{\text{cusp}}(\phi)$ exactly**
- **We reproduced the 1-loop order of $\Gamma_{\text{cusp}}(\phi)$ from this TBA**

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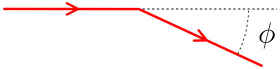

$$\langle W_{\rm cusp}\rangle = e^{-\Gamma_{\rm cusp}(\phi)\log(\frac{\Lambda_{\rm IR}}{\Lambda_{\rm UV}})}$$

$$\Gamma_{\rm cusp}(\phi) = -\lambda\left(\frac{1}{\cos\frac{\phi}{2}}-1\right) - \lambda^2\left(\frac{1}{\cos\frac{\phi}{2}}-1\right)\log\left(\cos\frac{\phi}{2}\right)^2 + \mathcal{O}(\lambda^3)$$

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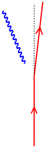
- **We proposed a TBA system to compute $\Gamma_{\rm cusp}(\phi)$ exactly**
- **We reproduced the 1-loop order of $\Gamma_{\rm cusp}(\phi)$ from this TBA**

Introduction and Motivations

- $\Gamma_{\text{cusp}}(\phi)$ encodes physical data of ABJM gauge theory
- In the small ϕ limit, it gives the bremsstrahlung function

$$\Gamma_{\text{cusp}}(\phi) \simeq -\phi^2 B(\lambda)$$

$$\text{Radiated energy:} \quad E = 2\pi B \int dt\, \dot{v}^2$$



- $B(\lambda)$ is known from a localization computation

$$B(\lambda) = \frac{\kappa}{64\pi}\, {}_2F_1\left(\tfrac{1}{2}, \tfrac{1}{2}; 2; -\tfrac{\kappa^2}{16}\right) \qquad \lambda = \frac{\kappa}{8\pi}\, {}_3F_2\left(\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}; 1; \tfrac{3}{2}; -\tfrac{\kappa^2}{16}\right)$$

(Lewkowycz, Maldacena 13) (Bianchi, Griguolo, Leoni, Penati,Seminara, 14)

(Bianchi, Griguolo, Mauri, Penati, Preti, Seminara, 17) (Bianchi, Preti, Vescovi 18)

Computing the same function from integrability, would provide a direct **derivation of the interpolating function** $h(\lambda)$, which enters all integrability-based results for ABJM

Introduction and Motivations

- Integrability can be useful in $d > 2$ QFT as well.
- Spectrum of single trace operators is an integrable problem in $\mathcal{N} = 4$ super Yang-Mills

  (Minahan, Zarembo 02) (Beisert, Staudacher 03, 05) (Beisert, Kristjansen, Staudacher 03) (Beisert 05) (Beisert, Eden, Staudacher 06) (Gromov, Kazakov, Vieira 09) (Gromov, Kazakov, Kozak, Vieira 09) (Arutyunov, Frolov 07, 08, 09) (Bombardelli, Fioravanti, Tateo 09) (Cavaglia, Fioravanti, Tateo 10) (Gromov, Kazakov, Leurent, Volin 13)

  Apologies for the many omissions!
- Spectrum of single trace operators is an integrable problem in ABJM ($\mathcal{N} = 6$ Chern-Simons-matter)

  (Minahan, Zarembo 08) (Gaiotto, Giombi, Yin 08) (Gromov, Vieira 08) (Ahm, Nepomechie 08) (Gromov, Mikhaylov 08) (Bombardelli, Fioravanti, Tateo 09) (Gromov, Levkovich-Maslyuk 09) (Gromov, Sizov 14) (Cavaglià, Fioravanti, Gromov, Tateo 14) (Bombardelli, Cavaglià, Fioravanti, Gromov, Tateo 17)

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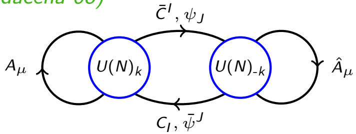
Introduction and Motivations

- Wilson loops in $\mathcal{N} = 4$ super Yang-Mills can be described with integrability tools as well *(Drukker, Kawamoto 06) (Drukker 12) (Correa, Maldacena, Sever 12) (Gromov, Sever 12) (Gromov, Levkovich-Maslyuk 15) (Correa, Leoni, Luque 18)*

- Why these ideas were not immediately and straightforwardly extended to ABJM Wilson loops?

