Nonequilibrium Transport and Thermalization in Strongly Disordered 2D Electron Systems

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Samples: IBM T. J. Watson Research Center, USA
Thermalization in isolated quantum many-body systems

• When an isolated quantum system is prepared far out of equilibrium:

Does it reach thermal equilibrium at long times?

Thermalizing

• Ergodic; system acts as a heat bath for its subsystems (exchange of energy and particles)

• Equilibrium statistical mechanics

• Quantum information lost with time

Nonthermalizing

• Some memory of initial state retained at long times

• Integrable systems

• Many-body-localized systems

• Quantum many-body scar states

Disorder + interactions: Can localization survive?

Anderson localization

Single particle
Conductivity $\sigma = 0$

Many-body localization (MBL)

Conductivity $\sigma = 0$ at $0 < T < T_{cr}$

• No particle transport but spreading of quantum information

Dynamical phase diagram

Thermal

$W_c$

MBL

Disorder $W$

[Fig. from Abanin et al., Rev. Mod. Phys. 91, 021001 (2019)]
Effect of the range of interactions on MBL? Role of dimensionality?

• MBL: strong disorder + short-range interactions (+ highly nonequilibrium conditions)

• MBL with longer-range interactions? Longer range interactions generally favor thermalization

Thermalizes as time $t \rightarrow \infty$ and system size $L \rightarrow \infty$

• MBL in dimensions $D > 1$? (In $D=1$?)

• MBL in mixed dimensionality systems?

MBL in synthetic many-body systems

- Ultracold atoms in optical lattices
- Trapped ions
- Superconducting qubits
- Spins of NV centers in diamond

[Fig. from https://physics.aps.org/articles/v4/78]

[Choi et al., Science 352, 1547 (2016)]
MBL in a disordered 2D bosonic optical lattice

- Ultracold atoms in optical lattices
- Ground state in the absence of disorder: Mott insulator
- Track time evolution of the initial out-of-equilibrium state (density step)

Density asymmetry (imbalance) vs time for different disorder strength $\Delta$

Imbalance persists at long times for high enough disorder $\Rightarrow$ MBL

Transition to MBL when all characteristic energy scales are comparable

[Choi et al., Science 352, 1547 (2016)]
MBL in synthetic many-body systems

- Ultracold atoms in optical lattices
- Trapped ions
- Superconducting qubits
- Spins of NV centers in diamond

[Fig. from https://physics.aps.org/articles/v4/78]

- Systems remain isolated only up to some time; time scales: $\sim 10^{-3}$ s
- Finite-size effects

[Choi et al., Science 352, 1547 (2016)]
MBL in real, solid-state materials? MBL in Coulomb interacting systems?

- Inevitable coupling to phonons $\Rightarrow$ MBL lost when coupled to a bath

![Graph of $a$-InO films](image)

- Vanishing conductivity at nonzero $T$?
- Other interpretations possible
  [Humbert et al., Nat. Commun. 12, 6733 (2021)]

- Other materials?
- Power-law interactions
- Nonequilibrium dynamics?

Transport in a thermally isolated, disordered 2D electron system (2DES) in Si: dynamics following a quantum quench

Effect of the Coulomb interaction range on thermalization of conductivity (interactions are 3D ⇒ mixed dimensionality system)

Results - effects of the Coulomb interaction range:
- No effect on equilibrium transport
- Striking effect on the dynamics

![Diagram showing MBL, prethermal (MBL-like), thermal (glassy) regimes with 1/r^3 and 1/r interaction ranges.](image-url)
Si MOSFET:

- **Basic building block of modern electronics:** well-developed, mature technology (device aspects well-understood, good contacts even at low $T$...)

