Nonequilibrium Transport and Thermalization in Strongly Disordered 2D Electron Systems

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Thermalization in isolated quantum many-body systems

- When an isolated quantum system is prepared far **out of equilibrium**:

Does it **reach thermal equilibrium** at long times?

Thermalizing

- Ergodic; system acts as a heat bath for its subsystems (exchange of energy and particles)
- Equilibrium statistical mechanics
- Quantum information lost with time

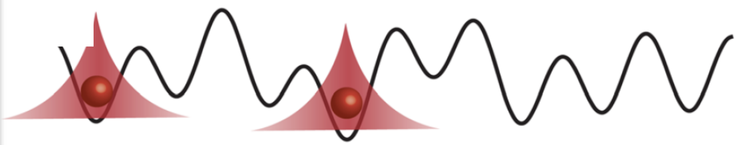
Nonthermalizing

- Some memory of initial state retained at long times
- Integrable systems
- Many-body-localized systems
- Quantum many-body scar states

[Polkovnikov *et al*., Rev. Mod. Phys. 83, 863 (2011); Nandkishore & Huse, Annu. Rev. Condens. Matter Phys. 6, 15 (2015); Mori *et al*., J. Phys. B: At. Mol. Opt. Phys. 51, 112001 (2018); Abanin *et al.*, Rev. Mod. Phys. 91, 021001 (2019); Gopalakrishnan & Parameswaran, Phys. Rep. 862, 1 (2020); …]

Disorder + interactions: Can localization survive?

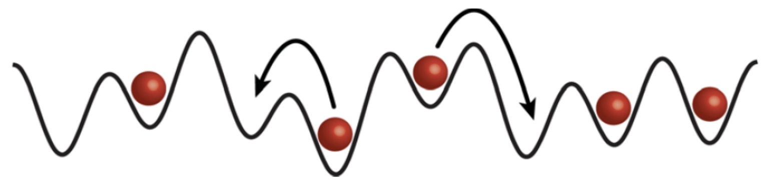
Anderson localization



Single particle

Conductivity $\sigma = 0$

Many-body localization (MBL)



Conductivity $\sigma = 0$ at $0 < T < T_{cr}$

[Figs. from Abanin *et al.*, Rev. Mod. Phys. 91, 021001 (2019)]

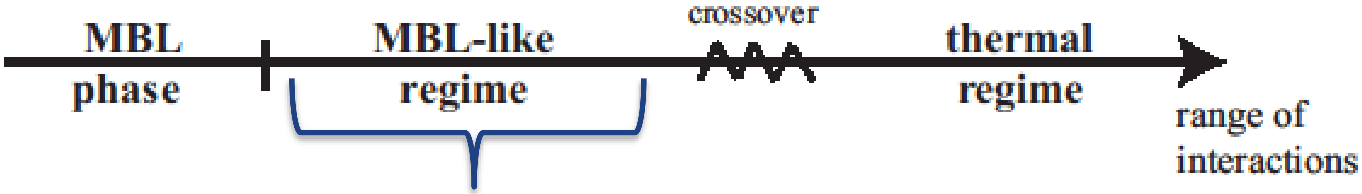
- No particle transport but spreading of quantum information

Dynamical phase diagram

Effect of the range of interactions on MBL? Role of dimensionality?

- MBL: strong disorder + short-range interactions
  (+ highly nonequilibrium conditions)

- MBL with longer-range interactions?
  Longer range interactions generally favor thermalization



Thermalizes as time $t \longrightarrow \infty$
and system size $L \longrightarrow \infty$

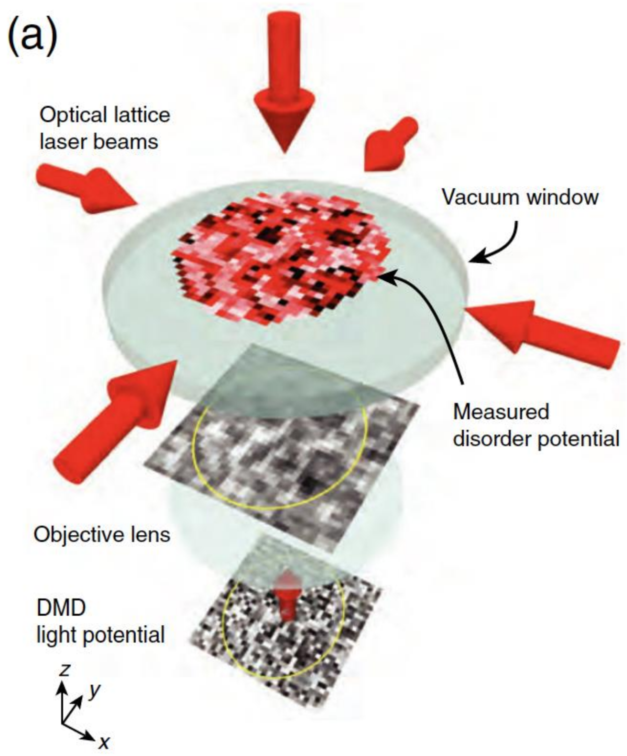
- MBL in dimensions D > 1? (In D=1?)

- MBL in mixed dimensionality systems?

[Gornyi *et al.*, Phys. Rev. Lett. 95, 206603 (2005); Basko *et al*., Ann. Phys. 321, 1126 (2006); Nandkishore & Sondhi, Phys. Rev. X 7, 041021 (2017); Tikhonov & Mirlin, Phys. Rev. B 97, 214205 (2018); Gopalakrishnan & Huse, Phys. Rev. B 99, 134305 (2019); Sajna & Polkovnikov, Phys. Rev. A 102, 033338 (2020)…]

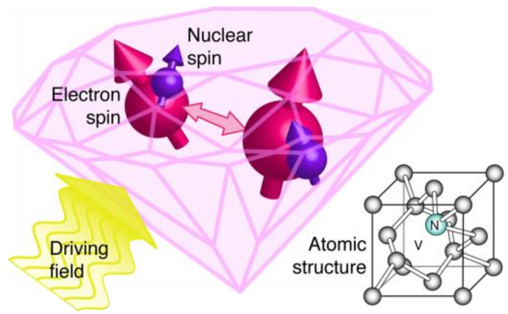
MBL in synthetic many-body systems

- Ultracold atoms in optical lattices



[Choi *et al.*, Science 352, 1547 (2016)]

- Trapped ions
- Superconducting qubits
- Spins of NV centers in diamond

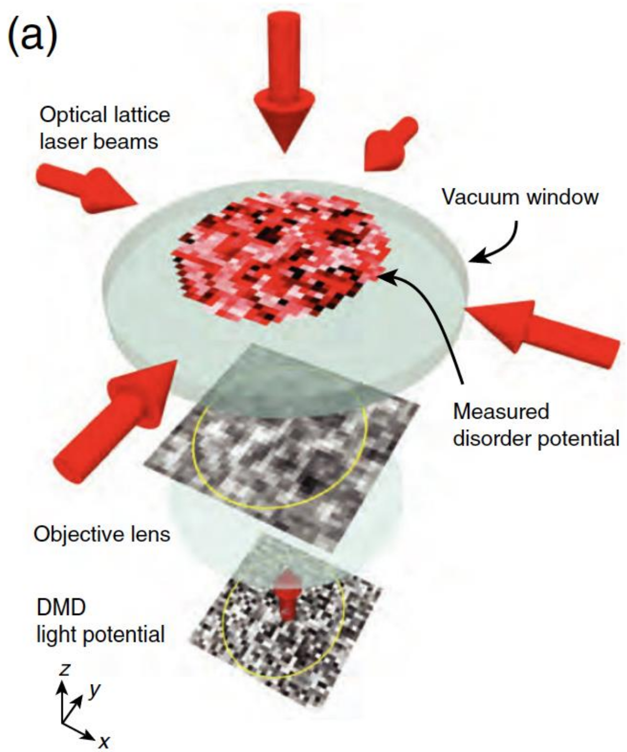


[Fig. from https://physics.aps.org/articles/v4/78]

MBL in a disordered 2D bosonic optical lattice

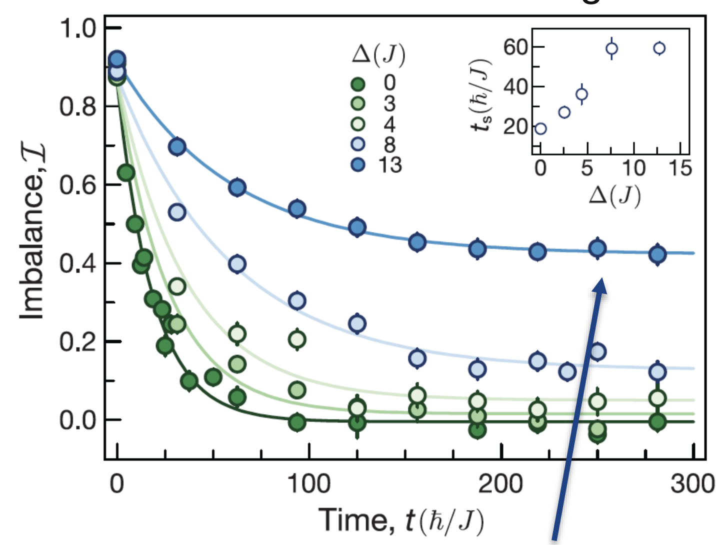
- Ultracold atoms in optical lattices

- Ground state in the absence of disorder: Mott insulator
- Track time evolution of the initial out-of-equilibrium state (density step)



[Choi *et al.*, Science 352, 1547 (2016)]

Density asymmetry (imbalance) vs time for different disorder strength **Δ**

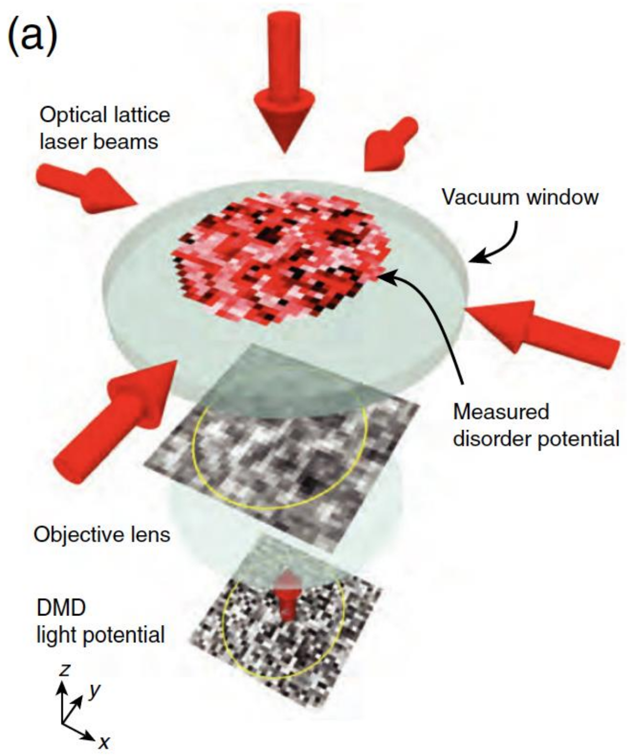


Imbalance persists at long times for high enough disorder ⇒ **MBL**

Transition to MBL when all characteristic energy scales are comparable

MBL in synthetic many-body systems

- Ultracold atoms in optical lattices



[Choi *et al.*, Science 352, 1547 (2016)]

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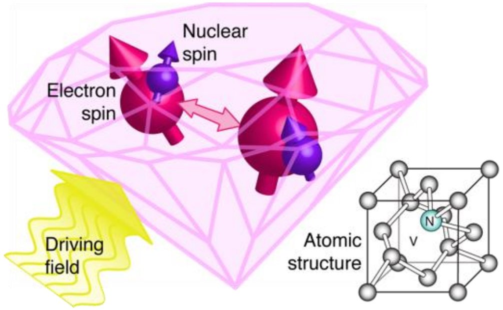