ABJM theory

It is an $\mathcal{N}=6$ Chern-Simons-matter theory *(Aharony, Bergman, Jafferis, Maldacena 08)*



Holographic dual of type IIA string theory in $AdS_4 \times \mathbb{CP}^3$ (when k and N large)

⊛ Single trace operators alternate one type of matter with the other

$$\mathrm{tr}[C_1 \bar{C}^2 C_1 \bar{C}^2 C_1 \bar{C}^2 C_1 \bar{C}^2] \equiv |\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\rangle$$

⊛ The spectrum of single trace operators is described in terms of integrable alternating spin chains

$$|p\rangle_A = \sum_n e^{ipn} |\uparrow\uparrow\uparrow\uparrow\underbrace{\downarrow}_{n}\uparrow\uparrow\uparrow\rangle \qquad |p\rangle_B = \sum_n e^{ipn} |\uparrow\uparrow\uparrow\uparrow\uparrow\underbrace{\downarrow}_{n}\uparrow\uparrow\rangle$$

Scale dimension of operators $\leftrightarrow$ Energy of spin chain states.

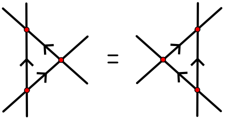
Similarities and Differences with $\mathcal{N} = 4$ SYM

- An $SU(2|2)$ underlying symmetry constraints the bulk $2 \to 2$ S-matrix and dispersion relation of magnons
- In ABJM two types of magnons: $\boxslash_A \oplus \boxslash_B$

$$S^{AA} = S_0^{AA}(p, q, h(\lambda))\hat{S}(p, q, h(\lambda)) \quad S^{AB} = S_0^{AB}(p, q, h(\lambda))\hat{S}(p, q, h(\lambda))$$

$\hat{S}$: same $SU(2|2)$ matrix part than in

$\mathcal{N} = 4$ SYM *(Beisert 05)* $\Rightarrow$ Yang-Baxter Eq. ✓



S_0^{AA} & S_0^{AB}: dressing factors constrained by crossing symmetry

$$S_0^{AA}(x_1, x_2) = \frac{x_1^+ - x_2^-}{x_1^- - x_2^+} \frac{1 - \frac{1}{x_1^+ x_2^-}}{1 - \frac{1}{x_1^- x_2^+}} \sigma(x_1, x_2) \qquad S_0^{AB}(x_1, x_2) = \sigma(x_1, x_2)$$

σ is the square root of the BES function appearing in $\mathcal{N} = 4$ SYM

(Ahn, Nepomechie 08)

Similarities and Differences with $\mathcal{N} = 4$ SYM

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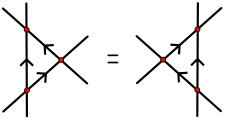

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Similarities and Differences with $\mathcal{N} = 4$ SYM

- The dispersion relation of magnons

$$E(p) = \frac{1}{2}\sqrt{1 + 16 h(\lambda)^2 \sin^2\left(\frac{p}{2}\right)} \qquad \begin{cases} h(\lambda) \text{ not fixed by symmetry} \\ h(\lambda) \text{ in all integrability-based results} \end{cases}$$

- In $\mathcal{N} = 4$ SYM

$$h(\lambda)^2 = \frac{\lambda}{16\pi^2}$$

computing $B(\lambda)$ from localization *(Correa, Henn, Maldacena, Sever 12)*
from integrability *(Gromov, Sever 12)*

A windfall in $\mathcal{N} = 4$ SYM!! Had it been different for weak and strong coupling regimes, **BMN limit would not possible!**

