Effective models for dynamical scalarization

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<u>Based on:</u> M. Khalil, <u>R. Mendes</u>, N. Ortiz, J. Steinhoff, PDR **106**, 104016 (2022)

- Testing General Relativity
- Limitations of the pN expansion: the rise of nonperturbative effects
- Accommodating nonperturbative effects: pK, pD & all that
- Effective action approach to dynamical scalarization
- Going beyond the adiabatic approximation

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- Tests of basic principles of a (metric) theory of gravity
 - Universality of free fall
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MICROSCOPE Mission: First Results of a Space Test of the Equivalence Principle

Pierre Touboul et ol. Phys. Rev. Lett. 119, 231101 - Published 4 December 2017

Physics see Synopus, Solide Texts of the Ecolymence Principle



MicroSCOPE - CNES

- Tests of basic principles of a (metric) theory of gravity
 - Universality of free fall
 - Local Lorentz invariance
 - Local position invariance

• Tests in the post-Newtonian regime
• Parametrized post-Newtonian (PPN)
formalism

$$\phi/c^2 = GM/Rc^2 \ll 1, \quad v/c \ll 1$$

$$g_{00} = -1 + 2U - 2\beta U^{2} - 2\xi \Phi_{W} + (2\gamma + 2 + \alpha_{3} + \zeta_{1} - 2\xi)\Phi_{1} + 2(3\gamma - 2\beta + 1 + \zeta_{2} + \xi)\Phi_{2} + 2(1 + \zeta_{3})\Phi_{3} + 2(3\gamma + 3\zeta_{4} - 2\xi)\Phi_{4} - (\zeta_{1} - 2\xi)A - (\alpha_{1} - \alpha_{2} - \alpha_{3})w^{2}U - \alpha_{2}w^{i}w^{j}U_{ij} + (2\alpha_{3} - \alpha_{1})w^{i}V_{i} + 0(\epsilon^{3})$$
$$g_{0i} = -\frac{1}{2}(4\gamma + 3 + \alpha_{1} - \alpha_{2} + \zeta_{1} - 2\xi)V_{i} - \frac{1}{2}(1 + \alpha_{2} - \zeta_{1} + 2\xi)W_{i} - \frac{1}{2}(\alpha_{1} - 2\alpha_{2})w^{i}U - \alpha_{2}w^{j}U_{ij} + 0(\epsilon^{2.5})$$
$$g_{ij} = (1 + 2\gamma U)\delta_{ij} + 0(\epsilon^{2})$$

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Parameter	Effect	Limit	Remarks
$\gamma - 1$	time delay	$2.3 imes 10^{-5}$	Cassini tracking
	light deflection	4×10^{-4}	VLBI
$\beta - 1$	perihelion shift	3×10^{-3}	$J_2 = 10^{-7}$ from helioseismology
	Nordtvedt effect	$2.3 imes 10^{-4}$	$\eta_N = 4\beta - \gamma - 3$ assumed
ξ	Earth tides	10^{-3}	gravimeter data
α_1	orbital polarization	10^{-4}	Lunar laser ranging
		$2 imes 10^{-4}$	PSR J2317+1439
α_2	spin precession	$4 imes 10^{-7}$	solar alignment with ecliptic
α_3	pulsar acceleration	4×10^{-20}	pulsar \dot{P} statistics
η_N	Nordtvedt effect	$9 imes 10^{-4}$	lunar laser ranging
ζ1		$2 imes 10^{-2}$	combined PPN bounds
ζ2	binary acceleration	4×10^{-5}	\ddot{P}_p for PSR 1913+16
ζ3	Newton's 3rd law	10^{-8}	lunar acceleration
ζ4		-	not independent (see Equation (58))

Table 4: Current limits on the PPN parameters. Here η_N is a combination of other parameters given by $\eta_N = 4\beta - \gamma - 3 - 10\xi/3 - \alpha_1 + 2\alpha_2/3 - 2\zeta_1/3 - \zeta_2/3$.

