ICTP SAIFR | International Centre for Theoretical Physics
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Gravitational Waves meet Amplitudes in the Southern Hemisphere

Towards 5PN in compact binary dynamics

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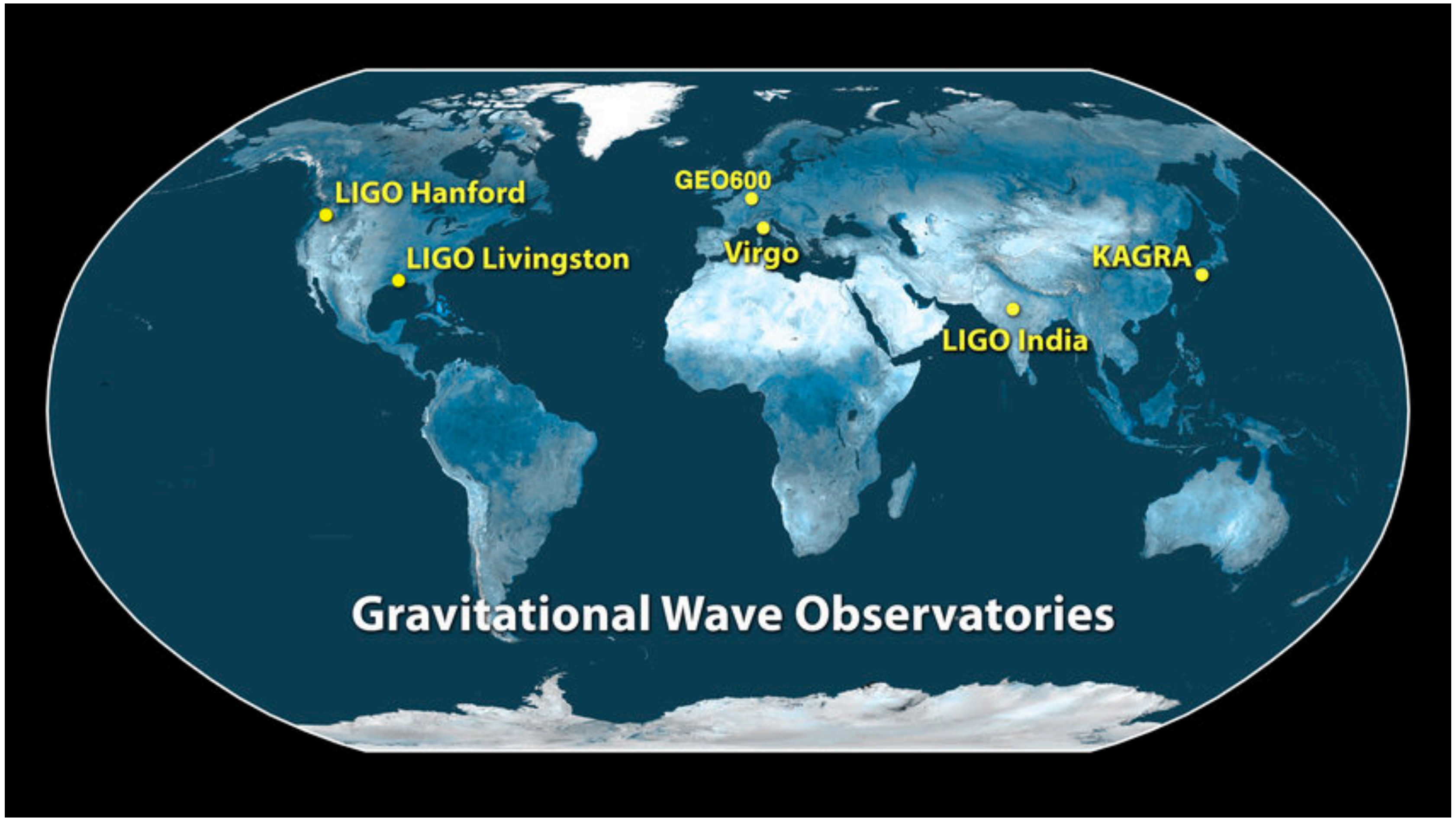
GWSC GRAVITATIONAL WAVE SCIENCE CENTER

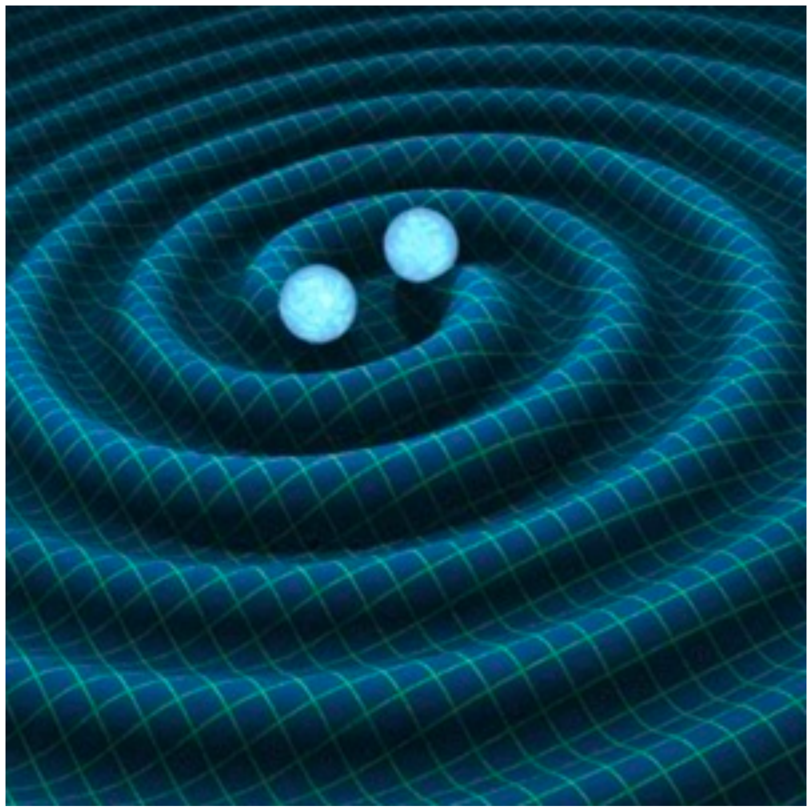
Southern emisphere GW observatories are less necessary than in the electromagnetic case