- In ABJM it is a non-trivial function

$$h(\lambda)^2 = \begin{cases} \lambda^2 + \cdots & \text{for } \lambda \ll 1 \\ 2\lambda + \cdots & \text{for } \lambda \gg 1 \end{cases}$$

Similarities and Differences with $\mathcal{N}=4$ SYM

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A proposal for $h(\lambda)$ in ABJM

An exact integrability-based computation of the slope function is written in terms of integrands **similar** to the ones appearing in matrix model results *(Gromov, Sizov 14)*

Gromov and Sizov propose the identification

$$\kappa(\lambda) \equiv 4 \sinh\left(2\pi h(\lambda)\right)$$

which would imply

$$\lambda = \frac{\sinh\left(2\pi h\right)}{2\pi}\, {}_3F_2\left(\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}; 1; \tfrac{3}{2}; -\sinh^2\left(2\pi h\right)\right)$$

An exact integrability-based computation of the bremsstrahlung function would provide a direct derivation of the interpolating $h(\lambda)$

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An exact integrability-based computation of the bremsstrahlung function would provide a direct derivation of the interpolating $h(\lambda)$

Cusp anomalous dimension from integrability

Method:

1. Insert a chain of fields of length L at a point in the WL
2. WL sets open boundaries: determine the reflection matrix
3. Rotate one of the $R^a{}_b$ to introduce a cusp angle
4. Incorporate finite size effects with a Thermodynamic Bethe Ansatz
5. The vacuum energy in the $L \to 0$ limit gives the cusp anomalous dimension

This was successfully done in $\mathcal{N} = 4$ SYM

(Correa, Maldacena, Sever 12) (Drukker 12)

Wilson loop's open spin chain

- 1/2 BPS Wilson loop *(Drukker, Trancanelli 09)*

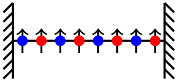
$$W = \mathrm{tr}(\mathcal{W}) = \left(e^{i\int dt\ L}\right) \qquad L(t) = \begin{pmatrix} A_t - \frac{2\pi i}{k}\mathcal{M}^I_J C_I \bar{C}^J & -i\sqrt{\frac{2\pi}{k}}\eta\bar{\psi}^1_+ \\ -i\sqrt{\frac{2\pi}{k}}\bar{\eta}\psi_1^+ & \hat{A}_t - \frac{2\pi i}{k}\mathcal{M}^I_J \bar{C}^J C_I \end{pmatrix}$$

$$\mathcal{M} = \mathrm{diag}(-1,1,1,1) \qquad \eta\bar{\eta} = 2i$$

W is susy invariant, $\mathcal{W}$ is susy covariant

$$\left.\begin{array}{ll} W = \mathrm{tr}\left(e^{i\int dt\ L}\right) & SU(1,1|3) \\ C_1\bar{C}^2 C_1\bar{C}^2 C_1\bar{C}^2 C_1\bar{C}^2 & SU(2|2) \end{array}\right\} \quad SU(1|2) \text{ in common}$$

$$\longrightarrow\!\!\!\!-(C_1\bar{C}^2 C_1\bar{C}^2 C_1\bar{C}^2 C_1\bar{C}^2)\!\!\longrightarrow\!\!\!\!- \quad \longleftrightarrow$$



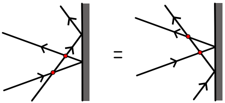
- Magnon impurities can reflect on the boundary. Bootstrapping with the $SU(1|2)$ symmetry, the reflection matrix is constrained

$$R^{A(B)}(p) = R_0^{A(B)}(p)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{-ip/2} & 0 \\ 0 & 0 & 0 & -e^{ip/2} \end{pmatrix}$$

Matrix part is analogous to the one for Giant Graviton open boundaries in $\mathcal{N} = 4$ SYM and in ABJM $[SU(2|1)]$

(Hofman, Maldacena 06) (Chen, Ouyang, Wu 18)

