1 - Vector mesons to probe quark axial current
2 - Quark-meson mixings: flavor symmetry breaking/sea quarks

Fabio L. Braghin

Instituto de Física - Universidade Federal de Goiás
braghin@ufg.br

POETIC23 - Inst. Principia / ICTP- SAIFR - IFT-UNESP
May 5th 2023
Talk based on:


* F.L.B., Flavor-dependent corrections for the U(3) NJL coupling constant, Phys. Rev. D 103, 094028 (2021),


Principia Institute + ICTP-SAIFR (workshop)
1 Motivations/context

2 Vector meson coupling to constituent quark axial current
   Quark-antiquark interaction - dynamical calculation
   Relation to $g_A$ and form factor

3 Quark-antiquark mesons + sea-quarks in improved NJL model
   Quark polarization in the NJL model - FSB
   A calculation on U(5) NJL
   Pion strangeness content leading to Meson Mixing
   Quark-antiquark states of light scalars

4 Summary

* Few slides presented in the talk have been withdrawn.
(Low energy) QCD effective models: **global hadron properties**
* Dynamical Chiral Symmetry Breaking: \(< \bar{q}q >\) masses/couplings

* Some models ”Near-exausted” (?) resources: still phenomenology and test-model

**Nambu-Jona-Lasinio (NJL) model**: low energy QCD/Quark model
\(\sim\) punctual interactions \(G_0 \sim 1/M^2_G\) or \(1/\Lambda^2\) valence quarks
Usually improvements rely on further free parameters

Strangeness content of nucleon (electromag. \(\sim 5\%\))
Charm content of nucleon: Brodsky, Hoyer, Peterson/many (1%)
LHCb-NNPDF: evidence \(3\sigma\) c.l. (?)

**Outcome → quarks/meson mixings: sea quarks**
* Flavor symmetry breaking (FSB)
* Spin content of the nucleon (hadrons) from axial current
* Pion and axial mesons (unstables) to nucleon: axial charge (nucleon or constituent quark)
* Roughly: **axial mesons** as chiral partners ($\rho - A_1$ and $\omega - f_1$)
* Non-central collisions: vector mesons production

- Straightforward dynamical (one-loop polarization) calculation of **leading meson couplings to constituent quarks**

_F.L.B., Phys. Rev. D105, 054009 (2022); Phys. Rev. (2019); E (2023)

* Vector mesons probe/couples to axial current
* If yes, Even a photon could probe the axial current (by VMD)
Vector mesons couplings to axial current
(dynamically generated)

\[ Z[\eta, \bar{\eta}] = N \int D[\bar{\psi}, \psi] \exp i \int d^4x \left[ \bar{\psi} (i\slashed{\partial} - m) \psi - \frac{g^2}{2} \int_y j^\beta_\mu(x) \tilde{R}^{\mu\nu}_{\beta\alpha}(x - y) j^\alpha_\nu(y) + \bar{\psi} \eta + \bar{\eta} \psi \right] \]

Color quark current \( j^\mu_\alpha = \bar{\psi} \lambda_\alpha \gamma^\mu \psi \),
\( i, j, k = 0, \ldots (N_f^2 - 1) \) for U(\( N_f = 2 \)), \( \alpha, \beta \ldots = 1, \ldots (N_c^2 - 1) \)

Fierz transformation \( \rightarrow \) all flavor-Dirac channels
Auxiliary fields: suitable for quark-antiquark states
Leading couplings: meson-constituent quarks

Expansion of quark determinant (some ambiguities-symmetries)

