Geometric phases, von Neumann algebras and AdS/CFT

Johanna Erdmenger

Julius-Maximilians-Universität Würzburg

Overview

- Geometry and entanglement
- Wormholes and factorization in AdS/CFT
- Berry phase in quantum mechanics
- Relation to von Neumann algebras
- 2d CFTs and their gravity dual

Berry phases for wormholes: Motivation

- Wormholes provide relation between entanglement and geometry in AdS/CFT
  van Raamsdonk; Maldacena, Susskind
- Concept of wormhole also present in simple quantum mechanics H. Verlinde
- Berry phase provides geometrical picture of how degrees of freedom entangle
- Factorization puzzle - Fibre bundle approach, missing information
- von Neumann algebras: Definition of trace requires vanishing Berry phase
- Also for AdS_3/CFT_2

Talk based on

- Berry phase in quantum mechanics and wormholes

Nogueira, Banerjee, Dorband, J.E., Meyer, van den Brink
arXiv:2109.06190, PRD

- Berry phases in AdS_3/CFT_2

Banerjee, Dorband, J.E., Meyer, Weigel arXiv:2202.11717, JHEP

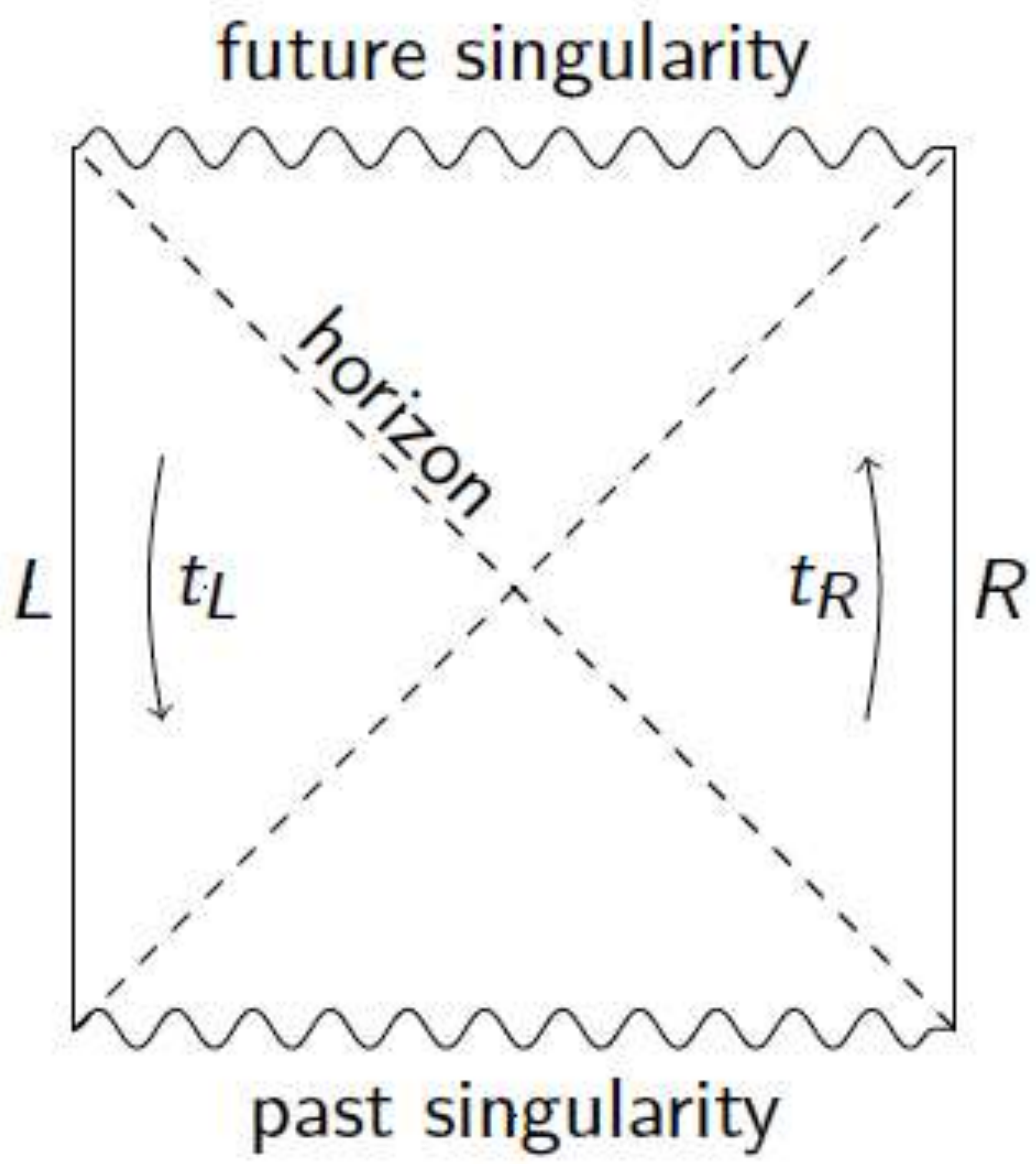
- Berry phases and von Neumann algebras

Banerjee, Dorband, J.E., Weigel arXiv:2306.00055

I. Black holes and wormholes in AdS/CFT and the factorization puzzle

Eternal AdS black hole

- Global coordinates (Kruskal)
- Non-traversable wormhole
- Singularity in time coordinate: Time-like Killing vector switches sign at horizon



Eternal AdS black hole

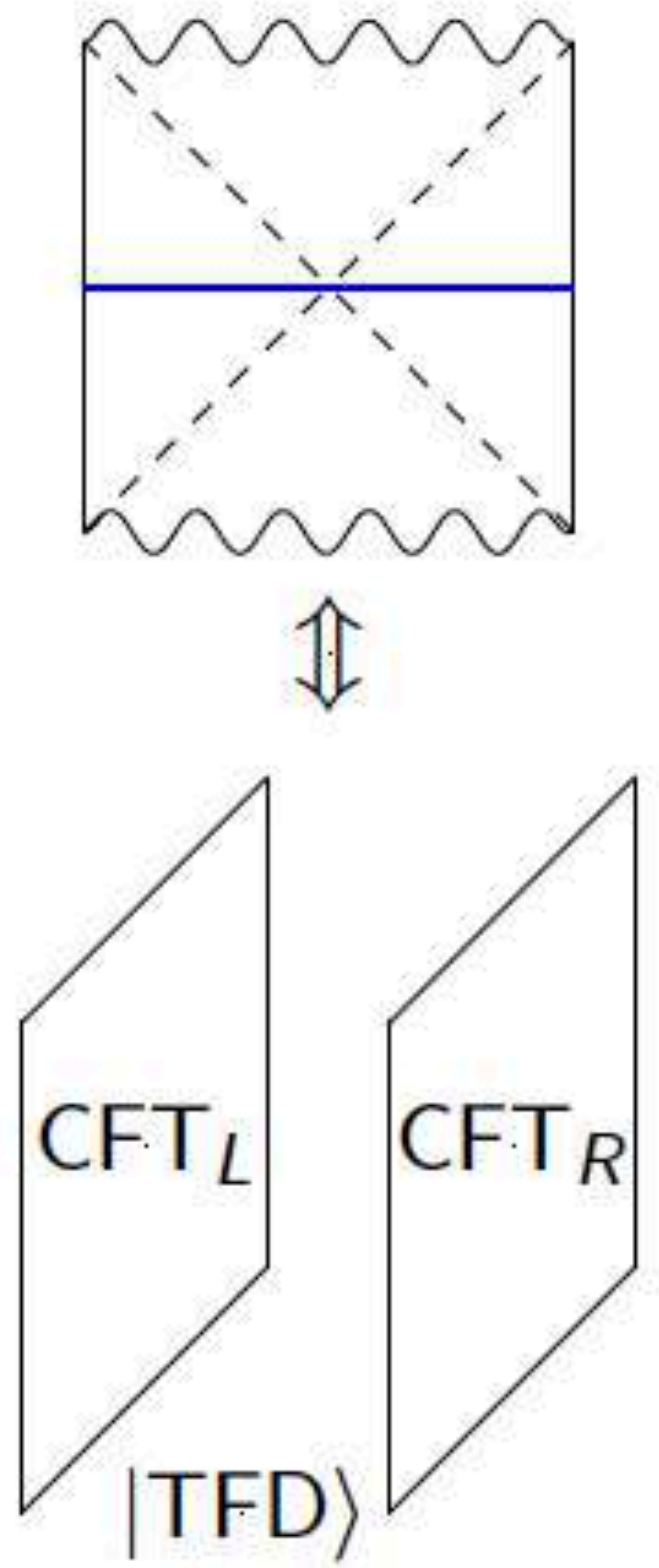
- Eternal black hole in AdS spacetime is dual to two copies of the boundary CFT, entangled in the TFD state