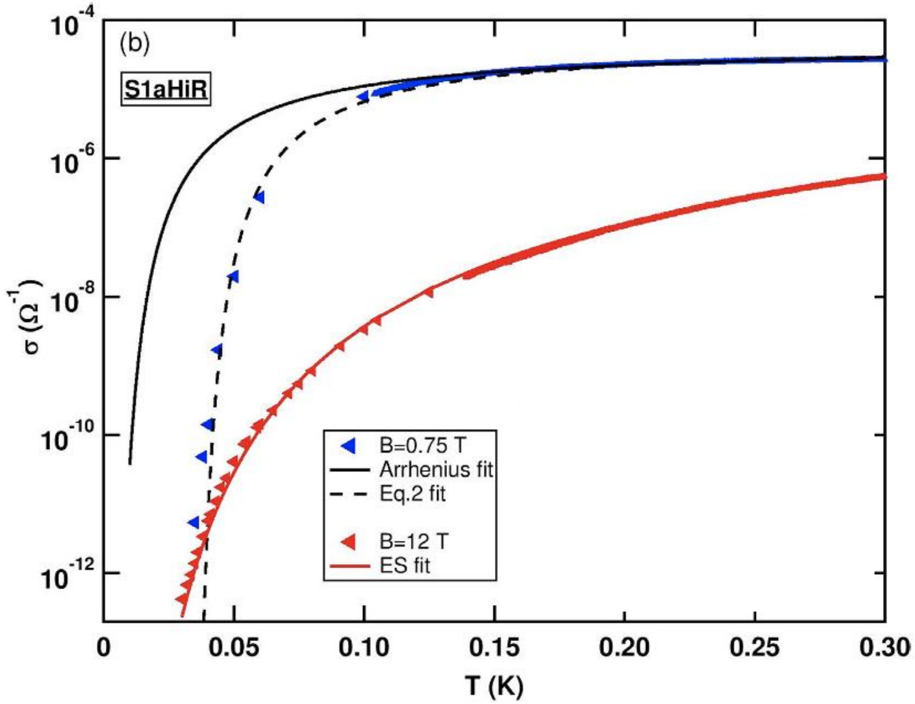
[Fig. from https://physics.aps.org/articles/v4/78]

- Systems remain isolated only up to some time; time scales: ~ 10^{-3} s
- Finite-size effects

MBL in real, solid-state materials? MBL in Coulomb interacting systems?

- Inevitable coupling to phonons ⇒ MBL lost when coupled to a bath

_a_-InO films



[Fig. from Ovadia *et al.,* Sci. Rep. 5, 13503 (2015); see also: Tamir *et al.,* Phys. Rev. B 99, 035135 (2019); Ovadyahu, Phys. Lett. 108, 156602 (2012); Ovadyahu, Phys. Rev. B 91, 035113 (2015)]

- Vanishing conductivity at nonzero *T?*
- Other interpretations possible

  [Humbert et al., Nat. Commun. 12, 6733 (2021)]

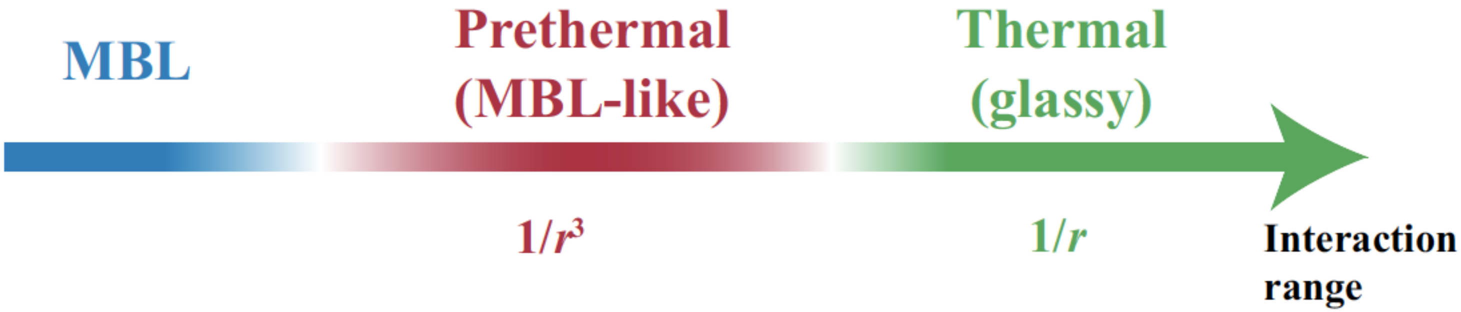
- Other materials?
- Power-law interactions
- Nonequilibrium dynamics?

Outline

- **Transport in a thermally isolated, disordered 2D electron system (2DES) in Si: dynamics following a quantum quench**

- **Effect of the Coulomb interaction range on thermalization of conductivity** (interactions are 3D ⇒ mixed dimensionality system)

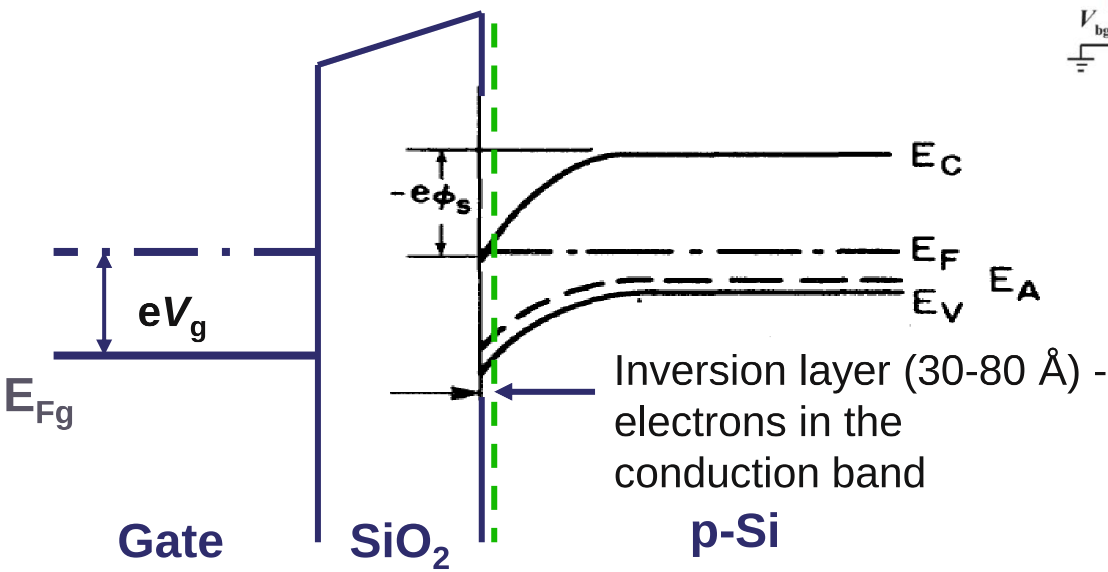
- **Results - effects of the Coulomb interaction range:**
  - **No effect on equilibrium transport**
  - **Striking effect on the dynamics**



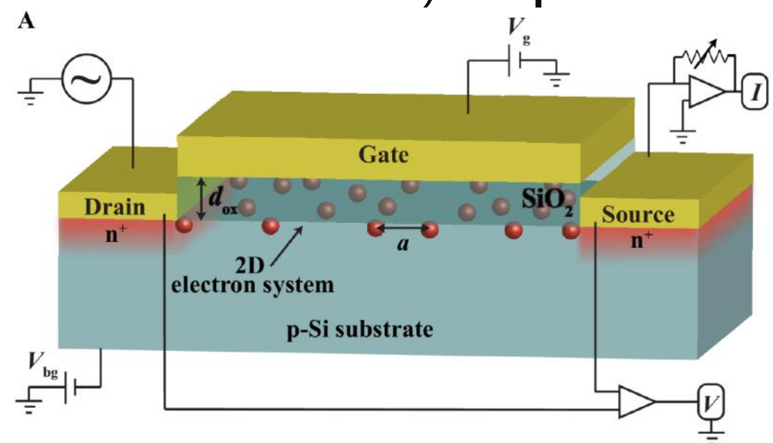
Two-dimensional electron system in Si MOSFETs: An excellent candidate for observing MBL

Si MOSFET:

- **Basic building block of modern electronics:** well-developed, mature technology (device aspects well-understood, good contacts even at low *T*…)
- Discovery of 2D behavior of electrons in 1966!
  [Fowler, Fang, Howard, Stiles, PRL 16, 901 (1966)]



MOSFET (metal-oxide-semiconductor field-effect transistor) – **capacitor!**



Electric field effect: conductivity $\sigma(V_g)$

Vary density n_s using V_g

$$n_s = C_{ox}(V_g - V_T)/e$$

- **Total density** in a 2DES changes quickly, within the device time constant $\tau = RC$ (~5 ns at most, for our devices)

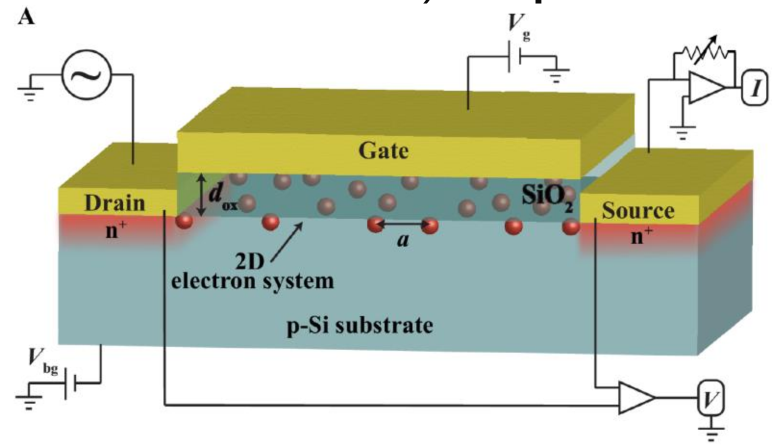
Two-dimensional electron system in Si MOSFETs: An excellent candidate for observing MBL

Si MOSFET:

- Electron **density n_s varied easily** up to three orders of magnitude by changing gate voltage V_g

⇒ a) Can be prepared far out of equilibrium
b) Can study thermalization dynamics across the quantum metal-insulator transition (MIT)

MOSFET (metal-oxide-semiconductor field-effect transistor) – **capacitor**!