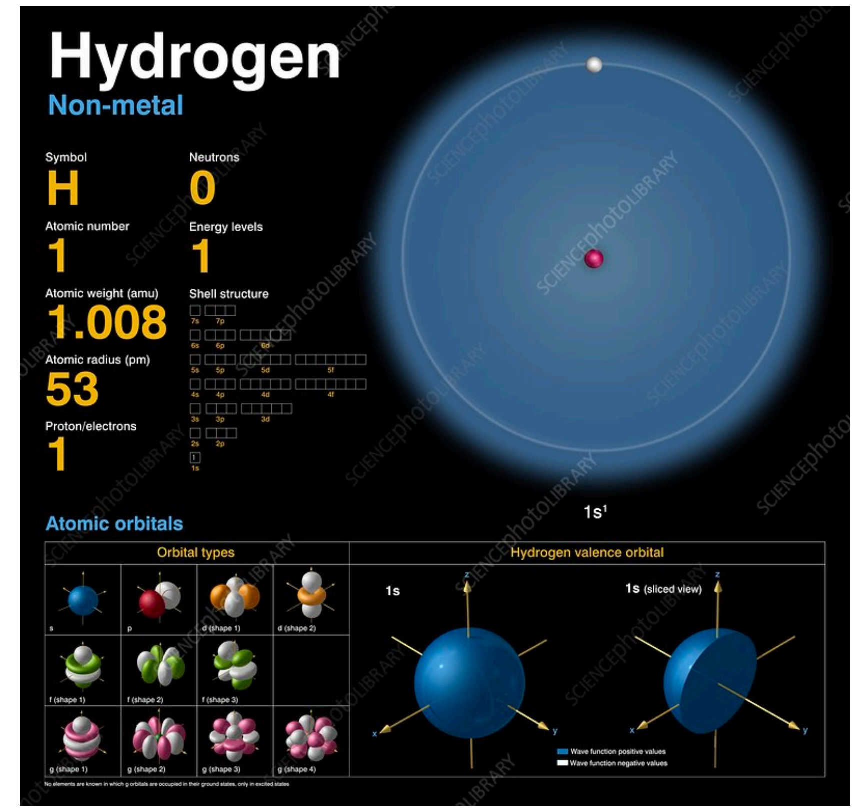


It is even more important to have programs like this to grow a strong and global community of GW/GR enthusiasts

…before we start:

Compact binaries are the new hydrogen atom





Simple and rich of features at the same time

Instrumental to improve our understanding of certain dynamical regimes/configurations

ok let's be pragmatic

we need accurate WF's
(especially the **phase**)

balance equation:

circular orbit

conservative dynamics

radiative sector

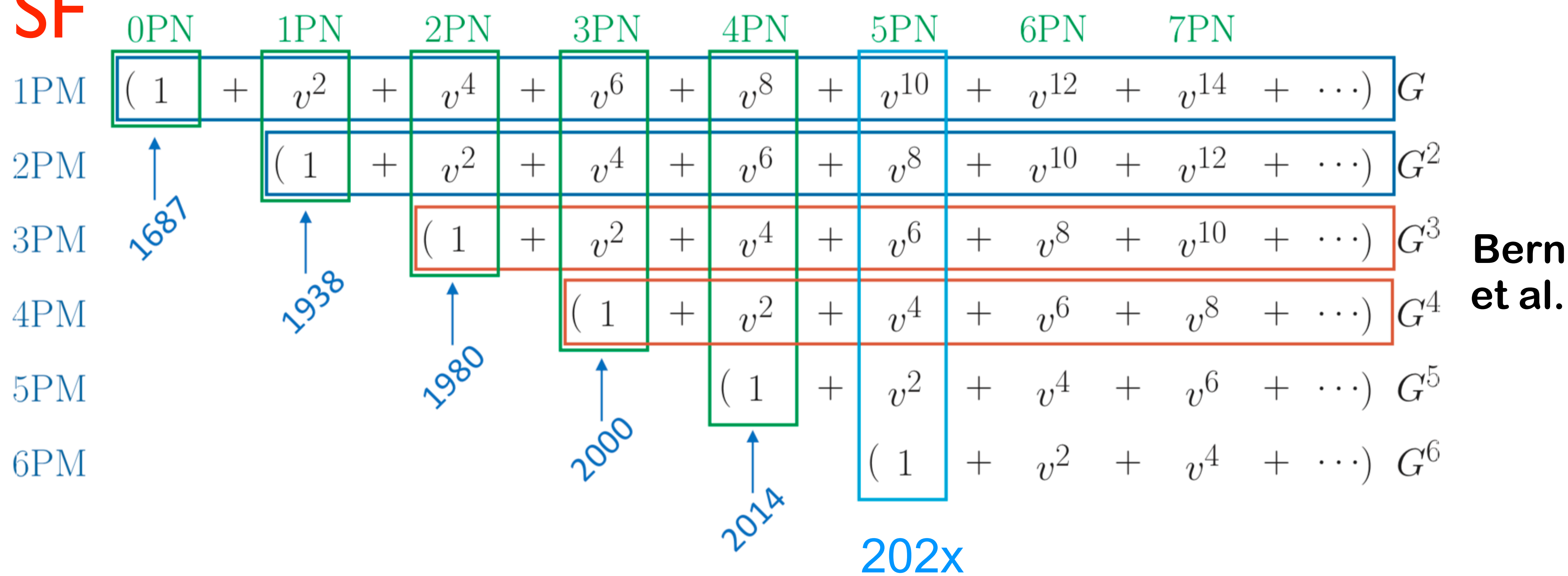
$$\dot{E}(\omega) = P_{GW}(\omega)$$

(Schott terms can be put on either side!)

