Large N Partition Functions, Holography, and Black Holes

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The Large N Limit of Superconformal field theories and supergravity

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Abstract

We show that the large N limit of certain conformal field theories in various dimensions include in their Hilbert space a sector describing supergravity on the product of Anti-deSitter spacetimes, spheres and other compact manifolds. This is shown by taking some branes in the full M/string theory and then taking a low energy limit where the field theory on the brane decouples from the bulk. We observe that, in this limit, we can still trust the near horizon geometry for large N. The enhanced supersymmetries of the near horizon geometry correspond to the extra supersymmetry generators present in the superconformal group (as opposed to just the super-Poincare group). The 't Hooft limit of 3+1 $\mathcal{N} = 4$ super-Yang-Mills at the conformal point is shown to contain strings: they are IIB strings. We conjecture that compactifications of M/string theory on various Anti-deSitter spacetimes is dual to various conformal field theories. This leads to a new proposal for a definition of M-theory which could be extended to include five non-compact dimensions.

ABJM@15

WIS/12/08-JUN-DPP

$\mathcal{N} = 6$ superconformal Chern-Simons-matter theories, M2-branes and their gravity duals

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We construct three dimensional Chern-Simons-matter theories with gauge groups $U(N) \times U(N)$ and $SU(N) \times SU(N)$ which have explicit $\mathcal{N} = 6$ superconformal symmetry. Using brane constructions we argue that the $U(N) \times U(N)$ theory at level k describes the low energy limit of N M2-branes probing a $\mathbf{C}^4/\mathbf{Z}_k$ singularity. At large N the theory is then dual to M-theory on $AdS_4 \times S^7/\mathbf{Z}_k$. The theory also has a 't Hooft limit (of large N with a fixed ratio N/k) which is dual to type IIA string theory on $AdS_4 \times \mathbf{CP}^3$. For $k = 1$ the theory is conjectured to describe N M2-branes in flat space, although our construction realizes explicitly only six of the eight supersymmetries. We give some evidence for this conjecture, which is similar to the evidence for mirror symmetry in $d = 3$ gauge theories. When the gauge group is $SU(2) \times SU(2)$ our theory has extra symmetries and becomes identical to the Bagger-Lambert theory.

June 2008

Motivation

Supersymmetric localization allows for the exact calculation of physical observables in supersymmetric QFTs.

Apply this tool to SCFTs with holographic duals in string and M-theory.

$$Z_{\mathrm{CFT}}[J] = Z_{\mathrm{string/M}}[\phi]$$

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Goal: Describe recent progress on these topics for 3d SCFTs with AdS_4 duals in M-theory.

Plan

- Motivation ✓
- The ABJM theory on S^3
- The ABJM topologically twisted index
- Holography and black holes
- Other 3d $\mathcal{N} = 2$ SCFTs

The ABJM theory on S^3

ABJM and holography

The ABJM theory: $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ CS-matter theory with two pairs of bi-fundamental chirals $(A_{1,2}, B_{1,2})$ and superpotential

$$\mathcal{W} = \mathrm{Tr}(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1)\,.$$

For $k > 2$ it has $\mathcal{N} = 6$ supersymmetry and $\mathrm{SU}(4)_R \times \mathrm{U}(1)_b$ global symmetry. Describes N M2-branes on $\mathbb{C}^4/\mathbb{Z}_k$.

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- In the limit of fixed k and large N, the ABJM theory is dual to the M-theory background $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$

$$(L/\ell_{\mathrm{P}})^6 \sim k\,N\,.$$

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- At large k and fixed 't Hooft coupling $\lambda = N/k$ the theory is dual to type IIA string theory on $\mathrm{AdS}_4 \times \mathbb{CP}^3$

$$k\,g_{\mathrm{st}} = L/\ell_{\mathrm{s}} \sim \lambda^{1/4}\,.$$

  Perturbative type IIA string theory at large k and small g_{st}, i.e. fixed λ and large N.

