Holography with broken spacetime symmetries and its transition into adulthood

Holography@25

School: June 5-13, 2023, Workshop: June 14-17, 2023

São Paulo, Brazil

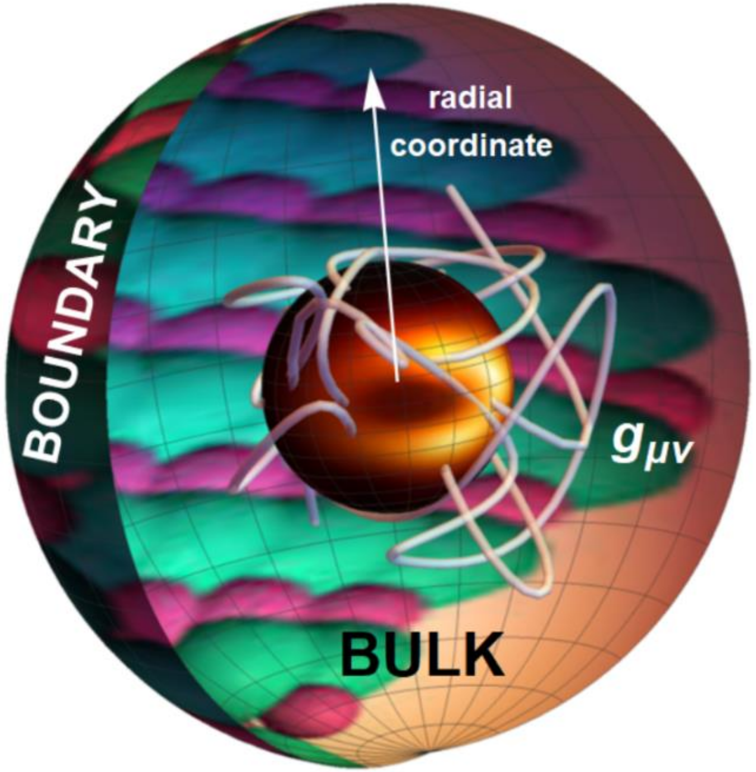
ICTP-SAIFR/IFT-UNESP

TheoryLab

上海交通大學
Shanghai Jiao Tong University

Matteo Baggioli

Jiao-Tong University Shanghai



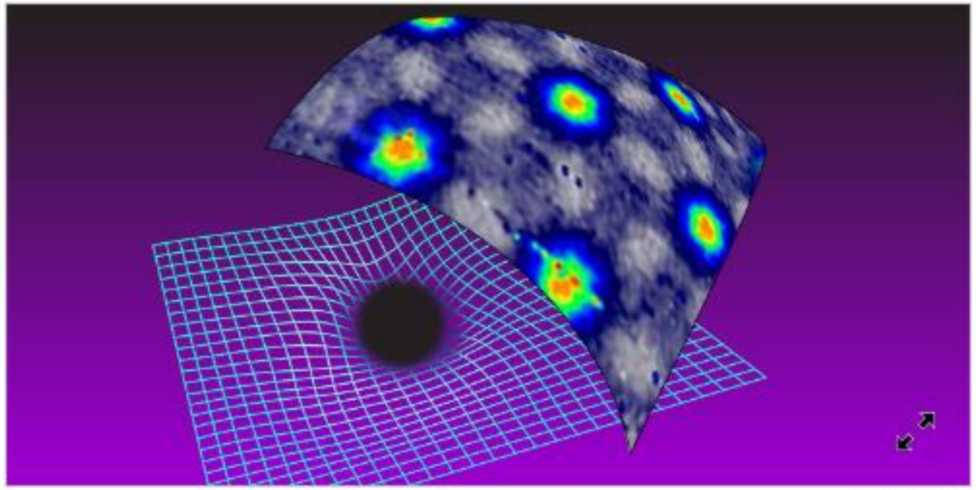
ALL ROADS LEAD TO ~~ROME~~
Holography

RESEARCH NEWS

Holographist by Trade

April 14, 2020 • *Physics* 13, 57

Tackling condensed-matter problems with string theory is becoming a dedicated profession.

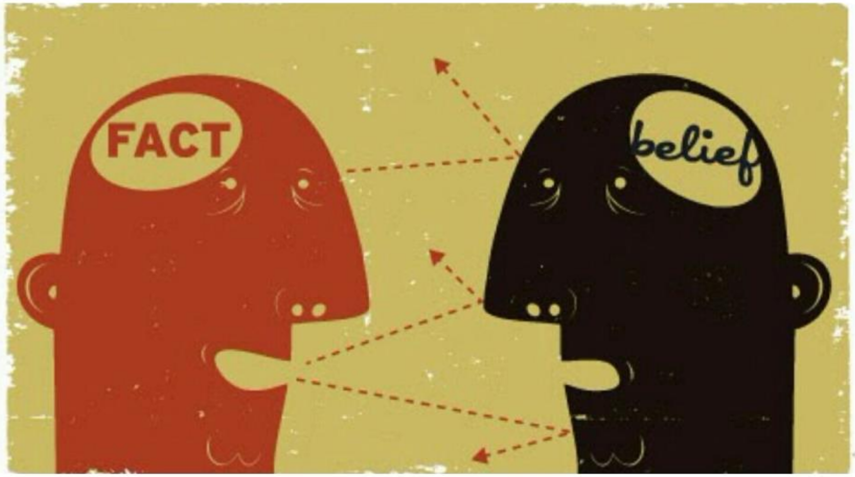

APS/Carin Cain

Holographists use a string theory correspondence that relates black holes and condensed-matter systems, such as metals and superconductors.

APS https://physics.aps.org/articles/v13/57

This is a personal (and therefore biased) historical reconstruction of the facts

References are not complete [and obviously my works are over-emphasized ☺]



1998

Lectures on holographic methods for condensed matter physics

Sean A. Hartnoll

2008

Gravity Dual of Spatially Modulated Phase

Shin Nakamura, Hirosi Oguri, Chang-Soon Park

2009 (+ Donos-Gauntlett 2011)

Optical Conductivity with Holographic Lattices

Gary T. Horowitz, Jorge E. Santos, David Tong

2012 (beginning of my PhD)

2013 (+ Andrade-Withers 2013)

Holography without translational symmetry

David Vegh

2013-2022

many authors

Colloquium: Hydrodynamics and holography of charge density wave phases

Matteo Baggioli and Blaise Goutéraux

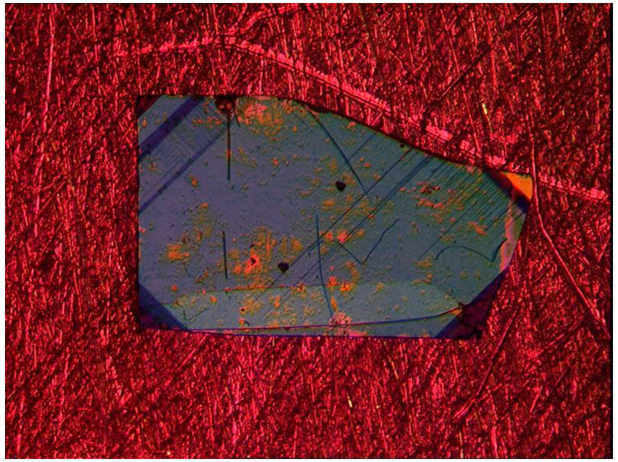
Rev. Mod. Phys. **95**, 011001 – Published 4 January 2023

2023

2023

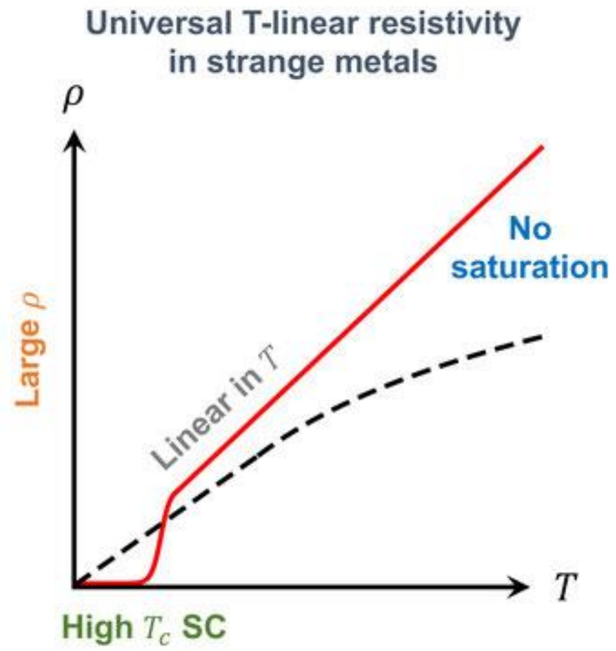
Holography@25

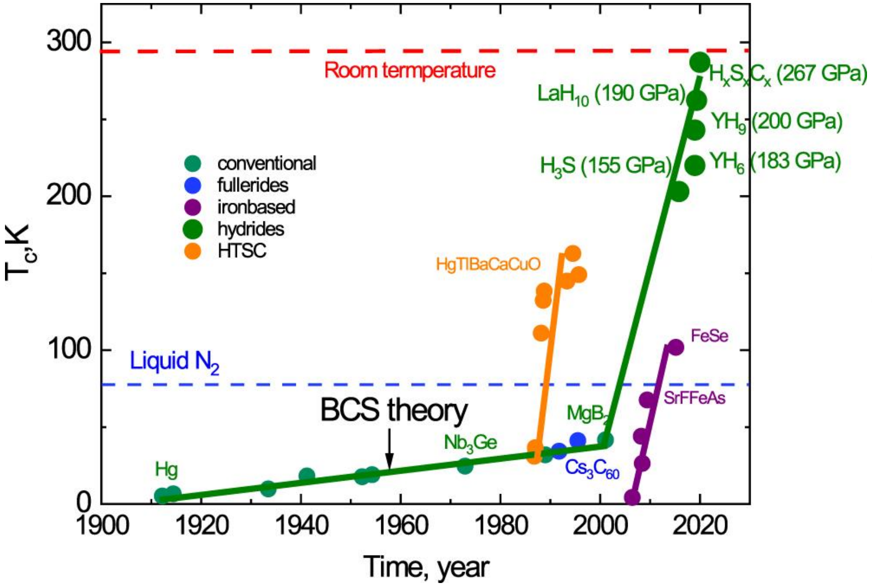
Original motivation

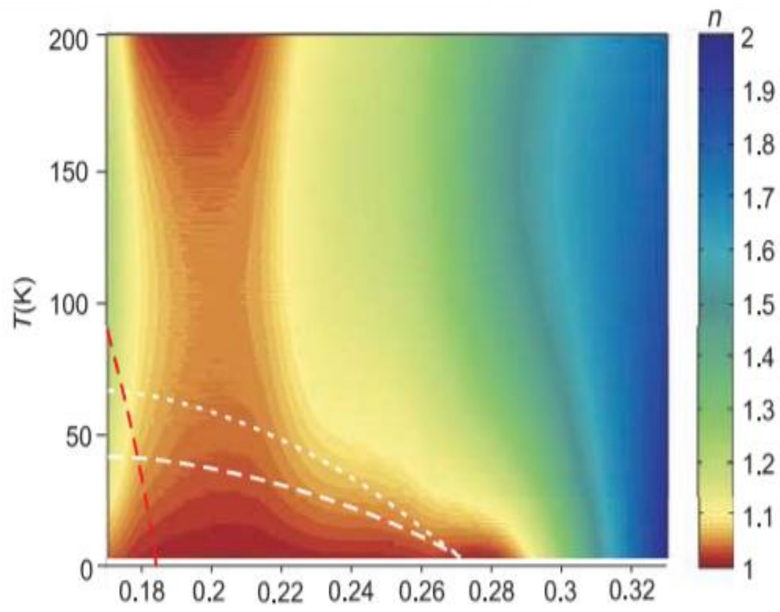


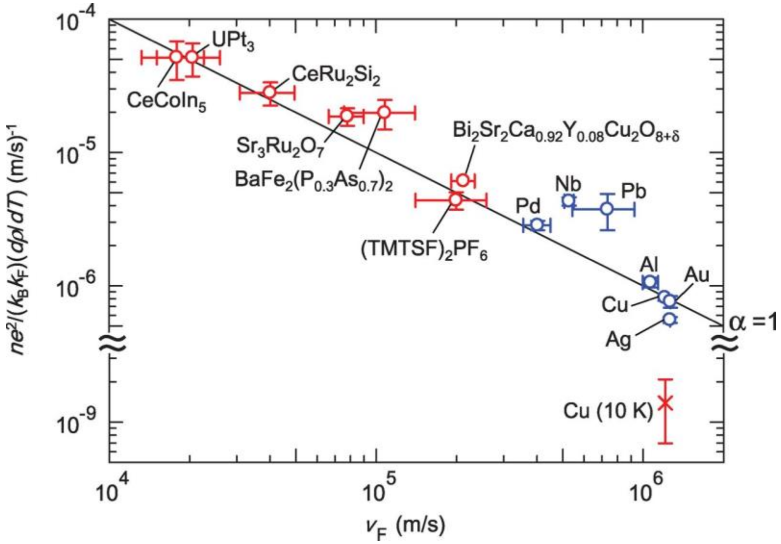
Strange metals
High-Tc superconductivity

