Dynamical Quantum Chaos in Many-Body Systems: An experimental quest for the origin of irreversibility from Loschmidt Echoes to Out of Time Order Correlators

IFEG CONICET UNC

UNC Universidad Nacional de Córdoba

Horacio M. Pastawski **FaMAF-IFEG**

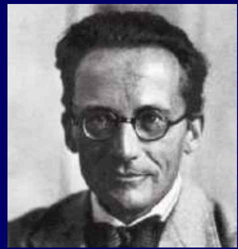
órgano: “Entanglement” de Walter Thirring (Viena)

Gabriel Villot (Santa Fe): **"Los libros de la buena memoria"** 1,20 x 1,20 m

inspiration: an old story from Wien

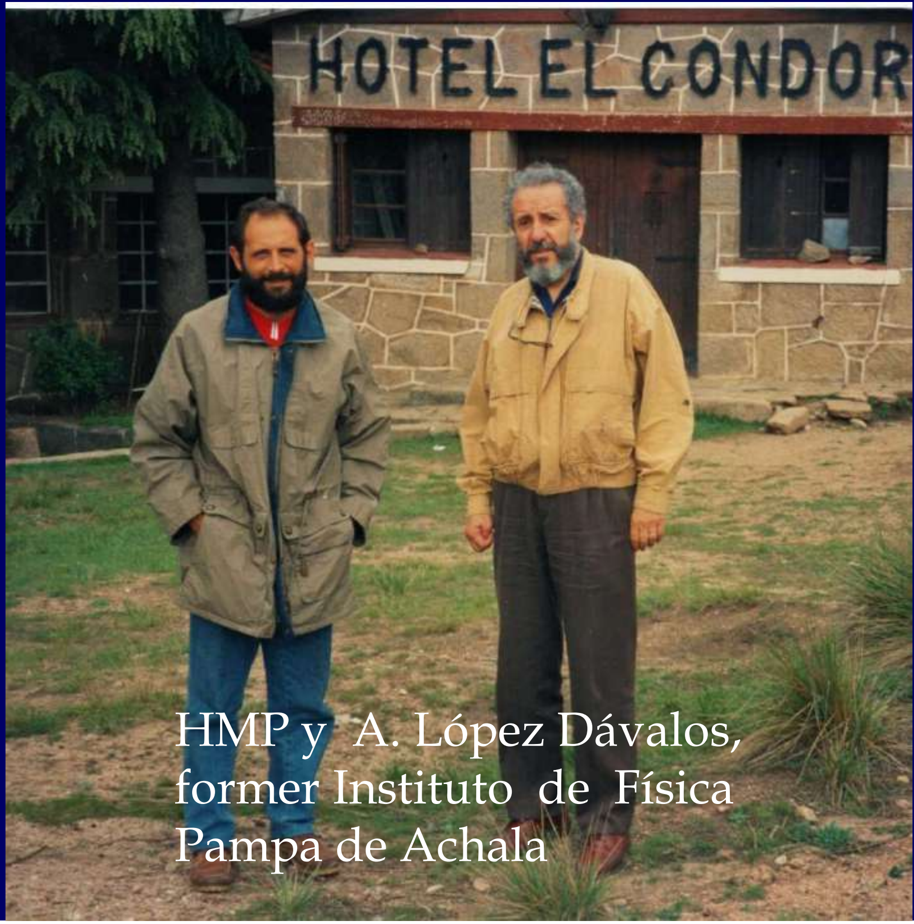
L.Boltzmann

F.Hasenöhrl



E.Schrödinger

W.Thirring

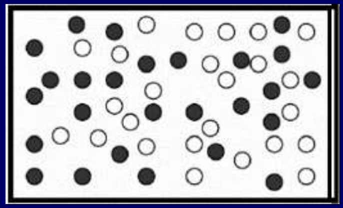


HMP y A. López Dávalos,
former Instituto de Física
Pampa de Achala

G. Beck y E. Gaviola

Boltzmann's controversies

Boltzmann (1872): Any system, *Stoßzahlansatz* molecular chaos without external influence, goes to irreversibly to maximum disorder (2nd law of thermodynamics, entropy increases)



Zermelo-Poincare: ***Mechanics is cyclic***...!

Loschmidt: Loschmidt: Mechanics is reversible: a system can go from "apparent" disorder to order ***"simply"*** by reversing the momentum of each particle.

Boltzmann: then you do it…!!

…a Loschmidt's daemon…!

…"…a being whose faculties are so sharpened that he can follow every molecule in its course, such a being whose attributes are essentially finite as our own…." J. C Maxwelll

2 *usually daemon* **:** an attendant power or spirit **:** **GENIUS**

spanish: geniecillo o diablillo

Boltzmann's arguments

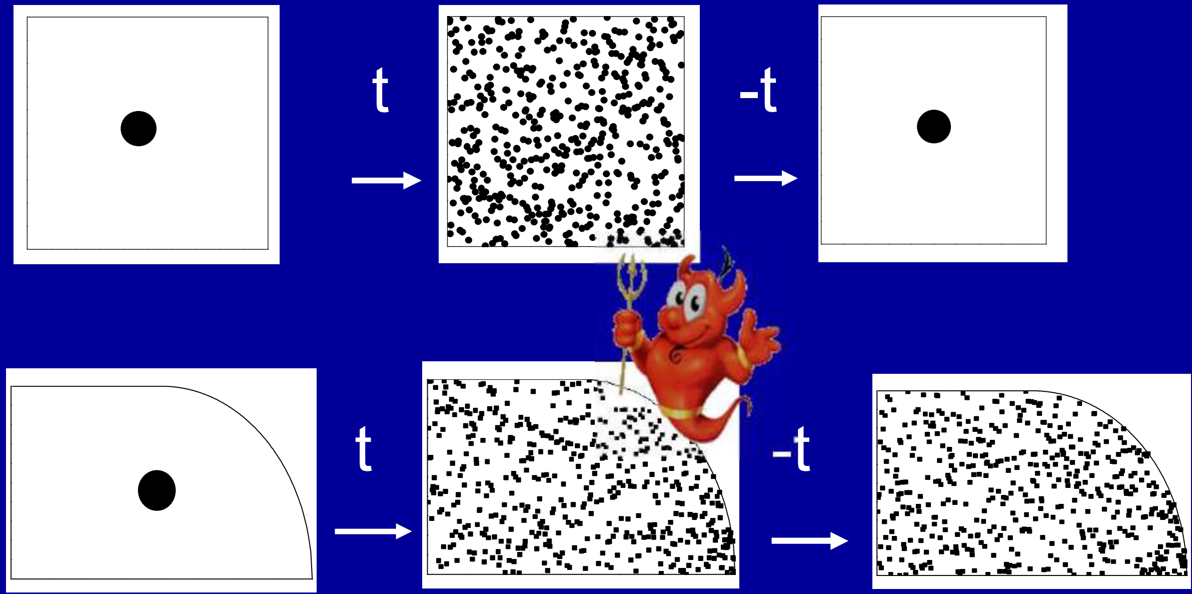
Stoßzahlansatz

- Stosszahlansatz: "chaos" instability of molecular dynamics.
- Entropy increase is a particularity of the initial state.

100 years later…chaos…!

Poincaré, Kolmogorov, Lorentz, → Chirikov, Zaslawski, Ruelle,Casati

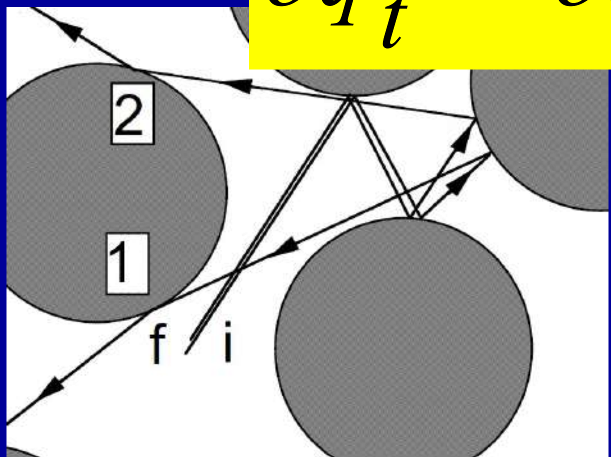
Conceptual progress: Any small mistake leads to exponential divergence of trajectories and practical irreversibility.



Integrable systems (rare) are insensitive to errors.

Nonintegrable systems (chaotic) are very hypersensitive to errors.

$$\delta q_t \propto \delta q_0 \exp[\lambda t] \quad \text{if} \quad \delta q_t \leq R$$



IC that differ infinitesimally produce trajectories that divergenge exponentially (***Butterfly effect***) but then diffusion takes over.

$$\delta q_t \propto \sqrt{2\,dDt} \quad \text{if} \quad \delta q_t > R$$

What is the Lo
daemon i
erime

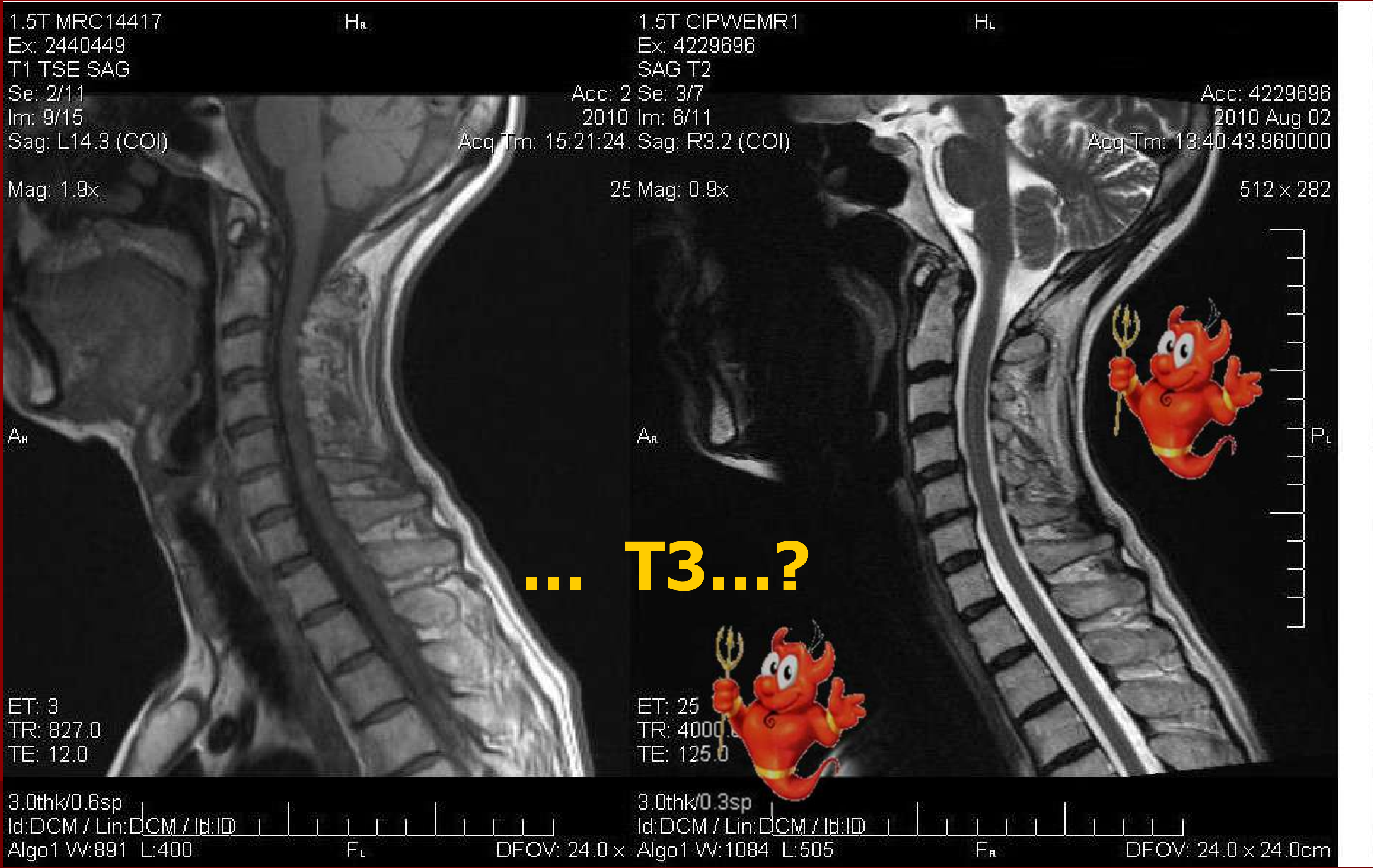
Hahn the"Wizard":
spin echo,

Hahn's runners

$$H \rightarrow -H$$

Hahn echo decays in a time scale $T_2 \simeq \hbar/J$ of spins interaction

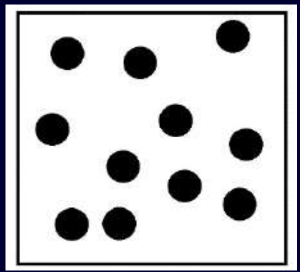
thermalization **T1** vs **T2** reversibility of spin precesion

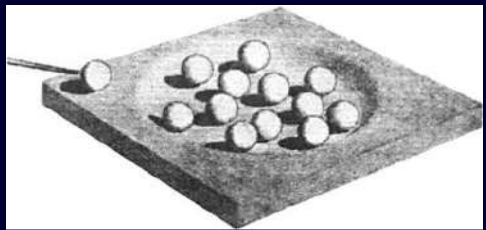


VELOCITY-INVERSION AND IRREVERSIBILITY IN A DILUTE GAS OF HARD DISKS

J. ORBAN * and A. BELLEMANS

PHYSICS LETTERS 22 May 1967





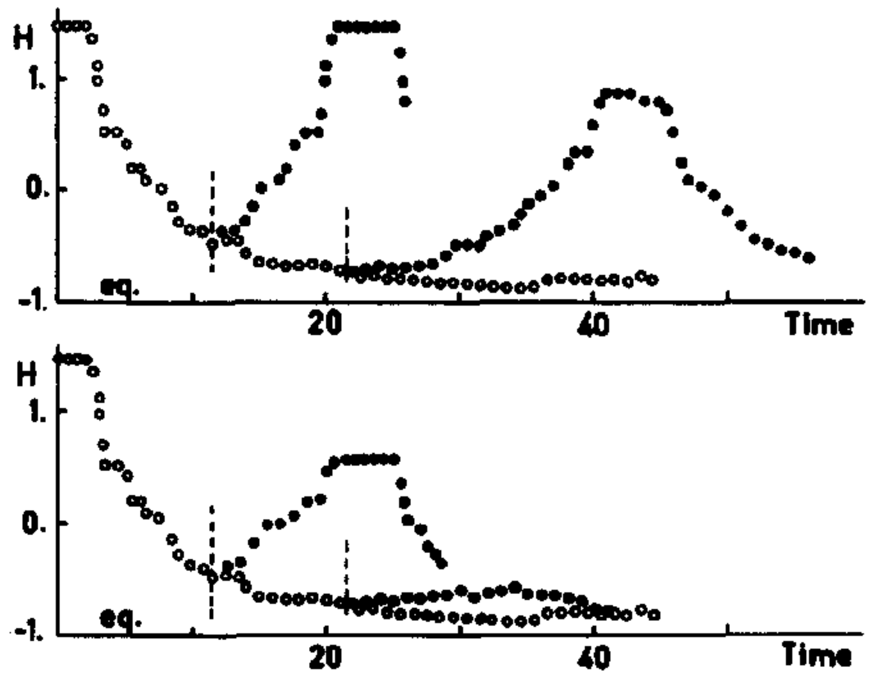
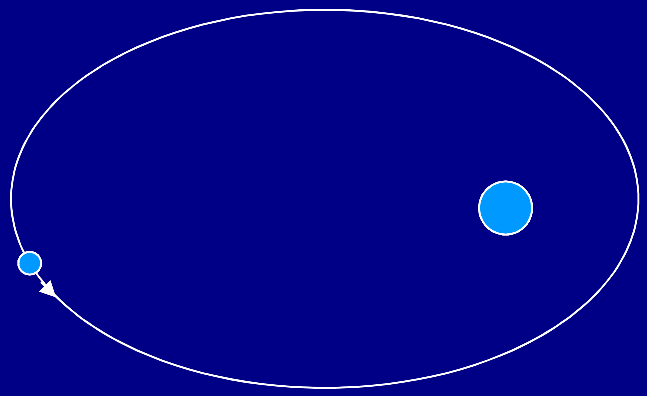


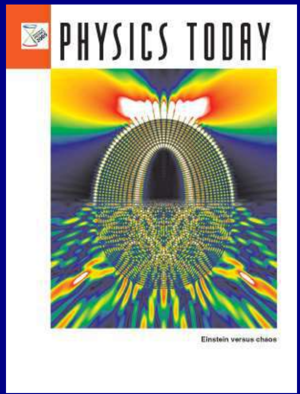
Fig. 1. Plot of H with time (in arbitrary units) showing the kinetic (o) and the anti-kinetic (●) evolutions for velocity-inversions taking place at 50 or 100 collisions, with random errors 10^{-5} and 10^{-2} respectively.

chaos in the quantum world?

$$\boxed{H = T + V = cte. = \varepsilon}$$



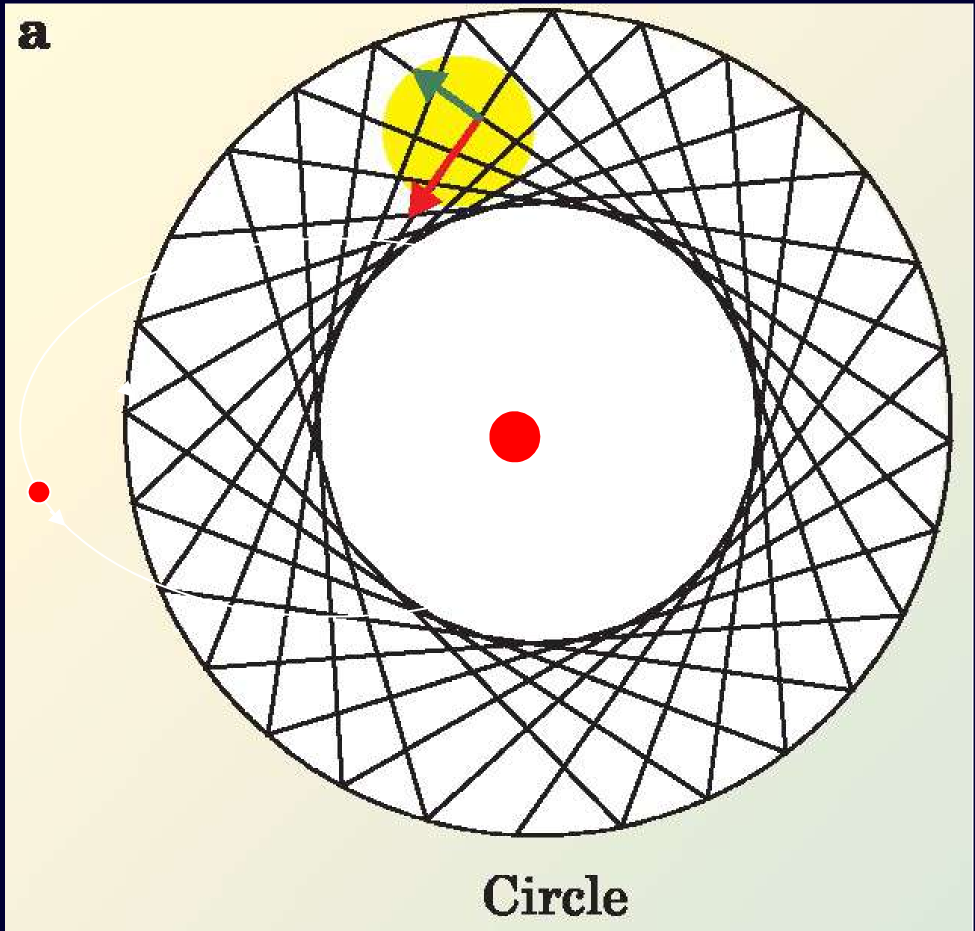
$$\int_0^{T_i} p_i \frac{dq_i}{dt}\, dt = n_i h \quad i = 1, \ldots, d,$$

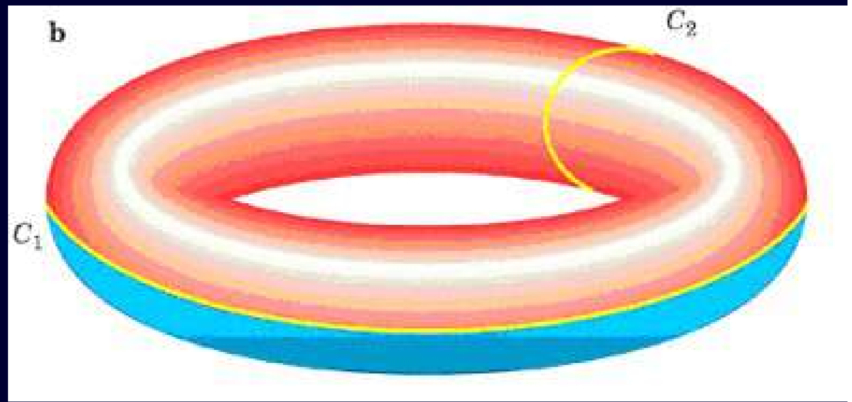


non-separable variables?

chaos in the quantum domain ….?

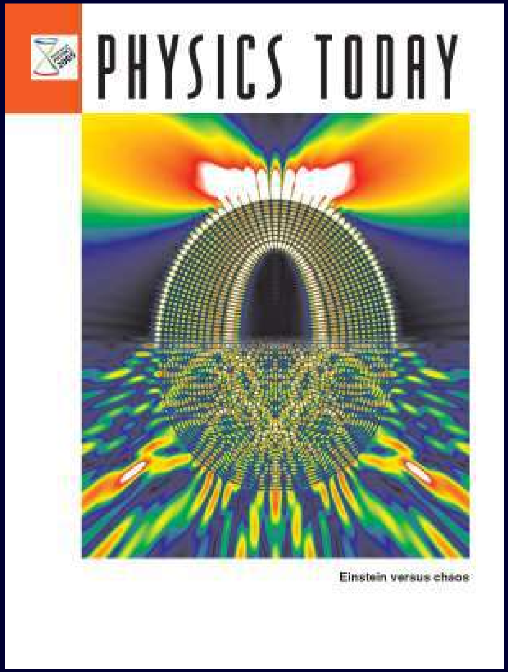
$$\boxed{H = T + V = cte. = \varepsilon}$$

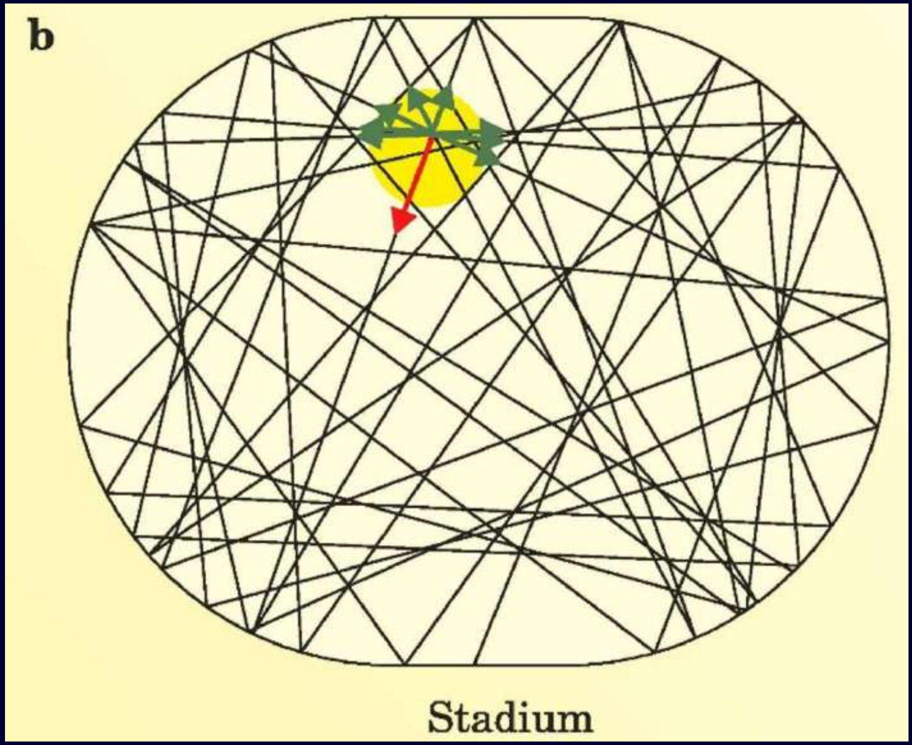




$$\boxed{\frac{1}{2\pi}\oint_{C_i} \boldsymbol{p}\cdot \mathrm{d}\boldsymbol{q} = n_i\hbar \quad i = 1,\ldots,d \rightarrow \varepsilon_{1,\ldots,d}}$$

but….





