Scaling Similarity in Large N quantum mechanics

Juan Maldacena

Institute for Advanced Study

ICTP Saifr, June 2023

The duality between AdS_5xS^5 and super-Yang-Mills can be obtained by taking the near horizon limit of D3 branes in ten dimensions.

Holography for D p-branes

- What if we take the near horizon limit of Dp branes, for $p \neq 3$?

Holography for D p-branes

- We also get a duality between a geometry and a super Yang Mills theory with 16 supercharges. The theory is not scale invariant.

Itzhaki, JM, Sonnenschein, Yankielowicz

- In the gravity limit, the solution is rather simple
  - It has dilaton that depends on the radius
  - The Einstein metric is Weyl equivalent (conformal) to $AdS_{2+p} \times S^{8-p}$

Boonstra, Skenderis, Townsend

Why are we returning to this topic?

The D0 case is particularly interesting.

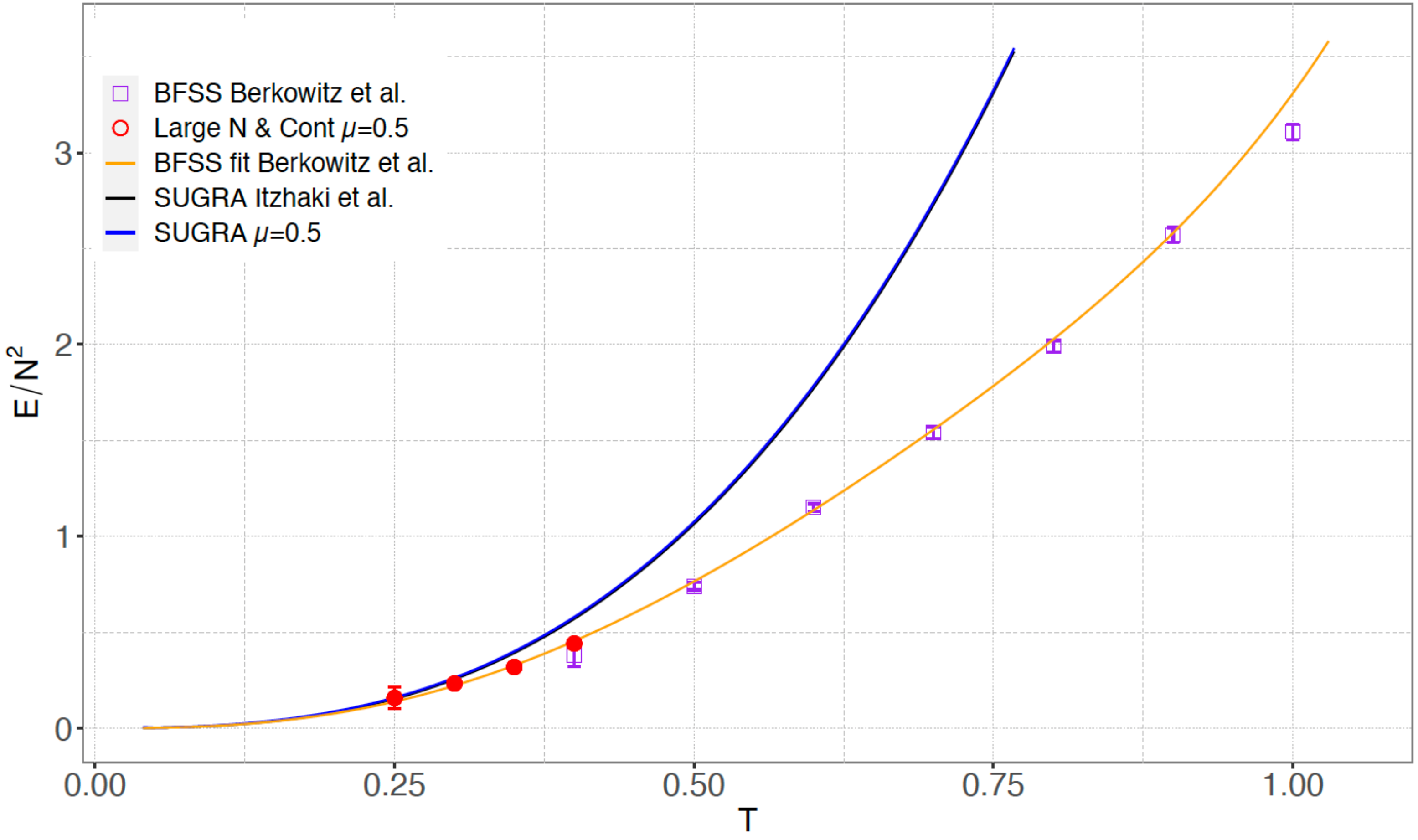
The matrix model is the simplest quantum mechanical theory* that has a bulk Einstein gravity dual†.