PN is the phenomenologically relevant perturbative scheme

PN, PM, SF

Energy:



0SF 1SF 2SF 3SF 4SF

0PN	1	x^0
1PN	$1+\nu$	x
2PN	$1+\nu+\nu^2$	x^2
3PN	$1+\nu+\nu^2+\nu^3$	x^3
4PN	$1+\nu+\nu^2+\nu^3+\nu^4$	x^4

$$x \equiv \left(\frac{GM\omega}{c^3}\right)^{2/3}$$

(even more striking for the scattering angle)

2PN PN iteration

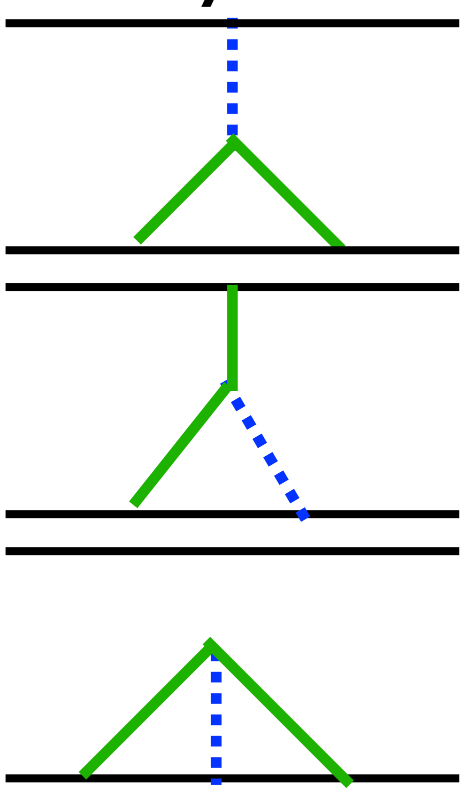
Problems in inverting Poisson operators

3PN ADM, PN iteration, surface integrals

First divergencies: Hadamard and dimensional regularisations

4PN ADM+SF, Fokker lagrangian, EFT

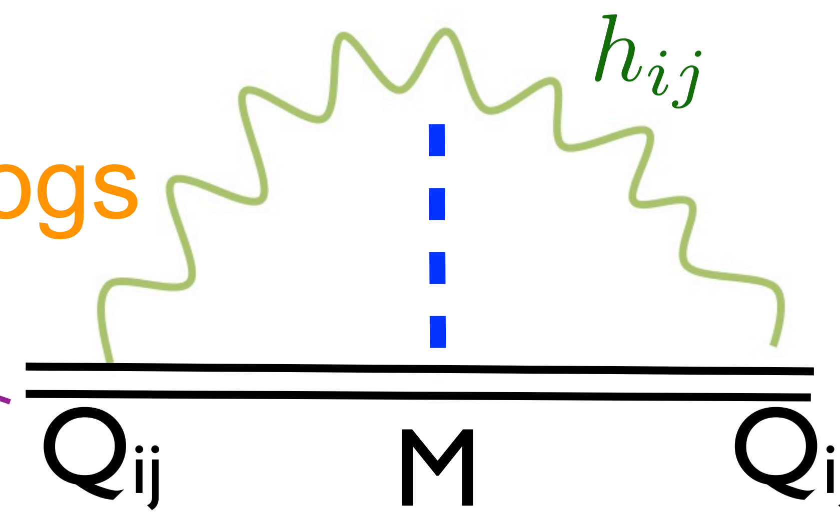
Physical IR divergencies killed by tail contributions



$$\frac{1}{5}G^2M\left(\frac{1}{\epsilon}-\frac{1}{\epsilon}-\frac{41}{30}\right)\dddot{Q}_{ij}^{2} + \text{logs}$$