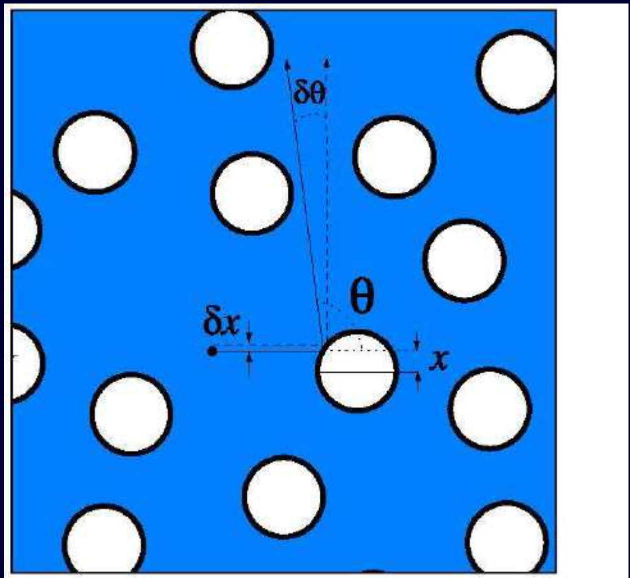
One notices immediately that type (b) [motion] excludes the quantum condition we have formulated. . . . On the other hand, classical statistical mechanics deals essentially only with type (b); because only in this case is the microcanonical ensemble of one system equivalent to the time ensemble. EINSTEIN

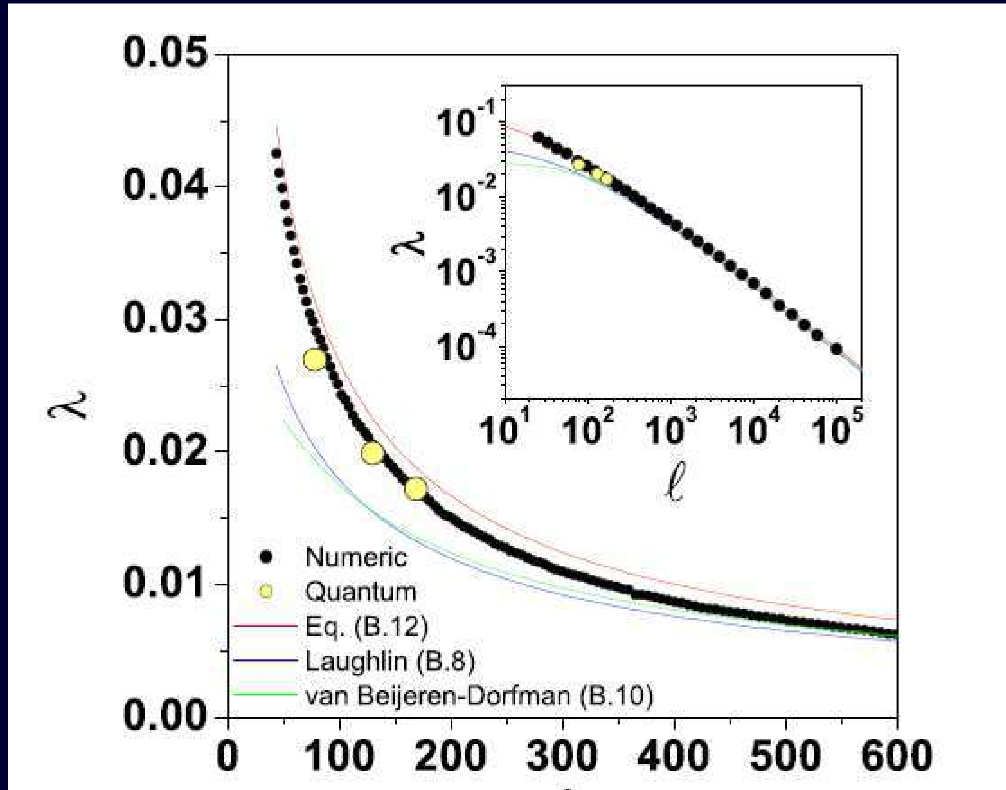
How to reconcile CHAOS,

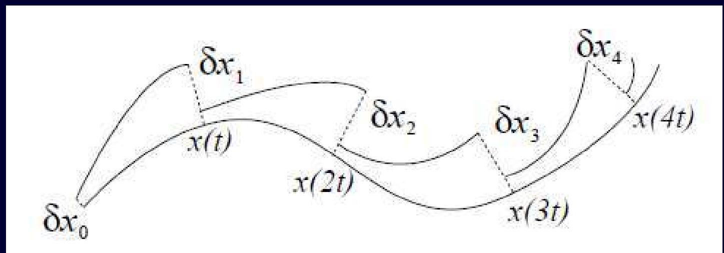
i.e. dynamical instability,

with energy quantization?

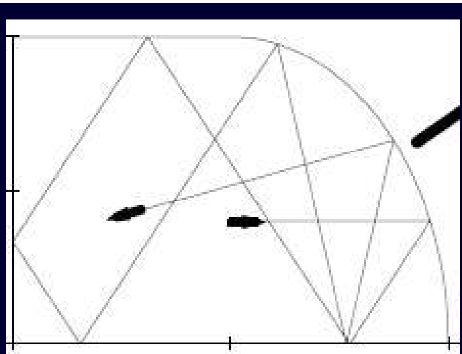
Instability of Classical Dynamics.







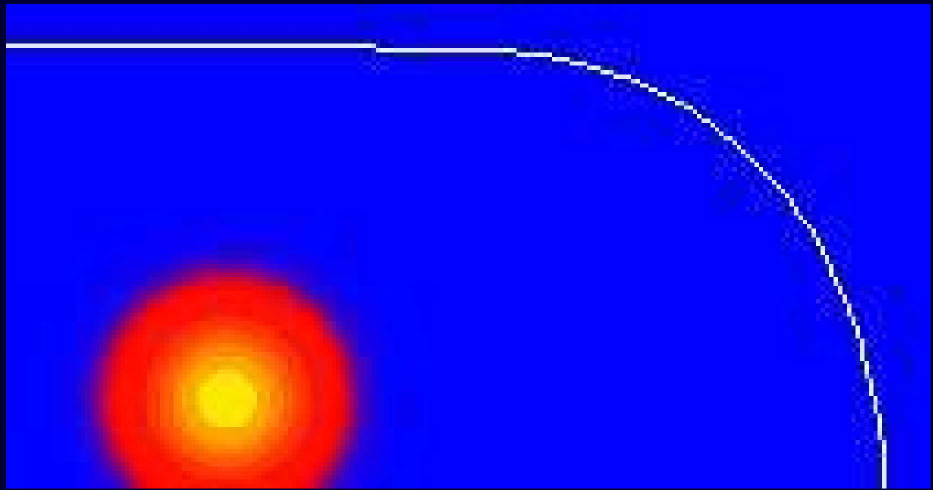
$$\frac{\lambda}{v} = \frac{1}{\ell}\ \ln\left[\frac{\ell}{R}\right] + \frac{\pi}{R} + \sqrt{\frac{4}{R^2} - \frac{1}{\ell^2}}\ \left(\arcsin\left[\frac{R}{2\ell}\right] - \frac{\pi}{2}\right)$$

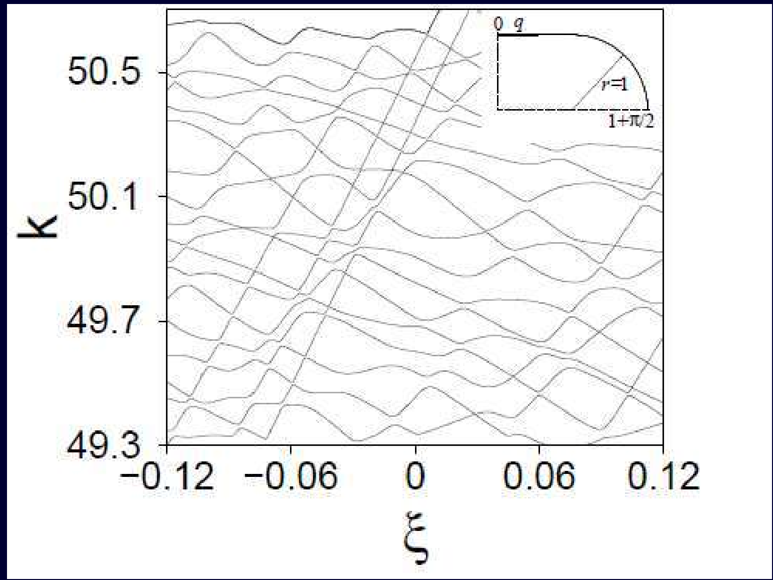


100+ years later…

Casati, Shepelyansky, Chirikov, Guarneri, Izrailev

But…., what about <u>Quantum</u> Mechanics….?





www.lanais.famaf.unc.edu.ar/loschmidt

QM is NOT sensitive to errors in the initial condition (IC).

--drawback ?????

VOLUME 56, NUMBER 23 PHYSICAL REVIEW LETTERS 9 JUNE 1986

Dynamical Stability of Quantum "Chaotic" Motion in a Hydrogen Atom

G. Casati,[a] B. V. Chirikov, I. Guarneri,[b] and D. L. Shepelyansky
Institute of Nuclear Physics, 630090 Novosibirsk, Union of Soviet Socialist Republics

$$H = p^2/2 - 1/x + \epsilon x \cos(\omega t), \quad x > 0,$$

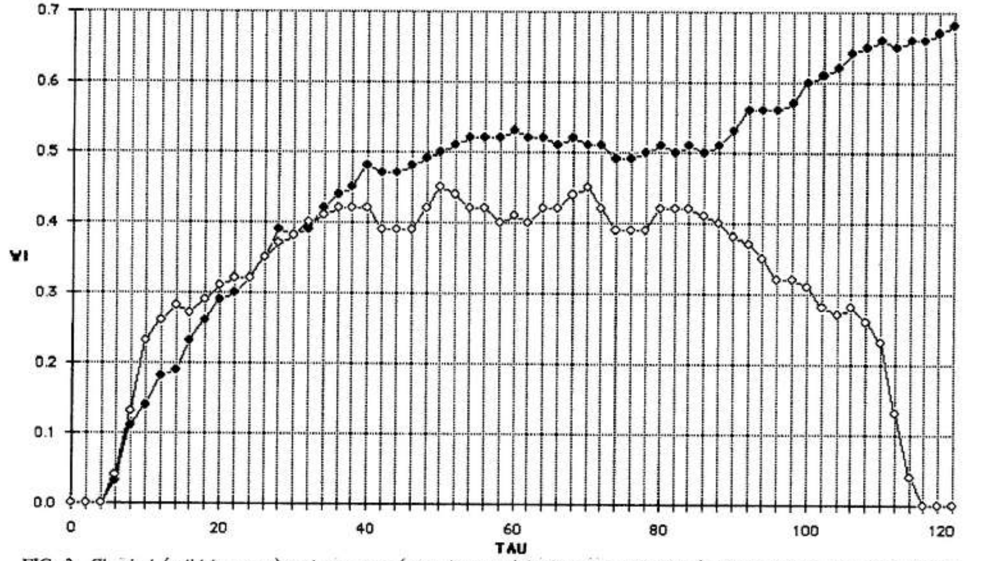


FIG. 3. Classical (solid lozenges) and quantum (open lozenges) ionization probability (excitation above the unperturbed level $n = 150$) as a function of time τ for the case of Fig. 2. Notice the perfect specular symmetry of the quantum curve about the time of reversal $\tau = 60$.

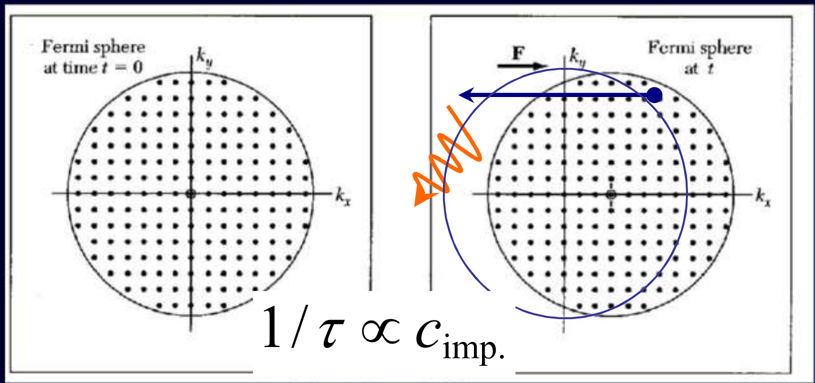
my personal scientific story

- Electronic Transport in Disordered Metals and Superconductors (w/ A.López-Dávalos & F.De la Cruz)
- Anderson Localization & Quantum Transport (w/ J.F.Weisz & M.C.G.Passeggi)
- Spin diffusion = Many body Dynamics (w/ P.R.Levstein & R.Calvo)
- Decoherence in time dependent quantum transport (w/ J.L.D'Amato, P.A. Lee and B.Altshuler).
- Quantum Chaos → Parametric correlations (w/ A.Szafer&B.Altshuler)

is electron transport irreversible?

1) Boltzmann: Fermi sphere "exact" semiclassical dynamics under $V(x)=eEx$

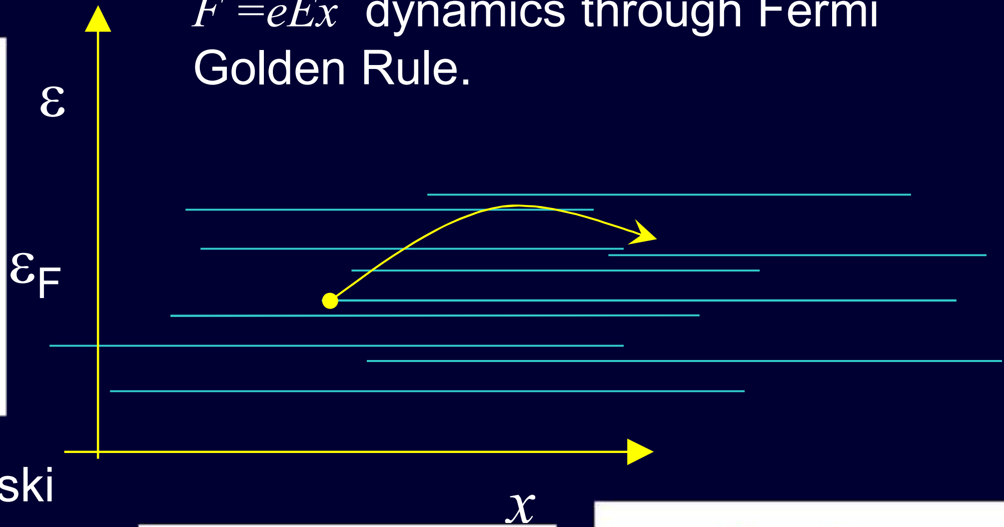
Impurity Scattering as Perturb.



$$1/\tau \propto c_{\text{imp.}}$$

2)Kubo: H_o+V_{imp} "exact" eigenstates

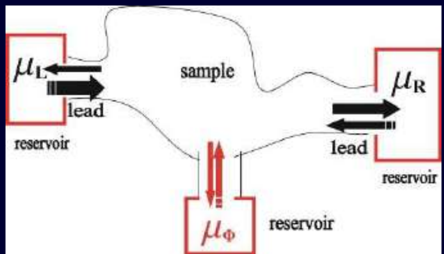
$F=eEx$ dynamics through Fermi Golden Rule.



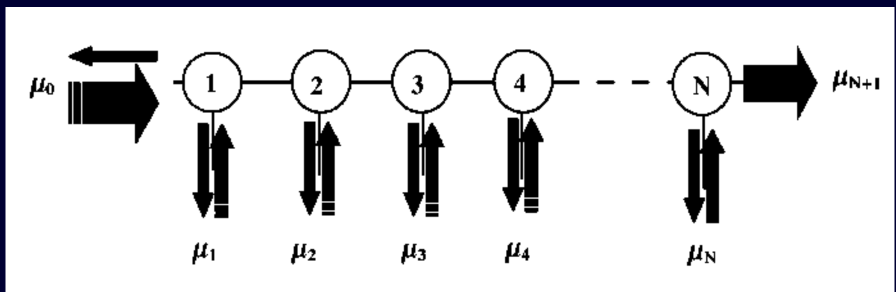
$$\delta\varepsilon = \eta \simeq c\,k_B T$$

$$\sigma = \lim_{\eta\to 0}\lim_{N\to\infty}\sigma_{N,\eta}$$

3) Landauer-Büttiker→D'Amato-Pastawski



$$T_{RL} = 2\Gamma_R \left|G^R_{RL}\right|^2 2\Gamma_L$$



$$\sigma = \overbrace{\frac{ne^2\tau}{m}}^{\text{Drude}} = \overbrace{e^2 D N_o}^{\text{Einstein}}$$

Quantum chaos/Diffusion enables dissipation.

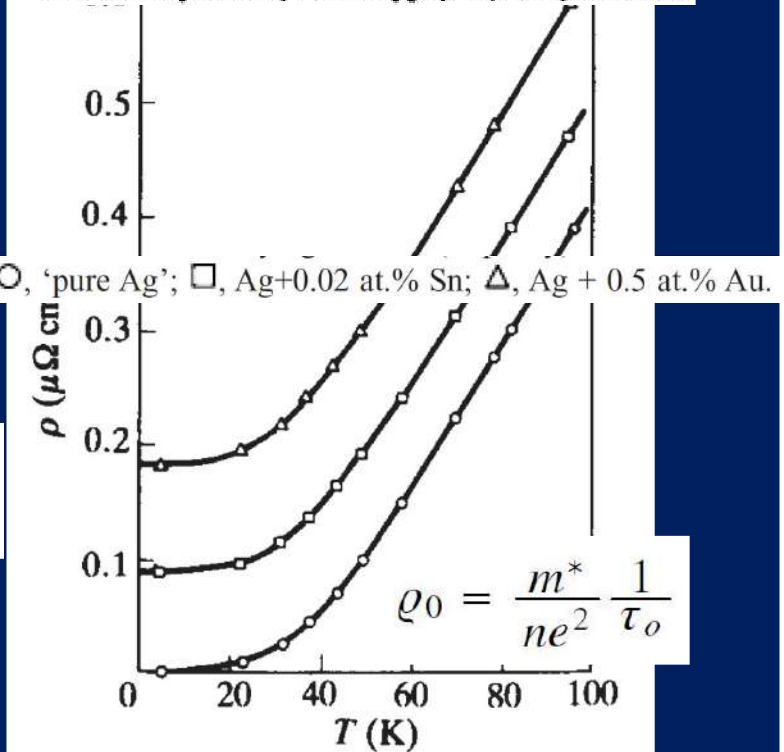
Reduce temperature→ less environment + errors, but… dissipation can not be reduced…!!

$\eta = 1/\tau_\Sigma$ decay rate (decoherence) scales with $k_B T$

analogy with residual resistance, (R. Laughlin)

ELECTRICAL RESISTIVITY AS QUANTUM CHAOS

Nuclear Physics B (Proc. Suppl.) 2 (1987) 213-224



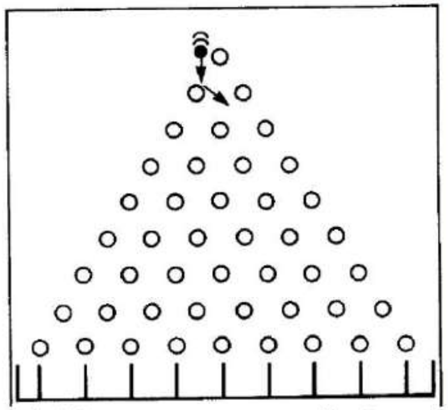
Universal stability towards decoherence in quantum diffusive 1D chains
F.S. Lozano-Negro, E. Alvarez Navarro, N. C. Chávez, F. Mattiotti, F. Borgonovi, H.M. Pastawski, G. L. Celardo arXiv:2307.05656v1

At low temp. dissipation is NOT affected by environment/ only by $\mathcal{H}$ dynamics !!!!

ELECTRICAL RESISTIVITY AS QUANTUM CHAOS

R. B. LAUGHLIN

Nuclear Physics B (Proc. Suppl.) 2 (1987) 213-224
North-Holland, Amsterdam



crystalline array of impenetrable spheres

$$\frac{1}{\tau} = \frac{v\pi R^2}{\Omega}$$

$$\Delta v \rightarrow \Delta v + \alpha\frac{v\Delta x}{R}$$

$$1/\tau \propto c_{\text{imp.}}$$

$$\Delta x \rightarrow \Delta x + \Delta v\tau$$

$$\frac{\alpha v\tau}{R} \geq 1$$

$$\lambda = \frac{1}{\tau}\,\ell n\left[1 + \frac{\alpha v\tau}{R}\right]$$

$$\langle v(0)v(t)\rangle = \langle v(0)v(0)\rangle\, e^{-t/\tau} \quad ,$$

τ^{-1} identified with λ, one obtains the Drude expression

$$\sigma(\omega) = \frac{\rho\tau e^2}{m}\left[\frac{1}{1 - i\omega\tau}\right] \quad .$$

SOVIET PHYSICS JETP VOLUME 28, NUMBER 6 JUNE, 1969

QUASICLASSICAL METHOD IN THE THEORY OF SUPERCONDUCTIVITY

A. I. LARKIN and Yu. N. OVCHINNIKOV

$$\langle\langle p_z(0)\,p_z(t)\rangle\rangle = {}^1/_3\, p_0^2 \exp(-|t|\,/\,\tau_{tr})$$

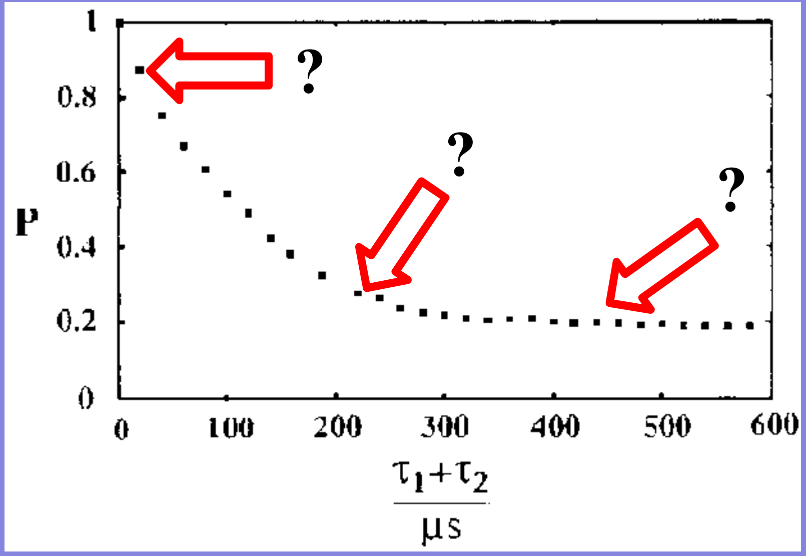
$$\langle p_z^2(0)\,p_z^2(t)\rangle = \frac{p_0^4}{9}\left[1 + \frac{4}{5}\exp\left\{-|t|\left(\frac{1}{\tau} - \frac{1}{\tau_2}\right)\right\}\right]$$

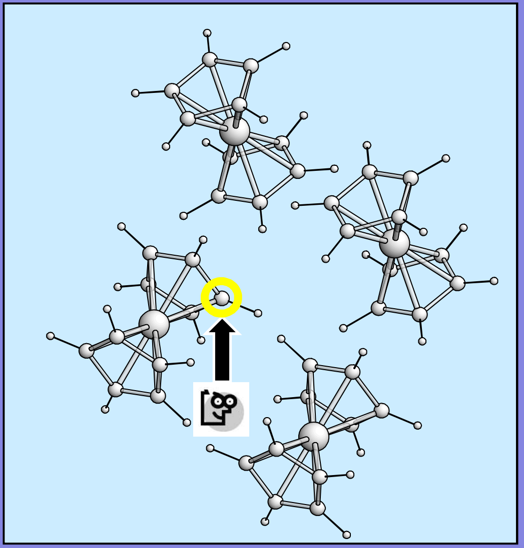
$$\langle [p_z(t)\,p_z(0)]^2\rangle = h^2\left\langle\left(\frac{\partial \mathbf{p}_z(t)}{\partial z(0)}\right)^2\right\rangle,$$

$$\langle p_z(0)\,p_z(t)\,p_z(0)\,p_z(t)\rangle = p_0^4 \exp\left\{-\frac{2|t|}{\tau_{tr}}\right\}\left[\frac{1}{5} + |t|\left(B - \frac{2}{5\tau_{tr}}\right)\right]$$

Thus, even for quasiclassical scattering of particles by impurities, the commutator of momentum operators at different moments of time increases exponentially with the time.