- Discovery of 2D behavior of electrons in 1966!
  [Fowler, Fang, Howard, Stiles, PRL 16, 901 (1966)]

**MOSFET** (metal-oxide-semiconductor field-effect transistor) – **capacitor**!

Electric field effect: conductivity $\sigma(V_g)$

Vary density $n_s$ using $V_g$

$$n_s = C_{ox} \left( V_g - V_T \right)/e$$

- **Total density** in a 2DES changes quickly, within the device time constant $\tau = RC$ (~5 ns at most, for our devices)
Two-dimensional electron system in Si MOSFETs: An excellent candidate for observing MBL

**Si MOSFET:**
- Electron density $n_s$ varied easily up to three orders of magnitude by changing gate voltage $V_g$

$\Rightarrow$ a) Can be prepared far out of equilibrium
b) Can study *thermalization dynamics across the quantum metal-insulator transition* (MIT)

- **Weak electron-phonon coupling at low $T$**  
  ($T \lesssim 1.6$ K for our samples in the regime of interest)

$\Rightarrow$ Heat transfer by *electron diffusion through contacts* and leads

Coupling further reduced by placing samples and leads in *vacuum*

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MOSFET (metal-oxide-semiconductor field-effect transistor) – capacitor!

Si: $T_{\text{Debye}} = 645$ K

Two-dimensional electron system in Si MOSFETs: An excellent candidate for observing MBL

Si MOSFET:

- **Screening the Coulomb interaction** within the 2DES by reducing the oxide thickness $d_{ox}$ (distance from the gate)

  **Coulomb interaction in the presence of the gate**

  $\sim |1/r - 1/(r^2 + 4d_{ox}^2)^{1/2}|$

  Coulomb interaction of an electron with an image charge of another electron in a 2DES

  At large distances $r \gg 2d_{ox}$, interaction $\sim 1/r^3$

  Realized at low enough densities, such that the mean electron separation

  $$2a = 2(\pi n_s)^{-1/2} \gg 2d_{ox}$$

  [Widom & Tao, PRB 38, 10787 (1988); Ho et al., PRB 80, 155412 (2009); Skinner & Shklovskii, PRB 82, 155111 (2010); Skinner & Fogler, Phys. Rev. B 82, 201306(R) (2010); Fregoso & Sá de Melo, PRB 87, 125109 (2013)]
2D electron system (2DES) in Si: Our samples

- We focus on the case of **strong disorder** (smooth random potential due to Na\(^+\) ions in the oxide; frozen below ~150-200 K); \(\mu_{\text{peak}} \approx 0.05 - 0.06 \text{ m}^2/\text{Vs}\)

**Long-range Coulomb interaction**

- **Thick oxide:**
  - \(d_{\text{ox}} = 50 \text{ nm}\)
  - \(5.3 \lesssim d_{\text{ox}}/a \lesssim 8.0\)

- **Coulomb interaction:**
  \[\sim 1/r\]

  Mean carrier separation:
  \[2a = (\pi n_s)^{-1/2}\]

**Screened Coulomb interaction ("short-range")**

- **Thin oxide:**
  - \(d_{\text{ox}} = 6.9 \text{ nm}\)
  - \(0.7 \lesssim d_{\text{ox}}/a \lesssim 1.5\)

  For \(d_{\text{ox}}/a \ll 1\), screened Coulomb interaction:
  \[\sim 1/r^3\]
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  \[ \sim 1/r^3 \]

- Thermalization in \(\text{In}_x\text{O films}\) (Anderson insulators) and screening with the nearby metallic plane: \(d/a \sim 1.5 - 2.5\) (samples in liquid \(^4\text{He}\))
  \[\Rightarrow\] No effect on the equilibration time
  
2D electron system in Si: Our samples

Sample dimensions:
Length $L$ [µm] $\times$ width $W$ [µm]
$\sigma = G/(W/L)$, $G$ is conductance

- Measured devices on several chips in each set of MOSFETs

Conductivity $\sigma$ vs electron density $n_s$ in thin-oxide MOSFETs at $T=4.2$ K