J. Maldacena, [hep-th/0106112]

- TFD state is the purification of a thermal state of one CFT

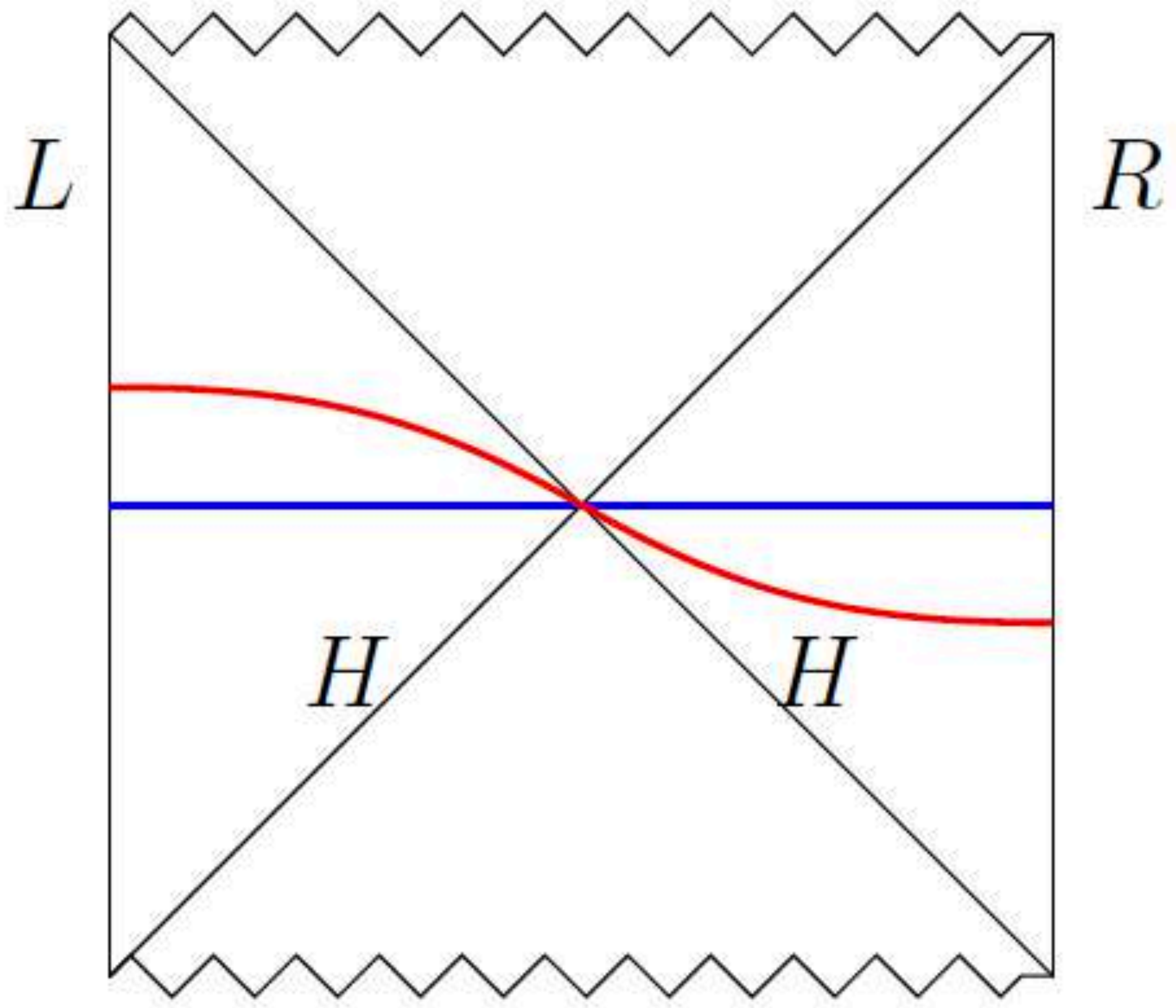
$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}}\sum_n e^{-\frac{\beta}{2}E_n}\,|n\rangle_L\,|n\rangle_R^*\,,$$

$$\mathrm{tr}_R\,|\mathrm{TFD}\rangle\langle\mathrm{TFD}| = \frac{1}{Z}e^{-\beta H_L} = \rho_\beta$$



ER = EPR

Van Raamsdonk 2010; Maldacena, Susskind 2013

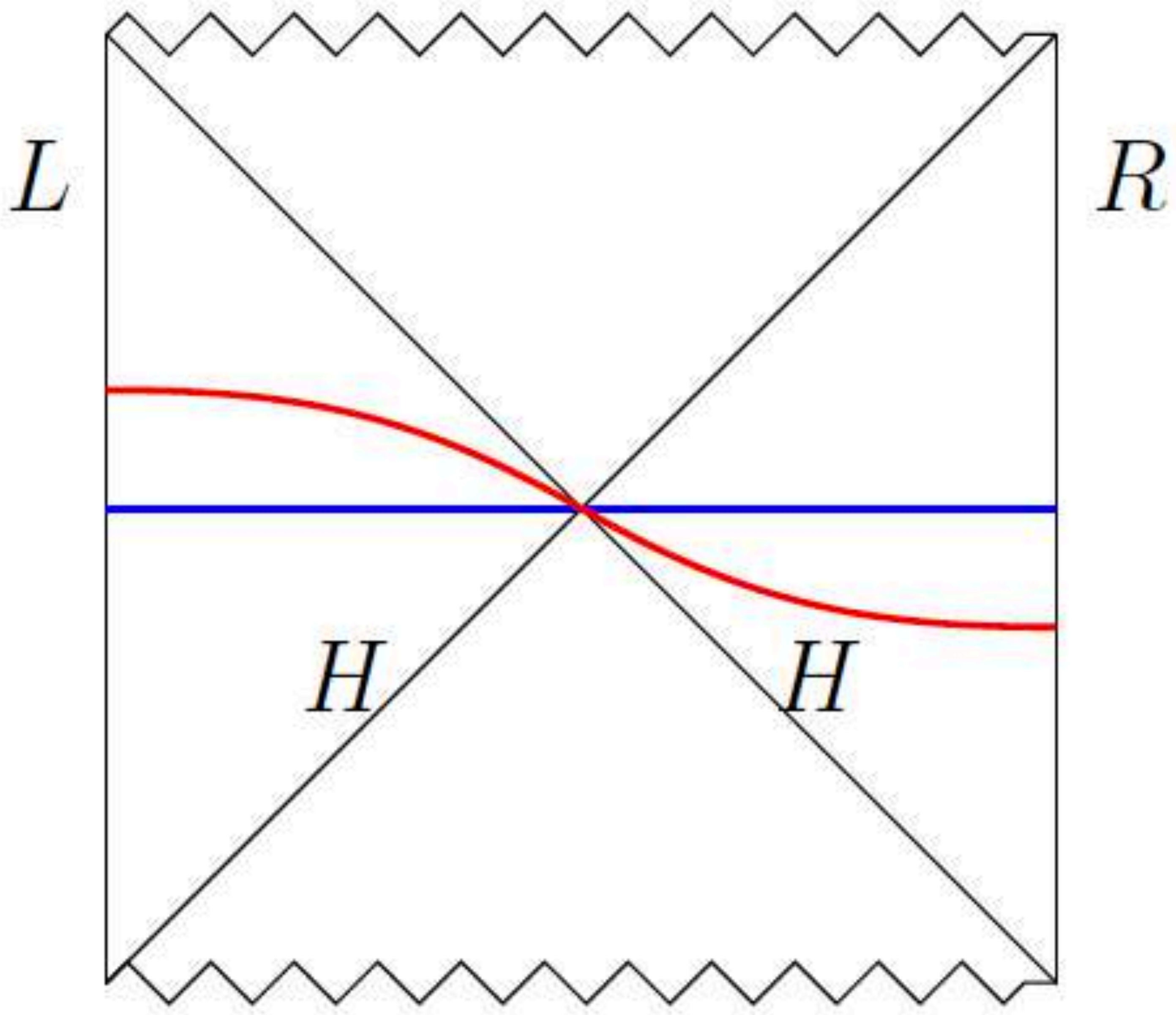


- Relation between entanglement and geometry
- EPR: Einstein-Podolsky-Rosen entanglement
- ER: Einstein-Rosen bridge (wormhole)
- Two entangled CFTs with EPR correlation are connected through a wormhole (ER bridge)

Factorization puzzle

Maldacena+Maoz '13; Harlow '16

- The two CFTs have disjoint Hilbert spaces since there is no interaction between them, $\mathcal{H}_L \otimes \mathcal{H}_R$
- The wormhole Hilbert space does not factorize
- Apparent contradiction?



Wormholes in quantum mechanics

H. Verlinde 2021
2003.13117
2105.02129

$$Z(\beta) = \mathrm{tr}(e^{-\beta H})$$

$$Z(D) = \int [dX]\, e^{\int_D \Omega - \oint_{\partial D} H dt}$$

$D =$ 

generalized coordinates and momenta X^a, symplectic form $\Omega = \frac{1}{2}\omega_{ab} dX^a \wedge dX^b$

Exact symplectic structure: $\Omega = d\alpha$, $\int_D \Omega = \oint_{\partial D} \alpha$

$$Z(\beta) = Z(D)$$

If symplectic structure is non-exact:

$$\langle Z(\beta)^n \rangle = Z(\Sigma_n)$$

$\Sigma_n =$  .