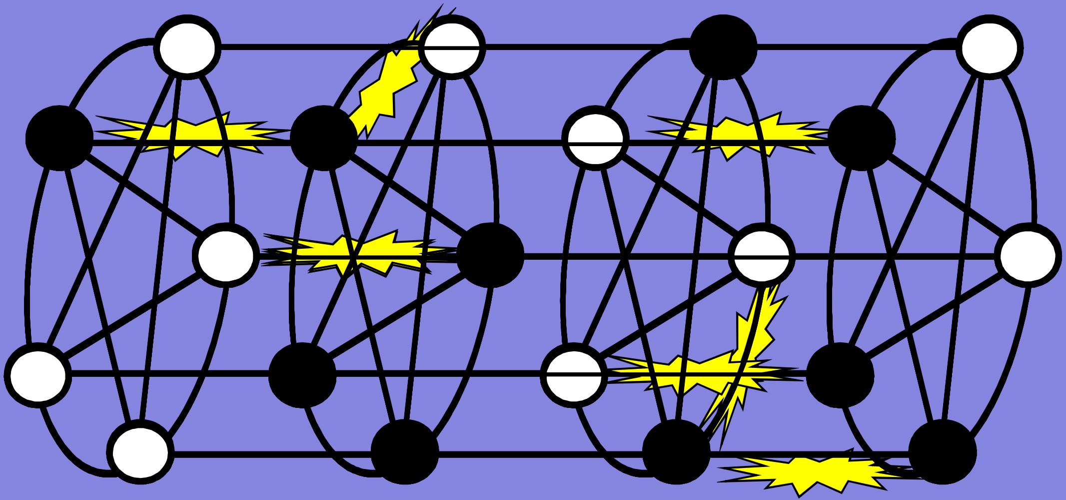
spin diffusion in a Ferrocene crystal





many-body interactions→ spin "diffusion"

Zhang, Meier y Ernst *Phys.Rev.Lett.*1992

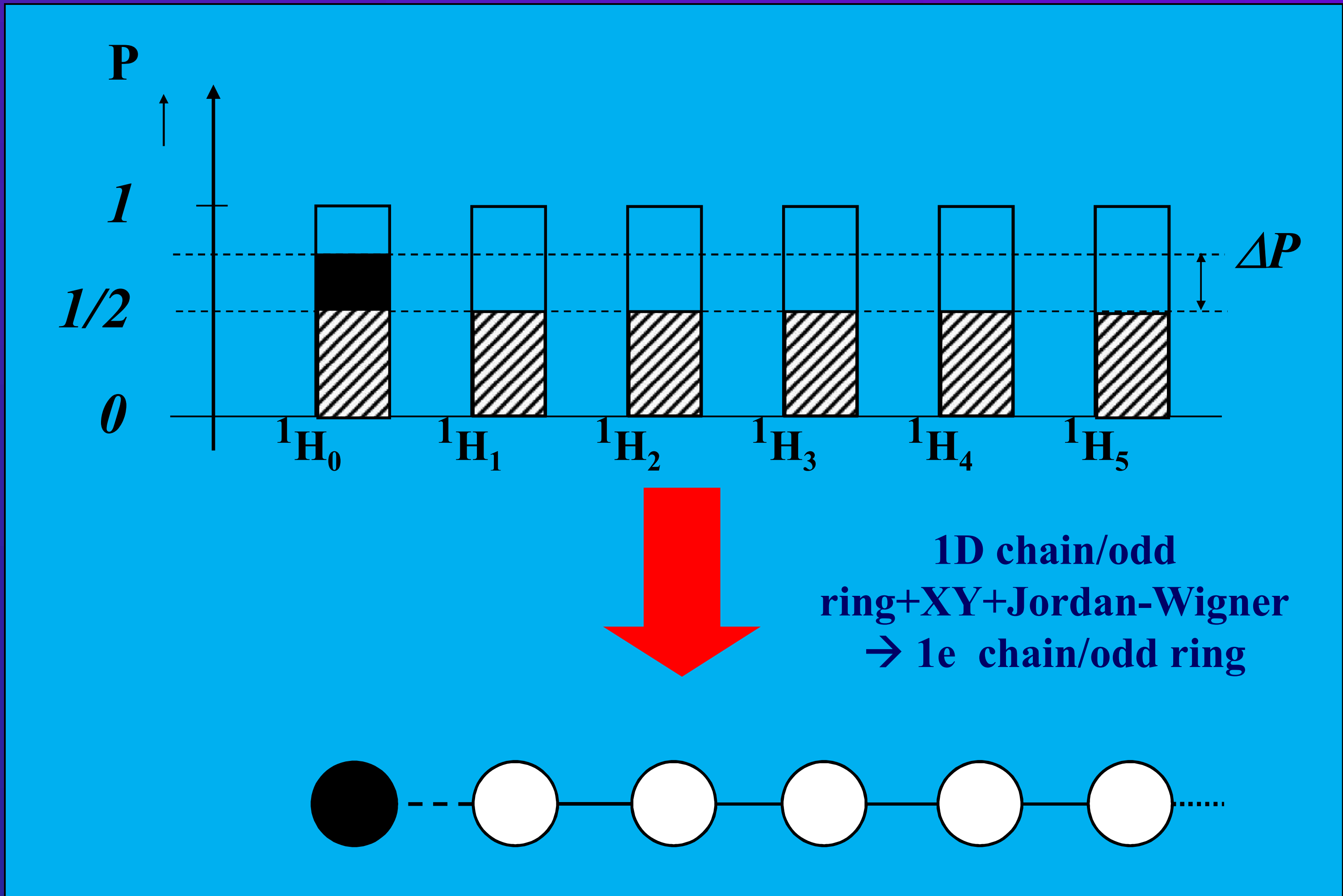


1) Short time? 2) Long time? 3) Poincaré recurrence?

Relevant Energy Scales

$$E_{thermal} \ggg E_{Zeeman}(H_0) \ggg E_{Zeeman}(H_1) >> E_{dip}$$

$$k_B T \approx 10^4 \hbar\omega_0 \approx 10^8\, \hbar\omega_1 \approx 10^9 d_{ij} \quad (T \approx 300\ ^oK)$$

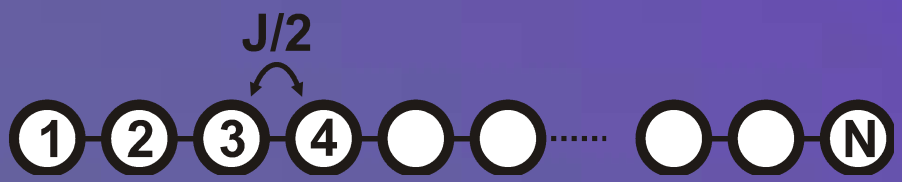


Spin dynamics at high temperature $k_B T \ggg$ J

$$\hat{H} = \sum_{i=1}^{N-1} \frac{1}{2} J \left(\hat{S}_i^+ \hat{S}_{i+1}^- + \hat{S}_i^- \hat{S}_{i+1}^+ \right)$$

Jordan-Wigner transformation (JWT)

$$\hat{H}_e = \sum_{i=1}^{N-1} \frac{1}{2} J \left(\hat{c}_i^\dagger \hat{c}_{i+1} + \hat{c}_i \hat{c}_{i+1}^\dagger \right)$$



Initial State (local excitation)

$$J \ \hat{S}_n^z \hat{S}_{n+1}^z \longrightarrow \frac{J}{2} \hat{c}_n^+ \hat{c}_n \hat{c}_{n+1}^+ \hat{c}_{n+1}$$

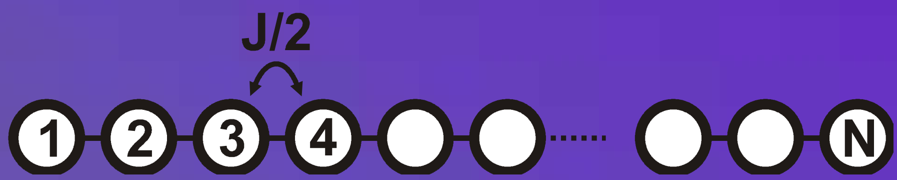
many-body

time ordered correlation

$$P_{n1}(t) = \frac{\langle \Psi | \overbrace{e^{+\mathrm{i}\hat{H}t/\hbar} \hat{S}_n^- e^{-\mathrm{i}\hat{H}t/\hbar}}^{\hat{S}_n^-(t)} \hat{S}_1^+ | \Psi \rangle}{\langle \Psi | \hat{S}_n^- \hat{S}_1^+ | \Psi \rangle}$$

$$\begin{aligned} \hat{c}_n^+ |\mathrm{vac}\rangle &= |n\rangle \\ P_{n1}(t) &= \left| \langle n | e^{-\mathrm{i}\hat{H}t/\hbar} |1\rangle \right|^2 \end{aligned}$$

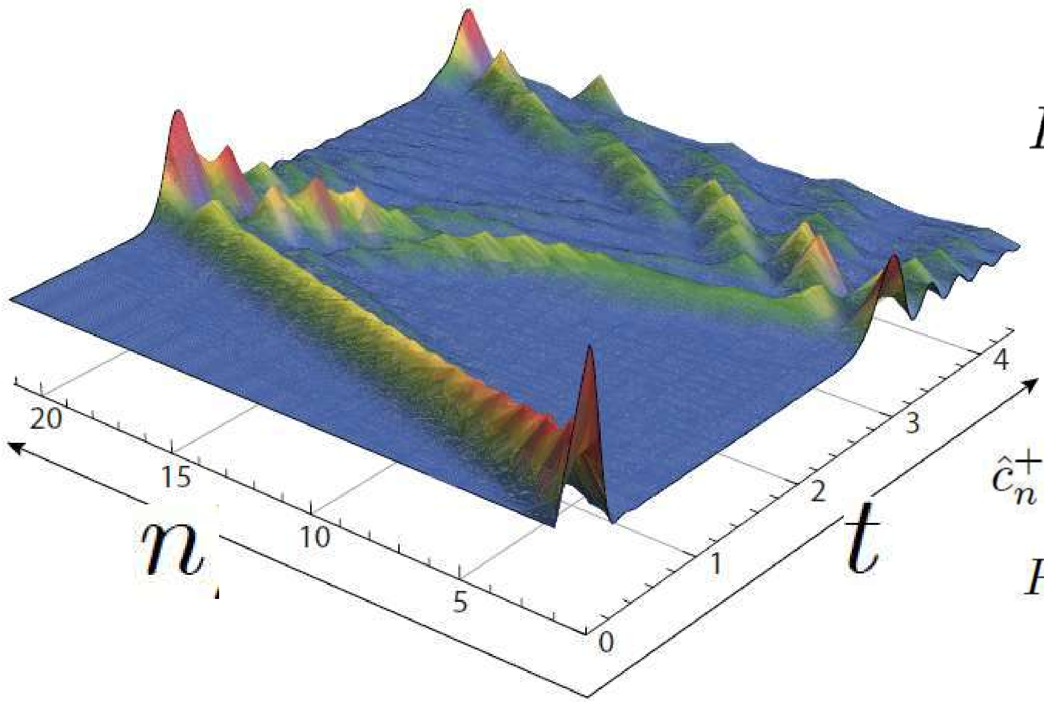
Poincaré cycles in spin dynamics …?



Initial State (local excitation)

time ordered correlation

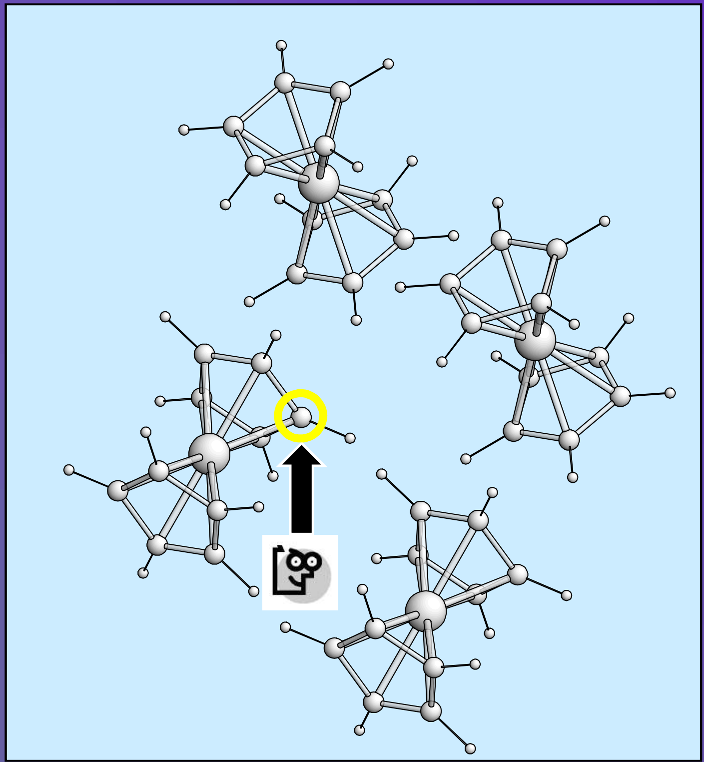
$$P_{n1}(t) = \frac{\langle\Psi|\overbrace{e^{+\mathrm{i}\hat{H}t/\hbar}\hat{S}_n^- e^{-\mathrm{i}\hat{H}t/\hbar}}^{\hat{S}_n^-(t)}\hat{S}_1^+|\Psi\rangle}{\langle\Psi|\hat{S}_n^-\hat{S}_1^+|\Psi\rangle}$$



$$E_i = 0 \quad \forall i \in \{1, ..., N\}$$

$$\begin{aligned} \hat{c}_n^+|\mathrm{vac}\rangle &= |n\rangle \\ P_{n1}(t) &= \left|\langle n| e^{-\mathrm{i}\hat{H}t/\hbar}|1\rangle\right|^2 \end{aligned}$$

many-spin dynamics➔ quantum spin “diffusion”

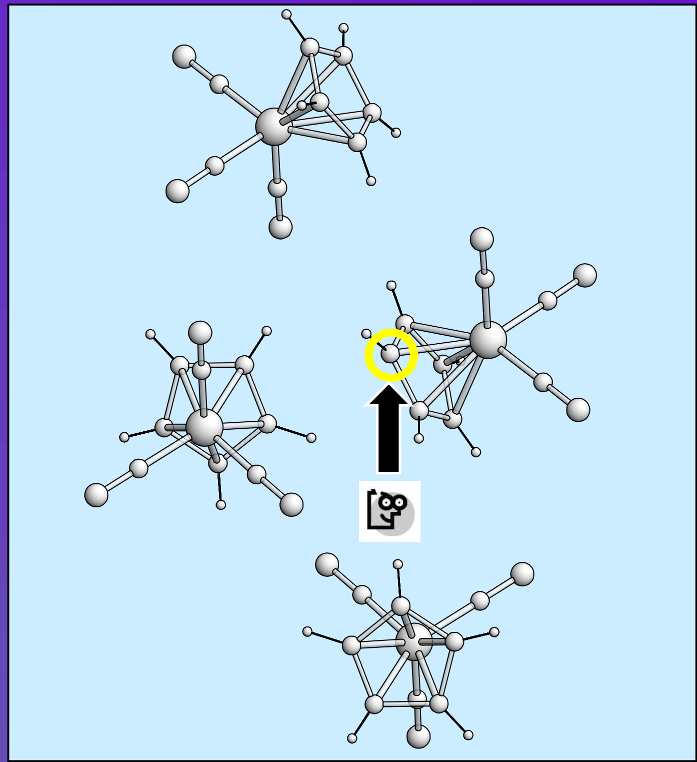


a ^{13}C

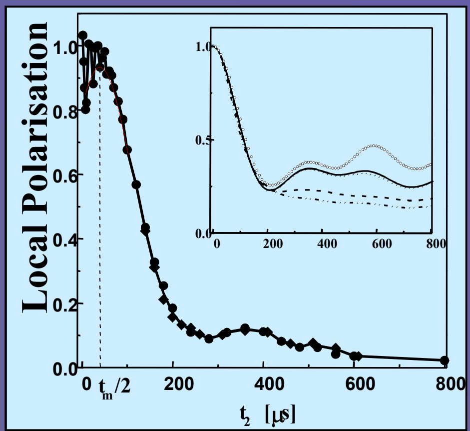
“spies”

the ^{1}H

spin



HMP, Levstein, Usaj, Phys.Rev.Lett. **75**, 4310 (1995)

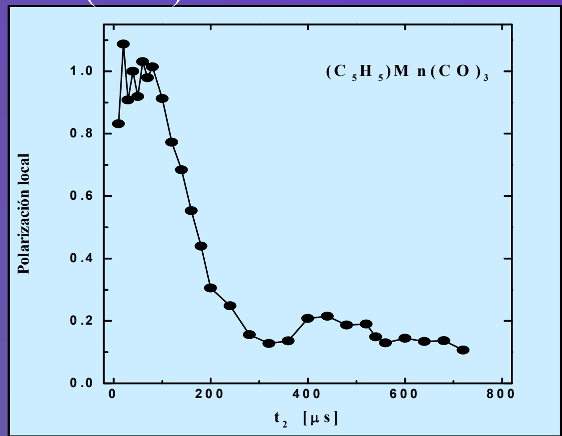


finite ring size

➔mesoscopic echoes

(Poincaré recurrences)

+…decoherence

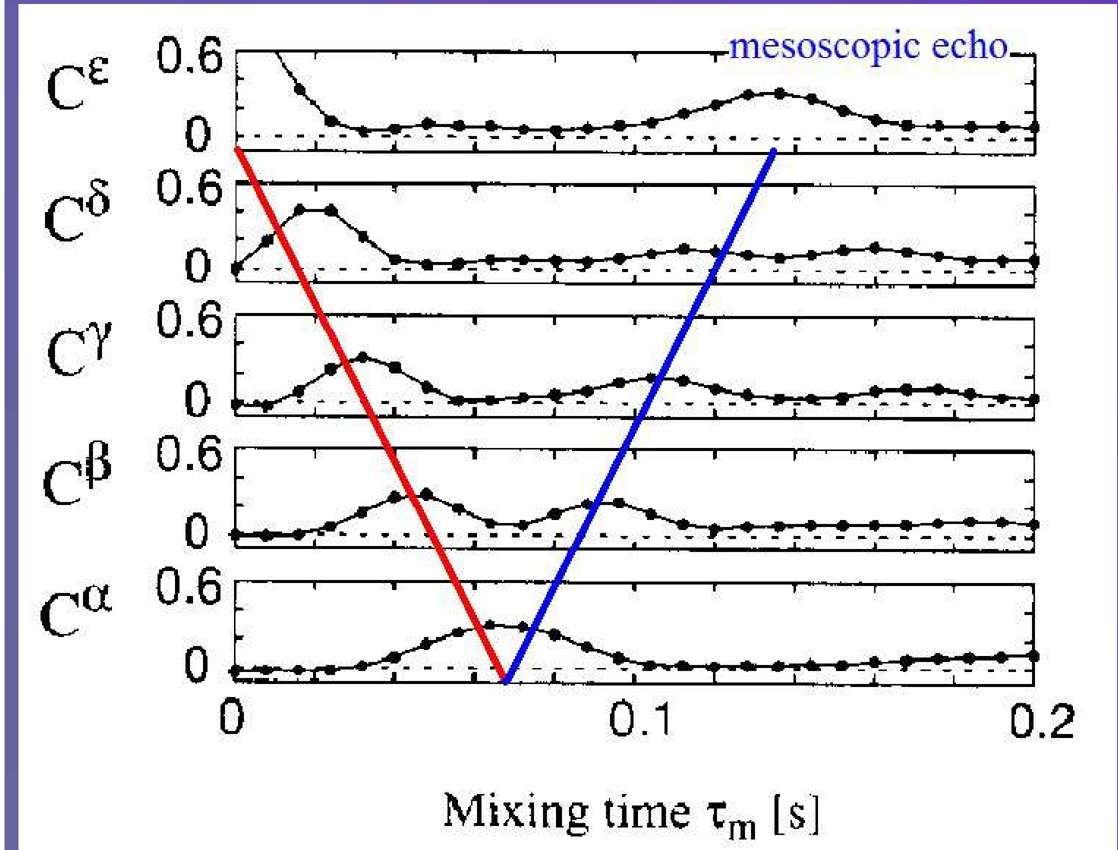


a SPIN EXCITATION behaves as QUANTUM WAVE

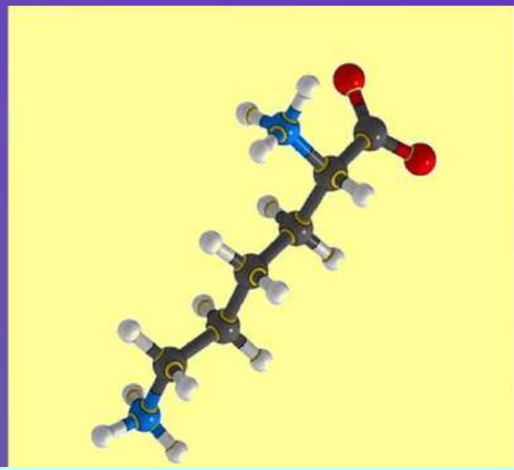
Time-resolved observation of spin waves in a linear chain of nuclear spins

Z.L. Mádi, B. Brutscher, T. Schulte-Herbrüggen, R. Brüschweiler, R.R. Ernst

Laboratorium für Physikalische Chemie, ETH Zentrum, 8092 Zürich, Switzerland



The study described in this letter has been inspired by discussions with Professor H.M. Pastawski and Professor P.R. Levstein who calculated nuclear spin wave evolution under a 'planar' or 'XY' Hamiltonian [3].



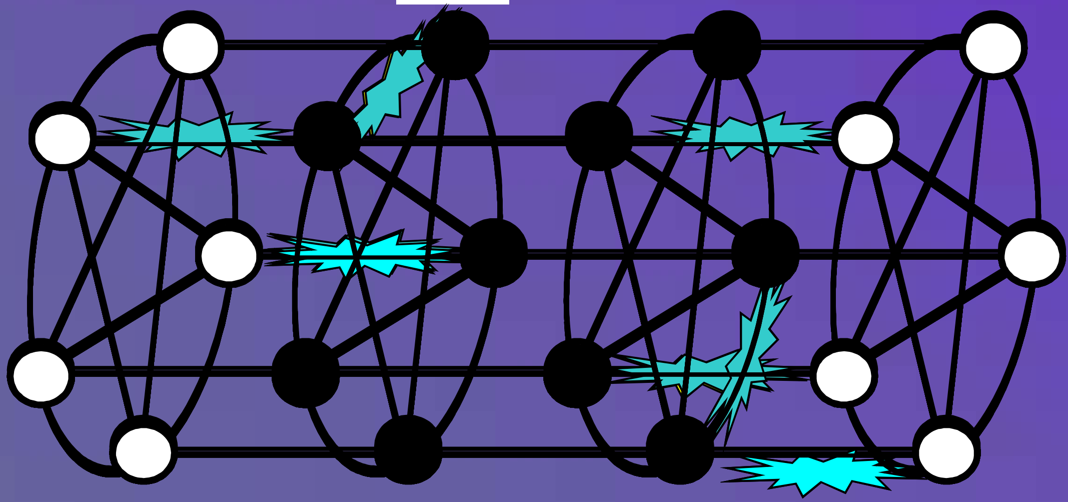
HMP, Usaj, Levstein, Chem. Phys. Lett. (1996)

time reversal in a MANY-BODY SYSTEM…?

