SYK and Quantum de Sitter

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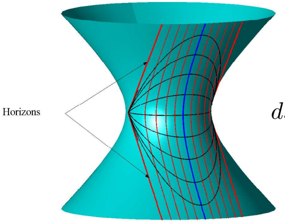
São Paolo, June 14, 2023

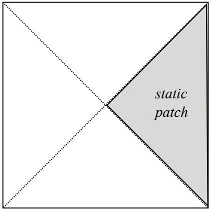
Pure Einstein gravity in 2+1-D de Sitter space:

$$S = \frac{1}{16\pi G}\int_{\mathcal{M}} d^3x\sqrt{-g}\,(R-2\Lambda) + \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^2x\sqrt{\gamma}K$$



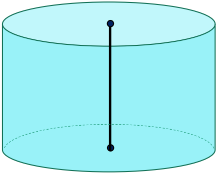
Static patch:

$$ds^2 = (1-\Lambda r^2)dt^2 + \frac{dr^2}{1-\Lambda r^2} + r^2 d\theta^2$$



Schwarzschild de Sitter thermodynamics:

$$ds^2 = -(1 - 8GE - r^2)dt^2 + \frac{dr^2}{(1 - 8GE - r^2)} + r^2 d\phi^2$$



$$T_{\rm SdS} = \frac{\sqrt{1-8GE}}{2\pi}. \qquad S_{\rm SdS} = \frac{A_H}{4G} = \frac{\pi}{2G}\sqrt{1-8GE}.$$

$$\frac{dS_{\rm SdS}}{d(-E_{\rm dS})} = \frac{1}{T_{\rm SdS}}$$

Can we reproduce this from a microscopic theory?

SYK model

coupled to gravity
double scaled

dS_3 gravity

SL(2,C) CS
theory

*

Virasoro CFT

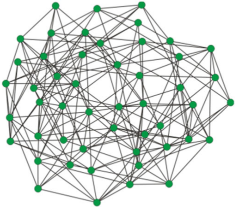
coupled to gravity
$c_+ + c_- = 26$

SYK model = 1D many body QM with maximal chaos

$$H_{\mathrm{SYK}} = i^{p/2} \sum_{i_1 \ldots i_p} J_{i_1 \ldots i_p} \psi_{i_1} \ldots \psi_{i_p} \qquad \{\psi^i, \psi^j\} = \delta^{ij}$$

random couplings

N majorana variables



$$\left\langle J^2_{i_1 \ldots i_p} \right\rangle = \mathcal{J}^2 \frac{2^{p-1} p!}{p^2 N^{p-1}}.$$

$$G(\tau_1, \tau_2) = \frac{1}{N} \sum_i \psi_i(\tau_1) \psi_i(\tau_2)$$

Double scaled SYK model has also been exactly solved:

$$H_{\text{SYK}} \;=\; i^{p/2} \sum_{i_1 \ldots i_p} J_{i_1 \ldots i_p} \psi_{i_1} \ldots \psi_{i_p}$$

N -> ∞
p -> ∞

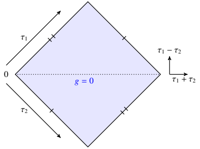
$p^2/N = \lambda$ = fixed

$$G(\tau_1, \tau_2) \;=\; \frac{1}{N} \sum_i \psi_i(\tau_1) \psi_i(\tau_2) \qquad\qquad G(\tau_1, \tau_2) \;=\; e^{\frac{1}{p}\, g(\tau_1, \tau_2)}$$

$$S_{\text{eff}} = \frac{N}{8p^2} \int d\tau_1 d\tau_2 \left[\partial_{\tau_1} g \partial_{\tau_2} g - 4\mathcal{J}^2 \exp g(\tau_1, \tau_2) \right].$$

Dynamical mean field theory reduces to 2D Liouville theory

with complex central charge!