II. Berry phases and von Neumann algebras in quantum mechanics

Berry Phase

- Time-dependent Schrödinger eq.

$$i\hbar\frac{\partial|\psi\rangle}{\partial t} = H(\lambda(t))|\psi\rangle$$

- Ground state

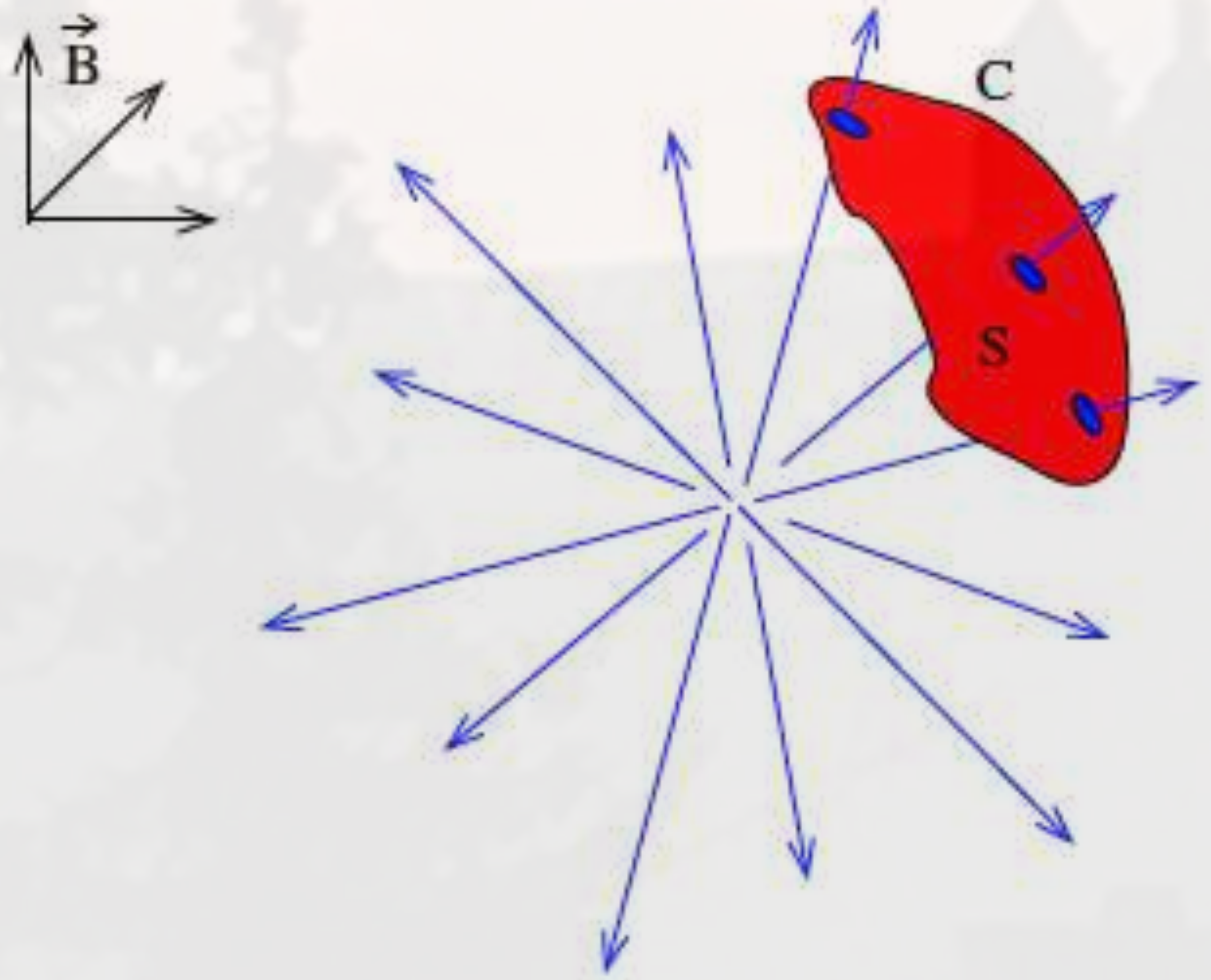
$$|\psi(t)\rangle = U(t)\,|n(\lambda(t))\rangle$$

- Berry connection

$$\mathcal{A}_i(\lambda) = -i\langle n|\frac{\partial}{\partial\lambda^i}|n\rangle$$

- Berry phase

$$\dot{U} = -i\mathcal{A}_i\,\dot{\lambda}^i U$$



$$e^{i\gamma} = \exp\left(-i\oint_C \mathcal{A}_i(\lambda)\,d\lambda^i\right)$$

Review: Lectures by D. Tong

Berry phase

- Maurer-Cartan form: Connection on a group manifold *M* defined for any group element σ
- $$A_{\mathrm{MC}} = \sigma^{-1}\mathrm{d}\sigma$$
- Berry connection: Ground state expectation value of the Maurer-Cartan form

$$A_{\mathrm{B}}(\lambda) = \mathrm{i}\langle\psi_0|\, A_{\mathrm{MC}}\,|\psi_0\rangle$$

- Berry curvature: $F_{\mathrm{B}}(\lambda) = \mathrm{i}\langle\psi_0|\,\omega_{\mathrm{KK}}\,|\psi_0\rangle$
- Berry phase: $$\Phi_{\mathrm{B}} = \int_0^{2\pi}\int_0^{\pi} F_{\mathrm{B}}$$

$$\omega_{\mathrm{KK}} = \mathrm{d}A_{\mathrm{MC}}$$

Kirillov-Kostant symplectic form

von Neumann algebras

von Neumann 1930

Jefferson; Liu, Leutheusser; Witten; Chandrasekaran, Penington, Witten

Concept of algebraic QFT
for classifying operator algebras w. r. t. entanglement properties

Type I - density matrix and trace (as in quantum mechanics),

admits irreducible representations

Type II - trace prescription, but does not act irreducibly

Type III - no trace prescription (eg. free QFTs)

von Neumann algebras

Zur Algebra der Funktionaloperationen und Theorie der normalen Operatoren.

Von

J. v. Neumann in Berlin

Einleitung.

1. Die vorliegende Arbeit zerfällt in zwei, im wesentlichen unabhängige, Teile. Der erste (§§ I—III) ist der Untersuchung der linearen und beschränkten Operatoren (d. h. Matrizen) des Hilbertschen Raumes $\mathfrak{H}$ gewidmet, indem die algebraischen Eigenschaften des von ihnen gebildeten (nicht-kommutativen) Ringes $\mathcal{B}$ betrachtet werden. Den Gegenstand des zweiten Teiles hingegen bilden diejenigen, nicht notwendig überall (in $\mathfrak{H}$) sinnvollen und beschränkten Operatoren, die die sogenannte Hilbertsche Spektraldarstellung mit komplexen Eigenwerten zulassen (vgl. die ausführlichere Explizierung dieser Begriffe im § 4 der Einleitung). Dies sind die als „normal" zu bezeichnenden Operatoren, die bisher nur im Beschränkten betrachtet wurden [1]), und für die wir eine neue allgemeinere Definition geben werden (vgl. am vorhin angeführten Orte).

J. v. Neumann, *Zur Algebra der Funktionaloperationen und Theorie der Normalen Operatoren*, *Mathematische Annalen* **102** (1930) 370–427.

Two-spin system: Berry phase and type I Von Neumann algebra

- Coupled spins in external magnetic field:
  Electronic Zeeman interaction in hydrogen atom $H = J\boldsymbol{S}_1 \cdot \boldsymbol{S}_2 - 2\mu_B B S_{1z}$

- Ground state $|\psi_0\rangle = -\frac{\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}}{\sqrt{2}}|\uparrow\downarrow\rangle - \frac{\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}}{\sqrt{2}}|\downarrow\uparrow\rangle$; $\tan\alpha = 2\mu_B\frac{B}{J}$

- Projective Hilbert space $\mathbb{CP}^3$ (= SU(4)/U(3))

Schmidt decomposition $|\psi_0\rangle = \sum_{i=\uparrow,\downarrow} \kappa_i |i, \tilde{i}\rangle$

$\kappa_\uparrow = \sqrt{\frac{1-\sin\alpha}{2}}$ $\kappa_\downarrow = \sqrt{\frac{1+\sin\alpha}{2}}$

Two-spin system: Berry phase and type I Von Neumann algebra

Entanglement entropy

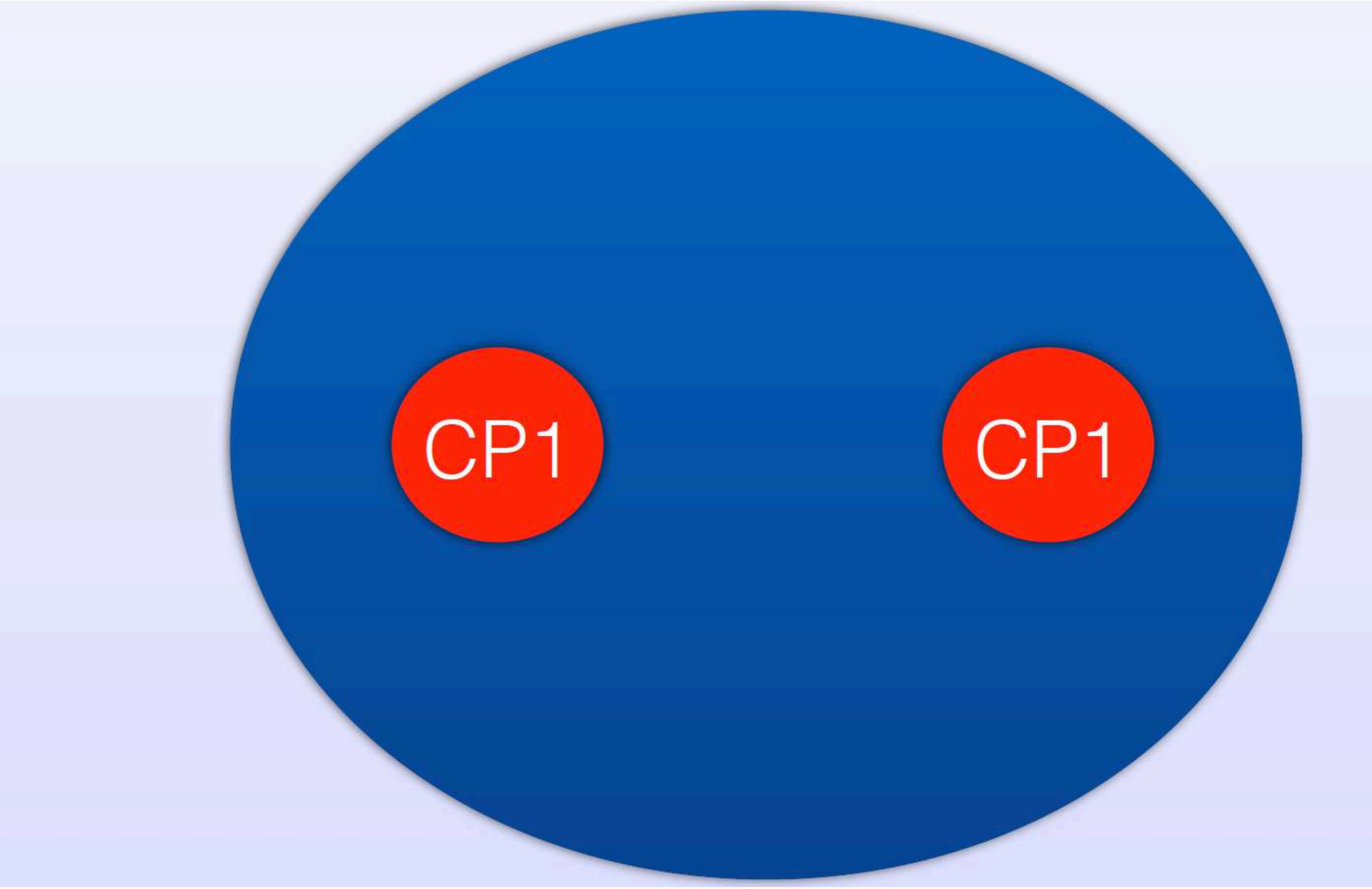
$$S_{\mathrm{EE}} = -\sum_{i=\uparrow,\downarrow} \kappa_i^2 \ln \kappa_i^2 = \sin\alpha \ln \frac{1-\sin\alpha}{\cos\alpha} - \ln \frac{\cos\alpha}{2}$$