[a posteriori,
probably not the biggest success …]

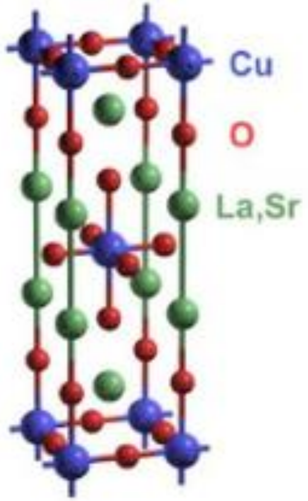


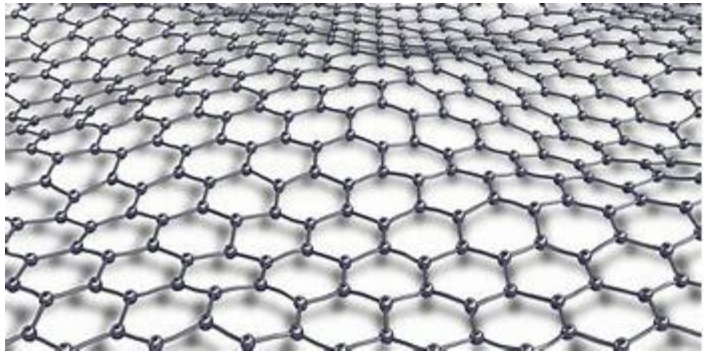


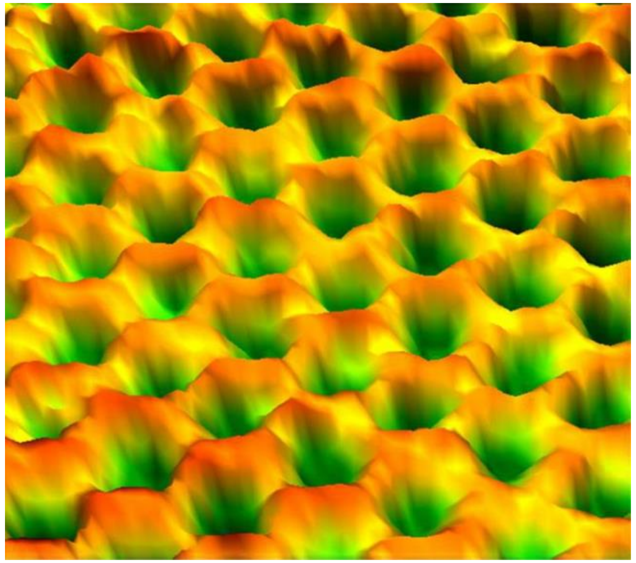


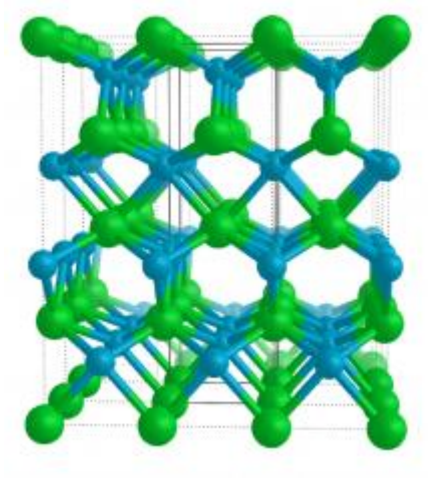


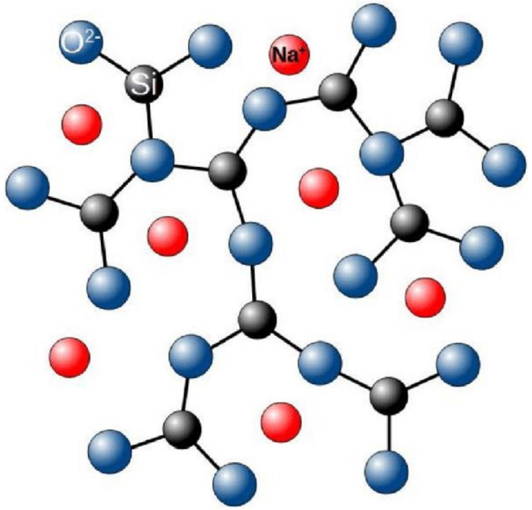
Condensed matter systems

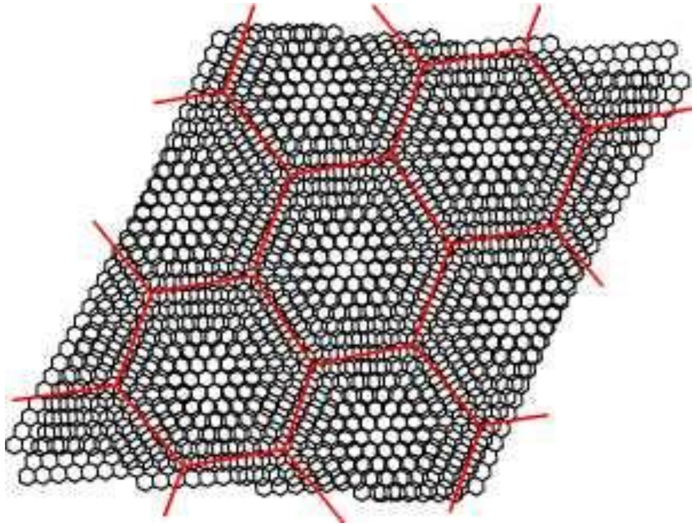


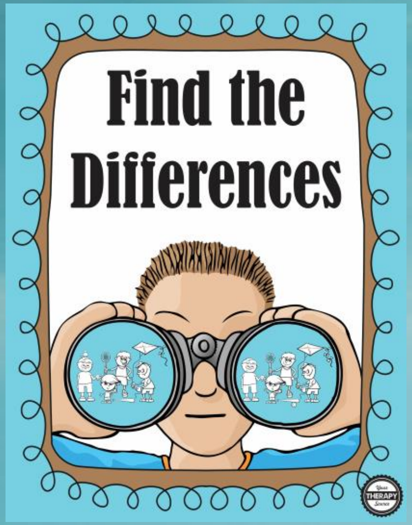








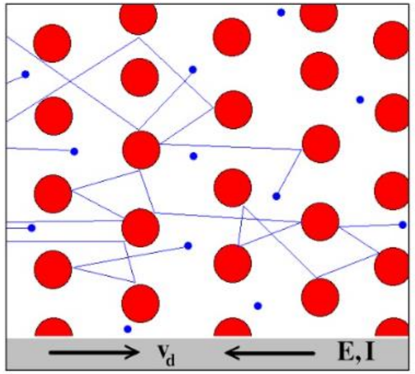




This was holography in 5th grade
(~ 2008)

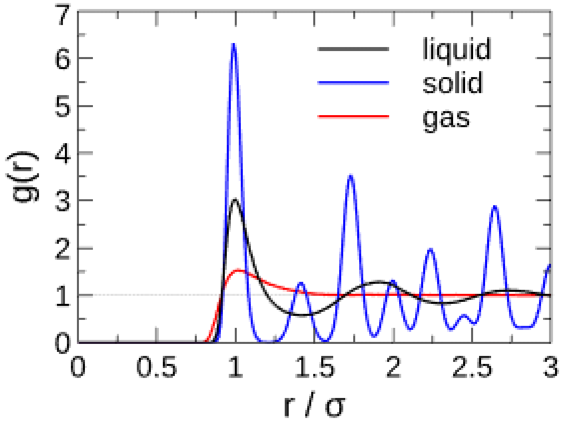
Translational invariance is broken

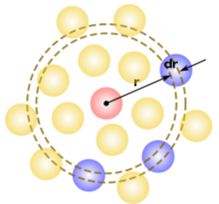
- ❖ Explicitly



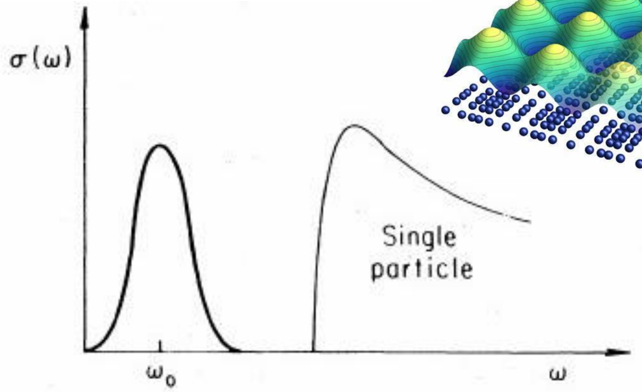
Paul Karl Ludwig Drude (1863–1906)

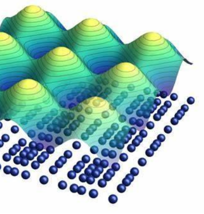
- ❖ Spontaneously





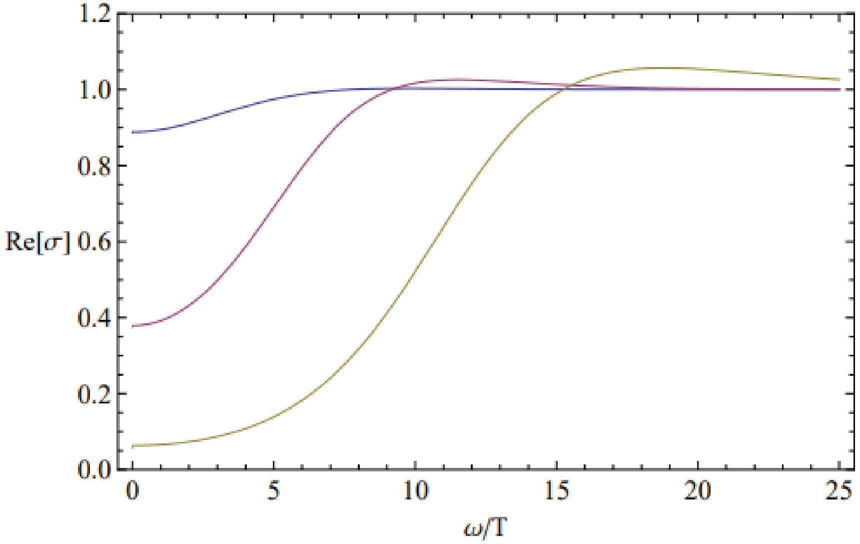
- ❖ Pseudo Spontaneously

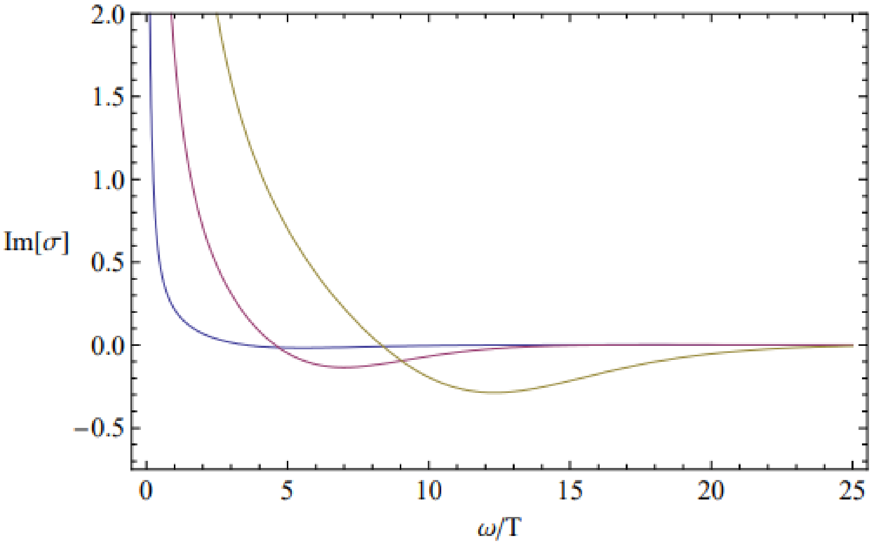


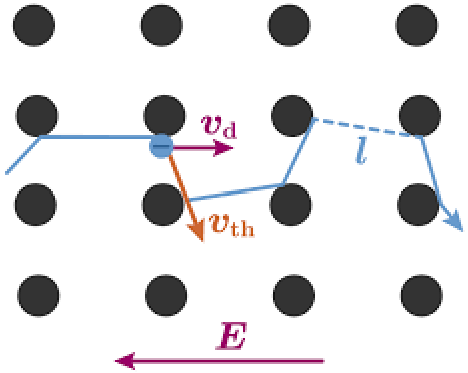


- How did “we” make holography compatible with broken translations ?
- What did “we” learn ?
- Where did “we” fail ?
- What’s next ?