IR

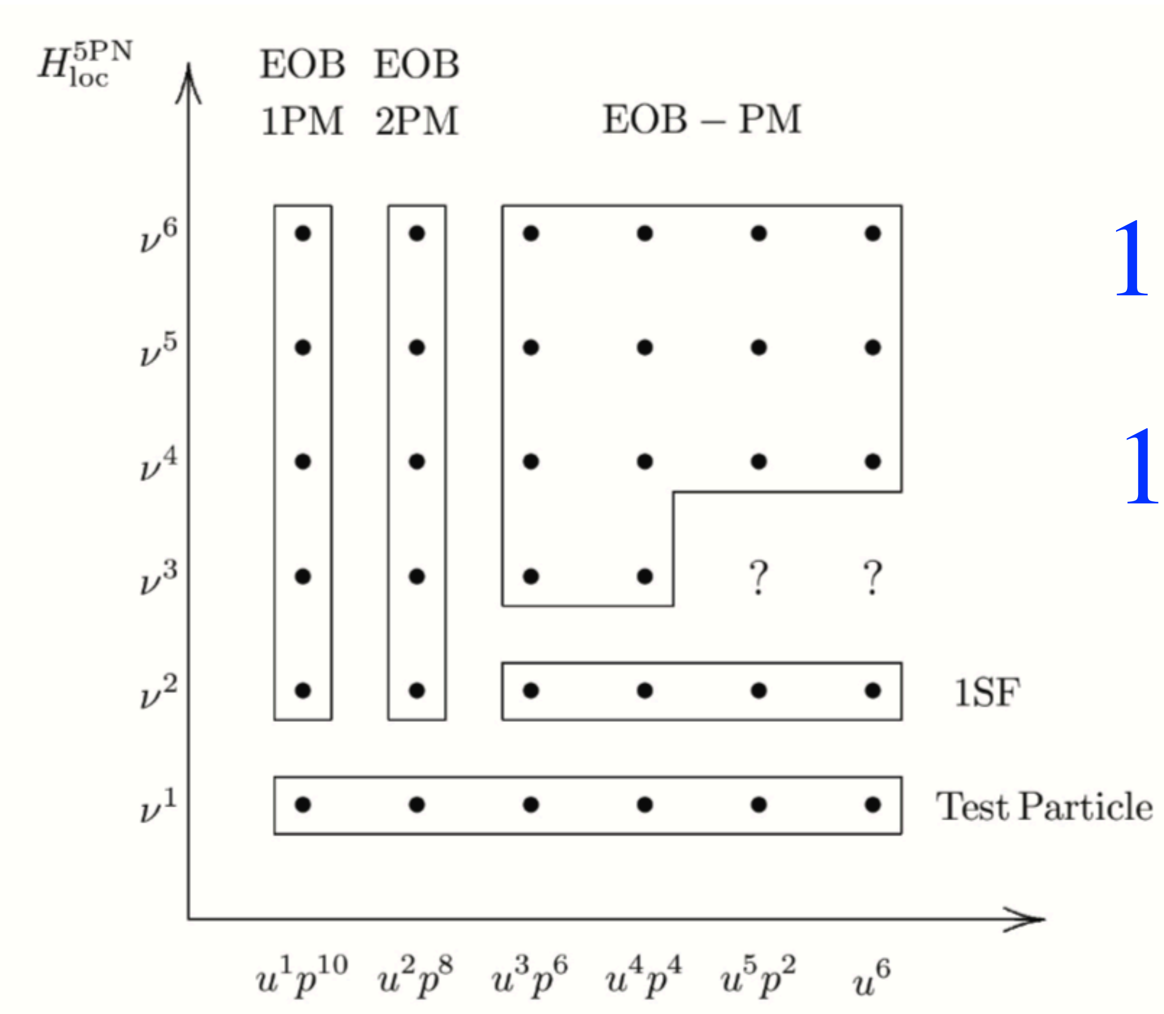
UV



First appearance of highly nontrivial 4-loop integrals: resort to particle physics technology

5PN, Tuttifrutti

Bini et al.



from 1909.02375

test particle limit

1SF redshift z translated into H_{EOB}

1PM and 2PM amplitude results

scattering angle properties

34 out of 36 coefficients of local c.o.m. Hamiltonian

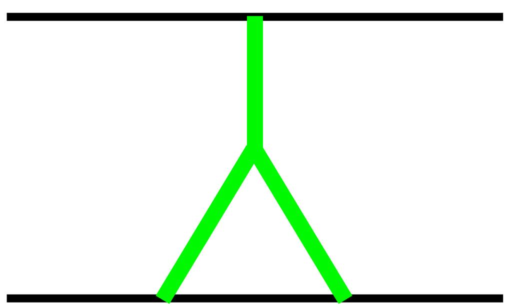
(and 147 out of 151 6PN local terms)

5PN, ab initio

method of regions:

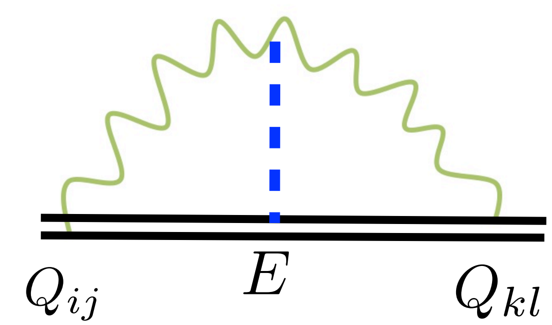
$$\frac{i}{\mathbf{k}^2 - k_0^2} \longrightarrow \frac{i}{\mathbf{k}^2} \sum_{n \geq 0} \left(\frac{k_0^2}{\mathbf{k}^2} \right)^n$$

Potential modes

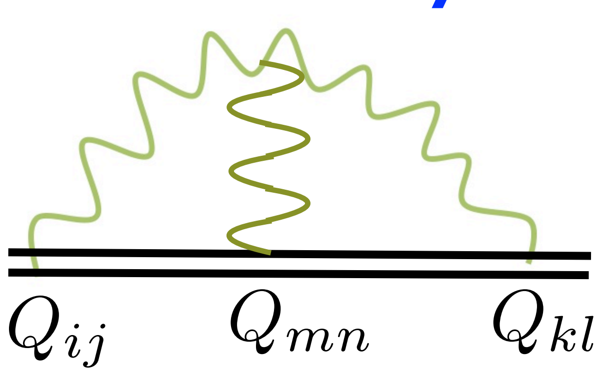


(including finite size effects)

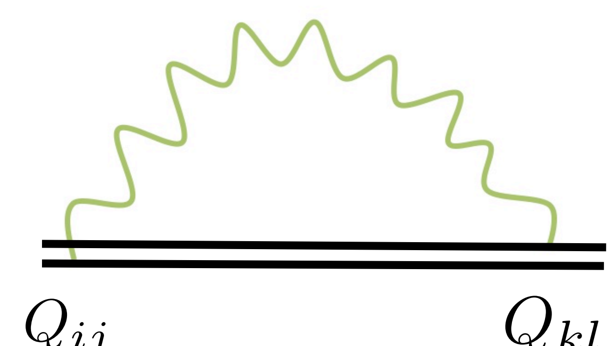
Tails



Memory



Radiation reaction



squared

5PN, potential modes

Blumlein et al.

