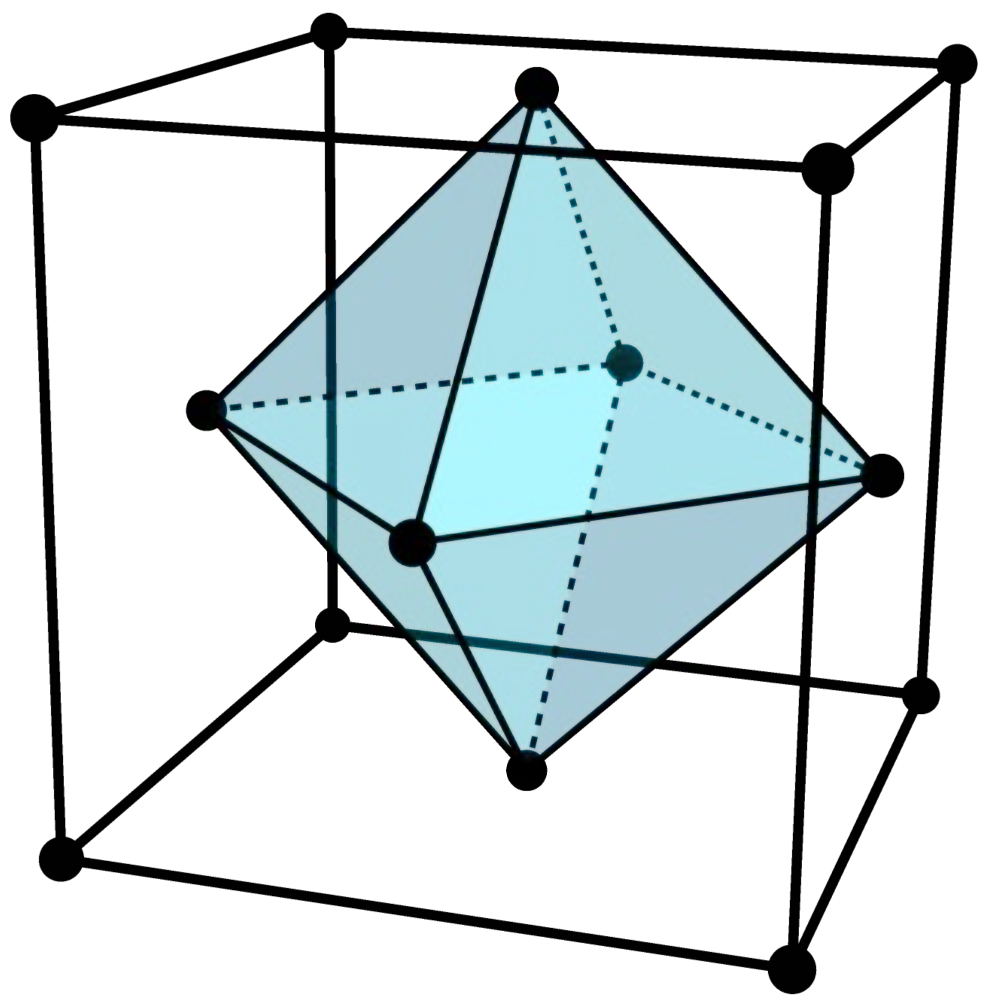
Boundary modes in fracton models

Rodrigo Pereira

IIP, Natal, Brazil



Workshop on Strong Electron Correlations in Quantum Materials, ICTP-SAIFR, June 22, 2023

My story with Eduardo #TBT

- First met in 1999 during my undergrad at Unicamp.

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Revista Brasileira de Ensino de Física, vol. 24, no. 2, Junho, 2002

March 2002

Introdução à Teoria Quântica de Campos: do Oscilador Harmônico ao Campo Escalar Livre

Rodrigo Gonçalves Pereira e Eduardo Miranda
Instituto de Física Gleb Wataghin, Unicamp, C. P. 6165, CEP 13083-970 - Campinas, SP

Recebido em 8 março, 2002. Aceito em 8 de maio, 2002.

My story with Eduardo #TBT

Brazilian Journal of Physics, vol. 33, no. 1, March, 2003

Brazilian School on Statistical Mechanics, São Carlos, March 2002.

Introduction to Bosonization

E. Miranda

Instituto de Física Gleb Wataghin, Unicamp,

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Received on 27 August, 2002

“The highly pedagogical notes of Eduardo Miranda, which are self-contained and assume just some basic knowledge of second quantization, are of particular relevance as an introduction to the use of modern bosonization techniques.” [BJP (2003)]

My story with Eduardo #TBT

- First met in 1999 during my undergrad at Unicamp.
- Masters 2002-2004. Worked on 1D Kondo lattice, magnetic domain walls and impurities in Luttinger liquids.
- I keep going back to the lessons I learned from Eduardo. This talk is an example.

Outline

- Topological phases with boundaries
- Fractons
- Gapped boundaries of the Chamon model
- Effective field theory for boundary modes

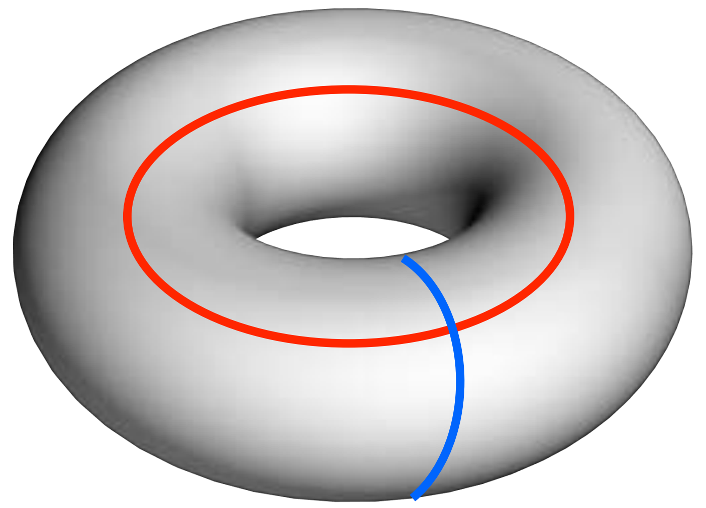
In collaboration with **Weslei Fontana (Natal)**

[arXiv:2210.09867, to appear in SciPost]

Gapped edges of topological phases

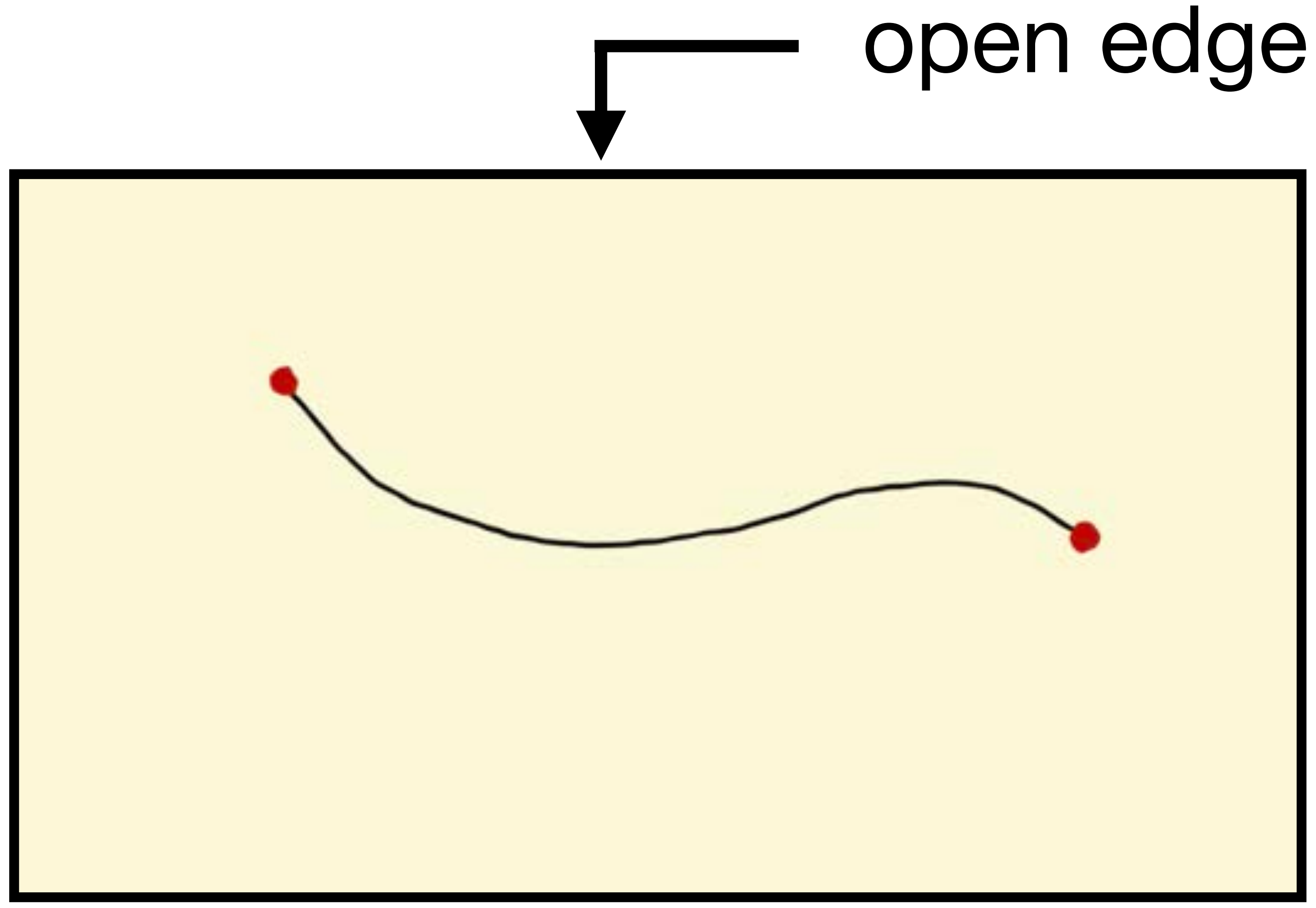
Topological-ordered phases without symmetries, e.g. fractional quantum Hall states and quantum spin liquids.

Long-range entanglement. Ground state degeneracy depends on topology of closed surface in real space.
Fractionalization: elementary excitations are anyons.



Gapped edges classified by condensation of anyons with self/mutual bosonic statistics: Lagrangian subgroups of the set of quasiparticles.

[Kapustin & Saulina, NPB (2011); Levin, PRX (2013); Wang & Wen, PRB (2015)]



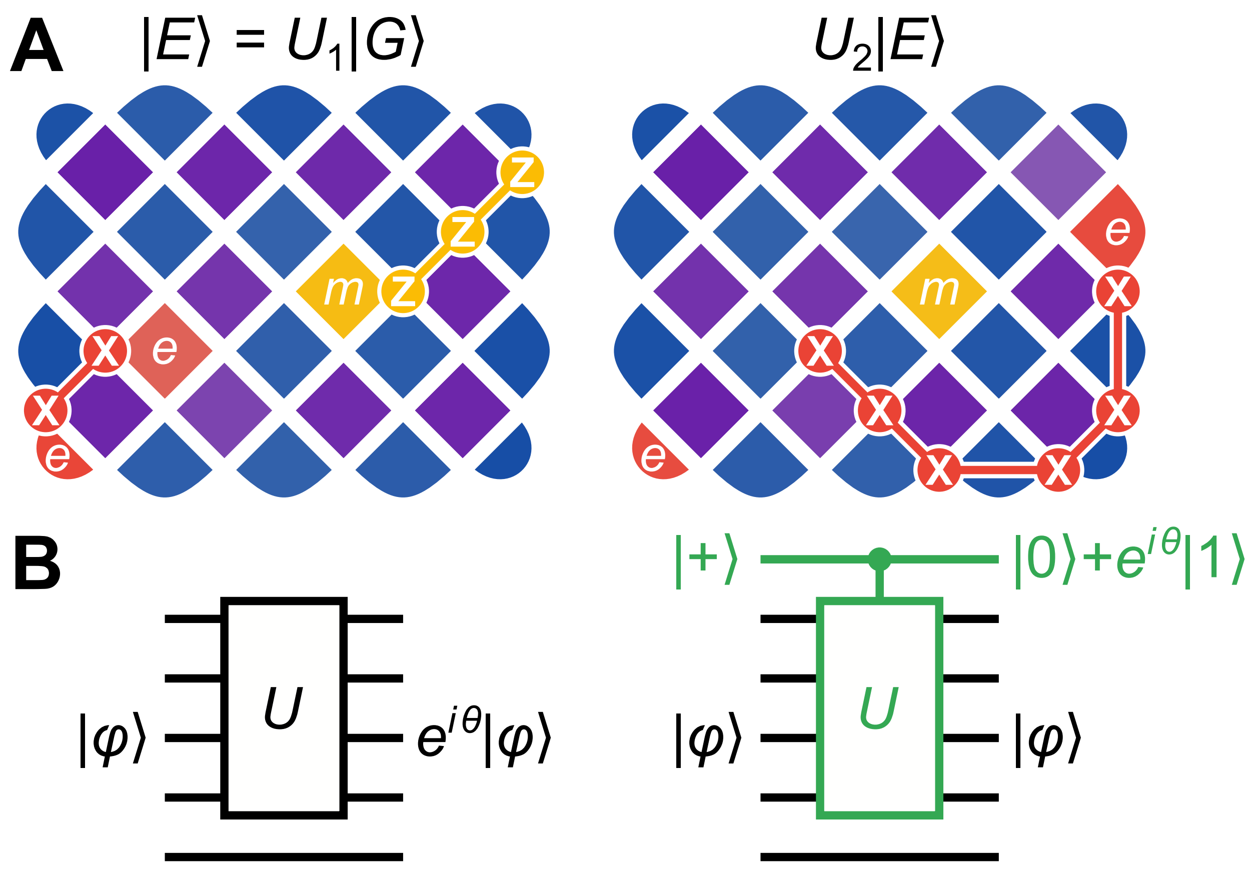
Example: Z_2 topological order

Present in Anderson's resonating valence bond state.