No entanglement (J=0) $S_{EE} = 0$ Entanglement orbit $\mathbb{C}P^1 \times \mathbb{C}P^1$

Maximal entanglement
(J very large) $S_{EE} = \ln 2$ $\frac{SU(2)}{\mathbb{Z}_2} = \mathbb{R}P^3$

Intermediate entanglement $\mathbb{C}P^1 \times \mathbb{R}P^3$

State spaces - entanglement orbits



- Projective Hilbert space CP3 = SU(4)/U(3)
- CP1 x CP1: Two single-qubit subspaces of CP3

No entanglement: ground state lies in CP1xCP1
Describes local (one-sided) properties - factorization

Maximal entanglement:
States in non-diagonally embedded submanifold of CP3

$$\frac{SU(2)}{\mathbb{Z}_2} = \mathbb{RP}^3$$

Two-spin system: Berry phase and type I Von Neumann algebra

Berry phase from symplectic volume of entanglement orbit

Reduced density matrix from $P = \begin{bmatrix} \sqrt{\frac{1-\sin\alpha}{2}} & 0 \\ 0 & \sqrt{\frac{1+\sin\alpha}{2}} \end{bmatrix}$

Other points in the orbit $Q = uP, \quad u = e^{-\mathrm{i}\frac{\phi}{2}\sigma_z} e^{-\mathrm{i}\frac{\theta}{2}\sigma_y} e^{\mathrm{i}\frac{\phi}{2}\sigma_z}$

Connection on the orbit $A = \mathrm{i}\,\mathrm{tr}\left(Q^\dagger \mathrm{d}Q\right) = \frac{\sin\alpha}{2}(1-\cos\theta)\mathrm{d}\phi$

Symplectic form $\Omega = \mathrm{d}A = \frac{\sin\alpha}{2}\sin\theta \mathrm{d}\theta \wedge \mathrm{d}\phi$

Berry phase

$$V_{\text{symp}} = \int \Omega = \frac{\sin\alpha}{2} V(S^2) = 2\pi\sin\alpha = \Phi_G$$

vanishes for maximally entangled state

Two-spin system: Berry phase and type I Von Neumann algebra

Trace functional

$$f(ca) = cf(a) \quad \text{for} \quad c \in \mathbb{C}, a \in \mathcal{A} \quad \text{and}$$
$$f(a+b) = f(a) + f(b) \quad \text{for} \quad a, b \in \mathcal{A}.$$

Cyclicity?

$$f(ab) = f(ba)$$

$$f_0(a_L) = \langle \psi_0 | a_L | \psi_0 \rangle \qquad a_L = a_{L,n}\sigma_n, \quad b_L = b_{L,n}\sigma_n, \quad n \in \{0, x, y, z\}, \quad a_{L,n}, b_{L,n} \in \mathbb{R}$$

$$f_0([a_L, b_L]) = 2\mathrm{i} \sin \alpha \left(a_{L,y} b_{L,x} - a_{L,x} b_{L,y} \right)$$

$$f_0([a_L, b_L]) \propto \Phi_G$$

Trace functional on commutator proportional to geometric phase

vanishes for maximally entangled state

Banerjee, Dorband, J.E., Weigel arXiv:2306.00055

Quantum tomography

Rainer Blatt and David Wineland, "Entangled states of trapped atomic ions," Nature **453**, 1008–1015 (2008).

A quantum state is reconstructed using measurements on an ensemble of identical quantum states

Density matrix

For $|\Psi_\phi\rangle = (|00\rangle + \exp(i\phi)|11\rangle)/\sqrt{2}$, the measurement of the product

of Pauli matrices $\sigma_x \otimes \sigma_x$ allows to reconstruct the phase

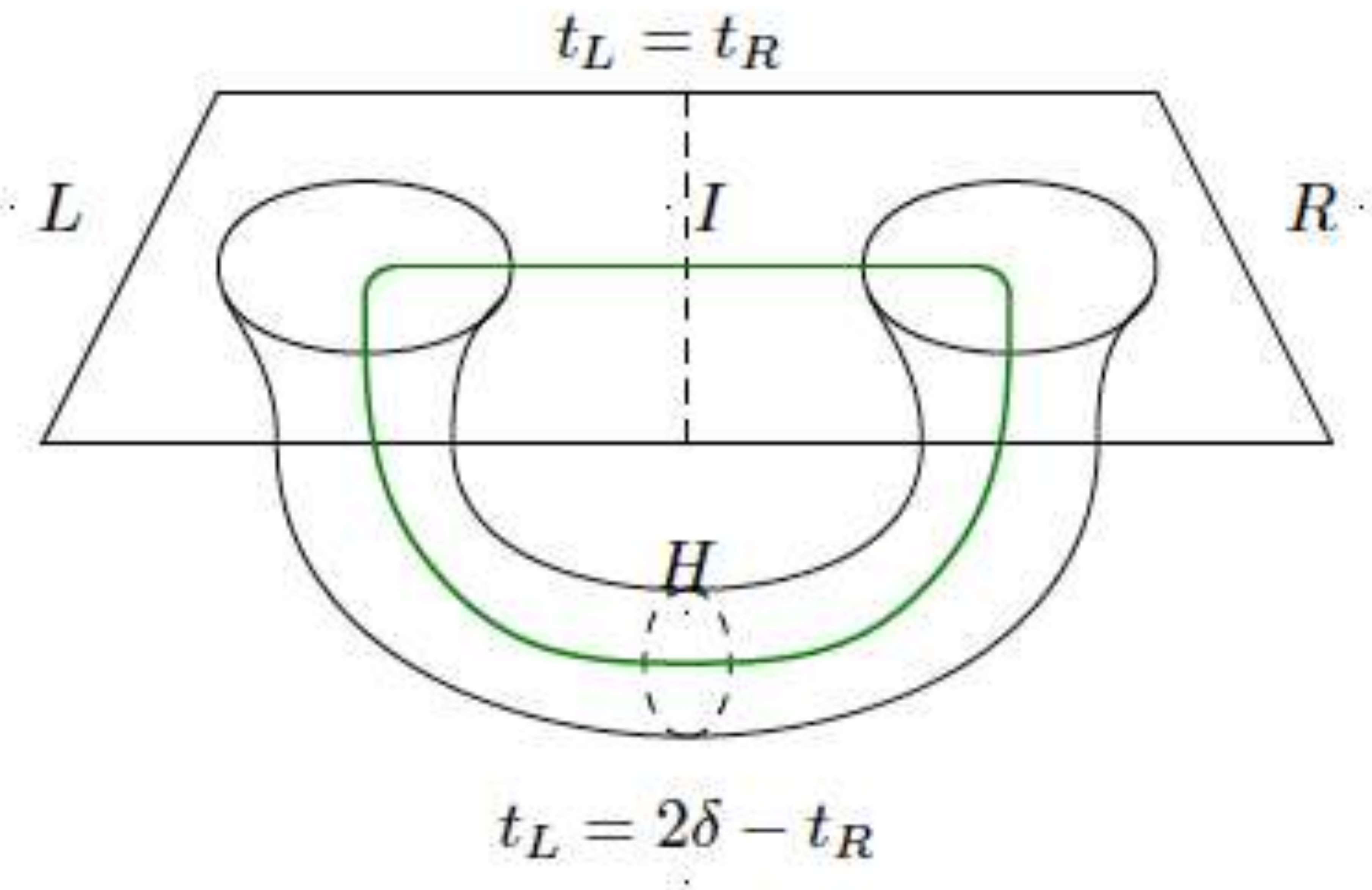
$$\langle\Psi_\phi|\sigma_x \otimes \sigma_x|\Psi_\phi\rangle = \cos\phi$$