ABJM on S^3

The path integral on S^3 can be computed by supersymmetric localization and reduces to a matrix model [Kapustin-Willett-Yaakov]

$$Z(N,k) = \frac{1}{N!^2} \int \frac{d^N\mu}{(2\pi)^N} \frac{d^N\nu}{(2\pi)^N} \, \exp\left[\frac{\mathrm{i}k}{4\pi} \sum_{i=1}^{N} (\mu_i^2 - \nu_i^2) \right] \frac{\Pi_{i<j} \left[2\sinh(\frac{\mu_i - \mu_j}{2}) \right]^2 \left[2\sinh(\frac{\nu_i - \nu_j}{2}) \right]^2}{\Pi_{i,j} \left[2\cosh(\frac{\mu_i - \nu_j}{2}) \right]^2}$$

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Three methods have been used to study $Z(N,k)$ at large N

1. Map to CS theory on $S^3/\mathbb{Z}_2$ (or topological strings on $\mathbb{P}^1 \times \mathbb{P}^1$) and solve with large N techniques. Applies at large N, fixed N/k. [Drukker-Mariño-Putrov]
2. Study the large N limit at fixed k numerically. [Herzog-Klebanov-Pufu-Tesileanu]
3. Map the problem to a free Fermi gas on the real line with non-standard kinetic term. Valid at large N and finite k. [Mariño-Putrov]

ABJM at large N - An Airy tale

At large N and fixed k the S^3 partition function of the ABJM theory is [Mariño-Putrov], [Fuji-Hirano-Moriyama]

$$Z_{S^3} = e^{\mathcal{A}(k)} C^{-\frac{1}{3}} \mathrm{Ai}[C^{-\frac{1}{3}}(N - B)] + \mathcal{O}(e^{-\sqrt{N}})\,,$$

with

$$B = \frac{k}{24} + \frac{1}{3k}\,, \qquad\qquad C = \frac{2}{\pi^2 k}\,,$$

and

$$\begin{aligned}\mathcal{A}(k) &= \frac{2\zeta(3)}{\pi^2 k}\left(1 - \frac{k^3}{16}\right) + \frac{k^2}{\pi^2}\int_0^\infty \frac{x \log\left(1 - e^{-2x}\right)}{e^{kx} - 1}\, dx \\ &= -\frac{\zeta(3)}{8\pi^2}k^2 + 2\zeta'(-1) + \frac{1}{6}\log\frac{4\pi}{k} + \sum_{n\geq 2}\left(\frac{2\pi}{k}\right)^{2n-2}\frac{(-4)^{n-1}|B_{2n}B_{2n-2}|}{n(2n-2)(2n-2)!}\,.\end{aligned}$$

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\end{aligned}$$

The large N expansion takes the explicit form

$$-\log Z_{S^3} = \frac{2}{3\sqrt{C}}N^{\frac{3}{2}} - \frac{B}{\sqrt{C}}N^{\frac{1}{2}} + \frac{1}{4}\log N - \mathcal{A}(k) + \frac{1}{4}\log\frac{32}{k} + \mathcal{O}(N^{-\frac{1}{2}}) \,.$$

ABJM at large N - An Airy tale

This can be reorganized *à la 't Hooft* into a type IIA string theory expansion

$$F_{S^3} = -\sum_{\mathrm{g}\geq 0} (2\pi \mathrm{i}\lambda)^{2\mathrm{g}-2} F_{\mathrm{g}}(\lambda)\, N^{2-2\mathrm{g}}\,.$$

The genus g type IIA free energies can be computed systematically (up to $e^{-\sqrt{\lambda}}$ corrections) and read (agrees with topological string results)