$\Rightarrow$ Boundary Yang-Baxter Eq. ✓



- So far, everything looked great. However, at this point, the project stalled for years

- **Why?**

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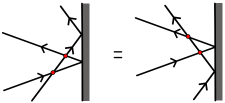


⊛ So far, everything looked great. However, at this point, the project stalled for years

⊛ **Why?**

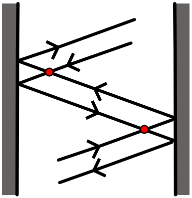
In my case, I lacked the precise weak coupling spin chain picture, which is needed to pin down the correct the dressing factors R_0^A and R_0^B to fully determine the reflection matrix

Determining the Dressing Factors

- Combining particle+antiparticle we can form an $SU(2|2)$ singlet

- Its reflection on the boundaries must be trivial

$\Rightarrow$ Boundary crossing condition *(Hofman, Maldacena 06)*



spectral parameters

$$x^+ + \frac{1}{x^+} - x^- - \frac{1}{x^-} = \frac{i}{h(\lambda)}$$

$$\frac{x^+}{x^-} = e^{ip}$$

$$\left(R_0^A(p)\right)^2 \left(R_0^B(\bar{p})\right)^2 = \left(\frac{\frac{1}{x^+} + x^+}{\frac{1}{x^-} + x^-}\right)^2 \frac{1}{\sigma^2(p, -\bar{p})}$$

It resembles the boundary crossing conditions for Wilson loop in $\mathcal{N} = 4$ SYM $\rightarrow$ we can use the $\mathcal{N} = 4$ SYM dressing factor to solve the ABJM crossing condition

Determining the Dressing Factors

$$R_0^{A/B}(p) = \pm\, r^{A/B}(p) \underbrace{\left[\frac{1}{\sigma_{\text{bdry}}(p)\sigma(p,-p)} \left(\frac{1+\frac{1}{(x^-)^2}}{1+\frac{1}{(x^+)^2}} \right) \right]^{\frac{1}{2}}}_{\mathcal{N}=4 \text{ SYM dressing factor}}$$

$$\sigma_{\text{bdry}}(p) = e^{i\chi(x^+)-i\chi(x^-)}$$

$$i\chi(x) = \begin{cases} i\Phi(x) = \oint_{|z|=1} \frac{dz}{2\pi i} \frac{1}{x-z} \log \left\{ \frac{\sinh[2\pi h(z+\frac{1}{z})]}{2\pi h\left(z+\frac{1}{z}\right)} \right\} & \text{if} \quad |x|>1 \\ i\Phi(x) + \log \left\{ \frac{\sinh[2\pi h(x+\frac{1}{x})]}{2\pi h\left(x+\frac{1}{x}\right)} \right\} & \text{if} \quad |x|<1 \end{cases}$$

(Drukker 12) (Correa, Maldacena, Sever 12)

⊛ At strong coupling, the scattering phase of magnons is related to the time delay of sine-Gordon solitons (during the reflection). This is a classical string theory computation in $AdS_2 \times S^2$, completely identical to the one in $AdS_5 \times S^5$.

Already captured by $\left[\cdots\right]^{1/2} \Rightarrow r^{A/B}(p) \sim \mathcal{O}(1)$ for $\lambda \gg 1$

Determining the Dressing Factors

- We are left with

$$r^{A}(p)r^{B}(\bar{p}) = \frac{\frac{1}{x^{+}} + x^{+}}{\frac{1}{x^{-}} + x^{-}}$$

Many ways of solving this. We look for the solution consistent with weak coupling behaviours

Determining the Dressing Factors

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Many ways of solving this. We look for the solution consistent with weak coupling behaviours

- For this is crucial to have the correct spin chain description at weak coupling. Which one is the insertion $\mathcal{O}$ to be interpreted as BPS reference state?