$H \rightarrow -H+\Sigma$

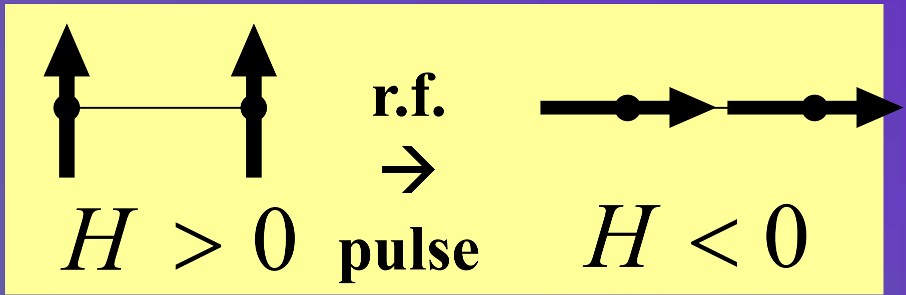
(spins coupled through dipolar interaction)

$$M(t)=\frac{\langle\Psi|\, e^{+\mathrm{i}\hat{H}t/\hbar}e^{-\mathrm{i}(\hat{H}+\hat{\Sigma})t/\hbar}\,\hat{S}_0^-\, e^{+\mathrm{i}(\hat{H}+\hat{\Sigma})\;t/\hbar}e^{-\mathrm{i}\hat{H}t/\hbar}\,\hat{S}_0^+\,|\Psi\rangle}{\langle\Psi|\,\hat{S}_0^-\,\hat{S}_0^+\,|\Psi\rangle}$$



exp[-iHt] → exp[+iHt]

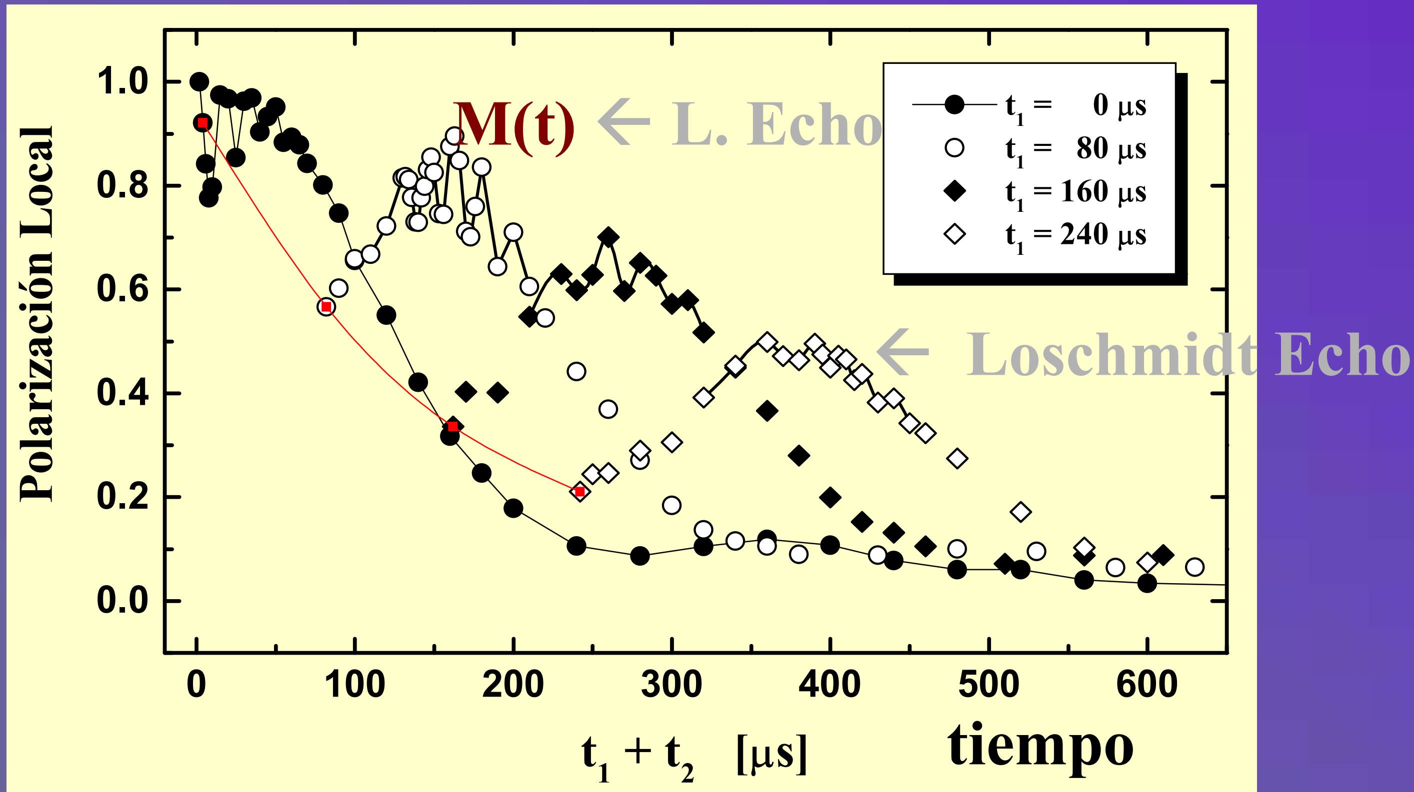
$$H \to -H + \Sigma$$



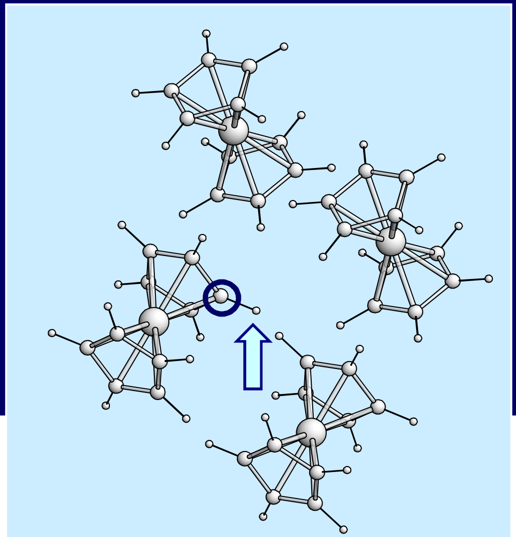
Loschmidt Echo family:

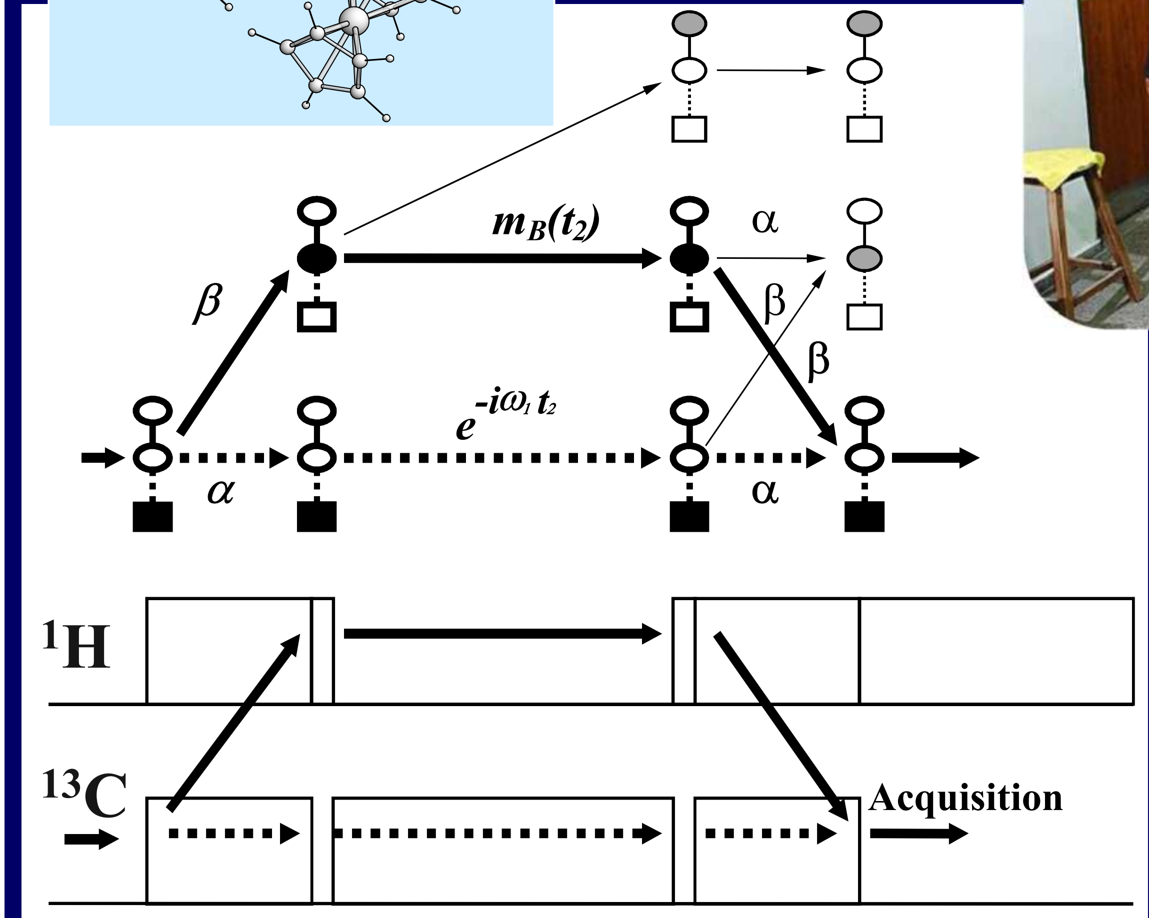
-one body: Hahn echo (1950)

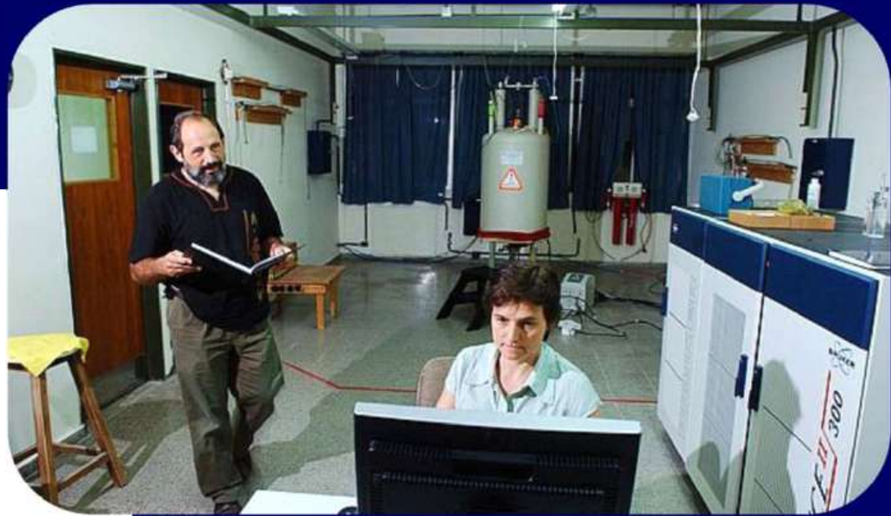
-many-body: Magic Echo (1970, Multiple Quantum Coherences (1985), Polarization Echo (1992), REPE (1998), PRLecho (2016)



Our “cat state”: we verified that two simultaneous alternatives “parallel words”



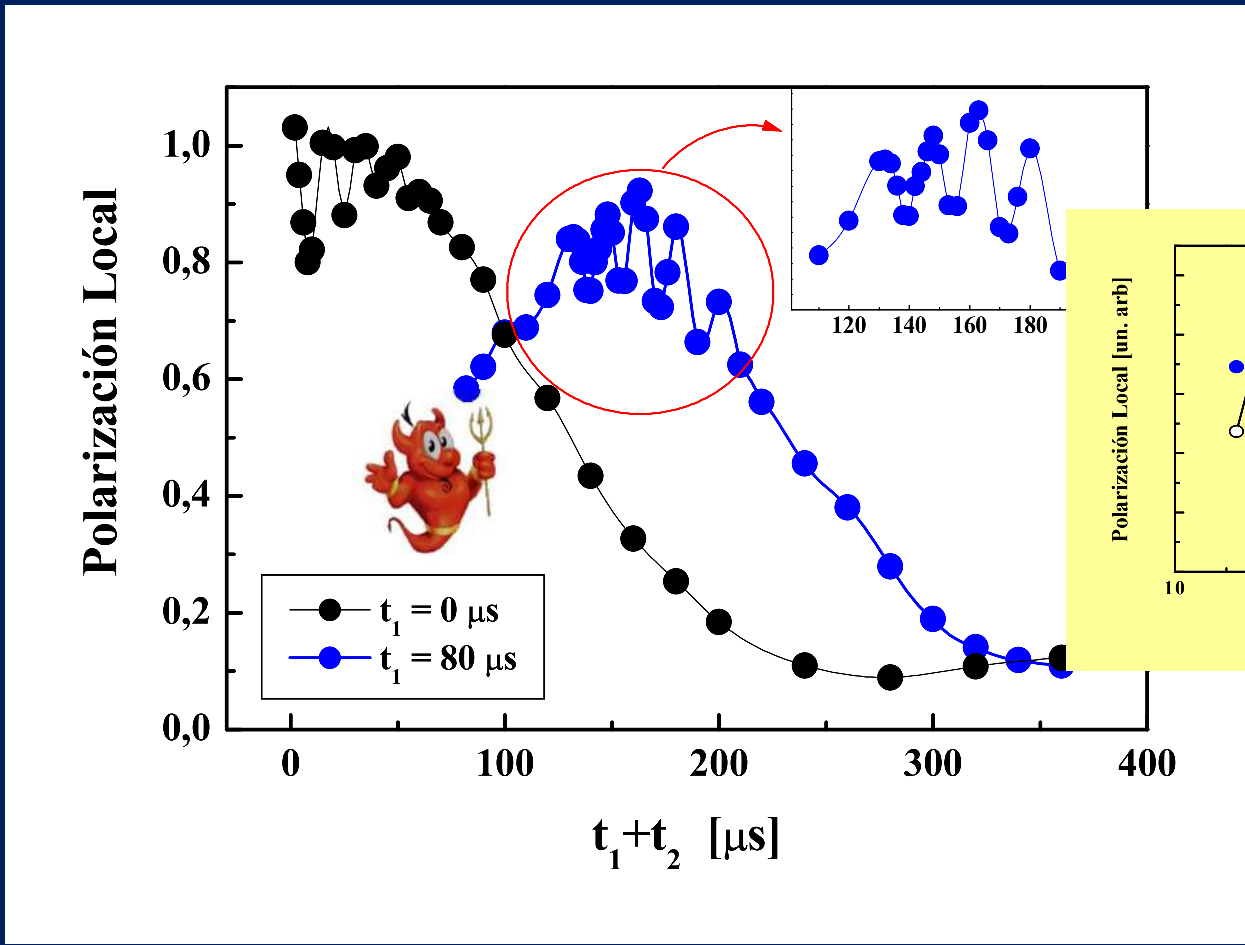


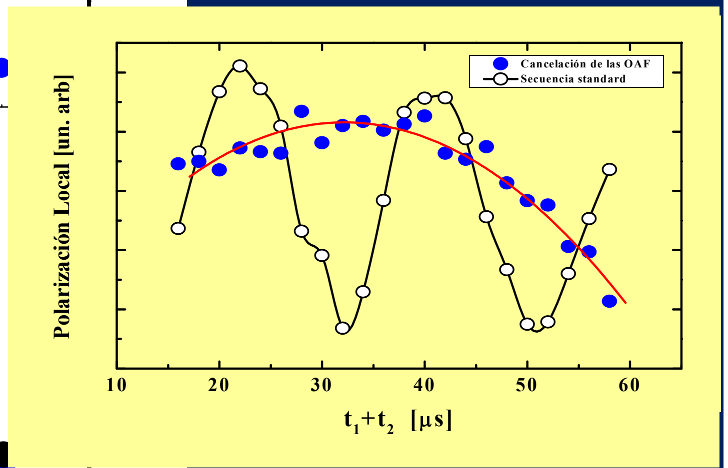


Patricia Levstein

Pastawski, Levstein, Usaj Phys. Rev. Lett. (1995); Levstein, Pastawski, Usaj, J. Chem. Phys. (1998),

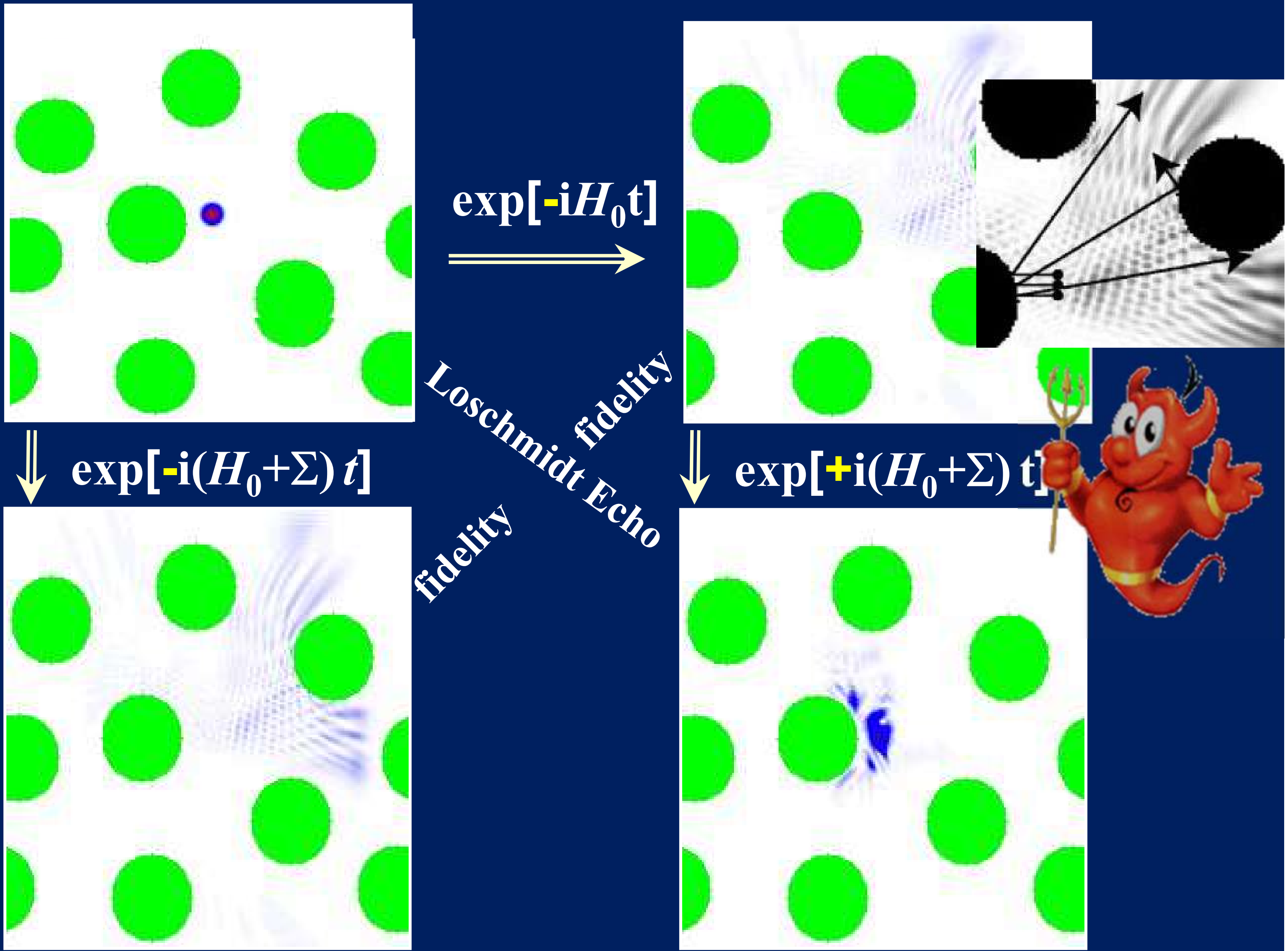
We used time reversal (Loschmidt echo) to monitor: the multi-espín superposition state





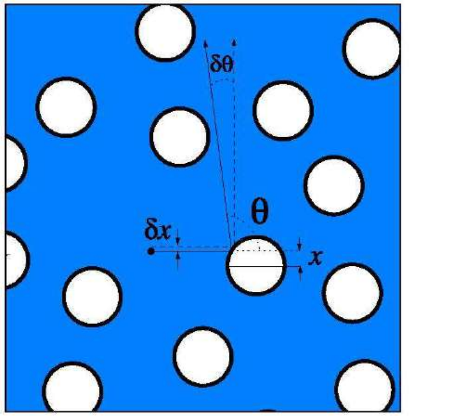
Pastawski, Levstein, Usaj Phys. Rev. Lett. (1995); Levstein, Pastawski, Usaj, J. Chem. Phys. (1998), Pastawski et. al. Phys. A (2000)

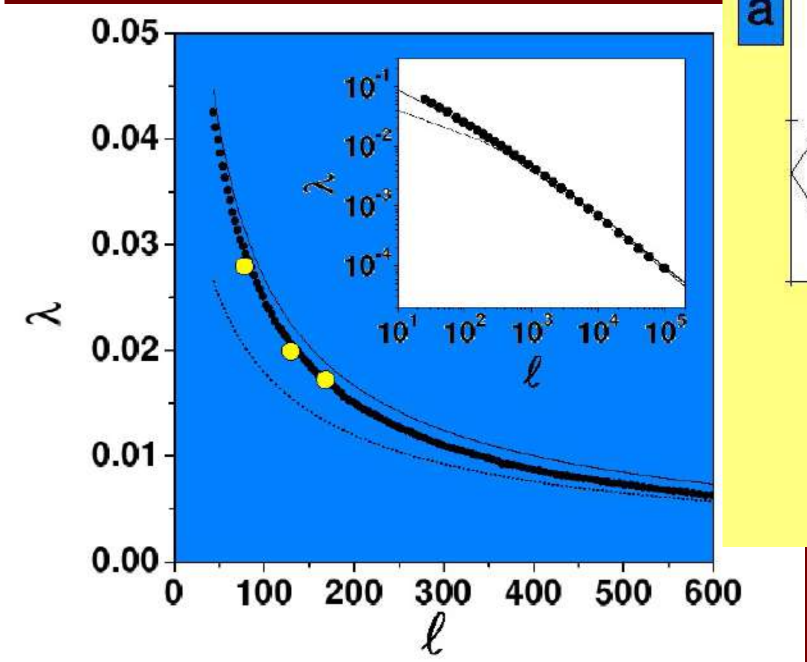
Loschmidt Echo: $M(t)=|\langle \phi_0|\exp[+i(H_0+\Sigma)t]\exp[-iH_0t]|\phi_0\rangle|^2$



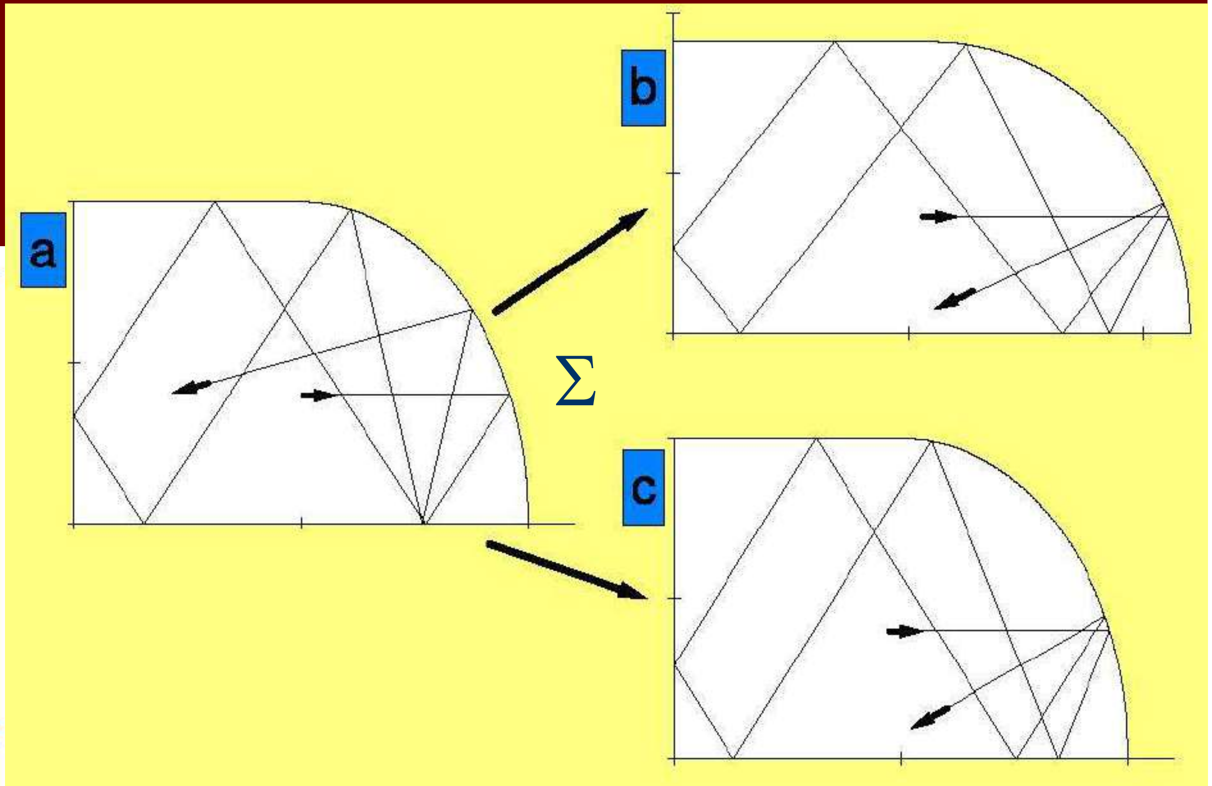
Classical chaos: H_o and its perturbation:

change m* effective mass ←→ dilation.