$$\begin{aligned}
F_0(\lambda) &= \frac{4\pi^3\sqrt{2}}{3}\,\hat{\lambda}^{\frac{3}{2}} + \frac{\zeta(3)}{2}\,,\\
F_1(\lambda) &= \frac{\pi}{3\sqrt{2}}\,\hat{\lambda}^{\frac{1}{2}} - \frac{1}{4}\log\hat{\lambda} + \frac{1}{6}\log\lambda + \frac{1}{12}\log\frac{\pi^2}{32} + 2\zeta'(-1) - \frac{1}{2}\log 2\,,\\
F_2(\lambda) &= \frac{5\,\hat{\lambda}^{-\frac{3}{2}}}{96\pi^3\sqrt{2}} - \frac{\hat{\lambda}^{-1}}{48\pi^2} + \frac{\hat{\lambda}^{-\frac{1}{2}}}{144\pi\sqrt{2}} - \frac{1}{360}\,,\\
F_3(\lambda) &= \frac{5\,\hat{\lambda}^{-3}}{512\pi^6} - \frac{5\,\hat{\lambda}^{-\frac{5}{2}}}{768\pi^5\sqrt{2}} + \frac{\hat{\lambda}^{-2}}{1152\pi^4} - \frac{\hat{\lambda}^{-\frac{3}{2}}}{10368\pi^3\sqrt{2}} - \frac{1}{22680}\,,
\end{aligned}$$

where

$$\hat{\lambda} = \lambda - \frac{1}{24}\,.$$

ABJM on S^3 and holography

These results are prime targets for string/M-theory and AdS$_4$ holography!

$$-\log Z_{S^3} = \frac{\pi\sqrt{2k}}{3}\, N^{\frac{3}{2}} - \frac{\pi}{3\sqrt{2}}\left(\frac{1}{8}k^{\frac{3}{2}} + k^{-\frac{1}{2}}\right) N^{\frac{1}{2}} + \frac{1}{4}\log N + \dots .$$

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From the on-shell action of the AdS$_4 \times X_7$ supergravity solution one finds

$$F_{S^3} = -\log Z_{S^3} = \sqrt{\frac{2\pi^6}{27\mathrm{vol}(X_7)}} N^{\frac{3}{2}} \,.$$

Plug in $\mathrm{vol}(S^7/\mathbb{Z}_k) = \frac{\pi^4}{3k}$ to find a match with the localization result.

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The $\log N$ term is obtained by summing over the KK modes around AdS$_4 \times S^7$. [Bhattacharyya-Grassi-Mariño-Sen]

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Note: Obtain the full Airy function using supersymmetric localization in supergravity on AdS$_4$?[Dabholkar-Drukker-Gomes]

Adding sources

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The ABJM S^3 partition function with a $\mathrm{U}(1) \times \mathrm{U}(1)$ invariant squashing and real mass deformation takes the form [NPB-Hong-Reys], [Nosaka], [Hatsuda], [Hristov], [Chester-Kalloor-Sharon]

$$Z_{S^3}(N, k, \Delta) = e^{\mathcal{A}(k,\Delta)} C_k^{-\frac{1}{3}} \mathrm{Ai}[C_k^{-\frac{1}{3}}(N - B_k)] + \mathcal{O}(e^{-\sqrt{N}})$$

with

$$C_k = \frac{2}{\pi^2 k} \frac{(b + b^{-1})^{-4}}{\prod_{a=1}^4 \Delta_a}\,, \quad B_k = \frac{k}{24} - \frac{1}{12k} \sum_{a=1}^{4} \frac{1}{\Delta_a} + \frac{1 - \frac{1}{4}\sum_a \Delta_a^2}{3k(b + b^{-1})^2 \prod_{a=1}^4 \Delta_a}\,,$$

and

$$\Delta_1 = \frac{1}{2} - \mathrm{i}\,\frac{m_1 + m_2 + m_3}{b + b^{-1}}\,,\ \Delta_2 = \frac{1}{2} - \mathrm{i}\,\frac{m_1 - m_2 - m_3}{b + b^{-1}}\,,$$
$$\Delta_3 = \frac{1}{2} + \mathrm{i}\,\frac{m_1 + m_2 - m_3}{b + b^{-1}}\,,\ \Delta_4 = \frac{1}{2} + \mathrm{i}\,\frac{m_1 - m_2 + m_3}{b + b^{-1}}\,,$$

such that $\sum_a \Delta_a = 2$.