- Weak electron-phonon coupling at low *T*
($T \lesssim 1.6$ K for our samples in the regime of interest)

Si: $T_{\mathrm{Debye}} = 645$ K

⇒ Heat transfer by **electron diffusion through contacts** and leads

Coupling further reduced by placing samples and leads in **vacuum**

[Zieve *et al.*, Phys. Rev. B 57, 2443 (1998); Altshuler *et al.*, Physica E 9, 209 (2001)]

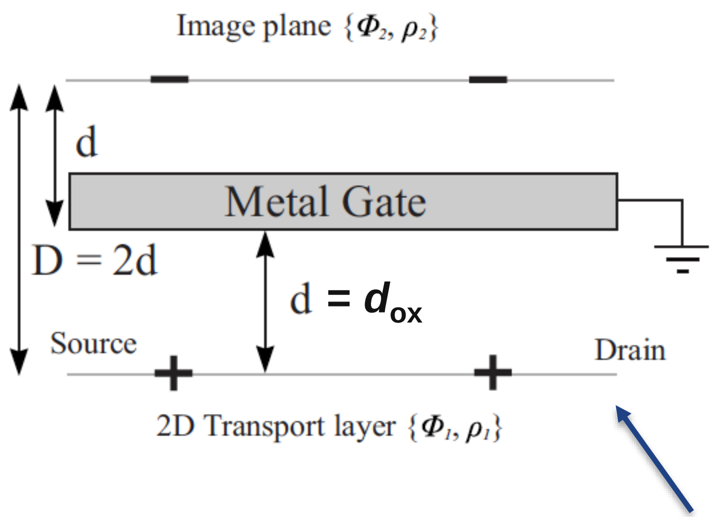
Two-dimensional electron system in Si MOSFETs: An excellent candidate for observing MBL

Si MOSFET:

- **Screening the Coulomb interaction** within the 2DES by reducing the oxide thickness d_{ox} (distance from the gate)

Coulomb interaction in the presence of the gate

$$\sim\left|1/r - \underbrace{1/(r^2 + 4d_{ox}^2)^{1/2}}_{\text{Coulomb interaction of an electron with an image charge of another electron in a 2DES}}\right|$$



[Widom & Tao, PRB 38, 10787 (1988); Ho et al., PRB 80, 155412 (2009); Skinner & Shklovskii, PRB 82, 155111 (2010); Skinner & Fogler, Phys. Rev. B 82, 201306(R) (2010); Fregoso & Sá de Melo, PRB 87, 125109 (2013)]

At large distances $r \gg 2d_{ox}$, interaction $\sim 1/r^3$

Realized at low enough densities, such that the mean electron separation

$$2a = 2(\pi n_s)^{-1/2} \gg 2d_{ox}$$

2D electron system (2DES) in Si: Our samples

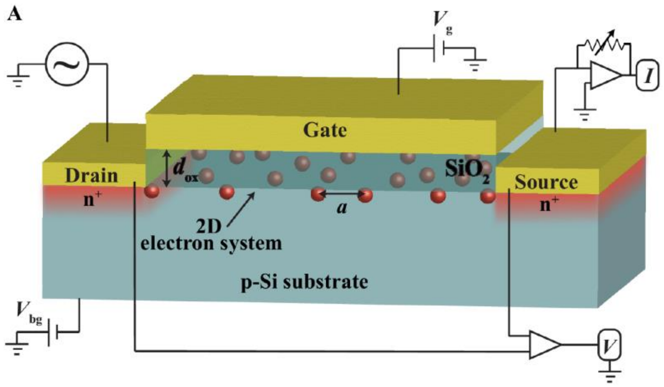
- We focus on the case of **strong disorder** (smooth random potential due to Na^+ ions in the oxide; frozen below ~150-200 K); $\mu_{\text{peak}} \approx 0.05 - 0.06$ m²/Vs

Long-range Coulomb interaction

- **Thick oxide:**
  - d_{ox} = 50 nm
  - $5.3 \lesssim d_{ox}/a \leq 8.0$
- Coulomb interaction:

$$\sim 1/r$$

Mean carrier separation: $2a = (\pi n_s)^{-1/2}$



The two sets of devices are otherwise identical

Screened Coulomb interaction ("short-range")

- **Thin oxide:**
  - d_{ox} = 6.9 nm
  - $0.7 \lesssim d_{ox}/a \lesssim 1.5$
- For $\frac{d_{ox}}{a} \ll 1$, screened Coulomb interaction:

$$\sim 1/r^3$$

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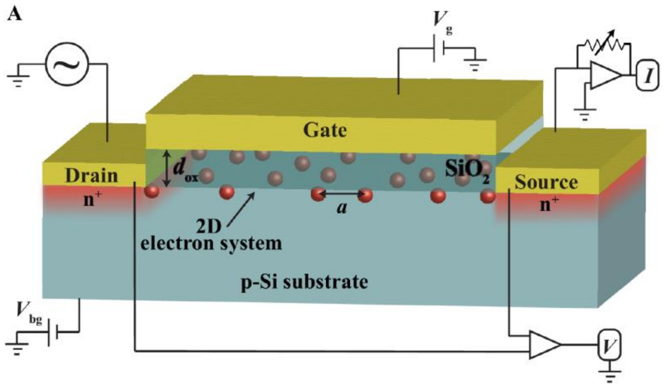

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- For $\frac{d_{ox}}{a} \ll 1$, screened Coulomb interaction:

$$\sim 1/r^3$$

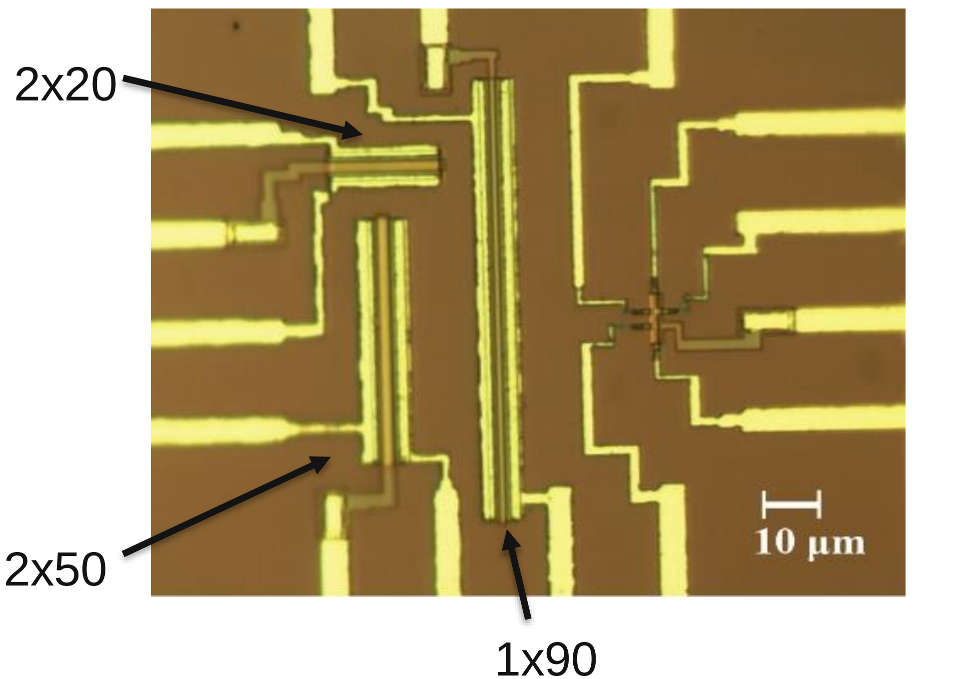
- Thermalization in **In_xO films** (Anderson insulators) and screening with the nearby metallic plane: $d/a \sim$ **1.5 - 2.5** (samples in liquid ^{4}He)

⇒ No effect on the equilibration time

[Ovadyahu, Phys. Rev. B 99, 184201 (2019)]

2D electron system in Si: Our samples

Top view of a Si chip with four devices

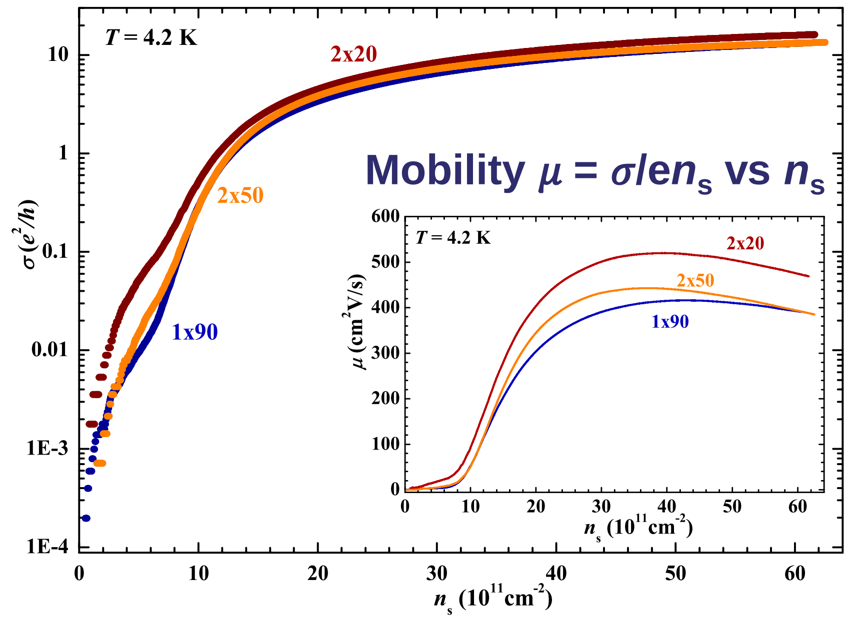


Sample dimensions:
Length L [μm] x width W [μm]
$\sigma = G/(W/L)$, G is conductance

- Measured devices on several chips in each set of MOSFETs

Conductivity σ vs electron density n_s in thin-oxide MOSFETs at T=4.2 K

$\mu_{\text{peak}} \approx 0.05 - 0.06$ m^2/Vs $\Rightarrow$ strong disorder

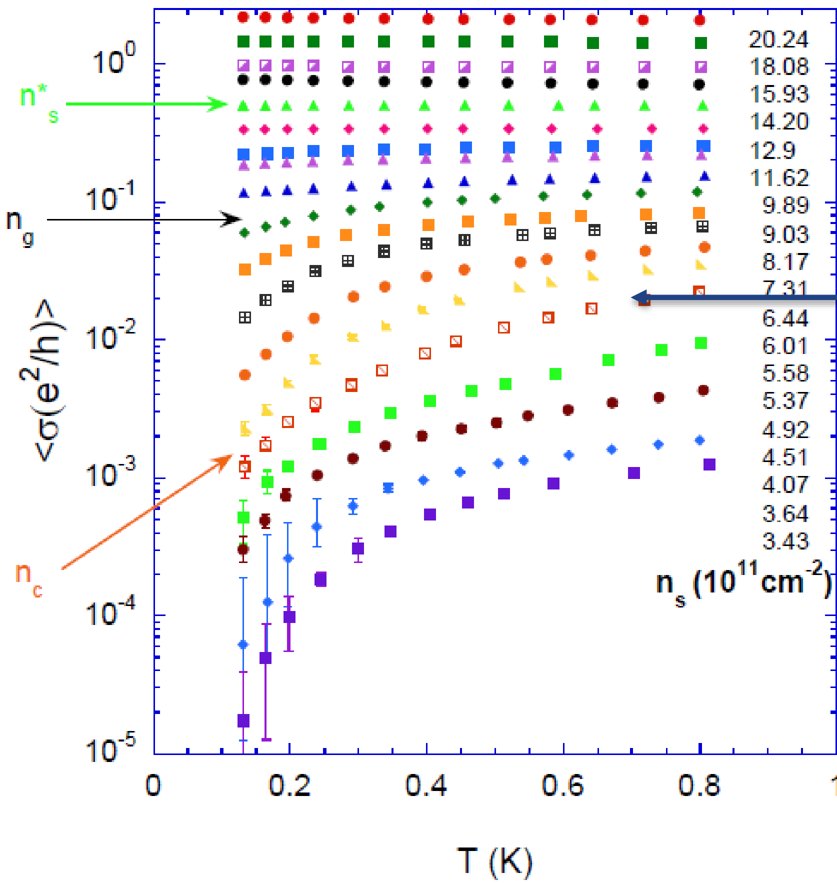


Drude: $\sigma = (2e^2/h)\,(k_F l)$

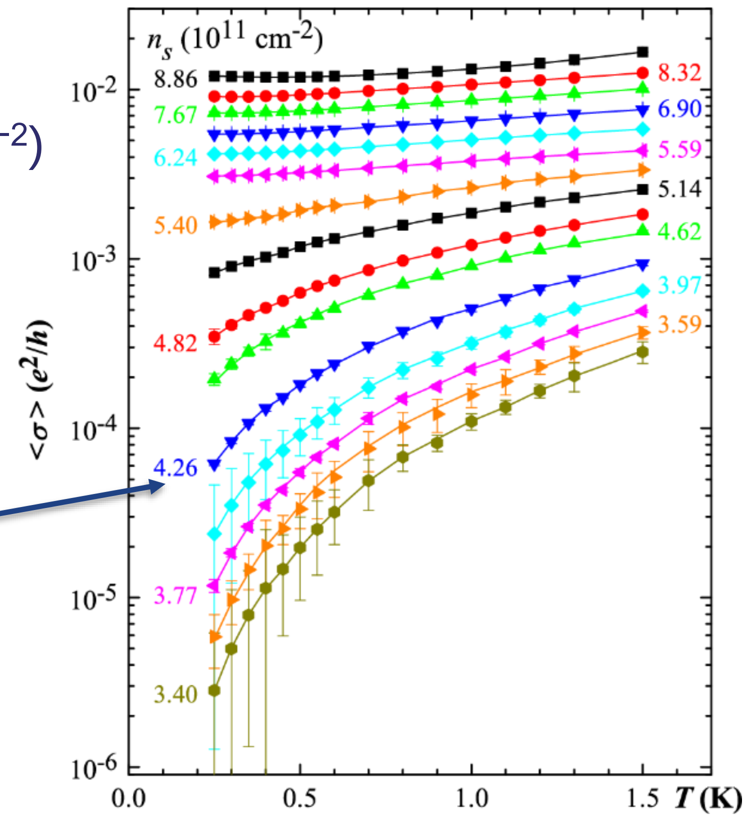
(k_F -Fermi wavevector, l – mean free path)