STEP 1: relaxing momentum and get finite conductivity





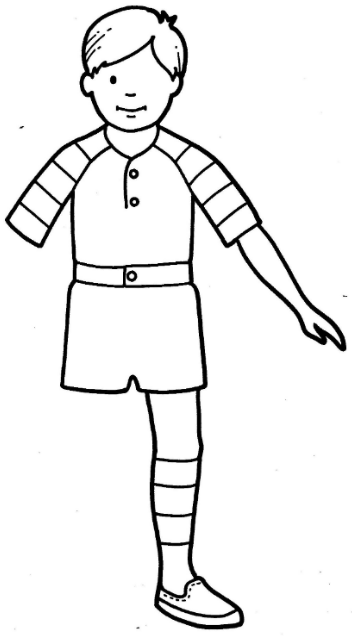


$$\sigma(\omega) = \sigma_0 + \frac{\rho^2}{\varepsilon + P}\frac{1}{1/\tau - i\omega}$$

This is what Holography could do in 5th grade

What is the problem ? $\tau \to \infty$

The DC conductivity is infinite
The system is translational invariant
Not so useful for real cond-mat systems



Solution #1

[Submitted on 2 Apr 2012 (v1), last revised 3 Aug 2012 (this version, v2)]

Optical Conductivity with Holographic Lattices

Gary T. Horowitz, Jorge E. Santos, David Tong

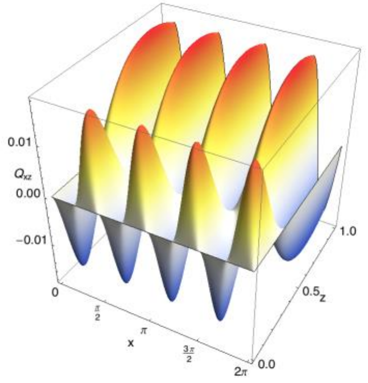
2 A Holographic Lattice

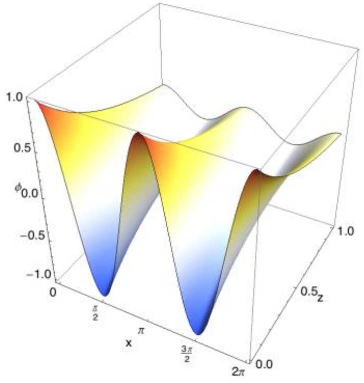
The minimal ingredients necessary to compute conductivity in a holographic framework are provided by Einstein-Maxwell theory in AdS_4. To this we add a neutral scalar field Φ which we will use to source the lattice. We work with the Lagrangian,

$$S = \frac{1}{16\pi G_N}\int d^4x\,\sqrt{-g}\left[R + \frac{6}{L^2} - \frac{1}{2}F_{ab}F^{ab} - 2\nabla_a\Phi\nabla^a\Phi - 4V(\Phi)\right], \qquad (2.1)$$

$$\Phi \to z\phi_1 + z^2\phi_2 + \mathcal{O}(z^3)$$

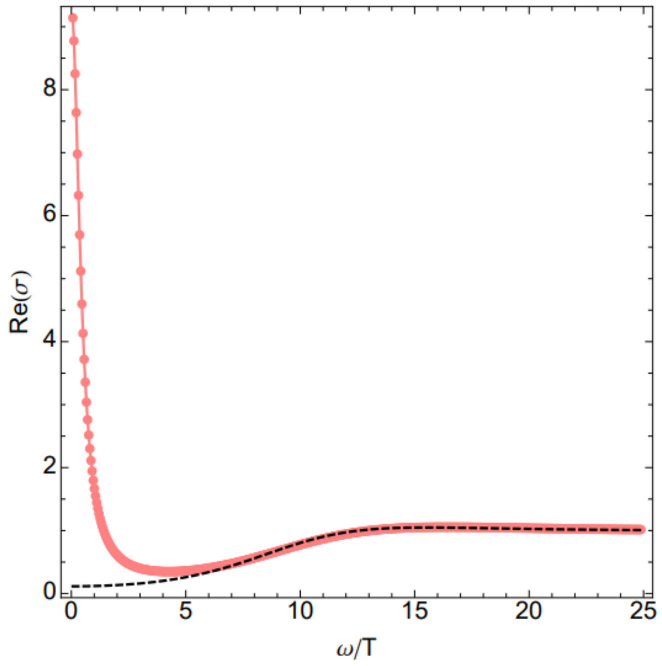
$$\phi_1(x) = A_0\cos(k_0 x)\,.$$

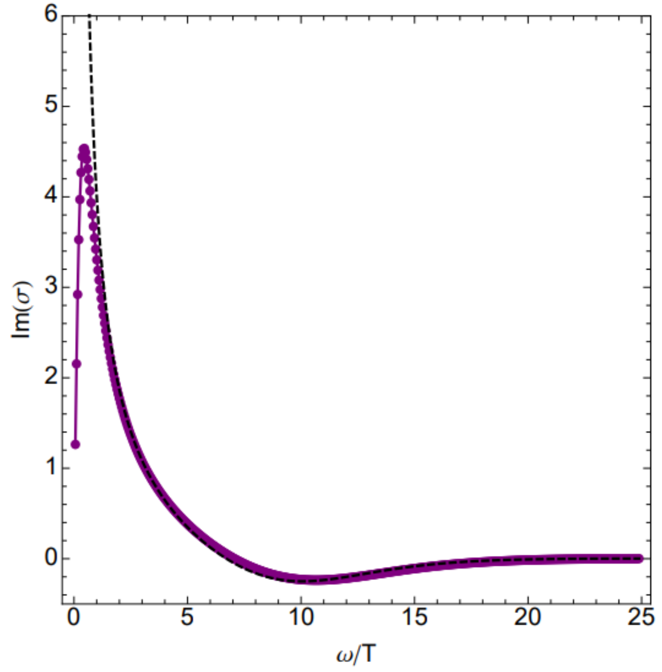




Drude physics from Holography

Only 1 issue:





Solution #2 : the first holographic revolution

[Submitted on 3 Jan 2013 (v1), last revised 15 Jan 2013 (this version, v2)]

Holography without translational symmetry

David Vegh

$$\frac{d}{dt}\vec{p}(t) = e\vec{E} - \frac{\vec{p}(t)}{\tau} \longleftrightarrow \mathcal{L}_I = \sqrt{-g}\, m^2 (\delta g_{tx})(\delta g^{tx})$$

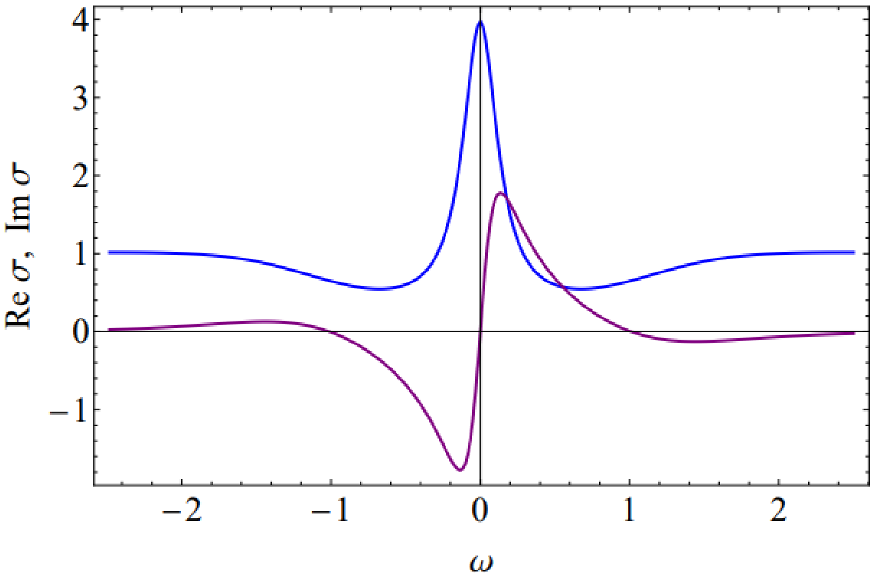
Momentum relaxation

Massive graviton

dRGT massive gravity: $$S = \frac{-1}{2\kappa^2}\int d^4x\,\sqrt{-g}\left[R + \Lambda - \frac{L^2}{4}F^2 + m^2\sum_{i=1}^{4} c_i\,\mathcal{U}_i(g,f)\right]$$

Breaking the (spatial components) of the stress-energy tensor conservation

Lorentz violating massive gravity in AdS



Solution #2 : a more convenient way, the axion model

[Submitted on 20 Nov 2013 (v1), last revised 7 Dec 2013 (this version, v2)]

A simple holographic model of momentum relaxation

Tomas Andrade, Benjamin Withers

Birth of the "homogeneous" models

$$S_0 = \int_M \sqrt{-g}\left[R - 2\Lambda - \frac{1}{2}\sum_I^{d-1}(\partial\psi_I)^2 - \frac{1}{4}F^2\right] d^{d+1}x$$

$$\boxed{\psi_I = \alpha_{Ia} x^a,}$$

$$\sigma_{DC} = r_0^{d-3}\left(1 + (d-2)^2\frac{\mu^2}{\alpha^2}\right)$$

Analytic DC conductivities

Advantages:

- Simple & analytic
- Doing the job
- Drude Physics (and more)

[Submitted on 18 Jun 2014 (this version), latest version 14 Oct 2014 (v4)]

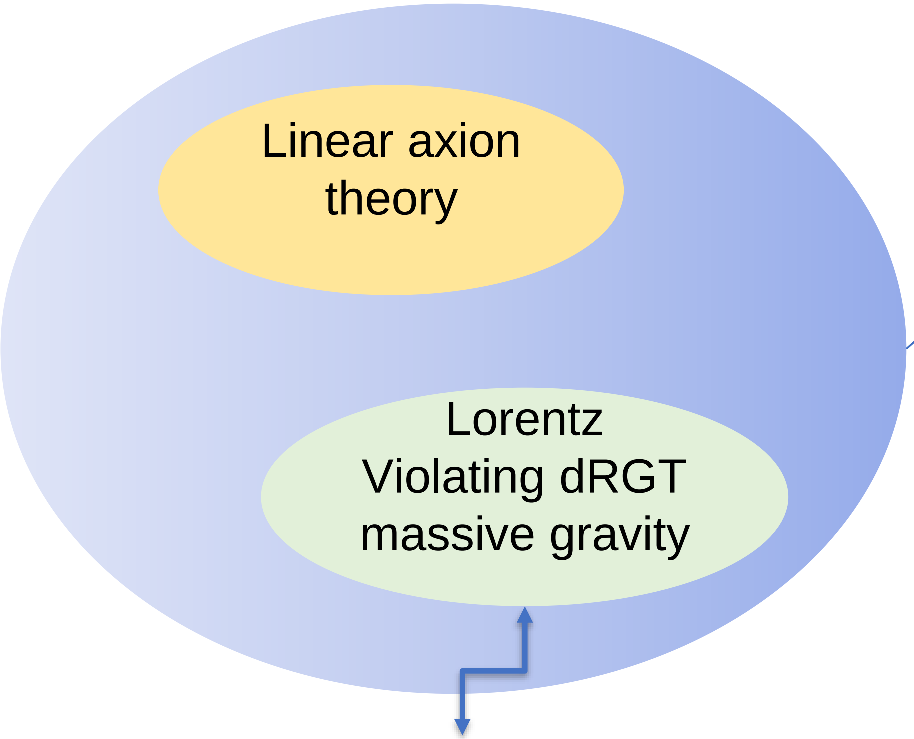
Thermoelectric DC conductivities from black hole horizons

Aristomenis Donos, Jerome P. Gauntlett

Disadvantages:

- What is this?
- No "real" lattice
- What is missing?