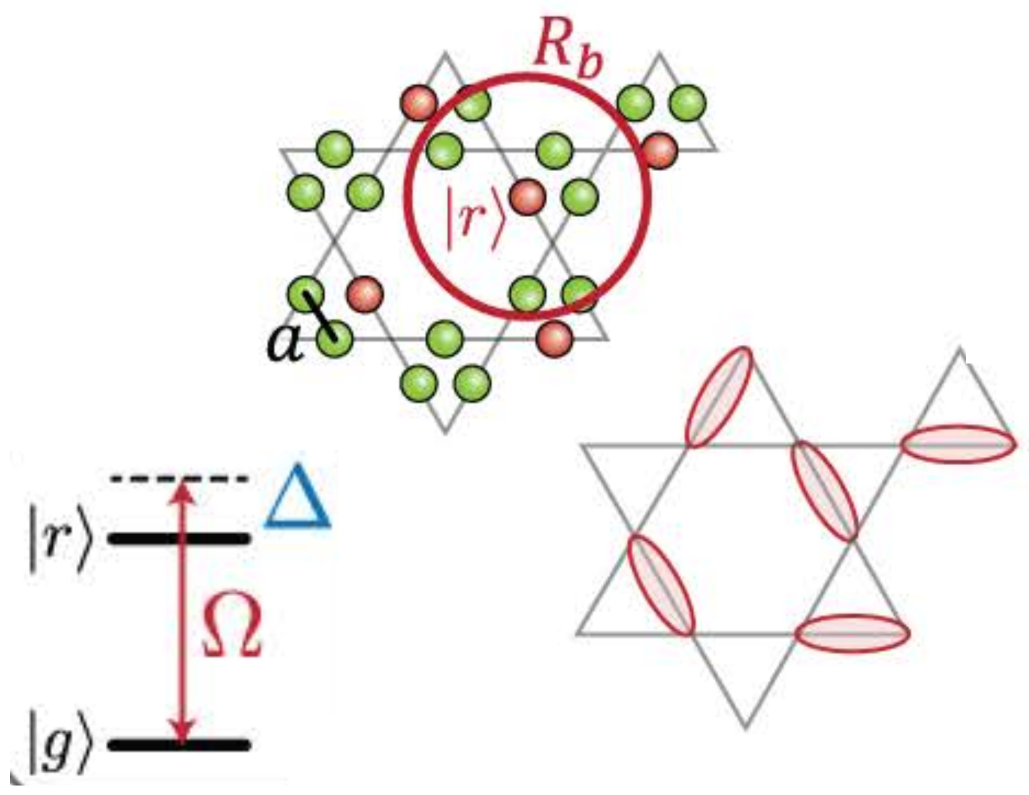
[Anderson (1973); Kivelson, Rokhsar & Sethna (1987)]

[Kivelson & Sondhi, "50 years of quantum spin liquids", arXiv:2305.18103]

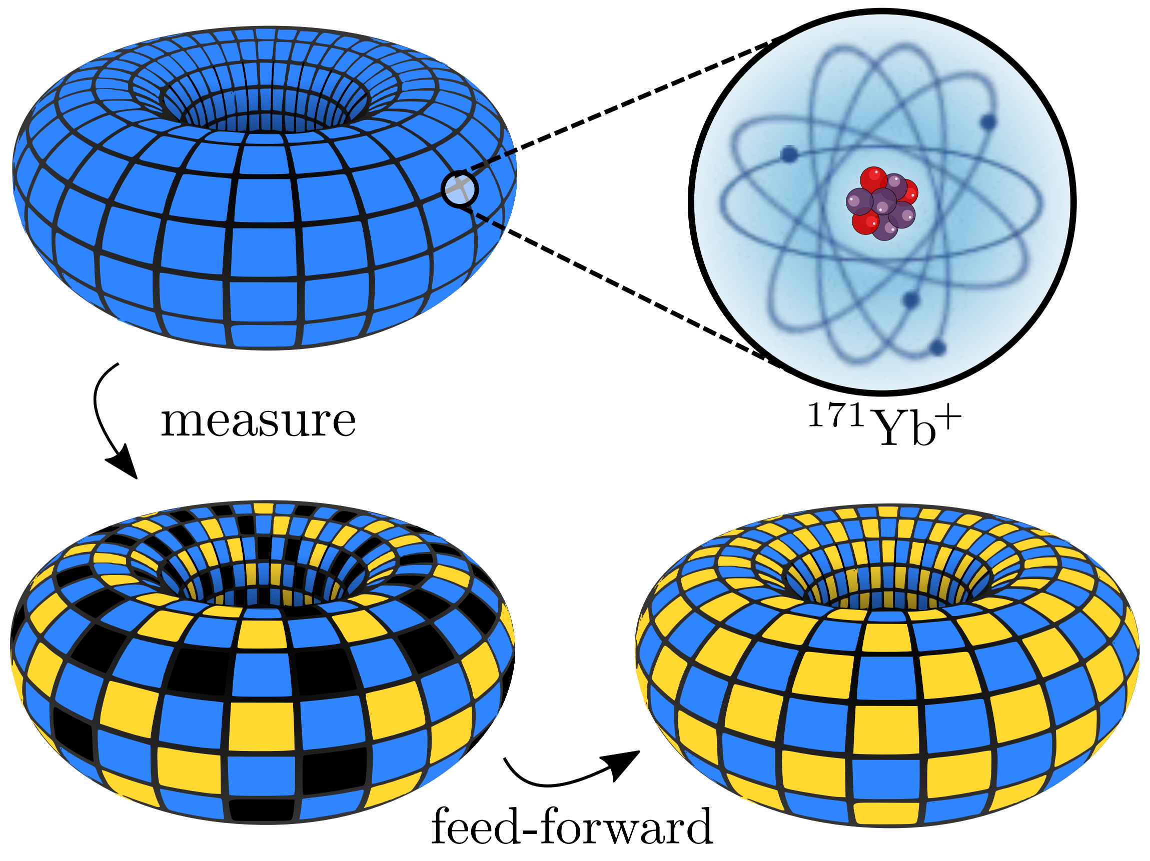
Realized in quantum simulation platforms: superconducting qubits, Rydberg atoms, trapped ions.



[Satzinger et al. (Google Quantum AI), Science (2021)]



[Semeghini et al., Science (2021)]



[Iqbal et al. (Quantinuum), arXiv:2302.01917]

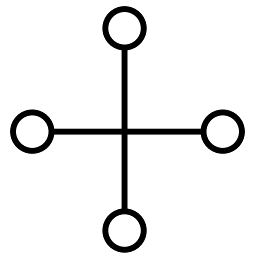
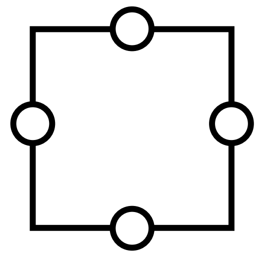
Anyons in the Z_2 spin liquid

Toric code: *e* and *m* particles have bosonic self-statistics, mutual semionic statistics; bound state of *e* and *m* is a fermion.

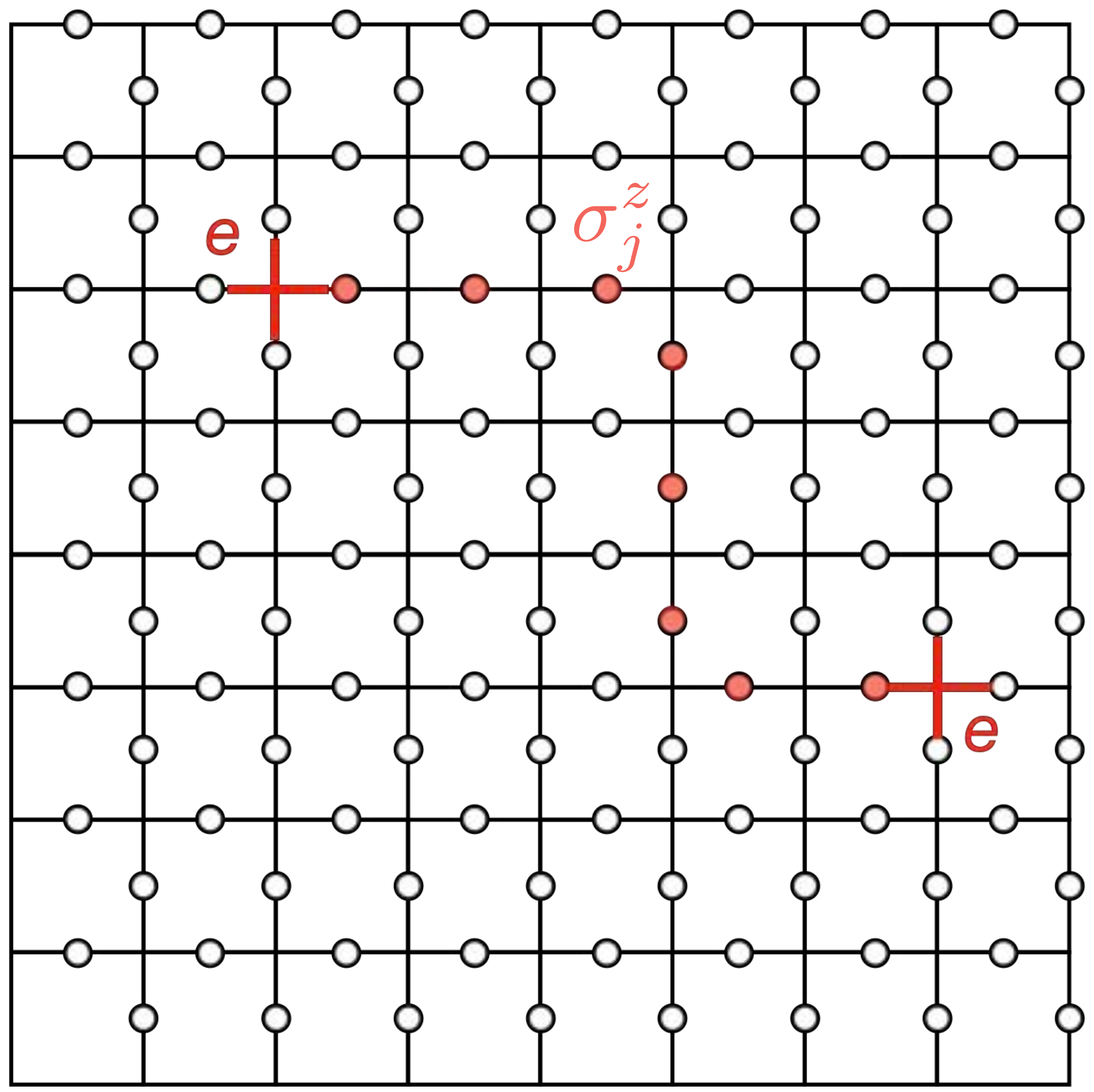
[Kitaev, arXiv:quant-ph/9707021]

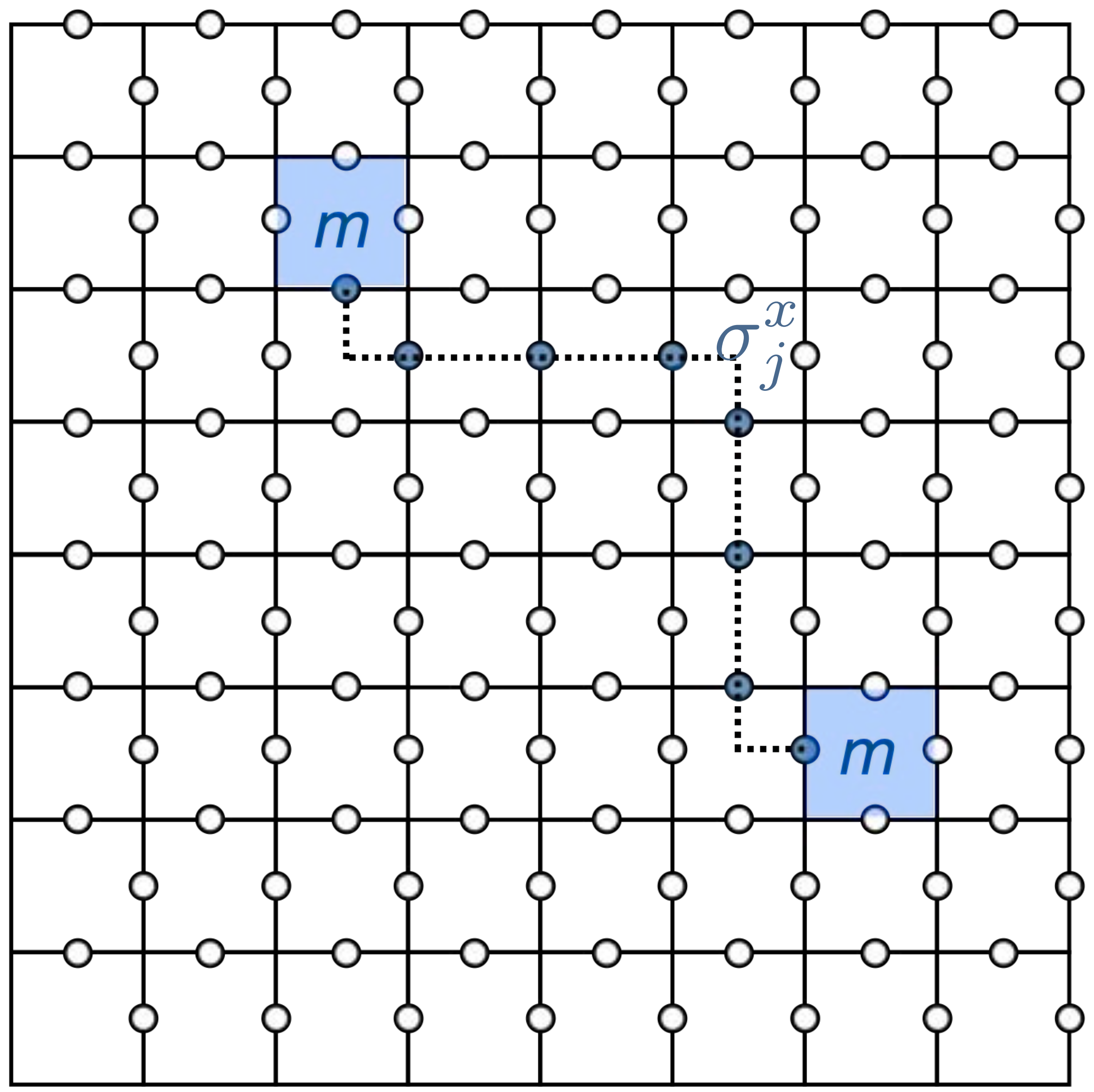
Stabilizers:

$$A_s = \prod_{j \in s} \sigma_j^x \qquad B_p = \prod_{j \in p} \sigma_j^z$$

star s  plaquette p 

$$H = -J \sum_s A_s - J \sum_p B_p$$



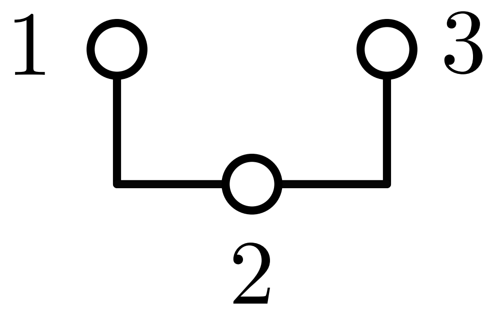


Two types of gapped edges

Open boundaries (surface codes): *e* particles condense on rough edges and *m* particles on smooth edges. Different boundary phases separated by quantum phase transition (described by Ising CFT). [Bravyi & Kitaev, arXiv:quant-ph/9811052]

Boundary stabilizers:

$$B'_p = \sigma^z_1 \sigma^z_2 \sigma^z_3$$



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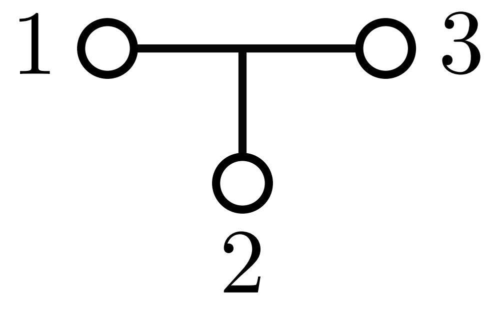