Equilibrium transport in a strongly disordered 2DES: Same temperature dependence of the conductivity

Long-range Coulomb interaction



Screened ("short-range") Coulomb interaction



2D MIT

n_c (10^{11} cm^{-2})

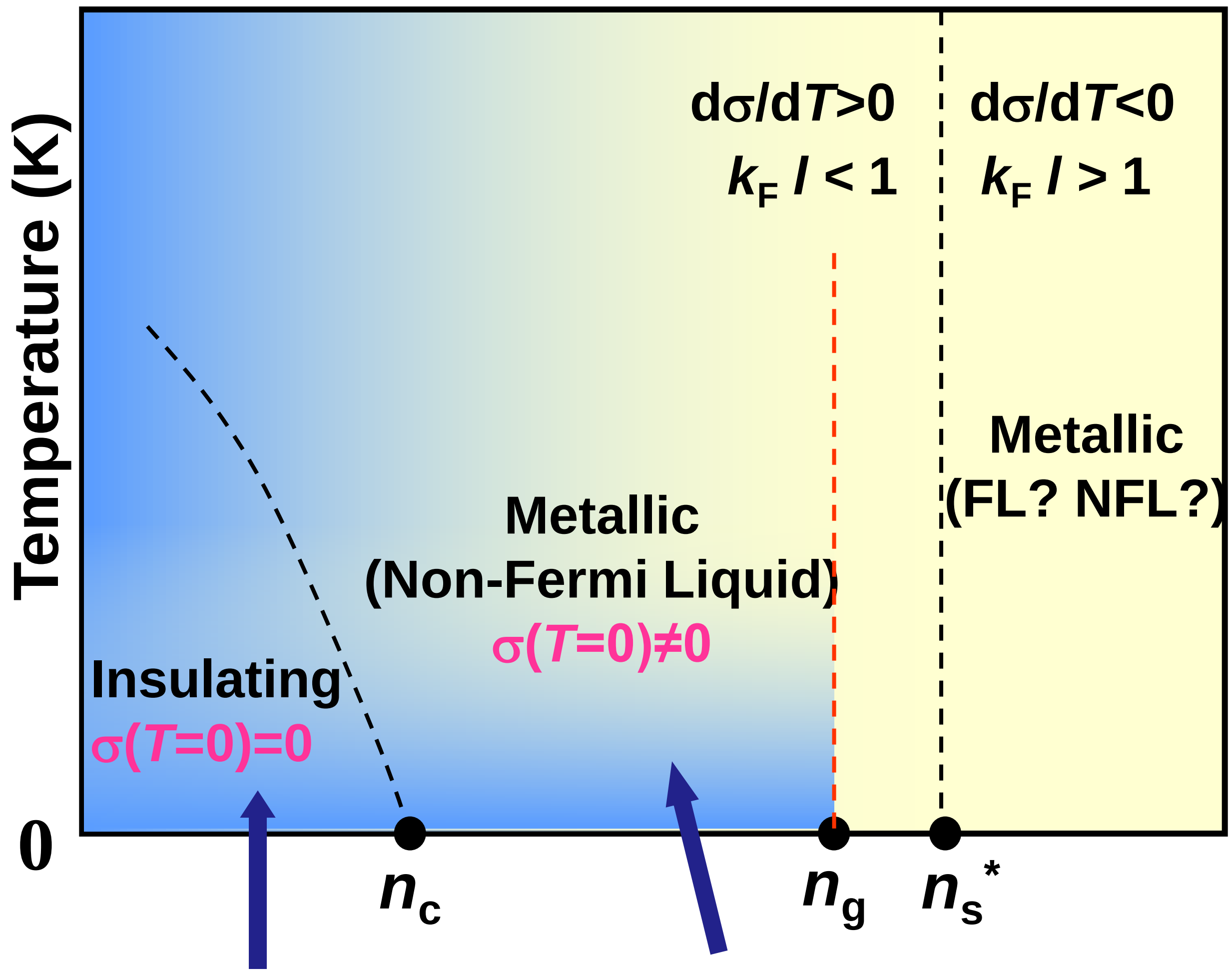
(5.0 ± 0.3)

(4.2 ± 0.2)

[S. Bogdanovich & D. Popović, Phys. Rev. Lett. 88, 236401 (2002)]

[P. V. Lin & D. Popović, Phys. Rev. Lett. 114, 166401 (2015)]

Phase diagram of a strongly disordered 2DES in Si: Equilibrium transport



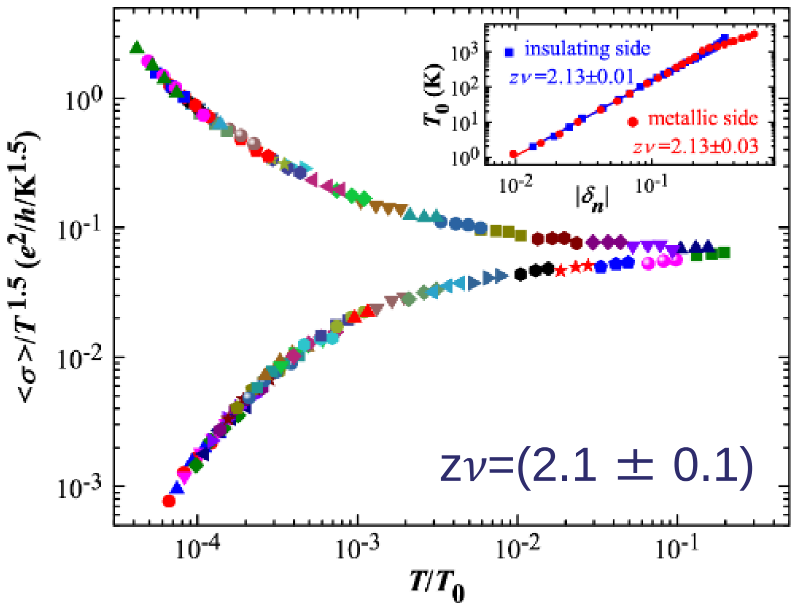
Exponential localization

"Bad" metal: $\sigma(n_s,T)=\sigma(n_s,T=0)+b(n_s)T^{3/2}$

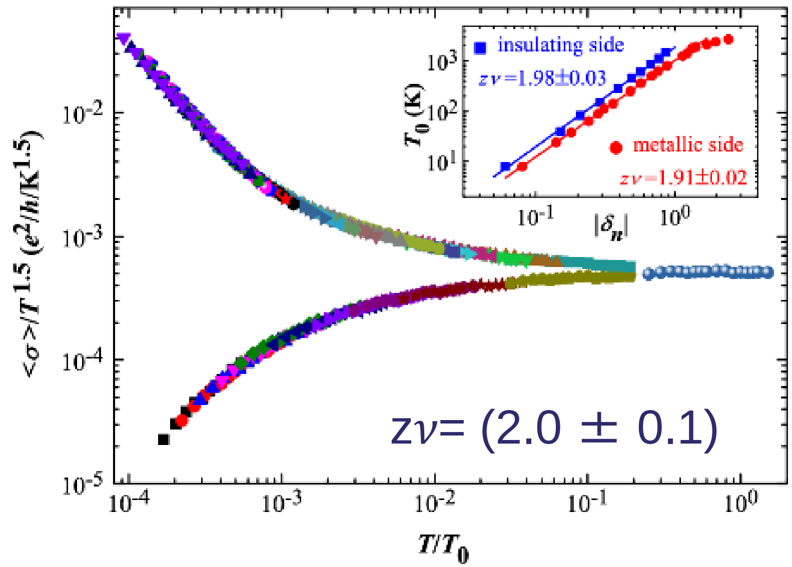
Reviews in book chapters:
D. P. in
a) "Conductor-Insulator Quantum Phase Transitions",
edited by Dobrosavljević, Trivedi, Valles (Oxford Univ. Press, 2012)
b) "Strongly Correlated Electrons in Two Dimensions", edited by Kravchenko (Pan Stanford Publishing, 2017)

Equilibrium transport: Scaling near the 2D MIT is the same

Long-range Coulomb interaction



Screened (“short-range”) Coulomb interaction



$$\sigma(n_s, T) = \sigma_c(T) f(T/T_0)$$
$$\sigma_c(T) = \sigma(n_s = n_c) \propto T^x$$

- Critical exponents are the same
- 2D MIT is disorder-dominated

[P. V. Lin & D. Popović, Phys. Rev. Lett. 114, 166401 (2015)]

Energy scales

- Fermi energy (temperature) T_F [K] =7.31 n_s [10^{11} cm^{-2}]
- Critical density for the MIT: n_c (10^{11} cm^{-2}) ~ 5.0 $\Leftrightarrow$ $T_F \sim 35$ K
- Onset of localization at n_c: disorder $W \sim T_F(n_c)$ $\Leftrightarrow$ $W \sim 35$ K
- Coulomb energy E_C:

Thick oxide:

$r_s = E_C / E_F \propto n_s^{-1/2} \sim 4$

Thin oxide:

Screening by the gate: $r_s \lesssim 1$ $(E_C \sim E_F)$

[Widom & Tao, Phys. Rev. B 38, 10787 (1988)]

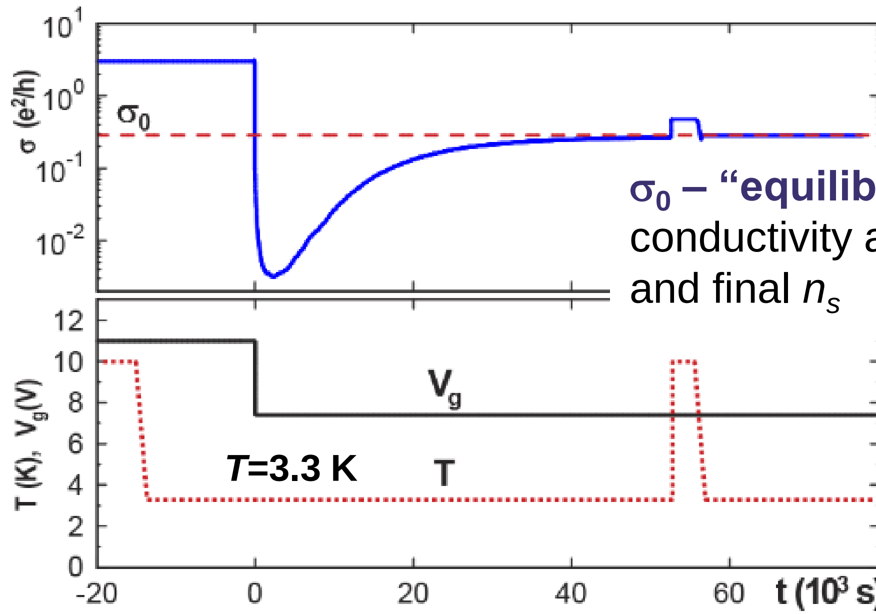
- **All energy scales (W, E_F, and E_C) are comparable**