*as opposed to QFT

† as opposed to higher spin gravity theories

There have been very interesting numerical simulations at finite temperature.

Latest montecarlo simulations



Monte Carlo String/M-theory Collaboration (MCSMC)
- Pateloudis, Bergner, Hanada, Rinaldi, Schaefer, Vranas, Watanabe, Bodendorfer

- Berkowitz, Rinaldi, Hanada, Ishiki, Shimasaki, Vranas

They have numerically computed the first 8-derivative corrections at tree and loop level.

Not reproduced from gravity.
Do not have the explicit form
of the 8 derivative correction as a function of $R_{\mu\nu\rho\sigma}, F_2, \phi$

Reproduced from gravity

Hanada, Hyakutake, Nishimura, Takeuchi

They are more advanced than analytic computations!

In the distant future → quantum simulated?

The number of qubits is roughly similar to that necessary for factoring large integers (breaking RSA code)

Holography for D p-branes

- Gravity solution

$$ds_E^2 = z^a \left[\frac{-dt^2 + dx_p^2 + dz^2}{z^2} + \# d\Omega_{8-p}^2 \right]$$

$$e^{-2\phi} \propto N^2 z^b \ , \qquad F_{8-p} \propto N \omega_{8-p}$$

$$a = -\bar{\theta}/8 \ , \qquad \bar{\theta} \equiv \frac{(3-p)^2}{(5-p)} \ , \qquad b = -\frac{(7-p)(3-p)}{(5-p)}$$

Itzhaki, JM, Sonnenschein, Yankielowicz

Boonstra, Skenderis, Townsend

Scaling similarity

$$ds^2 = z^a \left[\frac{-dt^2 + dx_p^2 + dz^2}{z^2} + \# d\Omega_{8-p}^2 \right]$$

$$e^{-2\phi} \propto N^2 z^b \ , \qquad F_{8-p} \propto N \omega_{8-p}$$

$$a = -\bar{\theta}/8 \ , \qquad \bar{\theta} \equiv \frac{(3-p)^2}{(5-p)} \ , \qquad b = -\frac{(7-p)(3-p)}{(5-p)}$$

Similarity:

$$t, \ x, \ z \longrightarrow \lambda t, \ \lambda x \ , \lambda z \ , \qquad I \longrightarrow \lambda^{-\bar{\theta}} I$$

Classical gravity action

-Hyperscaling violation exponent.

Dong, Harrison, Kachru, Torroba, H. Wang

-Action scaling similarity exponent

Simple case of a scaling similarity

Power law potentials.