Leading meson-constituent quark couplings (form factors)

\[
\mathcal{L}_{j_A} = \left[ G_A(Q, K)Q_\mu \pi^i(Q) + G_{\bar{A}}(Q, K)\bar{A}^i_\mu(Q) \right] j^{\mu}_{A,i}(K, Q),
\]

\[
\mathcal{L}_{v-q} = g_{r1}(Q, K)V^\mu_i(Q)j^{V,i}_{\mu}(K, Q) + g_{A1}(Q, K)\bar{A}^\mu_i(Q)j^{A,i}_{\mu}(K, Q)
\]

\[
+ g_{v1}(Q, K)V^\mu(Q)j^{\mu}_i(K, Q) + g_{f1}(Q, K)\bar{A}^\mu(Q)j^{\mu}_{A}(K, Q), \tag{1}
\]

\(G_A(Q, K), g_{r1}(Q, K), g_{A1}(Q, K), g_{f1}(Q, K)\) are one loop integrals

Coupling constants \((K = Q = 0)\) or \((Q^2 = M_{\pi}^2)\) ..

Numerically: correct order of magnitude (renormalization=1-fit)
Quark determinant: Emergence of Gluon and pion clouds

* Gluon cloud: dressing to quark currents → constituent quarks

* Pion cloud from the Goldstone boson couplings to (all) quark currents

* Last part of the talk: Flavor symmetry breaking → emergence of diverse sea quark "cloud"
Wess Zumino Witten type coupling

Next leading terms

For isosinglet $V_\mu$ and isotriplet $V_\mu^i$ mesons

\[
\mathcal{L}_{\nu\lambda} = i\delta_{ij}\epsilon^{\rho\mu\nu} F_{\lambda\rho}\mathcal{F}_{\rho\mu}(Q)j_{\nu}^{A,j}(K, K + Q) + i\epsilon^{\rho\mu\nu} F_{\lambda\rho}\mathcal{F}_{\rho\mu}(Q)j_{\nu}^{A,j}(K, K + Q),
\]

(2)

\[
j_{\mu}^{A,i}(K, K + Q) = \bar{\psi}(K + Q)\gamma_{\mu}\gamma_{5}\sigma^{i}\psi(K) \text{ and}
\]

\[
j_{\mu}^{A}(K, K + Q) = \bar{\psi}(K + Q)\gamma_{\mu}\gamma_{5}\psi(K).
\]

* Polarized vector meson, transversal directions in $\epsilon^{\rho\mu\nu}$

\[
\mathcal{F}_{\rho\mu}(Q) = Q_\rho V_\mu^i(Q) - Q_\mu V_\rho^i(Q), \quad \mathcal{F}_{\mu}(Q) = Q_\rho V_\mu(Q) - Q_\mu V_\rho(Q).
\]
Chiral partners: axial mesons-vector current

For isosinglet $\bar{A}_\mu$ and isotriplet $\bar{A}^i_\mu$ mesons

$$\mathcal{L}_{vja-A} = i\epsilon^{\sigma\rho\mu\nu} F_{vja}^{\sigma}(K, Q) K_\sigma G^i_{\rho\mu}(Q) j^V_{\nu,i}(K, K + Q)$$
$$+ i\epsilon^{\sigma\rho\mu\nu} F_{vja}^{\sigma}(K, Q) K_\sigma G^i_{\rho\mu}(Q) j^V_{\nu}(K, K + Q), \quad (3)$$

$$j^V_{\mu,i}(K, K + Q) = \bar{\psi}(K + Q) \gamma_\mu \sigma^i \psi(K)$$
$$j^V_{\mu}(K, K + Q) = \bar{\psi}(K + Q) \gamma_\mu \psi(K).$$

$$G^i_{\mu\nu} = \partial_\mu \bar{A}^i_\nu - \partial_\nu \bar{A}^i_\mu, \quad G_{\mu\nu} = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu. \quad (4)$$
Axial pion coupling and the $\rho$ coupling

Couplings to the axial current (out of 8 structures Ball-Chiu)

$$\mathcal{L}_{j_A} = \left[ G_A Q_\mu \pi^i(Q) + G_A^{-1} \bar{A}_\mu^i(Q) + i F_{\nu j a} \epsilon_{\mu \nu \rho \sigma} K^\nu Q^\rho j_{i}(Q) \right]$$

$$\times j_{\mu i}(K, Q), \quad (5)$$

From the same method:

$$\frac{F_{\nu j a}(K, Q)}{G_A(K, Q)} = \frac{1}{4 M^* F} = \text{constant.} \quad (6)$$

Renormalization condition can be $G_A \sim 1$.