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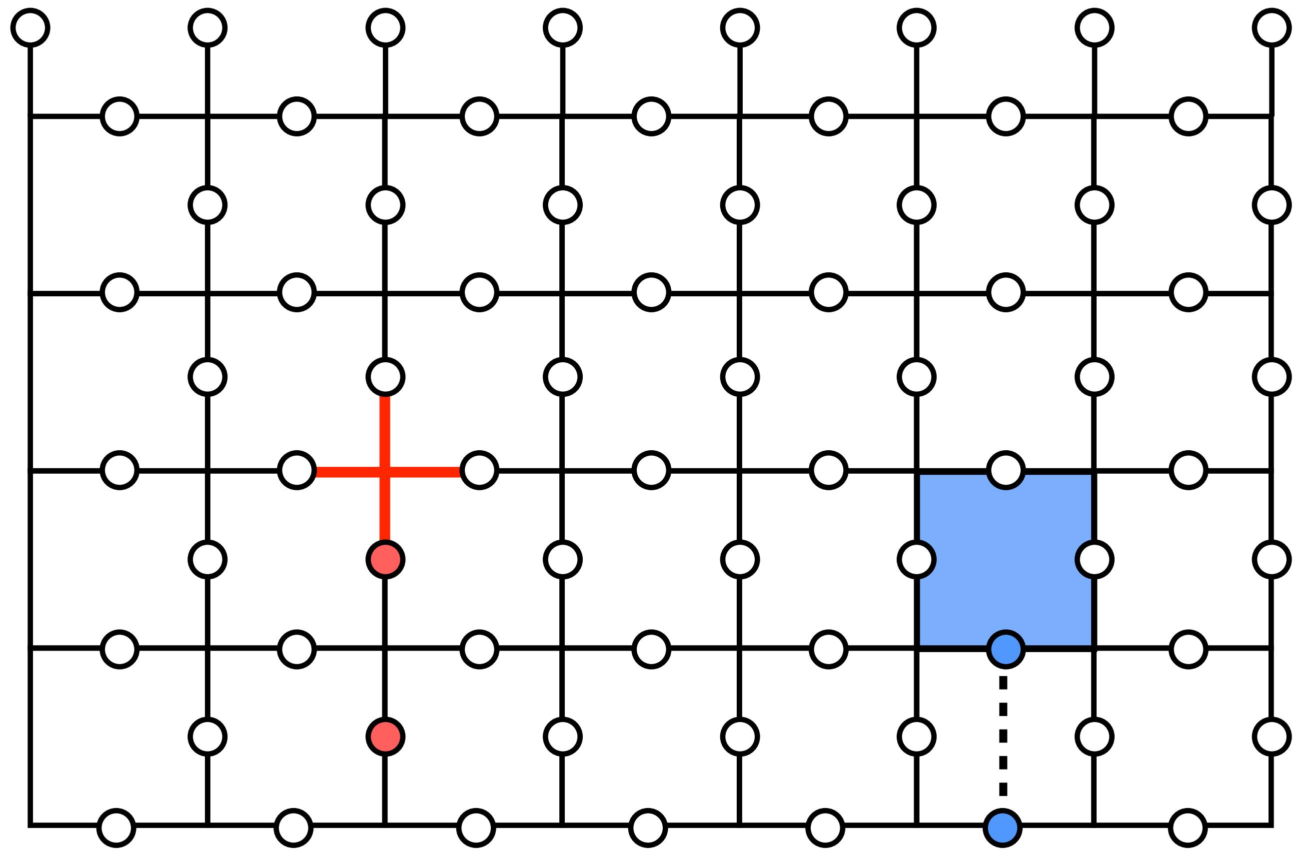

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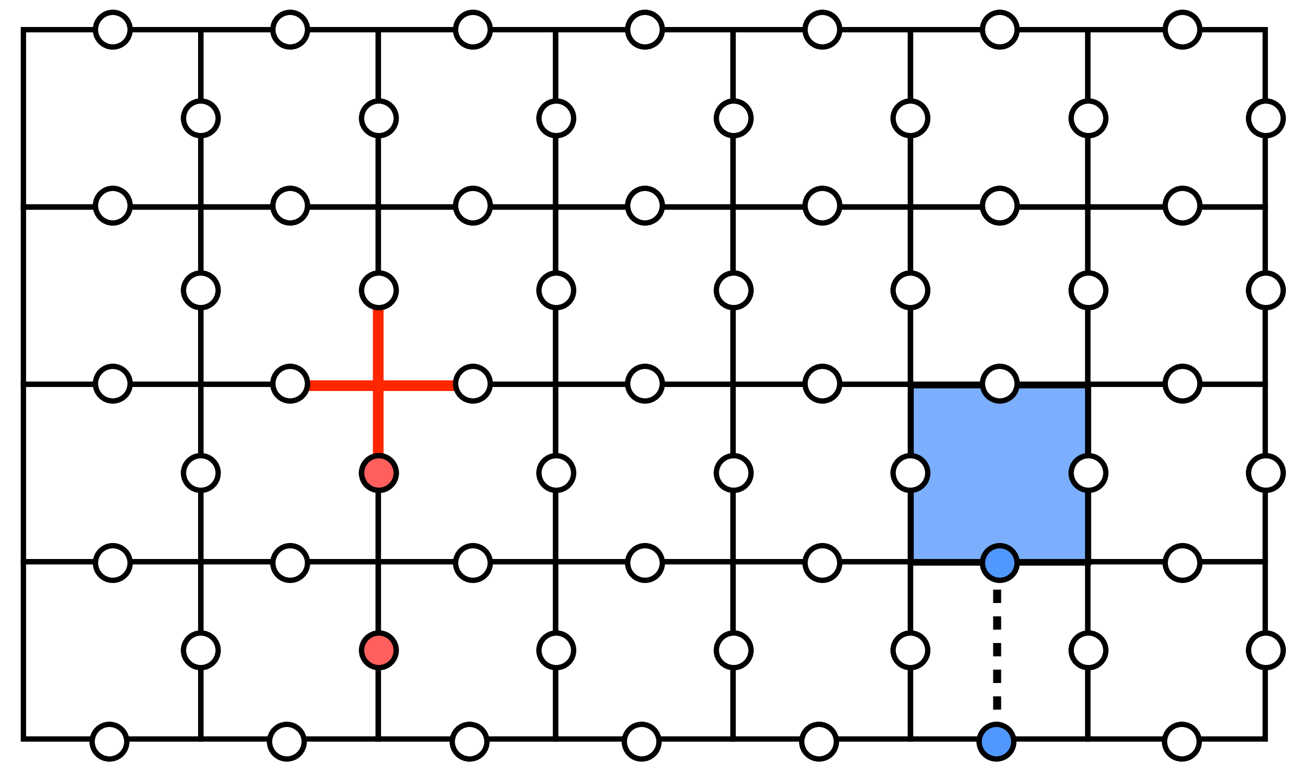

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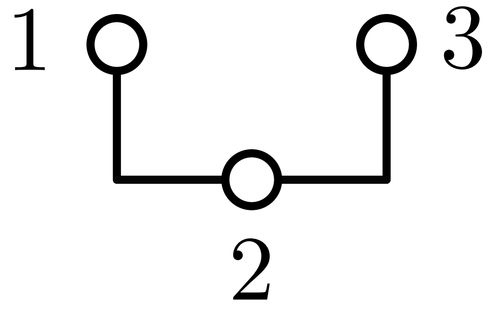

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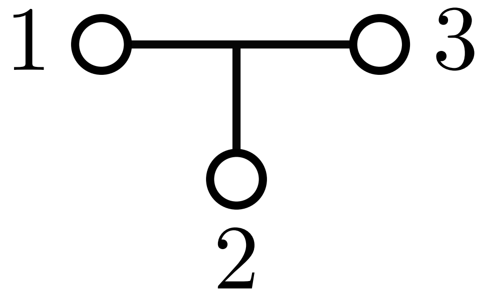

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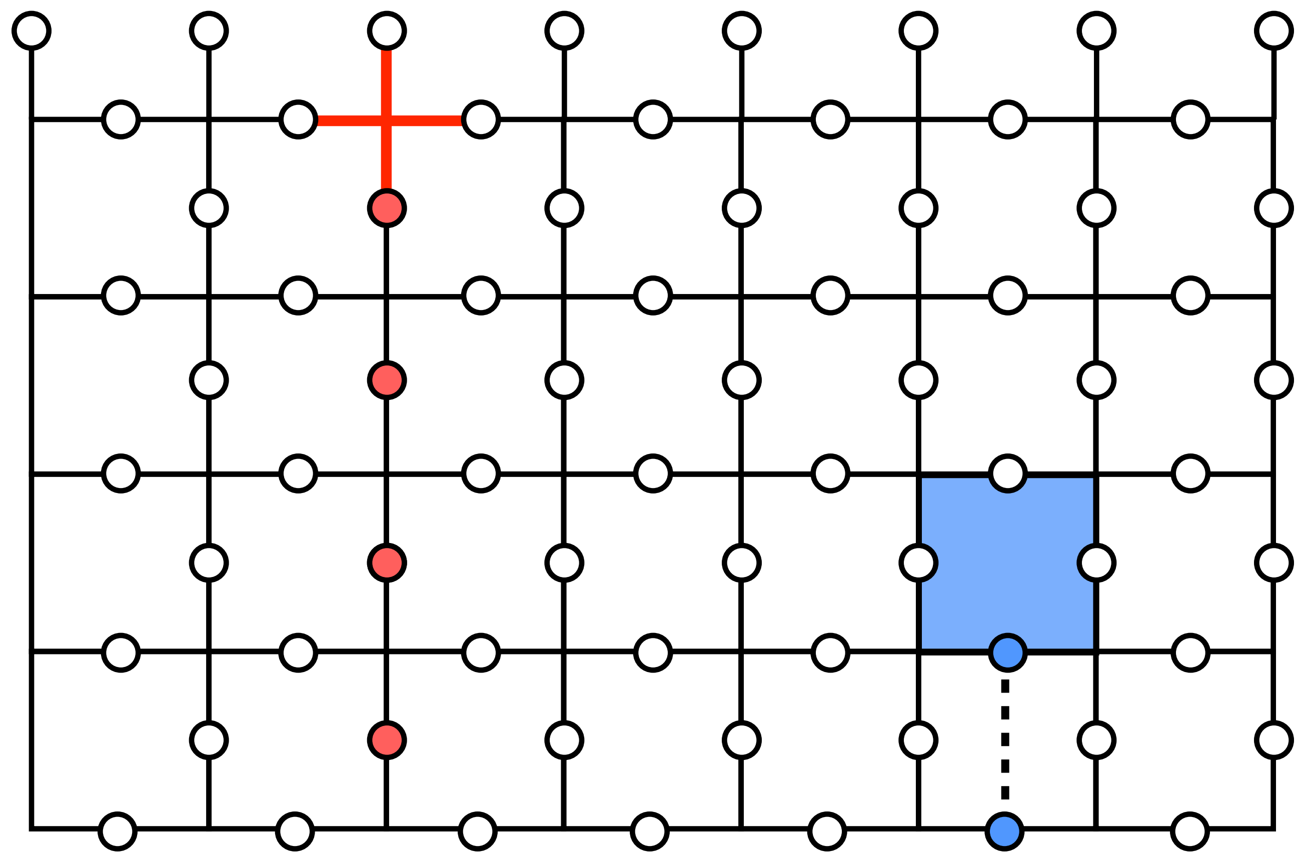

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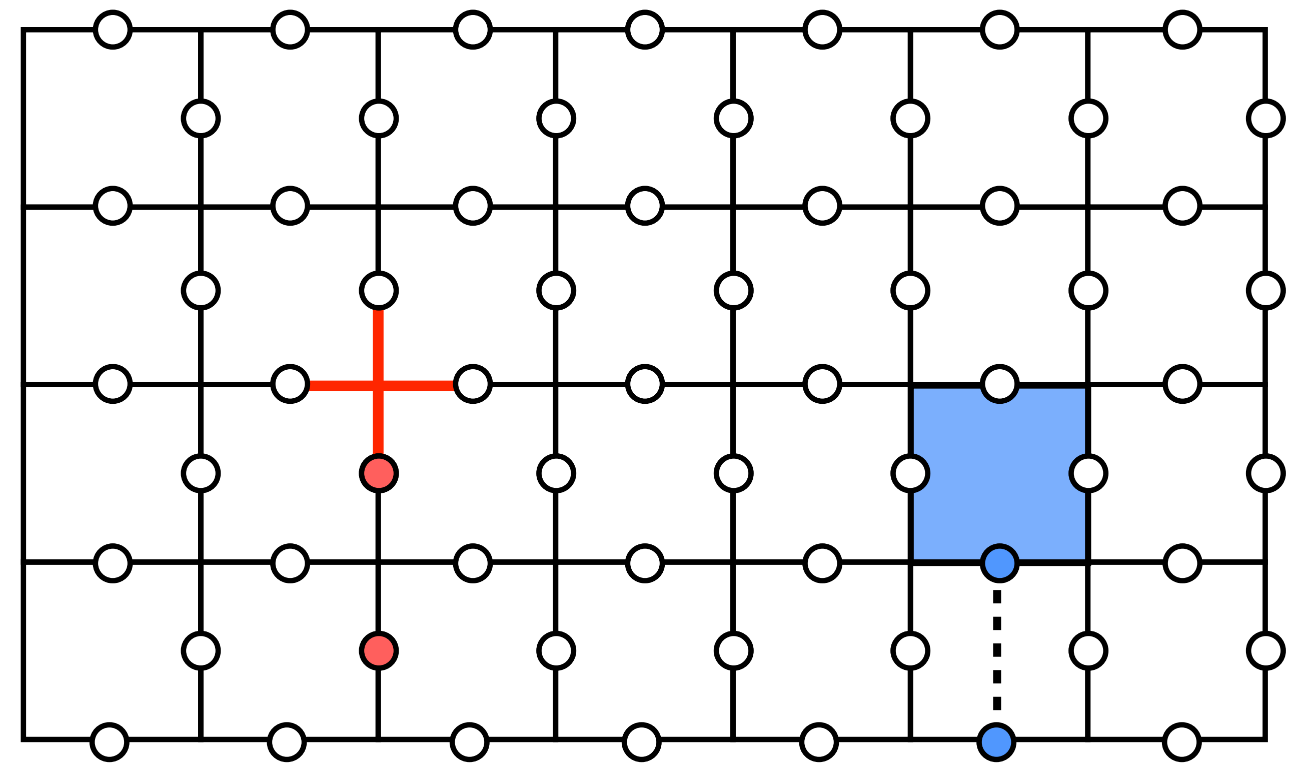

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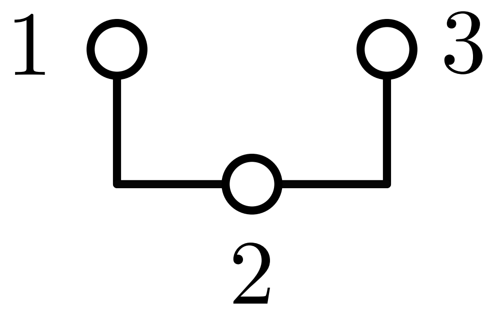

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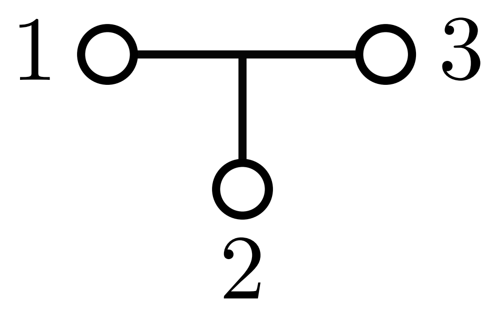

