The entropy of Hawking radiation

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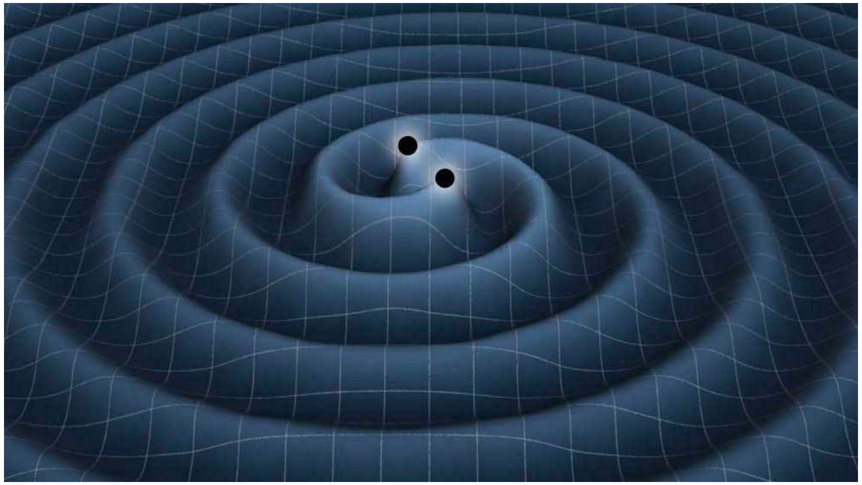
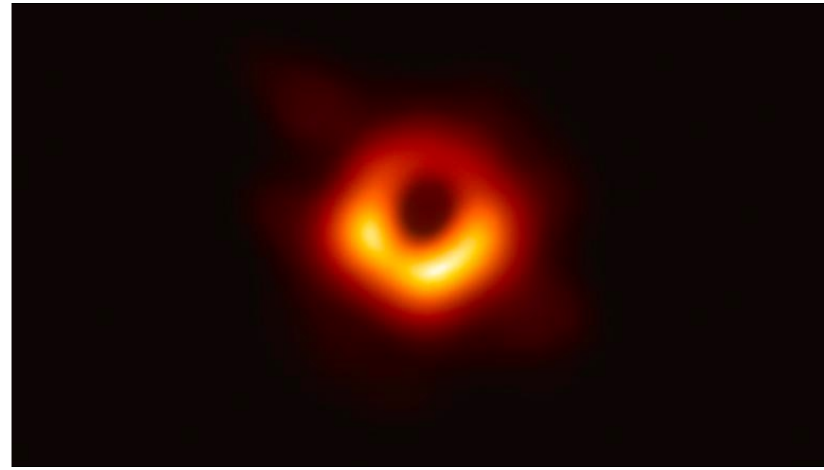
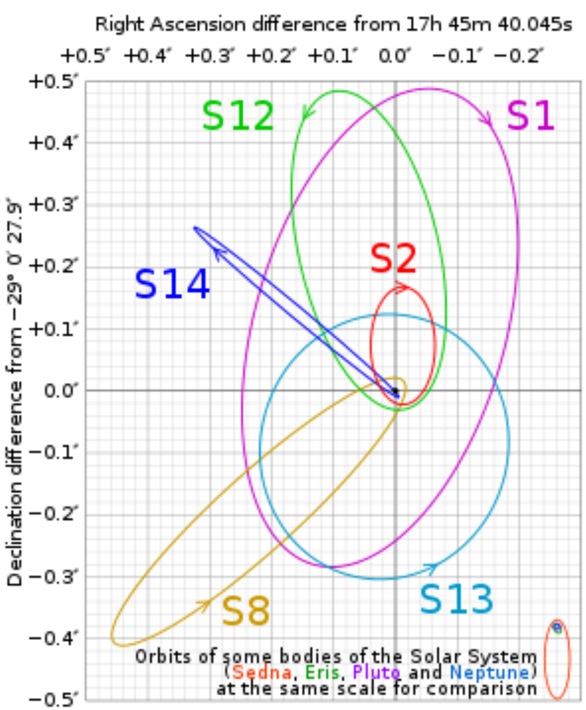
Black holes have been in the news







We will mainly talk about
<u>Quantum</u> aspects of black holes

We will discuss recent progress on the black hole information problem

Review: Almheiri, Hartman, JM, Shaghoulian, Tajdini
(this contains a more general list of references than this talk)

Two important papers in 2019: Penington
Almheiri, Engelhardt, Marolf, Maxfield

...many previous and follow up papers...

Another very interesting Development that I will not review → Saad, Shenker, Stanford

Outline

- Black hole entropy = area of horizon
- The fine grained gravitational entropy formula. Entropy = Minimal area
- Compute the entropy of radiation coming out of black holes.
- Get a result consistent with information conservation (as opposed to information loss).

This will not be historical, will hopefully be pedagogical…

Black hole from outside

Schwarzschild 1917

$$ds^2 = -(1 - \frac{r_s}{r})dt^2 + \frac{dr^2}{(1 - \frac{r_s}{r})} + r^2 d\Omega_2^2$$

$$r_s = 2G_N M/c^2$$

The most important result about quantum black holes:

Black holes are hot

- Black holes have a temperature.

Hawking

$$T = \frac{1}{4\pi r_s}$$

There can be white black holes!

Finite temperature and circles in Euclidean time

Thermal partition function for quantum systems:

$Z = Tr[e^{-\beta H}]$ = evolution in Euclidean time on a circle of length β

A theory on a Euclidean circle is related to a system is thermal equilibrium.

$$T = \frac{1}{\beta} = \frac{1}{\text{Length of Euclidean circle}}$$

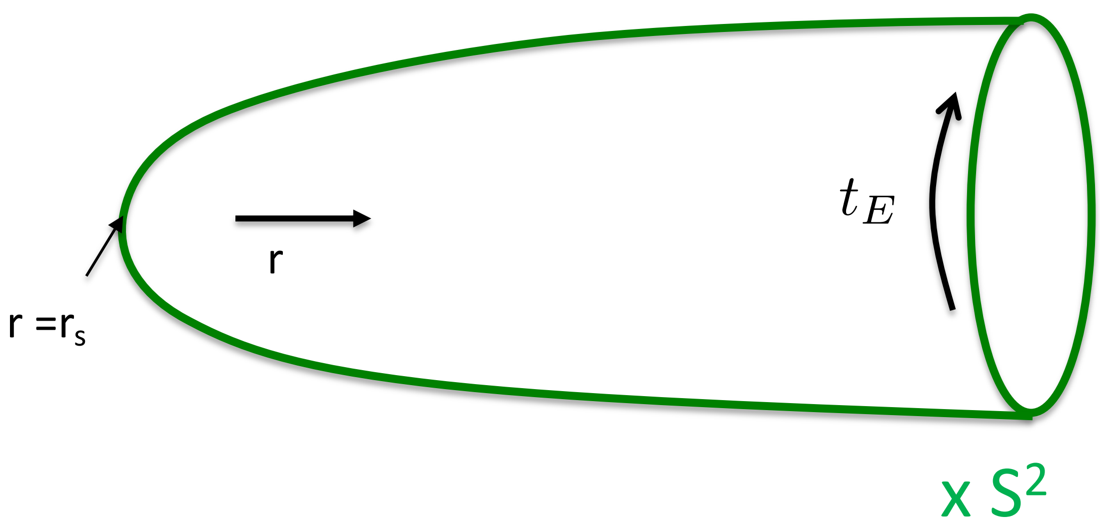
Euclidean black hole

$$ds^2 = -(1-\frac{r_s}{r})dt^2 + \frac{dr^2}{(1-\frac{r_s}{r})} + r^2 d\Omega_2^2$$

$$ds^2 = (1-\frac{r_s}{r})dt_E{}^2 + \frac{dr^2}{(1-\frac{r_s}{r})} + r^2 d\Omega_2^2$$

Hawking

$$t_E = t_E + \beta \ , \qquad \beta = 4\pi r_s ,$$ = inverse temperature far away



``cigar''

Gibbons
Hawking

Entropy

- Use first law:

$$dS = \frac{dE}{T} = \frac{dM}{T} \,, \qquad\qquad r_s = G_N M/c^2$$

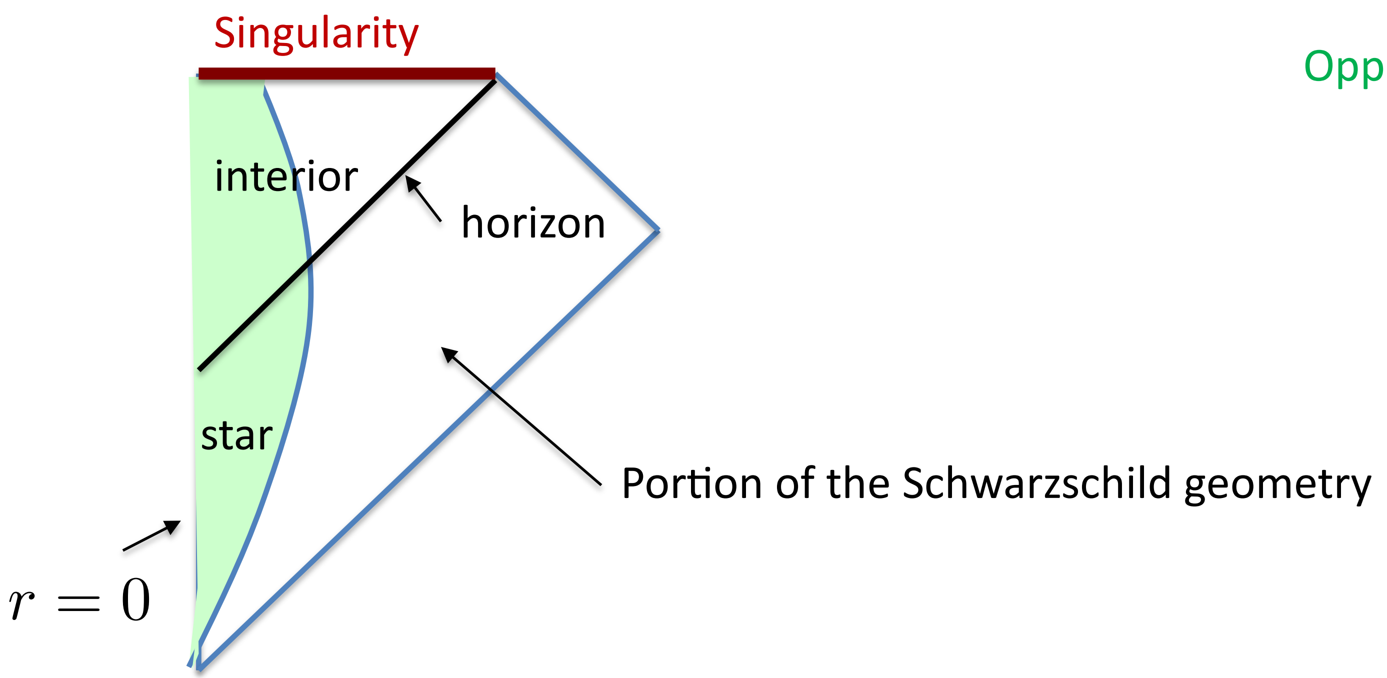
$$S = \frac{\text{Area}}{4G_N} = \frac{\text{Area}}{4l_p^2} = \frac{4\pi r_s^2}{4l_p^2}$$

A black hole is a thermodynamic object!