(Relativistic Consituent quark model - S.Weinberg + GT relation)

$$\frac{F_{\nu j a}(K, Q) \times |K||Q|}{G_V(K, Q)} \bigg|_{Q \sim K \sim 200-500 \text{MeV}} \sim 0.1. \quad (7)$$

What happens at high energies?
Can a Photon probe the axial current (Vector Meson Dominance)?
Witten’s procedure: quantization

\( \mathcal{L}_{vja} \) as a 5dim closed surface (Stoke’s theorem)

\[
n \Gamma = -\epsilon^{\sigma\rho\mu\nu} \frac{i}{240\pi^2} \int d^4K \ d^4Q \ F^{vja}(K, Q) K_\sigma F^i_{\rho\mu}(Q) j^A_{\nu}\iota(K, K + Q), \quad (8)
\]

\( n \) is an integer: \( \Gamma = \epsilon_{\sigma\rho\mu\nu} \Gamma^{\sigma\rho\mu\nu} \)

Quantized integrals (Sum over \( \mu\nu\rho\sigma \)) contain integrals of the type

\[
\Gamma_{(xyz0)} = -\frac{i}{240\pi^2} \int d^4K \ d^4Q \ F^{vja}(K, Q) K_x Q_y \\
\times \left[ \rho^-_{z}(Q)\bar{u}(K + Q)\gamma_0\gamma_5 d(K) + \rho^+_{z}(Q)\bar{d}(K + Q)\gamma_0\gamma_5 u(K) \right], \quad (9)
\]

* \( \rho^\pm_{z}(Q) = z\)-polarization component

From the last slide (one loop - rainbow ladder):

\[
\frac{F_{vja}(K, Q)}{G_A(K, Q)} = \frac{1}{4M^* F}. \quad (10)
\]

* Sum of \( \Gamma^{\sigma\rho\mu\nu} \rightarrow "\text{sum rule}" \)
**Figure:** Form factor $G_{vja}(K, Q)$ for effective gluon propagator (Tandy-Maris) as a function $Q^2$ for different values of $K$. Two effective masses $M^* = 0.33\text{GeV}$ and $M^* = 0.45\text{ GeV}$. 
\[ \Delta_A < r^2_\rho >= -6 \left. \frac{d \tilde{G}_{vja}}{dQ^2} \right|_{Q=0}, \quad < r^2_\rho > \simeq 0.28 - 0.56 \text{fm}^2 \]


\[ \sqrt{\Delta_A < r^2_\rho >} \sim \frac{\sqrt{< r^2 >_\rho}}{10} \]
II Improved- NJL model
NJL model and quark-antiquark Mesons

* Nambu-Jona-Lasinio model:

\[ \mathcal{L} = \bar{\psi}(i\partial - m_f)\psi + \frac{G_0}{2}[\!(\bar{\psi}\lambda_i\psi)^2 + (\bar{\psi}\gamma_5\lambda_i\psi)^2\!] \]

* Gluon exchange(s) and dynamics \( G_0 \sim \frac{1}{M_G^2}, \frac{1}{\Lambda^2} \) (flavorless)

* Current light quark masses \( m_f \) and generation of mass \( M_f = m_f + G_0 < \bar{q}q >_f \)

* Light meson multiplets (pseudoscalar, vector) reasonably well

* \( \eta - \eta' \) puzzle - \( U_A(1) \) anomaly - ‘t Hooft interaction

* Vacuum polarization also generates \( U(3) \) ‘tHooft int. for NJLmodel (without instantons) A.P.J., F.L.B., PRD90, 014049 (2014)
Quark model pseudoscalar mesons nonet