$\mathcal{O}(10^4)$ diagrams in EFT calculation

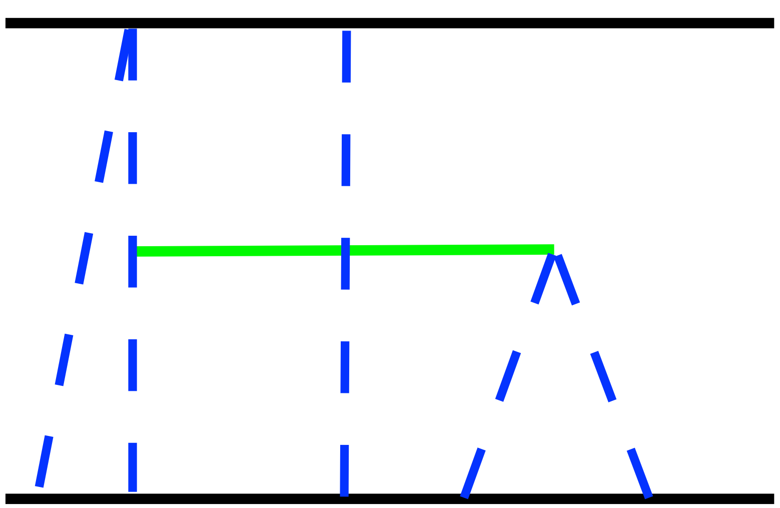
v^{12} Gv^{10} G^2v^8 G^3v^6 G^4v^4 | G^5v^2 | G^6

ok with 4PM

G^5v^2 → 1200/1830 factorizable diagrams → double checked

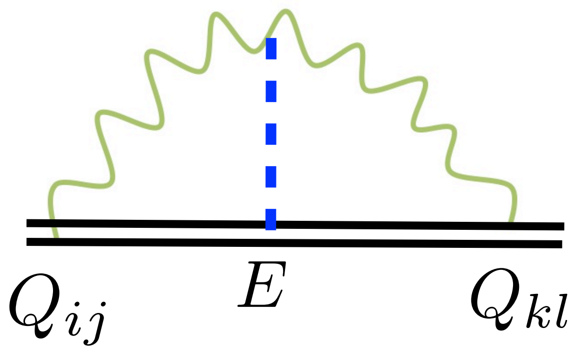
G^6 → all factorizable → double checked

factorizable diagram:



5PN tails

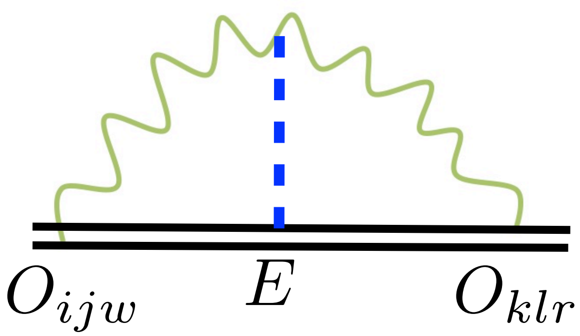
(tails computed for all multipoles)



NLO

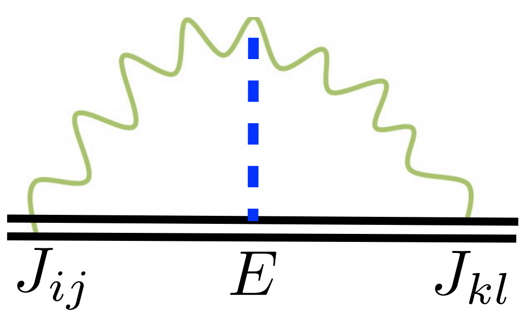
$$-\frac{1}{5}G^2E\left(\frac{1}{\epsilon_{UV}}-\frac{41}{30}\right)\dddot{Q}^2_{ij}$$

+ nonlocal terms



$$-\frac{1}{189}G^2M\left(\frac{1}{\epsilon_{UV}}-\frac{82}{35}\right)\ddddot{O}^2_{ijk}$$

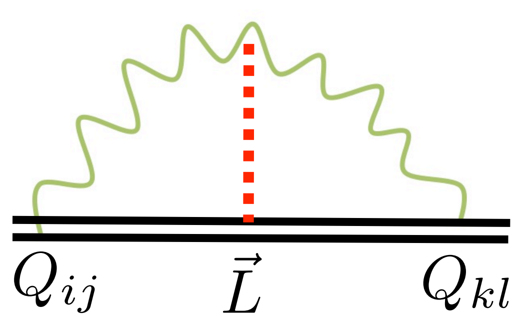
+ nonlocal terms



$$-\frac{16}{135}G^2M\left(\frac{1}{\epsilon_{UV}}-\frac{49}{20}\right)\dddot{J}_{i|jk}\left[\dddot{J}_{i|jk}+\dddot{J}_{j|ik}\right]$$

+ nonlocal terms

ok with 1SF scattering angle χ^{4PM}



~~$\frac{8}{15}$~~ $\frac{1}{30}G^2\epsilon_{ijk}L_k\ddddot{Q}_{il}\dddot{Q}_{jl}$ → $\chi^{4PM}\ \mathcal{O}(\nu^2)$