Quench dynamics in a disordered 2DES: Relaxations of σ after a large, rapid change of n_s

Experimental protocol

- Sample is warmed up to ~7-20 K to "reset" the sample
- At ~7-20 K, the gate voltage is set to some V_g^i where $k_F l \geq 1$
- The sample is **cooled to the measurement *T*** and allowed to equilibrate ($\Leftrightarrow$ σ saturates)
- Then *V*g, i.e. ***n*****s is quickly (<3 s) changed to a lower, final value**
- σ is measured continuously throughout the process

Long-range case (~1/*r*)



σ_0 – "equilibrium" conductivity at *T* and final n_s

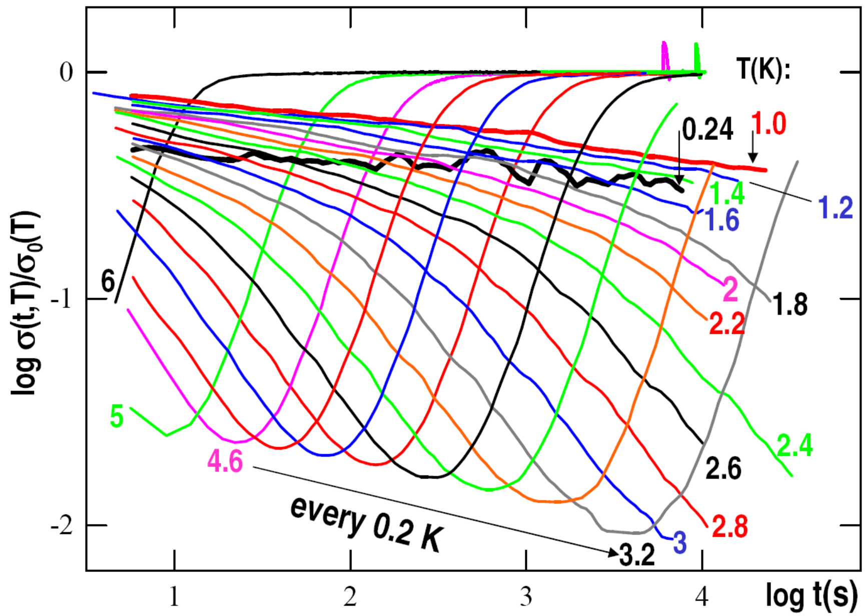
Time

Initial n_s(10^{11}cm^{-2}) =20.26 ; $k_F l$ ~1

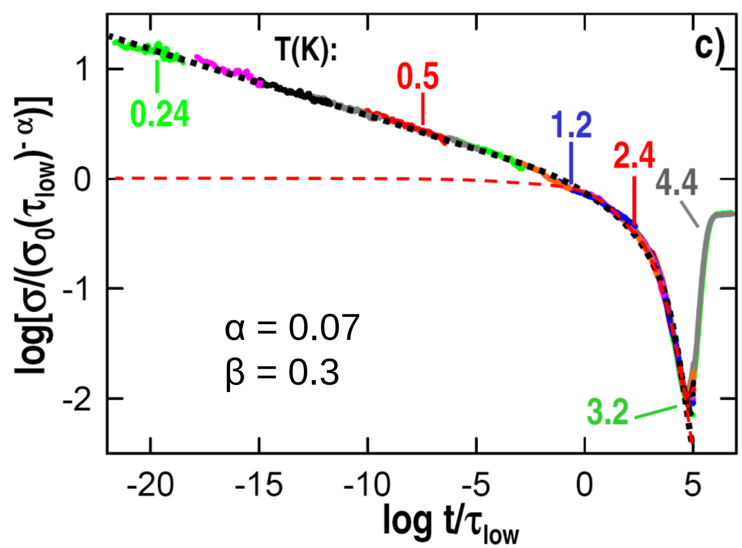
Final n_s(10^{11}cm^{-2}) = 4.74 $\geq n_c$

- Large perturbation: **Δ** $E_F \sim E_F$ ($k_B T \ll E_F$)

Quench dynamics in a high-disorder 2DES: Relaxations of σ after a large, rapid change of n_s



- Overshooting of equilibrium
- σ moves away from σ_0 at intermediate times
- Slow, **nonexponential** relaxation of σ at intermediate times (before the minimum)



- Dynamical scaling at **intermediate times:**

$$\sigma(t,T)/\sigma_0 \propto t^{-\alpha(n)} \exp\{-[t/\tau(n_s,T)]^{\beta(n)}\}$$

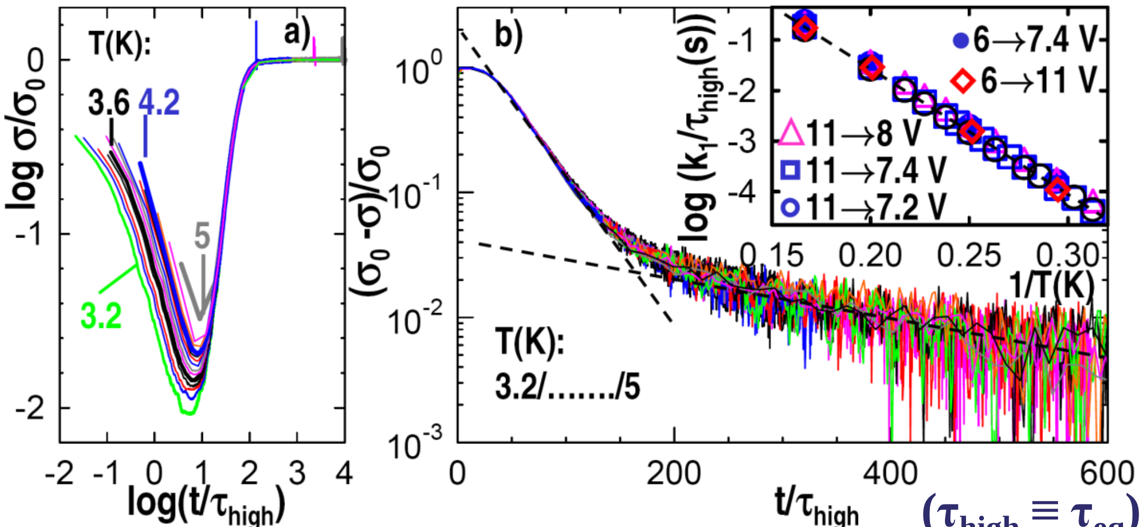
$$\alpha(n_s) < 0.4, 0.2 < \beta(n_s) < 0.45$$

Broad distribution of relaxation times

$$\boldsymbol{\sigma/\sigma_0 \propto t^{-\alpha} \quad \text{as } T \to 0}$$

Quench dynamics in a high-disorder 2DES: Relaxations of σ after a large, rapid change of n_s

Long times (approach to "equilibrium"):



($\tau_{high} \equiv \tau_{eq}$)

- Different T data collapse for times **after** the minimum
- The system reaches "equilibrium" after a long enough time t
- Approach to "equilibrium" **exponential in time**

- Thermalization at **long times**:

  $\tau_\sigma \propto \exp(E_A/T)$
- As $T \rightarrow 0$, $\tau_\sigma \rightarrow \infty$

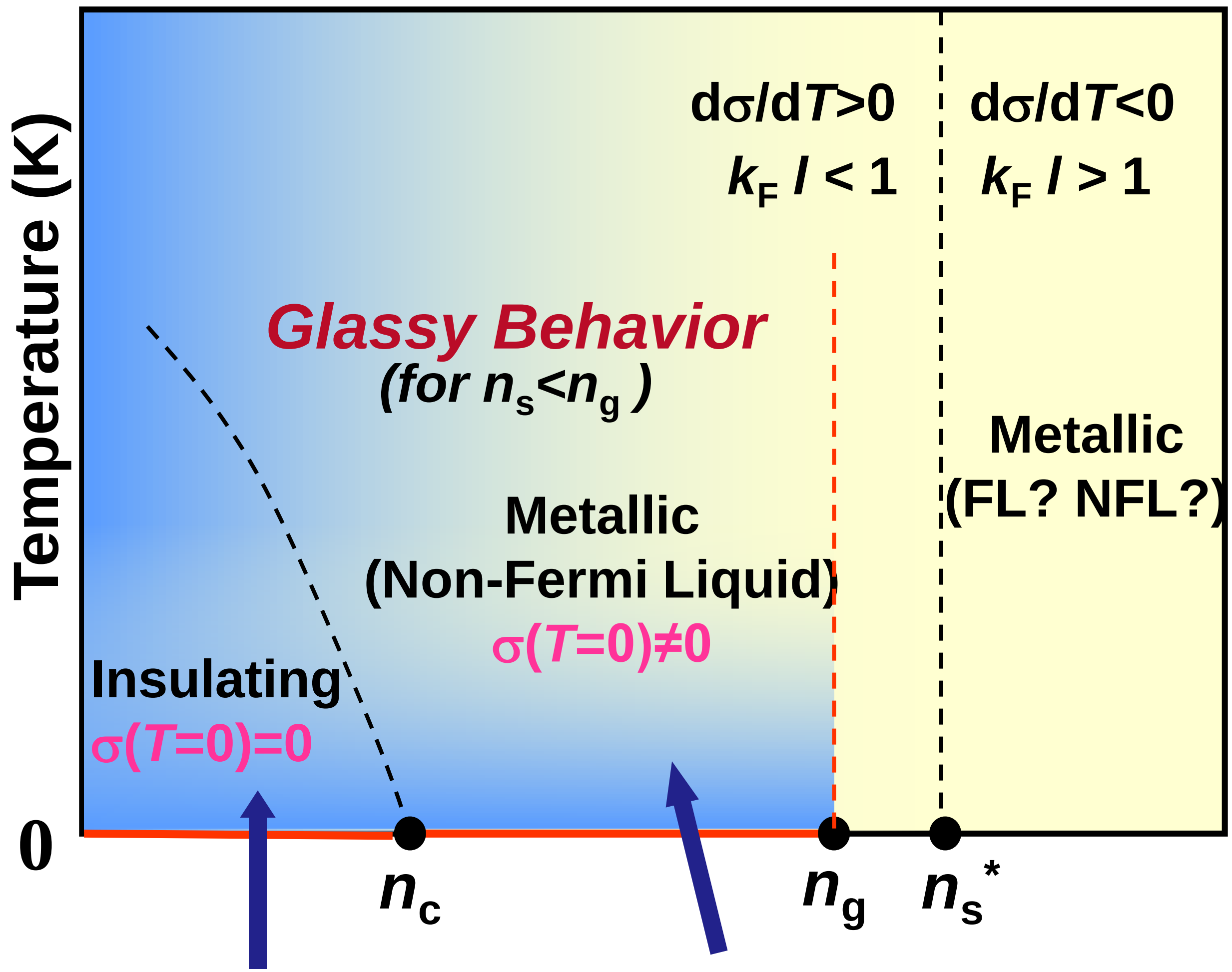
  (glass transition at T_g=0)

$E_A \approx 57$ K

[Jaroszyński & Popović, Phys. Rev. Lett. 96, 037403 (2006); also, Jaroszyński & Popović, Phys. Rev. Lett. 99, 046405 (2007); Jaroszyński & Popović, Phys. Rev. Lett. 99, 216401 (2007)]

Phase diagram of a strongly disordered 2DES in Si: Dynamics with long-range Coulomb interaction

Long-range Coulomb + disorder ⇒ **Frustration**



T=0 glass transition for $n_s < n_g$

$n_g \approx 7.5\times10^{11}\mathrm{cm}^{-2}$
$n_c \approx 5.0\times10^{11}\mathrm{cm}^{-2}$