This is surprising…

Let us discuss in more detail the geometry of a black hole

Geometry of a Black Hole made from collapse

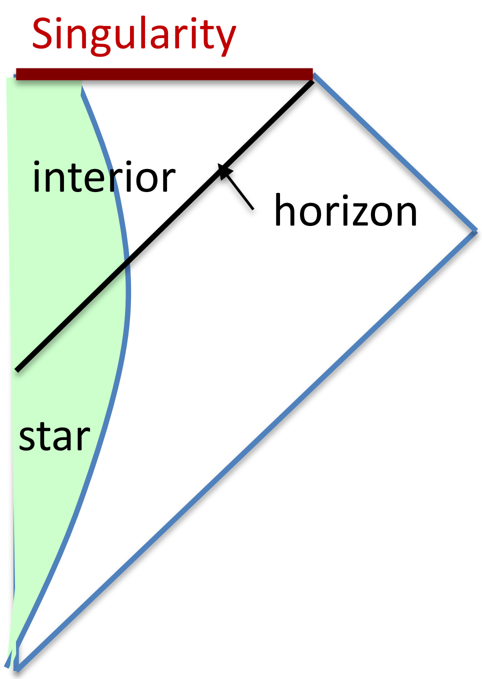


Oppenheimer Snyder 1939

Horizon Area law

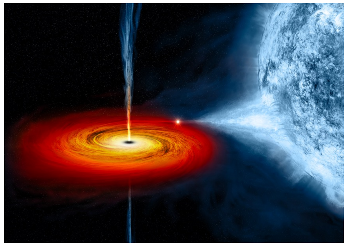
Area law: The area of a black hole horizon always increases. → 2^{nd} law of thermodynamics

Hawking



Starts with small area and it grows to larger area

Generalized entropy



$$S = \frac{\text{Area}_H}{4G_N} + S_{\text{matter}}$$

Bekenstein 70's

Question

- When a black hole emits Hawking radiation, it loses energy, so its area becomes smaller.
- What happens to the entropy ?

Question

- When a black hole emits Hawking radiation, it loses energy, so its area becomes smaller.
- What happens to the entropy ?

$$S = \frac{\text{Area}_H}{4G_N} + S_{\text{matter}} = \frac{\text{Area}_H}{4G_N} + S_{\text{QFT}}$$

Includes the entropy of quantum fields

Bombelli, Koul, Lee,Sorkin 1986

Obeys the 2nd Law

Wall 2010

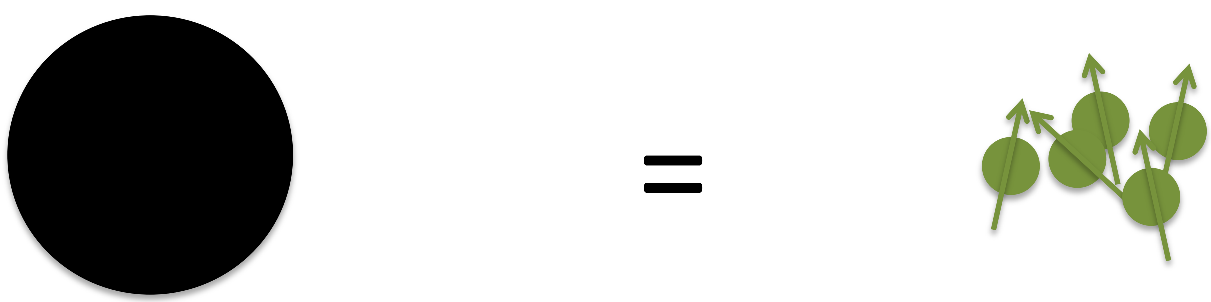
These results have inspired a very influential hypothesis:

A ``central dogma’’ in the study of quantum aspects of black holes

Black holes as quantum systems

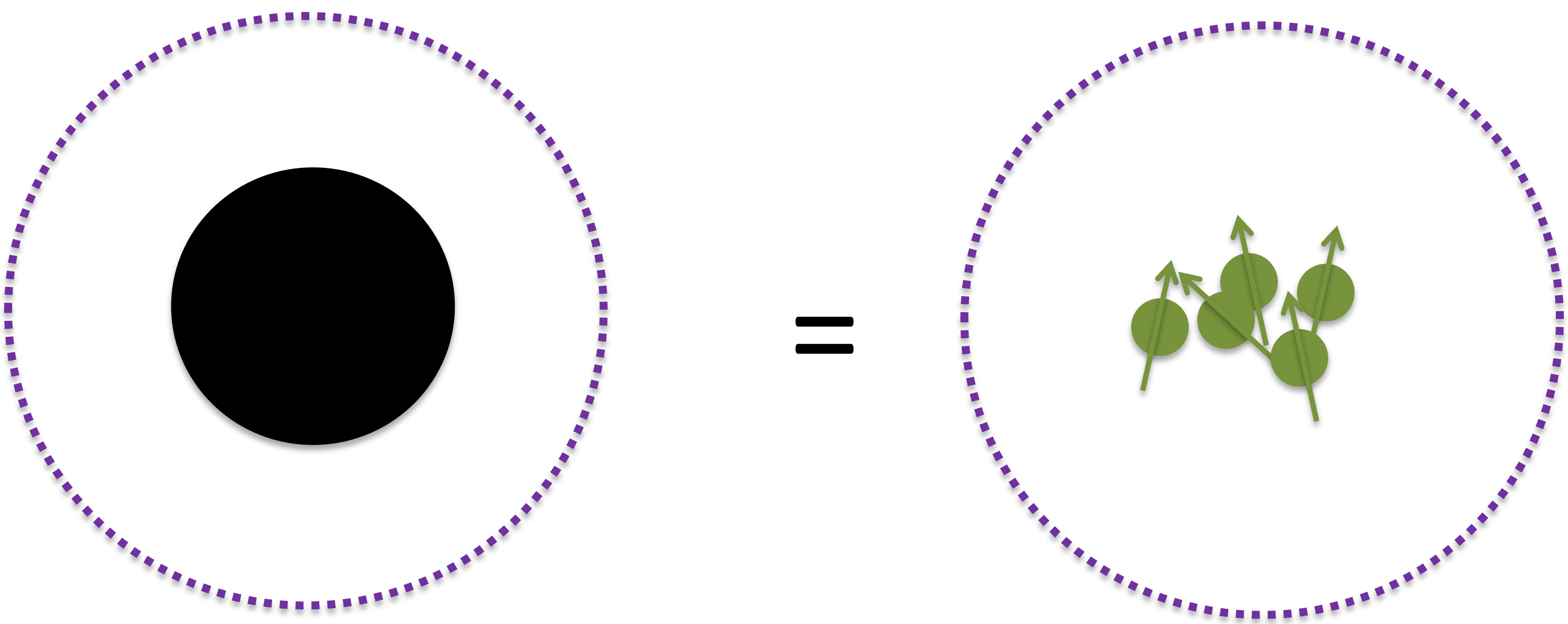
"Central dogma=central hypothesis"

- A black hole seen from the outside can be described as a quantum system with S degrees of freedom (qubits). S = Area/4 (l_p =1)
- It evolves according to unitary evolution, seen from outside.

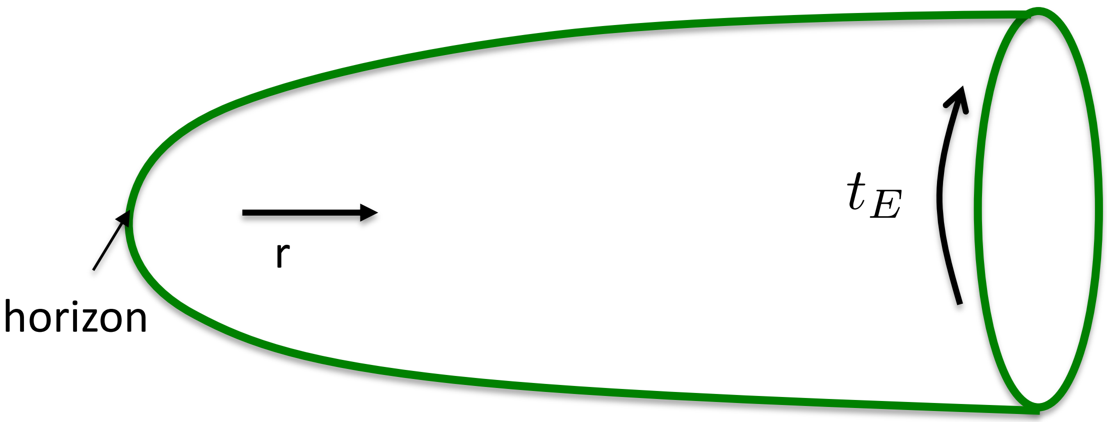


…in other words

- If one includes $A/4G_N$ "mysterious" qubits, then the black hole can be described as an ordinary quantum system.



Thermodynamics



``cigar''

Gibbons
Hawking

$$Z = Tr[e^{-\beta H}] \sim e^{-I_{grav}}$$

$$I_{grav} \propto -\frac{1}{G_N}\int \sqrt{g}R + \cdots$$

Tells us the answer but does not tell us what microstates we are counting

Evidence

1) Entropy counting

Special black holes, in special theories (supersymmetric) can be counted precisely using strings/D-branes → reproduce the Area formula. (+ also corrections to this formula)

Strominger Vafa

using results by

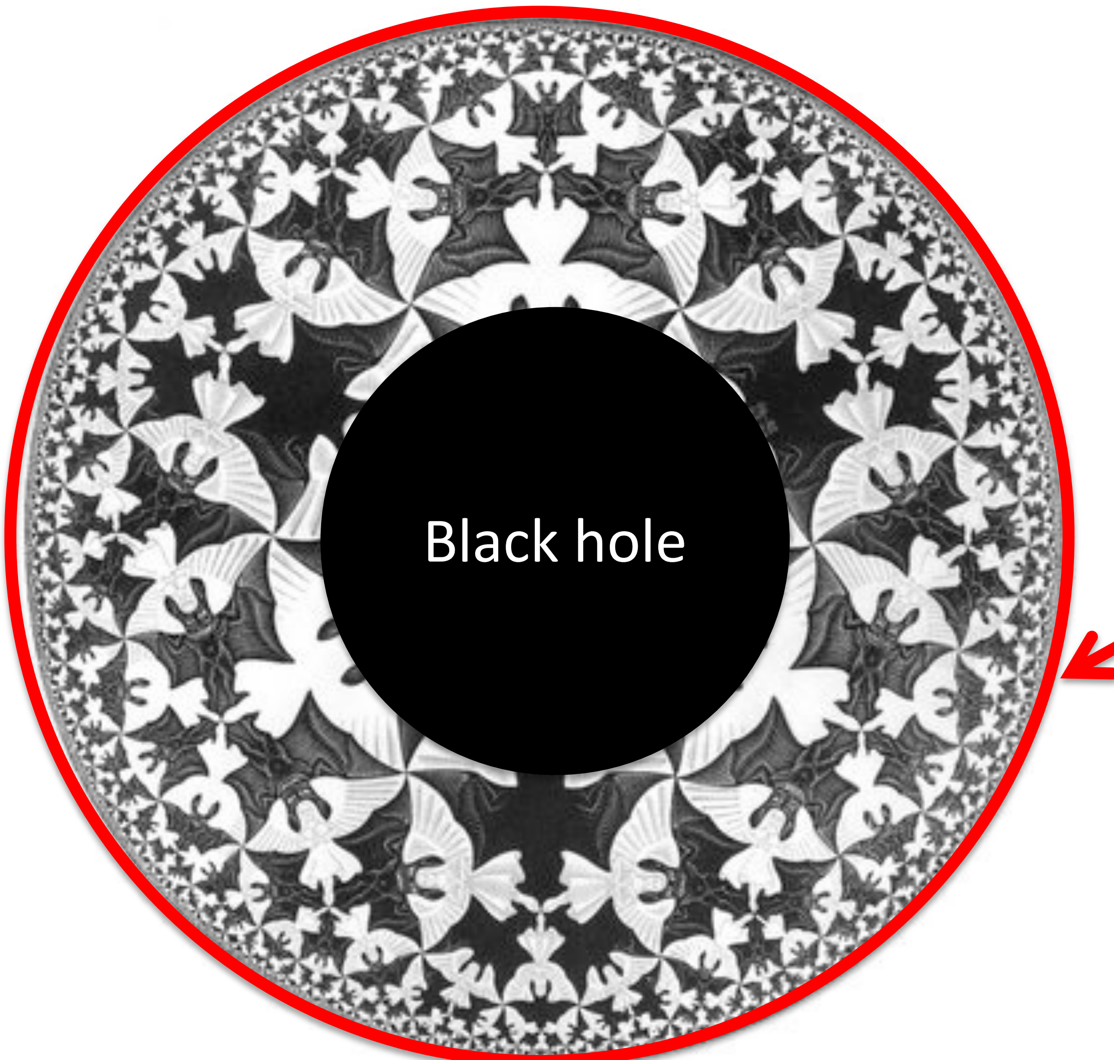
J. Polchinski

…

Sen

…

2) AdS/CFT...