Testing General Relativity

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Will (2006)

PHYSICAL REVIEW LETTERS

Laton Suggestion

New General Relativistic Contribution to Mercury's Perihelion Advance

Collections

Clifford M. Will Phys. Rev. Lett. 120, 191101 -- Published 8 May 2018



BepiColombo ESA/JAXA



- Universality of free fall
- Local Lorentz invariance
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Parametrized post-Newtonian (PPN) formalism

$$\phi/c^2 = GM/Rc^2 \ll 1, \qquad v/c \ll 1$$

Tests in the "strong gravity" regime $\phi/c^2 \sim 0.5$

Black holes

Neutron stars $\phi/c^2 \sim 0.2 - 0.3$

 $v/c \ll 1$ $v/c \leq 1$

Tests in the radiative regime



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Post-Newtonian expansion



Post-Newtonian expansion



Post-Newtonian expansion



$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[R - 2g^{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi \right] + S_m \left[\Psi_m; \frac{a(\phi)^2 g_{\mu\nu}}{g_{\mu\nu}} \right]$$

Universal coupling to "physical" metric $\tilde{g}_{\mu\nu} = a(\phi)^2 g_{\mu\nu}$

See also: Doneva et al. (2022), "Scalarization"

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Universal coupling to "physical" metric $\tilde{g}_{\mu\nu} = a(\phi)^2 g_{\mu\nu}$

• For a system of N <u>non-self-gravitating</u> point particles:

$$S_{m} = -\sum_{A=1}^{N} \int \widetilde{m}_{A} d\widetilde{s}_{A} = -\sum_{A=1}^{N} \int \widetilde{m}_{A} \sqrt{-\widetilde{g}_{\mu\nu}(x_{A})} dx_{A}^{\mu} x_{A}^{\nu}$$

$$S_{m} = -\sum_{A=1}^{N} \int \widetilde{m}_{A} a(\phi(x^{A})) ds_{A} = -\sum_{A=1}^{N} \int m_{A}(\phi(x^{A})) ds_{A}$$
Effective coupling:

$$m_{A}(\phi(x^{A}))$$

$$\alpha(\phi) = \frac{d \ln a(\phi)}{d\phi} = \frac{d \ln m(\phi)}{d\phi}$$

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[R - 2g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \right] + S_m \left[\Psi_m; \frac{a(\phi)^2 g_{\mu\nu}}{q_{\mu\nu}} \right]$$

Universal coupling to "physical" metric $\tilde{g}_{\mu\nu} = a(\phi)^2 g_{\mu\nu}$

• For a system of N <u>self-gravitating</u> point particles, Eardley (1975) suggests skeletonizing the action as

$$S_m = -\sum_{A=1}^N \int \underline{\widetilde{m}_A(\phi)} d\tilde{s}_A = -\sum_{A=1}^N \int m_A(\phi) ds_A$$

ield dependent binding energy

Body-dependent effective coupling:

$$\alpha_A(\phi) = \frac{d\ln m_A(\phi)}{d\phi}$$

* See also Damour and Esposito-Farèse (1992)

• Let:
$$\Phi = (g_{\mu\nu} - f_{\mu\nu}, \phi - \phi_0)$$

 σ : matter variables

• Total action: $S_{tot}^{GF}[\sigma, \Phi] = S_{\Phi}^{GF}[\Phi] + S_m[\sigma, \Phi]$ $S_{\Phi}[\Phi] + \text{GF terms}$

$$\frac{\delta S_{tot}^{GF}}{\delta \Phi} = \frac{\delta S_{\Phi}}{\delta \Phi} + \frac{\delta S_{m}}{\delta \Phi} = 0$$
$$\frac{\delta S_{tot}^{GF}}{\delta \sigma} = \frac{\delta S_{m}}{\delta \sigma} = 0$$

• Let:
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 σ : matter variables

- Total action: $S_{tot}^{GF}[\sigma, \Phi] = S_{\Phi}^{GF}[\Phi] + S_m[\sigma, \Phi]$ $S_{\Phi}[\Phi] + \text{GF terms}$
- Strategy:

• Fokker action:

$$S_{\rm F}[\sigma] = S_{tot}^{GF}[\sigma, \overline{\Phi}[\sigma]]$$

Following Damour & Esposito-Farèse (1996)



Following Damour & Esposito-Farèse (1996)