Adding sources

This result encodes integrated correlation functions of the ABJM theory on $\mathbb{R}^3$.

Expand at large N and use holography to constrain/compute the higher-derivative corrections to type II string theory and M-theory. [Chester-Pufu-Yin], [Binder-Chester-Pufu], ...

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Example:

Squashed S^3 partition function

$$F_{S_b^3} = \frac{\pi\sqrt{2k}}{12}\left[(b+b^{-1})\left[N^{\frac{3}{2}} + \left(\frac{1}{k} - \frac{k}{16}\right)N^{\frac{1}{2}}\right] - \frac{6}{k}N^{\frac{1}{2}}\right] + \frac{1}{4}\log N + \mathcal{O}(N^0)\,.$$

This captures integrated correlators of $T_{\mu\nu}$ for the ABJM theory in flat space!

$$C_T = \frac{32}{\pi^2}\left(\frac{\partial^2 F_{S_b^3}}{\partial b^2}\right)_{b=1} = \frac{64\sqrt{2k}}{3\pi}N^{\frac{3}{2}} + \frac{4(16-k^2)\sqrt{2}}{3\pi\sqrt{k}}N^{\frac{1}{2}} + \mathcal{O}(N^0)\,,$$

where

$$\langle T_{\mu\nu}T_{\rho\sigma}\rangle = \frac{C_T}{(48\pi)^2}\left(P_{\mu\rho}P_{\nu\sigma} + P_{\nu\rho}P_{\mu\sigma} - P_{\mu\nu}P_{\rho\sigma}\right)\frac{1}{\vec{x}^2}\,, \quad P_{\mu\nu} \equiv \delta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu\,.$$

What about the ABJM partition function on other 3-manifolds?

The ABJM topologically twisted index

TTI

The topologically twisted index (TTI) is a partition function of a 3d $\mathcal{N} = 2$ SCFT on $S^1 \times \Sigma_{\mathfrak{g}}$. Supersymmetry is preserved by Witten's topological twist on $\Sigma_{\mathfrak{g}}$. The 3d QFT is not topological. Using supersymmetric localization the path integral can be reduced to a matrix integral.[Benini-Zaffaroni], [Closset-Kim]

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For the ABJM theory the result is

$$
\begin{aligned}
Z_{S^1\times\Sigma_{\mathfrak{g}}}(N,k,\Delta,\mathfrak{n}) = {} & \frac{1}{(N!)^2} \sum_{\mathfrak{m},\tilde{\mathfrak{m}}\in\mathbb{Z}_N} \oint_{\mathcal{C}} \prod_{i=1}^{N} \frac{dx_i}{2\pi \mathrm{i} x_i} \prod_{j=1}^{N} \frac{d\tilde{x}_j}{2\pi \mathrm{i} \tilde{x}_j} \prod_{i=1}^{N} x_i^{k\mathfrak{m}_i} \prod_{j=1}^{N} \tilde{x}_j^{-k\tilde{\mathfrak{m}}_j} \\
& \times \left(\det \mathbb{B}(N,k,x,\tilde{x},\Delta)\right)^{\mathfrak{g}} \times \prod_{i\neq j}^{N} \left(1-\frac{x_i}{x_j}\right)^{1-\mathfrak{g}} \left(1-\frac{\tilde{x}_i}{\tilde{x}_j}\right)^{1-\mathfrak{g}} \\
& \times \prod_{i,j=1}^{N} \prod_{a=1,2} \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j} y_a}}{1-\frac{x_i}{\tilde{x}_j} y_a}\right)^{\mathfrak{m}_i-\tilde{\mathfrak{m}}_j+1-\mathfrak{g}-\mathfrak{n}_a} \prod_{a=3,4} \left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i} y_a}}{1-\frac{\tilde{x}_j}{x_i} y_a}\right)^{-\mathfrak{m}_i+\tilde{\mathfrak{m}}_j+1-\mathfrak{g}-\mathfrak{n}_a} .
\end{aligned}
$$