Clarifying an important point



$$V_{\mathrm{dRGT}} = -\beta_1 \sqrt{\frac{1}{2}\left(X + \sqrt{Z}\right)} - \beta_2 \sqrt{Z}$$

[Submitted on 30 Oct 2015]

Solid Holography and Massive Gravity

Lasma Alberte, Matteo Baggioli, Andrei Khmelnitsky, Oriol Pujolas

Most general Lorentz violating massive gravity theory in Stueckelberg formalism

"generalized axion theory"

$$X \equiv \frac{1}{2}\,\mathrm{tr}[\mathcal{I}^{IJ}] = \frac{1}{2}\partial_\mu\phi^I\partial^\mu\phi^I \; ; \qquad \boxed{\langle \phi^A \rangle = x^\mu \delta_\mu^A}$$

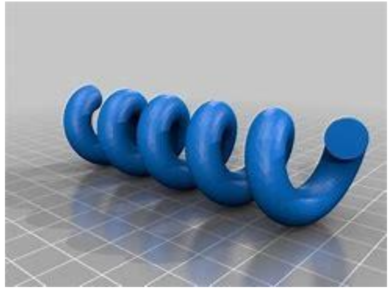
$$Z \equiv \det[\mathcal{I}^{IJ}] = \frac{1}{2}\left(\partial_\mu\phi^I\partial^\mu\phi^I\partial_\nu\phi^J\partial^\nu\phi^J - \partial_\mu\phi^I\partial^\mu\phi^J\partial_\nu\phi^I\partial^\nu\phi^J\right)$$

$$\boxed{S_\phi \equiv \int d^4x\sqrt{-g}\,\mathcal{L}_\phi = -\int d^4x\sqrt{-g}\,V(X,Z)\,.}$$

Retaining isotropy there is nothing more general you can do!

Well know fact in cosmology/gravity community

Solution #2 : more options, “same” physics



Helical lattices
[Donos, Hartnoll 2012]

$$\phi = e^{ikx_1}\varphi$$

Q-lattices
[Donos, Gauntlett 2013]

[Submitted on 30 Aug 2017 (v1), last revised 11 Nov 2017 (this version, v2)]

Conformal solids and holography

A. Esposito, S. Garcia-Saenz, A. Nicolis, R. Penco

“solidons”

$$\nabla_\mu J_I^{\mu\nu} = 0$$

Higher forms
[Grozdanov, Poovuttikul, 2018]

[these two models on the bottom only for spontaneous breaking]

Same IR physics

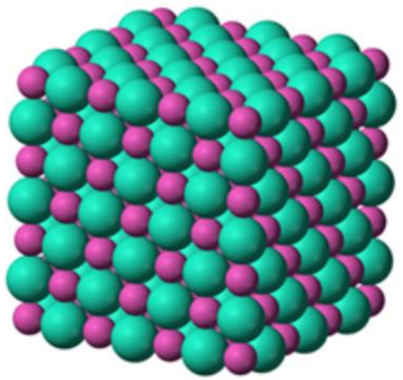
Equivalent effective descriptions

From solution #1 to solution #2

[Submitted on 14 Oct 2013 (v1), last revised 21 Feb 2014 (this version, v3)]

Holographic Lattices Give the Graviton a Mass

Mike Blake, David Tong, David Vegh



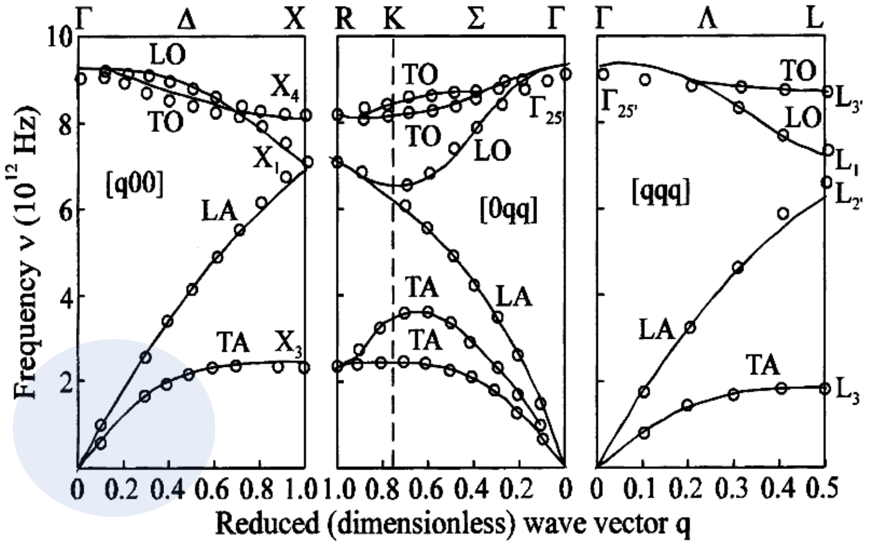
$$\phi_- = \epsilon \cos(k_L x)$$

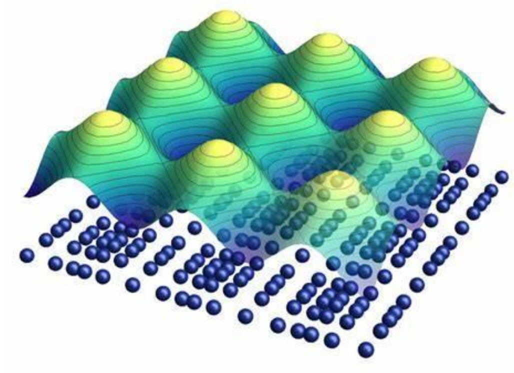
$$S_{\text{eff}} = \frac{1}{2}\int d^4x\sqrt{-g}M^2(r)\,g^{xx} \qquad M^2(r) = \frac{1}{2}\epsilon^2 k_L^2 \phi_0(r)^2$$

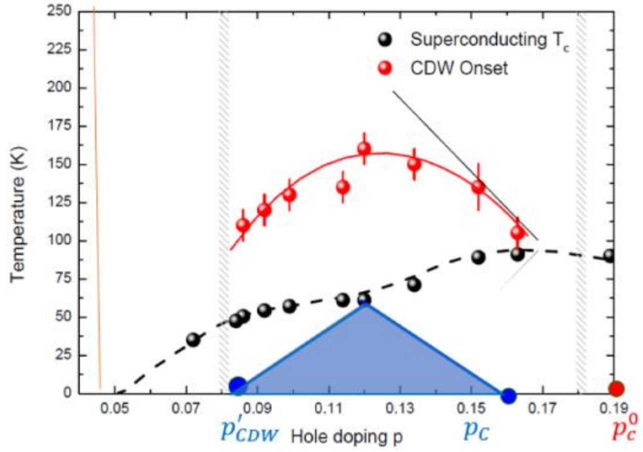
Drude rate: $\Gamma \sim M^2(r_h) \sim \epsilon^2 k_L^2 \phi_0(r_h)^2$

[In the hydro limit, inhomogeneities are irrelevant! So why bother? ☺]

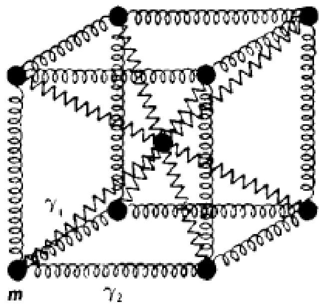
Still something missing







Phonons
Elasticity



Other spontaneous orders
Charge density waves

Spontaneous breaking of translational symmetry

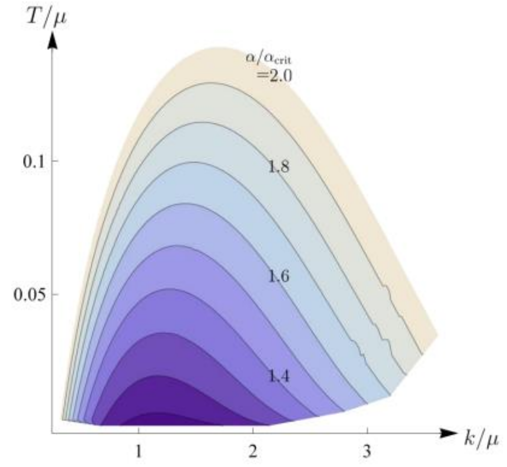
(= these materials are solid)

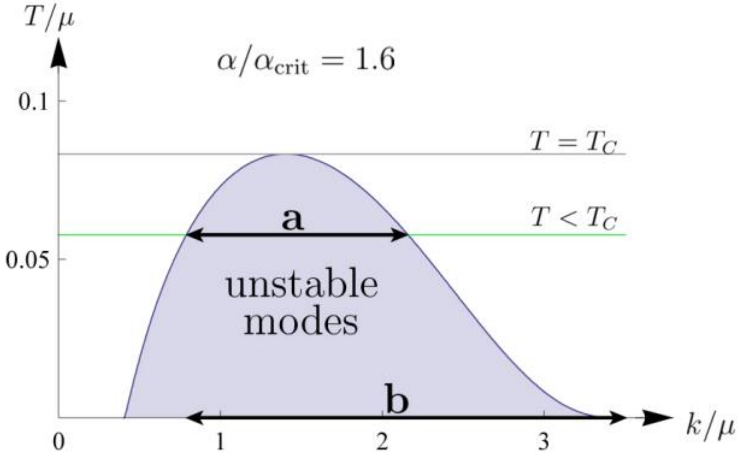
Solution #1: holographic stripes

[Submitted on 4 Nov 2009 (v1), last revised 26 Nov 2009 (this version, v2)]

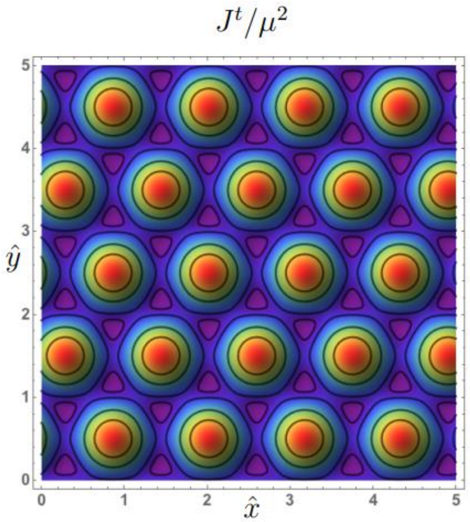
Gravity Dual of Spatially Modulated Phase

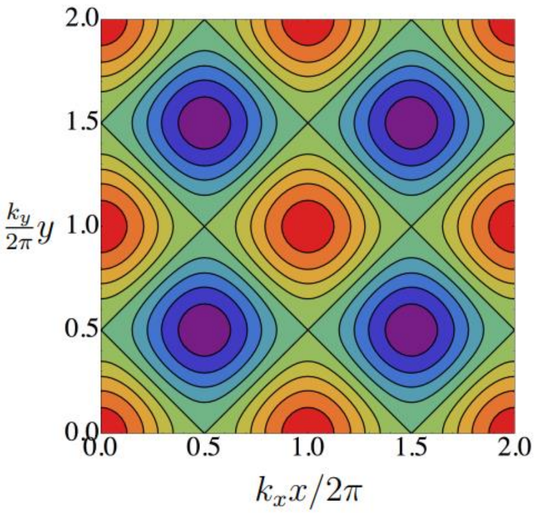
Shin Nakamura, Hirosi Ooguri, Chang-Soon Park





- Finite wave-vector instabilities
- Spontaneous formation of modulated structures
- “holographic charge density waves”





Lot of follow-up works:

- Different models
- More dimensions

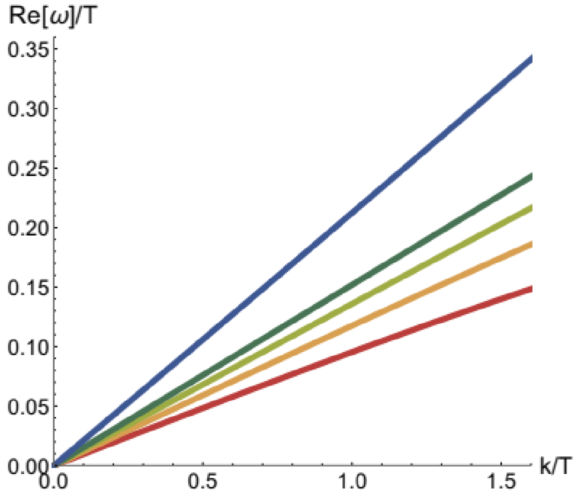
Solution #2: homogeneous models

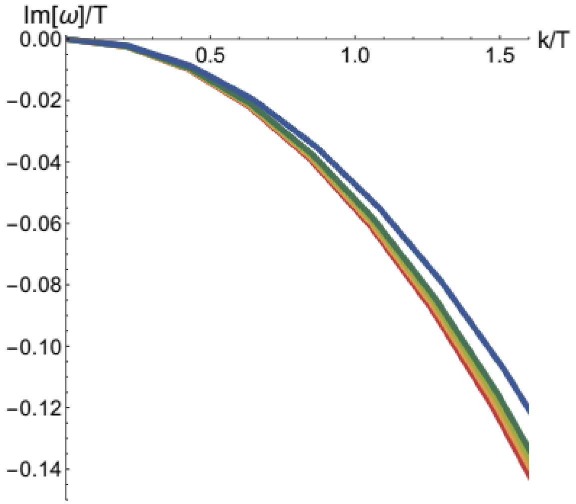
[same logic as before, periodic lattice structure does not (always) matter]

