TOPICS ON COHERENT STATES AND HIGH DIMENSION OPERATORS IN GAUGE/GRAVITY CORRESPONDENCE

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BPS coherent states, droplets and holes

- coherent states are widely used in various contexts of physics

- special types of coherent states in explicit gravity dual

Berenstein, Wang; Holguin, Wang; Lin; Holguin, Weng; Holguin; …

- with extra bosonic variables

$$F[\Lambda_Z] = \int dU \exp(\mathrm{Tr}(U\Lambda_Z U^{-1} a_Z^\dagger))\,|0\rangle$$

- generating function of multi-trace operators, by expanding the exponential

BPS coherent states, droplets and holes

- action of dilatation operator simplifies (on these bases)

- also in quiver cases, coulomb branches, and orbifold cases

- U(N), SO(N), Sp(N) cases

Berenstein, Wang; Holguin, Wang

Quarter and Eighth BPS coherent states

$$F[\Lambda_Y, \Lambda_Z] = \int dU \exp(\mathrm{Tr}(U \Lambda_Y U^{-1} a_Y^\dagger + U \Lambda_Z U^{-1} a_Z^\dagger)) \, |0\rangle$$

$$F[\Lambda_X, \Lambda_Y, \Lambda_Z] = \int dU \exp(\mathrm{Tr}(U \Lambda_X U^{-1} a_X^\dagger + U \Lambda_Y U^{-1} a_Y^\dagger + U \Lambda_Z U^{-1} a_Z^\dagger)) \, |0\rangle$$

- e.g. two-term and three-term HCIZs

- complete bases (albeit also-overcomplete due to coherent state); can be used in coherent state path integral

SL(2) sectors and their cousins

- coherent state in sl(2) sectors and their cousins

$$c_D^{\dagger n} a_Z^\dagger \leftrightarrow D_+^n Z, \quad c_D^\dagger a_Z^\dagger \leftrightarrow D_+ Z, \quad a_Z^\dagger \leftrightarrow Z.$$

$$K[R_Z, \Lambda_Z] = \int dU \exp(\mathrm{Tr}(U R_Z U^{-1} c_D^\dagger a_Z^\dagger a_Z)) \exp(\mathrm{Tr}(U \Lambda_Z U^{-1} a_Z^\dagger)) \left|0\right\rangle$$

- generating function of multi-trace products of forms

$$\mathrm{Tr}(D_+^{n_1} Z D_+^{n_2} Z ... D_+^{n_\alpha} Z ...)$$

- cousin psu(1,1|2) sectors

BPS coherent states and edge modes

- coherent state descriptions

Berenstein, Miller; Skenderis, Taylor; Simon; Balasubramanian et.al.; Lin, Zeng; …

$$\mathrm{Tr}(X^k) \quad \longleftrightarrow \quad t_k \quad \longleftrightarrow \quad a_k^\dagger, a_k.$$

$$\mathcal{H} = \bigotimes_k \mathcal{H}_k = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots$$

- states can be spanned by

$$\prod_k (t_k)^{w_k} := \prod_k \left(a_k^\dagger\right)^{w_k} |0\rangle$$

- coherent states written as

$$|Coh\rangle = \prod_k \exp(\Lambda_k \frac{t_k}{k})$$

- chiral field on the coherent states

$$\langle \hat{\phi}(\theta) \rangle_{|Coh\rangle} \rightarrow \phi(\theta)$$

- the shape of the droplet edge can be described by the chiral field $\phi(\theta)$ and

$$\langle\hat{\phi}(\theta)\rangle_{|Coh(\Lambda_k)\rangle} \to \phi_{\Lambda_k}(\theta)$$

- the chiral field is a displacement of the edge of the droplets, or a height function

- e.g. circularly shaped droplets, and more general cases (e.g. annular droplets, droplets with long strips)

Supermatrix coherent states

$$O=\int dU d\chi^\dagger d\chi e^{\mathrm{tr}(\chi\chi^\dagger)}\exp(\mathrm{tr}(\alpha U\Lambda_z U^\dagger Z+\alpha U\Lambda_y U^\dagger Y+\alpha U\Lambda_x U^\dagger X$$

$$-\beta\chi\Lambda_z^c\chi^\dagger Z-\beta\chi\Lambda_y^c\chi^\dagger Y-\beta\chi\Lambda_x^c\chi^\dagger X))$$

- $\Lambda_z, \Lambda_y, \Lambda_x$ are three $N\times N$ matrices. $\Lambda_z^c, \Lambda_y^c, \Lambda_x^c$ are three $T\times T$ matrices.
- contains particle-hole duality (e.g. between dual giants and giants)
- convolution ideas
- generating correlators of integrated operators from correlators of un-integrated operators

Berenstein, Wang; Holguin, Wang; Holguin, Weng; …

Supermatrix coherent states

- package coherent state parameter into supermatrix form

$$\int [dU_S] \exp(\mathrm{Str}(U_S \Lambda_S U_S^\dagger Z_S))$$

$$\Lambda_S = \begin{pmatrix} \alpha\Lambda & 0 \\ 0 & -\beta\Lambda^c \end{pmatrix}$$

- correlation functions can be calculated by using HCIZ, super-HCIZ type integrals, and auxiliary susy integrals