$$\mathcal{W} \overset{\text{susy}}{\longrightarrow} e^{-i\Lambda}\mathcal{W}e^{i\Lambda} \quad \Rightarrow \quad \delta^{\text{cov}}\mathcal{O} := \delta\mathcal{O} - i[\mathcal{O}, \Lambda]$$

$$\longrightarrow \begin{pmatrix} (C_1\bar{C}^2)^{\ell} & 0 \\ 0 & (\bar{C}^2 C_1)^{\ell} \end{pmatrix} \longrightarrow \qquad \text{Not BPS}$$

Determining the Dressing Factors

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$$\longrightarrow \begin{pmatrix} C_1\bar{C}^2 & -\frac{\eta}{2}\bar{\psi}^2_+ \\ 0 & \bar{C}^2 C_1 \end{pmatrix}^{\ell} \longrightarrow$$

BPS, but it looks like a $p = 0$ magnon state

Determining the Dressing Factors

- We are left with

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$$\longrightarrow \begin{pmatrix} 0 & (C_1\bar{C}^2)^{\ell}C_1 \\ 0 & 0 \end{pmatrix} \longrightarrow \qquad \text{BPS vacuum state}$$

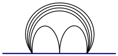
Determining the Dressing Factors

- $SU(2)$ sector for type B magnons

$$\mathbf{H}^B = \lambda^2 \sum_{n=0} (\mathbf{1} - \mathbf{P}_{n,n+1}) \qquad \to (C_1\, \bar{C}^3\, C_1\, \bar{C}^2\, C_1\, \bar{C}^2\, C_1 \cdots$$

$$|\psi(p)\rangle = \sum_{n=0} \left(e^{-ipn} + R(p)\, e^{ipn}\right) |n\rangle \quad \Rightarrow \quad R_0^B(p) = e^{-ip} + \mathcal{O}(\lambda^2)$$

- $SU(2)$ sector for type A magnons



$$\mathbf{H}^A = (\lambda + \lambda^2 \beta_0)\delta_{0,C_3} + \lambda^2 \sum_{n=0} (\mathbf{1} - \mathbf{P}_{n,n+1}) \qquad \to (C_3\, \bar{C}^2\, C_1\, \bar{C}^2\, C_1\, \bar{C}^2\, C_1 \cdots$$

- bulk terms order λ^2
- bdry terms start at order $\lambda \Rightarrow$ a state w/ energy $E_0 = \lambda + \cdots$

Perturbative resolution $\Rightarrow$ $R_0^A(p) = -1 + \mathcal{O}(\lambda^2)$

Determining the Dressing Factors

- All-loop proposal

$$r^{B}(p) = \frac{x^-}{x^+} \qquad r^{A}(p) = \frac{x^-}{x^+}\left(\frac{x^+ + \frac{1}{x^+}}{x^- + \frac{1}{x^-}}\right)$$

  - crossing equation ✓
  - weak & strong results ✓

- The type A state with energy order λ appears to be a boundary bound state

- The additional factor in $r^{A}(p)$ has a pole as $x^- \to i$, whose energy is

$$E_{\text{pole}} = \lambda + \mathcal{O}(\lambda^2)$$

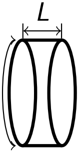
Recap: we found a solution of the boundary crossing condition that reproduces strong and weak coupling results

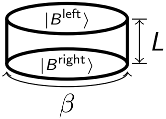
Cusp anomalous dimension from integrability

Method:

1. Insert a chain of fields of length L at a point in the WL ✓
2. WL sets open boundaries: determine the reflection matrix ✓
3. Rotate one of the $R^a{}_b$ to introduce a cusp angle ✓
4. → Incorporate finite size effects with a Thermodynamic Bethe Ansatz
5. → The vacuum energy in the $L \to 0$ limit gives the cusp anomalous dimension