Black hole in a box.
Evolving unitarity.

Hot fluid made out of
very strongly
interacting particles.

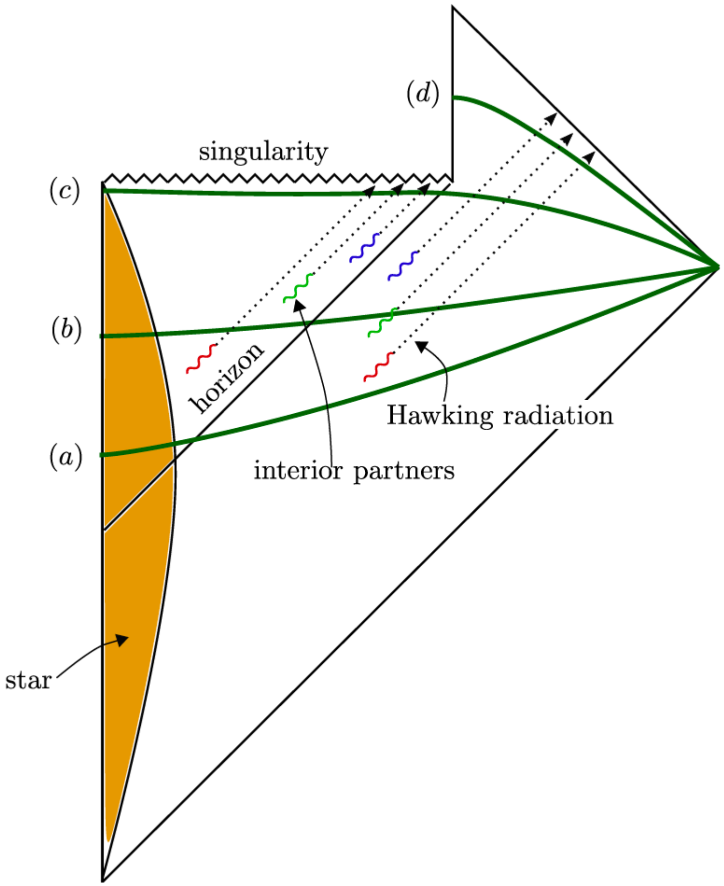
JM
Gubser, Klebanov, Polyakov
Witten

but

Hawking 1976 :

This can’t possibly be true!

Geometry of an evaporating black hole made from collapse

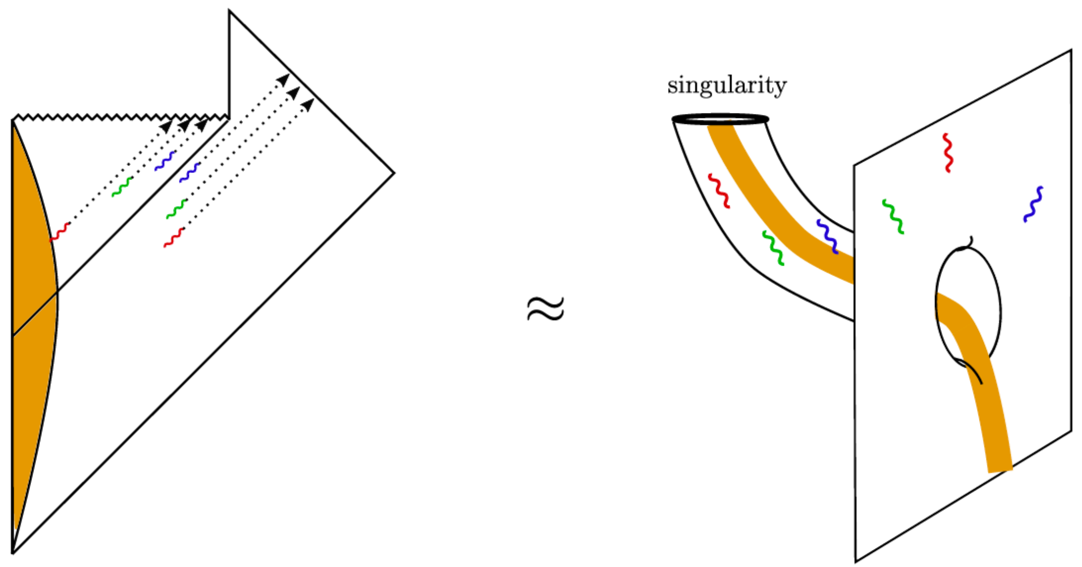


The radiation is entangled with partners of radiation.

Since we do not measure the interior we get a large entropy for the radiation.

A pure state seems to go a mixed state.

The skeptic's view:



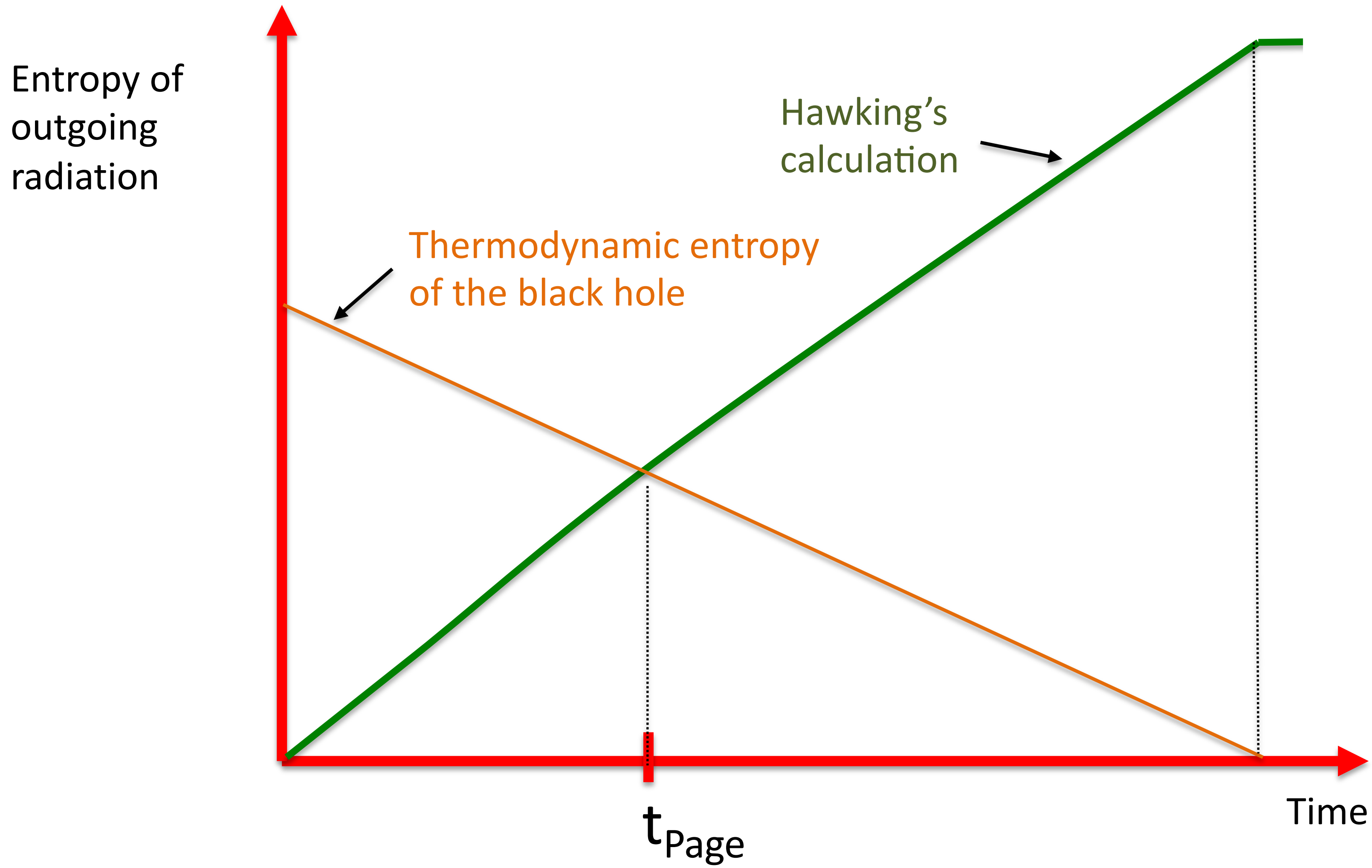
One universe splits of a ``teenage universe'' (big baby universe)

The state is pure if you include both universes, but not if you look only at the original universe.

A better statement of the problem

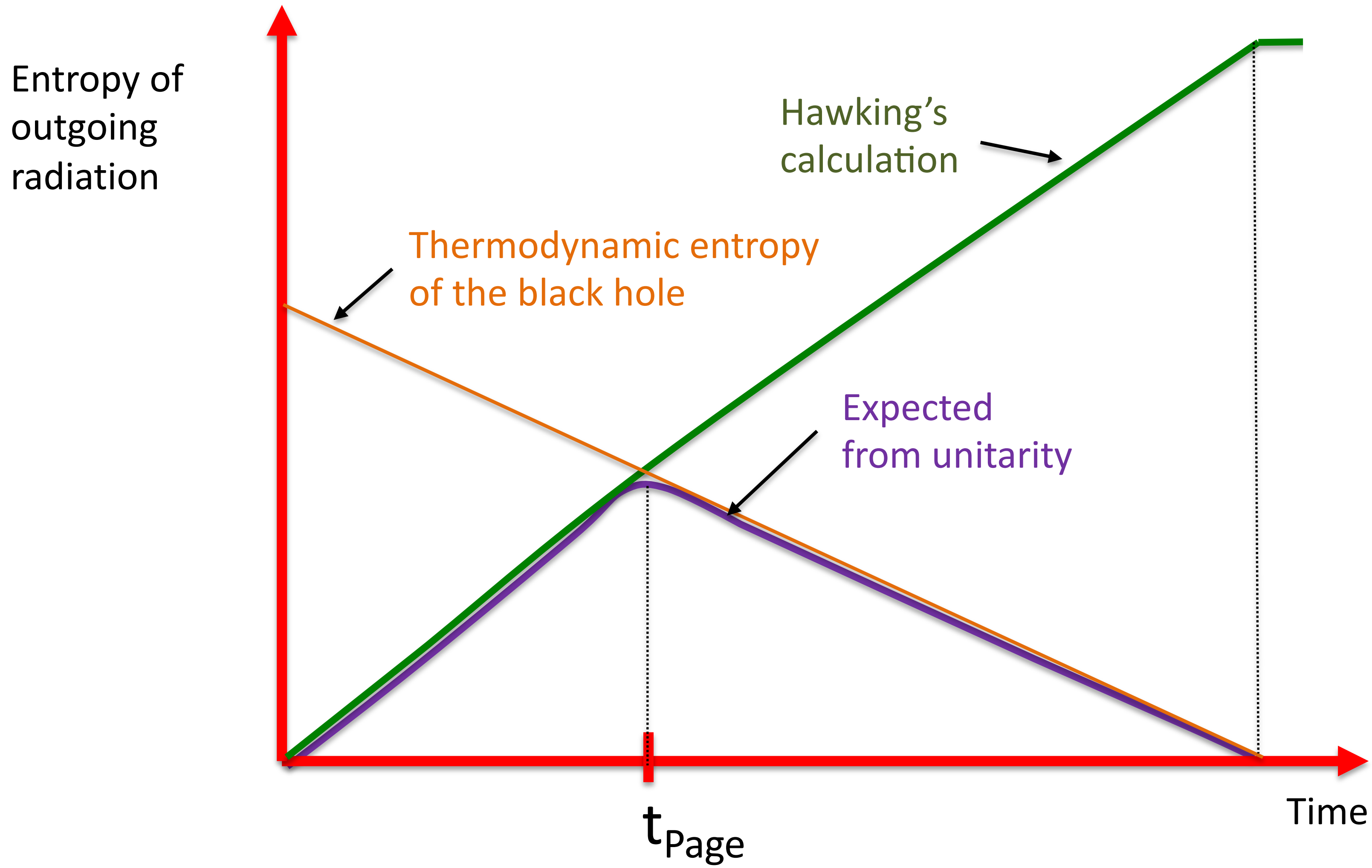
The Hawking curve

Compute the fine grained entropy of the radiation as it comes out of the black hole (formed by a pure state)