Dissipative contributions: in-in formalism

(irrelevant for potential modes *and tails*)

$$e^{iS_{eff}[x]} = \int \mathcal{D}[h] e^{iS_{tot}[x,h]}$$

$$S_{tot}[x,h] \to S_{tot}[x_1,h_1] - S_{tot}[x_2,h_2]$$

$$x_- = x_1 - x_2$$
$$x_+ = \frac{1}{2}(x_1 + x_2)$$
$$\langle h_- h_+ \rangle = G_{ret}$$

$$\left.\frac{\delta S_{eff}[x_\pm]}{\delta x_-}\right|_{x_-=0} = 0$$

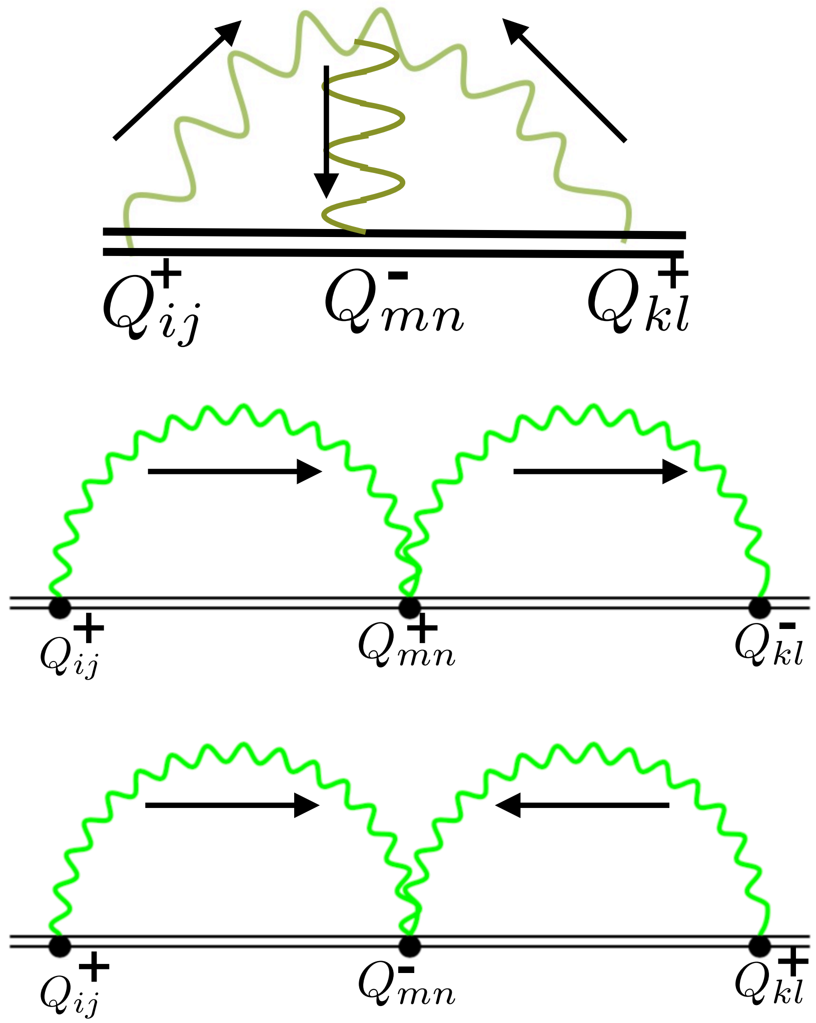
if $S_{eff}[x_\pm] = S_{eff}[x_1] - S_{eff}[x_2]$

the dynamics is conservative

and one can avoid using Keldish variables

Memory and double emission

5PN, finite and local-in-time

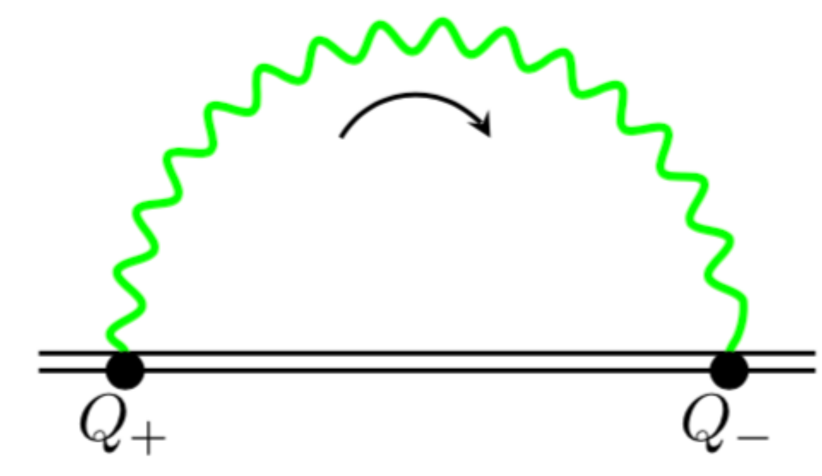


$$\longrightarrow \quad \chi^{4PM}\ \mathcal{O}(\nu^2)$$

To be recomputed taking into account of
Ward correction

Simple self-energy diagram

at 2.5PN does not contribute to the conservative dynamics, gives only dissipative effects



$$\mathbf{a}^i_{rr} \simeq \mathbf{a}^i_{BT} = -\frac{2}{5} G Q^{(5)}_{ij} x^j$$

But

$$\left(\text{radiation reaction}\right)^2$$

affects 5PN

$$\mathbf{a} = -\frac{GM}{r^3}\mathbf{r} + \cdots + \mathbf{a}_{rr} + \cdots + \mathbf{a}_{BT^2}$$

N

Burke Thorne 2.5PN

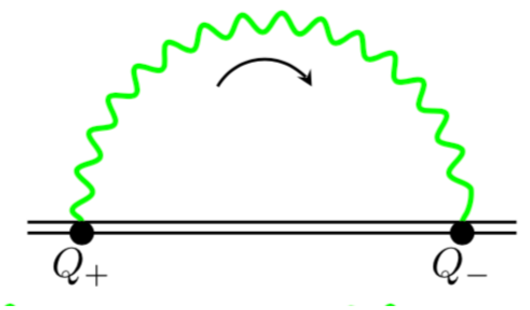
5PN

$$\mathbf{a}_{BT^2} = \frac{G^4 M^4 \nu^2}{r^6}\left[\left(c_1 v^4 + c_2 v^2 v_n^2 + c_3 v_n^4\right)\mathbf{r} + \left(c_4 v^2 + c_5 v_n^2\right) v_r \mathbf{v}\right] + \mathcal{O}\left(G^5\right)$$