- cases with more matrices

$$\int [dU_S] \exp(\mathrm{Str}(\Lambda_z^S U_S^\dagger Z_S U_S + \Lambda_y^S U_S^\dagger Y_S U_S + \Lambda_x^S U_S^\dagger X_S U_S))$$

$$\Lambda_z^S = \begin{pmatrix} \alpha\Lambda_z & 0 \\ 0 & -\beta\Lambda_z^c \end{pmatrix}, \Lambda_y^S = \begin{pmatrix} \alpha\Lambda_y & 0 \\ 0 & -\beta\Lambda_y^c \end{pmatrix}, \Lambda_x^S = \begin{pmatrix} \alpha\Lambda_x & 0 \\ 0 & -\beta\Lambda_x^c \end{pmatrix}$$

- quarter and eighth BPS cases

HHL and HHLL correlators

- field theory side using coherent states (for H) and single-trace operators (for L)

Holguin, Weng

- e.g.

$$|\lambda\rangle = \int_{\mathbb{CP}^{N-1}} d\varphi e^{\alpha\lambda\varphi^\dagger Z\varphi} |0\rangle$$

$$\langle\Lambda| = \langle 0| \int_{U(N)} dU e^{\alpha \mathrm{Tr}(\bar{Z}U^\dagger \bar{\Lambda} U)}$$

$$\mathcal{F}(\lambda, \Lambda, t) = \langle\Lambda| \, \mathrm{Tr}\left[\frac{1}{1-2t\tilde{Z}}\right] |\lambda\rangle$$

- matches gravity side and field theory side

Holguin, Weng; Bissi, Kristjansen,Young, Zoubos; Caputa, de Mello Koch, Zoubos; Lin; Komatsu, Jiang, Vescovi, Wu, Yang; Skenderis, Taylor; …

- new saddle point method

Berenstein, Wang; Holguin, Weng; de Mello Koch et.al.; Holguin

- reduced coherent states

String-added states

- adding string onto the coherent state. e.g.

$$\int dU \exp\left(\mathrm{Tr}(U\Lambda_Z U^{-1} a_Z^\dagger + U\Lambda_Y U^{-1} a_Y^\dagger)\right) \cdot$$

$$\langle\vec{v}|_{i_1} U^{-1} W_1 U \,|\vec{v}\rangle_{j_1} \langle\vec{v}|_{i_2} U^{-1} W_2 U \,|\vec{v}\rangle_{j_2} \ldots \langle\vec{v}|_{i_l} U^{-1} W_l U \,|\vec{v}\rangle_{j_l} \,|0\rangle$$

- the string part W can be labelled by BMN or spin-chain operators

- special cases: near BPS states; cases where W being BPS string

- special cases: interaction between the coherent state part and the string part

String-added states

- can have generating function (for adding strings)

$$\tilde{F}[t_k, s_k] = \int dU d\chi^\dagger d\chi e^{\mathrm{tr}(\chi\chi^\dagger)} \exp(\Gamma) \exp(\mathrm{tr}(\sum_k U t_k U^\dagger W_k - \sum_k \chi s_k \chi^\dagger W'_k))$$

$$\begin{aligned} \Gamma &= \mathrm{tr}(\alpha\Lambda_z U^\dagger Z U + \alpha\Lambda_y U^\dagger Y U + \alpha\Lambda_x U^\dagger X U - \beta\chi\Lambda_z^c\chi^\dagger Z \\ &\quad -\beta\chi\Lambda_y^c\chi^\dagger Y - \beta\chi\Lambda_x^c\chi^\dagger X) \end{aligned}$$

- taking differentials

$$\delta_{(t_k)^i_j} \tilde{F} = \int dU d\chi^\dagger d\chi e^{\mathrm{tr}(\chi\chi^\dagger)} \exp(\Gamma)\ {}_i \left(U^\dagger W_k U\right)_j$$

- open and closed strings

New techniques

- HCIZ, super HCIZ
- auxiliary field integral method; auxiliary susy integral
- spin-matrix theory

Baiguera, Harmark, Lei, and friends …

New techniques

• centrally extended su(2|2) symmetry

Beisert; Berenstein, Holguin; de Mello Koch et.al.; Suzuki; …

• permutation centralizer algebras; Brauer algebra bases; global symmetry bases; multi restricted Schur bases

Brown, Heslop, Ramgoolam, de Mello Koch, Kimura, Ben Geloun, Kemp, and friends …

• permutations, analytic combinatorics, algebraic combinatorics

New techniques

- localization
- coherent state formalism
- new saddle point method (e.g. in particular involving heavy states and generating functions)
- enhanced symmetry (from giants)

Outlooks

- near BPS black holes; near BPS states
- giant graviton expansion of superconformal index
- giants and coherent states in N=1, N=2 theories, and other theories

e.g. Martelli, Sparks; Basu, Mandal; Berenstein, Hartnoll; Pasukonis, Ramgoolam

- NATD, superstar

e.g. Lozano, Nunez, Zacarias, Sfetsos, Thompson, Macpherson, Roychowdhury, Gaiotto, Maldacena, …

- collective field theory method and Hamiltonian

  e.g. Jevicki, Das, Rodrigues, Mandal, Berenstein, Hartnoll, Dhar, Suryanarayana, …

- bootstrap, HHLL, HHL, spectrum of H

- fuzzball microstates, microstructure

- code subspace

  e.g. Bahiru, Belin, Papadodimas, Sarosi, Vardian; Berenstein, Miller; Belin, Withers; …

- Wilson operators, line operators, defect operators

- flat space limit

Thank you