The Hawking curve vs. the Page curve

Compute the fine grained entropy of the radiation as it comes out of the black hole (formed by a pure state)



The problem involves understanding fine grained entropy

Two notions of entropy

- Fine grained entropy or von Neuman entropy. Remains constant under unitary time evolution.
(sometimes called ``entanglement'' entropy)

$$S = -Tr[\rho \log \rho]$$ ★

- Coarse grained entropy = thermodynamic entropy, or Boltzmann entropy. Obeys 2nd law. Arises from ``sloppiness''

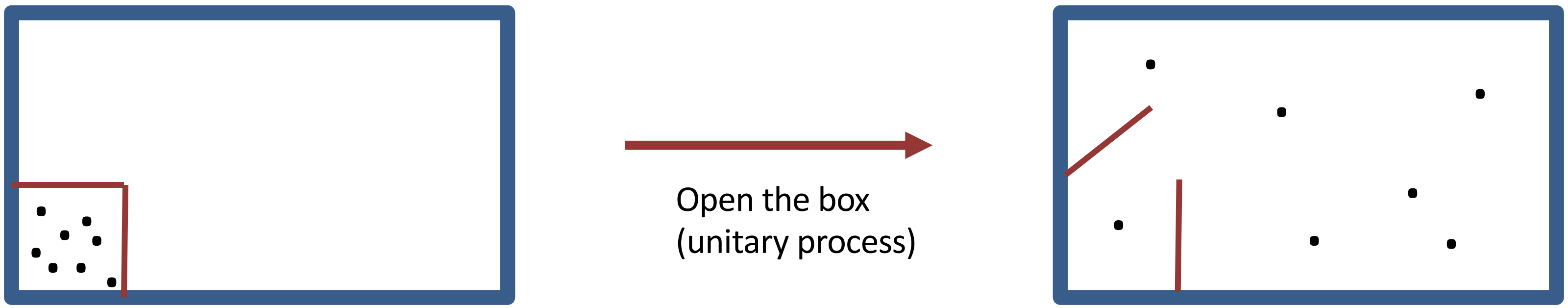
It is subtle to define it precisely. Here we are only mentioning to distinguish it from the star of our show. ★

$$S = \max_{\tilde{\rho}} \left(-Tr[\tilde{\rho} \log \tilde{\rho}] \right) \ , \qquad Tr[O\tilde{\rho}] = Tr[O\rho]$$

↑ Subset of observables, ``simple observables''

The difference in an example

Two kinds of entropy



$S_{in} = -Tr[\rho_{in} \log \rho_{in}]$ → $\rho_f = U^{-1} \rho_{in} U$

Von Neumann or fine grained entropy

Fine grained final entropy: $S_{fin} = -Tr[\rho_{fin} \log \rho_{fin}] = \mathrm{S_{in}}$

Coarse grained, thermodynamic or Boltzmann entropy: $S_{Thermodynamic} > \mathrm{S_{in}}$

For the moment we will be talking about the entropy of the black hole as seen from the outside.

This is the entropy of the quantum system that appeared in our "central hypothesis"

The horizon area computes thermodynamic entropy

How can we compute the fine grained one ?

Fine grained gravitational entropy

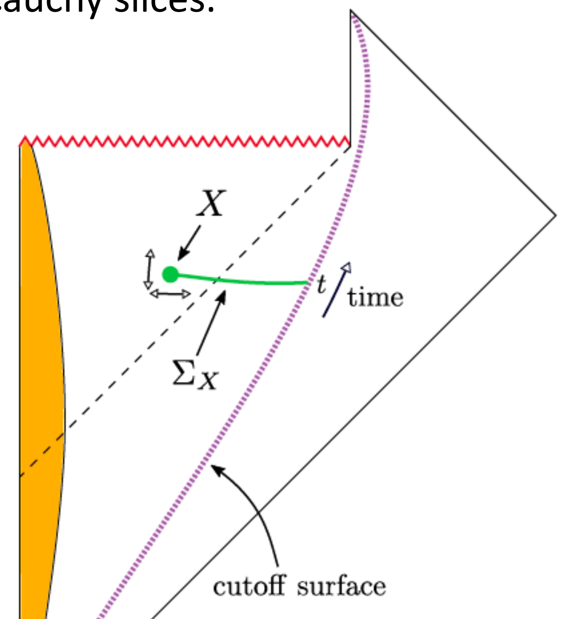
Ryu-Takayanagi 2006
Hubeny, Rangamani, Takayanagi 2007
Faulkner, Lewkowycz, JM 2013
Engelhardt, Wall 2014

$$S = \min_X \left\{ \text{ext}_X \left[\frac{\text{Area(X)}}{4G_N} + S_{\text{semi-cl}}(\Sigma) \right] \right\}$$

The final surface is called minimal quantum extremal surface.

Also maxi-min: minimize along a spatial slice (Cauchy slice) and then maximize among all possible Cauchy slices.

Wall; Akers, Engelhardt, Penington, Usatyuk.



We are allowed to take the surface to the inside. It depends on the geometry of the interior

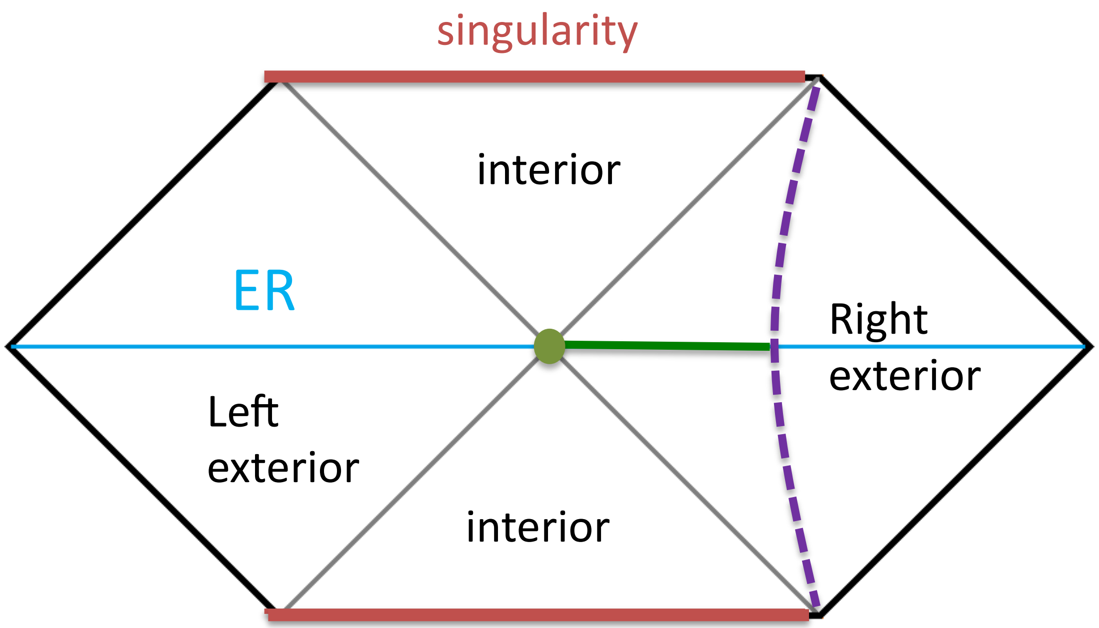
We will discuss its derivation later

For now we will just use it

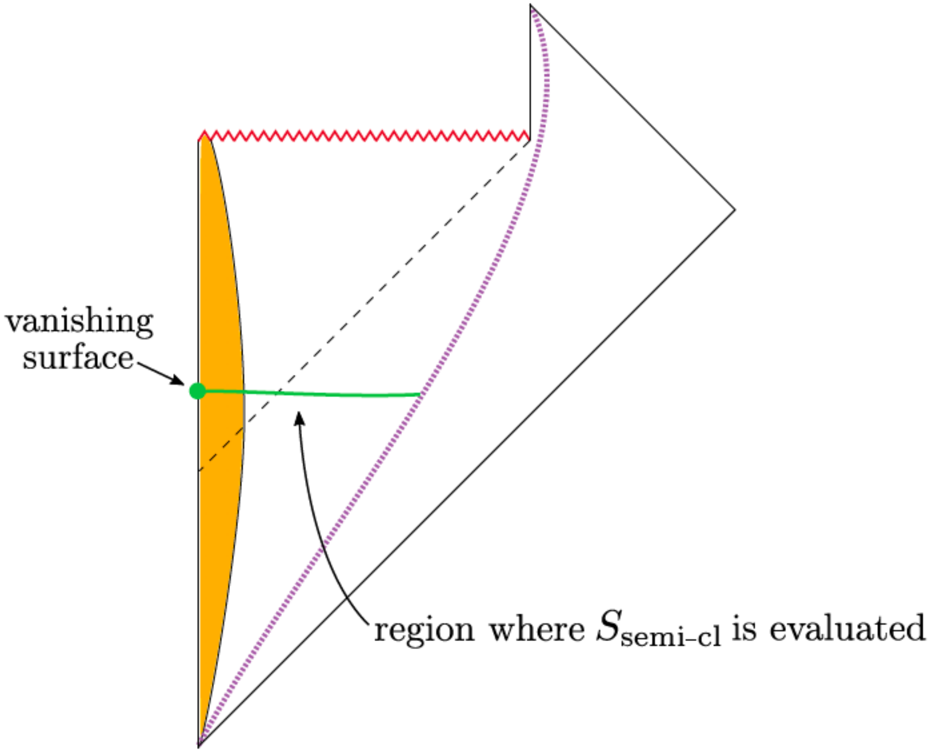
We should be surprised by the claim that there is a formula for the fine-grained entropy

Examples

Full Schwarzschild solution = Two entangled black holes



Vanishing surface

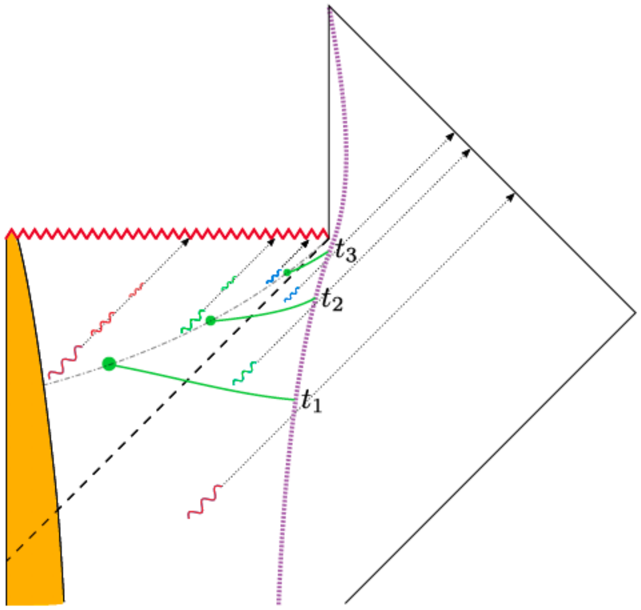


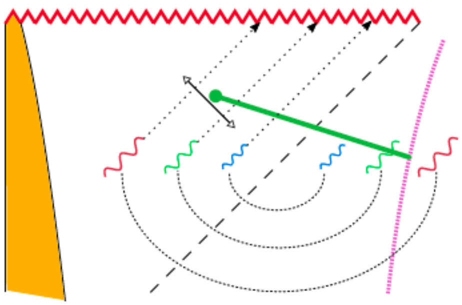
S = Entropy of star , which is zero if it was in a pure state.

New quantum extremal surface

Penington
Almheiri, Engelhardt, Marolf, Maxfield, 2019

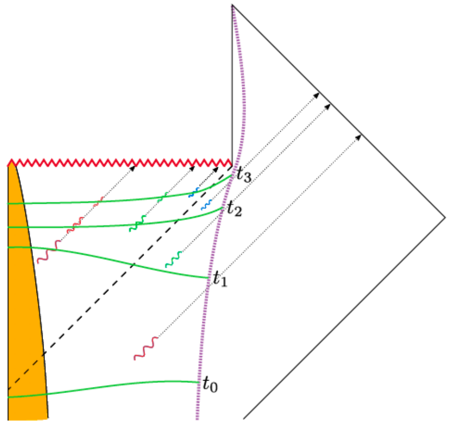
First appears about a scrambling time (r_s log S) after the black hole forms.