Conservative and dissipative terms

balance equation $$\dot{E} = \mu \mathbf{a} \cdot \mathbf{v}$$ ambiguity due to Schott terms

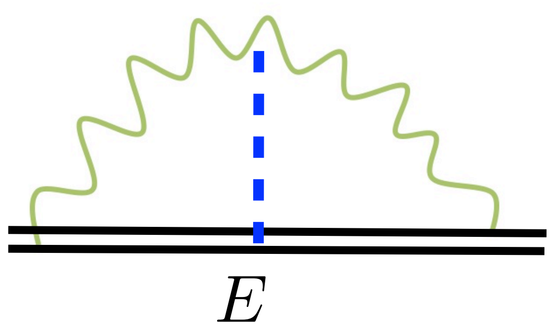
e.g.



$$\dot{E} = \mu \mathbf{a}^{BT} \cdot \mathbf{v} \simeq -\frac{G}{5} Q_{ij}^{(5)} \dot{Q}_{ij} = -\frac{G}{5} \dddot{Q}_{ij}^2 + \dot{V}_{Schott}$$

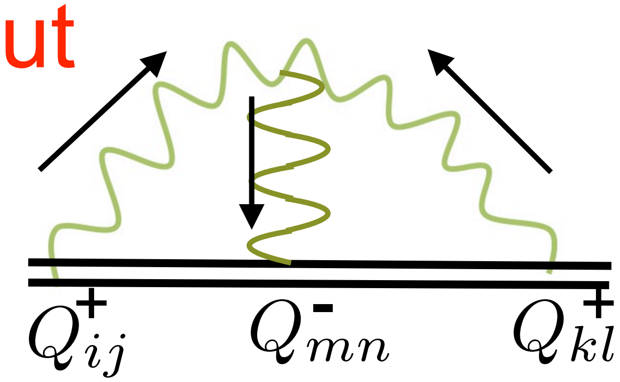
$$V_{Schott} = -\frac{G}{5} \left[Q_{ij}^{(4)} \dot{Q}_{ij} - Q_{ij}^{(3)} \ddot{Q}_{ij} \right]$$

half-integer PN, do not get mixed with conservative part



dissipative term is nonlocal, still do not get mixed

But



5PN local dissipative, mixes with conservative potential contributions

and the same for $\mathbf{a}_{BT^2}$

5PN dynamics missing elements

Tuttifrutti: two 2FS coefficient — and dissipative?

PM: 5PM and 6PM — but no new 5-loop integrals at 5PN; and memory?

EFT: memory — scattering angle test

New features:

- memory, nonlinear radiation reaction
- mixing of conservative and dissipative terms

What to expect at 6PN?

partial Tuttifrutti results

potential done with both EFT and PM up to G^4

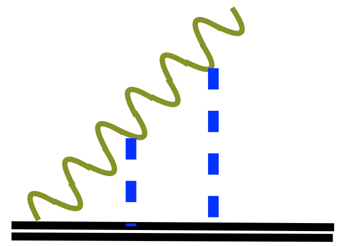
5 and 6-loop integrals at $\mathcal{O}(G^6)$

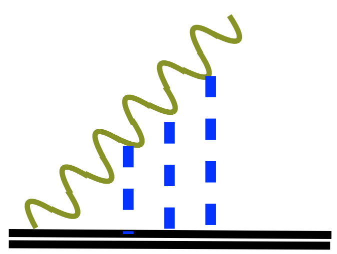
GW radiation

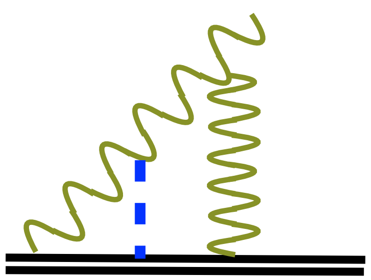
Blanchet et al.

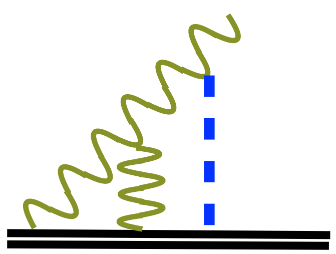
done in MPM formalism up to 4.5PN

- General solution in far zone (radiative moments)
- Matching with near zone (source moments)
- Knowledge of 4PN source dynamics
- IR and UV divergencies involved at 3PN (tails-of-tails)
- Subtle gauge issues
- Tail-of-tail-of-tail, memory-of-tail and tail-of memory









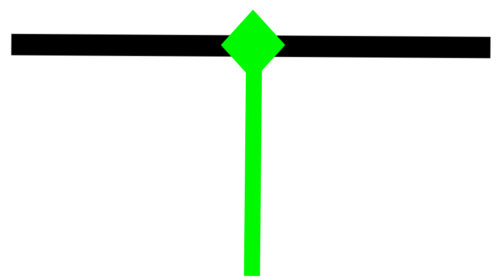
GW phase known at 4.5PN

in principle extendable up to 5PN or even 5.5PN

demonstrative 2PN calculation in EFT

Spin

EFT: $\vec{S}$ $(\sim +1\text{PN})$



Φ_{GW} aligned

	N	1PN	1.5PN	2PN	2.5PN	3PN	3.5PN	4PN	4.5PN	5PN
S^0	✓	✓	✓	✓	✓	✓	✓	✓	✓	??
S			✓	0	✓	✓	✓	?	??	?
SS				✓	0	✓	✓	?	?	??
S^3							✓	0	?	?
S^4								?	0	?