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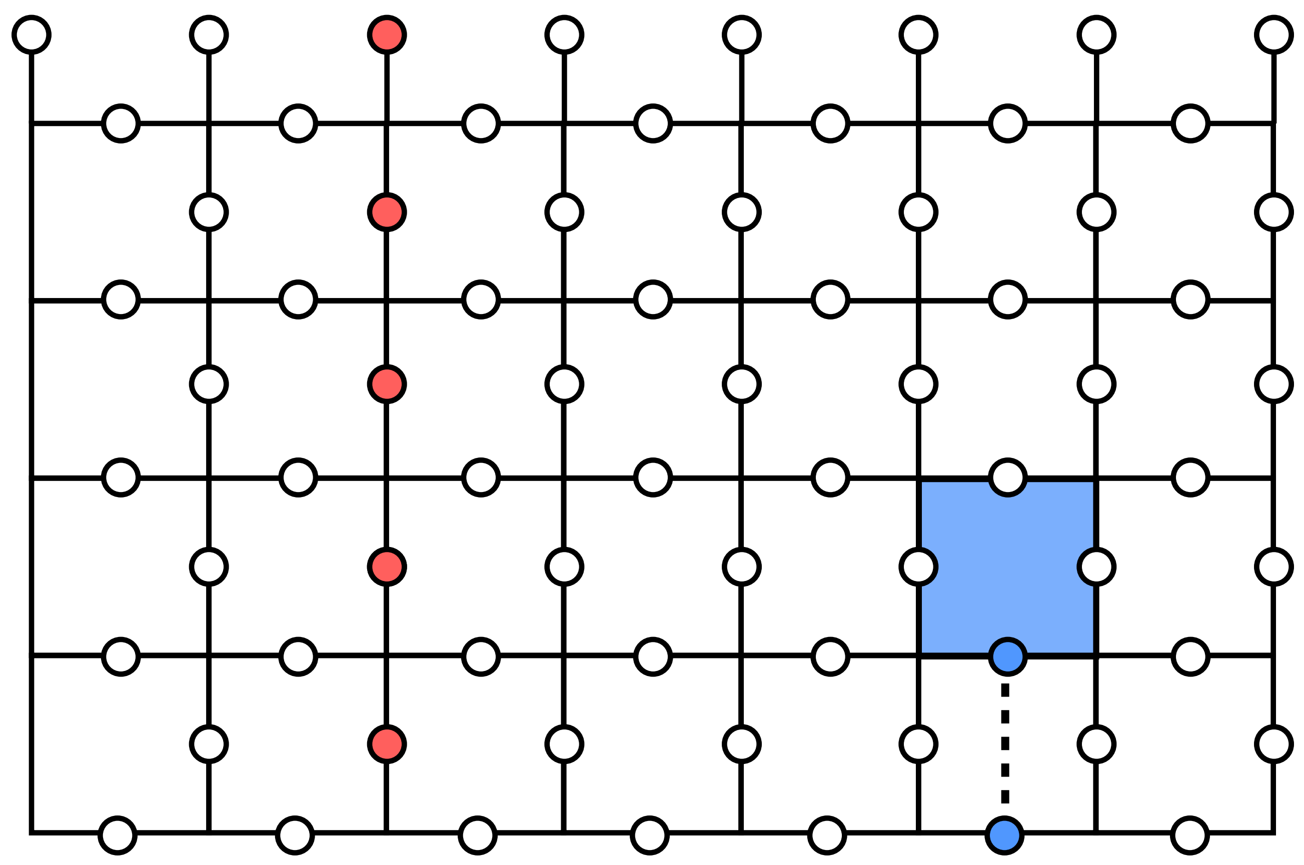

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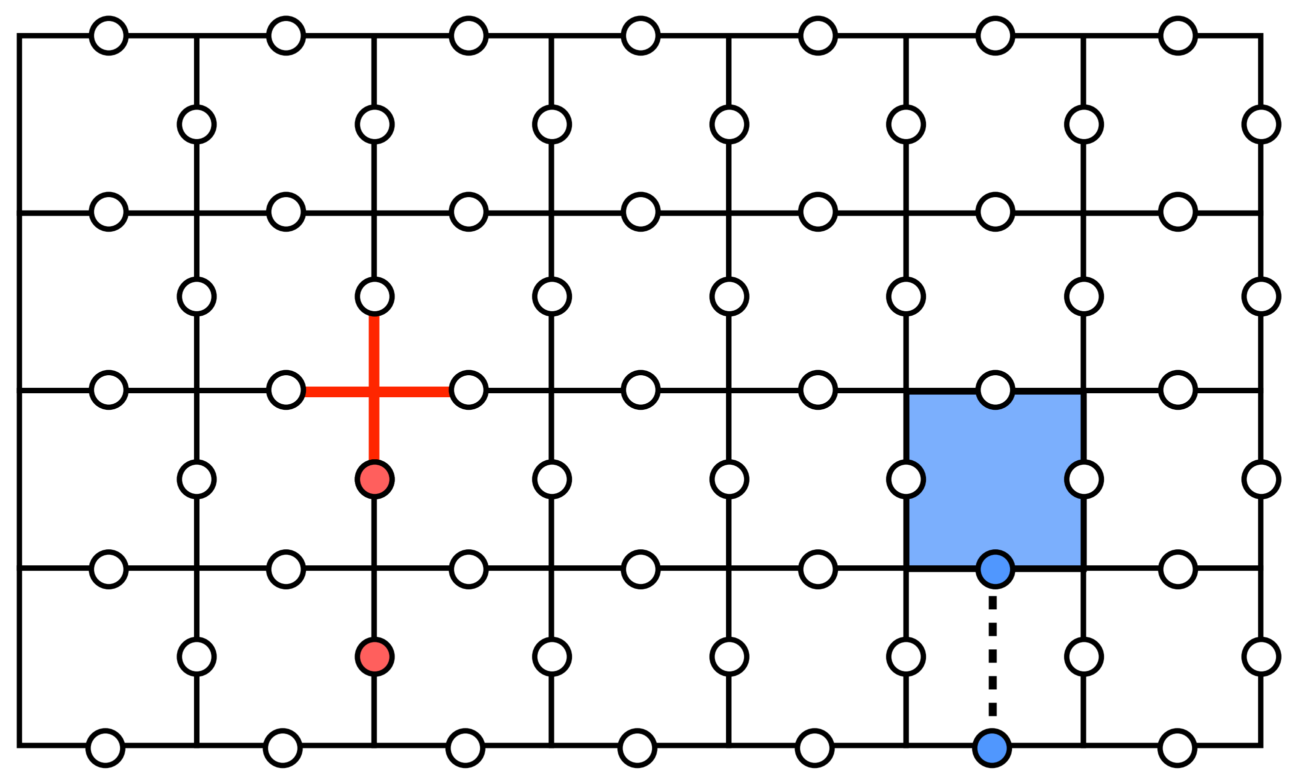

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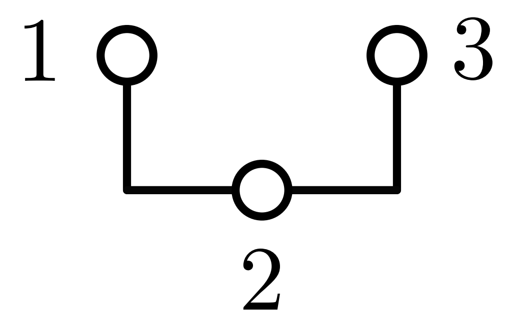

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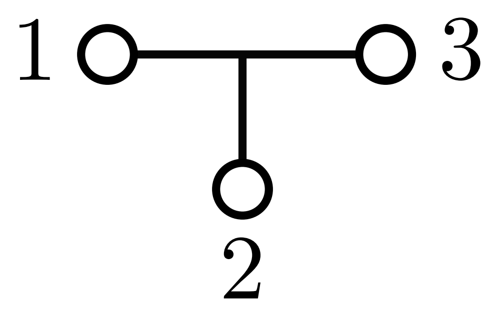

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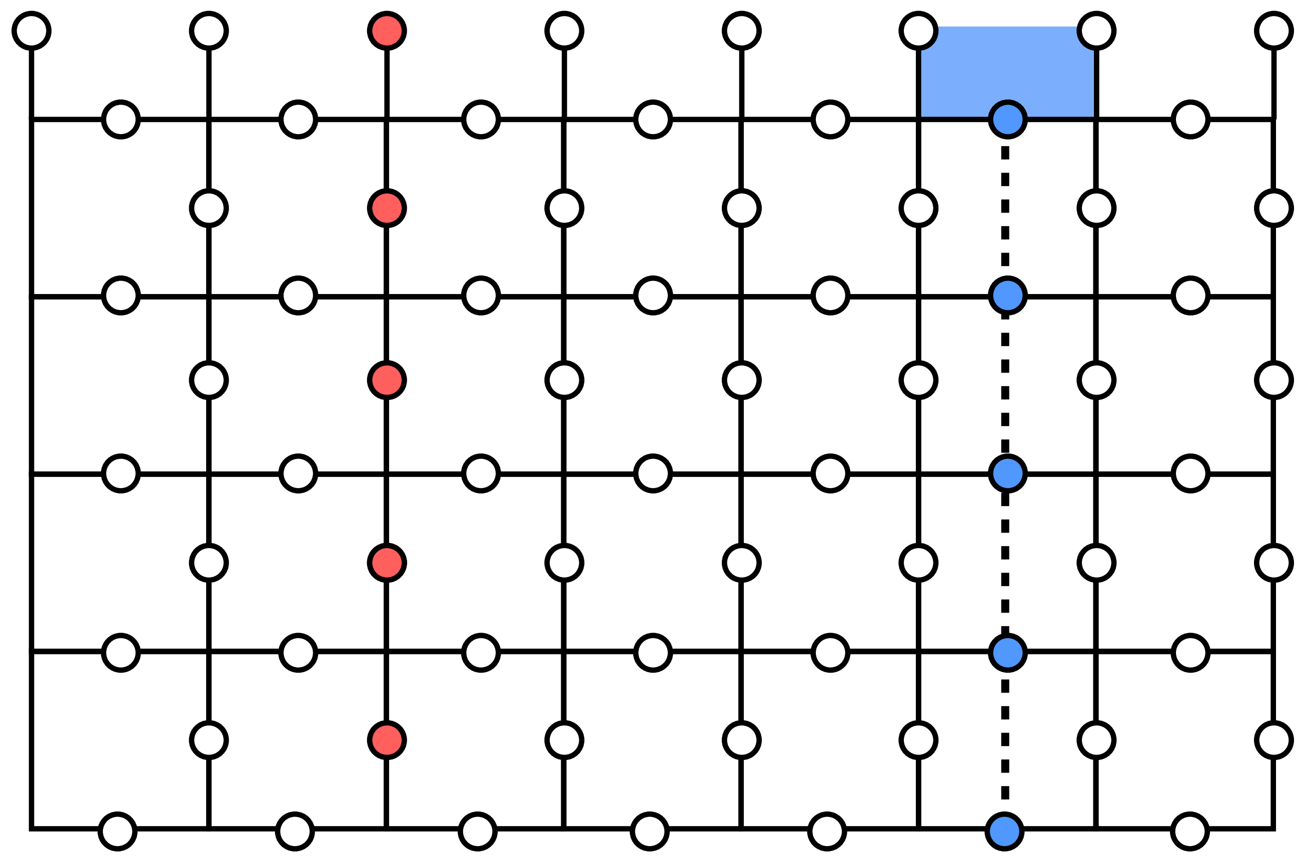

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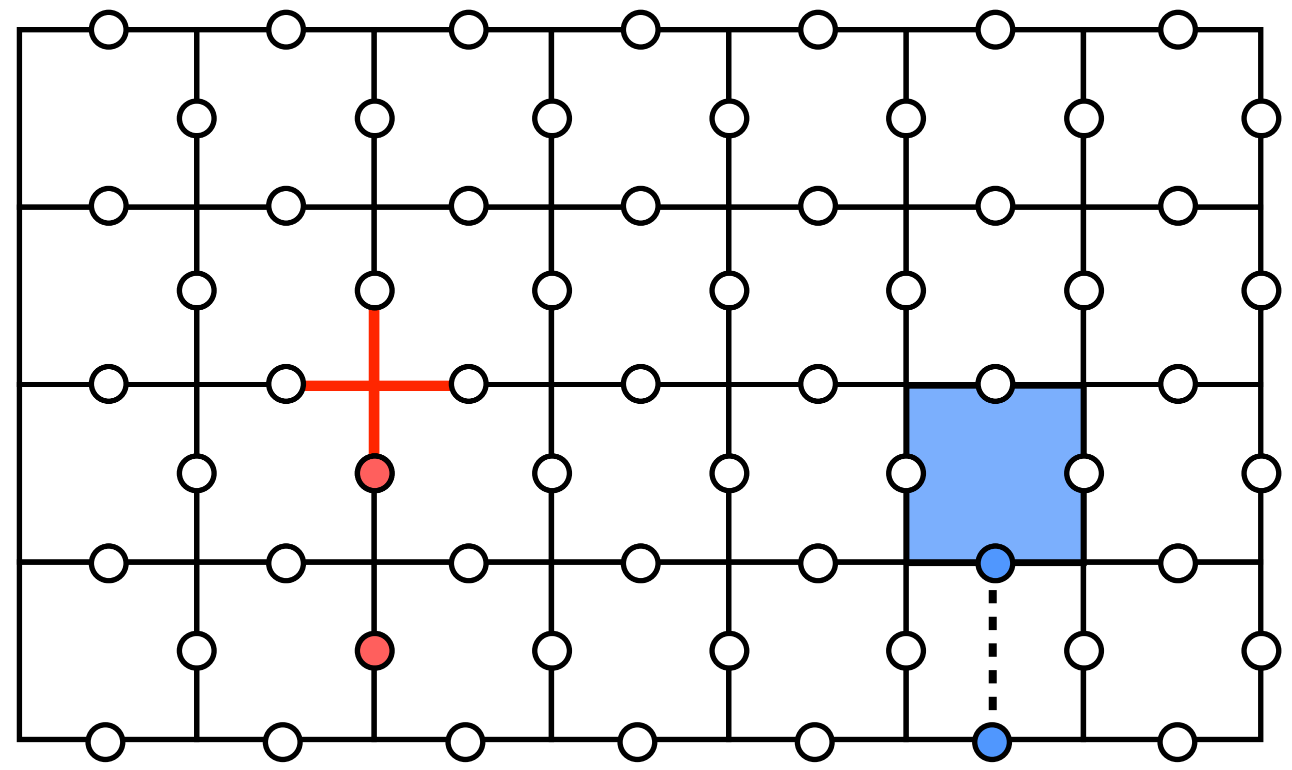

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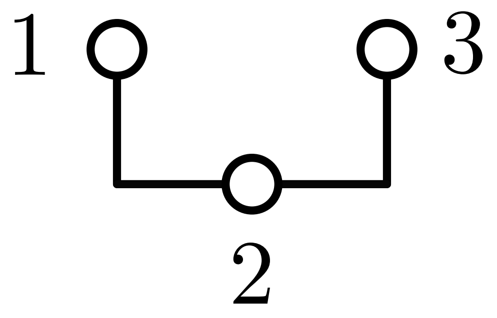

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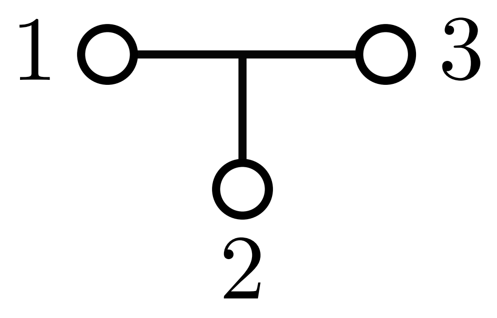