$\mu_{\text{peak}} \approx 0.05 - 0.06$ m$^2$/Vs $\Rightarrow$ strong disorder

Mobility $\mu = \sigma/en_s$ vs $n_s$

Drude: $\sigma = (2e^2/h) \left( k_F l \right)$

($k_F$ - Fermi wavevector, $l$ – mean free path)
Equilibrium transport in a strongly disordered 2DES: Same temperature dependence of the conductivity

Long-range Coulomb interaction

Screened ("short-range") Coulomb interaction

2D MIT

\( n_c (10^{11} \text{ cm}^{-2}) \)

(5.0 ± 0.3)

\( n_s (10^{11} \text{ cm}^{-2}) \)

(4.2 ± 0.2)


Phase diagram of a strongly disordered 2DES in Si: Equilibrium transport

\[ \frac{d\sigma}{dT} > 0 \quad k_F \ l < 1 \]

\[ \frac{d\sigma}{dT} < 0 \quad k_F \ l > 1 \]

**Insulating** \( \sigma(T=0)=0 \)

**Metallic** (Non-Fermi Liquid) \( \sigma(T=0)\neq0 \)

**Metallic** (FL? NFL?)

Exponential localization

“Bad” metal: \( \sigma(n_s, T)=\sigma(n_s, T=0)+b(n_s) T^{3/2} \)

**Reviews** in book chapters:
D. P. in
**Equilibrium transport:**

**Scaling near the 2D MIT is the same**

- Critical exponents are the same
- 2D MIT is disorder-dominated

\[
\sigma(n_s, T) = \sigma_c(T) f(T/T_0)
\]

\[
\sigma_c(T) = \sigma(n_s = n_c) \propto T^x
\]

\[z_v = (2.1 \pm 0.1)\]

\[z_v = (2.0 \pm 0.1)\]

Energy scales

- Fermi energy (temperature) \( T_F \) [K] = 7.31 \( n_s \) [10^{11} \text{ cm}^{-2}]

- Critical density for the MIT: \( n_c \) (10^{11} \text{ cm}^{-2}) \approx 5.0 \Leftrightarrow T_F \approx 35 \text{ K}

- Onset of localization at \( n_c \): disorder \( W \sim T_F (n_c) \Leftrightarrow W \sim 35 \text{ K}

- Coulomb energy \( E_C \):

  **Thick oxide:**

  \[ r_s = \frac{E_C}{E_F} \propto n_s^{-1/2} \approx 4 \]

  **Thin oxide:**

  Screening by the gate: \( r_s \ll 1 \) \( (E_C \approx E_F) \)

  [Widom & Tao, Phys. Rev. B 38, 10787 (1988)]

- **All energy scales** \( (W, E_F, \text{ and } E_C) \) are comparable
Quench dynamics in a disordered 2DES: Relaxations of $\sigma$ after a large, rapid change of $n_s$

Experimental protocol

- Sample is warmed up to ~7-20 K to “reset” the sample
- At ~7-20 K, the gate voltage is set to some $V_g^{i}$ where $k_F l \geq 1$
- The sample is cooled to the measurement $T$ and allowed to equilibrate ( $\Leftrightarrow \sigma$ saturates)
- Then $V_g$, i.e. $n_s$ is quickly (<3 s) changed to a lower, final value
- $\sigma$ is measured continuously throughout the process

- Large perturbation: $\Delta E_F \sim E_F$
  ($k_B T \ll E_F$)

Initial $n_s(10^{11}\text{cm}^{-2}) = 20.26$ ; $k_F l \sim 1$

Final $n_s(10^{11}\text{cm}^{-2}) = 4.74 \geq n_c$
Quench dynamics in a high-disorder 2DES: Relaxations of $\sigma$ after a large, rapid change of $n_s$

- Overshooting of equilibrium
- $\sigma$ moves away from $\sigma_0$ at intermediate times
- Slow, nonexponential relaxation of $\sigma$ at intermediate times (before the minimum)