Non degenerate quarks: $|u>, |d>, |s>$

$$\frac{P_a \lambda_a}{\sqrt{2}} = \begin{pmatrix} \frac{P_u}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{P_d}{\sqrt{2}} & K^0 \\ K^- & K_0 & \frac{P_s}{\sqrt{2}} \end{pmatrix}$$

$P_{1,2,3} \rightarrow$ pions

$P_{4,5,6,7} \rightarrow$ kaons

$$P_8 = \pi_8 = \frac{1}{\sqrt{3}} (\bar{u}u + \bar{d}d - 2\bar{s}s) \rightarrow \eta$$

$$P_0 = \pi_0 = \frac{\sqrt{2}}{\sqrt{3}} (\bar{u}u + \bar{d}d + \bar{s}s) \rightarrow \eta'$$

(11)
Improving NJL model: sea quarks manifest

Two reasons for improved NJL model (flavor symmetry breaking)

** Usually mean field NJL model for fixed $G_0$:
1) Gap equations for DChSB, one-loop
2) Mesons from Bound state equations, one-loop

** QCD Lagrangian: flavor symmetry breaking in $m_f$
1) in a "GOOD" effective model, this flavor breaking SHOULD be present in all parameters.. (EFT, Weinberg "theorem" 1979)

* So, one step further **
- One loop level for the coupling constant - calculated
- NJL coupling constant with flavor symmetry breaking
Quark-polarization: fundamental and NJL model

Figure: Polarization in the NJL model, solid lines are quarks, $P = 0$

Figure: Wiggly lines with a dot = (dressed) gluon propagator.
* The dots in the vertices = running quark-gluon coupling constant.
* Need to (re)normalize resulting strength of interactions.

Resulting interaction $G_{ij}$: flavor-dependent
Gap/Bound state equations $G \rightarrow$ coupled equations

One obtains (that plugs into the BSE) $i, j = 0, \ldots N_f^2 - 1$:

$$G_{ij} = G_{ij}(M_u^*, M_d^*, M_s^*). \quad (12)$$

Standard NJL gap equations $f = u, d, s$ (U(3) flavor)

$$\begin{align*}
(G_0) \quad & M_f - m_f = G_0 \text{Tr}(S_{0,f}(0)) \quad (13) \\
\end{align*}$$

By neglecting ALL mixing interactions $G_{i \neq j}$ and $G_{f_1 \neq f_2}$

$$\begin{align*}
\quad (G_{ij}) \quad & M_f^* - m_f = G_{ff} \text{Tr}(S_{0,f}(0)). \quad (14) \\
\end{align*}$$

with $S_{0f}(k) = 1/(k - M^*)$

Chiral condensates $\rightarrow$ sea quark-antiquark degrees of freedom

Coupled equations: $G_{ij}$ and $M_f^*$ perturbatively/self consistently

To fix parameters of the model (to fit observables), meson masses
First and second mixings: Coupling constants

Coupling constants in the fundamental representation (quarks) $G_{ff}$
Different from the ones of adjoint representation (mesons) $G_{ij}$

\[2G_{uu} = 2 \frac{G_{00}}{3} + G_{33} + \frac{G_{88}}{3},\]
\[2G_{dd} = 2 \frac{G_{00}}{3} + G_{33} + \frac{G_{88}}{3},\]
\[2G_{ss} = 2 \frac{G_{00}}{3} + 4 \frac{G_{88}}{3},\]  
(15)

where $G_{88}(M_f^*)$, $G_{00}(M_f^*)$ (for $f = u, d, s$)

Emergence of sea quarks in GAP eqs. (1-quark mixings)