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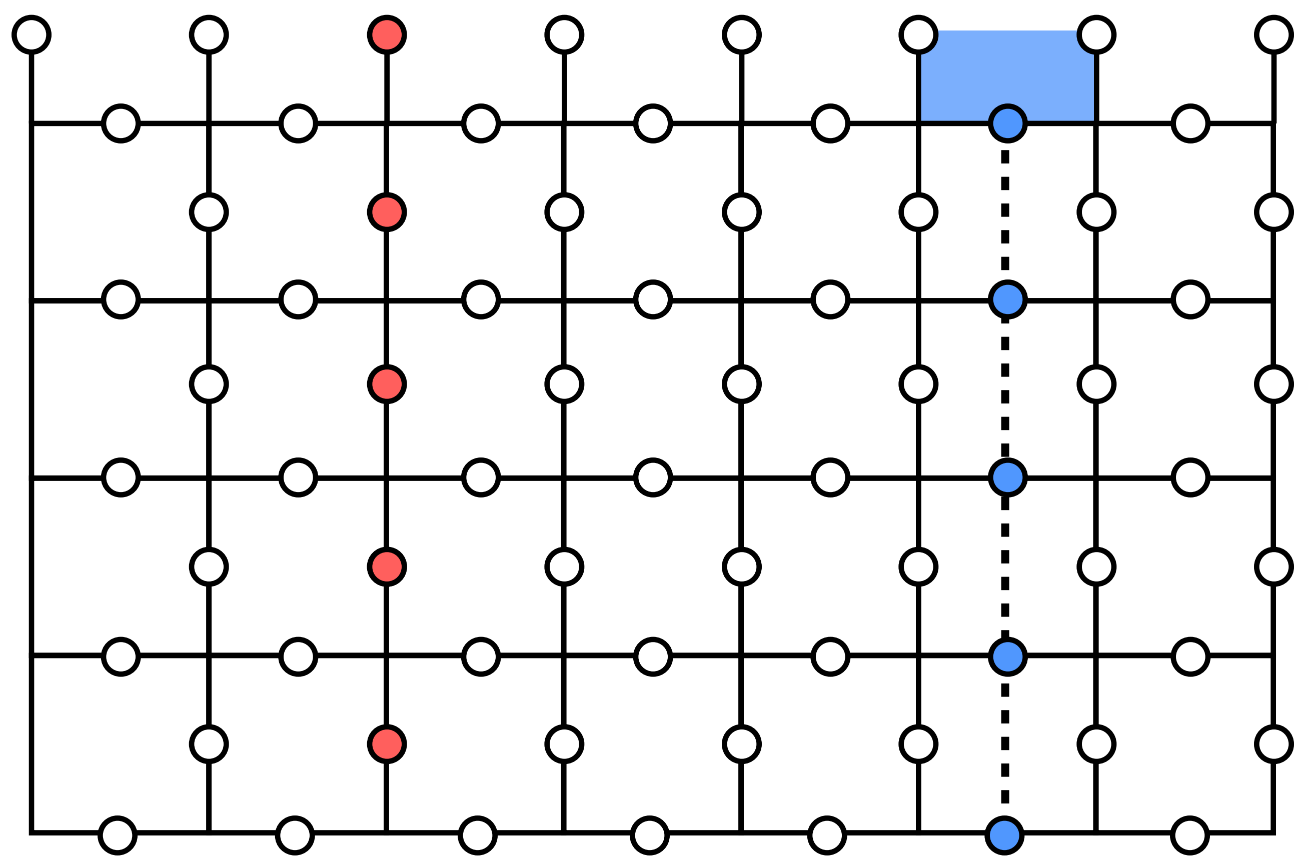

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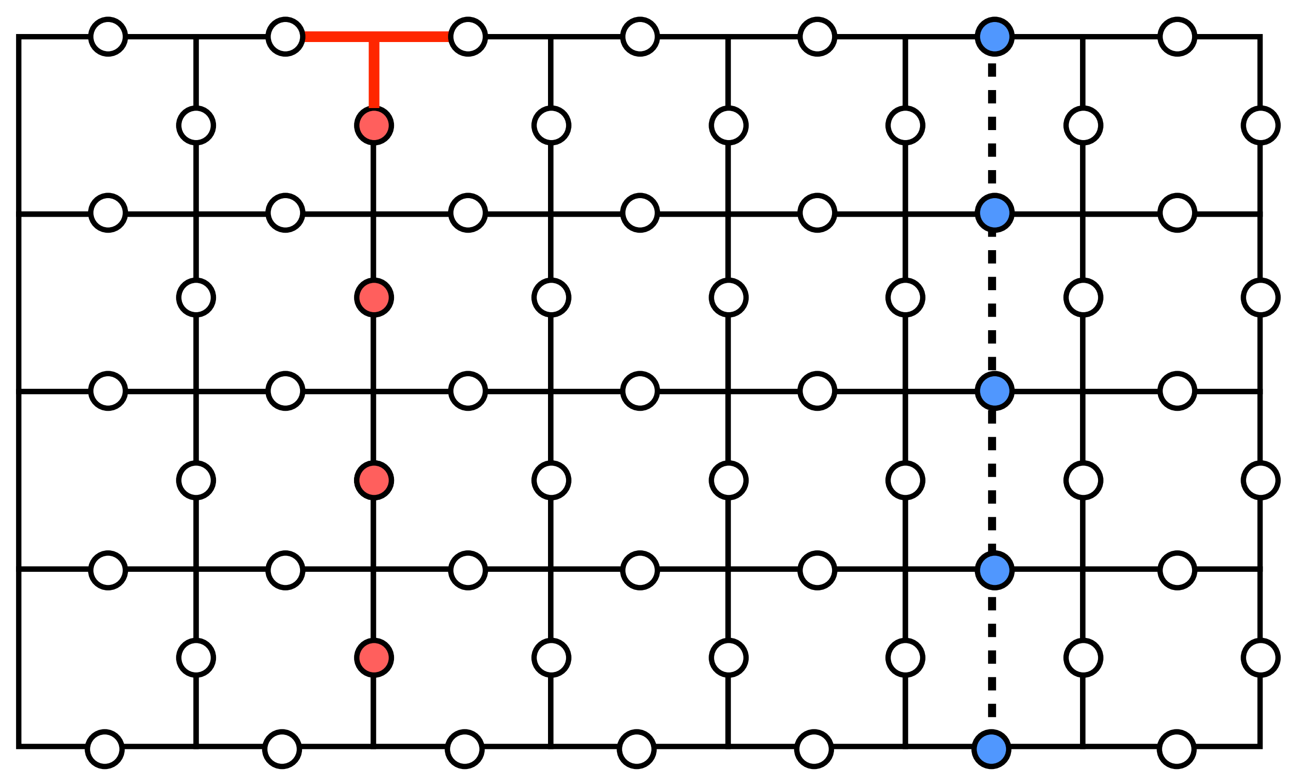


Fracton “topological” order

Restricted mobility: particles unable to move in isolation, move along certain directions by forming bound states (type-I, e.g. Chamon model, X-cube); or are completely immobile (type-II, e.g. Haah code).

[Chamon, PRL (2005); Bravyi, Leemhuis & Terhal, Ann. Phys. (2011); Haah, PRA (2011); Vijay, Haah & Fu, PRB (2015)]

Ground state degeneracy depends on geometric data; typically increases with system size.

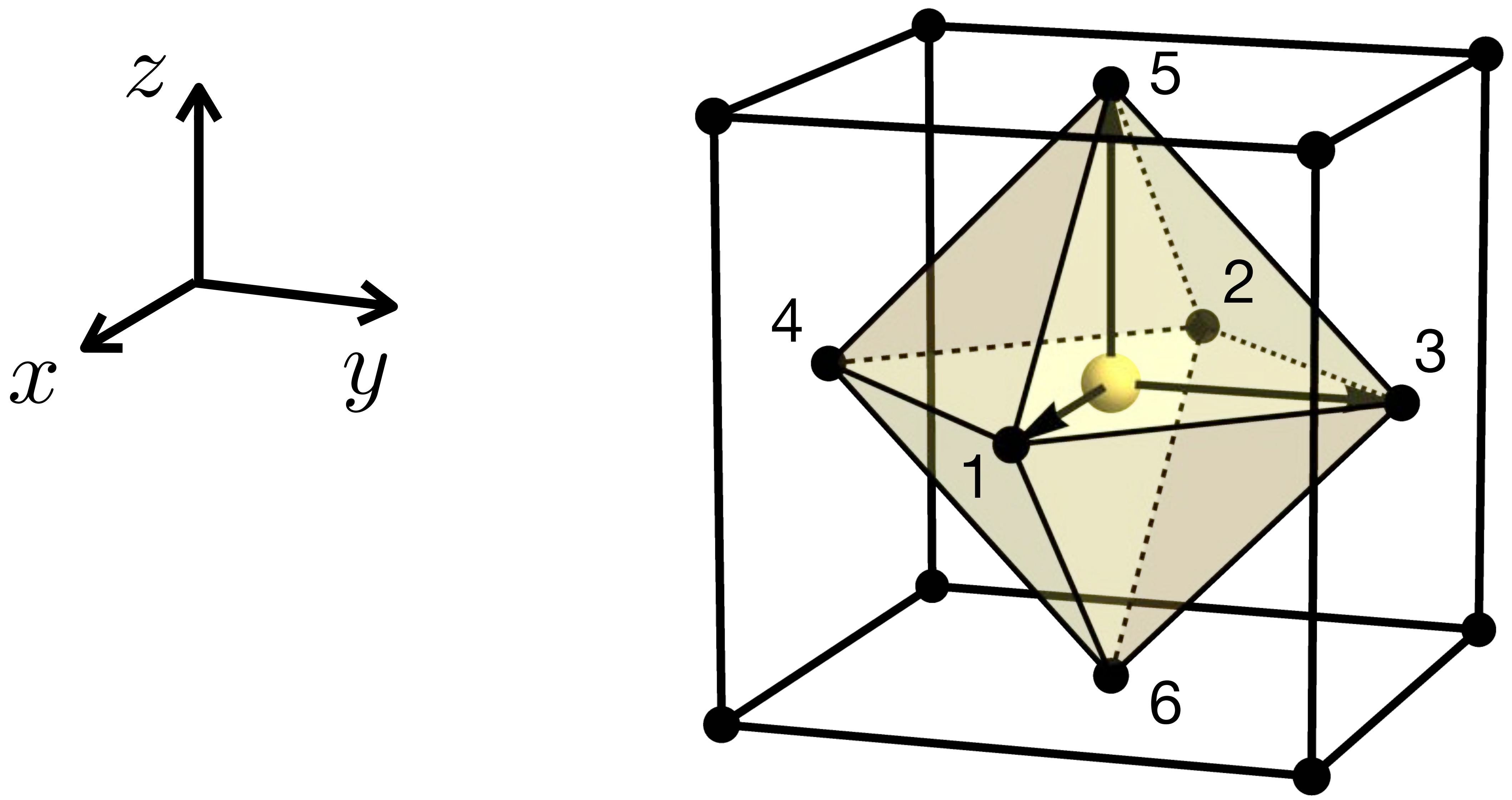
Not described by topological quantum field theories; effective field theories depend on microscopic details (UV-IR mixing).

Lattice models involve multi-spin interactions. Proposals with Majorana zero modes and quantum circuits with time evolution and measurements.

[You & von Oppen, PRR (2019); Verresen et al., arXiv(2021); Lu et al., PRX Quantum (2022)]

Chamon model

Stabilizers defined on octahedra in the fcc lattice. [Chamon, PRL (2005)]



$$\mathcal{O} = \sigma_1^x \sigma_2^x \sigma_3^y \sigma_4^y \sigma_5^z \sigma_6^z$$

$$H = -J \sum_I \mathcal{O}_I$$

sum over octahedra

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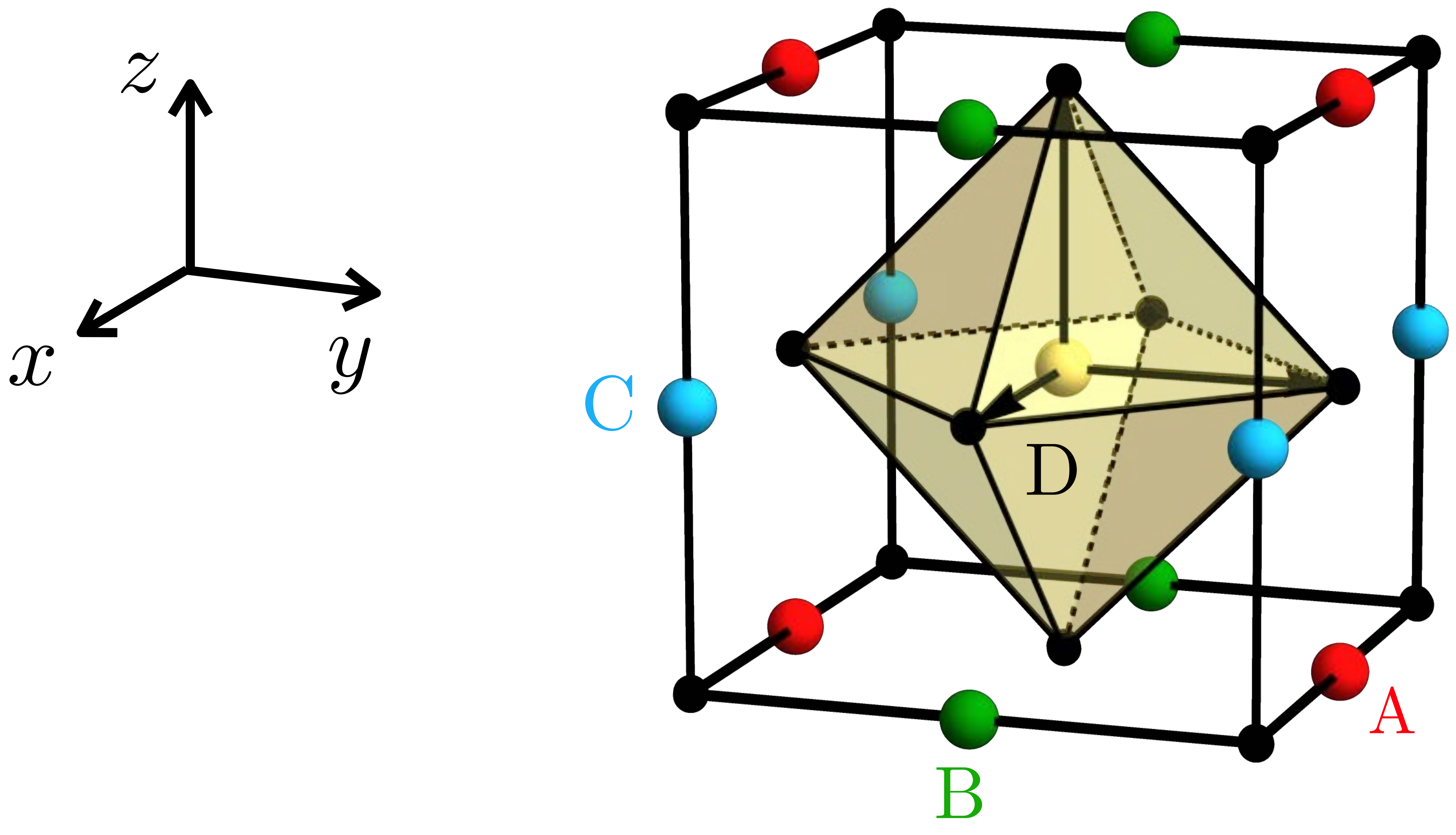

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sum over octahedra

Z_2 constraints on four sublattices:

$$\prod_{I \in \mathrm{A}} \mathcal{O}_I = \prod_{I \in \mathrm{B}} \mathcal{O}_I = \prod_{I \in \mathrm{C}} \mathcal{O}_I = \prod_{I \in \mathrm{D}} \mathcal{O}_I = 1$$