Reviews in book chapters:
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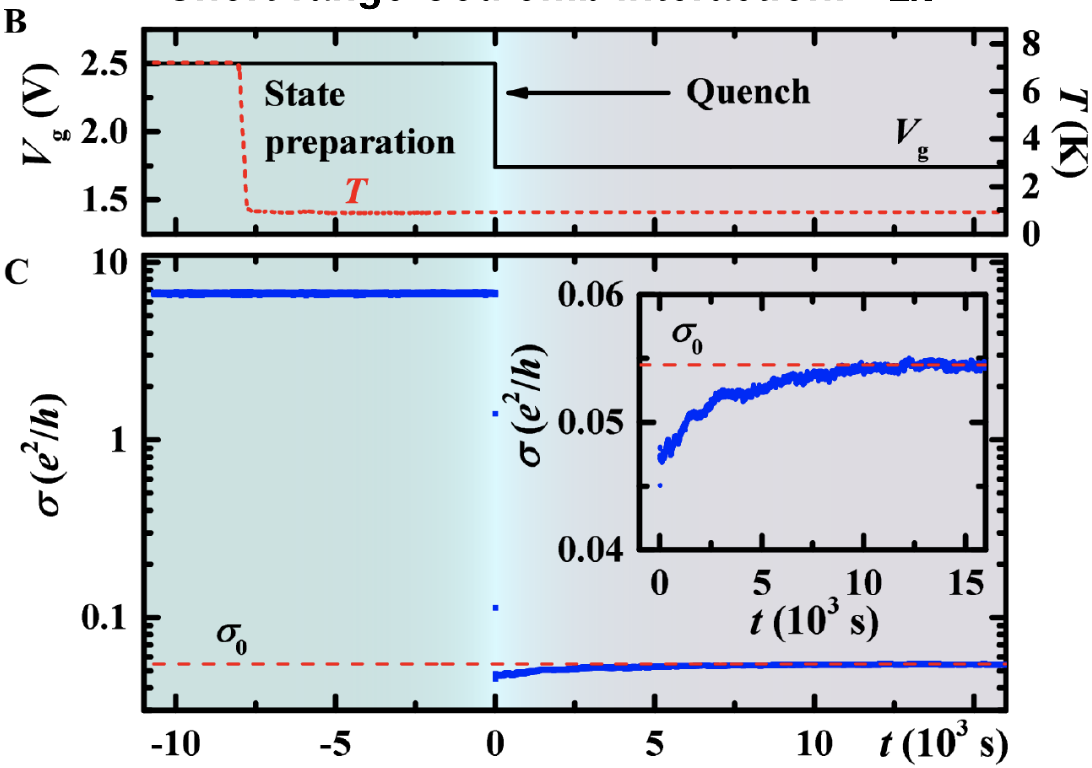
b) "Strongly Correlated Electrons in Two Dimensions", edited by Kravchenko (Pan Stanford Publishing, 2017)

Exponential localization;
Glassy insulator

"Bad" metal: $\sigma(n_s,T)=\sigma(n_s,T=0)+b(n_s)T^{3/2}$
Intermediate, glassy phase

Quench dynamics in a strongly disordered 2DES

Short-range Coulomb interaction: ~1/r^3



Initial n_s(10^{11}cm^{-2}) =32.2 ; $k_F l$ >1

Final n_s(10^{11}cm^{-2}) = 8.44 ≥ n_c ; T=0.92 K

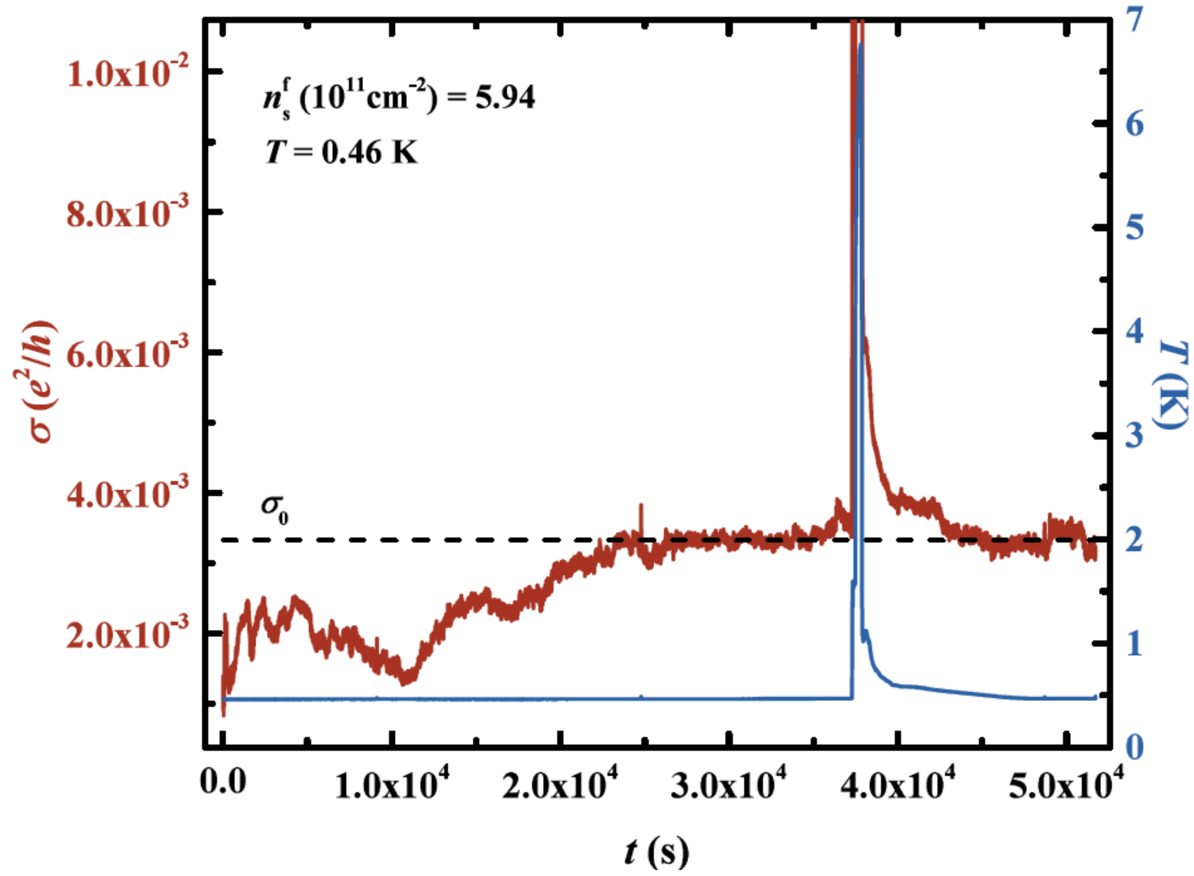
$n_c \approx 4.2\times10^{11}$cm^{-2}

Same experimental protocol for both sets of samples; samples and leads in vacuum

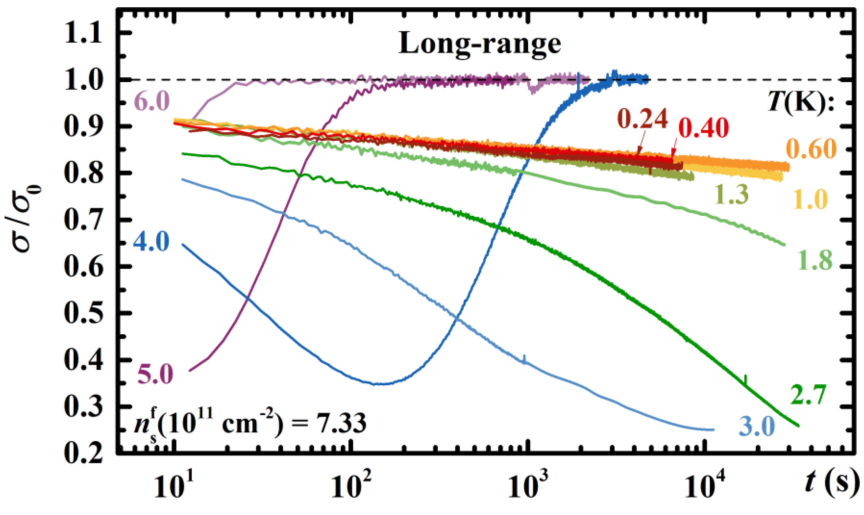
- Sample is warmed up to ~7-20 K to "reset" the sample
- At ~7-20 K, the gate voltage is set to some V_g^i where $k_F l \geq 1$
- The sample is **cooled to the measurement *T*** and allowed to equilibrate
- Then Vg, i.e. **n_s is quickly (<3 s) changed to a lower, final value**
- σ is measured continuously throughout the process

- Large perturbation: **Δ** E_F ~ E_F
  ($k_B T \ll E_F$)

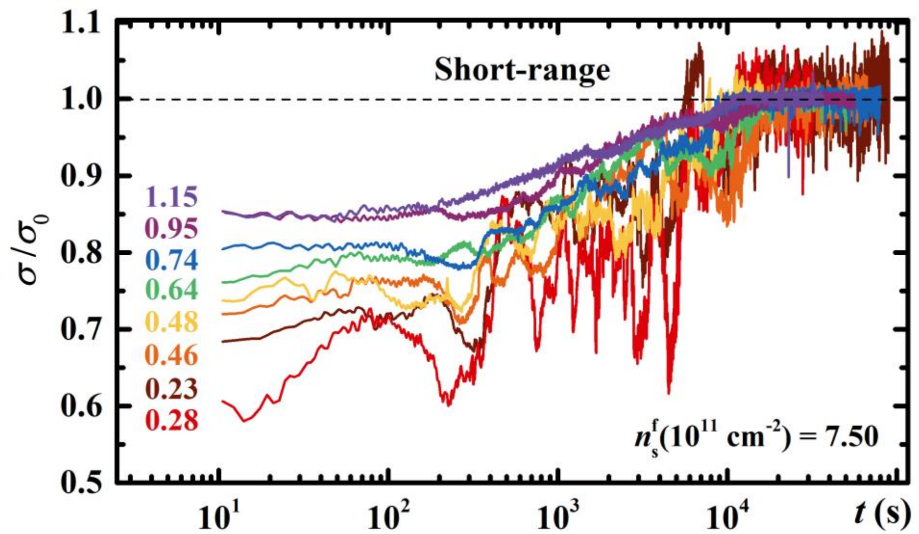
Short-range case: "Equilibrium" conductivity after n_s change and after cooling is the same



Quench dynamics in a disordered 2DES: Anomalously slow relaxations



n_c (10^{11} cm^{-2}) = (5.0 ±0.3)



n_c (10^{11} cm^{-2}) = (4.2 ±0.2)

Thick-oxide MOSFETs:

- Thermalization at **long times**:

  $\tau_\sigma \propto \exp(E_A/T)$

- As $T \to 0$, $\tau_\sigma \to \infty$

  (glass transition at T=0)

[Jaroszyński & Popović, Phys. Rev. Lett. 96, 037403 (2006); also Jaroszyński & Popović, PRL 99, 046405 (2007) and PRL 99, 216401 (2007)]

Thin-oxide MOSFETs:

Differences from the long-range case:

- σ does not go farther away from equilibrium with time for a given T; negligible relaxation
- No evidence of glassiness
- Thermalization time:

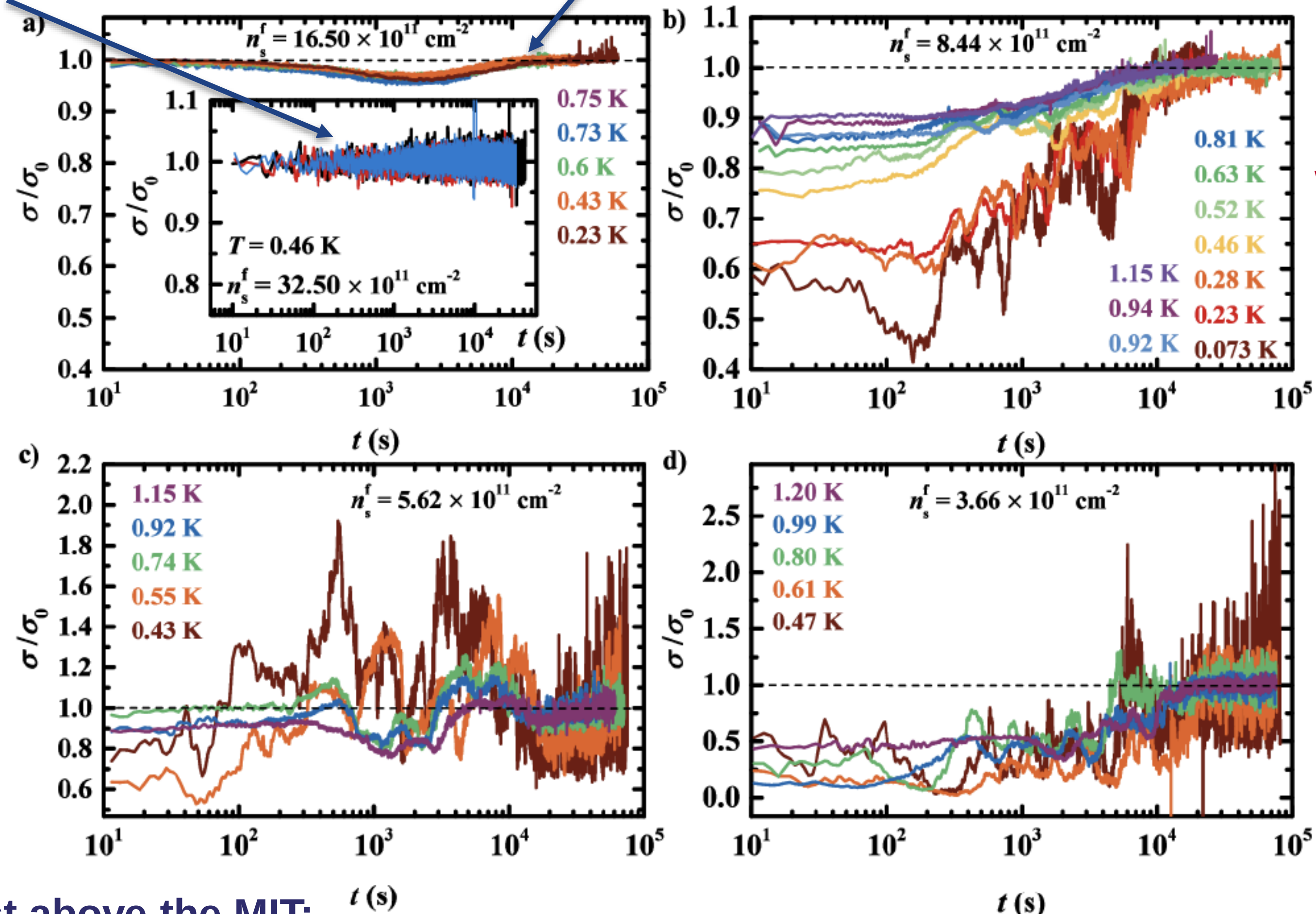
  $\tau_\sigma \sim 10^4$ **s**, similar for all T and n_s

- Observable when $k_F l < 1$

Quench dynamics with a short-range interaction: Relaxations at different electron densities

No relaxations when $k_F l > 1$

Relaxations appear when $k_F l \sim 1$



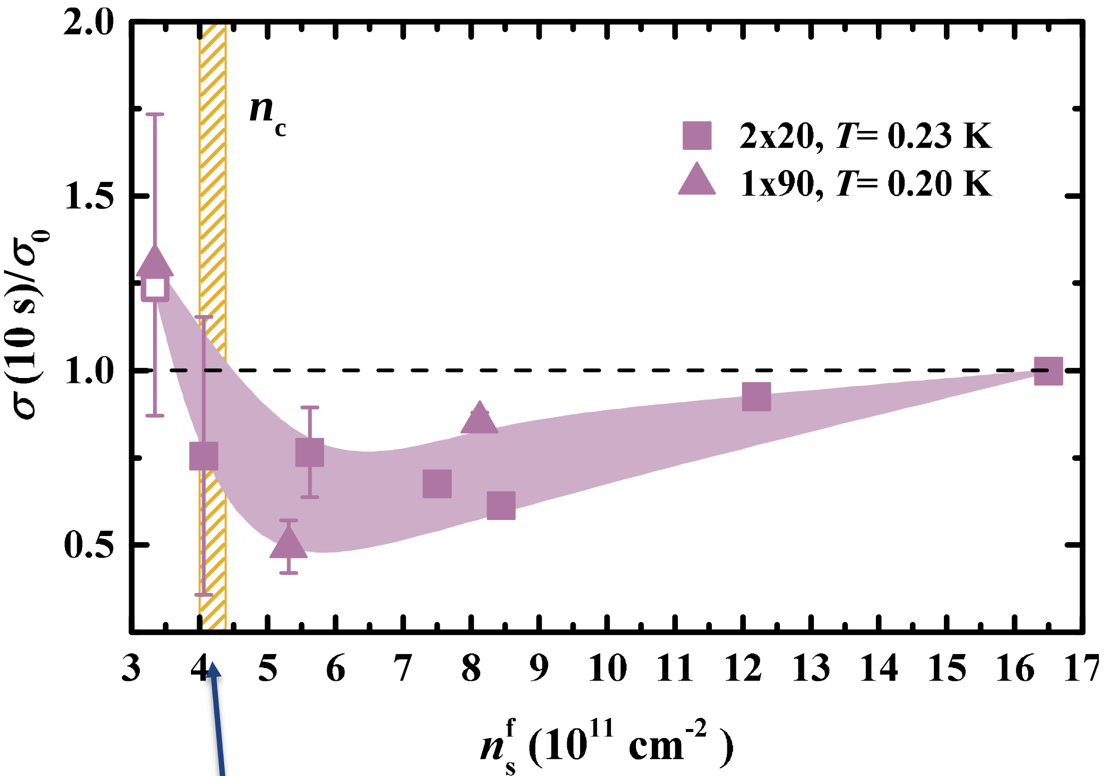
Metallic side: overshooting but weak time dependence at intermediate times

Insulating side: large noise; no relaxation at intermediate times

Just above the MIT: large noise

n_c (10^{11} cm^{-2}) = (4.2 ± 0.2)

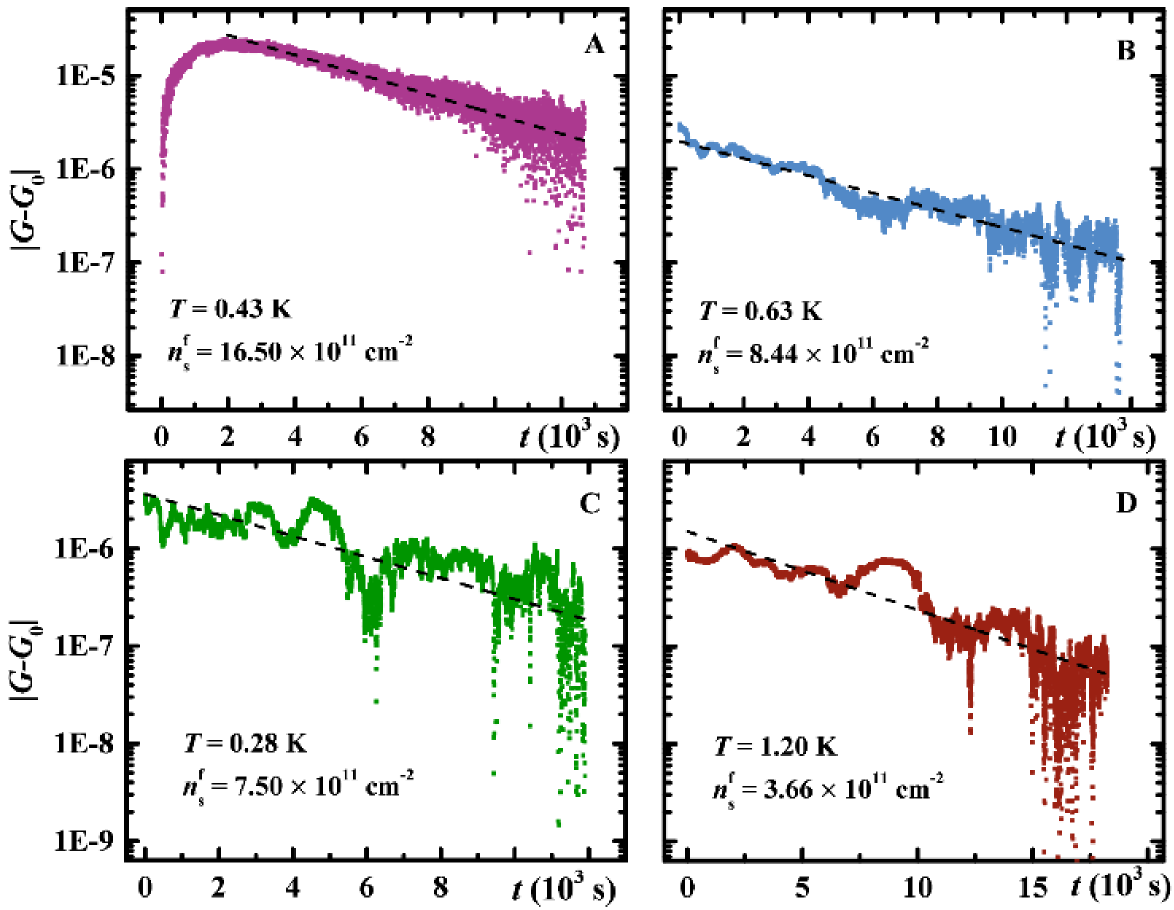
Quench dynamics with a short-range interaction: Initial amplitude of the relaxations at low *T*



Metal-insulator transition

- Initial amplitude σ/σ_0 at 10 s
- Deviations from σ_0 become **observable** on the metallic side of the MIT, **when $k_F l < 1$**
- As n_s is reduced, deviations from σ_0 become more pronounced and **peak just above the MIT; nonmonotonic** dependence on density

Approach to equilibrium in the short-range case: Exponential relaxations at long times