Here $y_a = e^{\mathrm{i}\pi\Delta_a}$ and supersymmetry imposes

$$
\sum_{a=1}^{4} \Delta_a = 2\,, \qquad \sum_{a=1}^{4} \mathfrak{n}_a = 2(1-\mathfrak{g})\,.
$$

TTI

Using (subtle) contour integration the TTI can be rewritten as

$$Z = \prod_{a=1}^{4} y_a^{-\frac{N^2}{2}\mathfrak{n}_a} \sum_{\{x_i,\tilde{x}_j\}} \left[\frac{1}{\det \mathbb{B}} \frac{\prod_{i=1}^{N} x_i^N \tilde{x}_i^N \prod_{i\neq j}^{N} (1-\frac{x_i}{x_j})(1-\frac{\tilde{x}_i}{\tilde{x}_j})}{\prod_{i,j=1}^{N} \prod_{a=1}^{2} (\tilde{x}_j - x_i y_a)^{1-\frac{\mathfrak{n}_a}{1-\mathfrak{g}}} \prod_{a=3}^{4} (x_i - \tilde{x}_j y_a)^{1-\frac{\mathfrak{n}_a}{1-\mathfrak{g}}}} \right]^{1-\mathfrak{g}}$$

Where x_i and $\tilde{x}_i$ are solutions to the following "Bethe Ansatz Equations"

$$e^{\mathrm{i}B_i} \equiv x_i^k \prod_{j=1}^{N} \frac{(1-y_3\frac{\tilde{x}_j}{x_i})(1-y_4\frac{\tilde{x}_j}{x_i})}{(1-y_1^{-1}\frac{\tilde{x}_j}{x_i})(1-y_2^{-1}\frac{\tilde{x}_j}{x_i})} = 1\,,$$

$$e^{\mathrm{i}\tilde{B}_j} \equiv \tilde{x}_j^k \prod_{i=1}^{N} \frac{(1-y_3\frac{\tilde{x}_j}{x_i})(1-y_4\frac{\tilde{x}_j}{x_i})}{(1-y_1^{-1}\frac{\tilde{x}_j}{x_i})(1-y_2^{-1}\frac{\tilde{x}_j}{x_i})} = 1\,,$$

and the Jacobian matrix $\mathbb{B}$ is given by

$$\mathbb{B} = \frac{\partial(e^{\mathrm{i}B_1},\cdots,e^{\mathrm{i}B_N},e^{\mathrm{i}\tilde{B}_1},\cdots,e^{\mathrm{i}\tilde{B}_N})}{\partial(\log x_1,\cdots,\log x_N,\log\tilde{x}_1,\cdots,\log\tilde{x}_N)}\,.$$

TTI at large N

The BAE solution in the large N limit takes the form [Benini-Hristov-Zaffaroni]

$$\log x_i = N^{\frac{1}{2}}\, t_i - \mathrm{i} v_i\,, \quad \log \tilde{x}_j = N^{\frac{1}{2}}\, t_j - \mathrm{i}\tilde{v}_j\,.$$

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Our approach: Use this solution as a starting point to numerically solve the BAE and calculate the index. The numerical results are very precise and led us to an analytic form for the TTI to all orders in N!

TTI at large N

The BAE solution in the large N limit takes the form [Benini-Hristov-Zaffaroni]

$$\log x_i = N^{\frac{1}{2}}\, t_i - \mathrm{i} v_i\,, \quad \log \tilde{x}_j = N^{\frac{1}{2}}\, t_j - \mathrm{i}\tilde{v}_j\,.$$