Landau, Lifshitz

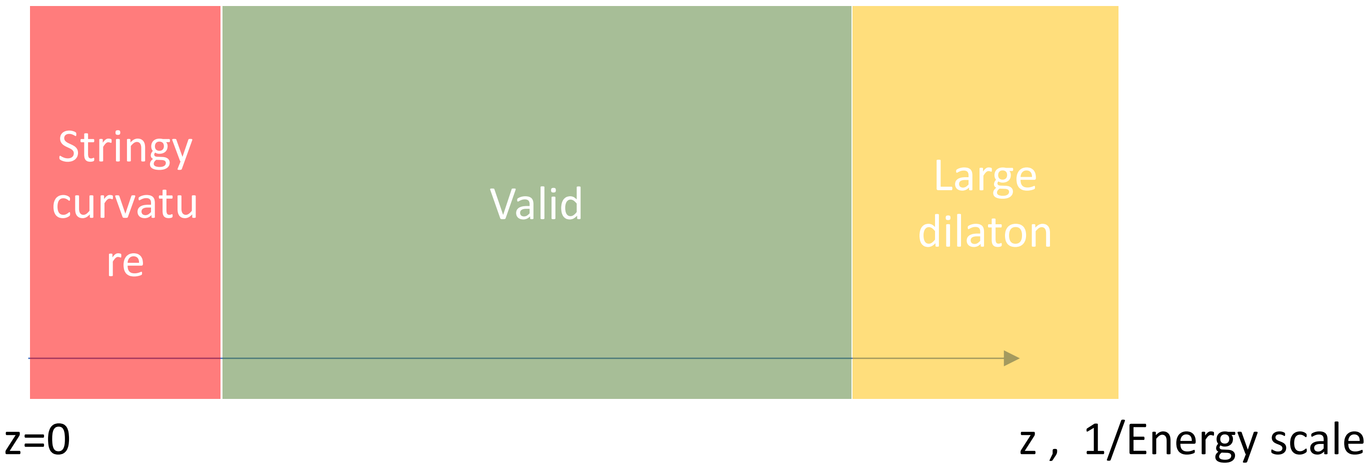
$$I = \int dt[\dot{x}^2 - x^n]$$

$$t \to \lambda t\ , \qquad x \to \lambda^{\frac{-2}{n-2}} x\ , \qquad I \to \lambda^{-\frac{n+2}{n-2}} I$$

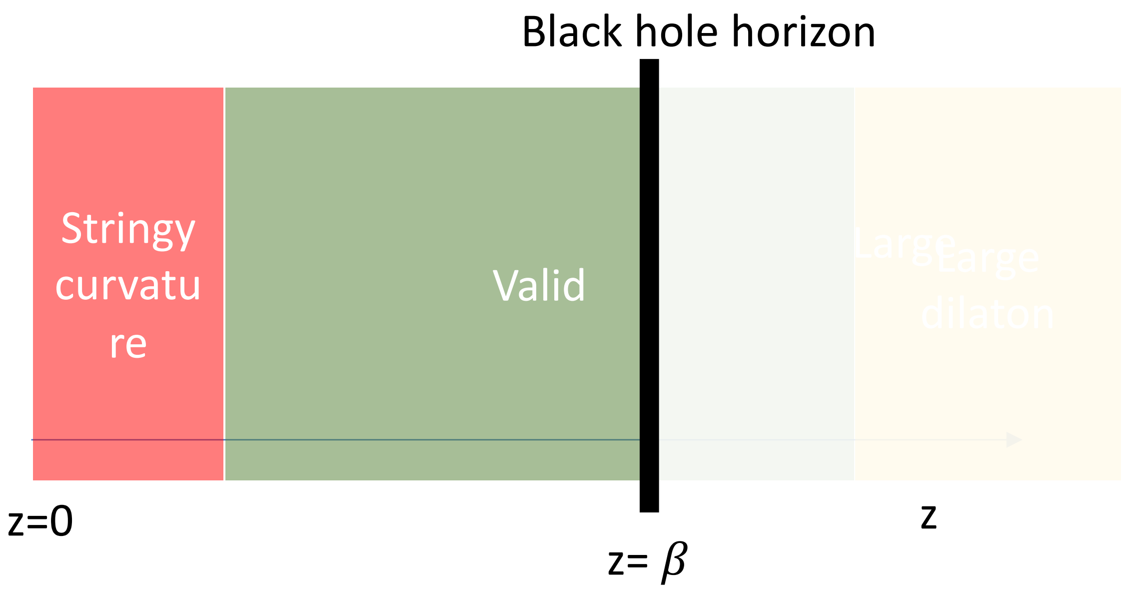
E.g. , for n=-1, the scaling of x, for n=-1, fixes the 3rd Kepler law.