Bulk properties

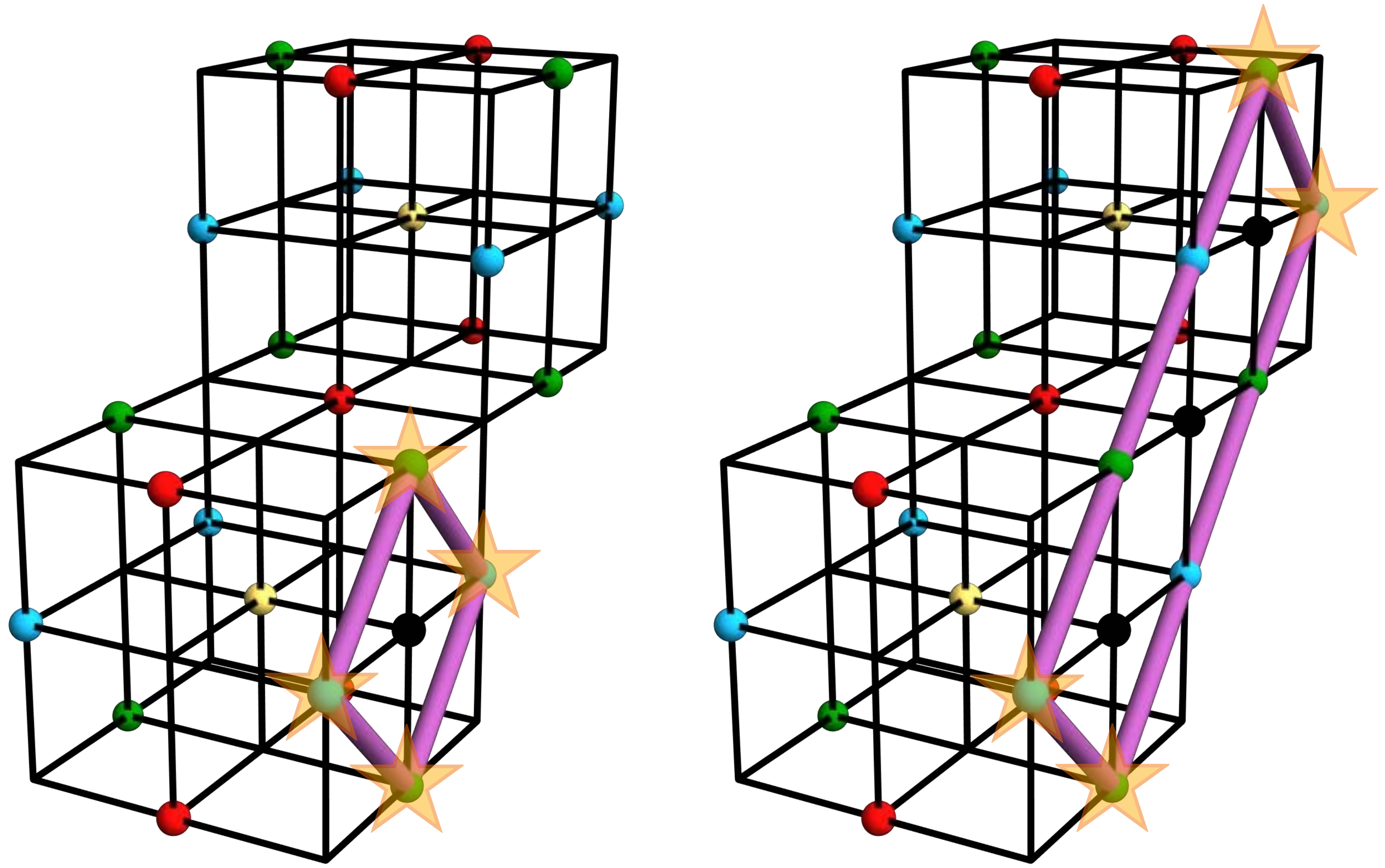
Ground state degeneracy on 3-torus with dimensions $L_x \times L_y \times L_z$:

$$\mathcal{O}_I|\Psi_0^{(\alpha)}\rangle = |\Psi_0^{(\alpha)}\rangle \quad \forall I \qquad \alpha = 1,\ldots,2^{4n} \qquad n = \gcd\left(\frac{L_x}{2},\frac{L_y}{2},\frac{L_z}{2}\right)$$

[Bravyi, Leemhuis & Terhal, Ann. Phys. (2011)]

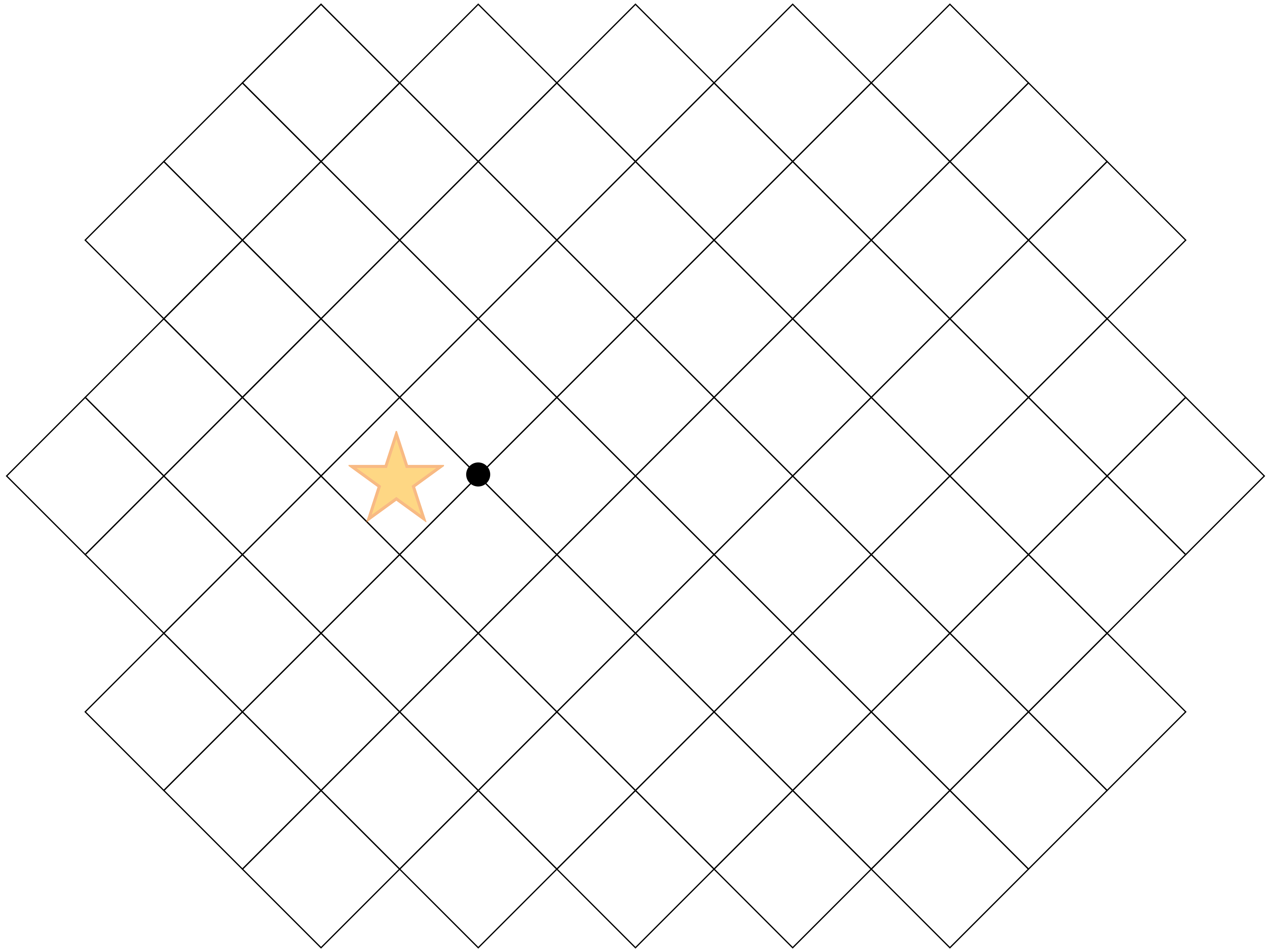
Defect: $\mathcal{O}_I = -1$

Spin operator creates four defects on a plane. Pair of defects (dipole) moves along rigid lines.



Single defects

Local operators cannot move single defect (fracton) without creating additional excitations.



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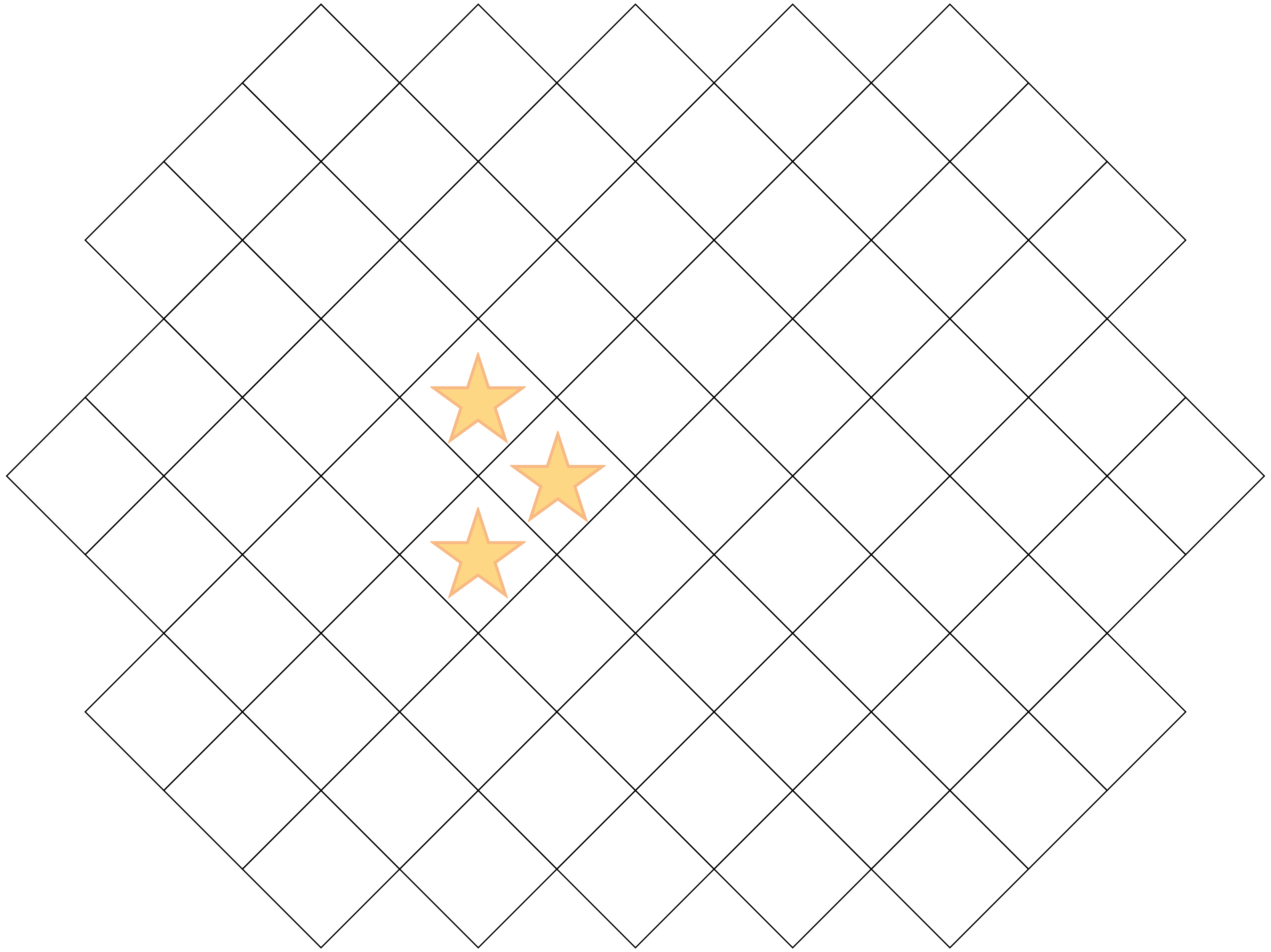
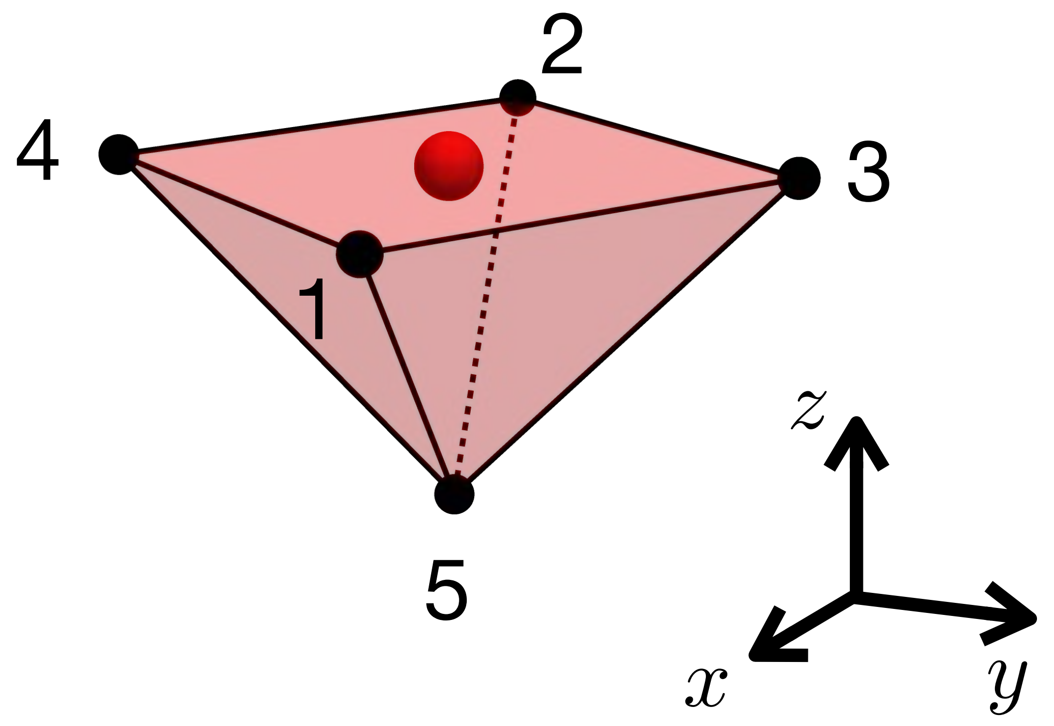


Chamon model with a boundary

Gapped boundaries of fracton models not classified by bulk braiding statistics.