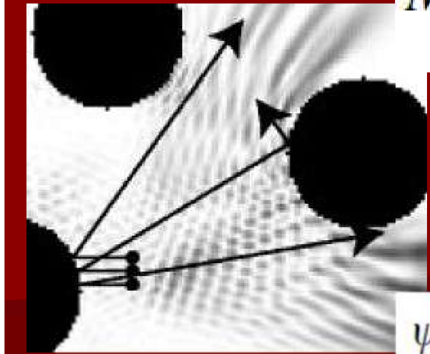


dilation

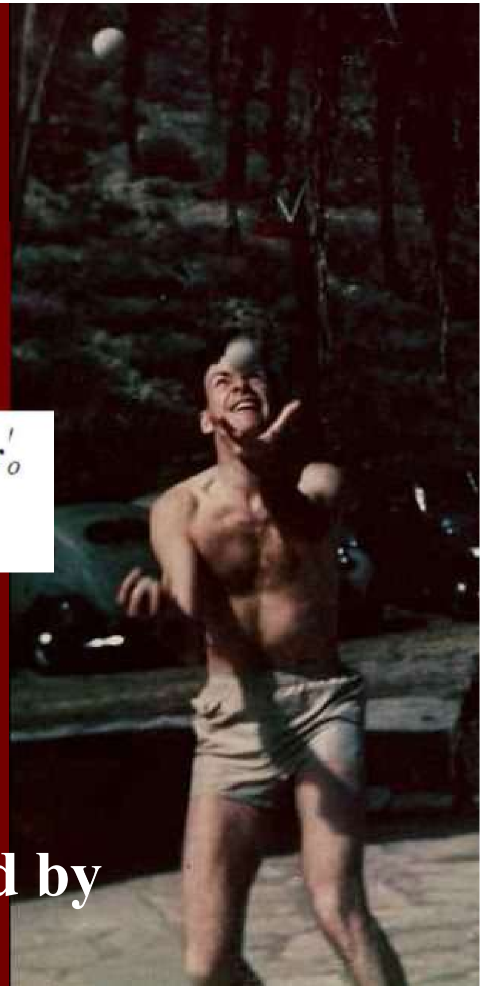


change m*

$$M(2t) = \int\int \phi(\mathbf{r}_o''')\psi_{LE}^*(\mathbf{r}_o''',2t) \times \phi^*(\mathbf{r}_o'')\psi_{LE}(\mathbf{r}_o'',2t)\mathrm{d}\mathbf{r}_o''\mathrm{d}\mathbf{r}_o''$$



Feynman (1942): ***all*** trajectories contribute to the **wave function**



$$\psi(\mathbf{r},t) = C \sum_{\gamma(\mathbf{r},t;\mathbf{r}_o',0)} \int \exp[\mathrm{i}(T_\gamma - V_\gamma)/\hbar] \underbrace{\phi(\mathbf{r}_o')}_{\text{initial packet}} \mathrm{d}\mathbf{r}_o'$$

$$\psi_{LE}(\mathbf{r}_o'',2t) = C^2 \sum_{\substack{\gamma_F(\mathbf{r}',t;\mathbf{r}_o',0)\\ \gamma_B(\mathbf{r}_o'',2t;\mathbf{r}',t)}} \int\int \exp[-\mathrm{i}(\tilde{T}_{\gamma_B} - \tilde{V}_{\gamma_B})/\hbar]\exp[\mathrm{i}(T_{\gamma_F} - V_{\gamma_F})/\hbar]\phi(\mathbf{r}_o')\mathrm{d}\mathbf{r}'\mathrm{d}\mathbf{r}_o'$$

$$M(2t) = \int\int \underbrace{\psi_{LE}^*(\mathbf{r}_o''',2t)\phi(\mathbf{r}_o''') \times \phi^*(\mathbf{r}_o'')\psi_{LE}(\mathbf{r}_o'',2t)\mathrm{d}\mathbf{r}_o''\mathrm{d}\mathbf{r}_o'''}_{\text{4 pathways towards 2 final points: } r_o'' \text{ and } r_o'''}$$

unstable trajectories→ reversibility fixed by chaos in H_o =Lyapunov

Q Dynamics unstable under Hamiltonian perturbations (Jalabert-Pastawski PhysRevLett 2001)

$M(t) \simeq \exp[-t/\tau_\phi]$ con $1/\tau_\phi = \min [\Gamma, \lambda]$

simplification…

IF spins in 1-d with XY interaction **THEN**
polarization echo M_{PE} becomes the Loschmidt echo:

$$M(t)=|\langle\psi_0|\exp[+i(H_0+\Sigma)t]\exp[-iH_0 t]|\psi_0\rangle|^2$$

Complex, <u>many</u>-body 10^{23} spin system $\approx$ •Single particle + •Chaotic dynamics

environment, pulse sequence errors, non secular terms. $\approx$ Unitary perturbation $H_0 \rightarrow -(H_0+V)$

•Peres (PRA 84) $\rightarrow$ QM: chaotic ← $M(t=\infty)$ *different* → integrable
•Shack and Caves (PRE 96) $\rightarrow$ Classical Mechanic:
• change in H ← *same Lyapunov divergence* → change IC :

Semiclassical theory

$$\psi\,(\mathbf{r},t) = \int dr'\,K(\mathbf{r},\mathbf{r}',t)\,\psi(\mathbf{r}',0) \simeq \sum_{s} \int dr'\,K_s\,(\mathbf{r},\mathbf{r}',t)\,\psi(\mathbf{r}',0)$$

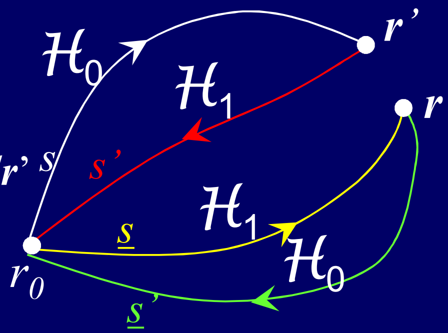
Localized state

$$K_s\,(\mathbf{r},\mathbf{r}',t) = C_s\, exp\,[i/\hbar\, S_s\,(\mathbf{r},\mathbf{r}',t) + i\mu_s]$$

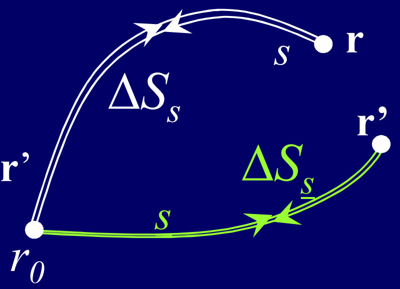
$$S_s\,(\mathbf{r},\mathbf{r}',t) = \int_0^t \mathrm{L}\,(\mathbf{q}_s(t'),\,\mathbf{q}_s'(t'),t')\,dt'$$

Only the classical paths s and s' ($\underline{s}$ and $\underline{s}'$) closer than $\hbar$ survive

$$M(t) = \sum_{\substack{s,\,s',\\ \underline{s},\,\underline{s}'}} \int dr \int dr'$$



$$M(t) \approx \sum_{s,\,\underline{s}} \int \mathrm{dr} \int \mathrm{dr'}$$

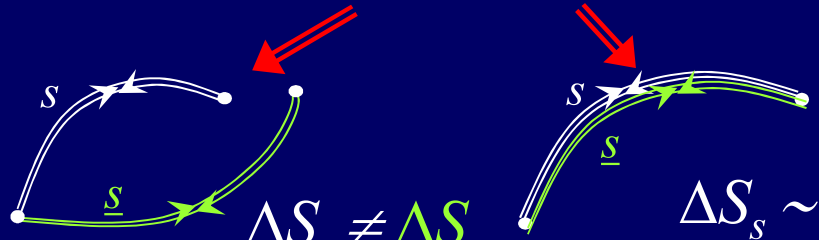


Assuming that the perturbation Σ does not change classical trajectories, only the quantum phase (action)

$$S_s^{\Sigma}(\mathbf{r},\mathbf{r}',t) = S_s^{0}(\mathbf{r},\mathbf{r}',t) + \Delta S_s\,(\mathbf{r},\mathbf{r}',t)$$

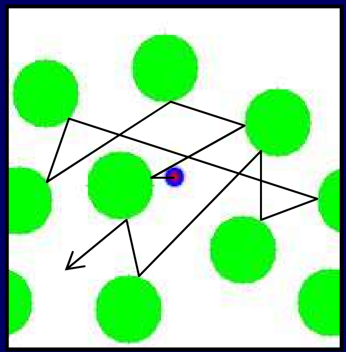
4 sums in M(t): split into
into paired terms:
ND non-diagonal - different paths
D diagonal - similar paths.
<averaging> ➔ simpler

$M(t)=M^{ND}(t)+M^{D}(t)$



$\Delta S_s \neq \Delta S_{\underline{s}}$ $\qquad$ $\Delta S_s \sim \Delta S_{\underline{s}}$

Information about the system and the perturbation is introduced in the form of distributions for ΔS_s



$M^{ND}(t) \cong \exp(-\Gamma t)$ $\qquad$ $\Gamma = \pi V^2 / \Delta$

$= V^2 m^2 v^4 \tau_e / 4\hbar^2$

Σ induced phase fluctuation rate FGR

$M^{D}(t) \cong \exp(-\lambda t)$

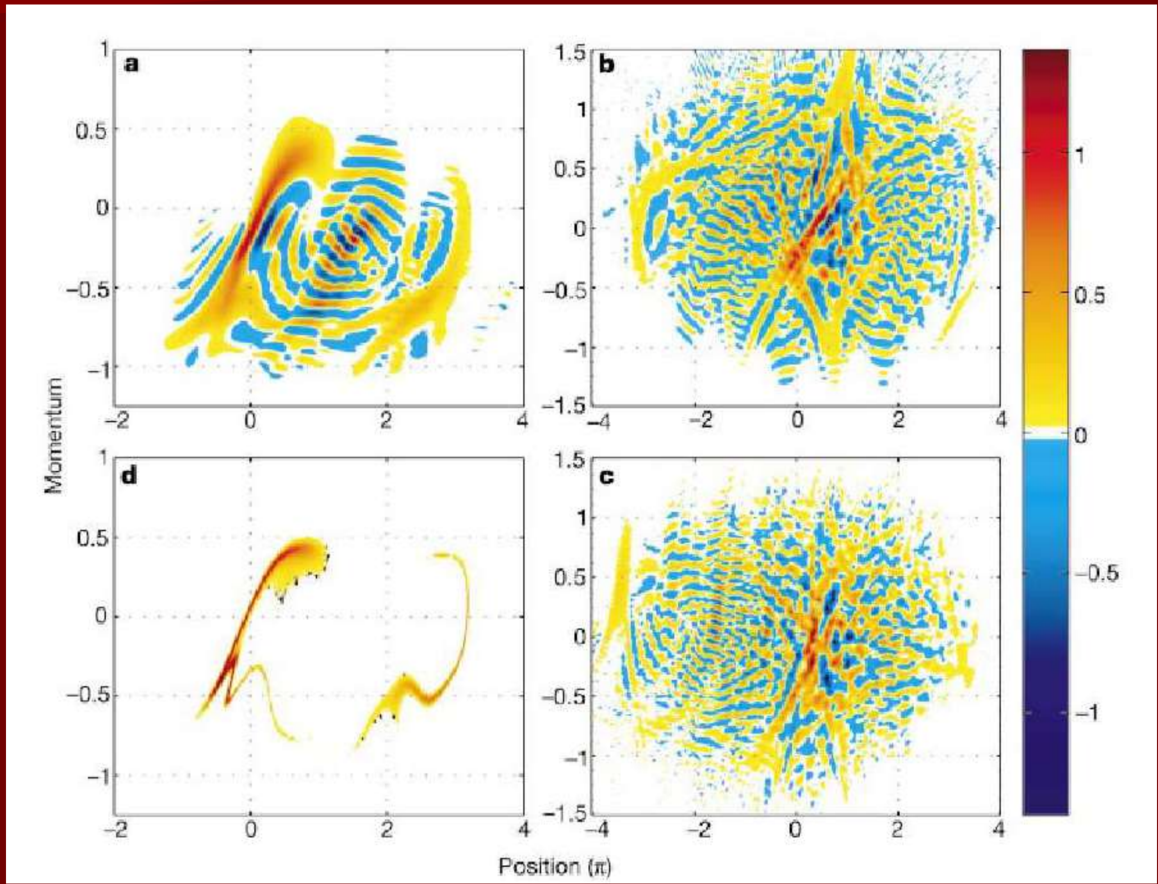
λ : Classical Lyapunov exponent!!!

The Loschmidt echo decays exponentially with the minimum between λ and Γ (Lyapunov vs. FGR)

Wigner's function

$$W(x, p_x, t) = G^<(x, p_x, t)$$
$$= \int d\delta x\ \psi^*(x+\delta x/2)\ \psi(x-\delta x/2)\ \exp[i\, p_x\, \delta x]$$

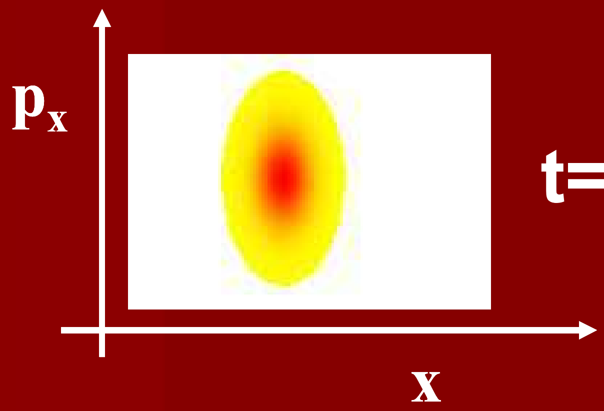
Wigner distribution



classical distribution

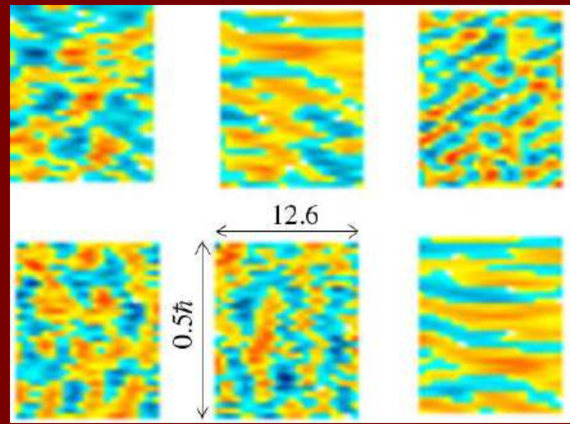
Wigner function $W(x, p_x, t) = G^{<}(x, p_x, t)$

$$= \int d\delta x \, dy \, \psi^*(x+\delta x/2;y)\, \psi(x-\delta x/2;y)\, \exp[i\, p_x . \delta x]$$

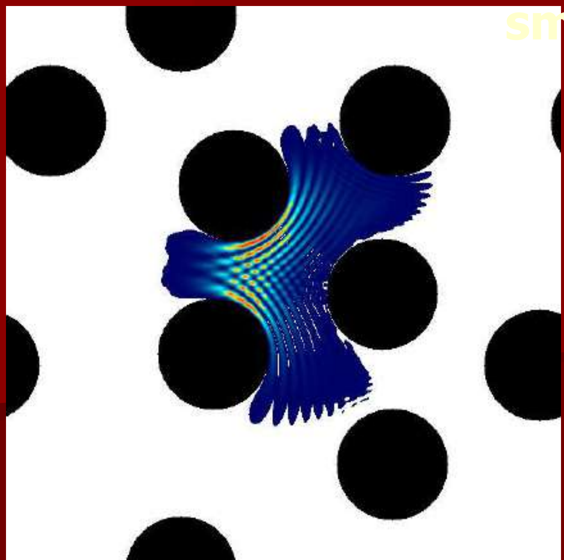


t=0

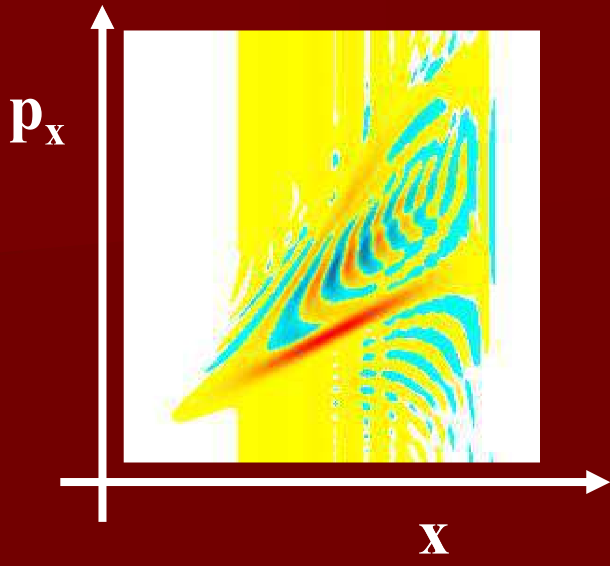
t » τ



small structure of Wigner patches → decoherence (Zure



← t>τ →



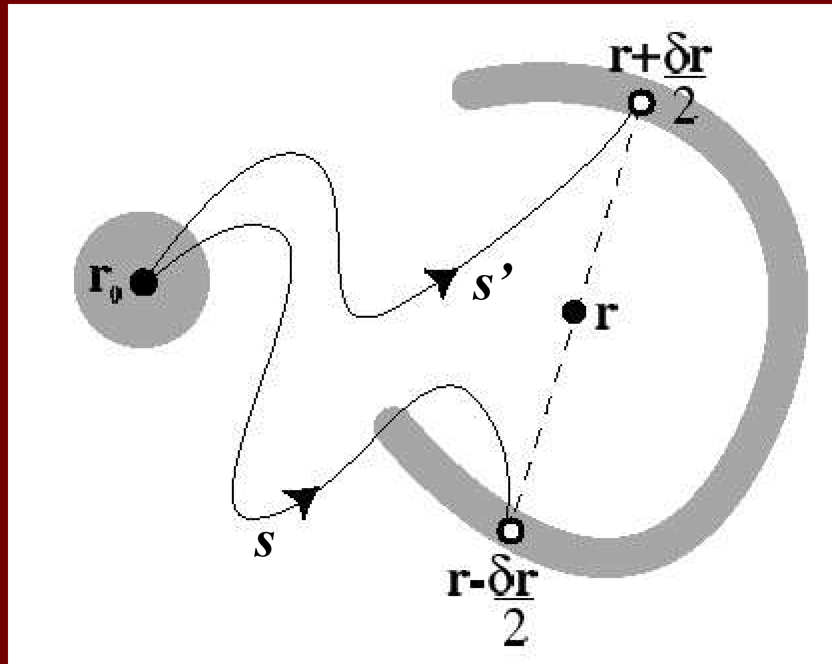
Wigner's semiclassical expansion:

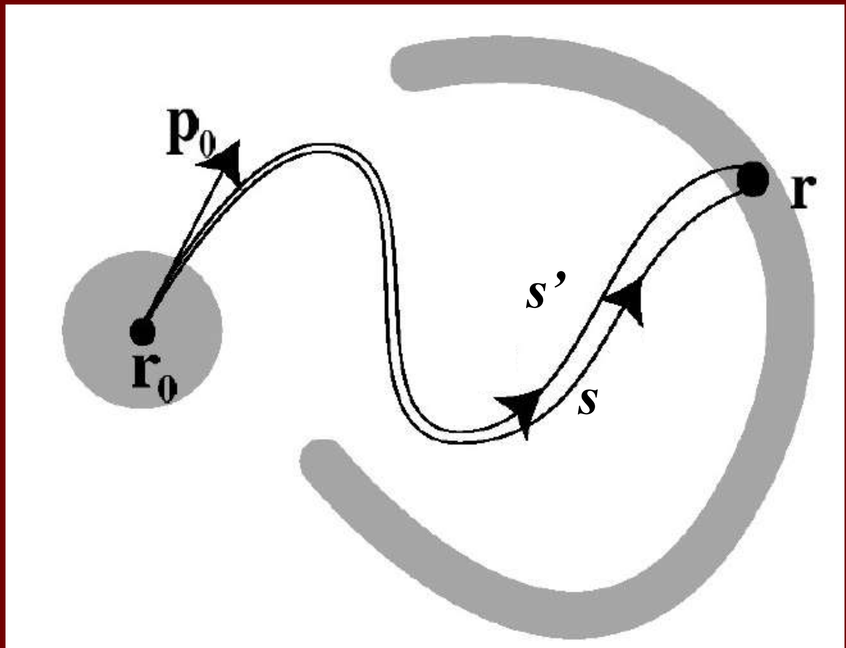
$$M(t) = \mathrm{Tr}[\tilde{G}^{<} G^{<}] = \mathrm{Tr}[\tilde{\rho}\,\rho]$$

NON-CLASSICAL point

mean of 2 CLASS. points

→ FermiGoldenRule



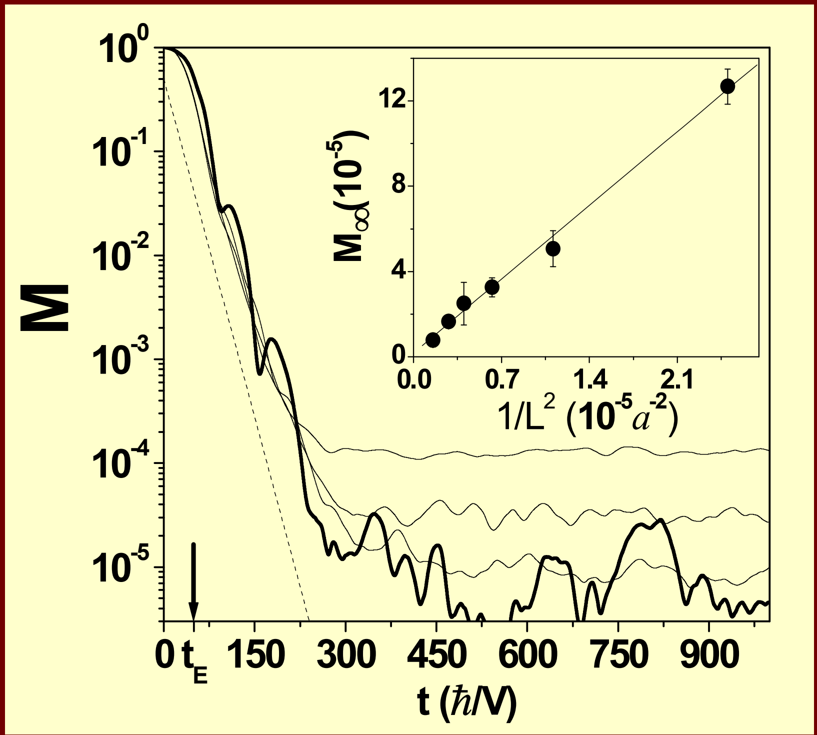


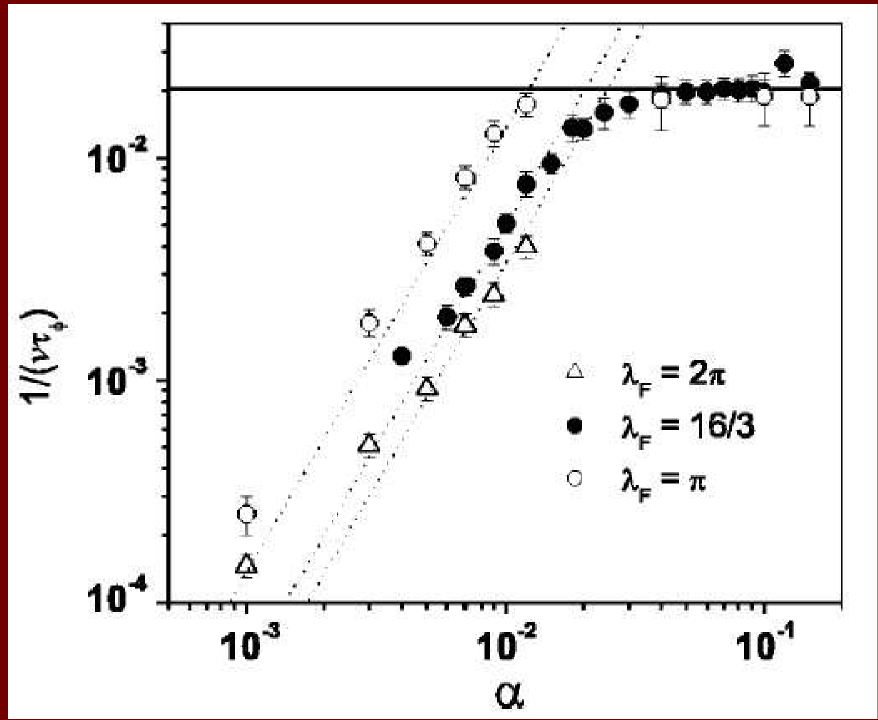
pairs ending in classical points →

perturbation independent Lyapunov + power law.