A similarity depends on a classical limit.

Validity of gravity solution (p<3)



Validity of gravity solution at finite temperature (p<3).



Valid for low, but not too low temperatures

$$\log Z = -I = -\beta F \propto V_p T^{p+\bar{\theta}}$$

Action similarity exponent → fixes the temperature dependence

Scaling symmetry of the gravity equations of motion

The equations of motion have a scaling symmetry.

It can be used to classify the small fluctuations = operators of the gravity theory.

Previously obtained by Sekino, Yoneya

As a separate trick, it is useful to note that these solutions can be also usefully viewed as coming from dimensional reduction from

$$AdS_{2+p+\bar{\theta}} \times S_{8-p}$$

Kanitscheider, Skenderis

Classify fields by their dimension Δ from the $AdS_{2+p+\bar{\theta}}$ point of view.

Useful trick (Side comment)

These solutions can be also usefully viewed as coming from dimensional reduction from

$$AdS_{2+p+\bar{\theta}} \times S_{8-p}$$

Kanitscheider, Skenderis

Classify fields by their dimension Δ from the $AdS_{2+p+\bar{\theta}}$ point of view.

What we will call Δ is the standard one in this higher dimensional AdS space.

Spectrum of dimensions from 11 dimensions.

- All these backgrounds can be uplifted to an M-theory plane wave background.

$$ds^2 = -dx^+ dx^- + \frac{1}{|y|^{7-p}}(dx^+)^2 + d\vec{x}_p^{\,2} + d\vec{y}_{9-p}^{\,2}$$

- Perturbations are just those of flat space far away. Simple powers of $y, \quad y^\ell$.

- Tracking the action of the similarity transformation → we can read off the dimensions. Similarity = rescaling + boost in 11 dimensions.

$$\Delta = \frac{7-p}{5-p}(b+2) + \frac{2\ell}{5-p}$$

E.g. $Tr[X^\ell]$ has dimension $\Delta = \frac{2\ell}{5-p}$

Spin of the field in the light cone directions in 11 dimensions, b=-2,-1,0,1,2. (only these values, for bosons)

One interesting question we can answer with this formula is the following

How many relevant operators do we have at strong coupling ?

Relevant operators at strong coupling

- We have no SO(9) invariant single trace relevant operator.

- We have a few double and triple trace SO(9) invariant operators.

e.g. $Tr[X^{(I}X^{J)}]Tr[X^{(I}X^{J)}]$, $\Delta = \frac{2}{5} \times 2 \times 2 = \frac{8}{5} < \frac{14}{5} = 1 + \bar{\theta}$, $\bar{\theta} = \frac{9}{5}$

- The coefficients of these should be fine tuned in order to simulate the model at very low energies.

At finite temperature we can use the $AdS_{2+p+\overline{\theta}}$ black branes.

Once the spectrum is fixed, we can write the wave equations.

We can get black hole quasinormal modes, etc. →

Simple ``spectrum'' of a black hole

Can we determine the scaling similarity exponent from the YM side?

We can determine it by performing a susy protected computation that is valid also at strong coupling

Two examples of SUSY protected computations

- Motion in the moduli space approximation, v^4 term is one loop exact.

Similar ideas : Smilga ; Morita, Shiba, Wiseman, Withers

$$I \propto \int dt \left[\dot{X}^2 + \frac{\dot{X}^4}{|X|^7} \right]$$

Similarity with exponent $\bar{\theta}$

- Sphere partition functions.

$$\log Z_{S^{p+1}} \propto N^2 R^{\bar{\theta}} \lambda^{\frac{p-3}{5-p}} \;, \qquad p > 0$$

Bobev, Bomans, Gautason, Minahan, Nedelin

Comparing the scaling of the susy protected computation with the one expected from gravity → get the exponent. Agreement!

Susy computations are consistent with the similarity.

Assuming we have the similarity → get the exponent.

Now we will switch to a different problem

What follows is based on work in progress with

Anna Biggs,   JM,   Vladimir Narovlansky

The SYK model is a quantum mechanical theory which develops a near <u>scale invariance</u> at low energies.