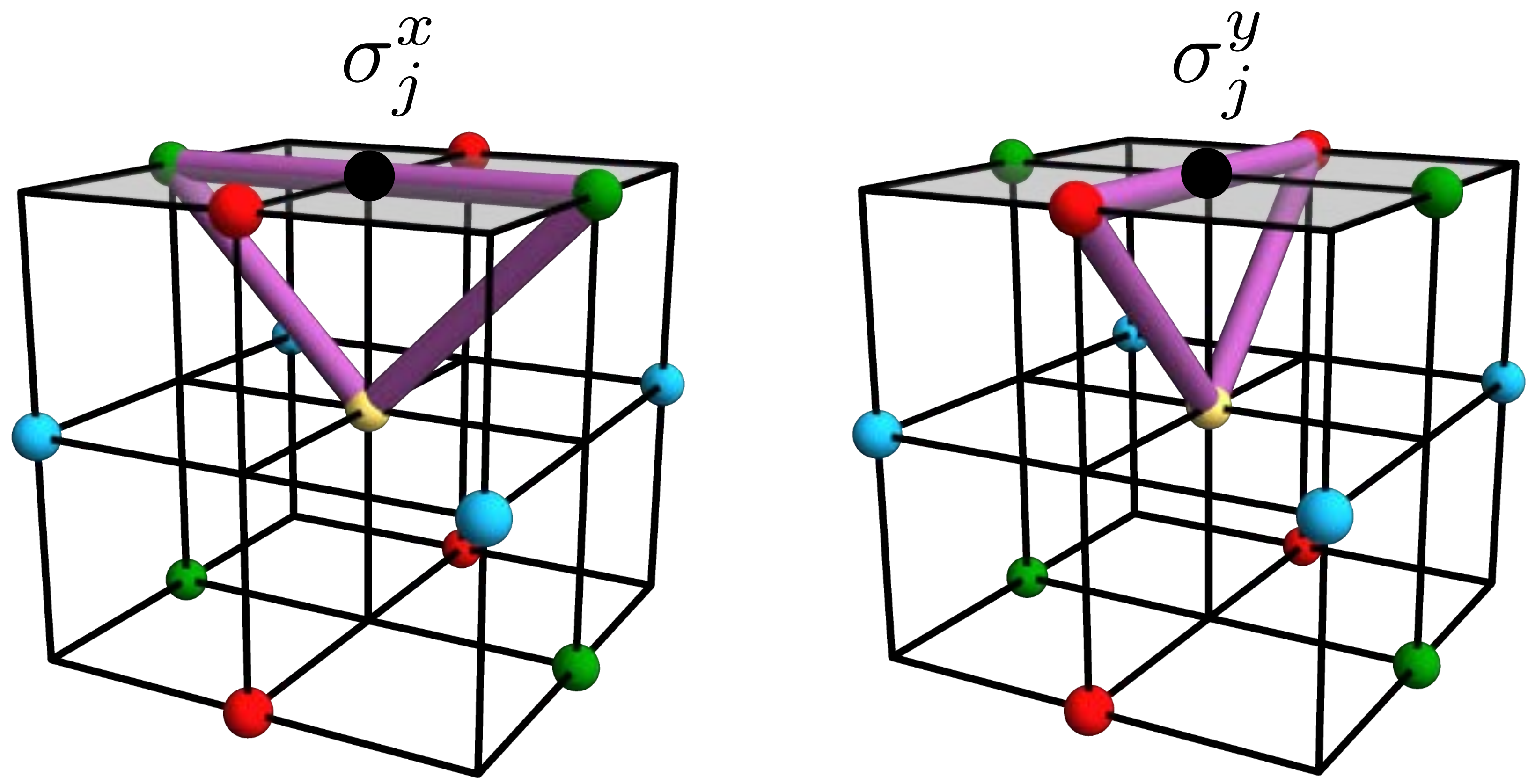
[Bulmash & Iadecola, PRB (2019); Manoj et al., SciPost (2021); Luo, Spieler, Sun & Karch, PRB (2022)]



Five-site stabilizers on (001) boundary: $\mathcal{O}'_I = \sigma_1^x \sigma_2^x \sigma_3^y \sigma_4^y \sigma_5^z$

Boundaries on AB and CD planes violate different constraints.
Spin operator on the boundary creates three defects.

In both cases boundary defects cost energy.
Boundary modes are gapped.



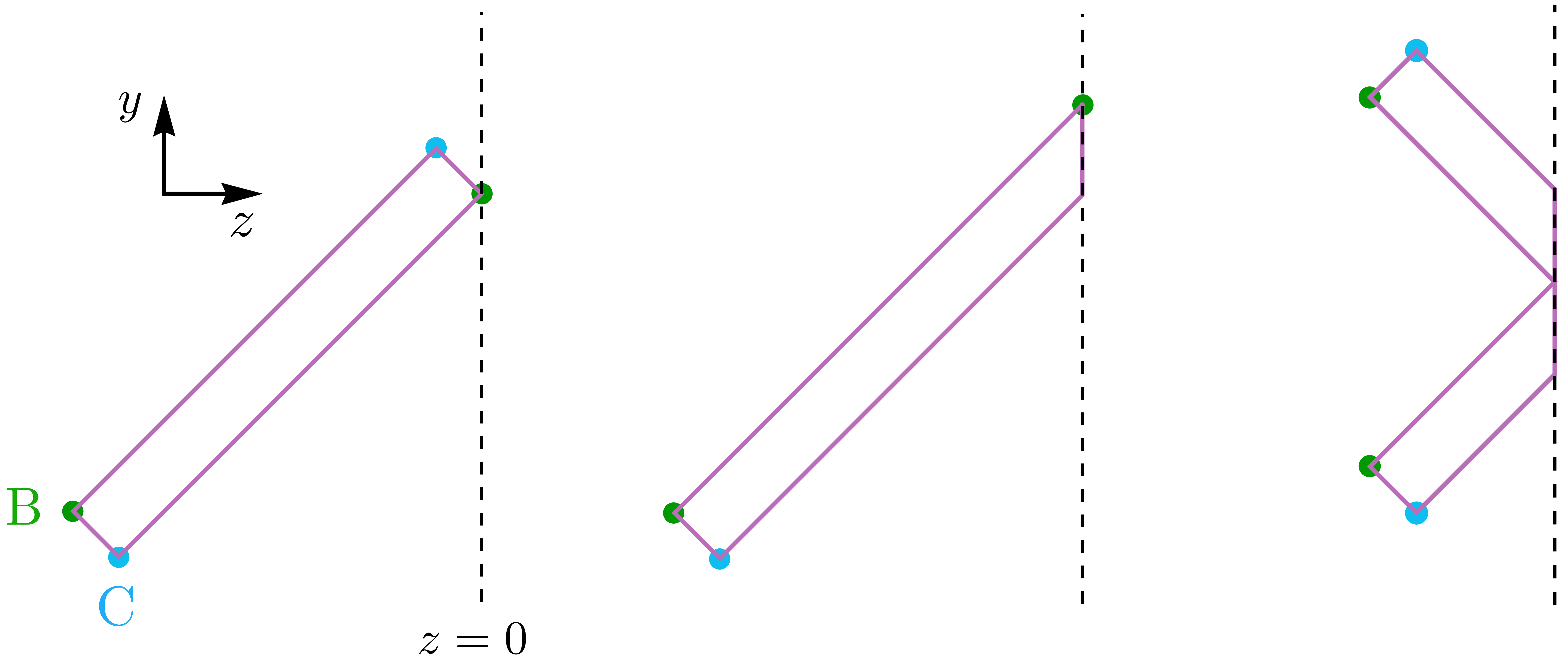
[Fontana & RP, arXiv (2022)]

Boundary processes

Dipoles can be converted into boundary fracton: $(b, c) \to b$ ✓ $(b, c) \to c$ ✗

$(a, d) \to a$ ✓ $(a, d) \to d$ ✗

Allowed processes depend on the parity of boundary plane coordinate *z*.



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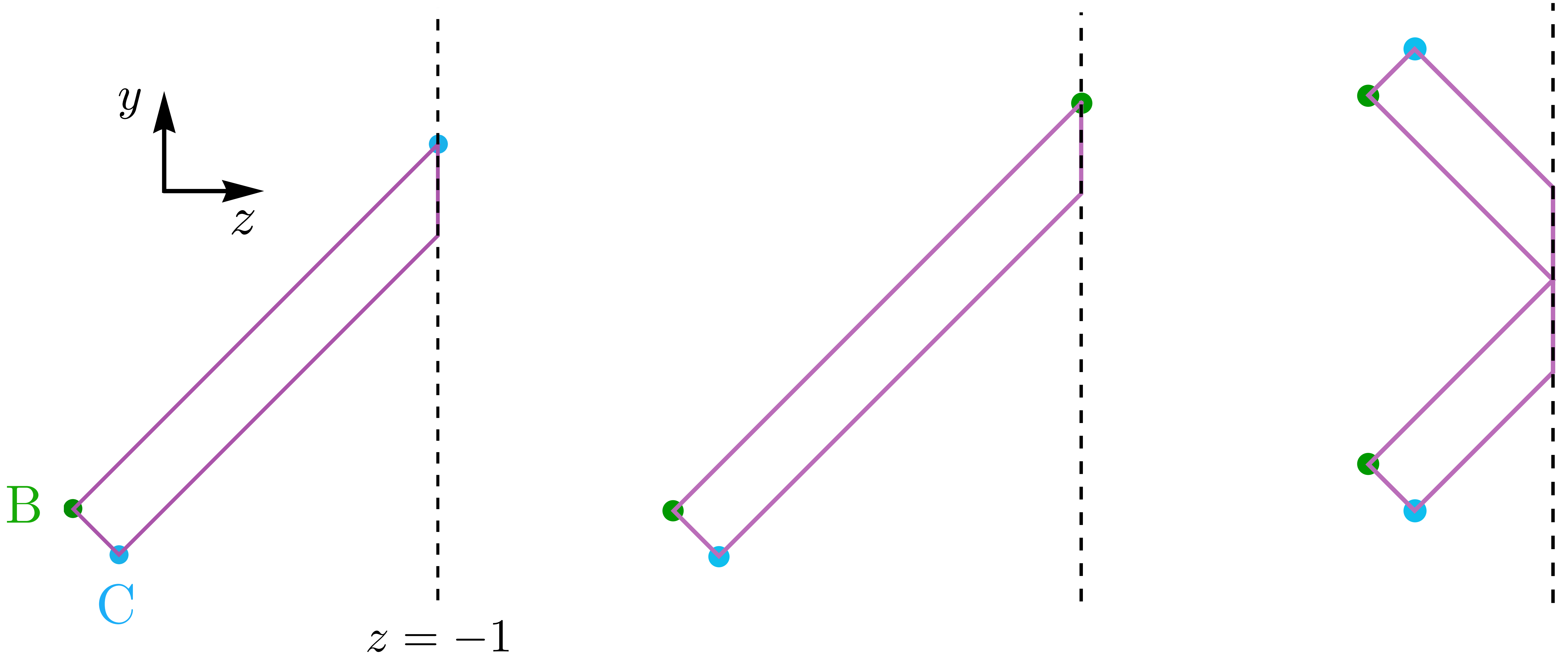


Effective field theory in the bulk

Write spin operators in terms of two bosonic fields. Ground state subspace described by effective Chern-Simons-like action with higher derivatives:

[Fontana, Gomes & Chamon, SciPost (2021)]

$$S = \sum_{l=1,2} \sum_{m,n=1,2} \int d^3r\, dt \left(\frac{1}{2\pi} K_{mn}\, A_m^{(l)} \partial_t\, A_n^{(l)} + \frac{1}{\pi} A_0^{(l)} K_{mn}\, D_m\, A_n^{(l)} \right)$$

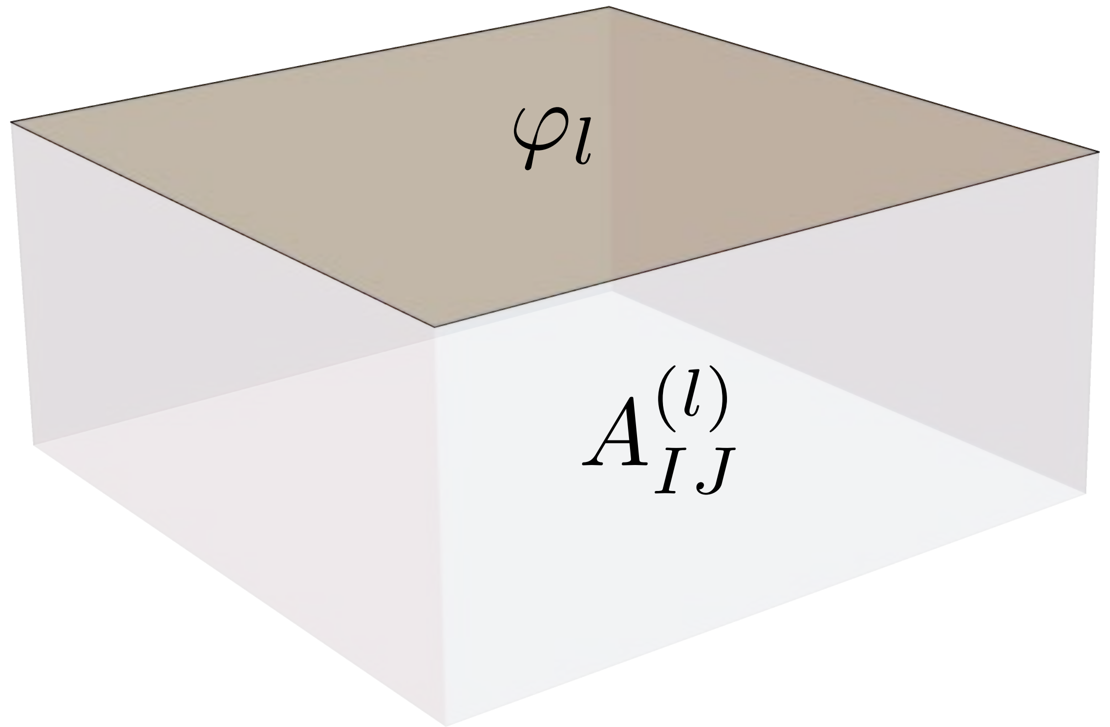
$$K = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad D_1 = \partial_x^2 - \partial_z^2 \qquad D_2 = \partial_y^2 - \partial_z^2$$

Add layer index *l* = 1,2 for (001) planes with odd/even z.

Gauge invariance: $$A_n^{(l)} \to A_n^{(l)} + \sqrt{a}\, D_n \zeta^{(l)} \qquad A_0^{(l)} \to A_0^{(l)} + \sqrt{a}\, \partial_t \zeta^{(l)}$$

Boundary theory

[Fontana & RP, arXiv (2022)]



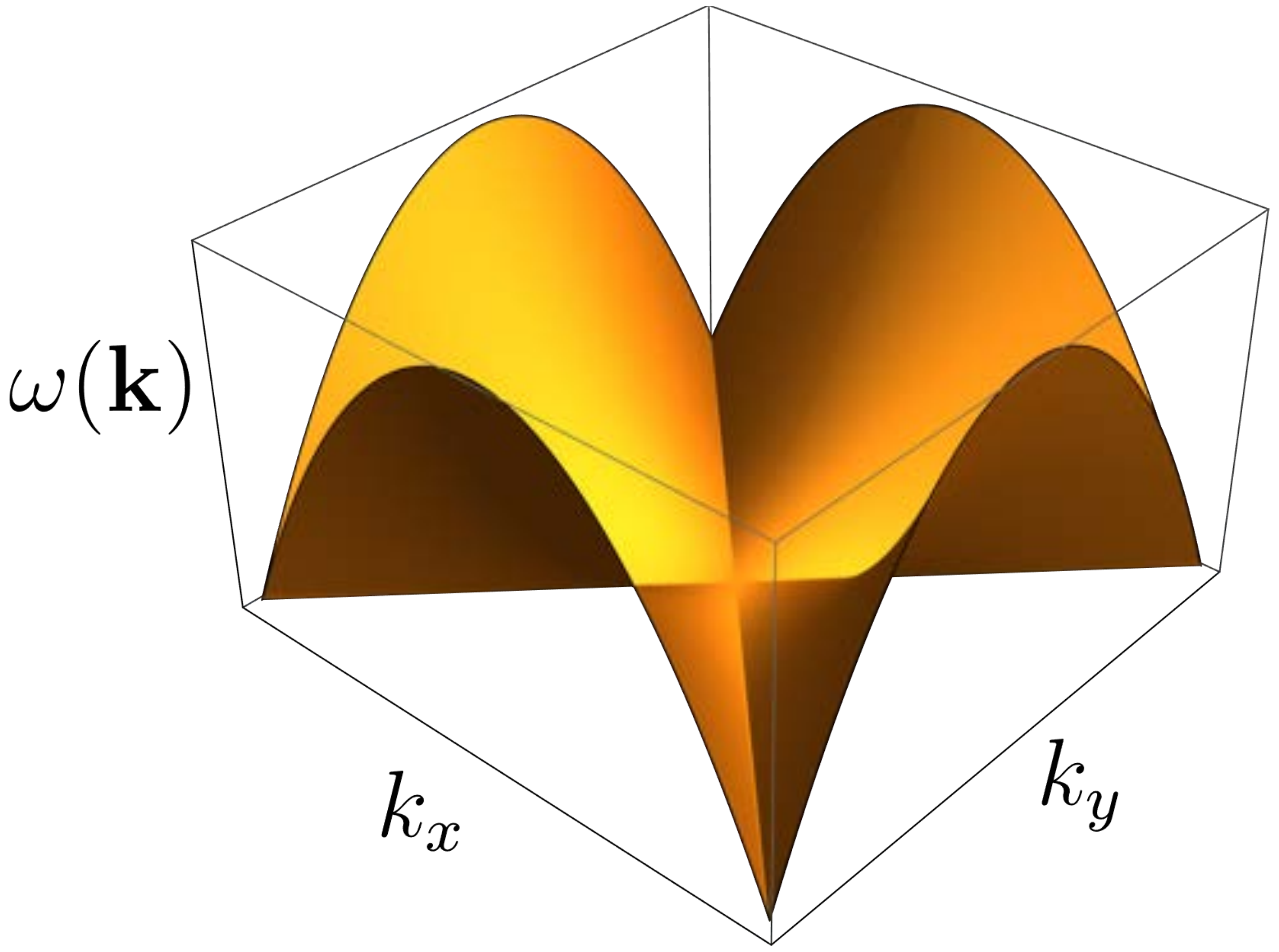
Analogy with QHE: gauge invariance requires physical degrees of freedom at the boundary. Write gauge fields as derivatives of two-component bosonic field.