Jalabert, HMP Phys.Rev.Lett. 2001; Cucchietti,HMP,Jalabert Phys.Rev.B 2004

Quantum Dynamical Chaos

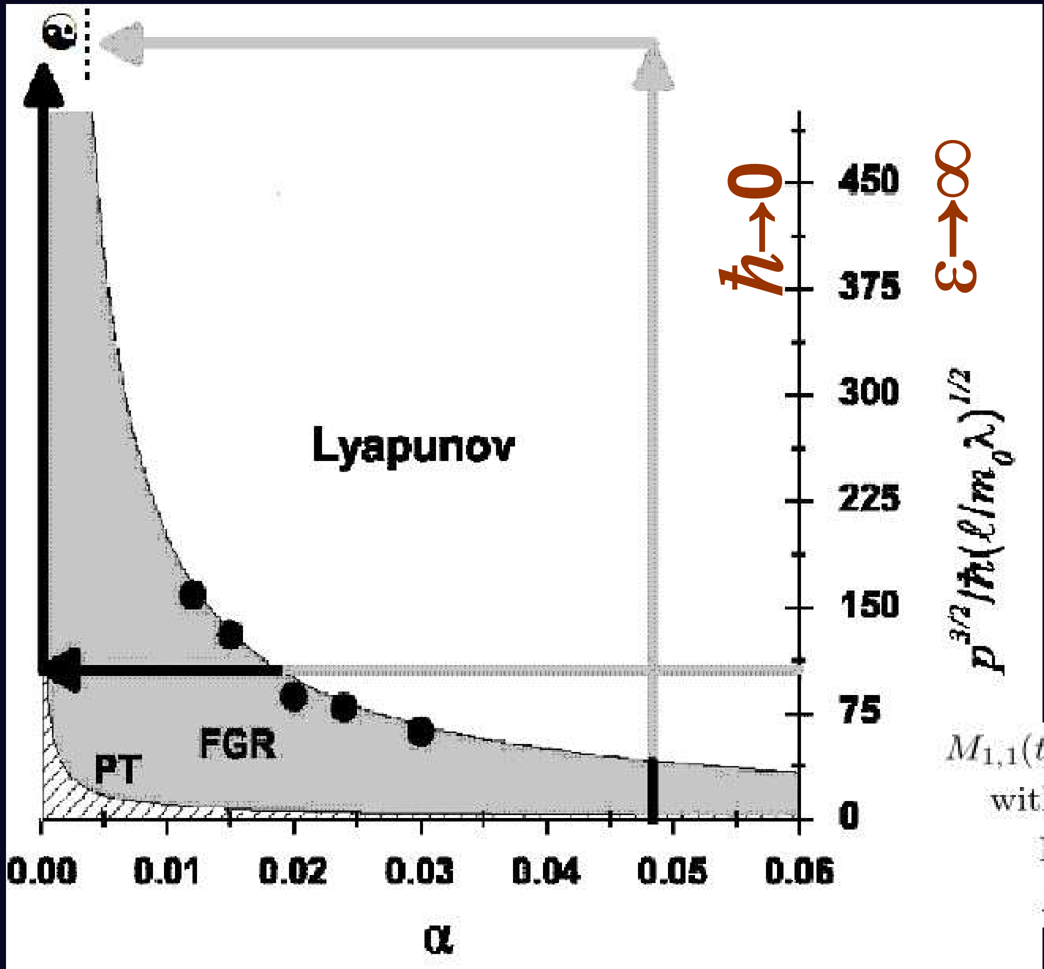




Jalabert, HMP PRL 2001

Jacquod, Silvestrov , Beenakker PRE2002,

Cucchietti, Wisniacki, HMP PRE 2002



$N \to \infty$

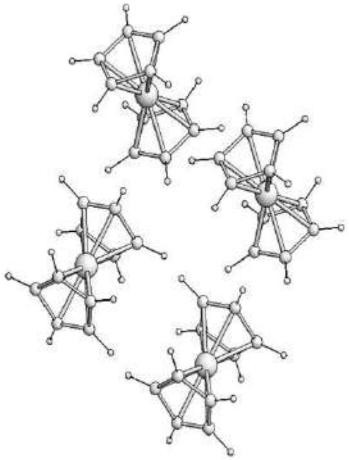
$$
\begin{aligned}
M_{1,1}(t) &\simeq \exp[-\kappa t] \\
\text{with} &\quad \kappa = \min\,[\Gamma, \lambda] \\
\Gamma &= 2\pi\,|\Sigma|^2 N_1 \\
\lambda &= \text{classical Lyapunov exponent}
\end{aligned}
$$

Any perturbation is stronger than the critical…! CLASSICAL OK

thermodynamic limit in many-body…?

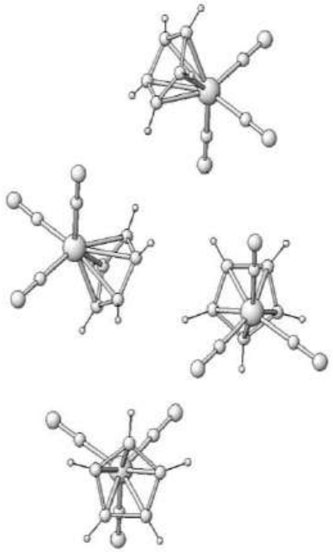
Central Hypothesis of Irreversibility: in many-body there is an Intrinsic Decoherence/Irreversibility because critical perturbation →0

our experiments: finite and infinite networks of nuclear spins

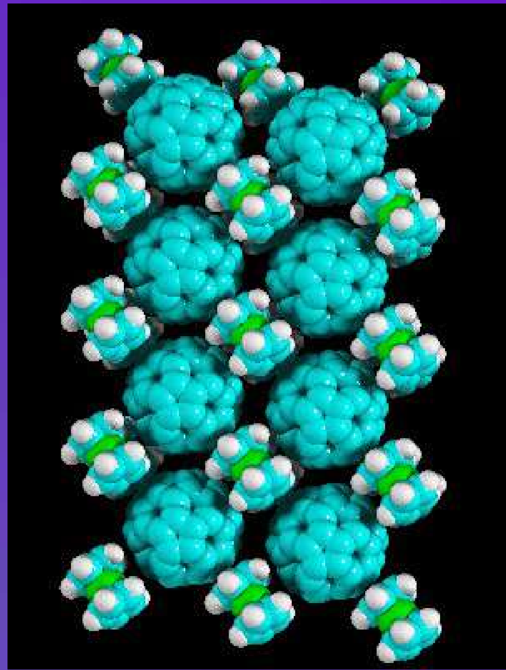


Ferrocene

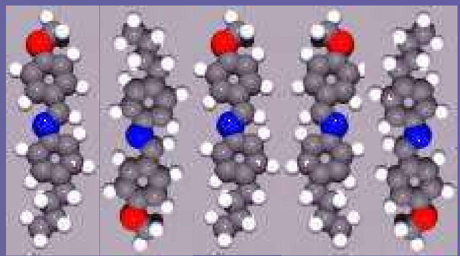
cobaltocene



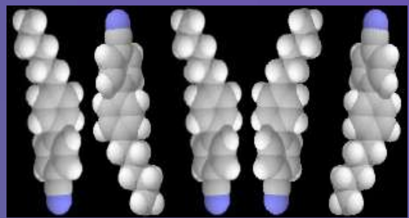
Cymantrene



$C_{60}(\text{ferrocene})_2$

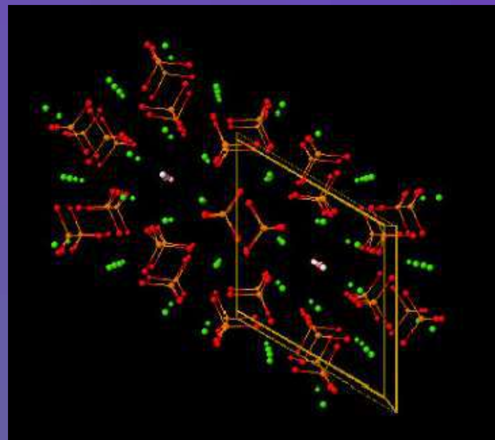


MBBA

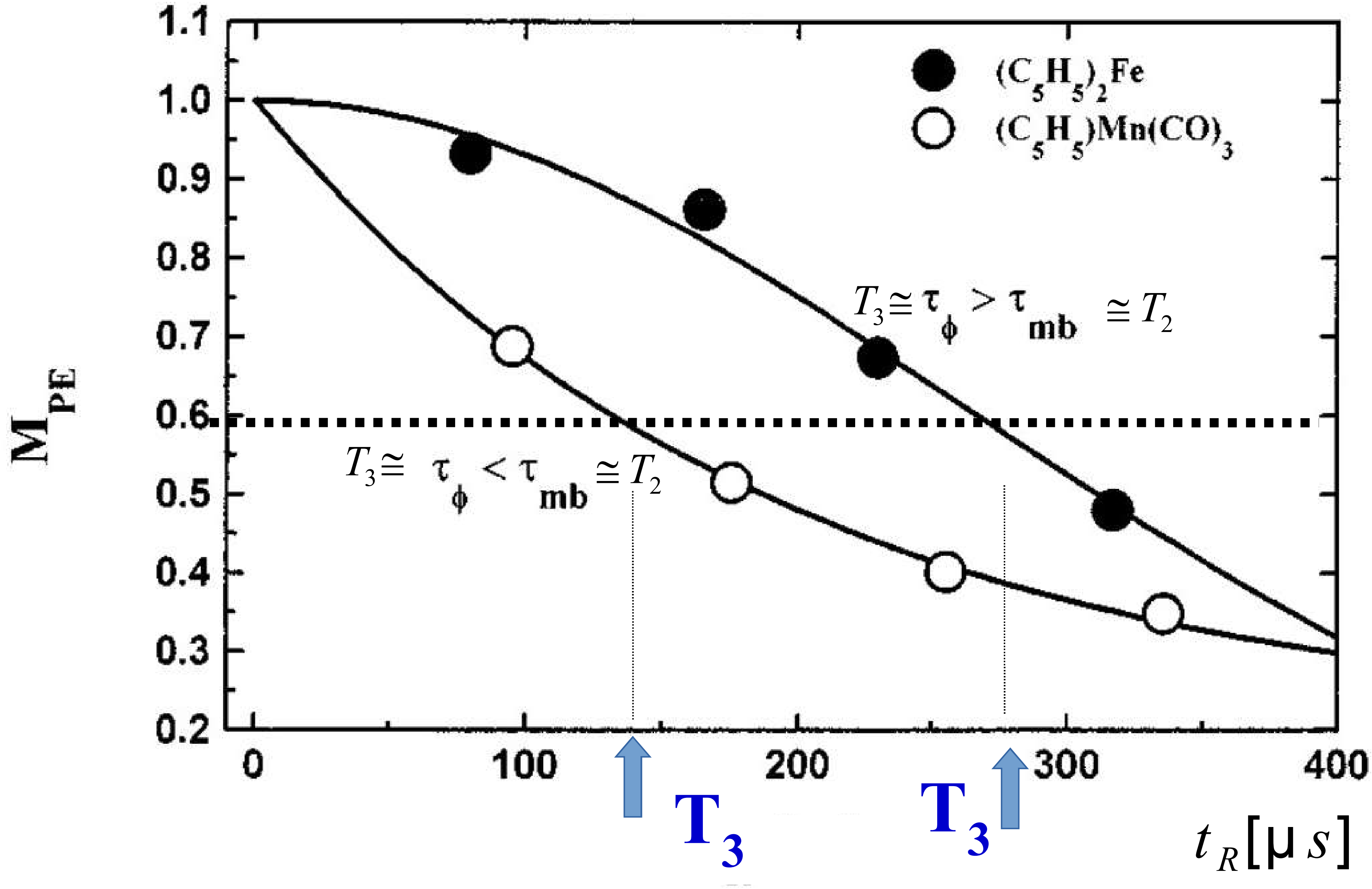


5CB

8CB



Hydroxyapatite
$Ca_{10}(PO_4)_6(OH)_2$
1-D spin chain



1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
M_{PE}
0
100
200
300
400
$(C_5H_5)_2Fe$
$(C_5H_5)Mn(CO)_3$
$T_3 \cong \tau_\phi > \tau_{mb} \cong T_2$
$T_3 \cong \tau_\phi < \tau_{mb} \cong T_2$
T_3
T_3
$t_R [\mu s]$

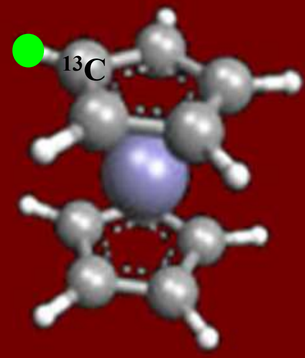
Polarization Echo Experiment

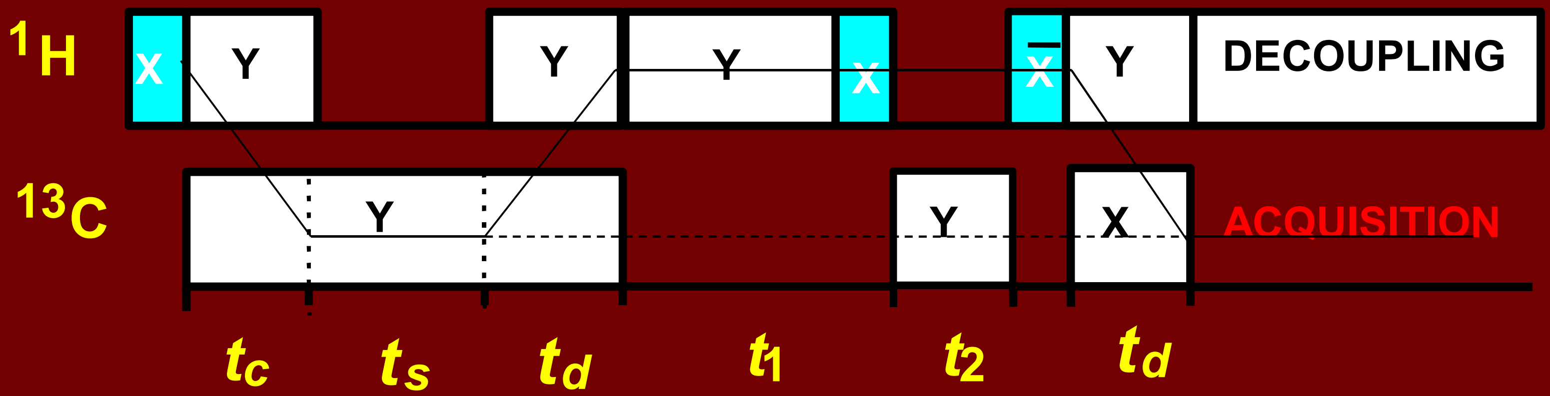
Creation of a non-equilibrium local polarization

Evolution of the system with H_{dip} $(0< t< t_R) \rightarrow P_{00}(t)$

Evolution of the system with $-H_{dip}$ $(t_R< t <2t_R)$

Detection of the local polarization at $2t_R \rightarrow M_{PE}(t_R)$





Change of the relative timescales

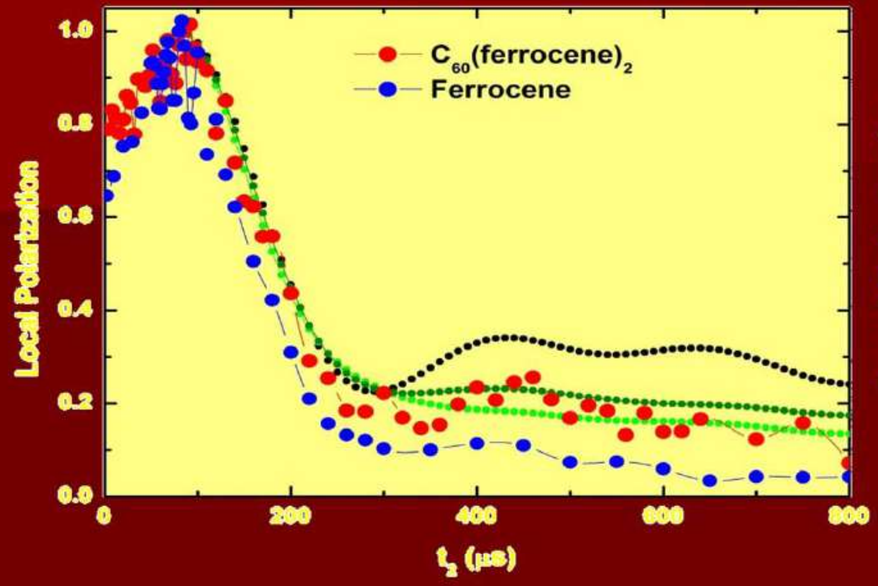
Intrinsic Spin Dynamics

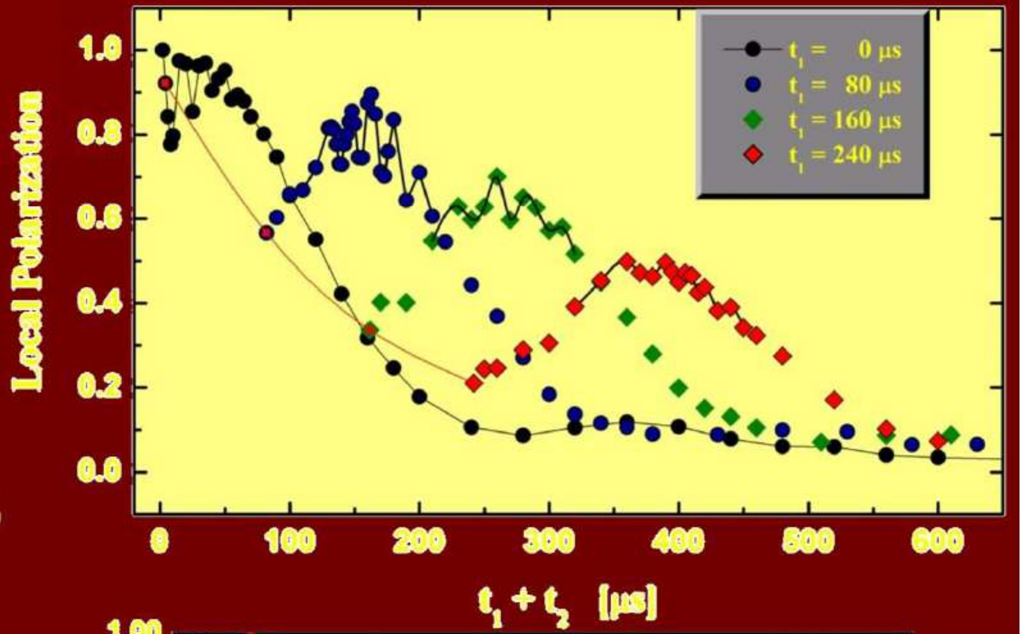
- Orientation
- Pulse sequence
- Coupling network (crystal)

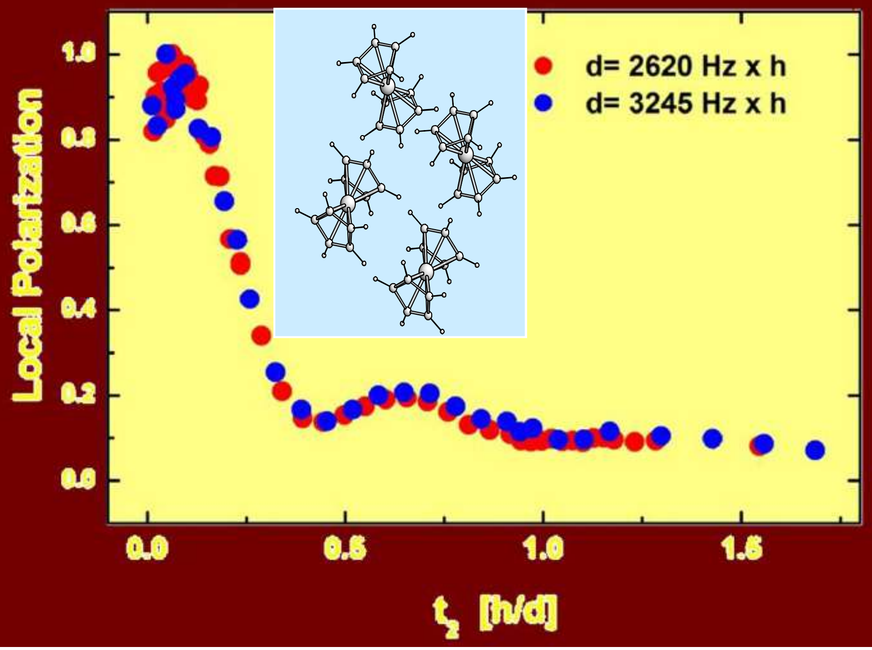
Coupling with the Environment

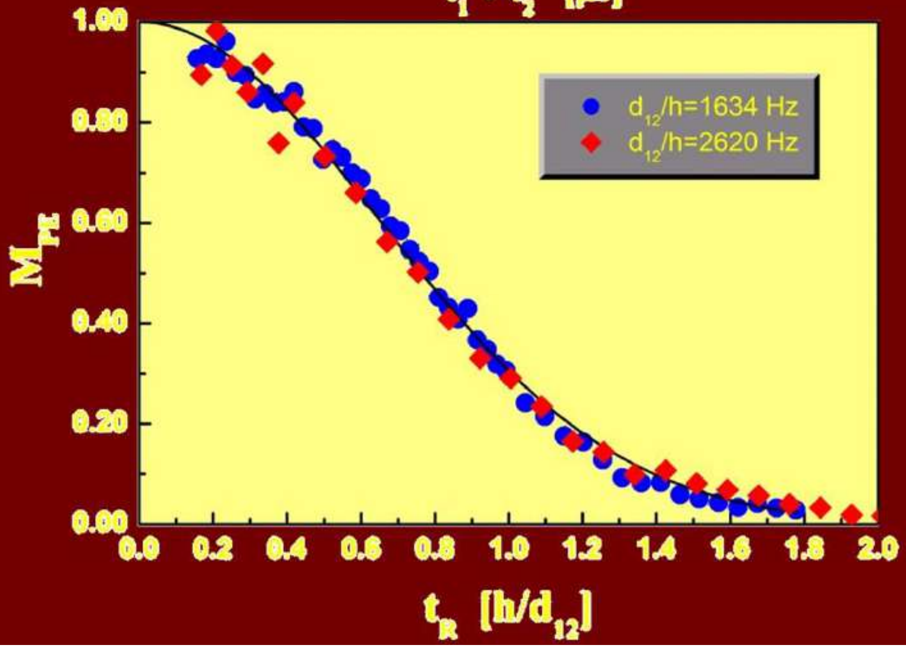
- Paramagnetic atoms
- Quadrupolar nuclei
- RF Power