Is there an SYK-like model that develops a scaling similarity at low energies?

We are studying such a model.

There is a model that had been considered before as a model for black holes.

Anninos, Anous, Denef

(In fact before SYK)

We will consider a slightly simplified version.

The models (two variants)

- $\mathcal{N}$=2,4 supersymmetric quantum mechanics with a random qth order superpotential.

$\mathcal{N}$=2 $\quad \int dt d^2\theta [D\phi^i \bar{D}\phi^i + W(\phi)] \ , \qquad W = \sum J_{ijk}\phi^i\phi^j\phi^k \qquad \phi$ is real

(different than than the $\mathcal{N}$=2 model considered by Fu, Gaiotto, JM, Sachdev)

$\mathcal{N}$=4 $\quad \int dt d^4\theta \phi^i \bar{\phi}^i + \int d^2\theta W(\phi) + \int d^2\bar{\theta} W(\bar{\phi}) \ , \qquad W = \sum J_{ijk}\phi^i\phi^j\phi^k \qquad \phi$ is complex

(We wrote the q=3 version)

Dynamical bosons + fermions

We will discuss the $\mathcal{N}=2$ one, the other is almost identical at the level of the large N equations.

We can find the large N equations for the two point functions

$$G_\phi(t,t') = \langle \phi^i(t)\phi^i(t')\rangle \ , \qquad G_\psi = \langle \psi^i\psi^i\rangle \ , \qquad G_F = \langle F^i F^i\rangle$$

Auxiliary fields

$$G_\phi(t,t') \to G_\phi(t-t') \ , \qquad \text{etc.}$$

The large N equations

Definitions of the self energies:

$$G_\phi(\omega)[\omega^2 - \Sigma_\phi(\omega)] = 1 \ , \qquad G_\psi(\omega)[-i\omega - \Sigma_\psi(\omega)] = 1 \ , \qquad G_F(\omega)[1 - \Sigma_F(\omega)] = 1$$

Self energies in terms of G (melon approximation) :

$$\Sigma_\phi = -2G_F G_\phi + 2G_\psi^2$$
$$\Sigma_\psi = 2G_\psi G_\phi$$
$$\Sigma_F = -G_\phi^2$$

Both sides are functions of (t,t'), or really t-t'.

(we specialized to q=3 and J=1 to avoid clutter)

Naïve low energy analysis

Set all functions to be power laws, $G_\phi \sim t^{-2\Delta}$, and similarly for the others.
Assume SUSY at short times.

$$G_\phi \propto \frac{1}{t^{2\Delta}} \ , \qquad G_\psi \propto \frac{1}{t^{2\Delta+1}} \ , \qquad G_F \propto \frac{1}{t^{2\Delta+2}}$$

Anninos, Anous, Denef

Insert in equations → Find $\Delta = 0$.

Not really a solution, some coefficients diverge as $\Delta \to 0$

These equations were considered by Lin, Shao, Wang, Yin, as an (uncontrolled) approximation to a different model, also inspired by BFSS.