Dynamical scaling at intermediate times:

$$\sigma(t, T)/\sigma_0 \propto t^{\alpha(n)} \exp\{-[t/\tau(n_s, T)]^{\beta(n)}\}$$

$\alpha(n_s) < 0.4, 0.2 < \beta(n_s) < 0.45$

Broad distribution of relaxation times

$$\sigma/\sigma_0 \propto t^{-\alpha} \text{ as } T \rightarrow 0$$
Quench dynamics in a high-disorder 2DES: Relaxations of $\sigma$ after a large, rapid change of $n_s$

- Different $T$ data collapse for times after the minimum.
- The system reaches “equilibrium” after a long enough time $t$.
- Approach to “equilibrium” exponential in time.

Long times (approach to “equilibrium”):

- Thermalization at long times:
  \[ \tau_{\sigma} \propto \exp \left( \frac{E_A}{T} \right) \]
- As $T \to 0$, $\tau_{\sigma} \to \infty$
  (glass transition at $T_g=0$)

$E_A \approx 57$ K

Phase diagram of a strongly disordered 2DES in Si: Dynamics with long-range Coulomb interaction

**Glassy Behavior** (for $n_s < n_g$)

- **Insulating**
  - $\sigma(T=0) = 0$

- **Metallic** (Non-Fermi Liquid)
  - $\sigma(T=0) \neq 0$

- **Metallic** (FL? NFL?)
  - $k_F l < 1$
  - $d\sigma/dT > 0$

- **Metallic**
  - $k_F l > 1$
  - $d\sigma/dT < 0$

- **Exponential localization; Glassy insulator**

- **“Bad” metal**:
  - $\sigma(n_s, T) = \sigma(n_s, T=0) + b(n_s) T^{3/2}$

- Intermediate, glassy phase

**$T=0$ glass transition for $n_s < n_g$**

- $n_g \approx 7.5 \times 10^{11} \text{cm}^{-2}$
- $n_c \approx 5.0 \times 10^{11} \text{cm}^{-2}$

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**Reviews** in book chapters:

Quench dynamics in a strongly disordered 2DES

Same experimental protocol for both sets of samples; samples and leads in vacuum

- Sample is warmed up to ~7-20 K to “reset” the sample
- At ~7-20 K, the gate voltage is set to some $V_g$ where $k_F l \geq 1$
- The sample is cooled to the measurement $T$ and allowed to equilibrate
- Then $V_g$, i.e. $n_s$ is quickly (<3 s) changed to a lower, final value
- $\sigma$ is measured continuously throughout the process

Initial $n_s(10^{11}\text{cm}^{-2}) = 32.2$ ; $k_F l > 1$

Final $n_s(10^{11}\text{cm}^{-2}) = 8.44 \geq n_c$ ; $T=0.92$ K

$n_c \approx 4.2 \times 10^{11}\text{cm}^{-2}$

- Large perturbation: $\Delta E_F \sim E_F$ ($k_B T \ll E_F$)
Short-range case: “Equilibrium” conductivity after $n_s$ change and after cooling is the same.

$n_s^f (10^{11}\text{cm}^{-2}) = 5.94$

$T = 0.46 \text{ K}$
Quench dynamics in a disordered 2DES: Anomalously slow relaxations

Thick-oxide MOSFETs:
- Thermalization at long times:
  \[ \tau_\sigma \propto \exp \left( \frac{E_A}{T} \right) \]
- As \( T \to 0 \), \( \tau_\sigma \to \infty \)
  (glass transition at \( T=0 \))


Thin-oxide MOSFETs:
- \( \sigma \) does not go farther away from equilibrium with time for a given \( T \); negligible relaxation
- No evidence of glassiness
- Thermalization time:
  \[ \tau_\sigma \sim 10^4 \text{ s}, \text{ similar for all } T \text{ and } n_s \]
- Observable when \( k_F l < 1 \)
Quench dynamics with a short-range interaction: Relaxations at different electron densities