Boundary TBA schematically

$\frac{1}{T} = \beta$ 

$\leftrightarrow$ 

Physical strip $\left\{ \begin{array}{c} p \leftrightarrow i\tilde{E} \\ E \leftrightarrow i\tilde{p} \end{array} \right\}$ Mirror theory

$$Z(L,\beta) = \mathrm{Tr}_{\mathrm{open}}[e^{-\beta H^{\mathrm{open}}_{B_l,B_r}}] = \langle B_l | e^{-LH_{\mathrm{closed}}} | B_r \rangle$$

- Analytic continuation of $R(p)$ gives the probability of emitting pairs of particles from the boundary state *(Ghoshal, Zamolodchivov 93)*

$$|B\rangle = \exp\left(\int_0^\infty \frac{d\tilde{p}}{2\pi} K^{a,b}(\tilde{p}) a_a^\dagger(-\tilde{p}) a_b^\dagger(\tilde{p}) \right) |0\rangle = \exp\left(\int_0^\infty \frac{d\tilde{p}}{2\pi} \right. $$ $$\left. \right) |0\rangle$$

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- In the $\beta \to \infty$ limit,

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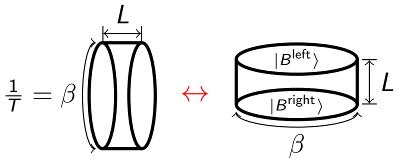

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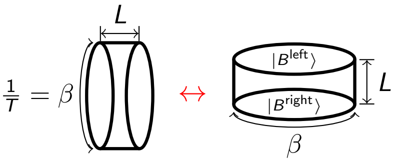

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Boundary TBA schematically

A TBA analysis shows that the vacuum energy is

$$\mathcal{E}_0(L) = -\frac{1}{2\pi}\sum_Q \int_0^\infty d\tilde{p}\, \log(1+Y_Q)$$

Y_Q are solutions to certain integral equations. Schematically

$$\log Y_Q = \log\left[\mathrm{Tr}(K_Q \bar{K}_Q)\right] - 2L\tilde{E}_Q + \mathrm{K}_{Q,Q'} * \log(1+Y_{Q'})$$

The asymptotic solution, giving the leading finite size correction, is

$$Y_Q \approx \mathbf{Y}_Q = e^{-2L\tilde{E}_Q}\mathrm{Tr}(K_Q \bar{K}_Q)$$

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TBA eqs for periodic and open boundary conditions look the same

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TBA in ABJM

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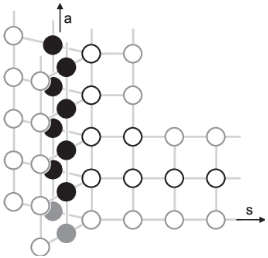
- Many Y-functions, one for each type of mirror particle



from (Gromov, Kazakov, Vieira)

$Y^{I}_{a,0}$, $Y^{II}_{a,0}$, for $a > 0$,
$Y_{a,1}$, $Y_{1,s}$ for $a > 0$ & $s > 0$,
$Y_{2,2}$

We shall assume that the TBA system for the ABJM Wilson loop is the same as for the periodic ABJM, after substracting the asymptotic solution

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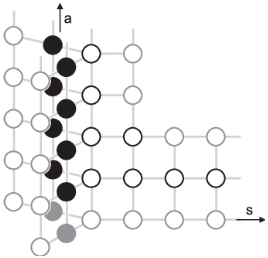

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$$Y_{2,2}$$

We shall assume that the TBA system for the ABJM Wilson loop is the same as for the periodic ABJM, after substracting the asymptotic solution