Screening in coupling constants $G_{ff}$ above

And resulting mixing interations (2-meson mixing interactions)

\[G_{i\neq j} \propto (M_{f_1} - M_{f_2})^{n=1,2}\]
\[G_{f_1 \neq f_2} \propto (M_{f_1} - M_{f_2})^{n=1,2}\]
By neglecting ALL mixing interactions $G_{i \neq j}$ - uncoupled equations:

$$1 - 2G_{ij} l_{12}^{ij} (P_0^2 = -M_\phi^2, \vec{P}^2 = 0) = 0,$$

(16)

rest frame of meson $\phi$, eg, pseudoscalar mesons (NG)

$G_{ij}$: defines the meson structure in the adjoint representation $E_g$.

$G_{11}, G_{22}, G_{33}$: pion structure

$G_{44}, G_{55}, \ldots$ kaon structure
\[ I_{f_1 f_2}^{ij}(P_0, \vec{P}) = Tr_{D,F,C} \int \frac{d^4 k}{(2\pi)^4} \lambda_i i \gamma_5 S_{0,f_1}(k + P/2) \lambda_j i \gamma_5 S_{0,f_2}(k - P/2), \quad (17) \]

* Since \( G_{ij}(M_u, M_d, M_s \ldots) \) strange/heavier quark-antiquark states (sea) contribute for the pion...

* Both fundamental and adjoint representations

Table: Sets of parameters: Lagrangian quark masses, ultraviolet cutoff and the quark effective masses obtained from an initial NJL-gap equation $G_0 = 10 \text{GeV}^{-2} \rightarrow$ fitting procedure neutral $\pi^0, K^0$

<table>
<thead>
<tr>
<th>set of parameters</th>
<th>$m_u$ (MeV)</th>
<th>$m_d$ (MeV)</th>
<th>$m_s$ (MeV)</th>
<th>$\Lambda$ (MeV)</th>
<th>$M_u$ (MeV)</th>
<th>$M_d$ (MeV)</th>
<th>$M_s$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>3</td>
<td>7</td>
<td>133</td>
<td>680</td>
<td>405</td>
<td>415</td>
<td>612</td>
</tr>
<tr>
<td>$V$</td>
<td>3</td>
<td>7</td>
<td>133</td>
<td>685</td>
<td>422</td>
<td>431</td>
<td>625</td>
</tr>
</tbody>
</table>
By varying freely $M_s^*$ in up-down gap equations
BSE of neutral pion: Strangeness in pions

* Normalization point at nearly \( M_s^* \sim 450 \) MeV \( G_{ij} \rightarrow G_0 \)
* "Physical point" \( M_s^* \sim 550 \) MeV
Strangeness in $F_\pi$

$F_\pi \simeq 102$ MeV at the "physical point"
(value obtained from the fixed parameters of the model)
U(5) NJL-model and cutoffs: mesons masses

* NJL model not expected to work for heavy hadrons, still, we did some calculation
* Vector interaction $\Lambda_f$ (Bashir et al, Serna et al, others)
* $\langle \bar{c}c \rangle$, $\langle \bar{b}b \rangle$ non zero: NJL-type model with DChSB


Non-covariant ultraviolet cutoff improves improves interpretation

$$|\vec{k}| \leq \Lambda_u \simeq \Lambda_s \simeq \Lambda_c \simeq \Lambda_b \sim 0.5\text{GeV}$$

1) heavy quarks: non covariant (non relativistic) anyway
2) light quarks: results similar to other regularizations