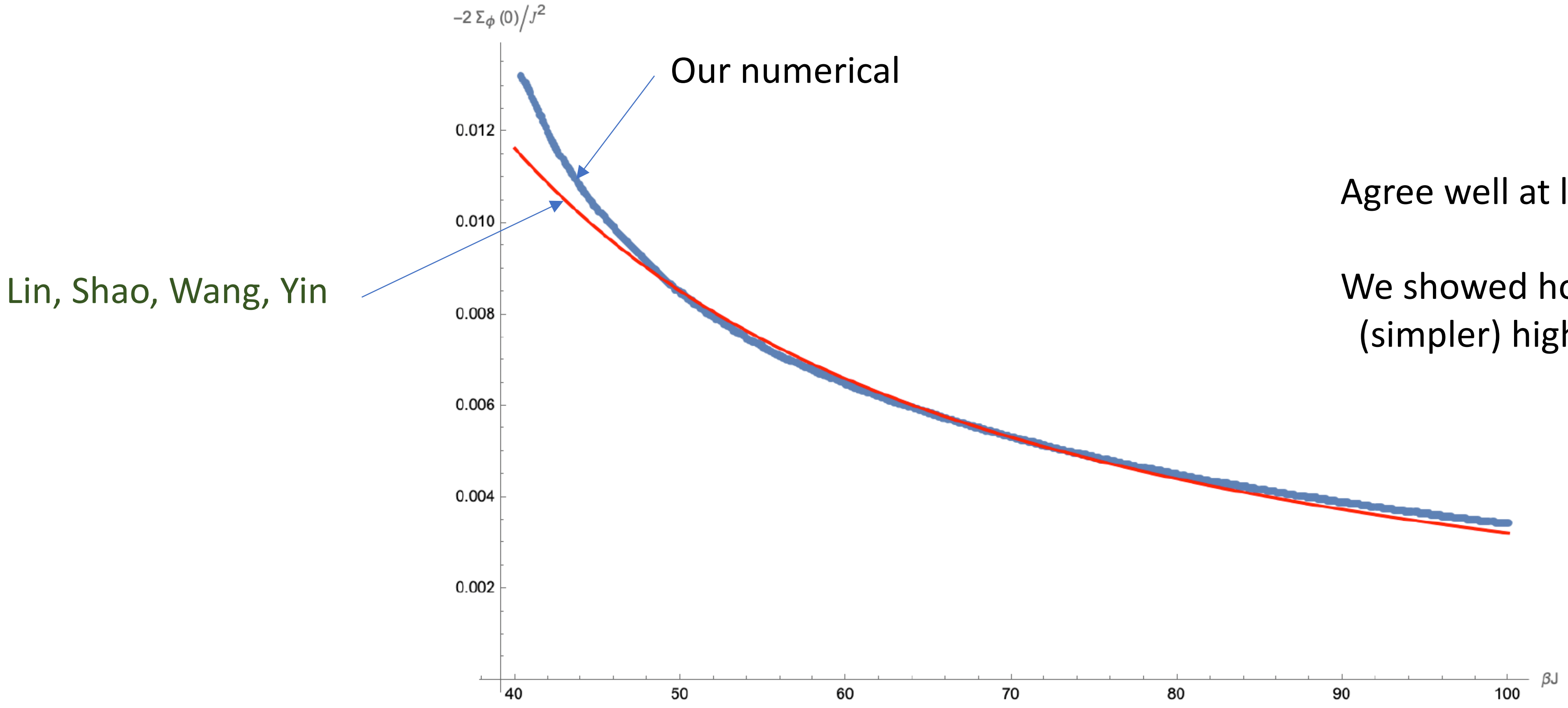
They solved them in a low temperature approximation, $\beta J \gg 1$. (non-trivial analysis)

Lin, Shao, Wang, Yin

They found:

$$\log Z = -\beta F \propto N T^{6/5}$$

We checked it vs a numerical solution



Agree well at low temperatures.

We showed how it interpolates to the (simpler) high temperature behavior.

We are trying to understand this model better.
Does it indeed display a scaling similarity?

As part of this process we developed a more systematic understanding of their approximation

The low temperature expansion again

$$G_\phi = \bar{G}_\phi + \delta G_\phi$$

Constant, independent of Euclidean time.
Much larger than the non-constant part.

Expand the effective action in this constant .

Expand the action

$$G_\phi = \bar{G}_\phi + \delta G_\phi$$

Constant

Expand the effective action in this constant .