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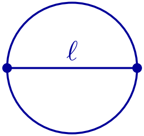
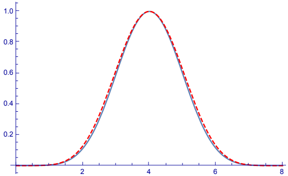


$$= \int dE_1\, dE_2 \rho(E_1)\rho(E_2)\, C_\ell(E_1, E_2)\, e^{-\beta E_1 - \tau E_{21}}$$

$$\rho(E) = \vartheta_1(2\theta, q)$$

$$C_\ell(E_1, E_2)^2 = \frac{1}{\vartheta_\ell(\pm\theta_1 \pm \theta_2, q)}$$

$$E = \frac{\mathcal{J} \cos\theta}{\sqrt{1-q}}, \qquad q = e^{-\lambda}$$



2+1 de Sitter gravity can be reformulated as an $SL(2,\mathbb{C})$ CS theory

$$S_E = i\kappa \int (AdA + \tfrac{2}{3}A^2) - i\kappa \int (\bar{A}d\bar{A} + \tfrac{2}{3}\bar{A}^3)$$

$$A = e + \omega \qquad\qquad \bar{A} = e - \omega$$

Quantum states of SL(2,C) CS theory are Virasoro conformal blocks:

$$S = \frac{i\kappa}{2\pi} \int d^2z \Big(\frac{1}{2}\partial\phi_+ \bar{\partial}\phi_+ + 2e^{\phi_+}\Big) - \frac{i\kappa}{2\pi} \int d^2z \Big(\frac{1}{2}\partial\phi_- \bar{\partial}\phi_- + 2e^{\phi_-}\Big)$$

$$c_\pm = 1 + 6Q_\pm^2, \qquad Q_\pm = b_\pm + \frac{1}{b_\pm} \qquad \pm\, i\kappa = \frac{1}{b_\pm^2}$$

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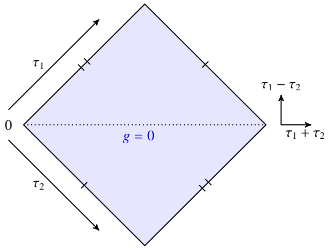
Quantum states of SL(2,C) CS theory are Virasoro conformal blocks:

$$\boxed{\begin{gathered} S = \frac{i\kappa}{2\pi}\int d^2z \Big(\frac{1}{2}\partial\phi_+\bar{\partial}\phi_+ + 2e^{\phi_+}\Big) - \frac{i\kappa}{2\pi}\int d^2z\Big(\frac{1}{2}\partial\phi_-\bar{\partial}\phi_- + 2e^{\phi_-}\Big) \\ b_\pm = e^{\pm i\pi/4}\beta \qquad ; \qquad c_\pm = 13 \pm i\Big(\beta^2 - \frac{1}{\beta^2}\Big) \end{gathered}}$$

2D gravity theory: $c_+ + c\_ = 26$ + bc ghost



$$Z = \int d\mu_B \, e^{-\beta\mu_B} \langle \mathrm{FZZT}(\mu_B) | e^{\frac{\pi i}{2}(L_0 + \bar{L}_0 - \frac{c}{12})} | C \rangle$$

$$= \int d\mu_B \, e^{-\beta\mu_B} \, \vartheta_1(2\theta, q)^2 \qquad \begin{array}{c} \mu_B = \mu \cos\theta \\ q = e^{-\pi\beta^2} \end{array}$$

Al.B.Zamolodchikov showed that for complex Liouville CFT

$$\boxed{\langle \theta_1 | V_\ell | \theta_2 \rangle_{\mathrm{DOZZ}} = \frac{1}{\vartheta_\ell(\pm\theta_1 \pm \theta_2, q)}}$$

Coupling the SYK model to gravity V2:

$$H \;=\; H_+ - H_- \qquad\qquad H_\pm \;=\; i^{p/2} \sum_{i_1\ldots i_p} J_{i_1\ldots i_p} \psi^\pm_{i_1} \ldots \psi^\pm_{i_p}$$