$$A^{(l)}_{IJ} = \sqrt{a}\, D_{IJ}\varphi_l \qquad [\varphi_l(\mathbf{r}), D_{xy}\varphi_{l'}(\mathbf{r}')] = i\pi(K^{-1})_{ll'}\delta(\mathbf{r}-\mathbf{r}') \qquad D_{xy} = \partial_x^2 - \partial_y^2$$

$$\boxed{S_{\rm bd} = \frac{1}{2\pi}\int d^2r\, dt\, \left(K_{ll'}\,\varphi_l\partial_t D_{xy}\varphi_{l'} - M_{ll'}D_{xy}\varphi_l D_{xy}\varphi_{l'}\right)}$$

Boundary modes are gapless. Dispersion has signature of UV-IR mixing.

$$\omega = \mu|k_x^2 - k_y^2| \qquad \mu = \sqrt{\det(M)}$$



Symmetries and charged operators

Conserved charges: $Q_l = \frac{1}{\pi}\int dxdy\, K_{ll'} D_{xy}\varphi_{l'}$ $\qquad D_{xy} = \partial_x^2 - \partial_y^2$

Charged operators: $\Psi_q(\mathbf{r}) \sim e^{iq_l\varphi_l(\mathbf{r})}$ $\qquad q = (q_1, q_2)$ $\qquad [Q_l, \Psi_q(\mathbf{r})] = -q_l\Psi_q(\mathbf{r})$

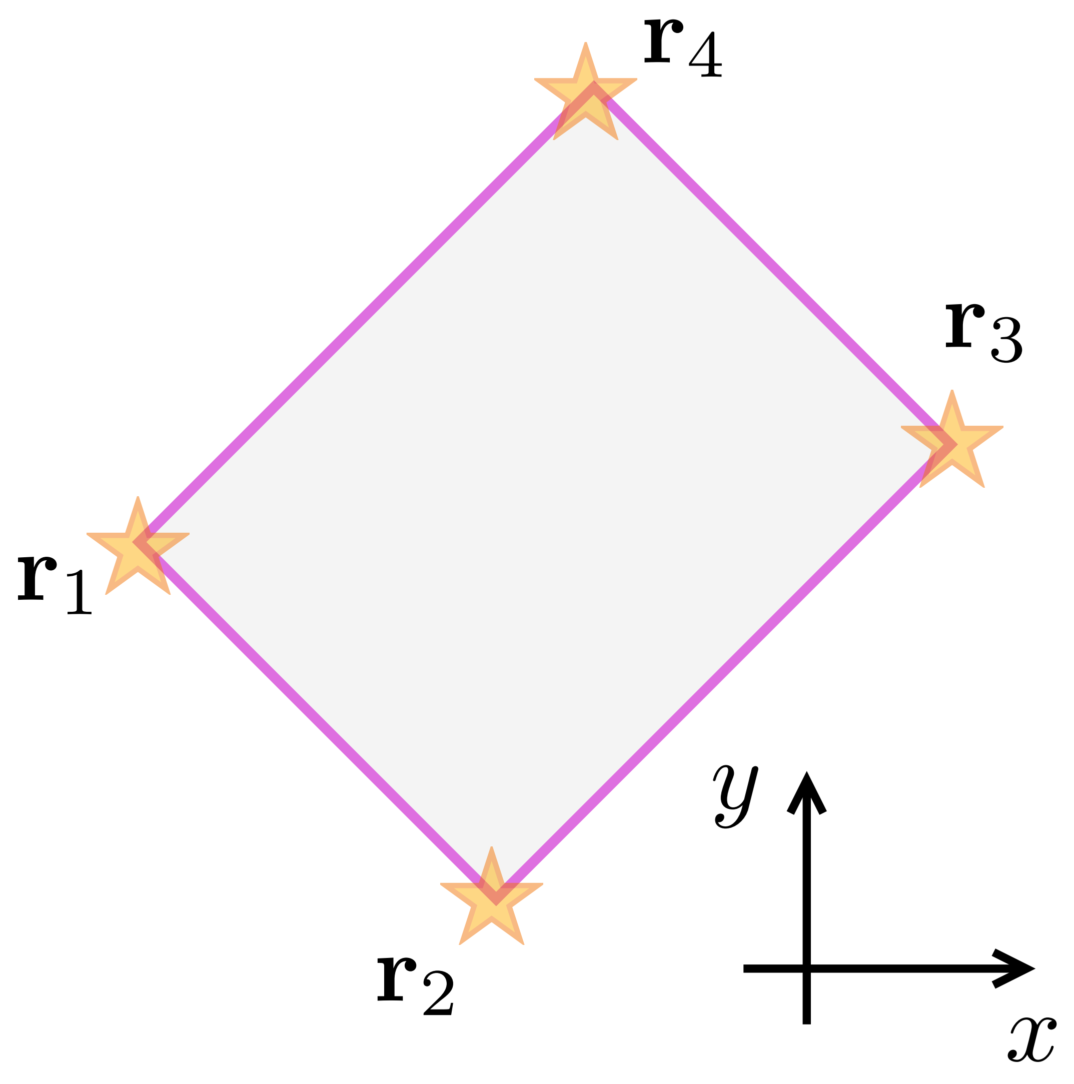
Dipoles created by exponentials of derivatives: $\Psi_q^\dagger(\mathbf{r}+\boldsymbol{\delta})\Psi_q(\mathbf{r}) \sim e^{-iq_l\boldsymbol{\delta}\cdot\nabla\varphi_l(\mathbf{r})}$

Boundary action has U(1) subsystem symmetries:

$$\varphi_l(x,y) \to \varphi_l(x,y) + f_l(x+y) + g_l(x-y)$$

Generalized neutrality condition:

$\left\langle \prod_\alpha e^{iq_l^\alpha\varphi_l(\mathbf{r}_\alpha)} \right\rangle \neq 0$ only if $\mathbf{r}_\alpha$ vertices of rectangular membrane.



Boundary condition

To match number of d.o.f. in the boundary theory, we impose that normal derivative must be linear combination of bosonic fields:

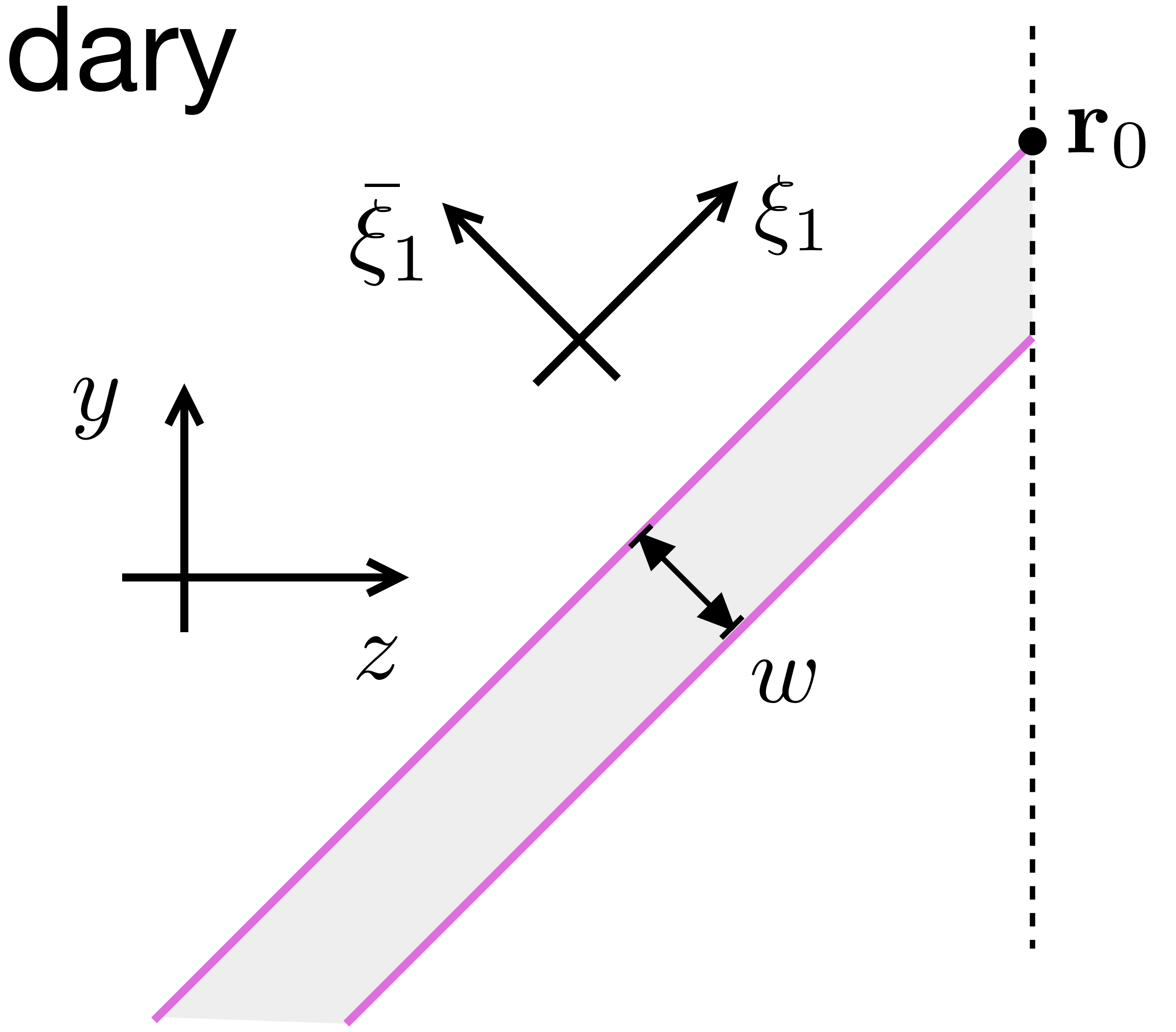
$$\boxed{a\partial_z \varphi_l \to c_{ll'}\varphi_{l'}}$$

(coefficients depend on the type of boundary)

Ribbon operators that extend to the boundary create boundary fracton at the end point.

$$W_l = e^{\frac{i}{\sqrt{a}} \oint d\xi_1 \int_{\bar{\xi}_1}^{\bar{\xi}_1+w} d\bar{\xi}_1 \, A_2^{(l)}}$$

$$W_l = e^{i\sqrt{2}w[\partial_z\varphi_l(\mathbf{r}_0)+\partial_y\varphi_l(\mathbf{r}_0)]} \sim e^{i\sqrt{2}w[c_{ll'}\varphi_{l'}(\mathbf{r}_0)+\dots]}$$



How to gap out the boundary modes?

Perturbations are similar to cosine terms in Luttinger liquid theory.

$$\delta H_q = -g_q \int dxdy\, \cos(q_l \varphi_l)$$

$$\sim \frac{g_q}{2} \int dxdy\, (q_l \varphi_l)^2$$

?

which leaves (353) unchanged. In this case, the Umklapp term becomes

$$H_{Umklapp} \to \frac{\Delta}{2\pi^2\alpha^2} \int dx \cos\left[4\sqrt{\pi g}\phi\,(x)\right]. \tag{356}$$

It is known from renormalization group arguments that for $g > g_{crit} = 1/2$, the Umklapp term is *irrelevant:* its effect on the low-energy sector is simply to renormalize the effective parameters u and g but, other than this, it can be *ignored*. However, if $g < 1/2$, then the Umklapp term is *relevant:* it is responsible for the *opening of a gap in the spectrum*. In this case, a Luttinger liquid description is no longer valid. The case $g = g_{crit} = 1/2$ (which coincides with the isotropic Heisenberg model as we saw) is a *marginal* case.

[Eduardo's "Introduction to bosonization"]

Correlations at weak coupling fixed point

Gapless boundary theory is scale invariant with dynamical exponent $z = 2$. But two-point correlation is (ultra) short-range:

can be small

$$\left\langle e^{iq_l\varphi_l(\mathbf{r})} e^{-iq_{l'}\varphi_{l'}(\mathbf{0})} \right\rangle = 0 \qquad (\mathbf{r} \neq 0) \quad \Rightarrow \quad \cos(q_l\varphi_l)$$

irrelevant at weak coupling

Dipole correlations can be long range:

$$\left\langle e^{iwq_l\,\partial_\xi\varphi_l(\bar{\xi})} e^{-iwq_{l'}\partial_\xi\varphi_{l'}(0)} \right\rangle = \left(\frac{1}{\alpha^2 + \bar{\xi}^2}\right)^{\eta_q} \quad \Rightarrow \quad \eta_q = \frac{(w/\alpha)^2}{8\pi\mu} q_l M_{ll'} q_{l'}$$

$$\cos(wq_l\partial_\xi\varphi_l)$$

can be relevant

Gapless phase is stable. BKT-type transition to non-fractonic phase.

Same theory appears in 2D model: XY plaquette model for Bose metal.

[Paramekanti, Balents & Fisher, PRB (2002); Seiberg & Shao, SciPost (2021); You & Moessner, PRB (2022); Grosvenor et al., PRB (2023)]

Strong coupling regime

Solvable lattice model is at strong coupling: vanishing dispersion $\mu \to 0$

Transition between gapped boundaries governed by competition between two cosine terms:

$$\delta H = -\int dxdy \left[g_1 \cos(q_0\varphi_1) + g_2 \cos(q_0\varphi_2)\right]$$

First-order boundary phase transition; mapping to 2D compass model/Xu-Moore model.

Weak coupling regime requires perturbations that enhance quantum fluctuations and layer mixing at the boundary.

Outlook: microscopic model for gapless phase? Boundary phase transitions?

[in collaboration with C. Chamon]

Summary

- Fracton phases represent a new type of “topological” order in which quasiparticles have restricted mobility.
- The Chamon model has two types of gapped (001) boundaries distinguished by broken parity constraints and processes that turn bulk dipoles into boundary fractons.
- Effective field theory predicts more boundary phases. Need better understanding of boundary phase transitions.

Conservation laws

Electric and magnetic fields vanish in the ground state manifold.

$$E_n^{(l)} = \partial_t A_n^{(l)} - D_n A_0^{(l)} = 0 \qquad B^{(l)} = D_1 A_2^{(l)} - D_2 A_1^{(l)} = 0$$

Subsystem symmetries: conservation of line/ribbon operators.

e.g. $$W_l = \exp\left[\frac{i\,q}{\sqrt{a}} \oint d\xi_1 \int_{\bar{\xi}_1}^{\bar{\xi}_1+w} d\bar{\xi}_1\, A_2^{(l)}\right]$$