Forward Evolution and Loschmidt Echo



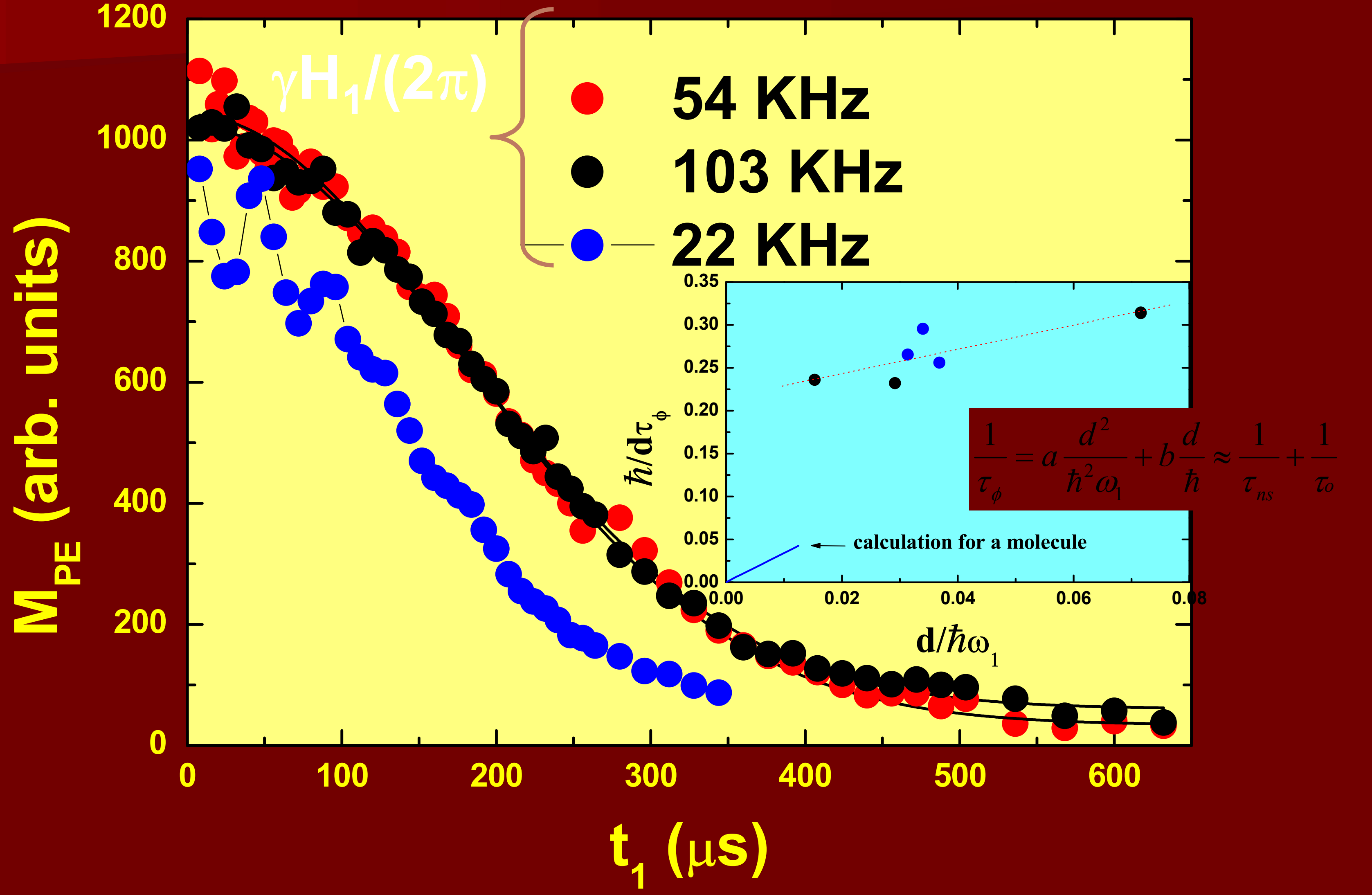




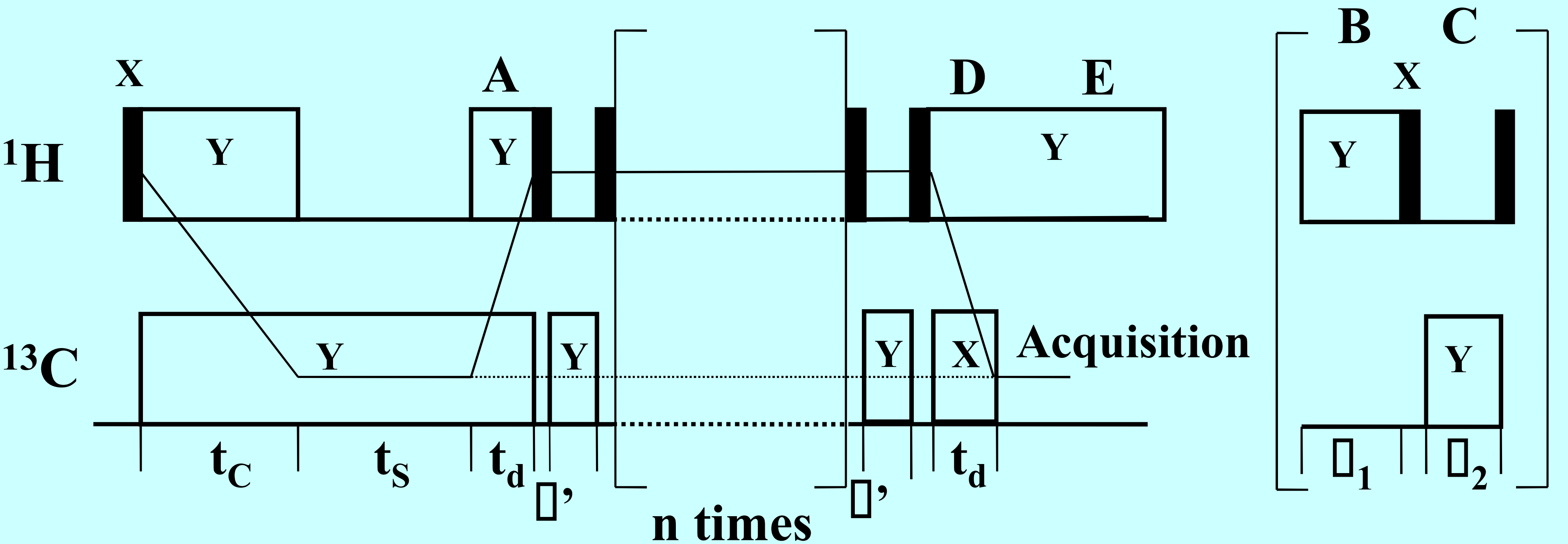


Variation of the perturbation Σ

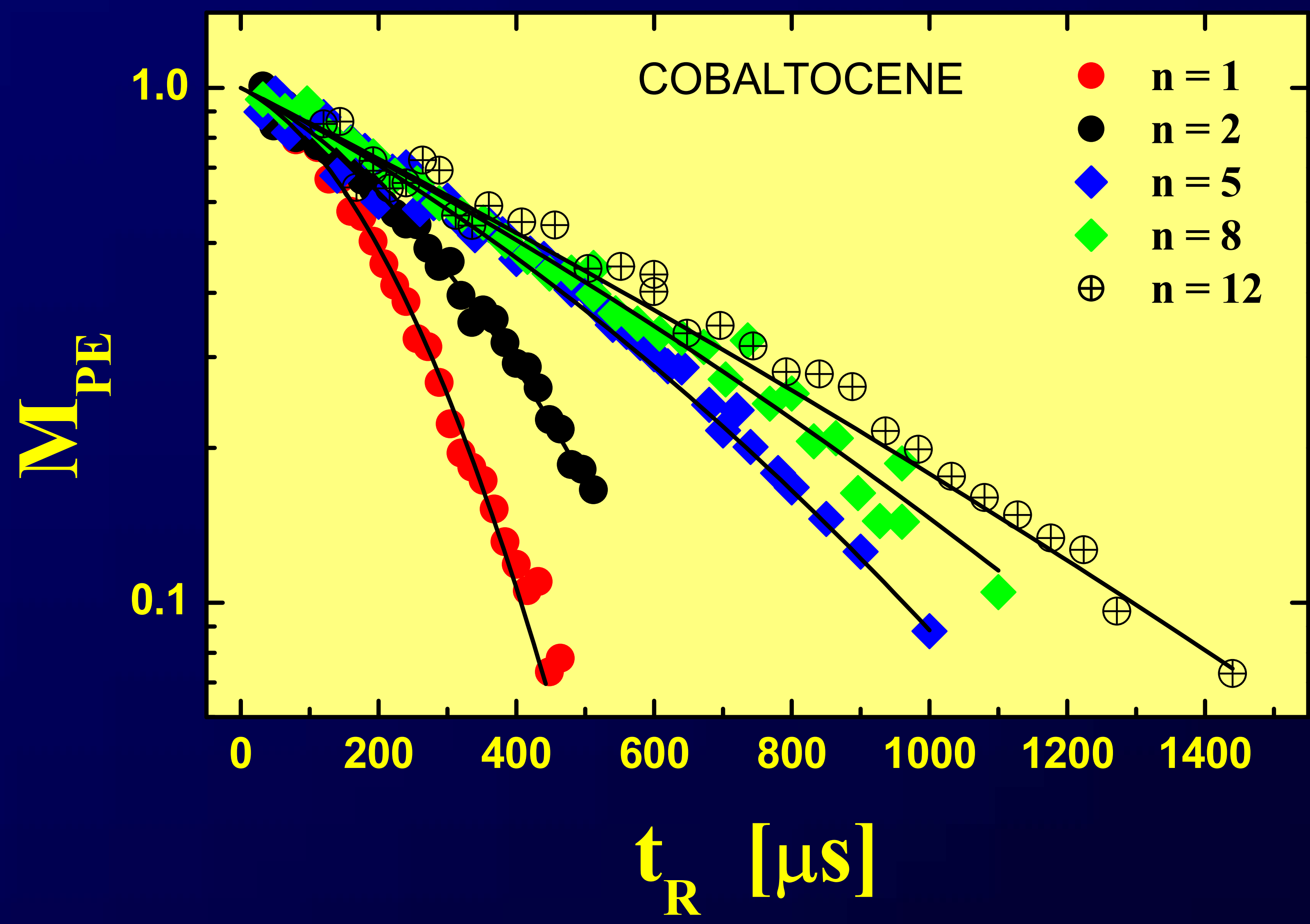
(Are the non-secular terms Σ responsible for the attenuation of the polarization echoes? They are proportional to $1/H_1$)



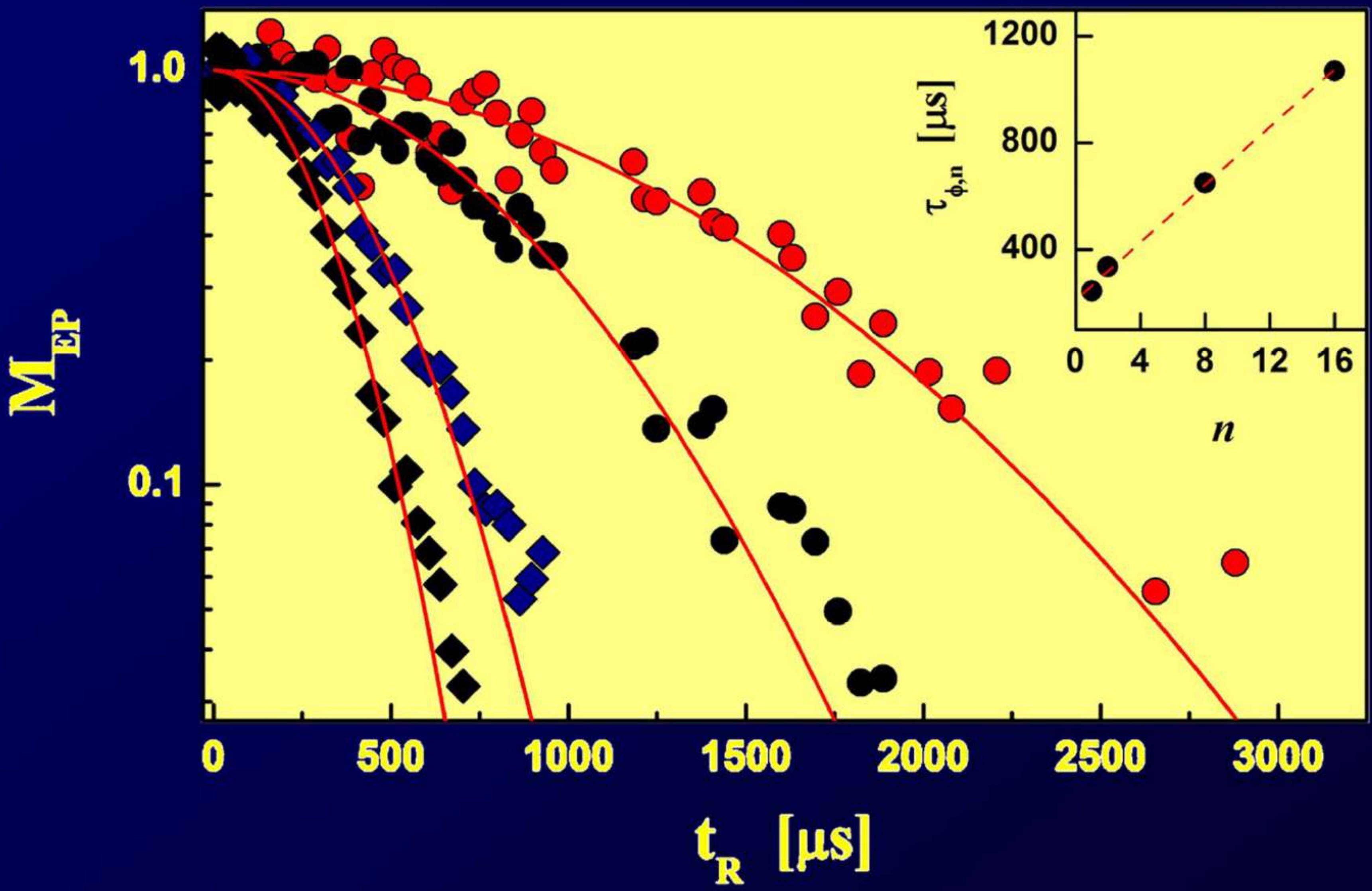
The REPE sequence



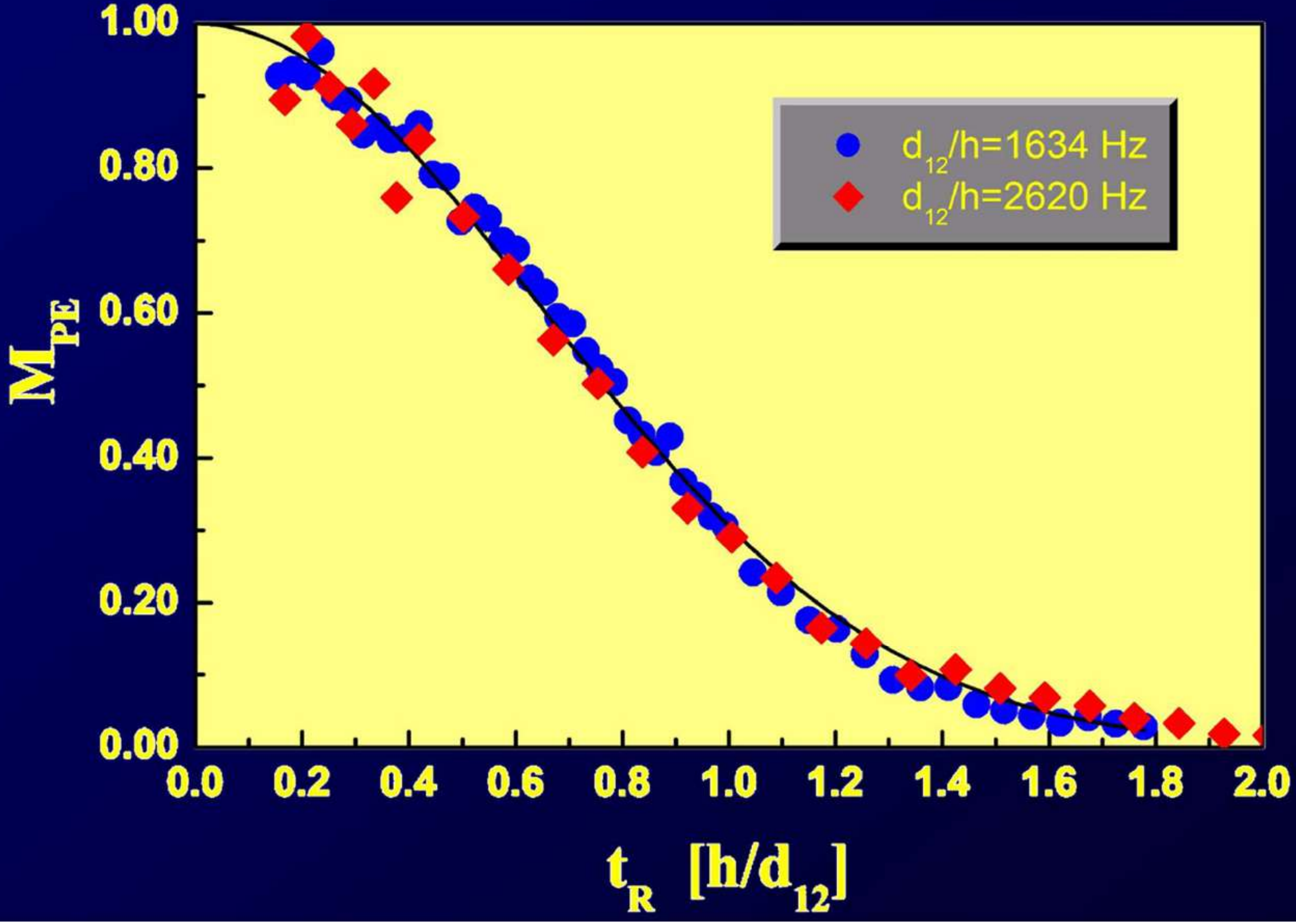
Reduced Evolution of the Polarization Echo

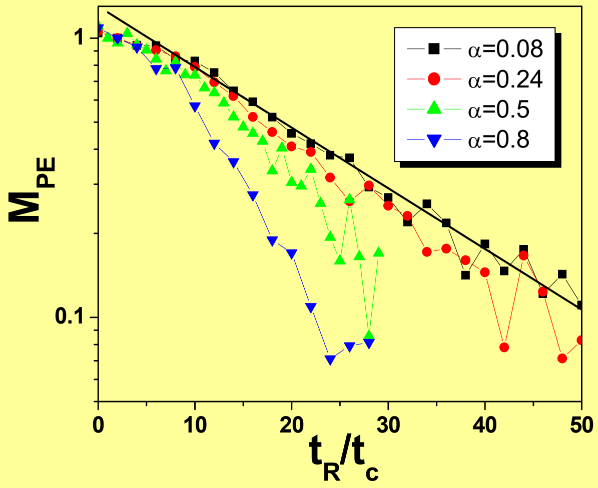


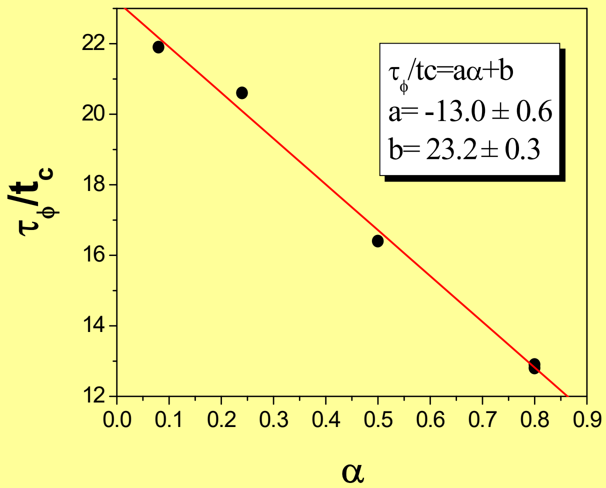
Reduced Evolution of the Polarization Echo



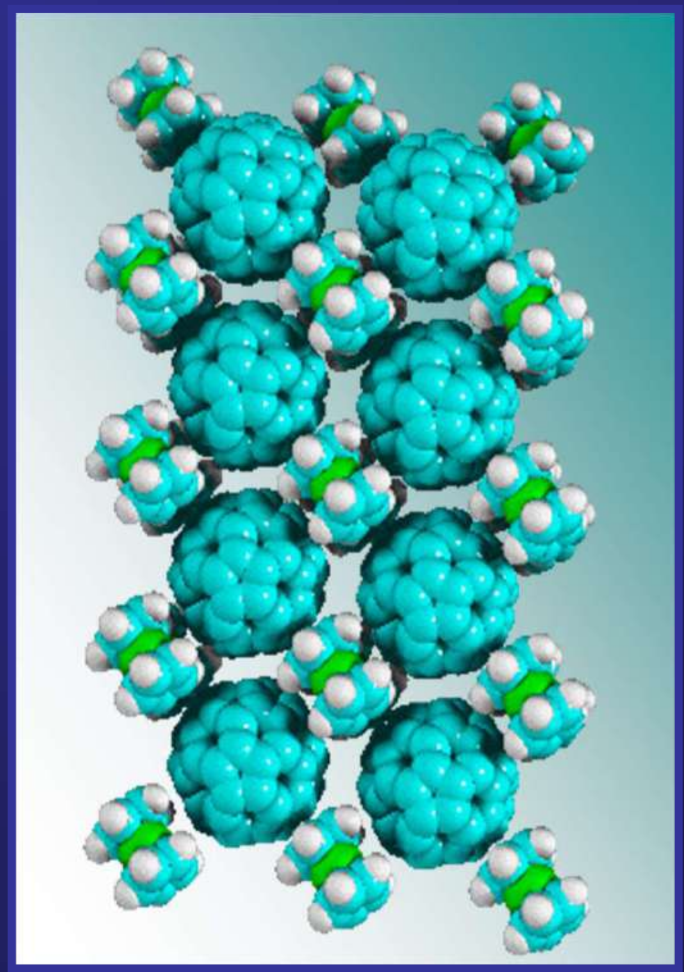
MPE for two orientations of a single crystal of ferrocene