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To write the result compactly define

$$\hat{N}_\Delta \equiv N - \frac{k}{24} + \frac{1}{12k}\sum_{a=1}^{4}\frac{1}{\Delta_a}\,,$$

in terms of which $F_{S^1\times\Sigma_\mathfrak{g}} = -\log Z_{S^1\times\Sigma_\mathfrak{g}}$, takes the simple form:

$$F_{S^1\times\Sigma_\mathfrak{g}} = \frac{\pi\sqrt{2k\Delta_1\Delta_2\Delta_3\Delta_4}}{3}\sum_{a=1}^{4}\frac{\mathfrak{n}_a}{\Delta_a}\left(\hat{N}_\Delta^{\frac{3}{2}} - \frac{\mathfrak{c}_a}{k}\hat{N}_\Delta^{\frac{1}{2}}\right) + \frac{1-\mathfrak{g}}{2}\log\hat{N}_\Delta - \hat{f}_0(k,\Delta,\mathfrak{n})\,,$$

where $\mathfrak{c}_a$ are given by

$$\mathfrak{c}_a = \frac{\prod_{b\neq a}(\Delta_a+\Delta_b)}{8\Delta_1\Delta_2\Delta_3\Delta_4}\sum_{b\neq a}\Delta_b\,.$$

The universal index

The universal index is defined by setting $\Delta_a = \frac{1}{2}$ and $\mathfrak{n}_a = \frac{1-\mathfrak{g}}{2}$. We then define $\hat{N} = N - \frac{k}{24} + \frac{2}{3k}$ to obtain

$$\frac{F_{S^1 \times \Sigma_{\mathfrak{g}}}}{1-\mathfrak{g}} = \frac{\pi\sqrt{2k}}{3}\left(\hat{N}^{\frac{3}{2}} - \frac{3}{k}\hat{N}^{\frac{1}{2}}\right) + \frac{1}{2}\log\hat{N} - \hat{f}_0(k)\,.$$

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No closed form expression for $\hat{f}_0(k)$ but at large k we find

$$\hat{f}_0(k) = -\frac{3\zeta(3)}{8\pi^2}k^2 + \frac{7}{6}\log k + \mathfrak{f}_0 + \sum_{n=1}^{5}\left(\frac{2\pi}{k}\right)^{2n}\frac{\mathfrak{f}_{2n}}{3^{n+2}} + \mathcal{O}(k^{-12})\,,$$

with $\{\mathfrak{f}_{2n}\} = \left\{-\frac{6}{5},\ \frac{19}{70},\ -\frac{41}{175},\ \frac{279}{700},\ -\frac{964636}{875875}\right\}$ and $\mathfrak{f}_0 = -2.096848299$.

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For low values of k, $\hat{f}_0(k)$ can be determined numerically with very good precision:

$$\hat{f}_0(1) = -3.045951311\,, \quad \hat{f}_0(2) = -1.786597534\,,$$
$$\hat{f}_0(3) = -1.386373044\,, \quad \hat{f}_0(4) = -1.306589553\,.$$

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We have checked these results with extensive numerical calculations to great accuracy. They are exact up to $e^{-\sqrt{N}}$ corrections.

The universal index

This result can be reorganized as a type IIA string theory expansion

$$F_{S^1\times\Sigma_\mathfrak{g}} = -\sum_{g\geq 0}(2\pi\mathrm{i}\lambda)^{2g-2}F_g(\lambda)\,N^{2-2g}\,.$$

For low genera we find

$$\begin{aligned}
\frac{F_0(\lambda)}{1-\mathfrak{g}} &= \frac{4\pi^3\sqrt{2}}{3}\,\hat{\lambda}^{\frac{3}{2}} + \frac{3\zeta(3)}{2}\,,\\
\frac{F_1(\lambda)}{1-\mathfrak{g}} &= \frac{2\pi\sqrt{2}}{3}\,\hat{\lambda}^{\frac{1}{2}} - \frac{1}{2}\log\hat{\lambda} - \frac{2}{3}\log\lambda + \mathfrak{f}_0\,,\\
\frac{F_2(\lambda)}{1-\mathfrak{g}} &= \frac{\hat{\lambda}^{-1}}{12\pi^2} - \frac{5\hat{\lambda}^{-\frac{1}{2}}}{36\sqrt{2}\pi} + \frac{2}{45}\,,\\
\frac{F_3(\lambda)}{1-\mathfrak{g}} &= \frac{\hat{\lambda}^{-2}}{144\pi^4} - \frac{\hat{\lambda}^{-\frac{3}{2}}}{162\sqrt{2}\pi^3} + \frac{19}{5670}\,,
\end{aligned}$$

where $\hat{\lambda} = \lambda - \frac{1}{24}$.