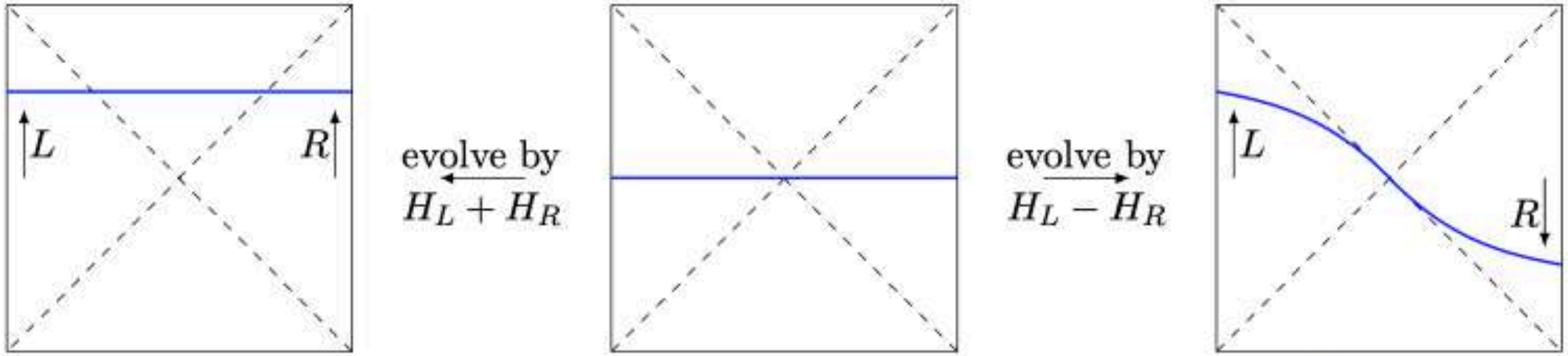
Berry phase and Von Neumann algebra for eternal black hole

No global Killing vector in
the presence of a wormhole

related to mass/temperature
of eternal black hole

Leads to non-exact symplectic form





$$\left|\mathrm{TFD}_\delta\right\rangle = e^{-i(H_L+H_R)\delta}\left|\mathrm{TFD}\right\rangle$$

Symmetry:
does not transform state

Wormhole Berry Phase

Time translations at each boundary $\mathrm{U}(1) \times \mathrm{U}(1)$

Bulk isometry from *H_L - H_R* $\mathrm{U}(1)$

Bulk moduli space of classical solutions $\dfrac{\mathrm{U}(1) \times \mathrm{U}(1)}{\mathrm{U}(1)} \sim \mathrm{U}(1) \sim S^1$

parametrized by δ

Berry connection $A_\delta = \mathrm{i}\langle \mathrm{TFD}| U^\dagger \partial_\delta U |\mathrm{TFD}\rangle$ $U = e^{\mathrm{i}(H_L + H_R)\delta}$

$\Phi_G^{(\mathrm{TFD})} = \int \mathrm{d}\delta \, A_\delta \neq 0$ proportional to geometric phase of bulk moduli space

Type II vs. type III von Neumann algebra for eternal black hole

Liu, Leutheusser; Witten; Chandrasekaran, Penington, Witten

Single-trace operators form generalised free field theory in large N limit

Type III von Neumann algebra

Applies to both CFT_L and CFT_R - both algebras have trivial center

Hamiltonian diverges with N^2 and cannot be included into algebra; therefore consider

$$H'_L = H_L - \langle H_L \rangle \qquad U = H'_L/N \qquad [U, \mathcal{O}] = \frac{1}{N}[H'_L, \mathcal{O}] = -\frac{\mathrm{i}}{N}\partial_t \mathcal{O} \overset{N\to\infty}{\to} 0$$

$$\mathcal{O} \in \mathcal{A}_{L,0}$$

Consider $\mathcal{A}_L = \mathcal{A}_{L,0} \otimes \mathcal{A}_U$ including the algebra of bounded functions of U

To construct $\mathcal{A}_R$, the same operator U has to be used

Both left and right algebra share the same center, proportional to the black hole mass

Type II vs. type III von Neumann algebra for eternal black hole

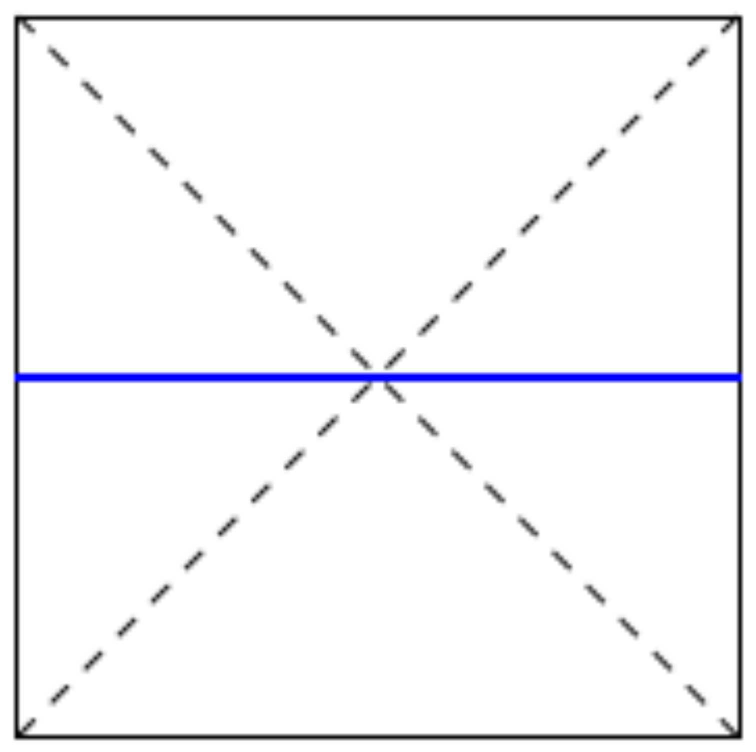
Including 1/N corrections gives rise to type II algebra with vanishing center

Allows for well-defined trace functional for a particular state Witten ‘21

In geometric phase approach:

Non-factorization -> non-zero geometric phase -> no trace definition -> type III vN algebra

Maximally entangled state -> geometric phase vanishes -> type II vN algebra



Banerjee, Dorband, J.E., Weigel arXiv:2306.00055

Missing information

When a symplectic form is non-exact, there is no global section for the entire system

i.e. different coordinate patches are necessary to cover the space

An observer in one coordinate patch has no information about the other

III. Berry phases in AdS_3/CFT_2

Banerjee, Dorband, J.E., Meyer, Weigel arXiv:2202.11717

AdS_3/CFT_2

Virasoro group: $\widehat{Diff}(S^1)$ group elements $(f(\phi), \alpha)$

generator: $T(\phi) = \sum_n L_n e^{in\phi}$ $\quad [L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12} m \left(m^2 - 1\right) \delta_{m+n,0}$

Highest weight-state $|h\rangle$:

CFT vacuum $|0\rangle$: invariant under L_{-1}, L_0, L_1, SL(2,R) symmetry

General $|h\rangle$ with $h > 0$ invariant under L_0, U(1) symmetry

Stabilizer group

AdS_3/CFT_2

In 3d, the bulk spacetime is completely fixed by

- Boundary metric: $g^{(0)}_{ij}$
- Expectation value $\langle h | T(\phi) | h \rangle$

Fefferman+Graham 1985
de Haro, Skenderis, Solodhukin 2000

$$ds^2 = \frac{dr^2}{r^2} + \left(\frac{1}{r^2} g^{(0)}_{ij} + g^{(2)}_{ij} + r^4 g^{(4)}_{ij} \right) dx^i dx^j$$

$$g^{(2)}_{ij} = -\frac{1}{2} R^{(0)} g^{(0)}_{ij} - \frac{6}{c} \langle h | T_{ij} | h \rangle \quad \text{and} \quad g^{(4)}_{ij} = \frac{1}{4} \left(g^{(2)} \left(g^{(0)} \right)^{-1} g^{(2)} \right)_{ij}$$

Symmetries: $\langle h | T(\phi) | h \rangle$ invariant under $SL(2,R)/U(1)$, becomes Killing symmetry of the bulk

Example: CFT in vacuum $\langle 0 | T(\phi) | 0 \rangle = -\frac{c}{24}$ $SL(2,R)$ symmetry

Dual to empty AdS $SL(2,R)$ Killing symmetry

Coadjoint orbits

- A Virasoro group element g acts on a gate X by the *adjoint transformation*

  $$\mathrm{Ad}_g(X) = \frac{d}{dt}\left(g \cdot e^{tX} \cdot g^{-1}\right)\Big|_{t=0}$$

  and on a state v by the *coadjoint transformation*

  $$\langle \mathrm{Ad}^*_g(v), X\rangle = \langle v, \mathrm{Ad}_{g^{-1}}(X)\rangle$$

  Coadjoint orbit: set of states v reachable through coadjoint transformations on fixed state v_0,

  $$O_{v_0} = \{v = \mathrm{Ad}^*_g(v_0) \mid g \in G\}$$

  for Virasoro group: coadjoint orbit = Verma module

Coadjoint orbits

- Coadjoint orbit has symplectic form

$$\omega = d\alpha = -d\langle v, \theta_g \rangle \ \text{where} \ \theta_g = \frac{d}{ds}\left[g^{-1}(t) \cdot g(s)\right]\Big|_{s=t}$$

- Geometric action on coadjoint orbits

$$S_{\text{geo}} = \int \alpha = -\int dt \langle \text{Ad}^*_{g(t)} v_0, \theta_{g(t)} \rangle$$