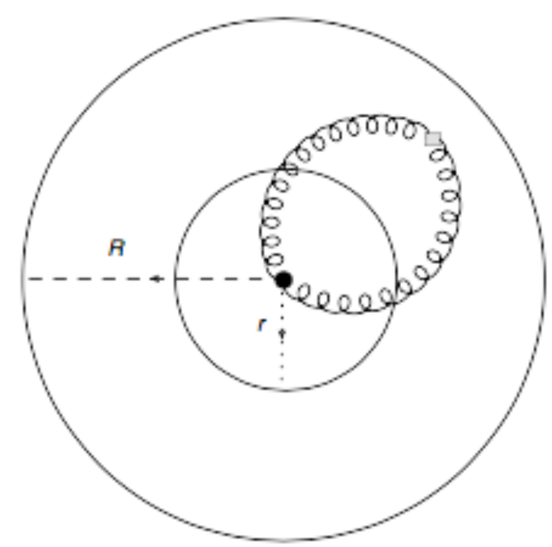

THE END

Renormalisation group analysis

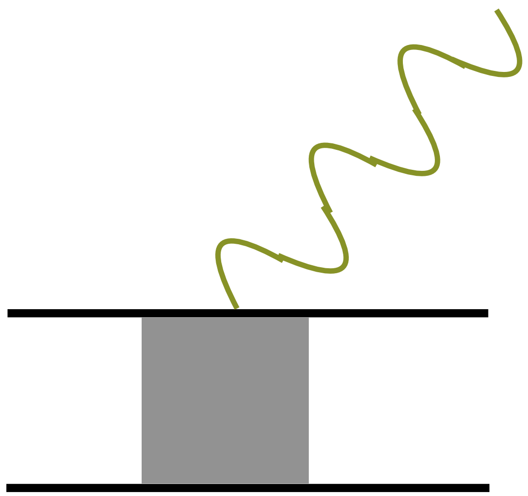
GR → PN → Multipoles ↔ ~~Near Zone~~

less predictive, more general:

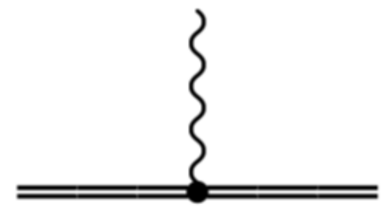
UV divergencies → renormalization → universal IR log's



Leading logs from renormalisation group analysis at arbitrary high PN order

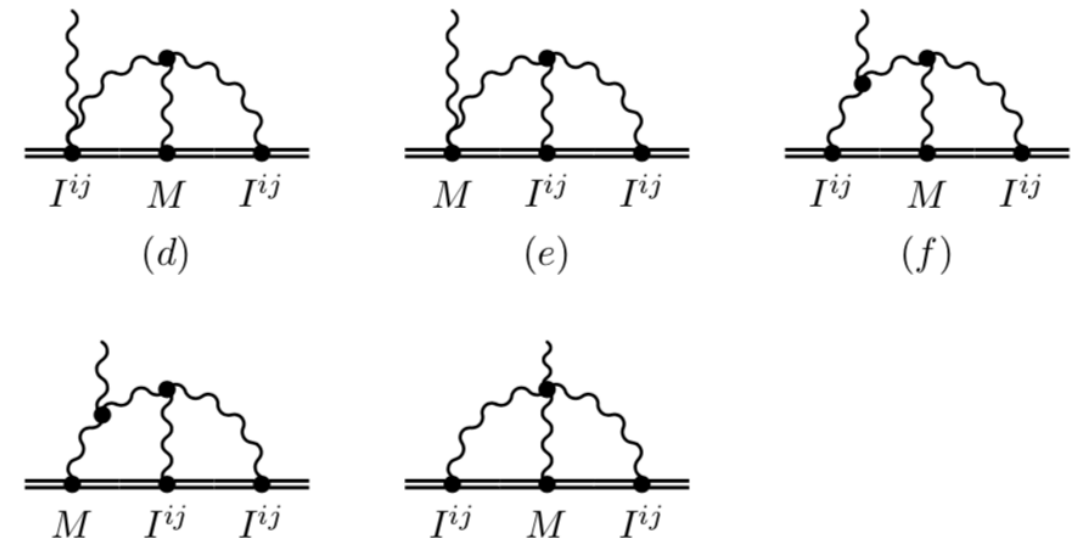


$T_{\mu\nu}$



+

Goldberger, Ross, Rotstein



$$E^{bare} = Z(\mu) E^{ren}(\mu)$$

$$\frac{\mathrm{d}\log E(\mu)}{\mathrm{d}\log\mu} = -\frac{2G^2}{5}\left[2Q^{(1)}_{ij}Q^{(5)}_{ij} - 2Q^{(2)}_{ij}Q^{(4)}_{ij} + Q^{(3)}_{ij}Q^{(3)}_{ij}\right]M$$

$$E_{\text{leading-}(\log)^n} = -\frac{M\nu\, x}{2}\left[\frac{64\nu}{15}\sum_{n=1}^{+\infty}\frac{6n+1}{n!}(4\beta_Q)^{n-1}x^{3n+1}(\log x)^n\right]$$

$$\beta_Q = -\frac{214}{105}$$

Perfect agreement

Kavanagh et al.

self-force $m_2 \ll m_1$ vs PN EFT + RG flow + 1st law

$$c^{10PN}_{\log^3} = \frac{445503488}{496125}$$

$$c^{13PN}_{\log^4} = -\frac{5017776128}{2083725}$$

$$\beta_Q = -\frac{214}{105}$$

$$c^{16PN}_{\log^5} = \frac{13315170733608}{27348890625}$$

$$\frac{64}{15}\frac{6n+1}{n!}(4\beta_Q)^{n-1}$$

$$c^{19PN}_{\log^6} = -\frac{68019046365134848}{8614900546875}$$

$$n = 3 \ldots 7$$

$$c^{22PN}_{log^7} = \frac{676660826650779320 32}{633195190195 3125}$$