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2. $S_{\rm F}[\sigma] = S_{tot}^{GF}[\sigma, \overline{\Phi}[\sigma]]$



• Newtonian limit:

 $G_{AB} = G[1 + (\alpha_A \alpha_B)_0]$ G_{A





• 2PN parameters: ϵ_{BCD}^{A} , ζ_{ABCD}

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- For any PN order: corrections proportional to α_{A0}^2 .

$$\alpha_{A0} = \alpha_A(\phi_0) = \frac{d \ln m_A(\phi)}{d\phi} \Big|_{\phi=\phi_0} = \frac{d \ln [\widetilde{m_A}(\phi)a(\phi)]}{d\phi} \Big|_{\phi=\phi_0}$$

→ constant if non-self-gravitating

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Expand in
$$C_A = \frac{Gm_A}{R_A c^2}$$
: $\alpha_{A0} = \alpha_0 [1 + A_1 C_A + A_2 C_A^2 + \cdots]$
(Damour & Esposito-Farèse, 1992) $\alpha_0 = d \ln a(\phi) / d\phi \Big|_{\phi_0}$

• Limitation of the post-Newtonian expansion: Non-perturbative effects make it inappropriate to expand

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[Damour & Esposito-Farèse (1992)]

• Limitation of the post-Newtonian expansion: Non-perturbative effects make it inappropriate to expand



 $\alpha_{A0} = \alpha_0 [1 + A_1 C_A + A_2 C_A^2 + \cdots]$

[Damour & Esposito-Farèse (1992)]

• Limitation of the post-Newtonian expansion: Non-perturbative effects make it inappropriate to expand $\alpha_{A0} = \alpha_0 [1 + A_1 C_A + A_2 C_A^2 + \cdots]$ Inner problem: Solve the TOV-like equations of hydrostatic equilibrium with BC $g_{\mu\nu} \xrightarrow{r \to r_{\text{match}}} \hat{g}_{\mu\nu} = f_{\mu\nu}$ $\phi \xrightarrow{r \to r_{\text{match}}} \hat{\phi}$ For $r/R \gg 1$: $g_{\mu\nu} = f_{\mu\nu} + \frac{2Gm_A(\hat{\phi})}{rc^2}\delta_{\mu\nu} + \cdots$ $\phi = \hat{\phi} - \frac{Gm_A(\hat{\phi})\alpha_A(\hat{\phi})}{rc^2} + \cdots$ r_{match} D

[Damour & Esposito-Farèse (1992)]

• Limitation of the post-Newtonian expansion: Non-perturbative effects make it inappropriate to expand

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[R. Mendes & T. Ottoni (2019)] For a nonminim

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Post-Keplerian expansion



Post-Keplerian expansion


Gravitational Waves Meet Amplitudes in the Southern Hemisphere

Post-Keplerian expansion

PSR J0348+0432

General Relativity





 Scalar-tensor theory of Damour and Esposito-Farèse



RAPID COMMUNICATIONS



color coding: scalar field amplitude

PHYSICAL REVIEW D 87, 081506(R) (2013)

Neutron-star mergers in scalar-tensor theories of gravity

Enrico Barausse,¹² Carlos Palenzuela,³ Marcelo Ponce,² and Luis Lehner^{4,5}
³Institut d'Astrophysique de Paris/CNRS, 98bis boulevard Arago, 75014 Paris, France
²Department of Physics, University of Gaelph, Gaelph, Ontario N1G 2W1, Canada
³Canadian Institute for Theoretical Astrophysics, Toronto, Ontario M5S 3H8, Canada
⁴Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada
³Canadian Institute for Advanced Research, Cosmology & Gravity Program, Canada (Received 21 December 2012; published 25 April 2013)

Scalar-tensor theories of gravity are natural phenomenological alternatives to General Relativity, where the gravitational interaction is mediated by a scalar degree of freedom, besides the usual tensor gravitons. In regions of the parameter space of these theories where constraints from both solar system experiments and binary-pulsar observations are satisfied, we show that binaries of neutron stars present marked differences from General Relativity in both the late-inspiral and merger phases. In particular, phenomena related to the spontaneous scalarization of isolated neutron stars take place in the late stages of the evolution of binary systems, with important effects in the ensuing dynamics. We comment on the relevance of our results for the upcoming Advanced LIGO/Virgo detectors.