Coadjoint orbit for Virasoro group

$$\mathcal{O}_{b_0} = \{b = \text{Ad}^*_{(f,\alpha)} b_0 | f \in \text{Diff}(S^1)\},$$

$$\text{Ad}^*_{(f,\alpha)} b_0 = f'^2 b_0 - \frac{c}{24\pi}\{f, \phi\}$$

Virasoro Berry phase

Oblak 1703.06142

generated by diffeomorphisms that change the CFT state

Berry phase for group manifold $A = i\langle\phi(t)|d|\phi(t)\rangle = i\langle\phi|\mathfrak{u}(\theta)|\phi\rangle$, $\mathfrak{u}(\theta) = U_g^\dagger dU_g$

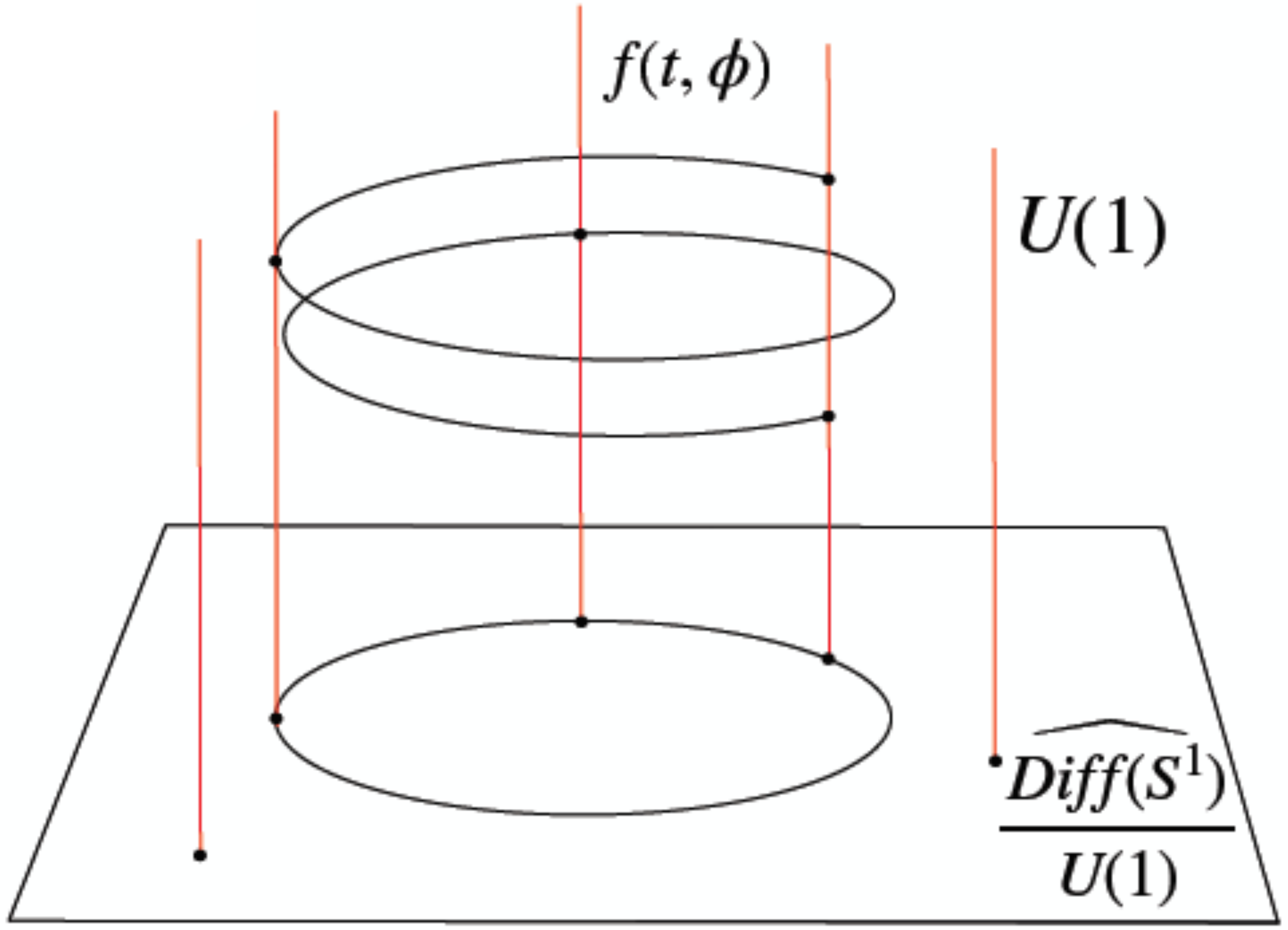
with central extension $A = \langle h|\mathfrak{u}(\theta)|h\rangle + c\langle h|\mathfrak{u}(m_\theta)|h\rangle$

(θ, m_θ): Maurer-Cartan form

Connection gives rise to Berry phase as before

Virasoro Berry phases probe the geometry of a particular coadjoint orbit

For states outside the coadjoint orbit, the stabilizer group leads to a phase



Virasoro Berry phase

For holographic CFTs: use fact that stress tensor is element of dual Lie algebra

$$2\pi b = 2\pi \mathrm{Ad}^*_{(f,\alpha)} b_0 = \langle h|\tilde{T}(\phi)|h\rangle = f'^2 \langle h|T(\tilde{\phi})|h\rangle - \frac{c}{12}\{f,\phi\},$$

$$b_0 = \frac{1}{2\pi}\langle h|T(\tilde{\phi})|h\rangle = \frac{1}{2\pi}\left(h - \frac{c}{24}\right)$$

Dual bulk geometries associated to Virasoro coadjoint orbits: Banados geometries

On-shell solutions to AdS3 gravity with Brown-Henneaux boundary conditions

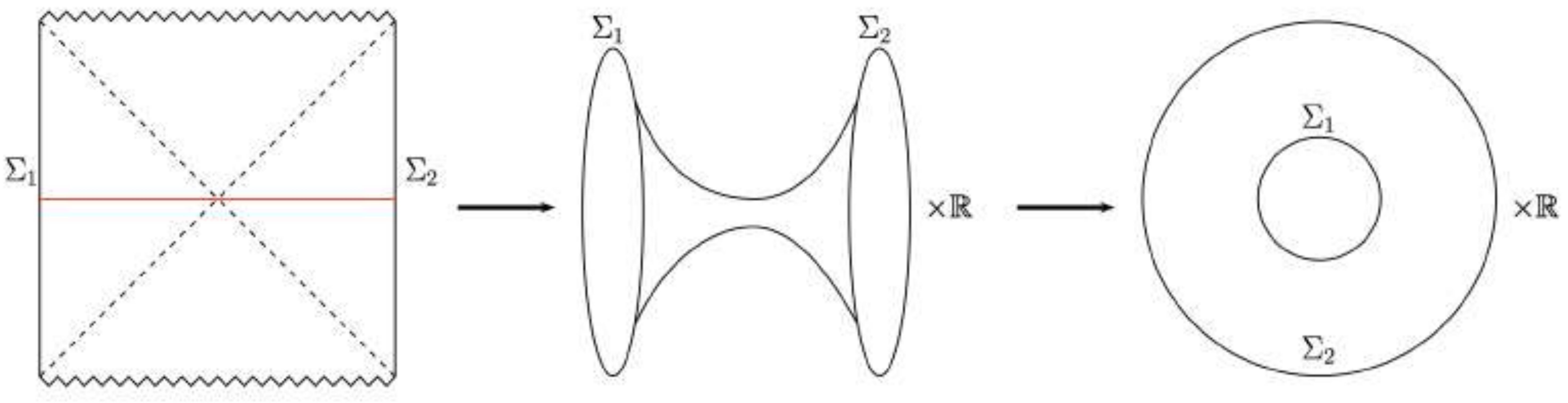
$$ds^2 = \ell^2 \frac{dr^2}{r^2} - \left(r dx^+ - \ell^2 \frac{L_-(x^-)\, dx^-}{r}\right)\left(r dx^- - \ell^2 \frac{L_+(x^+)\, dx^+}{r}\right)$$

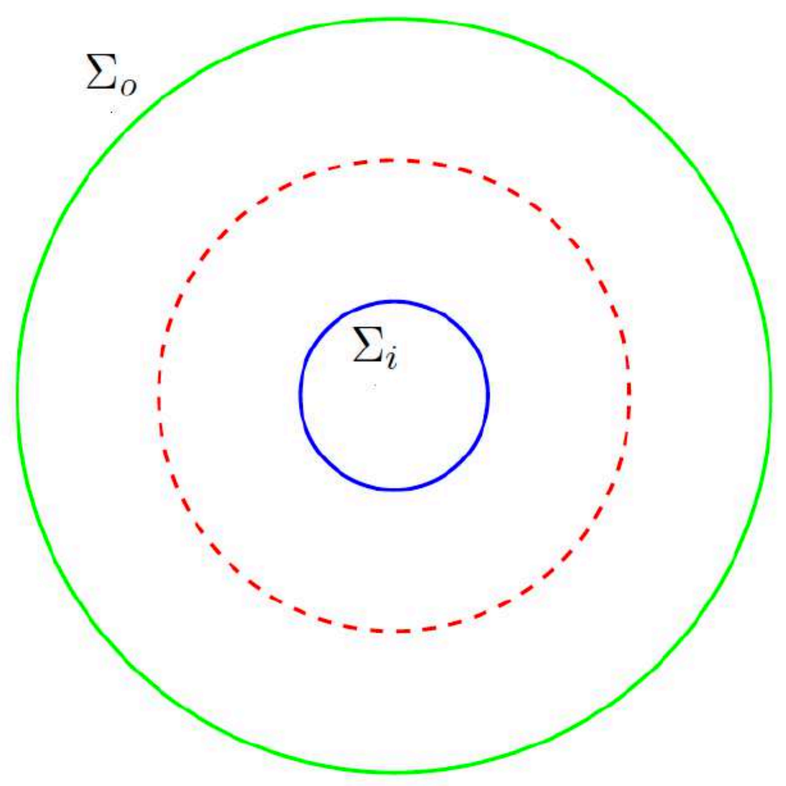
$$L(x^\pm) = \tfrac{6}{c}\langle h|\tilde{T}|h\rangle = \tfrac{12\pi}{c} b \text{ and } x^\pm = t \pm \phi$$