No relaxations when $k_F l > 1$
Relaxations appear when $k_F l \sim 1$

Metallic side: overshooting but weak time dependence at intermediate times

Insulating side: large noise; no relaxation at intermediate times

Just above the MIT: large noise

$n_c (10^{11} \text{ cm}^{-2}) = (4.2 \pm 0.2)$
Quench dynamics with a short-range interaction: Initial amplitude of the relaxations at low $T$

- Initial amplitude $\sigma/\sigma_0$ at 10 s
- Deviations from $\sigma_0$ become observable on the metallic side of the MIT, when $k_F l < 1$
- As $n_s$ is reduced, deviations from $\sigma_0$ become more pronounced and peak just above the MIT; nonmonotonic dependence on density

Metal-insulator transition
Approach to equilibrium in the short-range case: Exponential relaxations at long times

- Conductance relaxations
  \[ \Delta G(t) \propto \exp\left(-\frac{t}{\tau_\sigma}\right) \]
- Fit a wide range of \( n_s \) and \( T \)
  \[ \Rightarrow \text{Extract } \tau_\sigma \]
Quench dynamics with a short-range interaction: Thermalization time

- Long: $\tau_\sigma \sim 10^4$ s
- No dependence on density or temperature

What is the origin of this slow dynamics?
Thermal coupling to the environment sets the time scale for thermalization!

Short-range Coulomb interaction

Stronger coupling to the bath when in $^4$He vapor

τ$_\sigma$ reduced by an order of magnitude!

Prethermal, MBL-like
Effect of thermal coupling to the environment on thermalization time

Short-range Coulomb interaction

- $T < 2 \text{ K}$: weak electron-phonon coupling
  - a) Samples in vacuum: $\tau_{eq} \sim 10^4 \text{ s}$
    - weak thermal coupling to environment
  - b) Samples in $^4\text{He}$ vapor: $\tau_{eq} \sim 10^3 \text{ s}$
    - intermediate thermal coupling to environment

- $T > 2 \text{ K}$: cooling via phonons dominant
  - Strong thermal coupling to environment:
    - $\tau_{eq} < 200 \text{ s}$ (at 4.2 K in both vacuum and liquid $^4\text{He}$)

Long-range Coulomb interaction:

Glassy dynamics unchanged even at $T > 4.2 \text{ K}$ ⇒ insensitive to thermal coupling to environment

Exponential dependence of thermalization time on the coupling strength:

MBL-like dynamics
Quench dynamics and thermalization: Direct observation of MBL-like dynamics

Effect of interaction range in a 2DES for a fixed disorder strength:

- No difference in dc transport properties
- **Striking difference in nonequilibrium dynamics!** (when $k_F l < 1$; bad conductor)

- Screened Coulomb interaction leads to a **prethermal, MBL-like** regime at intermediate times
- Time scale: hours!

[L. J. Stanley et al., arXiv:2110.11473; under revision]
Quench dynamics and thermalization:
Direct observation of MBL-like dynamics

Effect of interaction range in a 2DES for a fixed disorder strength:

Power-law interactions: $\sim 1/r^\alpha$

- Our results provide constraints for theory and motivate further work

[Theory: Yao et al., PRL 113, 243002 (2014); Burin, PRB 92, 104428 (2015); Gutman et al., PRB 93, 245427 (2016)…]
Summary and outlook

• Strongly disordered 2D electron system in a Si MOSFET:
  ➢ Slow dynamics when $k_F l < 1$
  ➢ No glassy dynamics with screened Coulomb interaction ($\sim 1/r^3$)
  ➢ New, solid-state platform for studies of thermalization and MBL-like dynamics; time scales: hours!
  ➢ Building blocks for quantum information science?

[L. J. Stanley et al., arXiv:2110.11473; under revision]