DOE: 10.1103/PhysRevD.87.081506

PACS numbers: 04.30.-w, 04.25.D-, 97.60.Jd

RAPID COMMUNICATIONS



• The PK formalism avoids expanding _

→ capturing spontaneous scalarization

$$m_A(\phi_0) = \widetilde{m}_A a(\phi_0) \left[1 + A_1 \left(\frac{Gm_A}{R_A c^2} \right) + A_2 \left(\frac{Gm_A}{R_A c^2} \right)^2 + \cdots \right]$$

but still expands

$$m_{A}(\phi) = m_{A}(\phi_{0}) \left[1 + C_{1} \left(\frac{\phi - \phi_{0}}{\phi_{0}} \right) + C_{2} \left(\frac{\phi - \phi_{0}}{\phi_{0}} \right)^{2} + \cdots \right]$$

missing dynamical scalarization

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Same starting point

- Sennet, Shao, Steinhoff (2017)
- Khalil, Sennett, Steinhoff, Buonanno (2019)

$$S_m = -\sum_{A=1}^2 \int m_A(\phi) \sqrt{-u^\mu u_\mu} d\sigma$$

• Ist step: Reparametrizing in terms of the scalar charge/monopole:

$$Q_A = -\frac{dm_A}{d\phi} = -m_A \alpha_A$$

$$\phi = \phi_0 + \frac{Q}{r} + O\left(\frac{1}{r^2}\right)$$

- Sennet, Shao, Steinhoff (2017)
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• Ist step: Reparametrizing in terms of the scalar charge/monopole...

$$Q_A = -\frac{dm_A}{d\phi} = -m_A \alpha_A$$

... using a Legendre transformation

$$\overline{m}_{A}(Q) = m_{A}(\phi) + Q\phi$$

$$S_{m} = -\sum_{A=1}^{2} \int (\overline{m}_{A}(Q) - Q\phi) \sqrt{-u^{\mu}u_{\mu}} d\sigma$$

- Sennet, Shao, Steinhoff (2017)
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$$S_m = -\sum_{A=1}^2 \int m_A(\phi) \sqrt{-u^\mu u_\mu} d\sigma$$

 Ist step: Reparametrizing in terms of the scalar charge/monopole using a Legendre transformation:

$$\overline{m}_A(Q) = m_A(\phi) + Q\phi$$

- 2^{nd} step: Promote Q to an independent degree of freedom.
- 3rd step: Power-series expansion

$$\overline{m}_A(Q) = c^{(0)} + c^{(1)}Q + \frac{c^{(2)}}{2!}Q^2 + \frac{c^{(3)}}{3!}Q^3 + \frac{c^{(4)}}{4!}Q^4 + O(Q^5)$$

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• Often, S_{tot} is invariant under $\phi \to -\phi$ (or $Q \to -Q$): keep only even terms

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$$\overline{m}_A(Q) = m_A(\phi) + Q\widehat{\phi} \stackrel{\longrightarrow}{\longrightarrow} \widehat{\phi} = \phi - \phi_0$$

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$$\overline{m}_{A}(Q) = m^{(0)} - \phi_{0}Q + \frac{c^{(2)}}{2!}Q^{2} + \frac{c^{(4)}}{4!}Q^{4} + O(Q^{5})$$
• Often, S_{tot} is invariant under $\phi \to -\phi$ (or $Q \to -Q$): keep only even terms
• If $\phi_{0} \neq 0$: $c^{(1)} = -\phi_{0}$.