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where $\hat{\lambda} = \lambda - \frac{1}{24}$.

How can we derive this from type IIA string theory?

Holography and black holes

The universal black hole

The following is a 4d supersymmetric Euclidean solution of 4d $\mathcal{N}=2$ supergravity [Romans], [Benetti Genolini-Ipiña-Sparks], [NPB-Charles-Min], ...

$$ds_4^2 = U(r)d\tau^2 + \frac{dr^2}{U(r)} + r^2 ds^2_{\Sigma_\mathfrak{g}}\,, \qquad F = \frac{Q}{r^2}d\tau\wedge dr - \frac{\kappa}{g}\mathrm{vol}(\Sigma_\mathfrak{g})\,,$$

$$U(r) = \left(\sqrt{2}gr + \frac{\kappa}{2\sqrt{2}gr}\right)^2 - \frac{Q^2}{8r^2}\,, \qquad \kappa = 1, 0, -1.$$

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The solution is smooth for $g|Q| > \kappa$. The periodicity of τ is

$$\beta_\tau = \frac{\pi\sqrt{-\kappa + g|Q|}}{g^2|Q|}\,.$$

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The smooth Lorentzian black hole is obtained by taking $Q \to 0$ and exists only for $\kappa = -1$, i.e. $\mathfrak{g} > 1$. The regularized on-shell action for any β_τ is

$$I = -\frac{\pi}{4g^2 G_N}(\mathfrak{g} - 1)\,.$$

For the black hole solution this leads to $S_{\mathrm{BH}} = -I$.[Gibbons-Hawking],...

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This is nothing but the extremal magnetic Reissner-Nordström black hole in AdS_4!

The universal black hole

An 11d uplift of this solution is given by

$$\begin{aligned} ds_{11}^2 &= \tfrac{1}{4} ds_4^2 + ds_{\mathbb{CP}^3}^2 + \left(d\psi + \sigma_{\mathbb{CP}^3} + \tfrac{1}{4} A \right)^2 \,, \\ G_4 &= \tfrac{3}{8} \mathrm{vol}_4 - \tfrac{1}{4} \star_4 F \wedge J_{\mathbb{CP}^3} \,, \quad \text{with} \quad d\sigma_{\mathbb{CP}^3} = 2 J_{\mathbb{CP}^3} \,. \end{aligned}$$

Our QFT result for the TTI amounts to a prediction for the path integral of M-theory on this background to all orders in the $1/N$ expansion.

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$$F_{S^1\times\Sigma_{\mathfrak{g}}} = (1-\mathfrak{g})\frac{\pi\sqrt{2k}}{3}\left(N^{\frac{3}{2}} - \frac{32+k^2}{16k}N^{\frac{1}{2}}\right) + \frac{(1-\mathfrak{g})}{2}\log N + \dots\,.$$

The $N^{\frac{3}{2}}$ term comes from the 2-der on-shell action.[Benini-Hristov-Zaffaroni], [Azzurli-NPB-Crichigno-Min-Zaffaroni]

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An all-order prediction for the entropy of this black hole?

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An all-order prediction for the entropy of this black hole?

Similar results for other more complicated AdS$_4$ black hole solutions (ask me later!).

Other 3d $\mathcal{N} = 2$ SCFTs

Other examples

Similar results for the TTI of other 3d holographic SCFTs (no Airy function on S^3 for some of these!)