$$
\begin{aligned}
\log\left(\frac{Y_{1,1}}{\mathbf{Y}_{1,1}}\right) &= K_{m-1}\star\log\left(\frac{1+\bar{Y}_{1,m}}{1+Y_{m,1}}\,\frac{1+\mathbf{Y}_{m,1}}{1+\bar{\mathbf{Y}}_{1,m}}\right)+\mathcal{R}^{(01)}_{1m}\star\log(1+Y^{I}_{m,0})+\mathcal{R}^{(01)}_{1m}\star\log(1+Y^{II}_{m,0})\\
\log\left(\frac{\bar{Y}_{2,2}}{\bar{\mathbf{Y}}_{2,2}}\right) &= K_{m-1}\star\log\left(\frac{1+\bar{Y}_{1,m}}{1+Y_{m,1}}\,\frac{1+\mathbf{Y}_{m,1}}{1+\bar{\mathbf{Y}}_{1,m}}\right)+\mathcal{B}^{(01)}_{1m}\star\log(1+Y^{I}_{m,0})+\mathcal{B}^{(01)}_{1m}\star\log(1+Y^{II}_{m,0})\\
\log\left(\frac{\bar{Y}_{1,n}}{\bar{\mathbf{Y}}_{1,n}}\right) &= -K_{n-1,m-1}\star\log\left(\frac{1+\bar{Y}_{1,m}}{1+\bar{\mathbf{Y}}_{1,m}}\right)-K_{n-1}\circledast\log\left(\frac{1+Y_{1,1}}{1+\mathbf{Y}_{1,1}}\right)\\
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&\quad+\left(\mathcal{R}^{(01)}_{nm}+\mathcal{B}^{(01)}_{n-2,m}\right)\star\log(1+Y^{I}_{m,0})+\left(\mathcal{R}^{(01)}_{nm}+\mathcal{B}^{(01)}_{n-2,m}\right)\star\log(1+Y^{II}_{m,0})\\
\log\left(\frac{Y^{I}_{n,0}}{\mathbf{Y}^{I}_{n,0}}\right) &= \mathcal{T}^{||}_{nm}\star\log(1+Y^{I}_{m,0})+\mathcal{T}^{\perp}_{nm}\star\log(1+Y^{II}_{m,0})+\\
&\quad+\mathcal{R}^{(10)}_{n1}\circledast\log\left(\frac{1+Y_{1,1}}{1+\mathbf{Y}_{1,1}}\right)+\left(\mathcal{R}^{(10)}_{nm}+\mathcal{B}^{(10)}_{n,m-2}\right)\star\log\left(\frac{1+Y_{m,1}}{1+\mathbf{Y}_{m,1}}\right)\\
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\end{aligned}
$$

where the asymptotic solutions $\mathbf{Y}_{a,s}$ are obtained from the ABJM T-system and a Lüscher computation

Asymptotic solution

$$\mathbf{Y}^{I}_{a,0} = \mathbf{Y}^{II}_{a,0} = \left(\frac{z^{[-a]}}{z^{[+a]}}\right)^{2L} \frac{\varphi^{[-a]}}{\varphi^{[+a]}}\, \mathbf{T}_{a,1} \qquad x(u) + \tfrac{1}{x(u)} = \tfrac{u}{h} \qquad f^{[\pm a]} = f(u \pm i\tfrac{a}{2})$$

- The T-functions are

$$\mathbf{T}_{a,1} = 2(-1)^a \left[b_{0,a}\left(1 + \frac{u^{[a]}}{u^{[-a]}}\right) + 2\sum_{k=1}^{a-1} \frac{b_{k,a}\, u^{[a]}}{u^{[a-2k]}} \right]$$

$$b_{0,s} = \sin^2\tfrac{\phi}{2}\, P^{(0,1)}_{s-1}\left(1 - 2\cos^2\tfrac{\phi}{2}\right) \qquad b_{l,s} = b_{0,l}\, b_{0,s-l} \qquad b_{0,0} = 1$$

- $\varphi(u)$ can be fixed by comparison with a Lüscher correction

$$\mathbf{Y}^{I}_{a,0}(q) = e^{-2L\,\tilde{E}_a(q)}\, \mathrm{Tr}\left[R_{A,a}(q)\, \mathcal{C}\, R^{\phi}_{A,a}(-\bar{q})\, \mathcal{C}^{-1} \right]$$

$$\mathbf{Y}^{II}_{a,0}(q) = e^{-2L\,\tilde{E}_a(q)}\, \mathrm{Tr}\left[R_{B,a}(q)\, \mathcal{C}\, R^{\phi}_{B,a}(-\bar{q})\, \mathcal{C}^{-1} \right]$$