Berry phase for wormholes in AdS3

Henneaux, Merbis, Ranjbar arXiv:1912.09465

Two boundary CFTs for eternal AdS black hole





Holonomy $k_0 = \frac{1}{2\pi} \oint d\varphi A_\varphi$

Chern-Simons action

$$S_{[CS]} = \frac{k}{2\pi} \int dt dr d\varphi \left(A_\varphi \partial_t A_r + A_t F_{r\varphi} \right)$$

Abelian case: $A_\varphi = \partial_\varphi \mu + k_0$

Φ, Ψ Boundary values of μ

Boundary action for Φ, Ψ, k_0

Boundary action

$$S = \frac{k}{4\pi}\left(\int dt\left[\oint d\varphi\left(\partial_\varphi\Phi\partial_t\Phi\right) - H_\Phi\right] + \int dt\left[-\oint d\varphi\left(\partial_\varphi\Psi\partial_t\Psi\right) - H_\Psi\right] + 2\int dt\left[\oint d\varphi k_0\left(\partial_t\Phi - \partial_t\Psi\right) - H_0\right]\right),$$

where

$$H_\Phi = \int d\varphi\left(\partial_\varphi\Phi\right)^2,$$

$$H_\Psi = \int d\varphi\left(\partial_\varphi\Psi\right)^2,$$

$$H_0 = 2\pi\left(k_0\right)^2.$$

Conjugate momentum and symplectic form on phase space

Conjugate momentum for holonomy $\Pi_0 = -\frac{k}{2\pi}\oint d\varphi(\Phi - \Psi) = -\frac{k}{2\pi}\oint d\varphi\left(\int_{r_1}^{r_2} dr\, A_r\right)$

Symplectic form on boundary phase space $x = (\Phi, \Psi, \Pi_0, \Pi_\Phi, \Pi_\Psi, k_0)$

$$\omega = d\Pi_\Phi \wedge d\Phi + d\Pi_\Psi \wedge d\Psi + dk_0 \wedge d\Pi_0$$

Non-exact due to contribution of holonomy!

$$\frac{\widehat{LG}}{G} \times \frac{\widehat{LG}}{G} \longrightarrow \frac{\check{\widehat{LG}} \times \widehat{LG}}{G}$$

Berry phases for wormholes in AdS_3

A similar analysis may be performed for non-abelian SL(2,R) x SL(2,R) symmetry

Virasoro Berry phase

Symmetry transformations change states in the CFTs

Symplectic form on phase space can be mapped to symplectic form on Virasoro group manifold with coupling between both CFTs

Banerjee, Dorband, J.E., Meyer, Weigel arXiv:2202.11717

$$\frac{\mathrm{Diff}(S^1)}{S^1} \times \frac{\mathrm{Diff}(S^1)}{S^1} \longrightarrow \frac{\mathrm{Diff}(S^1)\times\mathrm{Diff}(S^1)}{S^1}$$

Symplectic form non-exact

Relation to geometric action

Reparametrize chiral boson

$$\Phi = k_0(f(t,\varphi) - \varphi) - \ln\left(-k_0 f'(t,\varphi)\right)$$

$$\Psi = k_0(\varphi - g(t,\varphi)) - \ln\left(k_0 g'(t,\varphi)\right)$$

Action becomes

$$S\left[k_0, \Phi, \Psi\right] = S^-_{\text{geo}}\left[f, b_0\right] - S^+_{\text{geo}}\left[g, b_0\right]$$

$$S^{\pm}_{\text{geo}}\left[h, b_0\right] = \int dt d\sigma \left(b_0 h' \partial_\pm h + \frac{k}{8\pi} \frac{h'' \partial_\pm h'}{(h')^2} \right)$$

Coupled Virasoro Berry phase $\quad b_0 = \frac{k}{8\pi} k_0(t)^2$

Henneaux et al
Jensen et al

$$b_0 = \frac{1}{2\pi} \langle h | T(\phi) | h \rangle$$ couples both Berry phases

Asymptotic dynamics of BTZ black hole is described by coupled Virasoro Berry phase

Both sides are coupled by the holonomy and connected by a radial Wilson line

Symmetry group is enhanced boundary no longer factorizes

$$\frac{\text{Diff}(S^1)}{S^1} \times \frac{\text{Diff}(S^1)}{S^1} \rightarrow \frac{\text{Diff}(S^1) \times \text{Diff}(S^1)}{S^1}$$

Conclusions and outlook

- Non-Factorization of wormhole Hilbert space due to non-exact symplectic form that results in non-zero Berry phase
- Mathematical structure also present in quantum mechanics and in CFT
- Relation between entanglement and geometry
- New possibilities for experimental study
- Relation to von Neumann algebras

Modular Berry phase

B. Czech

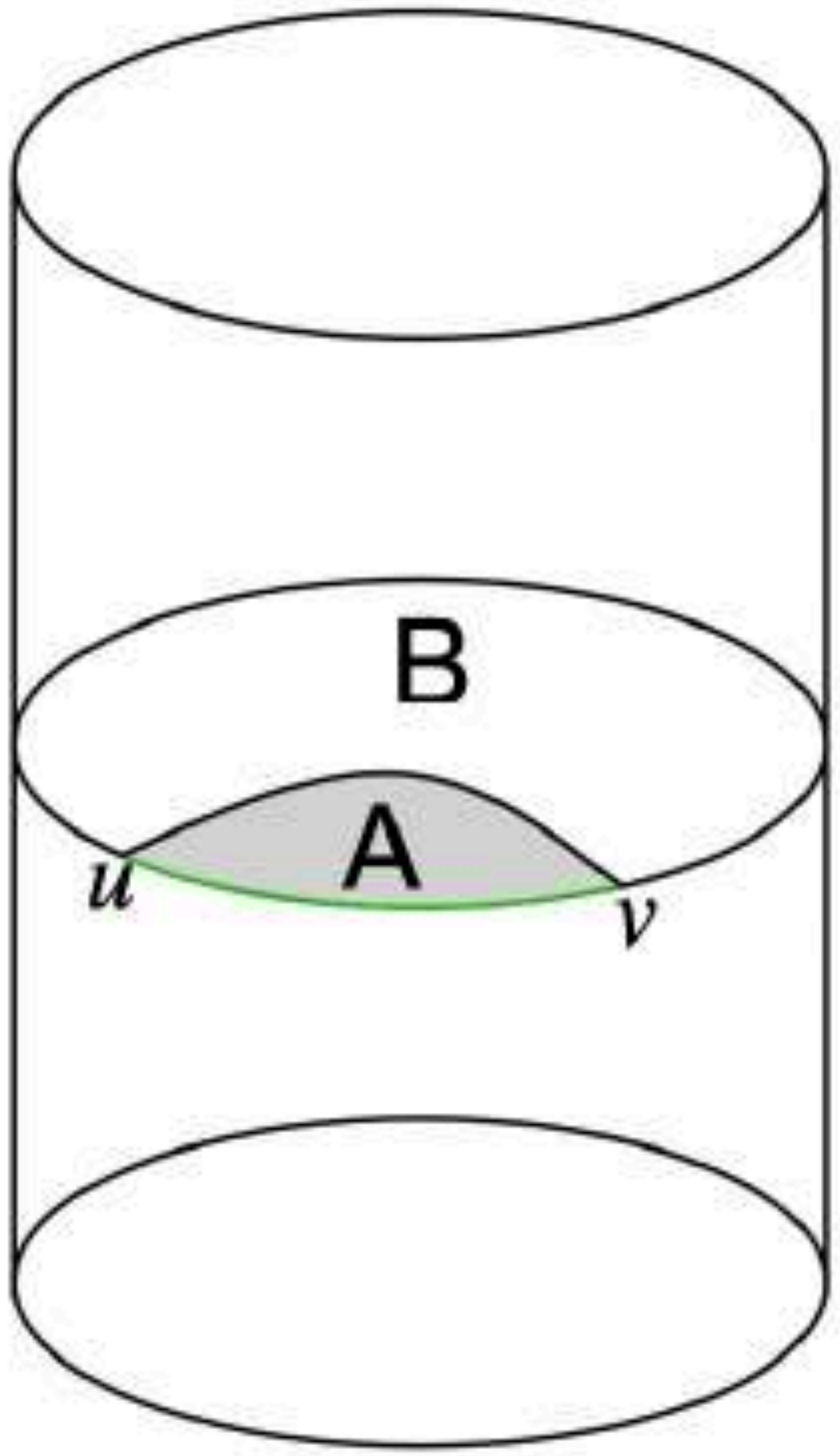
- Interval [u,v] on constant time slice in CFT with reduced density metric ρ_A dual bulk region bounded by RT geodesic

$$S(u,v) = \frac{l(u,v)}{4G_N}$$

- Modular Hamiltonian: $H_{\text{mod}} = -\log(\rho_A)$ generates abstract time evolution with respect to modular time parameter