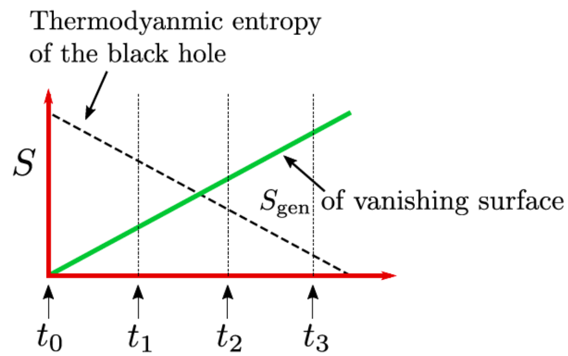


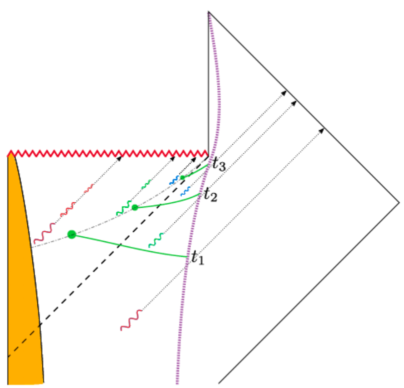


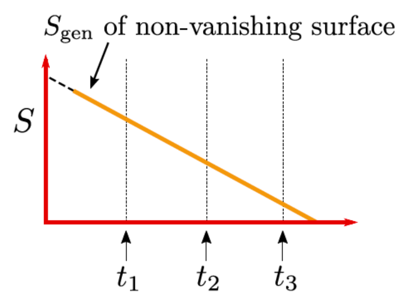
Its entropy is close to the area of the horizon at the time.

Two quantum extremal surfaces



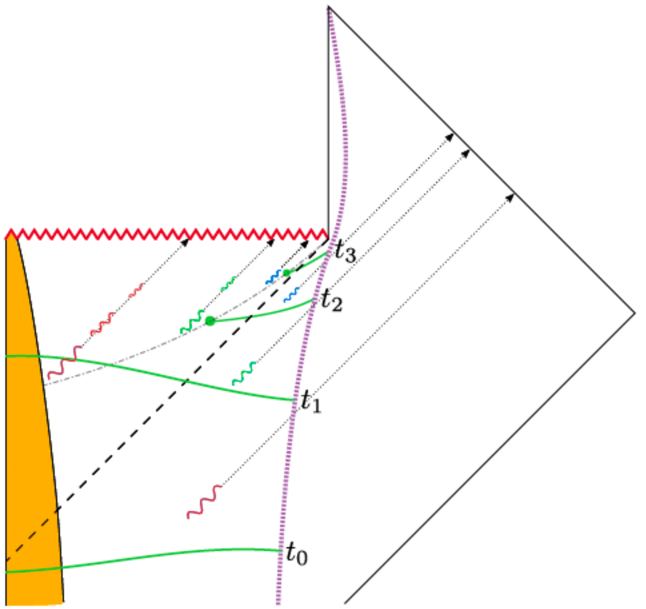


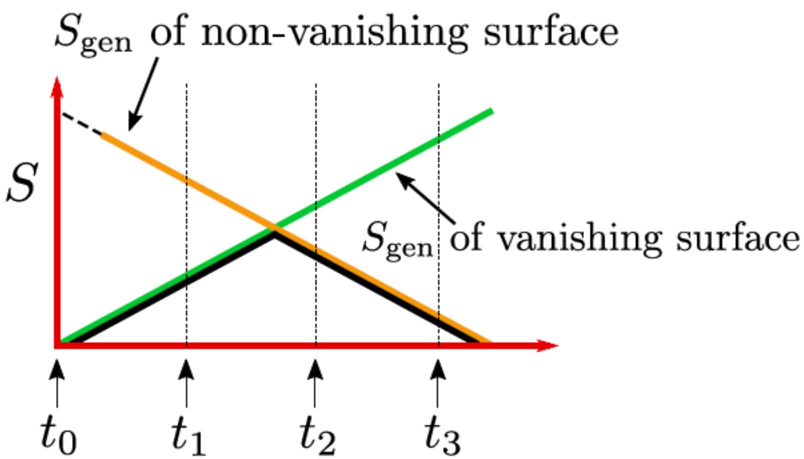




Penington
Almheiri, Engelhardt, Marolf, Maxfield

Choose the minimal one





Penington
Almheiri, Engelhardt, Marolf, Maxfield

…we get the Page curve for the black hole

but

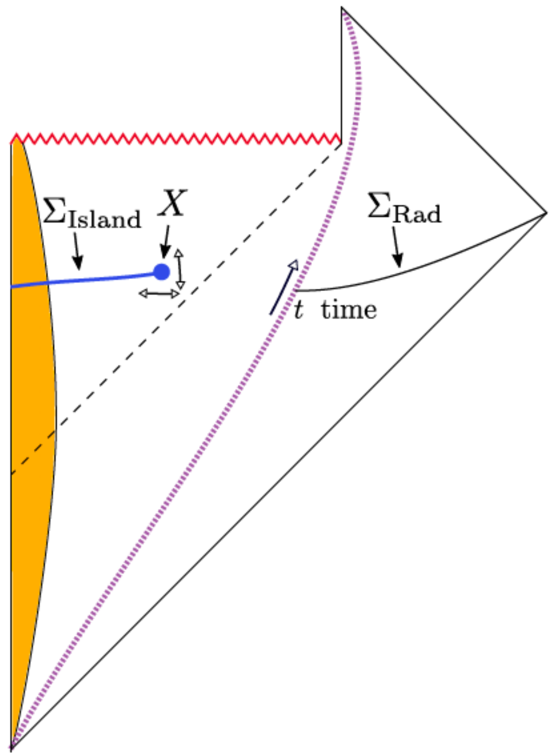
we really wanted the Page curve for the radiation!

- The radiation lives in a region where quantum gravity effects could be very small. (It could have left the AdS space, or it could be collected into a far away quantum computer).

- Since we obtained the state using gravity → we should apply the gravitational fine-grained entropy formula!

Penington; Almheiri, Mahajan, JM, Zhao

The "Island" formula

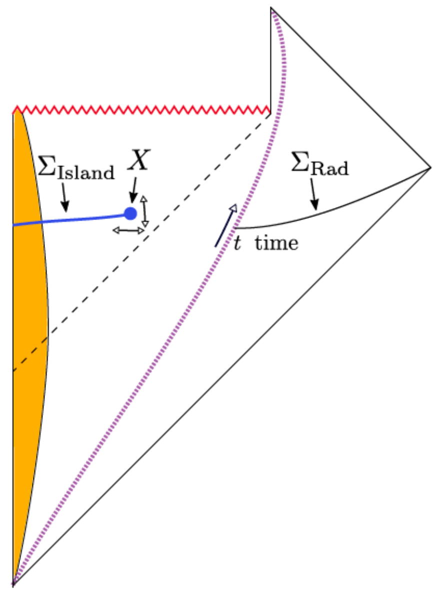


$$S_{\rm Rad} = \min_X \left\{ {\rm ext}_X \left[\frac{{\rm Area}(X)}{4G_N} + S_{\text{semi-cl}}[\Sigma_{\rm Rad} \cup \Sigma_{\rm Island}] \right] \right\}$$

We should view it as just a special case of the general gravitational fine-grained entropy formula

Penington; Almheiri, Mahajan, JM, Zhao

The "Island" formula

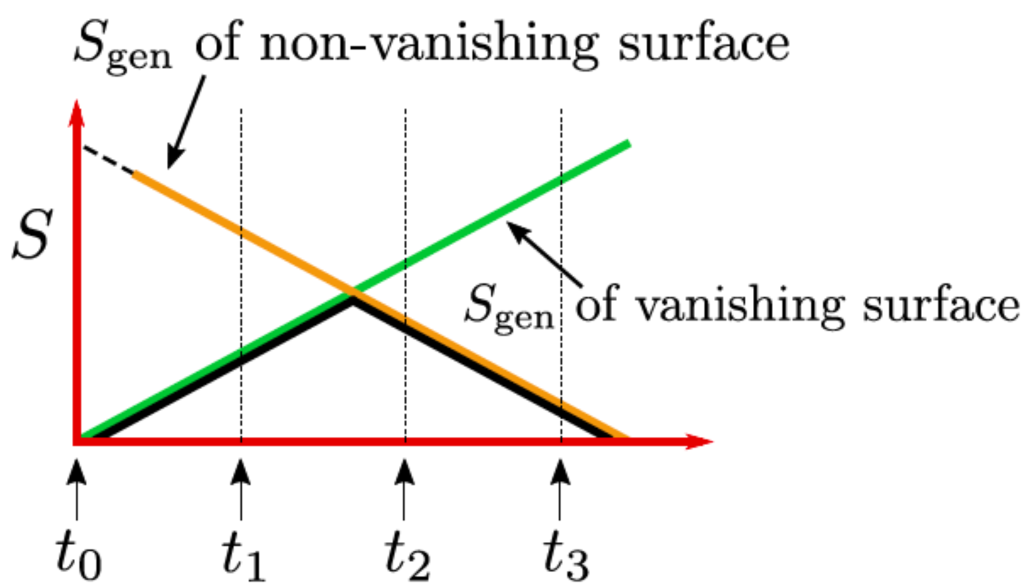


$$S_{\text{Rad}} = \min_X \left\{ \text{ext}_X \left[\frac{\text{Area}(X)}{4G_N} + S_{\text{semi-cl}}[\Sigma_{\text{Rad}} \cup \Sigma_{\text{Island}}] \right] \right\}$$

Entropy of the exact radiation state

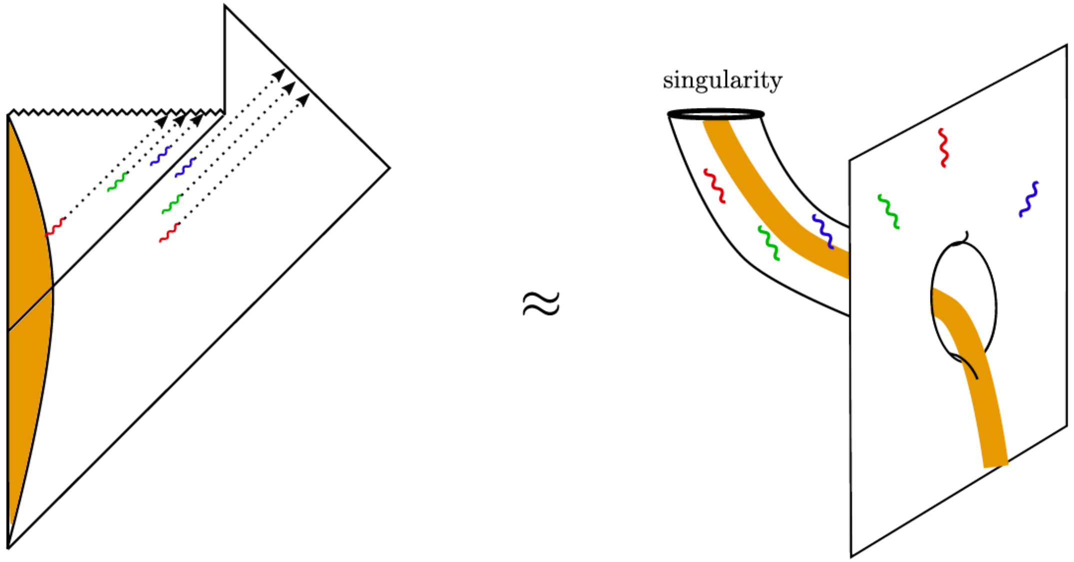
Entropy of the state in the semiclassical description

- If the initial matter state is pure, then the quantum extremal surfaces are the same as the ones we discussed before.
- Therefore we get the Page curve for radiation.