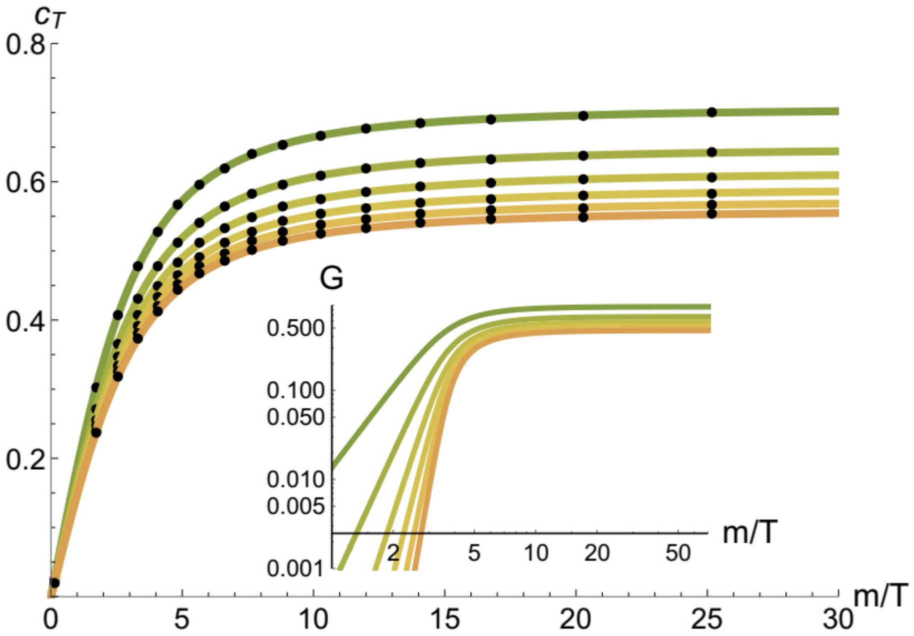
[Submitted on 8 Nov 2017]

Holographic Phonons

Lasma Alberte, Martin Ammon, Matteo Baggioli, Amadeo Jiménez-Alba, Oriol Pujolàs







$$\omega = c_T\, k - i\, D\, k^2$$

Habemus phonons !!

[more models and works after this]

(many names = no names)

$$c_T^2 = \frac{G}{\chi_{PP}}$$

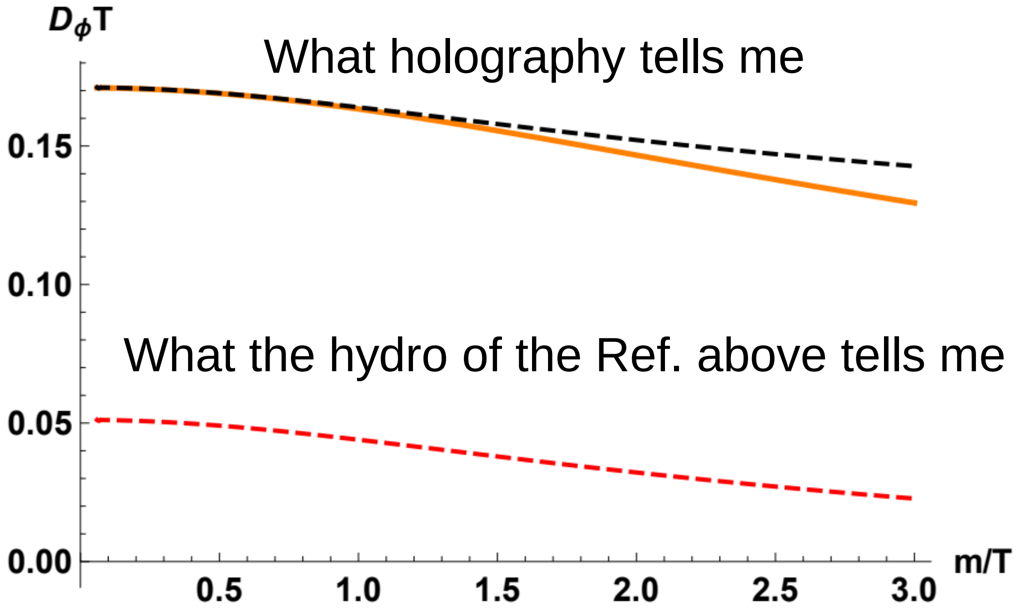
Lesson #1: hydrodynamics, we have a problem

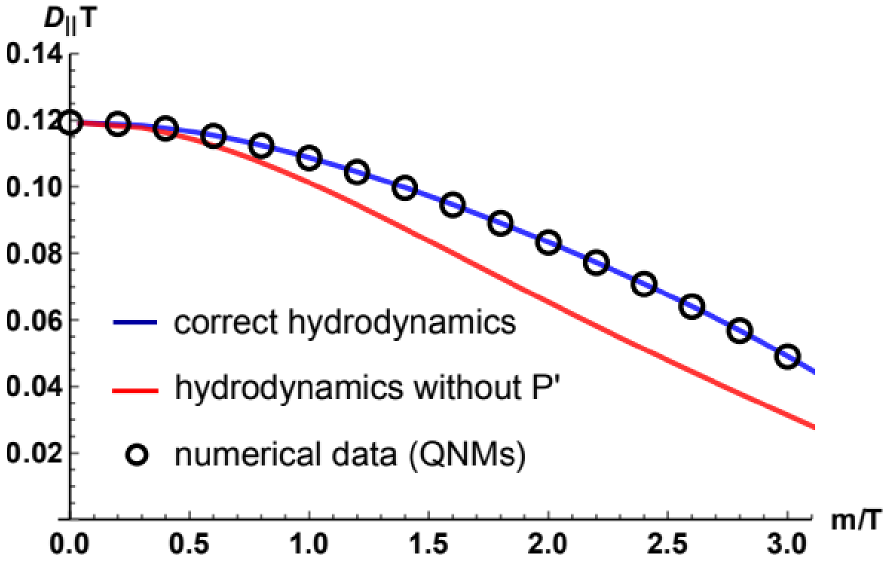
Reference for holographists

[Submitted on 16 Feb 2017 (v1), last revised 25 Sep 2017 (this version, v4)]

Theory of hydrodynamic transport in fluctuating electronic charge density wave states

Luca V. Delacrétaz, Blaise Goutéraux, Sean A. Hartnoll, Anna Karlsson





[Submitted on 22 May 2019 (v1), last revised 24 Jul 2019 (this version, v2)]

Longitudinal Sound and Diffusion in Holographic Massive Gravity

Martin Ammon, Matteo Baggioli, Seán Gray, Sebastian Grieninger

The hydrodynamic description is (~~wrong~~) incomplete

Holography driven EFT developments

[Submitted on 3 Aug 2019 (v1), last revised 14 Jan 2020 (this version, v3)]

Viscoelastic hydrodynamics and holography

Jay Armas, Akash Jain

Complete and correct hydrodynamic description matching with holographic models

arXiv:2001.05737 [pdf, other] hep-th cond-mat.soft cond-mat.str-el doi 10.1016/j.physletb.2020.135691

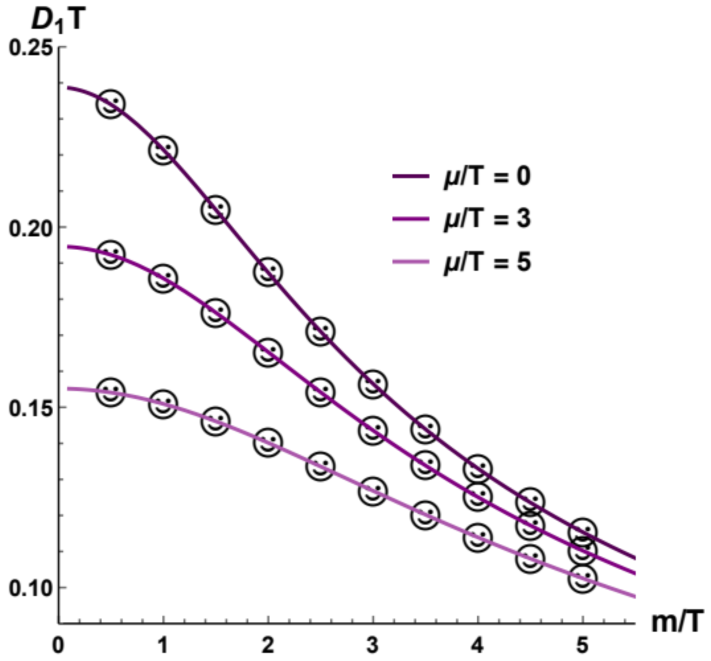
On the Hydrodynamic Description of Holographic Viscoelastic Models

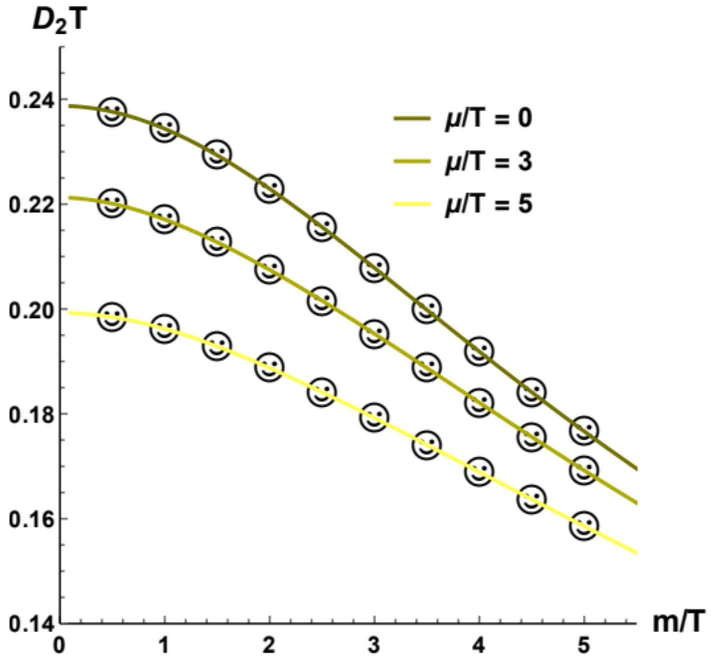
Authors: Martin Ammon, Matteo Baggioli, Seán Gray, Sebastian Grieninger, Akash Jain

arXiv:2005.01725 [pdf, other] hep-th cond-mat.mes-hall cond-mat.str-el doi 10.1007/JHEP09(2020)037

Magnetophonons & type-B Goldstones from Hydrodynamics to Holography

Authors: Matteo Baggioli, Sebastian Grieninger, Li Li



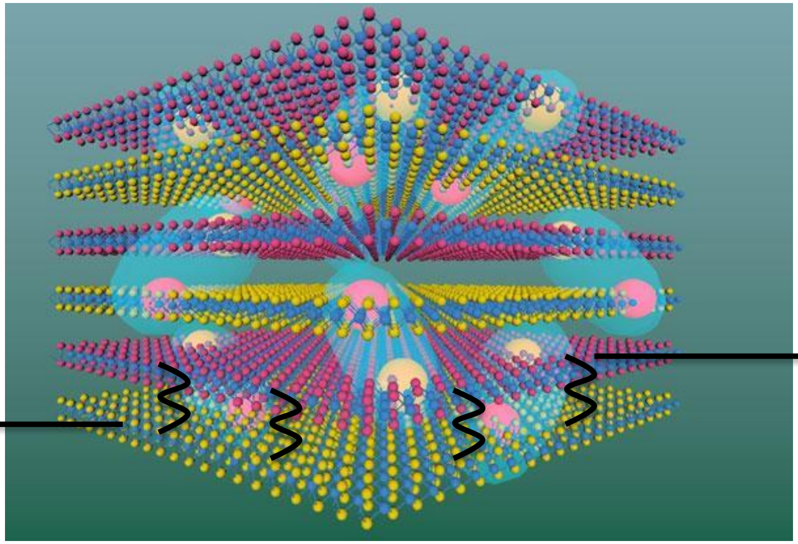


(Not the first nor the last time that holography corrects or helps the EFT/hydro side)

Holography
=
EFT 2.0

Ops, one more missing thing

Translational symmetry
is not exact !
The sliding mode is not
really a Goldstone mode