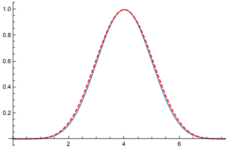
$$(H_+ - H_-)\big|\text{phys}\big\rangle \;=\; 0$$

Coupled system of two identical SYK models with equal energy constraint:

$$\big|\text{phys}\big\rangle \;=\; \sum_n \alpha_n \big|E_n\big\rangle, \qquad\qquad \big|E_n\big\rangle \;=\; \big|E_n\big\rangle_+ \big|E_n\big\rangle_-$$

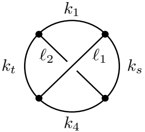
$$\big\langle E_1\big|V_\ell^{\text{phys}}\big|E_2\big\rangle \;=\; {}_+\big\langle E_1\big|V_\ell^+\big|E_2\big\rangle_+ \;\; {}_-\big\langle E_1\big|V_\ell^-\big|E_2\big\rangle$$

- What does de Sitter entropy represent?
- Do the usual rules of thermodynamics apply?
- What role does quantum chaos play in dS?
- What are good observables?
- ...

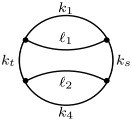


- Can we compute quasi-normal mode frequencies?

R-matrix



$$= R_{k_s k_t}\begin{bmatrix} k_4 & \ell_2 \\ k_1 & \ell_1 \end{bmatrix}$$



6j-symbol of SL(2,R) = SU(1,1)

=> BB

Mertens, Turiaci, HV
arxiv:1705.08508

6j-symbol of U_q (SL(2,R))

2+1 AdS Gravity

$\Longleftrightarrow$

6j-symbol of U_q (SU(1,1))

double scaled SYK

Jackson, McGough, HV
arxiv:1412.5205

Berkooz et al
arxiv:1811.02584

Coupling the SYK model to gravity V1:

$$S[\psi] = \sum_i \int d\tau \left(\psi_i \partial_\tau \psi_i - H_{\rm SYK}\right) \qquad\qquad H_{\rm SYK} = i^{p/2} \sum_{i_1\ldots i_p} J_{i_1\ldots i_p} \psi_{i_1} \ldots \psi_{i_p}$$

Promote couplings to time-dependent one forms with dynamical variance:

$$\Big\langle J_{i_1\ldots i_p}(\tau_1) J_{i_1\ldots i_p}(\tau_2) \Big\rangle = e^{h(\tau_1,\tau_2)}\, \mathcal{J}^2\, \frac{2^{p-1} p!}{p^2 N^{p-1}}.$$

$$ds_h^2 = e^{h(\tau_1,\tau_2)} d\tau_1 d\tau_2$$

=> Covariantizes the bilocal dynamical mean field theory!

Dynamical mean field theory in double scaling limit takes the form of a 2D Liouville CFT coupled to a dynamical background metric:

$$G(\tau_1, \tau_2) = \frac{1}{N} \sum_i \psi_i(\tau_1)\psi_i(\tau_2) \qquad\qquad G(\tau_1, \tau_2) = e^{\frac{1}{p}\, g(\tau_1,\tau_2)}$$

$$S_{\rm eff}[g,h] = \frac{N}{8p^2} \int d^2\tau \left[\partial_{\tau_1} g(\tau)\, \partial_{\tau_2} g(\tau) - g(\tau) R_h(\tau) + \mathcal{J}^2 e^{g(\tau)+h(\tau)} - 2\Lambda\, e^{h(\tau)}\right]$$

This can be rewritten as the difference of two Liouville CFTs!

$$S_{\rm eff}[g,h] = S_L[\varphi_+] - S_L[\varphi_-], \qquad \varphi_+ = g + h,\ \varphi_- = h$$

$$S_L[\varphi_\pm] = \frac{N}{8p^2} \int d^2\tau \left[\partial_{\tau_1}\varphi_\pm \partial_{\tau_2}\varphi_\pm + 2\Lambda\, e^{\varphi_\pm}\right]$$