• Newtonian-order solution:

- Sennet, Shao, Steinhoff (2017)
- Khalil, Sennett, Steinhoff, Buonanno (2019)

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(c^{-3}) \longrightarrow h_{00}(\vec{x}, t) = \frac{\overline{m}_A(Q_A)}{|\vec{x} - \vec{y}_A(t)|} + \frac{\overline{m}_B(Q_B)}{|\vec{x} - \vec{y}_B(t)|} + O(c^{-4})$$

$$h_{0i}(\vec{x}, t) = O(c^{-3})$$

$$h_{ij}(\vec{x}, t) = O(c^{-4})$$

$$\psi(\vec{x}, t) = \frac{Q_A}{|\vec{x} - \vec{y}_A(t)|} + \frac{Q_B}{|\vec{x} - \vec{y}_B(t)|} + O(c^{-4})$$

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$$h_{0i}(\vec{x}, t) = O(c^{-3})$$

$$h_{ij}(\vec{x}, t) = O(c^{-4})$$

• Inserting in the matter action one gets:

$$S \sim \int dt \left[-\overline{m}_A - \overline{m}_B + \frac{\overline{m}_A}{2} \vec{v}_A^2 + \frac{\overline{m}_B}{2} \vec{v}_B^2 + \frac{\overline{m}_A \overline{m}_B}{r} + \frac{Q_A Q_B}{r} \right]$$
$$v^i = \frac{dy^i}{dt} \qquad r = |\vec{y}_A - \vec{y}_B|$$

• Newtonian-order Hamiltonian:

- Sennet, Shao, Steinhoff (2017)
- Khalil, Sennett, Steinhoff, Buonanno (2019)

$$H = \overline{m}_A + \overline{m}_B + \frac{\vec{p}_A^2}{2\overline{m}_A} + \frac{\vec{p}_B^2}{2\overline{m}_B} - \frac{\overline{m}_A \overline{m}_B}{r} - \frac{Q_A Q_B}{r}$$

• Newtonian-order Hamiltonian:

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$$H = \overline{m}_A + \overline{m}_B + \frac{\vec{p}_A^2}{2\overline{m}_A} + \frac{\vec{p}_B^2}{2\overline{m}_B} - \frac{\overline{m}_A\overline{m}_B}{r} - \frac{Q_AQ_B}{r}$$

$$0 = \frac{\partial H}{\partial Q_A} = z_A \left(-\phi_0 + c_A^{(2)} Q_A + \frac{c_A^{(4)}}{6} Q_A^3 \right) - \frac{Q_B}{r} + O(Q_A^4)$$
$$z_A = \frac{\partial H}{\partial \bar{m}_A} = 1 - \frac{\vec{p}_A^2}{2\bar{m}_A^2} - \frac{\bar{m}_B}{r}$$

- Sennet, Shao, Steinhoff (2017)
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$$0 = \frac{\partial H}{\partial Q_A} = z_A \left(-\phi_0 + c_A^{(2)} Q_A + \frac{c_A^{(4)}}{6} Q_A^3 \right) - \frac{Q_B}{r} + O(Q_A^4)$$

To get an approximate solution:

•
$$z_A \approx 1$$

• $\phi_0 = 0$
• $\overline{m}_A = \overline{m}_B$
0 $= \frac{\partial H}{\partial Q} = -2Q \left[\frac{1}{r} - c^{(2)} - \frac{c^{(4)}}{6} Q^2 \right]$

- Sennet, Shao, Steinhoff (2017)
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$$0 = \frac{\partial H}{\partial Q_A} = z_A \left(-\phi_0 + c_A^{(2)} Q_A + \frac{c_A^{(4)}}{6} Q_A^3 \right) - \frac{Q_B}{r} + O(Q_A^4)$$

To get an approximate solution:
• $z_A \approx 1$
• $\phi_0 = 0$
• $\overline{m}_A = \overline{m}_B$

$$0 \le \frac{\partial^2 H}{\partial Q^2} = 2c^{(2)} - \frac{2}{r} + c^{(4)}Q^2$$

violated by $Q = 0$ if $c^{(2)} < 1/r$

$$0 = \frac{\partial H}{\partial Q} = -2Q \left[\frac{1}{r} - c^{(2)} - \frac{c^{(4)}}{6} Q^2 \right]$$

• Equation of motion for Q_A :