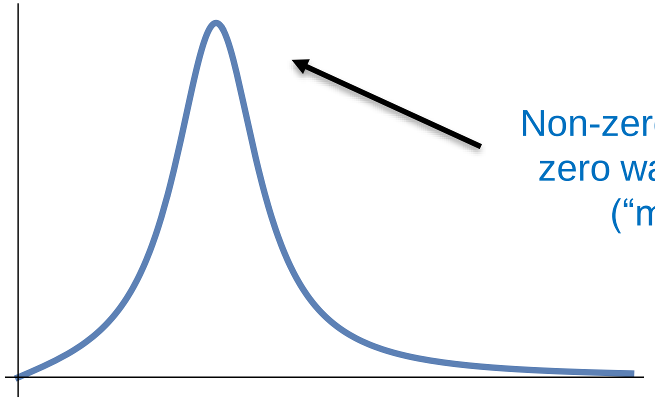
Pseudo-spontaneous
Breaking

(e.g. Pions)

Typical example:
pinned charge density waves

The dynamics of charge-density waves

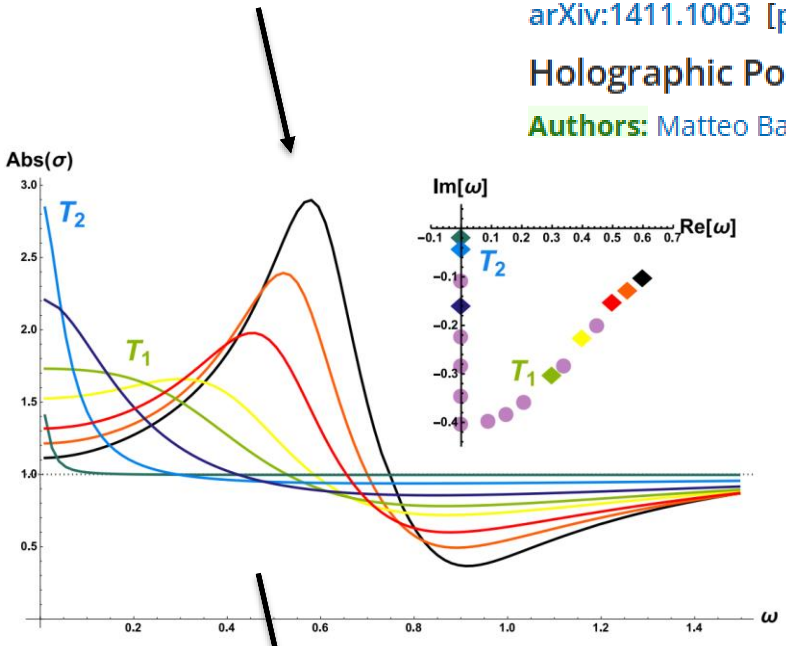
G. Grüner
Rev. Mod. Phys. **60**, 1129 – Published 1 October 1988



Non-zero energy at
zero wave-vector
("mass")

Impurities and disorder pin the would-be Goldstone mode
(more general: phason mode in incommensurate structures and aperiodic systems)

How we did it (without realizing it)



arXiv:1411.1003 [pdf, ps, other] hep-th cond-mat.str-el doi 10.1103/PhysRevLett.114.251602

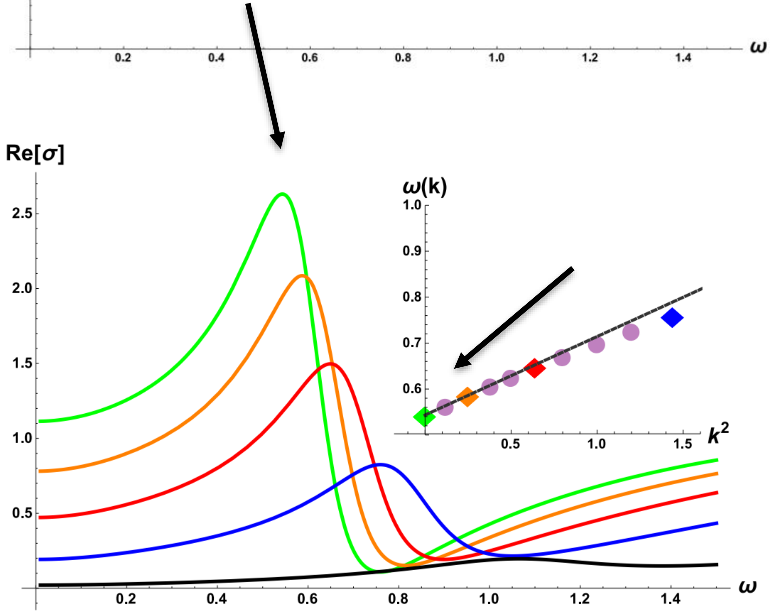
Holographic Polarons, the Metal-Insulator Transition and Massive Gravity

Authors: Matteo Baggioli, Oriol Pujolas

A pseudo-Goldstone mode appears

$$\omega = \omega_0 - i\Omega + \ldots$$

At that time, we did not understand it …
Things changed around 2017…



arXiv:1708.08306 [pdf, other] hep-th cond-mat.str-el gr-qc doi 10.1007/JHEP02(2018)085

Pinning of longitudinal phonons in holographic spontaneous helices

Authors: Tomas Andrade, Matteo Baggioli, Alexander Krikun, Napat Poovuttikul

arXiv:1708.08477 [pdf, other] hep-th doi 10.1007/JHEP01(2018)129

Black hole elasticity and gapped transverse phonons in holography

Authors: Lasma Alberte, Martin Ammon, Matteo Baggioli, Amadeo Jiménez, Oriol Pujolàs

General low-energy description

(1) Momentum relaxation rate
(2) Pinning frequency (or wave-vector)
(3) Goldstone (or “phase”) relaxation

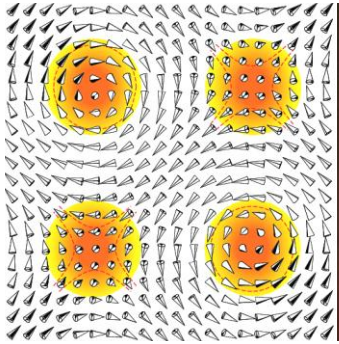
$$\dot{\delta\phi}^i = \underline{-\Omega\,\delta\phi^i} + O(\nabla^i)$$

$$\delta f^{(2)} = \frac{B+G}{2}\left(\nabla^i \delta\phi_i\right)^2 + \frac{G}{2}\left(\nabla \times \delta\phi\right)^2 + \underline{\frac{G\,q_o^2}{2}}\,\delta\phi_i\delta\phi^i$$

$$\dot{\pi}^i + \nabla_j \tau^{ji} = \underline{-\Gamma\pi^i} - \underline{G q_o^2}\delta\phi^i$$

(1) Is expected from explicit breaking of translations
(2) Is expected with spontaneous + explicit breaking (cfr. Pions)
(3) Is expected from topological defects (e.g., vortices in superfluids)

Our Goldstone relaxation is different!
No defects around!

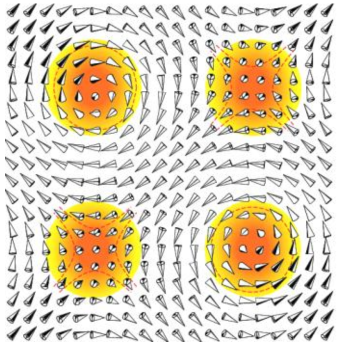


A simple way to see that it is different

arXiv:1904.05785 [pdf, other] hep-th doi 10.1007/JHEP09(2019)124

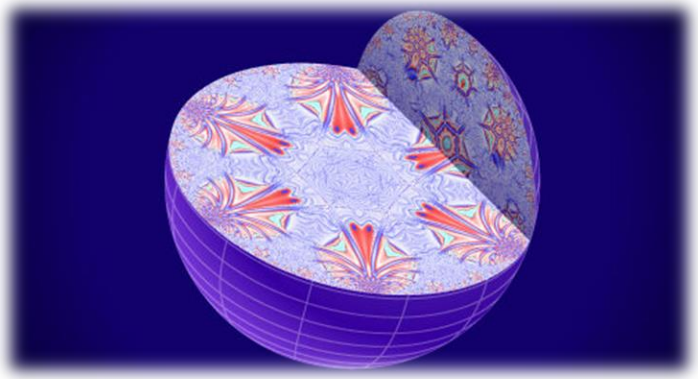
A Unified Description of Translational Symmetry Breaking in Holography

Authors: Martin Ammon, Matteo Baggioli, Amadeo Jiménez-Alba

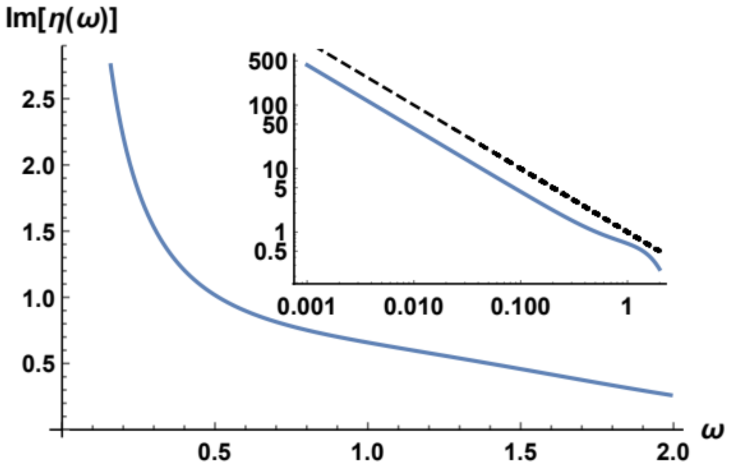


$$\eta(\omega) \equiv \frac{1}{i\,\omega}\,\mathcal{G}^{R}_{T_{xy}T_{xy}}(\omega, k=0)$$

$$\eta(\omega) = \frac{G}{\Omega - i\,\omega} + \eta + \ldots\,,$$



A different beast



A universal prediction from holography

arXiv:1812.08118 [pdf, other] hep-th cond-mat.str-el doi 10.1103/PhysRevLett.123.211602

Universal relaxation in a holographic metallic density wave phase

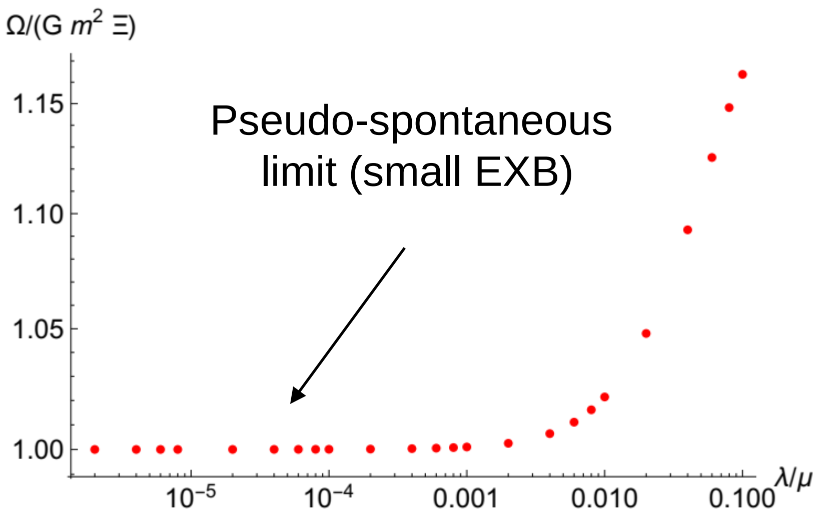
Authors: Andrea Amoretti, Daniel Areán, Blaise Goutéraux, Daniele Musso

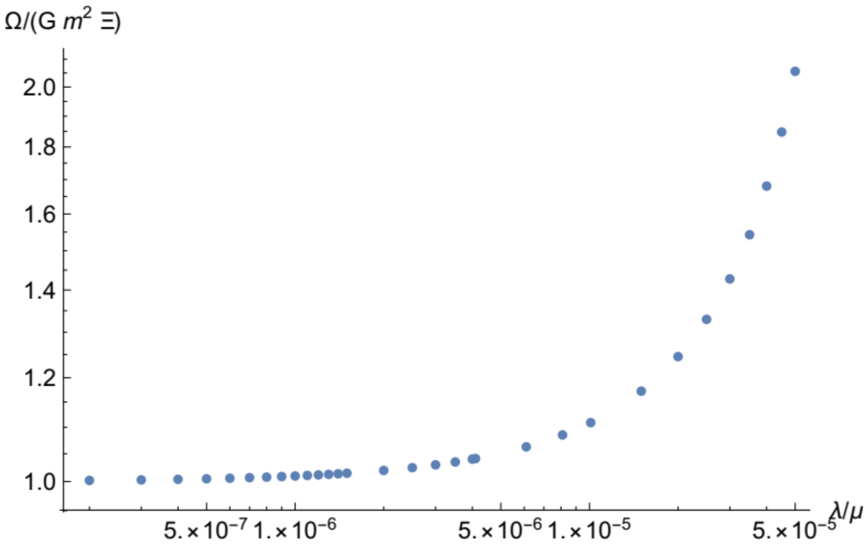
$$\Omega = G q_o^2 \xi$$

$$\lim_{\omega\to 0} \frac{1}{\omega} \mathrm{Im} G^R_{\partial_t\phi^i\partial_t\phi^i}(\omega, k=0)$$