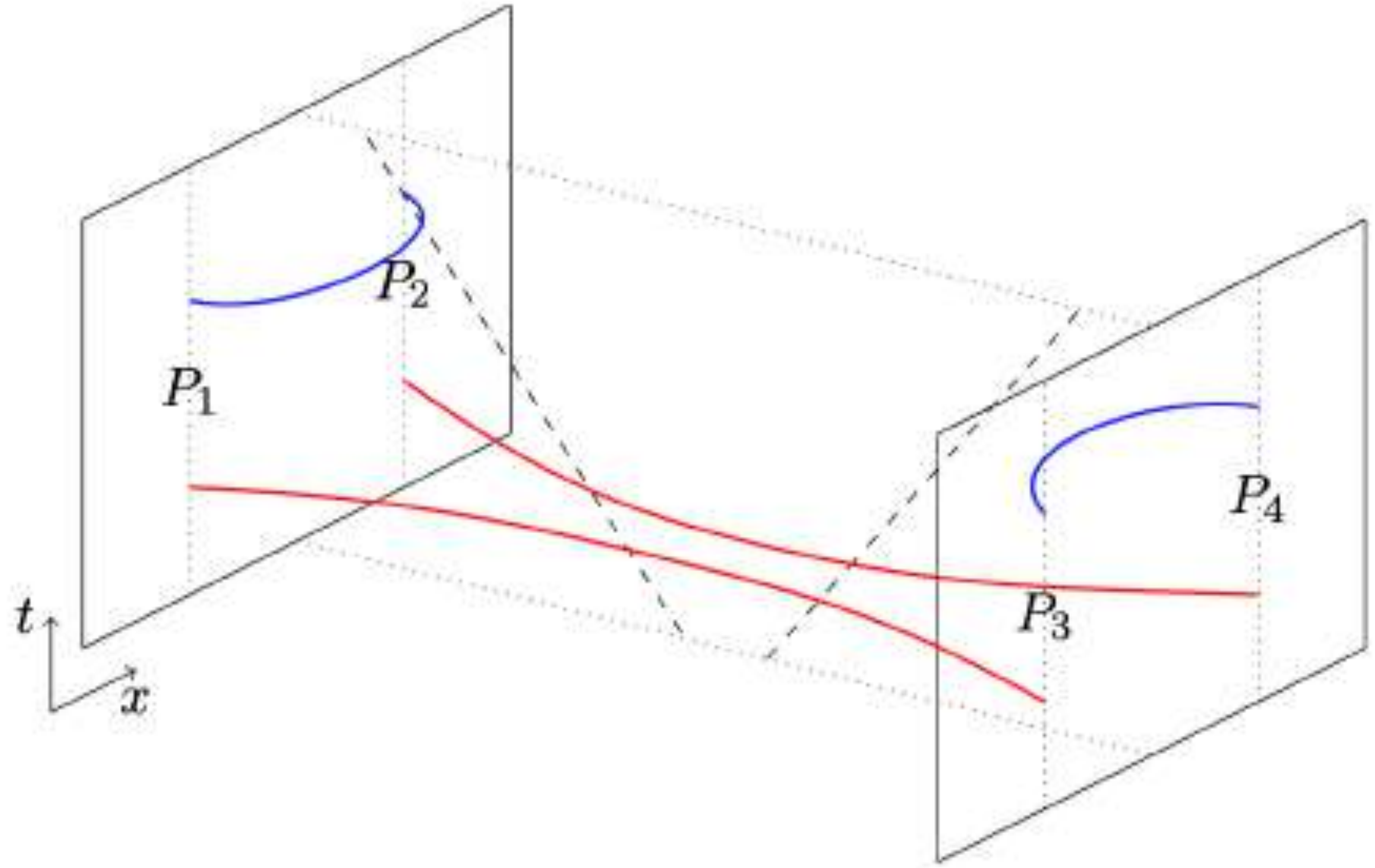
- Choice of modular time parameter is gauge symmetry for each interval

- Parallel transport of interval around closed loop leads to Berry phase



Modular Berry phase: Two-sided case

Transition in entanglement entropy (Hartman et al)



Change in Berry curvature

Gauge Berry phase

Proper diffeomorphisms correspond to gauge symmetries at the boundary

yield Killing charges and satisfy $\delta_\xi g_{\mu\nu} = 0$

Brown, Henneaux 1986
Compère; Mao, Seraj, Sheikh-Jabbari 2015

In presence of an eternal black hole, these charges may only be defined locally near the two boundaries

Example: Wormhole Berry phase for JT gravity

- Theory of 2d gravity with a scalar field (dilaton)

$$R+2=0 \quad \& \quad (\nabla_\mu \nabla_\nu - g_{\mu\nu})\,\Phi = 0$$

- Boundary conditions: $\gamma_{tt}|_{\partial M} = r_c^2 \quad \& \quad \Phi|_{\partial M} = \phi_b r_c$
- Diffeomorphisms given by time translations with

$$H_L = H_R = \Phi_H^2/\phi_B$$

Harlow, Jafferis '18

Example: Wormhole Berry phase for JT gravity

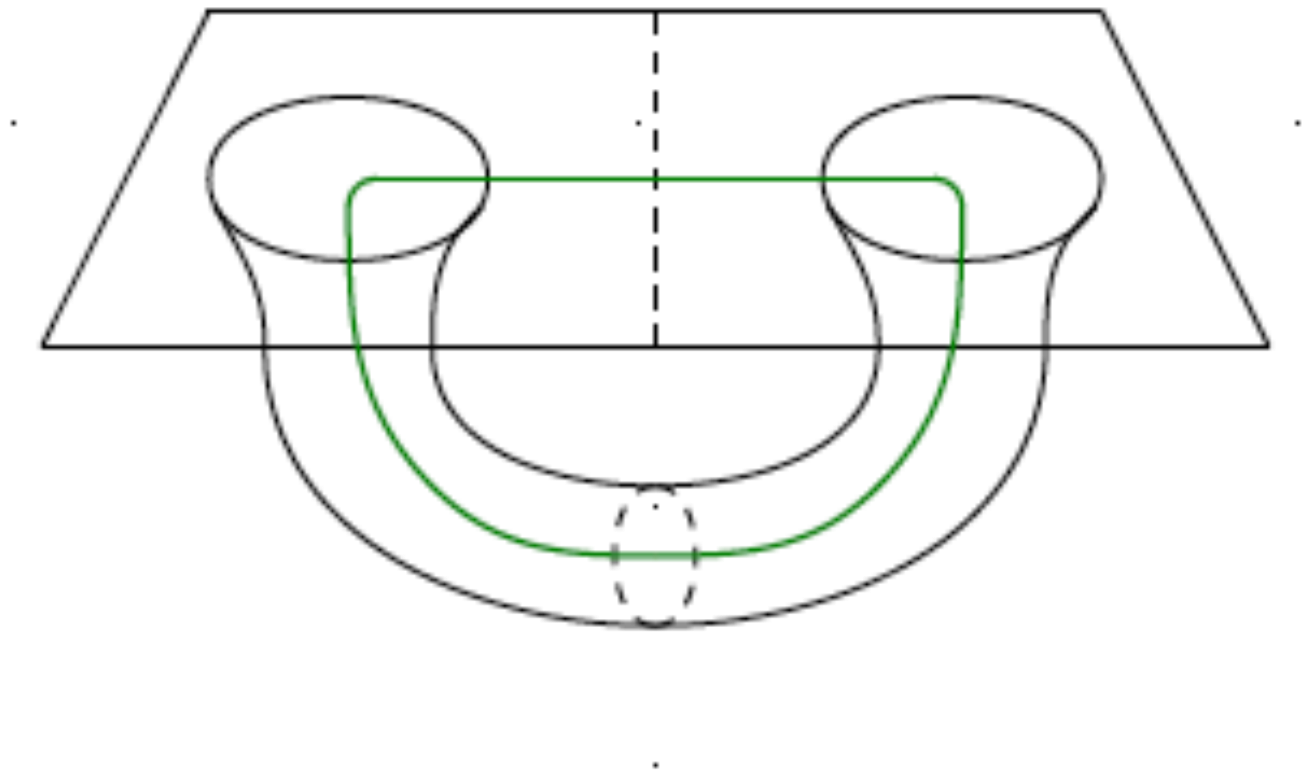
- Berry connection evaluates to:

$$A_\delta = 2\Phi_H^2/\phi_b$$

$\alpha = -2E\delta$ $\quad 2\pi$ periodic

Resulting Berry phase with winding number interpretation:

$$\Phi_{\mathrm{B}}^{\mathrm{JT}} = \oint A_\delta \mathrm{d}\delta = 2\frac{\Phi_H^2}{\phi_b}\int_0^{\frac{\pi}{E}} \mathrm{d}\delta = 2\pi$$



Wormhole Berry Phase

Non-zero Berry connection for diffeomorphism

$$u_0(\delta) = e^{-\mathrm{i}(H_L+H_R)\delta}$$

$$A_\delta = \mathrm{i}\langle \mathrm{TFD}_\alpha | \partial_\delta | \mathrm{TFD}_\alpha \rangle = \frac{2}{Z}\sum_n E_n e^{-\beta E_n}$$

$$|\mathrm{TFD}_\alpha\rangle = \tilde{u}_0(\delta)\,|\mathrm{TFD}_{\alpha=0}\rangle$$

Vanishing Berry connection for diffeomorphism

$$u_1(\delta) = e^{-\mathrm{i}(H_L-H_R)\delta}$$

$$A_\delta = \mathrm{i}\langle \mathrm{TFD}_{\alpha=0} | u_1^\dagger \partial_\delta u_1 | \mathrm{TFD}_{\alpha=0} \rangle = 0$$