The skeptic's complaint

- "This is just an accounting trick!"
- I have always said: "If you include the black hole interior, then the state is pure. The information problem arises because you do not have access to the interior!"



Gravity's accounting ``trick'' → ``oracle''

- It can be derived from the gravitational path integral. (Deriving the gravitational fine grained entropy formula)

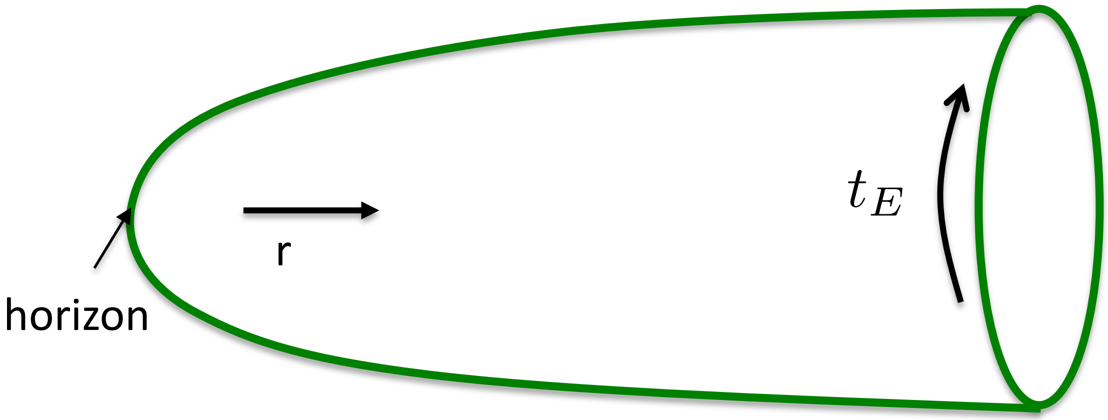
- Oracle: It gives us the true fine-grained entropy of the <u>exact</u> state, but using only the semiclassical state.

Deriving the fine grained entropy formula

- It is conceptually similar to the derivation of the black hole entropy using the Euclidean black hole.

Lewkowycz, JM ;
Faulkner, Lewkowycz, JM;
Dong, Lewkowycz, Rangamani;
Dong, Lewkowycz

Euclidean black hole



``cigar''

Gibbons
Hawking

$$\mathrm{Tr}[e^{-\beta H}] = Z_{\text{grav}} \sim e^{-I_{grav}} Z_{\text{semi-cl}}$$

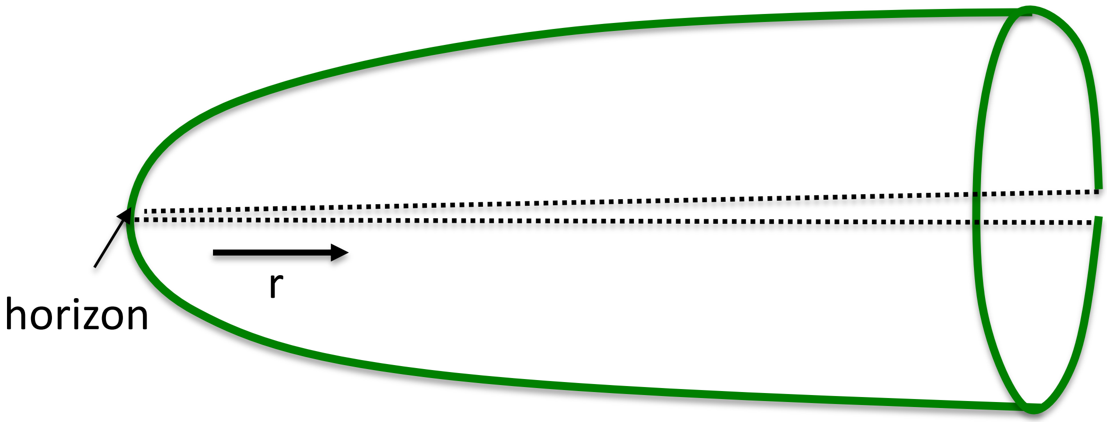
interpretation

$$I_{grav} \propto -\frac{1}{G_N} \int \sqrt{g} R + \cdots$$

$$S = (1 - \beta \partial_\beta) \log Z = \frac{\text{Area}}{4G_N} + S_{\text{semi-cl}}$$

Fix the temperature far away, gravity chooses the geometry dynamically.

Euclidean black hole



``cigar''

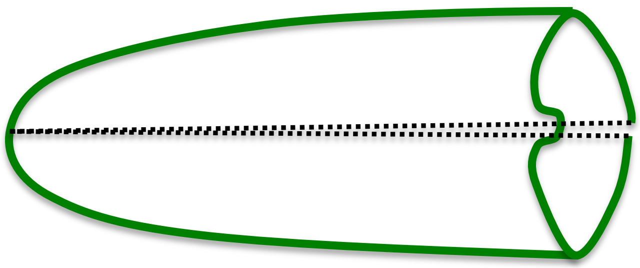
$$\tilde{\rho} \propto \rho$$

Gibbons
Hawking

$$S = (1 - n\partial_n) \log \mathrm{Tr}[\tilde{\rho}^n]|_{n=1} = (1 - n\partial_n) \log Z_n|_{n=1}$$

Computes the entropy if we can only compute the traces, but the actual density matrix itself

Other states prepared by a path integral.



$$\tilde{\rho} \propto \rho$$

Lewkowycz, JM
Faulkner, Lewkowycz, JM
Dong, Lewkowycz, Rangamani
Dong, Lewkowycz

Z$_3$ fixed point

$$\underbrace{\mathrm{Tr}[\tilde{\rho}^3]}_{\text{interpretation}}$$

Z_3 symmetry

$$S = (1 - n\partial_n)\mathrm{Tr}[\tilde{\rho}^n]|_{n=1} = (1 - n\partial_n)\log Z_n|_{n=1}$$

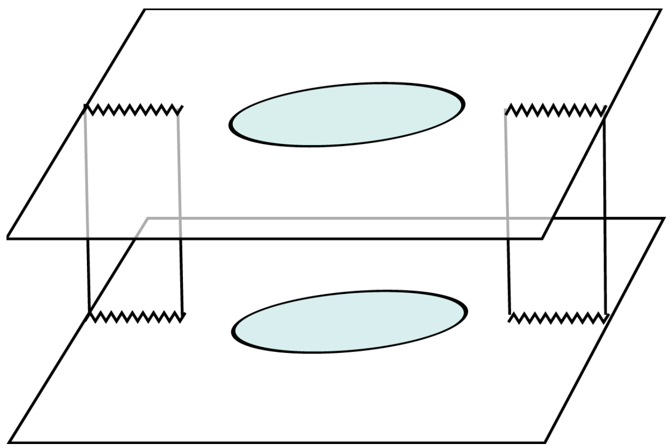
$$S = \min\left\{\mathrm{ext}\left[\frac{\mathrm{Area(X)}}{4G_N} + S_{\mathrm{semi-cl}}(\Sigma)\right]\right\}$$ = fine grained entropy formula

- In the same way that the Euclidean black hole gives us the entropy, this replica trick gives us the gravitational fine-grained entropy formula.

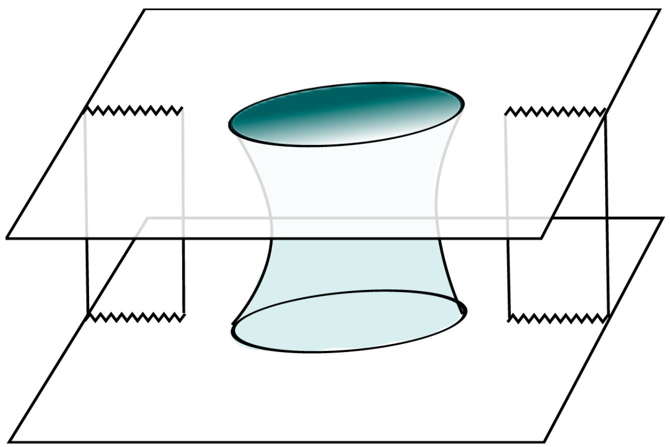
- If the state is prepared by a Euclidean path integral, and it has dynamical gravity only in some regions, then we should allow various topologies in that region.
- Interiors connected by ``replica wormholes'' → island formula.

Penington, Shenker, Stanford, Yang
Almheiri, Hartman, JM, Shaghoulian, Tajdini

Replica wormholes: n=2



Solution that gives Hawking's result



Replica wormhole, giving the Page answer when it dominates at late times.

- $Tr[\tilde{\rho}^{\mathrm{n}}]$ is given by these other non-trivial geometries.
- But the von Neuman entropy is given by a computation in the original semiclassical geometry → the fine grained entropy formula that we have seen above

So far, we only talked about entropy.

What is this telling us about the interior?

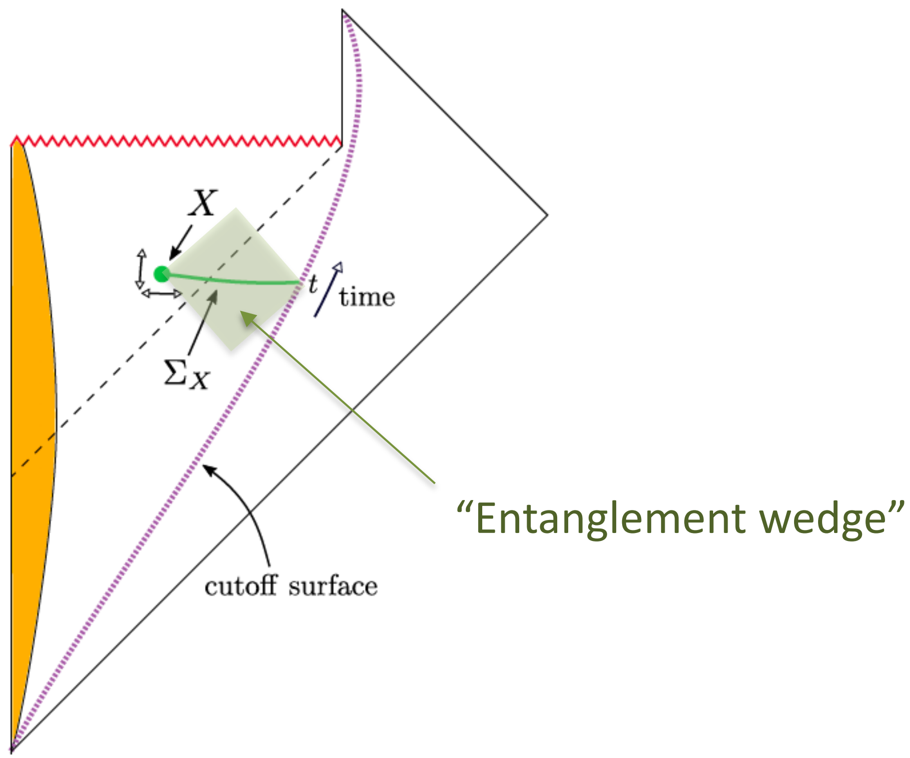
How do we describe the interior?

- The central dogma involves degrees of freedom that describe the black hole from the outside.

- What part of the interior do these degrees of freedom describe?
  - 1) All of the interior?
  - 2) None of the interior?
  - 3) Part of the interior? Which part?