$\bar{G}_\phi \gg \delta G_\phi$

$$G_f G_\phi^2 \to \delta G_f \delta G_\phi \bar{G}_\phi + \delta G_\phi^3 + \cdots$$

First term is quadratic

Solve exactly the quadratic terms

$$\frac{\log Z}{N} = \frac{1}{2}\log 2 + \frac{\pi}{\sqrt{8}\beta\bar{G}_{\phi}^{1/2}}$$

(for q=3, other values of q are similar)

Add the higher order terms perturbatively

$$\frac{\log Z}{N} = \frac{1}{2}\log 2 + \frac{\pi}{\sqrt{8}\beta \bar{G}_\phi^{1/2}} - \frac{1}{32\bar{G}_\phi^3}$$

Ground state entropy

Quadratic terms

Cubic terms (and quartic, for p>3)

Minimize with respect to $\bar{G}_\phi$ → determine $\bar{G}_\phi$ → find power $\log Z \propto T^{\frac{6}{5}}$

Scaling similarity

$$\frac{\log Z}{N} = \frac{1}{2}\log 2 + \underbrace{\frac{\pi}{\sqrt{8}\beta \bar{G}_\phi^{1/2}} - \frac{1}{32\bar{G}_\phi^3}}_{\text{Has a simple scaling similarity } \rightarrow \text{ extends to the approximate solution}}$$

Is the ground state entropy arising from BPS states?

The Witten index and the ground state entropy.

- Index for the $\mathcal{N}=2$ case: I =0 (we do not have an full argument, yet)

- Index for the $\mathcal{N}=4$ case: I = 2^N (or $(q-1)^N$ more generally)

- The large N analysis is essentially the same for the two cases. We should be compatible with the index in the second case.
- For the first maybe states are lifted at higher orders.

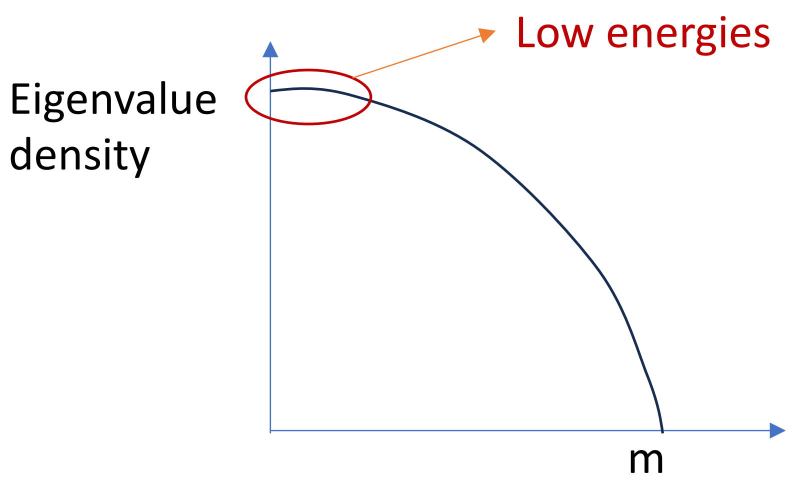
Interpretation

- It is puzzling that $\overline{\mathrm{G}}_{\phi}$ is growing without bound for small temperatures.

- For orientation we consider the q=2 case, which is exactly solvable.

q=2

- Harmonic oscillators with random mass matrix J_{ij} . Diagonalize it and get a distribution of masses.



Eigenvalue or frequency of the oscillator

At low energies, constant density → like modes of a 1+1 dimensional field.

Free energy $\log Z \propto TL,$ $L \propto N/m$ m = typical eigenvalue. 1/L ~ spacing between them

Free energy for q=2

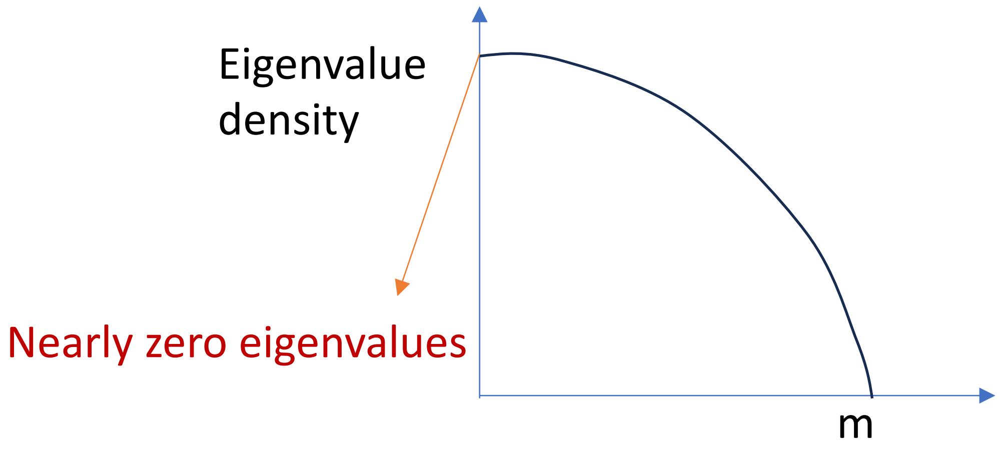
$$\frac{\log Z}{N} \propto \log\left[\frac{\beta\bar{G}_\phi}{1+\beta\bar{G}_\phi}\right] + \frac{\#}{\beta m}$$