Simple words: the Goldstone relaxation is not an independent parameter! It is slaved to the pinning frequency and properties of the spontaneous phase (Goldstone diffusivity and speed of sound)

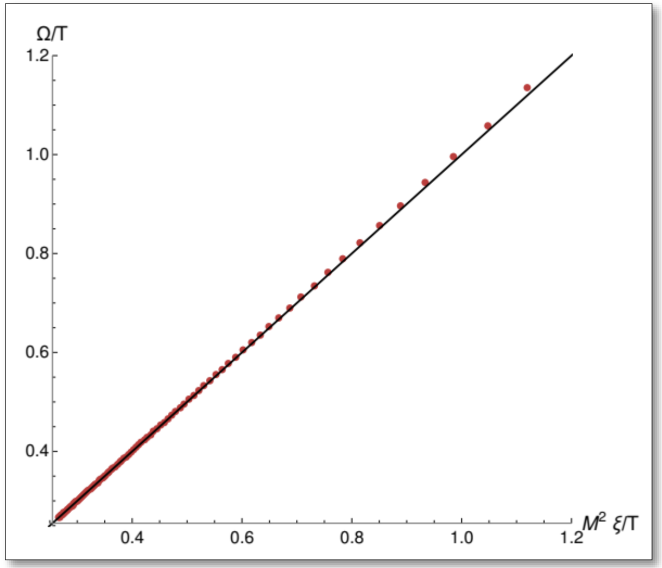
$$\Omega = \omega_0^2 \frac{D_\xi}{v_s^2}$$



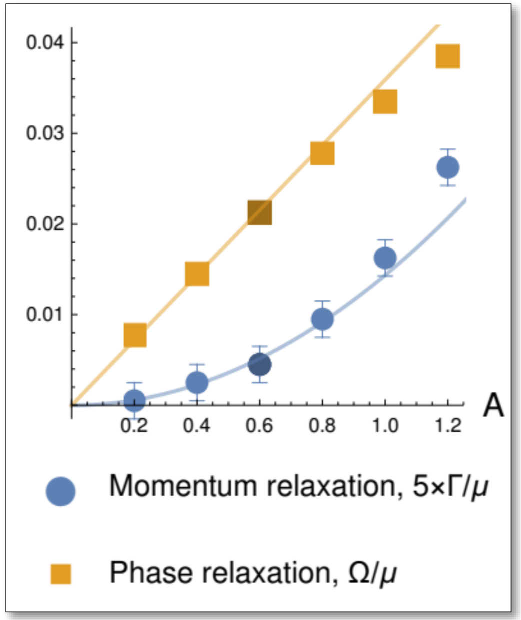


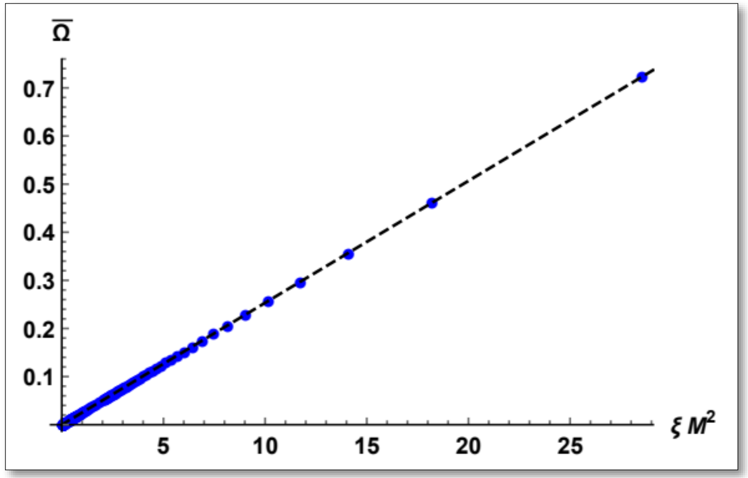
Similar to the Gell-Mann-Oakes-Renner (GMOR) relation [but for dissipative physics]

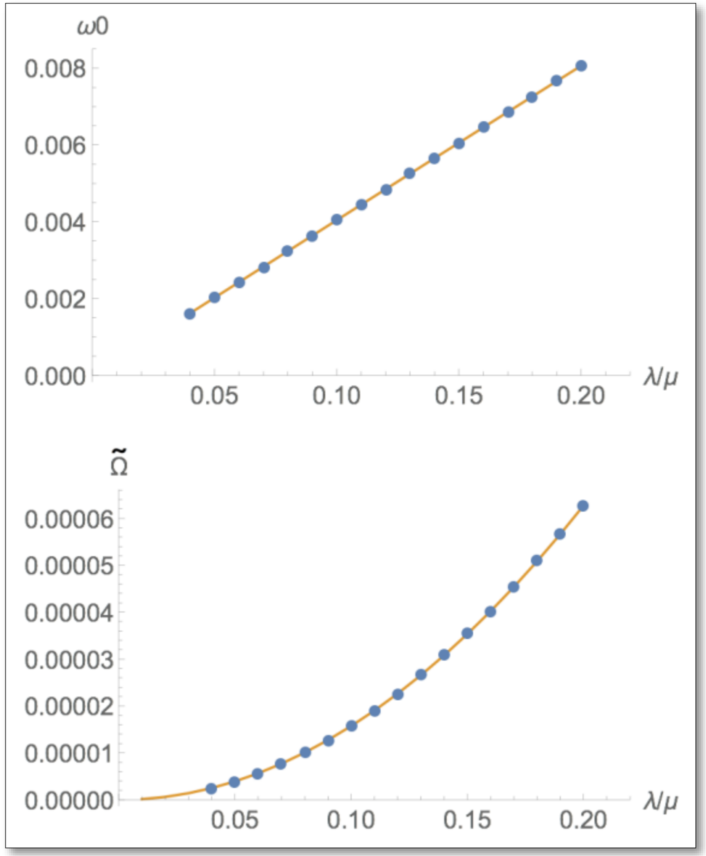
Yes, it is really universal



[I am not aware of anyone mentioning this relation before it was found in holographic models]







It has been later recognized also in QCD

[Submitted on 6 May 2020 (v1), last revised 7 Jul 2020 (this version, v2)]

Transport and hydrodynamics in the chiral limit

Eduardo Grossi, Alexander Soloviev, Derek Teaney, Fanglida Yan

It is notable that the two independent scalars comprising the superfluid expansion scalar, $\partial_\mu(f^2L^\mu)$ and $U\mathcal{M}^\dagger - \mathcal{M}U^\dagger$, must have the same dissipative coefficient, $\zeta^{(2)}$.

arising from entropy considerations, was not recognized in the linearized analysis of dissipation by Son and Stephanov [4], which leads to an additional transport coefficient in their theory[5].

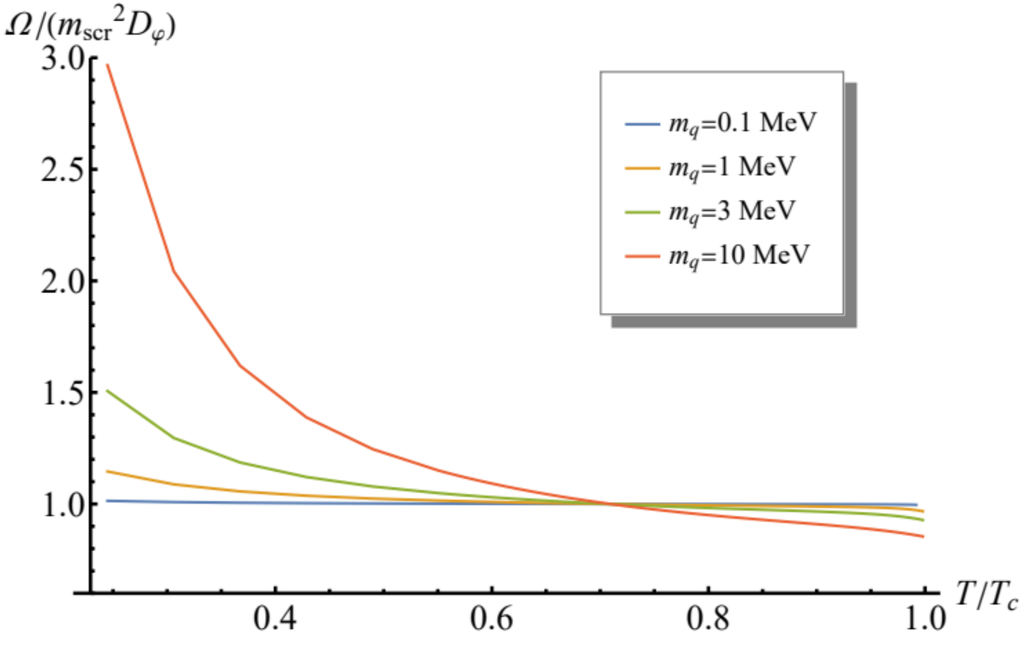
arXiv:2210.09088 [pdf, other] hep-ph hep-th doi 10.1007/JHEP12(2022)113

Pion dynamics in a soft-wall AdS-QCD model

Authors: Xuanmin Cao, Matteo Baggioli, Hui Liu, Danning Li

Verified explicitly in holographic models

$$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$$



It does not depend on the symmetry which is broken

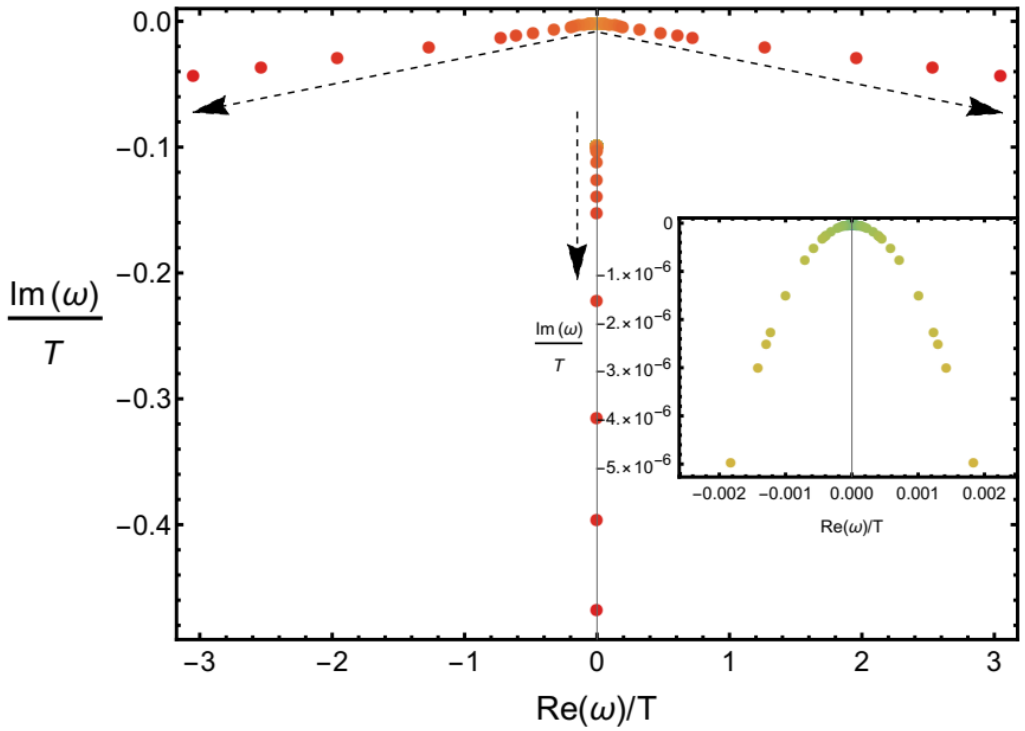
E.g.