Region described by the black hole degrees of freedom

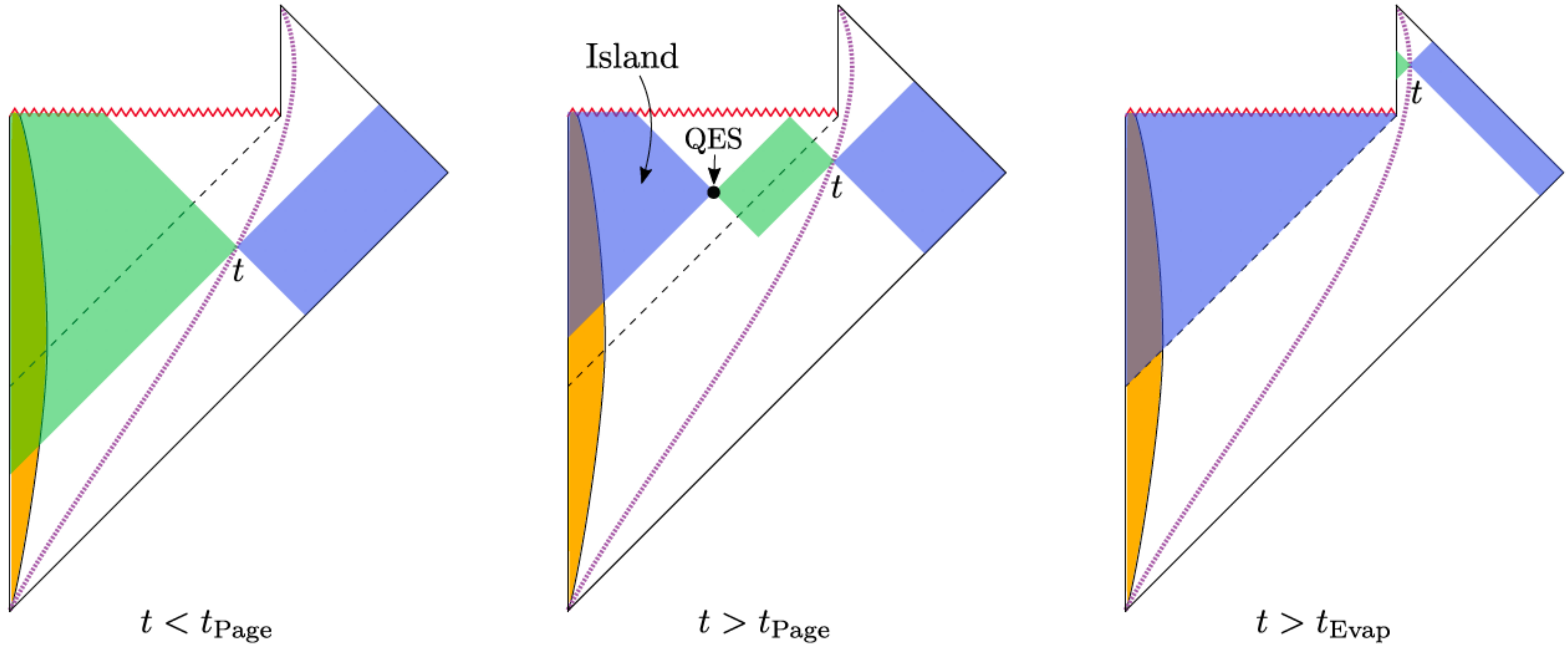
- Region that appears in the computation of the entropy.



Czech, Karczmarek, Nogueira, Van Raamsdonk, Wall, Headrick, Hubeny, Lawrence, Rangamani, Almheiri, Dong, Harlow, Jafferis, Lewkowycz, J.M., Suh, Wall, Faulkner….

Examples

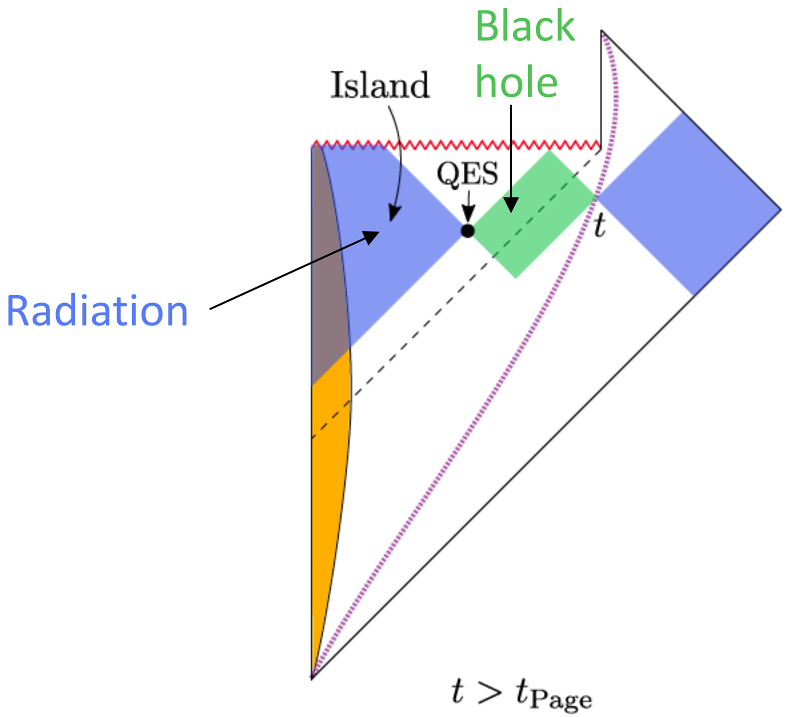
Entanglement wedge of the black hole in green. (black hole = quantum degrees of freedom describe the black hole from the outside)



Entanglement wedge of radiation in blue.
(At late times, it includes part of the black hole interior)

Describing the interior

Part of the interior belongs to the black hole degrees of freedom and another part of the interior to the radiation



- By performing (a very complicated) quantum operation on the radiation → we can extract information from the interior.
- We do not have a clear Lorentzian picture of how that happens.
- Speculation: the quantum computation that we need to do on the radiation creates is own spacetime, which connects through a wormhole to the interior.

Gao Jafferis Wall
Petz map: Almheiri, Dong, Harlow; Cotler, Hayden, Penington, Salton, Swingle, Walter; Chen, Penington, Salton; Penington, Stanford, Shenker, Yang

Jafferis, Lewkowycz, JM, Suh; Almheiri, Anous, Lewkowycz;…, Y. Chen

Conclusions

- We reviewed the gravitational fine-grained entropy formula.
- We applied it to the computation of the entropy of radiation and obtained results consistent with unitarity.
- At late times, most of the interior is part of the radiation. It is not part of the ``black hole degrees of freedom''.

What was Hawking's mistake?

- Not to use the fine-grained entropy formula.

(It was not known at the time)

- A lot of what we discussed was derived by thinking about aspects of AdS/CFT, which itself involves string theory.
- But you only need gravity as an effective theory to apply these formulas.

There is an amazingly deep connection between gravity and quantum mechanics!

Is the information puzzle solved?

- One aspect: computing the entropy, yes.
- Another aspect: Understanding what the state is, no.
- We can compute the entropy of the radiation but not its state (at least with the present understanding).
- As with black hole entropy, it is an accounting ``oracle''. The explicit gravitational representation of the states is still mysterious. But the semiclassical solution is representing some aspects of the state.

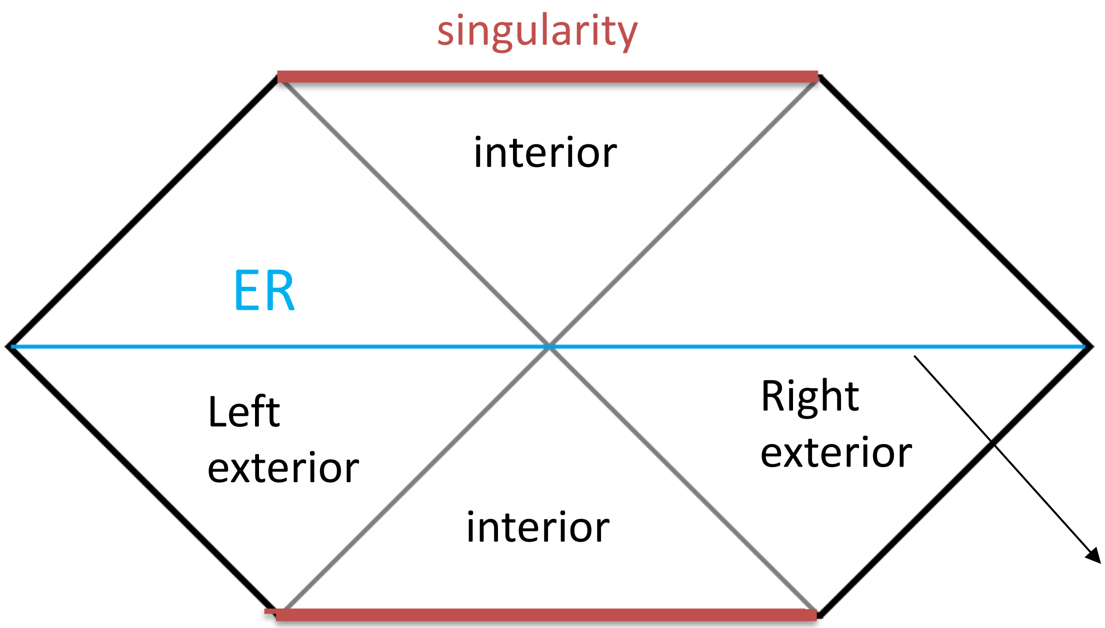
Future

- What further lessons is this teaching us about the interior? The singularity ?
- Implications for cosmology ?

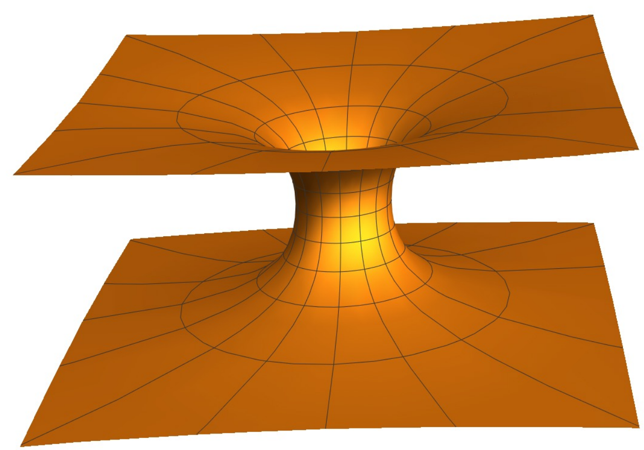
Thank you !

Extra slides

Full Schwarzschild solution



Eddington, Lemaitre, Einstein,
Rosen, Finkelstein,
Kruskal

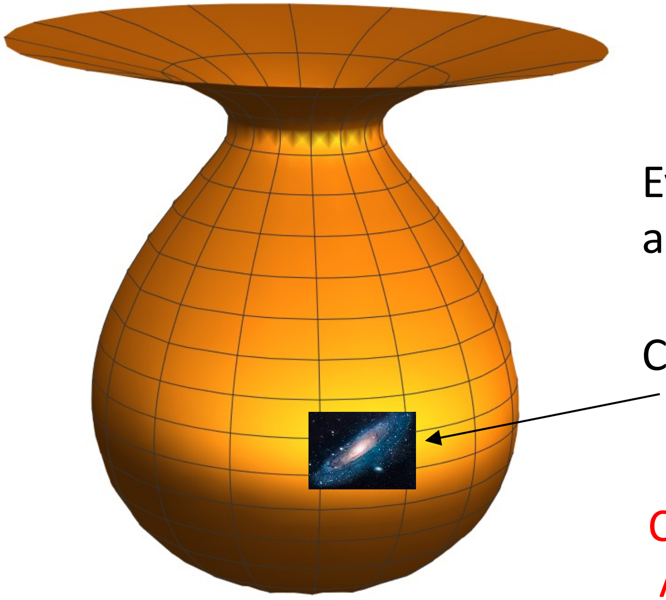


Vacuum solution. No matter.
Two exteriors, sharing the interior.

``Bags of Gold"

Wheeler

Initial slice:



Evolves to a black hole as seen from the outside and a black hole in a closed universe.