We find that $\bar{G}_\phi$ goes to infinity!

What is going on?

Interpretation for q=2

At large N there are eigenvectors of the mass matrix with almost zero eigenvalue and we get infinite vevs for the those components.



Lowest eigenvalues $\propto \frac{1}{N}$

Bar = average over couplings.

It is even worse because when we average over couplings. We get: $\overline{\langle \phi^2 \rangle} = \infty$

The situation is a bit different for $q>2$

For q=3

$$\frac{\log Z}{N} \propto \frac{1}{2}\log 2 + \frac{\#}{\beta \bar{G}_\phi^{1/2}}$$

Expand $\quad \phi^i = \bar{\phi}^i_\alpha + \delta\phi^i \;, \qquad \bar{\phi}^i \gg \delta\phi^i$

Constant in time

Small fluctuations, time dependent

Expand superpotential $\quad W \sim C_{ij}(\bar{\phi}^i)\delta\phi^i\delta\phi^j \;, \qquad C_{ij} = J_{ijk}\bar{\phi}^k$

Also looks like random masses. However, the mass scale is set by the vev of $\bar{\phi}^i$, which is becoming large along a very flat direction of the potential.

For q=3

$$\frac{\log Z}{N} \propto \frac{1}{2}\log 2 + \frac{\#}{\beta \bar{G}_\phi^{1/2}}$$

Same as what we had for q=2, but with $m = \left(\bar{G}_\phi\right)^{\frac{1}{2}}$

Prefers small $\bar{G}_\phi$ → implies that now this will not go to infinity. The further correction stabilizes it.

$$\frac{\log Z}{N} = \frac{1}{2}\log 2 + \frac{\pi}{\sqrt{8}\beta \bar{G}_\phi^{1/2}} - \frac{1}{32\bar{G}_\phi^3}$$

Because the model is close to quadratic → we expect no chaos or black hole like behavior!

In addition…

Anninos, Anous and Denef suggested that this model could display spin glass behavior.

We are exploring this question

This involves solving the model with n replicas and then taking $n \rightarrow 0$

We can similarly make a large $\overline{G_\phi}$ approximation

$$\left.\frac{\log Z}{N}\right|_{\text{before}} = \frac{1}{2}\log 2 + \frac{\pi}{\sqrt{8}\beta \bar{G}_\phi^{1/2}} - \frac{1}{32\bar{G}_\phi^3}$$

Independent of $\beta, \overline{G_\phi}$

X_n parametrizes the (rescaled) off diagonal components of G^{ab}

$$G_\phi^{ab} = \bar{G}_\phi X^{ab}$$

(X^{ab} has ones in the diagonal)

$$\frac{\log Z}{N} = n(\text{Same as before}) + F(X_n)$$

Same solution for the dynamical part.

Possibly a different ground state entropy. But we have not reached a conclusion yet….

Conclusions

- We discussed the scaling similarity of the Dp brane gravity solutions.
- We explained how it can be used to organize the spectrum of fluctuations.
- We explained how to get the similarity exponent from a susy computation.

- We analyzed an SYK-like model that displays a scaling similarity at large N.
- We explained some features of the solution of this model.

- We are still working on a possible spin glass phase… (have not found it yet).