- 3d $\mathcal{N}=4$ SYM $+\ N_f$ fundamentals. We have $\hat{N}=N+\frac{7N_f}{24}+\frac{1}{3N_f}$

$$\frac{F_{S^1\times\Sigma_\mathfrak{g}}}{1-\mathfrak{g}}=\frac{\pi\sqrt{2}}{3}N_f^{\frac{1}{2}}\left[\hat{N}^{\frac{3}{2}}-\left(\frac{N_f}{2}+\frac{5}{2N_f}\right)\hat{N}^{\frac{1}{2}}\right]+\frac{1}{2}\log\hat{N}-\hat{f}_0(N_f)\,,$$

- 3d $\mathcal{N}=2$ $V^{5,2}$ theory. We have $\hat{N}=N+\frac{k}{6}+\frac{1}{4k}$ and

$$\frac{F_{S^1\times\Sigma_\mathfrak{g}}}{1-\mathfrak{g}}=\frac{16\pi\sqrt{k}}{27}\left[\hat{N}^{\frac{3}{2}}-\left(\frac{9k}{16}+\frac{27}{16k}\right)\hat{N}^{\frac{1}{2}}\right]+\frac{1}{2}\log\hat{N}-\hat{f}_0(k)\,,$$

- 3d $\mathcal{N}=2$ Q^{111} theory. We have $\hat{N}=N+\frac{k}{6}$ and

$$\frac{F_{S^1\times\Sigma_\mathfrak{g}}}{1-\mathfrak{g}}=\frac{4\pi\sqrt{k}}{3\sqrt{3}}\left[\hat{N}^{\frac{3}{2}}-\left(\frac{k}{4}+\frac{3}{4k}\right)\hat{N}^{\frac{1}{2}}\right]+\frac{1}{2}\log\hat{N}-\hat{f}_0(k)\,,$$

- 3d $\mathcal{N}=3$ N^{010} theory. We have $\hat{N}=N+\frac{k}{12}+\frac{1}{3k}$ and

$$\frac{F_{S^1\times\Sigma_\mathfrak{g}}}{1-\mathfrak{g}}=\frac{4\pi\sqrt{k}}{3\sqrt{3}}\left[\hat{N}^{\frac{3}{2}}-\left(\frac{k}{4}+\frac{5}{4k}\right)\hat{N}^{\frac{1}{2}}\right]+\frac{1}{2}\log\hat{N}-\hat{f}_0(k)\,,$$

In each of these cases there is a BPS black hole solution for which the index accounts for the entropy.

Summary

- Presented exact results for the large N limit of the partition function of the ABJM theory on S^3 and $S^1 \times \Sigma_{\mathfrak{g}}$.
- Discussed how some of these results can be reproduced by string/M-theory via AdS/CFT.
- All order microscopic prediction for the entropy of the extremal AdS$_4$ magnetic Reissner-Nordström black hole.
- Generalization of these results to some other 3d $\mathcal{N} = 2$, $\mathcal{N} = 3$, and $\mathcal{N} = 4$ holographic SCFTs.

Outlook

- Extend to other 3d $\mathcal{N} = 2$ holographic SCFTs. [in progress]
- Analytic derivation of our results.
- Partition functions on other compact 3-manifolds.
  We can also calculate the SCI!
- Understand the shift in N from M-theory. [Bergman-Hirano], [in progress]
- Implications for the higher-derivative corrections to 4d and 11d supergravity?
- Supersymmetric localization in 4d supergravity?
- Derivation from (or lessons for) type IIA string theory and M-theory?
- OSV-type relation for AdS black holes?


thank you
merci
gracias
danke
obrigado
謝謝
teşekkür ederim
go raibh maith agat
ngiyabonga
спасибо
bedankt
dziękuję
sukriya
kop khun krap
terima kasih
감사합니다
arigatô
grazie
dakujem
mochchakkeram
tapadh leat
dank je
takk
taiku
sagolun
mauruuru
hvala
рахмат
Баярлалаа
faafetai lava
vinaka
спасибі
blagodaram
misaotra
matondo
paldies
grazzi
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хвала
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trugarez
мерси
shukriya
mercê
dhanyavadagalu
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ευχαριστώ
xièxie
rahmet
tanemirt
ありがとう
najis tuke
kam sah hamnida
rahmat
didi madloba
mèsi
dĕkuji
sobodi
enkosi
bayarlalaa
gràcie
kiitos
dankie
nanni
nandri
dhanyavad
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sulpáy
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dankon
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