The Stadard Model (a crash course)

SM ↔ Quantum Field Theory (QFT)

SM ↔ (Gauge) Symmetries

SM ↔ Spontaneus Sym. Breaking (SSB)

Matter Content
(Leptons, Quarks, Gauge Bosons, Higgs)

Symmetries (and their breaking)

$SU(3) \times \underbrace{SU(2)_L \times U(1)_Y}_{\to U(1)_{em}}$

$\xrightarrow[\text{fixes}]{\text{uniquely}}$ $\mathcal{L}_{SM}$ (Lagrangian Density)

⇓ QFT

Predictions
- Spectra
- Cross-Sections
- Bound States

Quantum Field Theory

Why? → Relativistic Quantum Mechanics does not work!
- → not causal! (see QFT I - video I)
- → Wave function describes the propagation of one particle states (more precisely: states with a definite number of particles)

Number of particles is not fixed anymore!

Inspired by: $\vec{E}, \vec{B} \rightleftharpoons$ number of photons

We will quantize a field theory:

Mechanics → Quantum Mechanics

$q_i(t), P_i(t) \xrightarrow{\text{Quant.}} \hat{q}_i(t), \hat{p}_i(t)$

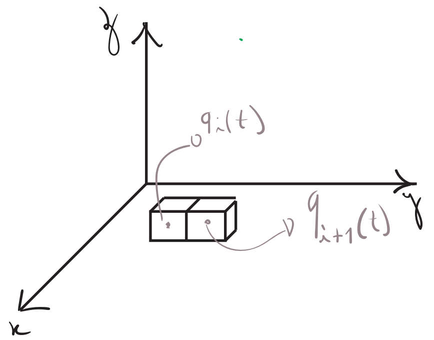
MECHANICS		QUANTUM MECHANICS
$q_i(t),\ p_i(t)$	$\xRightarrow{\text{QUANT.}}$	$\hat{q}_i(t),\ \hat{p}_i(t)$
$\{p_i, q_j\}_{P.B.} = -\delta_{ij}$	$\{\ ,\ \}_{P.B.} \Rightarrow -\frac{i}{\hbar}[\ ,\ ]$ $\downarrow$ 1	$[\hat{p}_i, \hat{q}_j] = -i\hbar\,\delta_{ij}$
$\dot{q}_i(t) = \{q_i, H\}_{P.B.} = \frac{\partial H}{\partial p_i}$		$\frac{d\hat{q}_i}{dt} = -i\hbar\,[\hat{q}_i, \hat{H}]$
$\dot{p}_i(t) = \{p_i, H\}_{P.B.} = -\frac{\partial H}{\partial q_i}$		$\frac{d\hat{p}_i}{dt} = -i\hbar\,[\hat{p}_i, \hat{H}]$

$$\{f, g\}_{P.B.} = \sum_i \left(\frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q_i}\right)$$

One could suspect from this unequal treatment of time and position (q), that this is not a good candidate for a relativistic theory

IT IS EASY TO GENERALIZE THAT TO A FIELD

$\phi(\vec{x}, t)$:



CLASSICAL FIELD THEORY (ClFT)

$$\{q_i(t)\} \sim \{\phi_i(t)\} \underset{\text{DISCRETE}}{\overset{\text{CONTINUUM}}{\rightleftarrows}} \phi(\vec{x}, t)$$

$$\{p_i(t)\} \sim \{\Pi_i(t)\} \rightleftarrows \Pi(\vec{x}, t)$$

$$\{\phi_i, \Pi_j\}_{P.B.} \Rightarrow -\frac{i}{\hbar}[\hat{\phi}_i, \hat{p}_j] \quad \Big\Downarrow \text{QUANT.}$$

$$\text{QFT} \Longrightarrow [\hat{\phi}_i, \hat{\Pi}_j] = i\,\delta_{ij} \rightleftarrows [\hat{\phi}(\vec{x}, t), \hat{\Pi}(\vec{y}, t)] = i\,\delta^3(\vec{x} - \vec{y})$$

IN THE ClFT, IF WE ASSUME LOCALITY:

$$S = \int dt\, L = \int dt \int d^3x\, \underbrace{\mathcal{L}(\phi, \partial_\mu \phi)}_{\text{LAGRANGIAN DENSITY}} = \int d^4x\, \mathcal{L}(\phi, \partial_\mu \phi)$$

$$\Pi(\vec{x}, t) \equiv \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$$

$$H = \int d^3x \left[\pi(\vec{x},t)\dot{\phi}(\vec{x},t) - \mathcal{L} \right] \equiv \int d^3x \, \mathcal{H}(\phi,\pi)$$

HAMILTONIAN DENSITY

EULER-LAGRANGE EQUATIONS GIVE US THE CLASSICAL EOM:

$$\boxed{\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right] = 0}$$

Building Lagrangians (Degrees of Freedom and Symmetries)

FIRST AND FOREMOST, WE WANT LORENTZ INVARIANTS, SO WE CHOOSE FIELDS THAT TRANSFORM UNDER SOME REPRESENTATION OF THE LORENTZ GROUP $(SO(1,3))$:

$$x_\mu \xrightarrow{\text{LORENTZ}} x'_\mu = \Lambda_\mu{}^\nu x_\nu$$

SCALARS: $\phi(x) \xrightarrow{\text{LORENTZ}} \phi'(x') = \phi(x)$ (Spin 0 particles)

SPINORS: $\psi(x) \xrightarrow{\text{LORENTZ}} \psi'(x') = M_D(\Lambda)\,\psi(x)$ (Spin 1/2 particles)

$$M_D(\Lambda) \equiv e^{-\frac{i}{2}\Theta_{\mu\nu}(\Lambda) S^{\mu\nu}} \qquad S^{\mu\nu} = \frac{i}{4}\left[\gamma^\mu, \gamma^\nu\right]$$

IMPORTANT $\rightarrow$ $M_D^{-1}(\Lambda)\gamma^\mu M_D(\Lambda) = \Lambda^\mu{}_\nu \gamma^\nu$

SO: $\gamma^\mu \partial_\mu \psi(x) \xrightarrow{\text{LORENTZ}} \gamma^\mu \partial'_\mu \psi'(x') = M_D(\Lambda)\gamma^\mu \partial_\mu \psi(x)$

Exercise 1: show this

VECTORS: $A_\mu(x) \xrightarrow{\text{LORENTZ}} A'_\mu(x') = \Lambda_\mu{}^\nu A_\nu(x)$ (Spin 1 particles)

WHICH THEN ALLOW FOR EASY BUILDING OF LORENTZ INVARIANTS:

$$(\phi)^n, \; A_\mu A^\mu, \; \underbrace{\overline{\psi}}_{}\psi, \; A^\mu \overline{\psi}\gamma_\mu \psi, \; F_{\mu\nu}F^{\mu\nu}, \; \ldots$$

$\psi^\dagger \gamma^0$: $\overline{\psi}(x) \xrightarrow{\text{LORENTZ}} \overline{\psi}(x) M_D^{-1}(\Lambda)$