То

- Sennet, Shao, Steinhoff (2017)
- Khalil, Sennett, Steinhoff, Buonanno (2019)

$$0 = \frac{\partial H}{\partial Q_A} = z_A \left(-\phi_0 + c_A^{(2)} Q_A + \frac{c_A^{(4)}}{6} Q_A^3 \right) - \frac{Q_B}{r} + O(Q_A^4)$$
get an approximate solution:
• $z_A \approx 1$
• $\phi_0 = 0$
• $\overline{m}_A = \overline{m}_B$
Stability criterion
 $0 \le \frac{\partial^2 H}{\partial Q^2} = 2c^{(2)} - \frac{2}{r} + c^{(4)}Q^2$
violated by $Q = 0$ if $c^{(2)} < 1/r$
 $0 = \frac{\partial H}{\partial Q} = -2Q \left[\frac{1}{r} - c^{(2)} - \frac{c^{(4)}}{6} Q^2 \right]$

- If $c^{(2)} < 0, Q = 0$ is always unstable: spontaneous scalarization!
- Even if $c^{(2)} > 0$, scalarization can occur at small orbital separations: $c^{(2)} < 1/r$: dynamical scalarization!

- Sennet, Shao, Steinhoff (2017)
- Khalil, Sennett, Steinhoff, Buonanno (2019)

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Stable solutions:

$$Q = \begin{cases} 0 & \text{for } 1/r \le c^{(2)} \\ \pm \sqrt{\frac{6}{c^{(4)}}} \sqrt{\frac{1}{r} - c^{(2)}} & \text{for } 1/r \ge c^{(2)} \end{cases}$$

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- Testing General Relativity
- Limitations of the pN expansion: the rise of nonperturbative effects
- Accommodating nonperturbative effects: pK, pD & all that
- Effective action approach to dynamical scalarization
- Going beyond the adiabatic approximation

• Alternative interpretation as a coarse-grained description:

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Dynamical short-length processes represented by dynamical variables on the worldline

• Alternative interpretation as a coarse-grained description:

$$\phi = \phi^{IR} + \phi^{UV}$$

$$\downarrow \lambda_{UV} \leq R$$

$$\downarrow \lambda_{IR} \geq R$$
spatially average

Spontaneous scalarization is triggered by the instability of the fundamental (monopolar) scalar mode (ϕ -mode)

Dynamical short-length processes represented by dynamical variables on the worldline



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Dynamical short-length processes represented by dynamical variables on the worldline

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Dynamical short-length processes represented by dynamical variables on the worldline

• Expected dynamics of the scalar mode:

$$c_{\dot{q}^{2}}\ddot{q} + V'(q) = \phi^{IR}(y) \qquad \longleftrightarrow S_{NS}^{crit} = \int d\tau \left[\frac{c_{\dot{q}^{2}}}{2}\dot{q}^{2} + \phi(y)q - m(q) + \cdots\right]$$

$$V(q) = \frac{c_{(2)}}{2}q^{2} + \frac{c_{(4)}}{4!}q^{4} + \cdots \qquad m(q) = c_{(0)} + V(q)$$

• Before: adiabatic approximation

- In Khalil, Mendes, Ortiz, Steinhoff (PRD 2022) we:
 - Includes the scalar mode dynamics;
 - Considers generic (eccentric) orbits;
 - Include radiation-reaction effects at the EOM level.

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• Computing the UV coefficients

 $c_{(2)}$ and $c_{(4)}$

- (i) Solve numerically the structure equations in the full theory, for a fixed baryon mass M_b and different values of ϕ_{∞}
- (ii) Compute $m(q) = m(\phi_{\infty}) \phi_{\infty}q(\phi_{\infty})$
- (iii) Fit a polynomial to $m(q) c_{(0)}$ and extract the quadratic $(c_{(2)})$ and quartic $(c_{(4)})$ coefficients.



• Computing the UV coefficients

C_q²:

- (i) Compute the quasinormal mode frequency of the ϕ -mode using NS perturbation theory.
- (ii) Contrast with the solution for a damped harmonic oscillator,

$$c_{\dot{q}^{2}}\ddot{q} + \dot{q} + c_{(2)}q = 0$$

$$q(t) = e^{-\omega_I t} [a \cos(\omega_R t) + b \sin(\omega_R t)]$$

$$\omega_R^2 + \omega_I^2 = \frac{c_{(2)}}{c_{\dot{q}^2}}$$





0.0

54

56

58

 $t/(10^3 M)$

60

62

50

60

Separation and scalar charge for an equal-mass binary with initial eccentricity e = 0.3 and $\phi_0 = 0$.

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A final recap!

Thank you!