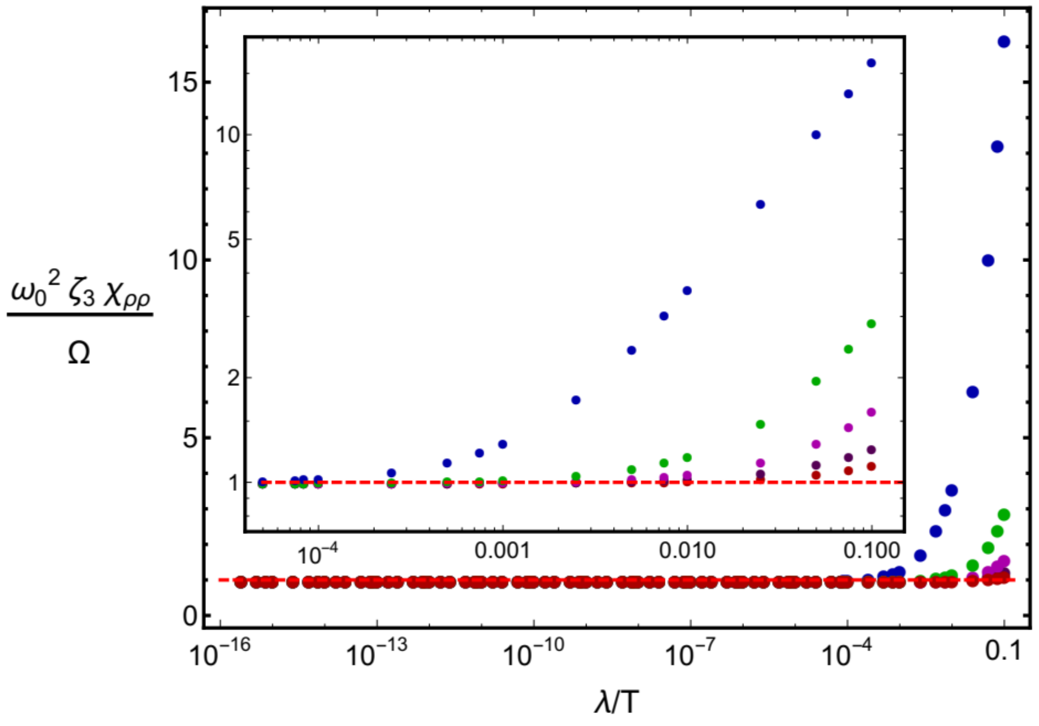
arXiv:2111.10305 [pdf, other] hep-th cond-mat.mes-hall cond-mat.str-el nucl-th doi 10.1007/JHEP03(2022)015

Pseudo-spontaneous $U(1)$ Symmetry Breaking in Hydrodynamics and Holography

Authors: Martin Ammon, Daniel Arean, Matteo Baggioli, Seán Gray, Sebastian Grieninger

In "superfluid" language
$$\Omega = \omega_0^2 \zeta_3 \chi_{\rho\rho} \qquad D_\xi = \zeta_3 \rho_s / \mu$$





Derived later from EFT/hydro

[Submitted on 6 May 2020 (v1), last revised 7 Jul 2020 (this version, v2)]

Transport and hydrodynamics in the chiral limit

Eduardo Grossi, Alexander Soloviev, Derek Teaney, Fanglida Yan

→ Entropy production in hydrodynamics

arXiv:2008.05339 [pdf, other] hep-th cond-mat.dis-nn cond-mat.mtrl-sci cond-mat.soft cond-mat.stat-mech doi 10.21468/SciPostPhys.9.5.062

Effective Field Theory for Quasicrystals and Phasons Dynamics

Authors: Matteo Baggioli, Michael Landry

→ Keldysh-Schwinger formalism : automatic !

arXiv:2111.13459 [pdf, other] hep-th cond-mat.str-el doi 10.1103/PhysRevLett.128.141601

Damping of Pseudo-Goldstone Fields

Authors: Luca V. Delacrétaz, Blaise Goutéraux, Vaios Ziogas

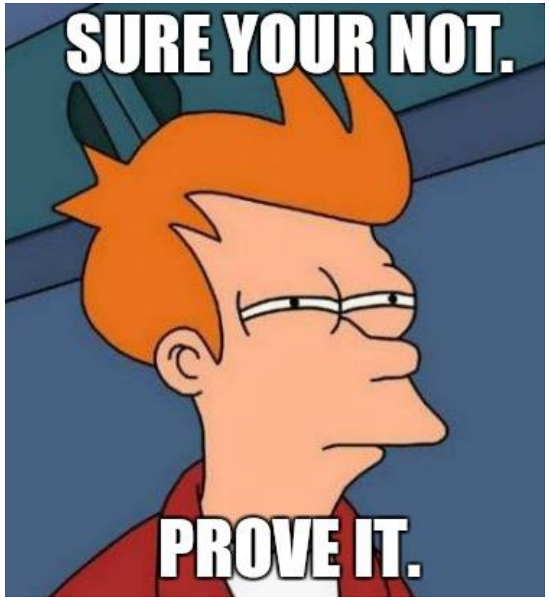
→ "locality" in hydrodynamics

arXiv:2112.14373 [pdf, ps, other] hep-th cond-mat.soft cond-mat.str-el hep-ph

Approximate symmetries, pseudo-Goldstones, and the second law of thermodynamics

Authors: Jay Armas, Akash Jain, Ruben Lier

→ Entropy production in hydrodynamics



Finally, a(nother) proposal for our beloved strange metals

arXiv:2111.13459 [pdf, other] hep-th cond-mat.str-el doi 10.1103/PhysRevLett.128.141601

Damping of Pseudo-Goldstone Fields

Authors: Luca V. Delacrétaz, Blaise Goutéraux, Vaios Ziogas

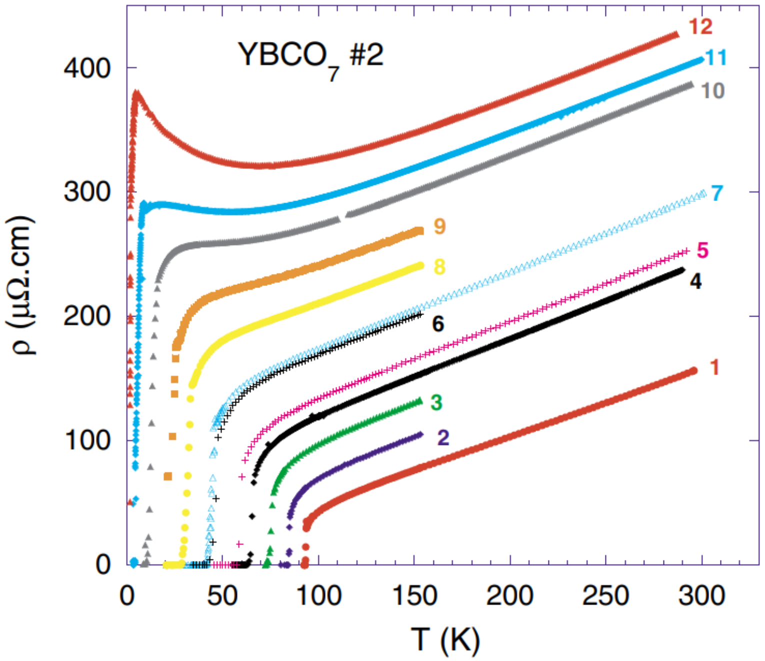
Restoring charge and heat fluctuations and assuming approximate invariance under Galilean boosts, the resistivity is (see appendix)

$$\rho_{\rm dc} = \frac{m^\star}{ne^2}\left(\Gamma + \frac{\omega_o^2}{\Omega}\right) = \frac{m^\star}{ne^2}\left(\Gamma + \frac{c_s^2}{D}\right)\,, \qquad (24)$$

+

$$D \simeq \frac{\hbar}{k_B T} v^2 \simeq \frac{\hbar}{k_B T} c_s^2$$

$$\rightarrow \quad \rho_{\rm dc} \simeq \frac{m^\star}{ne^2}\left(\Gamma + \frac{k_B T}{\hbar}\right)$$

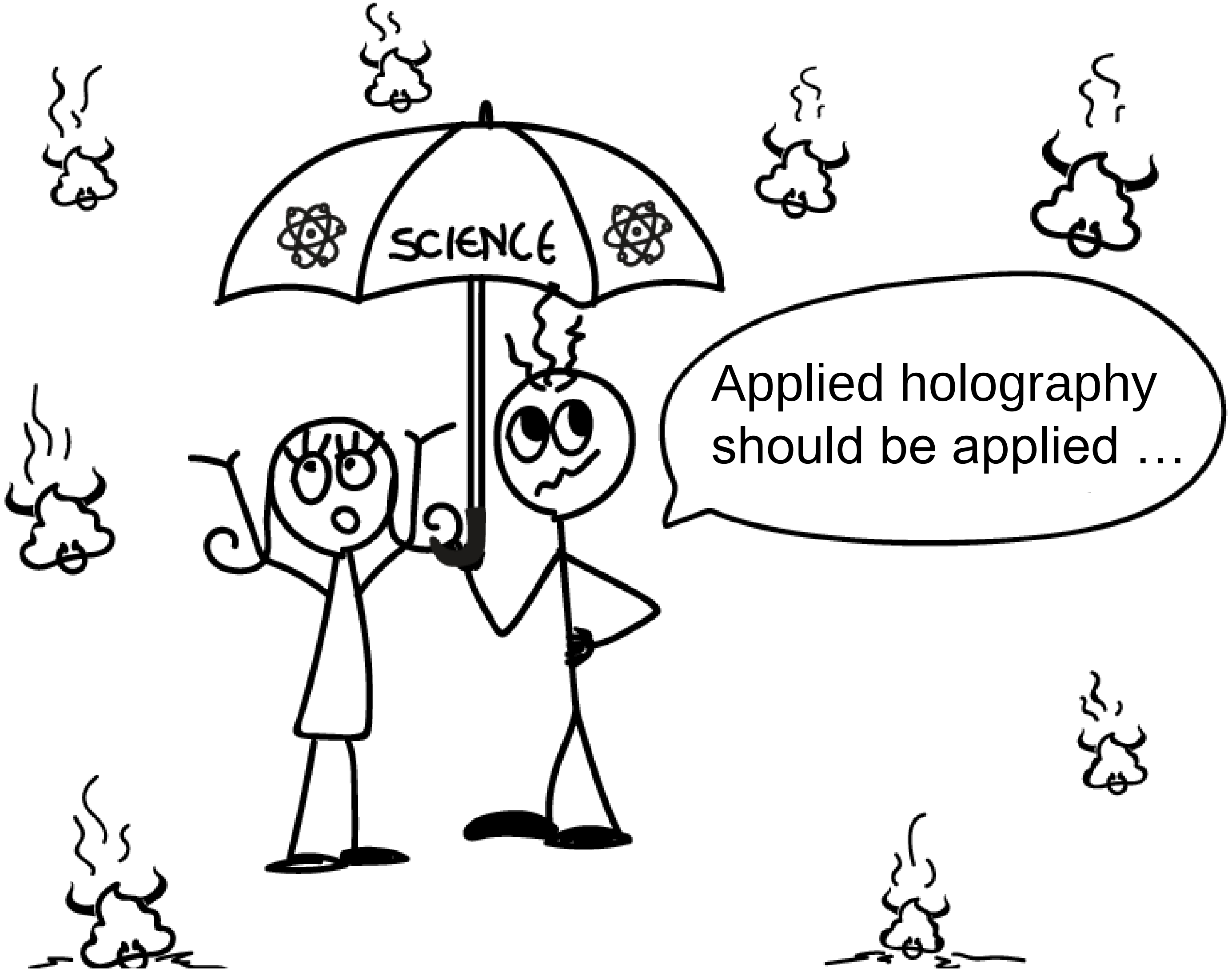


Electron irradiated
Optimally doped

Phys. Rev. Lett. 91, 047001, (2023)

Slope independent of momentum relaxation rate

What is next ?



https://thegrumpyscientist.com

Closing remarks

"Haters hate"

Critics are to build, not to destroy

arXiv:1603.09741 [pdf, ps, other] cond-mat.str-el hep-th doi 10.3952/physics.v56i3.3363
Demystifying the Holographic Mystique
Authors: D. V. Khveshchenko

arXiv:2011.11617 [pdf, ps, other] cond-mat.str-el hep-th
Die Hard Holographic Phenomenology of Cuprates
Authors: D. V. Khveshchenko

arXiv:2205.11478 [pdf, ps, other] cond-mat.str-el hep-th
SYK does not Transit Gloria Mundi just yet
Authors: D. V. Khveshchenko

Holography is certainly useful.
Holography has greatly contributed to EFT/hydrodynamics (even when not needed).
Holography has generated a new perspective in many condensed-matter problems (and not only).
Holography has prompted efforts from field theory, hydrodynamics and even experiments.
Is a big prediction missing? Maybe, but it is only 25 years old … Just on time for thinking seriously about its future

Holography@25

School: June 5-13, 2023, Workshop: June 14-17, 2023

São Paulo, Brazil

ICTP-SAIFR/IFT-UNESP