* 5 parameters → (7+4) or (21+4) PS mesons, (5 or 18) S meson
Masses $m_u = m_d$ within ~ 6% and 10%
<table>
<thead>
<tr>
<th>Set</th>
<th>2</th>
<th>3</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_\pi$ (MeV)</td>
<td>165(140) [-]</td>
<td>147(118) [-]</td>
<td>137†</td>
</tr>
<tr>
<td></td>
<td>{ 147(118) }</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_K$ (MeV)</td>
<td>505(494)[475]</td>
<td>512(501)[481]</td>
<td>495</td>
</tr>
<tr>
<td></td>
<td>{ 512(501) }</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_D$ (MeV)</td>
<td>1870(1863)[1869]</td>
<td>1868(1869)[1873]</td>
<td>1870</td>
</tr>
<tr>
<td></td>
<td>{ 1310(1378) }</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>{ 1469(1515) }</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_B$ (MeV)</td>
<td>5294(5275)[5288]</td>
<td>5279(5274)[5283]</td>
<td>5280</td>
</tr>
<tr>
<td></td>
<td>{ 4740(4831) }</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{B_s}$ (MeV)</td>
<td>5427(5392)[5418]</td>
<td>5421(5397)[5421]</td>
<td>5367</td>
</tr>
<tr>
<td></td>
<td>{ 4882(4954) }</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{B_c}$ (MeV)</td>
<td>6542(6460)[6504]</td>
<td>6539(6477)[6516]</td>
<td>6275</td>
</tr>
<tr>
<td></td>
<td>{ 5491(5595) }</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$G_0$  ($G_{ij}$)  [$G_{i\neq j} = 0$]  \{ $\bar{S}_c = \bar{S}_b = 0$ \}
**Table:** Probabilities of a meson with valence quark-antiquark structure to develop other types of sea quark/antiquark components from $G_{psqq} = Z_{ps}^2$

<table>
<thead>
<tr>
<th>Set</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Pr(\pi)$</td>
<td>2%</td>
<td>2%</td>
<td>2%</td>
<td>2%</td>
</tr>
<tr>
<td>$Pr(K)$</td>
<td>3%</td>
<td>3%</td>
<td>3%</td>
<td>5%</td>
</tr>
<tr>
<td>$Pr(D)$</td>
<td>4%</td>
<td>7%</td>
<td>7%</td>
<td>7%</td>
</tr>
<tr>
<td>$Pr(D_s)$</td>
<td>5%</td>
<td>4%</td>
<td>5%</td>
<td>5%</td>
</tr>
<tr>
<td>$Pr(B)$</td>
<td>9%</td>
<td>8%</td>
<td>9%</td>
<td>8%</td>
</tr>
<tr>
<td>$Pr(B_s)$</td>
<td>11%</td>
<td>7%</td>
<td>7%</td>
<td>7%</td>
</tr>
<tr>
<td>$Pr(B_c)$</td>
<td>6%</td>
<td>6%</td>
<td>6%</td>
<td>5%</td>
</tr>
</tbody>
</table>
\[ \frac{\Delta_{c,b} M_u}{M_u} > \frac{\Delta_{c,b} M_s}{M_s}, \]
Mixing matrix (Kroll, Feldmann et al)

\[
\begin{pmatrix}
\pi^0 \\
\eta \\
\eta'
\end{pmatrix} = M
\begin{pmatrix}
P_3 \\
P_8 \\
P_0
\end{pmatrix}
\]

Leading mixings:

\[
|\eta > = \cos \theta_{ps} |P_8 > - \sin \theta_{ps} |P_0 > ,
\]
\[
|\eta' > = \sin \theta_{ps} |P_8 > + \cos \theta_{ps} |P_0 > .
\]

(18)

\[
\theta_{ps} = \frac{1}{2} \arcsin \left( \frac{4 G^n_{08} \tilde{G}_{08}}{(M^2_\eta - M^2_{\eta'})} \right).
\]

(19)

\[
|\eta > = - (\epsilon_2 + \epsilon_1 \cos(\phi_{08}) |P_3 > + \sqrt{\frac{2}{3}} \cos(\phi_{08}) |P_8 > ,
\]
\[
|\pi_0 > = |P_3 > + \left( \sqrt{\frac{2}{3}} (\epsilon_1 + \epsilon_2 \cos(\phi_{08})) - \frac{\epsilon_2 S_{i\psi}}{\sqrt{3}} \right) |P_8 > .
\]
$\eta - \eta'$ mixing: usual basis