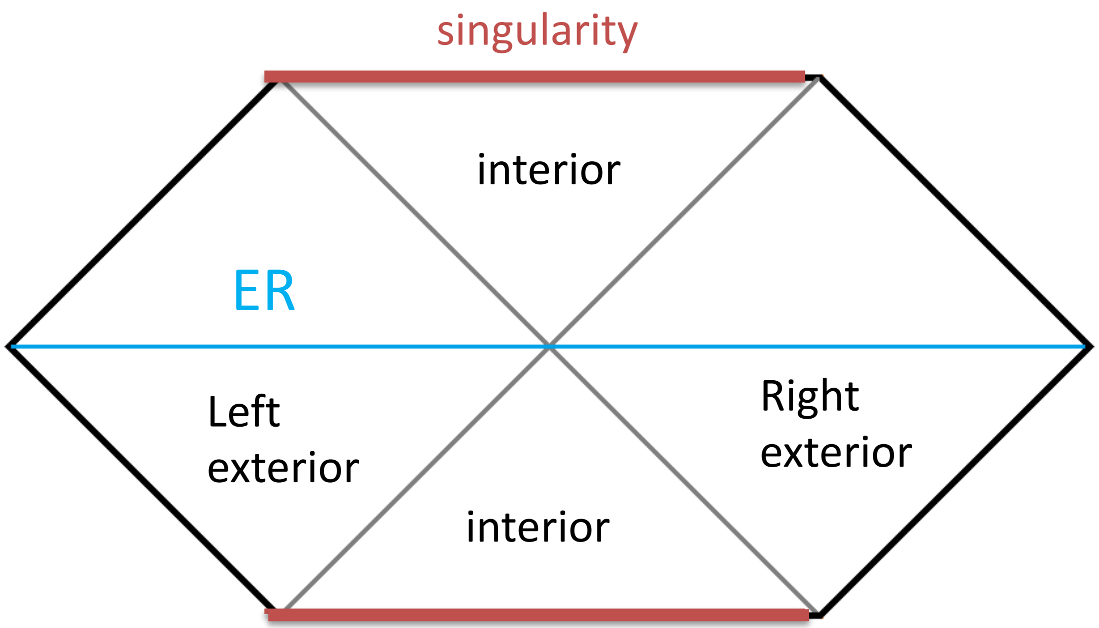
Can have arbitrarily large amount of entropy ``inside''

Counterexample to the statement that the Area entropy counts the entropy "inside" or the entropy of the ``interior''.

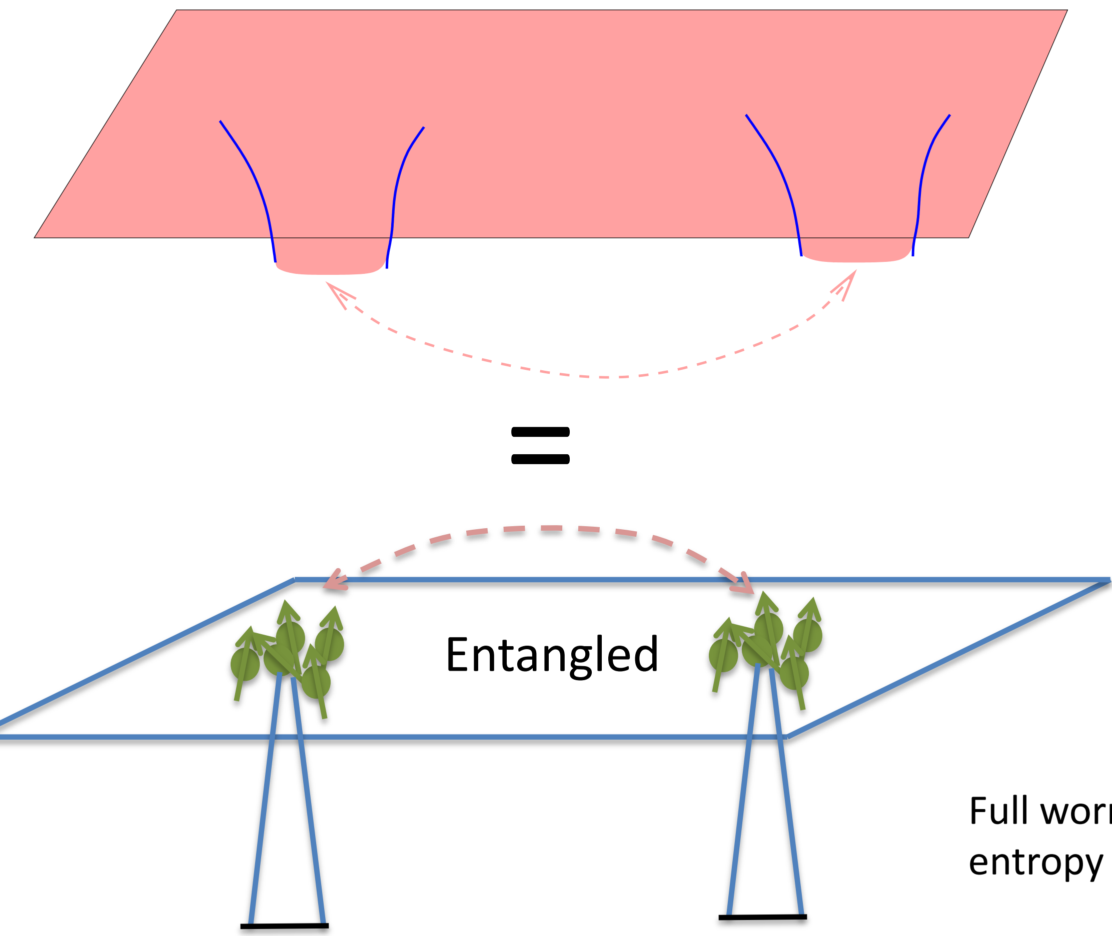
It is only a statement about the black hole as seen from the outside !

No statement has been made about the inside (yet).

Full Schwarzschild solution as a wormhole



Wormholes and entangled states



Connected through the interior

ER = EPR

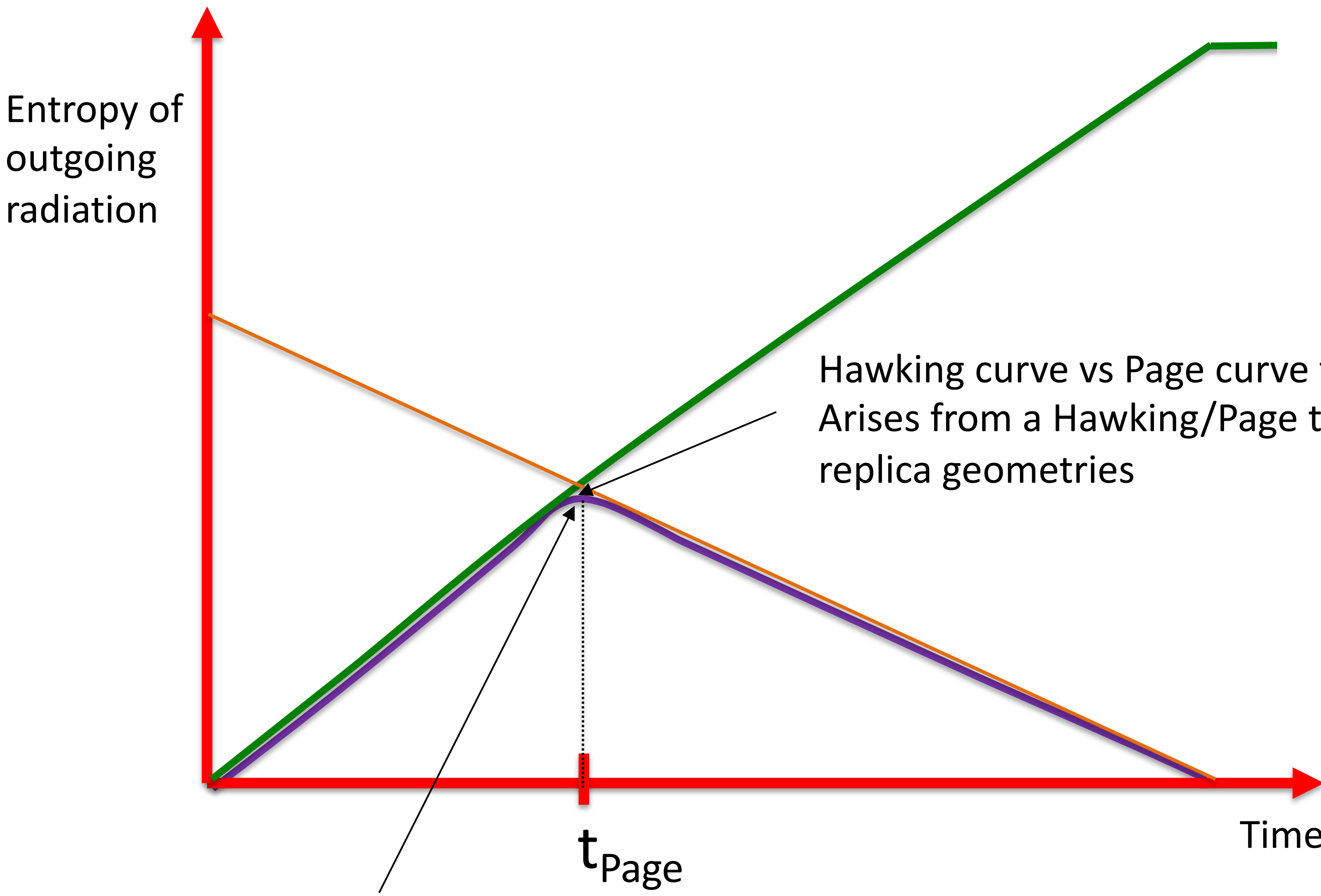
W. Israel
J.M.
JM Suskind

Full wormhole is in a pure state,
entropy of each black hole arises from entanglement

In a particular entangled state

$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

The Hawking curve vs. the Page curve



Hawking curve vs Page curve transition →
Arises from a Hawking/Page transition for the replica geometries

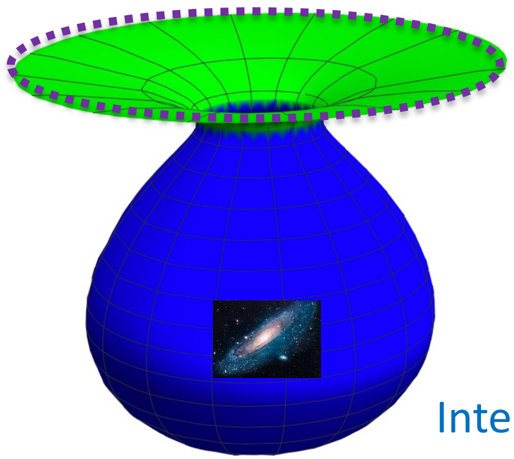
Gets smoothed out by considering the sum over non-replica symmetric geometries

Penington, Shenker, Stanford, Yang; Dong, H. Wang; Marolf, S. Wang, Z. Wang

``Bags of Gold”

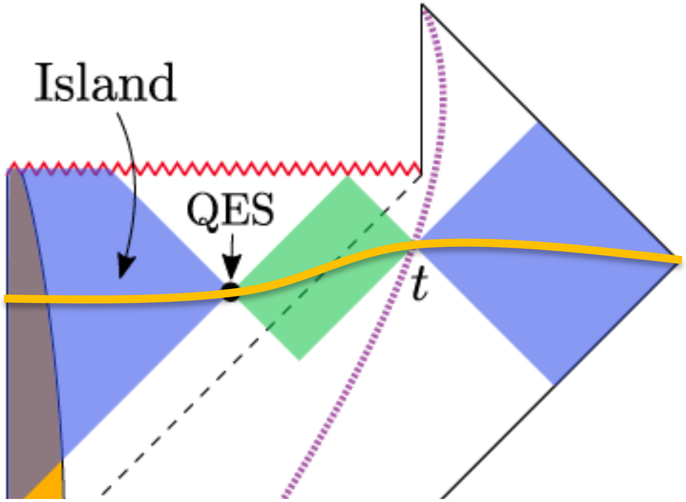
If there is a lot of entropy inside → the entanglement wedge of the black hole ends near the neck

Wall



Interior of the bag belongs to somebody else…

Bag of gold vs old black hole



The geometry and entropy on the orange slice is somewhat similar to the bag of gold.

Now the answer:

Entanglement wedge reconstruction hypothesis

- The quantum system describes everything that is included in its entanglement wedge.

- We can recover the state of a (probe) qubit inside the entanglement wedge.

- Recovery is state dependent (subspace dependent) and similar to quantum error correction.

Czech, Karczmarek, Nogueira, Van Raamsdonk, Wall, Headrick, Hubeny, Lawrence, Rangamani, Almheiri, Dong, Harlow, Jafferis, Lewkowycz, J.M., Suh, Wall, Faulkner….