$\mathcal{C}$: conj. matrix $\qquad R^{\phi} = \mathcal{S}^{-1}(\phi) R \mathcal{S}(\phi) \qquad \mathcal{S}(\phi)$: rot. matrix

- After fixing $\varphi(u)$

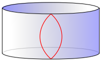
$$\mathbf{Y}^{I}_{a,0} = \mathbf{Y}^{II}_{a,0} = (-1)^{a+1} e^{-(2L+2)\tilde{E}_a(q)} \left(\frac{z^+ + \frac{1}{z^+}}{z^- + \frac{1}{z^-}} \right)^{1/2} \sigma_B^{1/2}(q)\sigma_B^{1/2}(-\bar{q})\mathbf{T}_{a,1}$$

which have to be replaced in

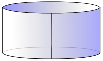
$$\mathcal{E}_0(L) = -\frac{1}{4\pi}\sum_{a=1}^{\infty}\int_0^{\infty} dq \log[1+Y^{I}_{a,0}] - \frac{1}{4\pi}\sum_{a=1}^{\infty}\int_0^{\infty} dq \log[1+Y^{II}_{a,0}]$$

- As $q \to 0$

$$\mathbf{Y}^{I}_{2n+1,0} = \mathbf{Y}^{II}_{2n+1,0} = \mathcal{O}(q^0)$$



$$\mathbf{Y}^{I}_{2n,0} = \mathbf{Y}^{II}_{2n,0} = \frac{16\, b^2_{0,n} h^2}{q^2} \left(\frac{h}{n}\right)^{4L+2} + \mathcal{O}(q^0)$$



Only $\mathbf{Y}^{\alpha}_{2n,0}$ contribute to the leading asymptotic solution

Using

$$\int_0^{\infty} dq \log\left[1+\frac{16\, b_{0,n}^2 h^2}{q^2}\left(\frac{h}{n}\right)^{4L+2}\right] \simeq 4\pi b_{0,n} h \left(\frac{h}{n}\right)^{2L+1}$$

The leading finite size correction becomes

$$\mathcal{E}_0(L) \simeq -2h^{2L+2}\sum_{n=1}^{\infty}\frac{b_{0,n}}{n^{2L+1}} = -2h^{2L+2}\sin^2\frac{\phi}{2}\sum_{k=0}^{\infty}\frac{P_k^{(0,1)}(-\cos\phi)}{(k+1)^{2L+1}}$$

$2L+1$ is number of fields in the insertion. Thus, the cusp anomalous dimension is obtained by setting $2L+1=0$

$$\Gamma_{\text{cusp}}(\phi) = -2h\,\sin^2\frac{\phi}{2}\sum_{k=0}^{\infty}P_k^{(0,1)}(-\cos\phi) = -h\left(\frac{1}{\cos\frac{\phi}{2}}-1\right)$$

This matches exactly the 1-loop result using $h(\lambda)=\lambda+\mathcal{O}(\lambda^2)$

Conclusions and Future Directions

- Another example of integrability being useful in $d > 2$

- ABJM cusp anomalous dimension from a BTBA system. It reproduces the 1-loop Γ_{cusp} and would provide the all-loop result

- This is the first step towards a direct derivation of the interpolating function $h(\lambda)$ appearing in all the integrability-based computations in ABJM

- This would require to solve the BTBA in the small angle limit ($\Gamma_{\text{cusp}} \simeq -\phi^2 B(\lambda)$)

* * *

- By going to higher orders and larger sectors in the perturbative spin chain $\Rightarrow$ one could further test the proposed dressing factors

- By iterating the BTBA eqs and comparing with the 2-loop result of Γ_{cusp} $\Rightarrow$ one could further test the proposed BTBA

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Thanks for your attention!