$\pi^0 - \eta$ mixing:

$$|\pi^0> = \frac{1}{\sqrt{2}} [1 + a_l] |\bar{u}u> - \frac{1}{\sqrt{2}} [1 - a_l] |\bar{d}d> - 2a_s |\bar{s}s>,$$

* Contributions for Up and down are different
* $<\bar{q}q|\hat{H}|\bar{q}q> \approx 2M_q \sim 900$ MeV.

$$\Delta_\eta m_{\pi^0} \approx 4a^2 M_s \sim 1 - 5$ MeV

$$\Delta_\eta M_{u,d}^* \sim \frac{3}{4} \Delta_\eta m_{\pi^0}.$$

* Meson -constituent quark couplings with mixings - on going

* Similarly: ChPT - eg Kaiser, (2007)- $\Delta_s M_\pi \sim 9 - 19$ MeV
Pion c-b content leading to mixing to $\eta_c, \eta_b$

Flavor eigenstates $P_3 = \frac{1}{\sqrt{2}} (\bar{u}u - \bar{d}d) \rightarrow \pi^0$

$$P_0 = \sqrt{\frac{2}{5}} (\bar{u}u + \bar{d}d + \bar{s}s + \bar{c}c + \bar{b}b) \rightarrow \eta'(958)$$

$$P_8 = \frac{1}{\sqrt{3}} (\bar{u}u + \bar{d}d - 2\bar{s}s) \rightarrow \eta(548)$$

$$P_{15} = \frac{1}{\sqrt{6}} (\bar{u}u + \bar{d}d + \bar{s}s - 3\bar{c}c) \rightarrow \eta_c(3415)$$

$$P_{24} = \frac{1}{\sqrt{10}} (\bar{u}u + \bar{d}d + \bar{s}s + \bar{c}c - 4\bar{b}b) \rightarrow \eta_b(9859)$$

From the mixing: $\pi^0 \sim P_3 + G_{30} P_0 + G_{38} P_8 + G_{3,15} P_{15} + G_{3,24} P_{24}$

such that mixing amplitude:

$$\langle \eta_c | \pi^0 \rangle \sim G_{3,15}, \quad \langle \eta_b | \pi^0 \rangle \sim G_{3,24}$$

Problem to identify: $M_{\pi^0} << M_{\eta_c} < M_{\eta_b}$

$\eta_c, \eta_b$ at rest $\rightarrow$ pions $K_{\pi^0} \sim (3375\text{MeV}) (9719\text{MeV})$
Quark-antiquark states of light scalars

Strong consequences of strangeness

Figure: Mesons $A_0$ and $\kappa$: inversion of hierarchy

\[ R_{a0f0} \equiv \frac{A_0^{0}(980) \rightarrow S_8 \rightarrow f_0(980)}{f_0(980) \rightarrow S_3 \rightarrow A_0^{0}(980)} \sim \left| \frac{a_{0,(8)}}{a_0} \right|^2 \left| \frac{f_{0,(3)}}{f_0} \right|^2 \equiv \left| \frac{A_8,a_0}{A_3,f_0} \right|^2. \]

Figure: Ratio of mixings $A_0 - f_0$ BESS-III: 0.4 or 0.97
One quark, one antiquark, one meson
Summary

- Vector mesons may probe axial quark current ($< r_\rho >^2_A$)
- FSB → Mixing effects → sea quark structures
- Meson mixings and strangeness/charm/bottom content

On going/planned:

- Flavor symmetry breaking/mixing effects in punctual vector effective interaction with B.El Bennich+F.Serna
- Flavor symmetry breaking/mixing effects in couplings, e.g. pion-constituent quarks and $j_{A,\mu}$
- Behavior of FSB with increasing energies/momenta (GPDs,PDFs)
- ...

Thank you for your attention!