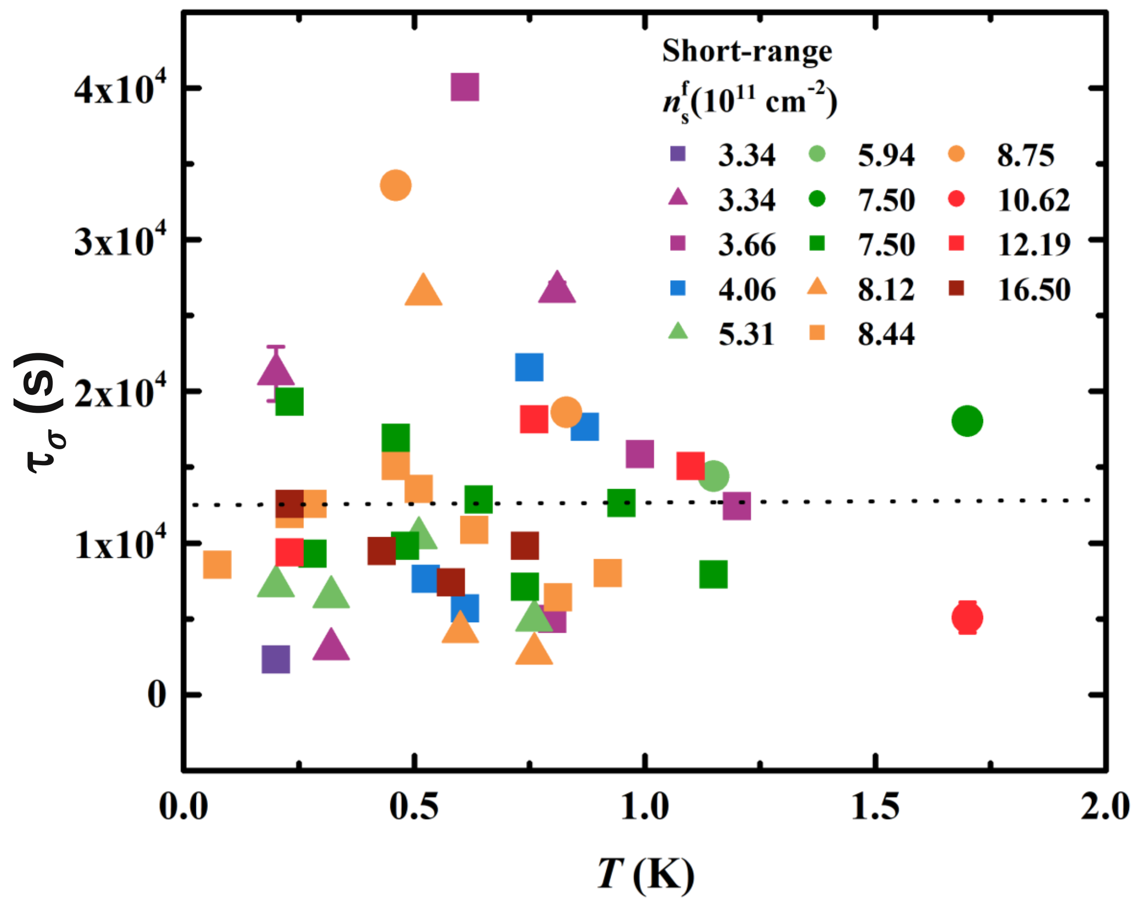
- Conductance relaxations

$$\Delta G(t) \propto \exp\left(-\frac{t}{\tau_\sigma}\right)$$

- Fit a wide range of n_s and T

$\Rightarrow$ Extract τ_σ

Quench dynamics with a short-range interaction: Thermalization time

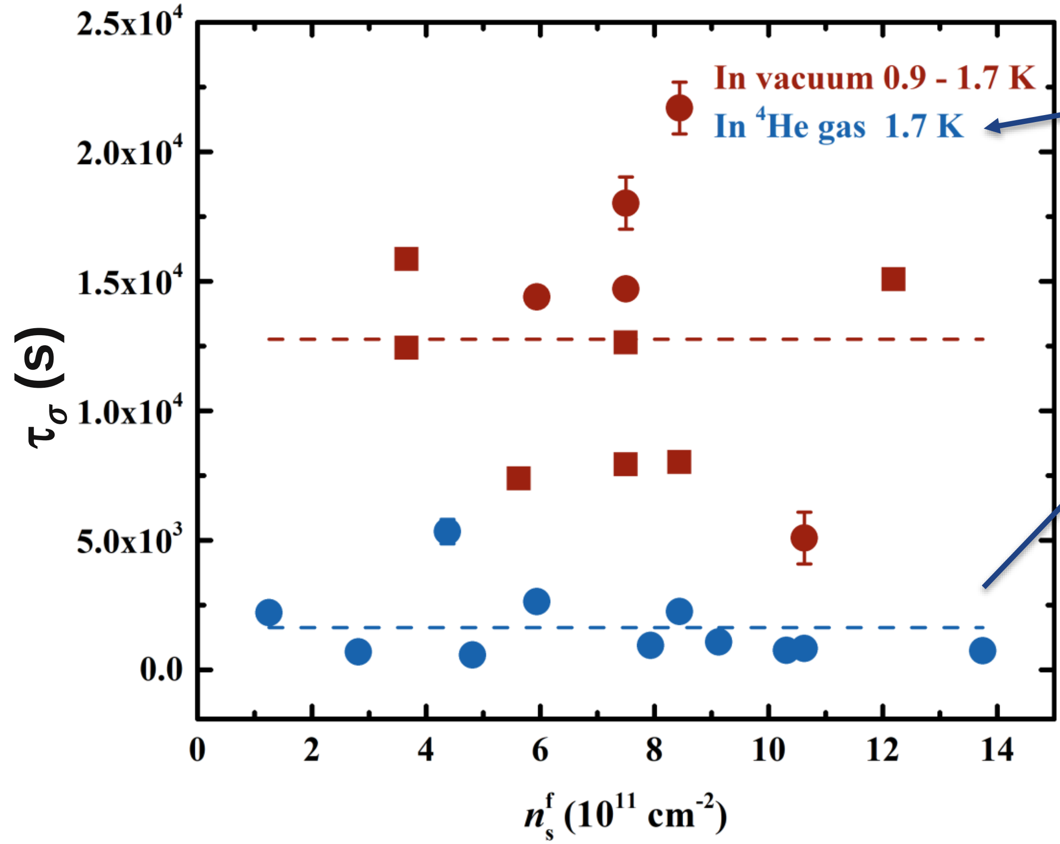


- **Long: $\tau_\sigma \sim 10^4$ s**
- **No dependence on density or temperature**

What is the origin of this slow dynamics?

Thermal coupling to the environment sets the time scale for thermalization!

Short-range Coulomb interaction



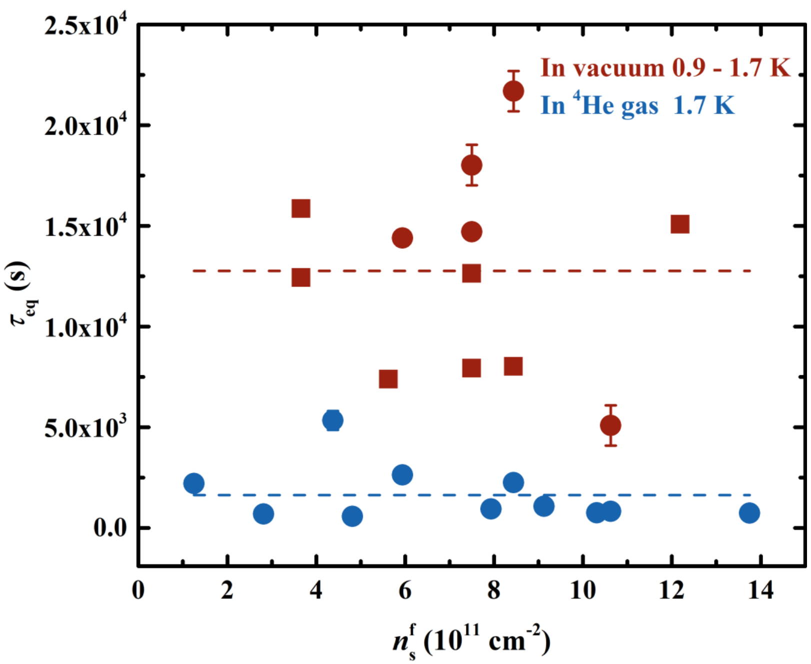
Stronger coupling to the bath when in ^{4}He vapor

τ_σ reduced by an order of magnitude!

Prethermal, MBL-like

Effect of thermal coupling to the environment on thermalization time

Short-range Coulomb interaction



- T < 2 K: weak electron-phonon coupling

a) Samples in vacuum: **τ_{eq} ~ 10^4 s**

weak thermal coupling to environment

b) Samples in ^{4}He vapor: **τ_{eq} ~ 10^3 s**

intermediate thermal coupling to environment

- T > 2 K: cooling via phonons dominant

Strong thermal coupling to environment: **τ_{eq} < 200 s** (at 4.2 K in both vacuum and liquid ^{4}He)

Long-range Coulomb interaction:

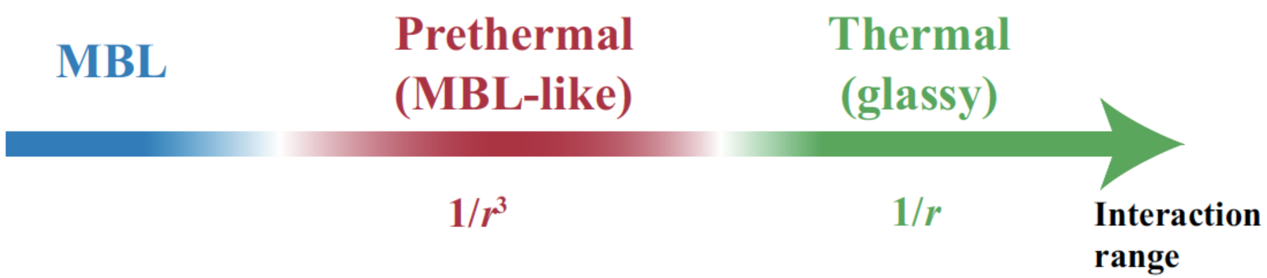
Glassy dynamics unchanged even at T > 4.2 K ⇒ insensitive to thermal coupling to environment

Exponential dependence of thermalization time on the coupling strength:
MBL-like dynamics

Quench dynamics and thermalization: Direct observation of MBL-like dynamics

Effect of interaction range in a 2DES for a fixed disorder strength:

- **No difference in dc transport properties**
- **Striking difference in nonequilibrium dynamics!** (when $k_F l < 1$; bad conductor)



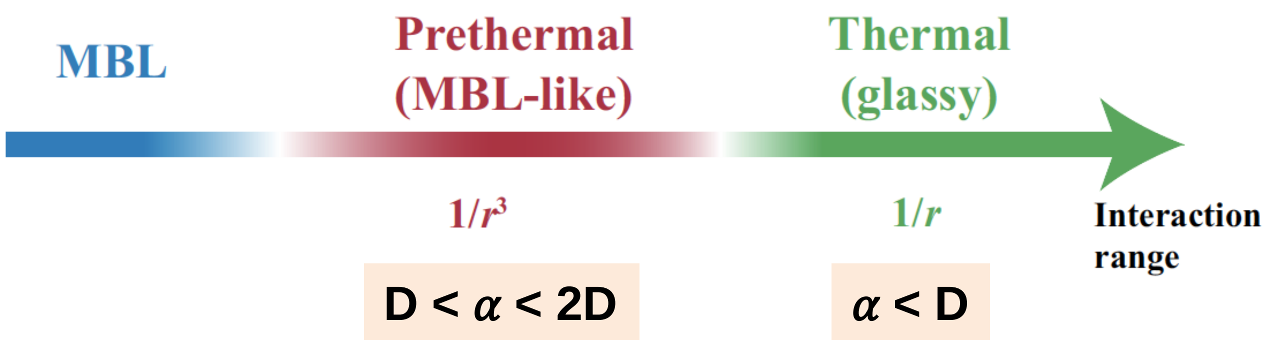
- **Screened Coulomb interaction** leads to a **prethermal, MBL-like** regime at intermediate times
- Time scale: hours!

[L. J. Stanley *et al.*, arXiv:2110.11473; under revision]

Quench dynamics and thermalization: Direct observation of MBL-like dynamics

Effect of interaction range in a 2DES for a fixed disorder strength:

Power-law interactions: ~ $1/r^{\alpha}$



[Theory: Yao et al., PRL 113, 243002 (2014); Burin, PRB 92, 104428 (2015); Gutman et al., PRB 93, 245427 (2016)…]

- Our results provide constraints for theory and motivate further work

Summary and outlook

- **Strongly disordered 2D electron system in a Si MOSFET:**
  - **Slow dynamics when $k_F l < 1$**
  - **No** glassy dynamics with screened Coulomb interaction (~1/r^3)
  - New, **solid-state platform** for studies of thermalization and **MBL-like** dynamics; **time scales: hours!**
  - **Building blocks for quantum information science?**

[L. J. Stanley *et al.*, arXiv:2110.11473; under revision]