2) Prove $R^3 = \kappa_5^2 N^2/(4\pi^2)$ and then using (13.8), prove (13.11).

3) Expand the action (13.17) + (13.18) up to 4th order in π^i's and σ.

4) Substitute (13.21) into (13.20) to find the topological charge B in terms of $F(r)$. What are the needed conditions on $F(r)$?

5) Consider an $AdS_6 \times S^4$ background in type IIA theory, sourced by a 4-form field strength $F_{(4)}$. Repeat the argument in the text to find a baryonic N-vertex in the corresponding $SU(N)$ field theory.

6) Consider the spin $j = 1$ representation of $SU(2)$. Construct the 9 independent $Y_{lm}(J_i)$'s and check that one can decompose a general 3×3 matrix in terms of them.

7) Consider an $SU(N)$ matrix theory with the potential

$$V = \mathrm{Tr}\left[\frac{1}{2}\sum_{i=1,2,3}\left(\frac{\mu X^i}{3}\right)^2 + \frac{\mu i}{3}\sum_{j,k,l=1}^{3} X^j X^k X^l - \frac{1}{4}\sum_{j,k=1,2}[X_j, X_k]^2\right]. \tag{13.45}$$

Does it have a fuzzy sphere ground state?

References and further reading

For more on Wilson loops, see any QCD textbook, e.g. [40]. The Wilson loop in AdS/CFT was defined and calculated in [41, 42]. I have followed here Maldacena's [41] derivation.

Exercises, Chapter 14.

1) Check that in the nonabelian case, for a closed square contour of side a, in a plane defined by $\mu\nu$, we have

$$\Phi_{\Box_{\mu\nu}} = e^{ia^2 F_{\mu\nu}} + \mathcal{O}(a^4). \tag{14.44}$$

2) Check that if a free relativistic string in 4 flat dimensions is stretched between q and $\bar{q}$ and we use the AdS/CFT prescription for the Wilson loop, $W[C] = e^{-S_{\rm string}[C]}$, we get the area law.

3) Consider a circular Wilson loop C, of radius R. Give an argument to show that $W[C]$ in $\mathcal{N} = 4$ SYM, obtained from AdS/CFT as in the rectangular case, is also conformally invariant, i.e. independent of R.

4) Check that if AdS_5 terminates at a fixed $U = U_m$ and strings are allowed to reach U_m and get stuck there, then we get the area law for $\langle W[C]\rangle$ at large interquark separation L (This is similar to what happens in the case of finite temperature AdS/CFT), by using the argument at the end of section 14.4 and calculating the scaling of the string areas at $U = U_m$ and at $U > U_M$ for $L \to \infty$.

5) Finish the steps left out in the calculation of the quark antiquark potential to get the final result for $V_{q\bar{q}}(L)$.

6) Verify that the Wilson loop (14.29) is 1/2 supersymmetric, substituting the $\mathcal{N} = 4$ SYM susy variations.

6) Near the boundary at $r = \infty$, the normalizable solutions (wavefunctions) of the massive AdS Laplacean go like $(x_0^\Delta \sim) r^{-\Delta}$ (where $\Delta = 2h_+ = d/2 + \sqrt{d^2/4 + m^2R^2}$). Substitute in the Polchinski-Strassler formula to obtain the r dependence of the integral at large r, and using that $r \sim 1/p$, estimate the hard scattering (all momenta of the same order, p) behaviour of QCD amplitudes.

physical interpretation is described in [45]. The type IIB maximally supersymmetric plane wave was found in [51] and it was shown to be the Penrose limit of $AdS_5 \times S^5$ in [52, 45].

Exercises, Chapter 17.

1) An Aichelburg-Sexl shockwave is a gravitational solution given by a massless source of momentum p, i.e. $T_{++} = p\delta(x^+)\delta(x^i)$. Find the function $H(x^+, x^i)$ defining the pp wave on a *UV brane*, with $e^{-\frac{2|y|}{R}}$ instead of $e^{\frac{2|y|}{R}}$ in (16.29).

2) If the null geodesic moves on S^5, one can choose the coordinates such that it moves on an equator, thus the Penrose limit gives the maximally supersymmetric pp wave. Show that if instead the null geodesic moves on AdS_5, the Penrose limit gives 10d Minkowski space (choose again $\rho = 0$)

3) The Killing spinor equation in 10 dimensional type IIB theory is

$$\mathcal{D}_M \epsilon = \nabla_M \epsilon + \frac{i}{24} F_{ML_1 \dots L_4} \Gamma^{L_1 \dots L_4} \epsilon. \tag{17.69}$$

Show that for a generic pp wave, we have 1/2 supersymmetry preserved, but for the maximally supersymmetric pp wave all the susy is preserved, and find the solution $\epsilon(x^+, x^i)$ in terms of an independent constant spinor parameter ψ.

4) Write down *all* the $\mathcal{N} = 4$ SYM fields (including derivatives) with $\Delta - J = 2$.

5) Check that, by cyclicity of the trace, the operator with 2 insertions of Φ^1, Φ^2 at levels $+n$ and $-n$ equals (up to normalization)

$$Tr[\Phi^1 Z^l \Phi^2 Z^{J-l}]. \tag{17.70}$$

6) Fill in the details of the diagonalization of the Cuntz Hamiltonian (17.52) to obtain the eigenenergies (17.61).

7) Prove the regularization (cut-off or dimensional regularization) of the integral (17.65).