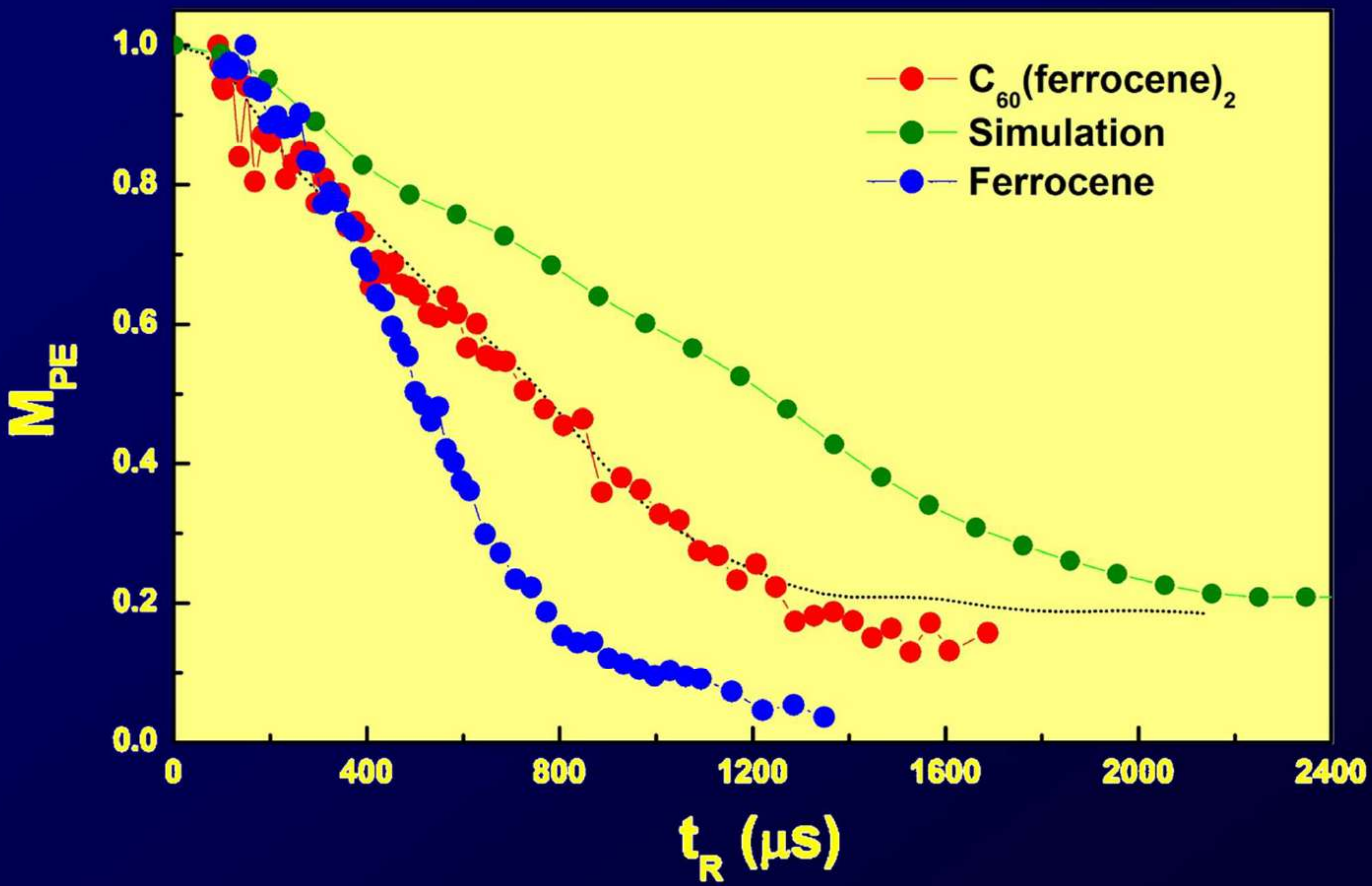




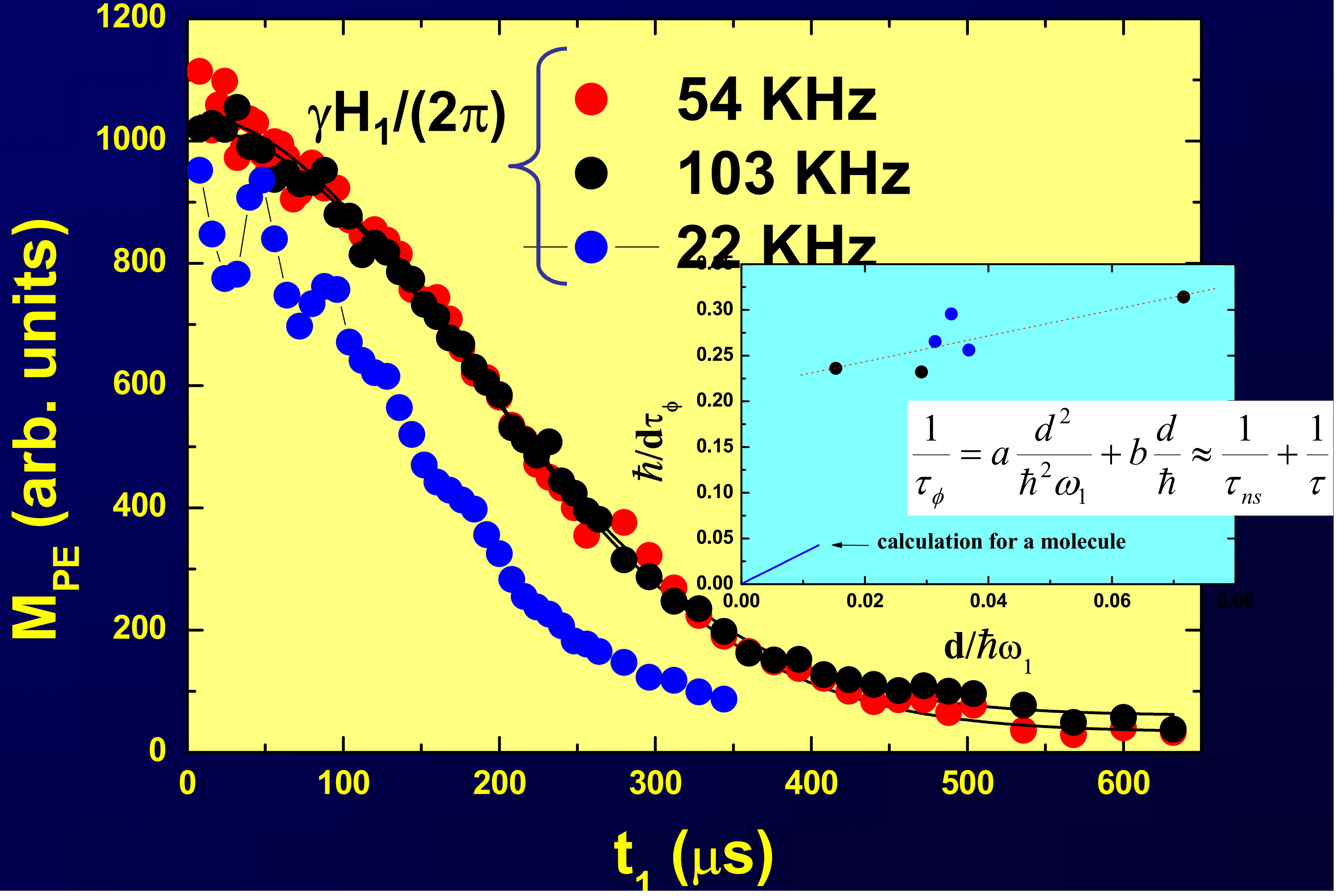
^{1}H-^{13}C interaction?



Diluting the dipolar network



Variation of the perturbation Σ

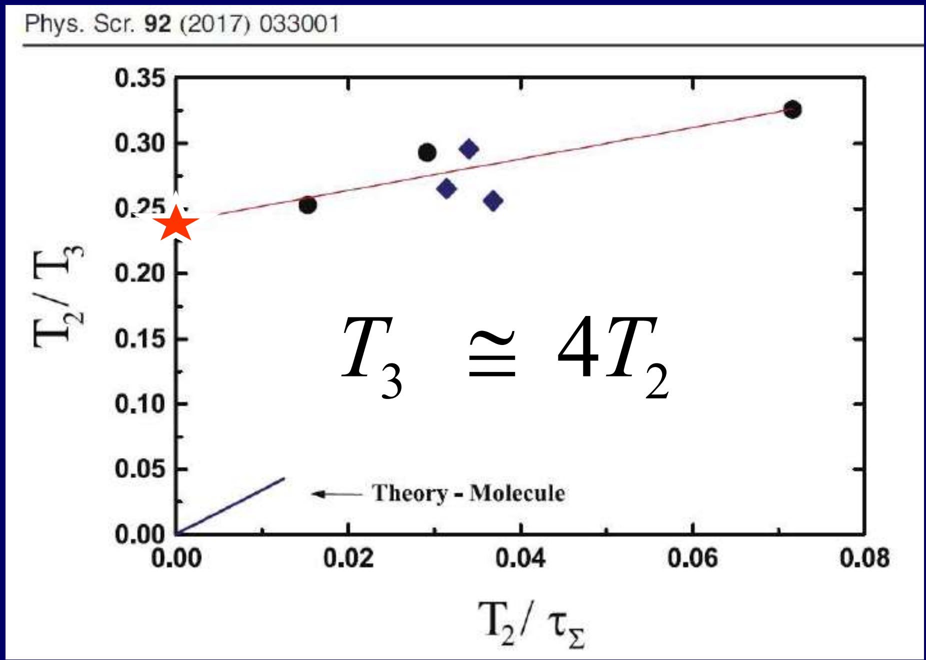


Are the non-secular terms Σ responsible for the attenuation of the polarization echoes?

They are proportional to $1/H_1$

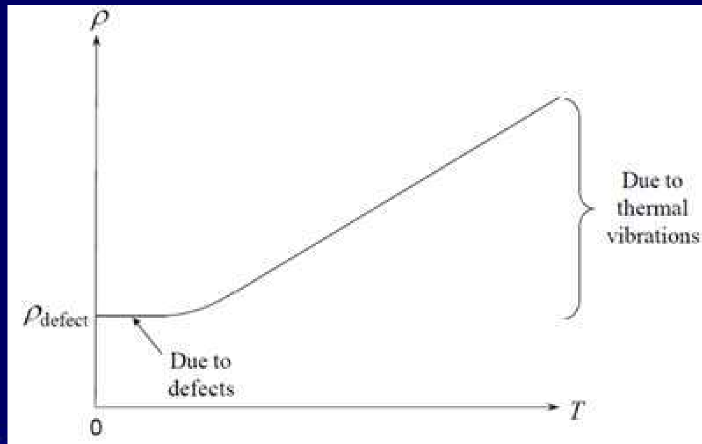
what controls the Loschmidt daemon efficiency in Quantum Mechanics?

environment $1/\tau_\Sigma$ + errors affect Polarization/Loschmidt echo, but…
$1/T_3$ can not be improved…!!



Decay rate $1/T_3 = 1/\tau_\phi$ scales with dynamics.

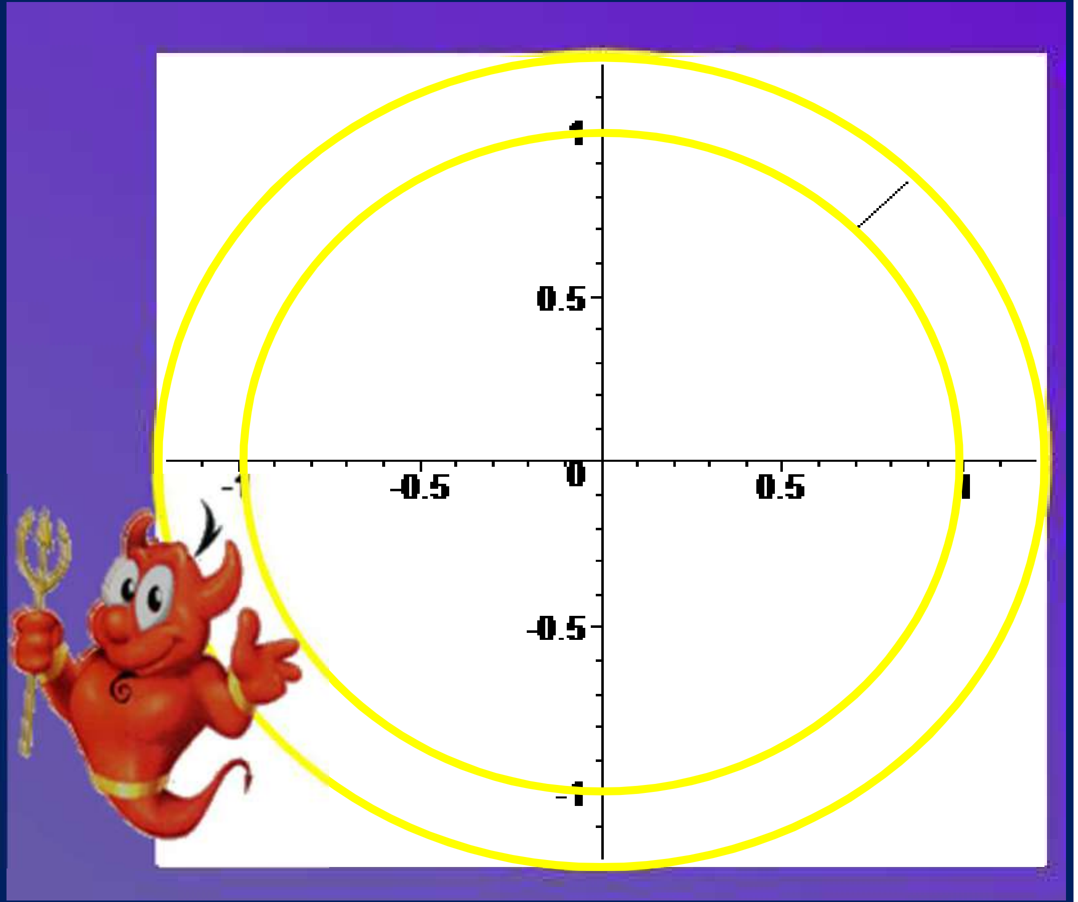
$$T_2 \lesssim T_3 \ll \tau_\Sigma.$$



Perturbation Independent Decay
NOT affected by errors only $\mathcal{H}_\sigma$ dynamics !!!!

rotating mixer

interlayer distance $\Delta R / n > \sqrt{2 D n \tau_o}$ blurring size



Our Tools: NMR - Loschmidt Echoes + MQC

1950 Hahn Echo (or Spin Echo) ↔ reverses one spin interactions

$$\mathcal{H} = -\gamma\hbar h_0 I^z$$

1972 Magic Echo (Rhim Pines Waugh) ↔ reverses many-spin interactions

$$\mathcal{H}_{zz} = \sum_{j<k} b_{jk}\frac{1}{2}\left(1-3\cos^2\theta\right)\left[3I_{jz}I_{kz} - \mathbf{I}_j \cdot \mathbf{I}_k\right]$$

1992 Polarization Echo (Zhang Meier Ernst) ↔ reverses many-spin interactions with local injection and detection of the polarization M_{PE}

Average Hamiltonian
Multiple quantum coherences

$$\begin{aligned}\mathcal{H}_{DQ} &= \frac{1}{3}\left(\mathcal{H}_{yy} - \mathcal{H}_{xx}\right) \\ &= -\frac{1}{2}\sum_{i<j} d_{ij}\left(I_i^+ I_j^+ + I_i^- I_j^-\right)\end{aligned}$$

Gabriela Pertovt, Santa Fe

Main Results

- Strong coupling with the environment → Exponential decay

$$M_{\mathrm{PE}}(t_{\mathrm{R}}) \propto \exp(-t/\tau_{\mathrm{env}})$$

- Weak coupling with the environment → Gaussian decay and $M_{PE}(t)$ scales with the dipolar interaction!!

Dynamically driven decoherence **Non-secular dipolar terms?**

Actually… H_{dip} → -(H_{dip} +Σ)

Then,

$$M(t) = \frac{\langle\Psi|\, e^{+\mathrm{i}\hat{H}t/\hbar} e^{-\mathrm{i}(\hat{H}+\hat{\Sigma})t/\hbar}\, \hat{S}_1^- \, e^{+\mathrm{i}(\hat{H}+\hat{\Sigma})t/\hbar} e^{-\mathrm{i}\hat{H}t/\hbar}\, \hat{S}_1^+ \,|\Psi\rangle}{\langle\Psi|\, \hat{S}_1^- \hat{S}_1^+ \,|\Psi\rangle}$$

Experiments varying Σ give $M_{\mathrm{PE}}(t_{\mathrm{R}}) \propto \exp\left\{(-t_{\mathrm{R}}/\tau_{\phi})^2\right\}$

with t_o independent on Σ !!! – perturbation independent regime!!

A subject that was becoming Hot

news & views

QUANTUM SIMULATION

Probing information scrambling

Quantum information encoded in one of many interacting particles quickly becomes scrambled. A set of tools for tracking this process is on its way.

Monika Schleier-Smith

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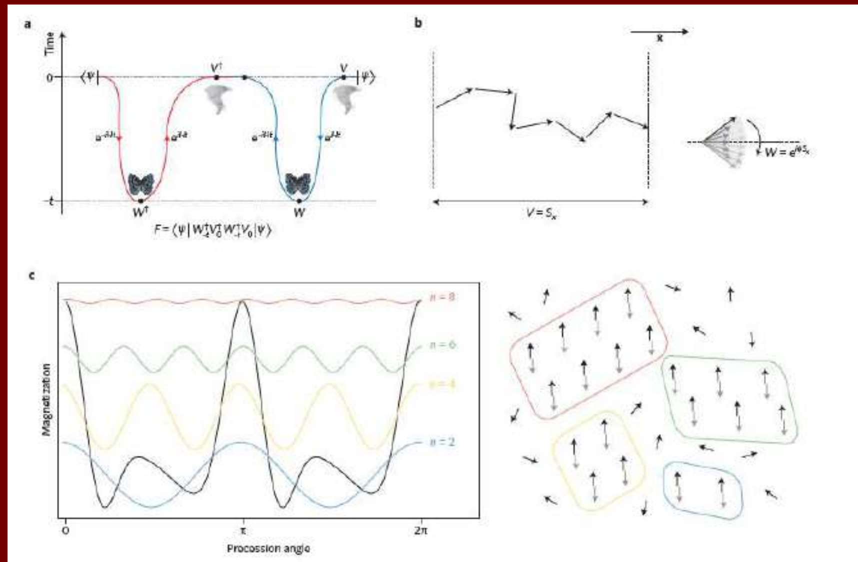
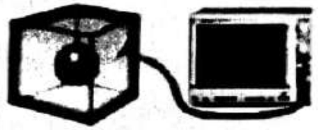
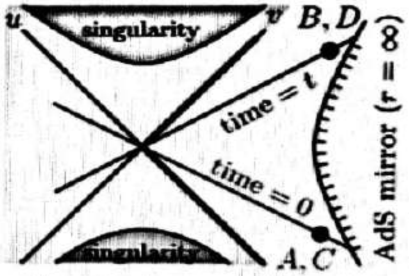


- T'Hooft conjectured that physics within a given volume has an alternative description in terms of some degrees of freedom on its boundary (*holographic principle*).
- Maldacena 1997: a precise variant of this conjecture, *AdS-CFT correspondence* (string theory in anti-de Sitter space vs. certain conformal field theories on its boundary).

- **The OTO correlators related to t'Hooft's effect are calculated in a well-defined setting: black hole in a "box" (actually, the anti-de Sitter space).**
- One considers correlators like $\langle D(t)C(0)B(t)A(0)\rangle$, where the operators A, B, C, D act near the space boundary.





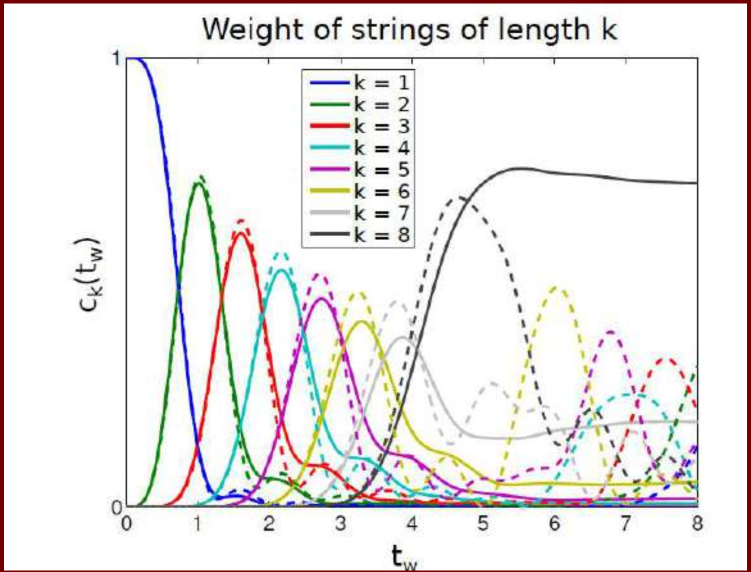
- The growth exponent $\varkappa$ equals the surface gravity, hence $\boxed{\varkappa = 2\pi T}$ due to Hawking's relation.

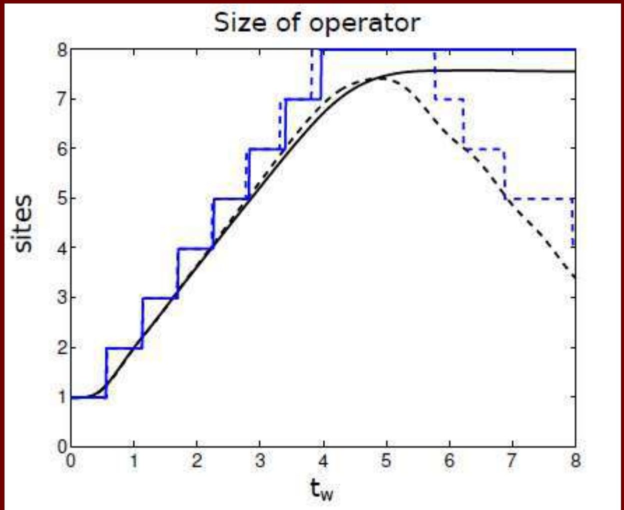
$$H = -\sum_i Z_i Z_{i+1} + g X_i + h Z_i,$$

Localized shocks

Daniel A. Roberts,[a] Douglas Stanford,[b,c] and Leonard Susskind[b]

$$\|[W_x(t_w), W_y]\| \leq c_0 \|W_x\| \; \|W_y\| \; e^{c_1 t_w - c_2 |x-y|},$$



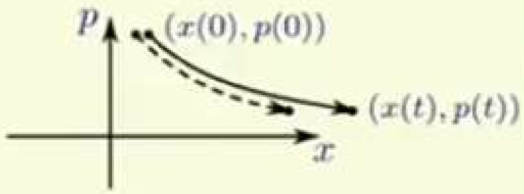


Maldacena/Kitaev: scrambling, chaos, OTOCs and Loschmidt echoes

Out-of-time-order (OTO) correlators

- First discussed by Larkin and Ovchinnikov (1969). Classically, they describe the divergence of phase space trajectories:

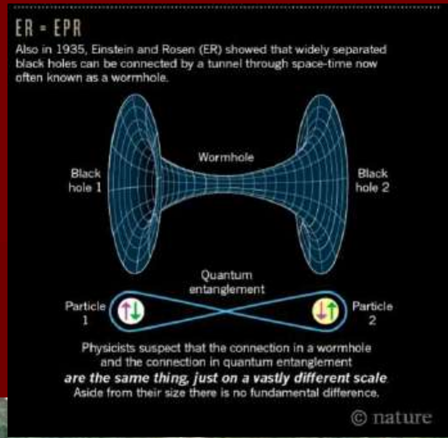
$$[p_j(t), p_k(0)] = i\hbar \frac{\partial p_j(t)}{\partial x_k(0)} \sim \boxed{\hbar e^{\varkappa t}}$$

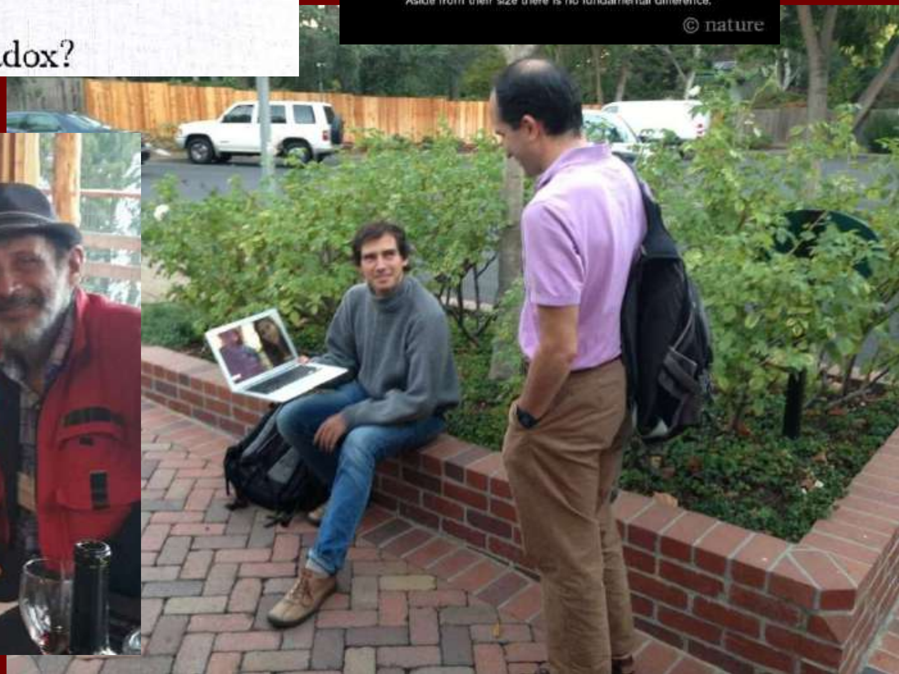


- Related to the Loschmidt echo

- For typical non-integrable systems with all-to-all interactions:
  - At early times (but after the two-point correlators have decayed): $\langle D(t)C(0)B(t)A(0)\rangle - \langle DB\rangle\langle CA\rangle \sim \frac{1}{N} e^{\varkappa t}$
  - At later times, the exponential growth saturates.
  - $\boxed{\varkappa \leqslant 2\pi T}$ (Shenker, Stanford, and Maldacena, 2015).

- Some properties of black holes and the SYK model are similar. In particular, the OTO correlators grow in time at the highest possible rate, $\varkappa = 2\pi T$.
- The model has some collective mode that is similar to a certain gravitational mode (t'Hooft's shock waves). Is there a more close relation, like the AdS-CFT correspondence? ($d+1$-dimensional anti-de Sitter space vs. d-dimensional conformal field theory.)
- What about the black hole information paradox?





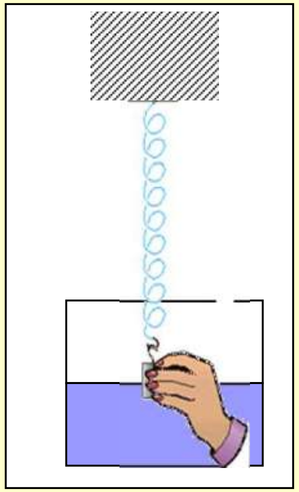
Loschmidt Echo: OTOCs are a particular case where perturbation is local in time and space

$$M(t) = \frac{\langle\Psi| e^{+\mathrm{i}\hat{H}t/\hbar} e^{-\mathrm{i}(\hat{H}+\hat{\Sigma})t/\hbar} \hat{S}_1^- e^{+\mathrm{i}(\hat{H}+\hat{\Sigma})t/\hbar} e^{-\mathrm{i}\hat{H}t/\hbar} \hat{S}_1^+ |\Psi\rangle}{\langle\Psi| \hat{S}_1^- \hat{S}_1^+ |\Psi\rangle}$$

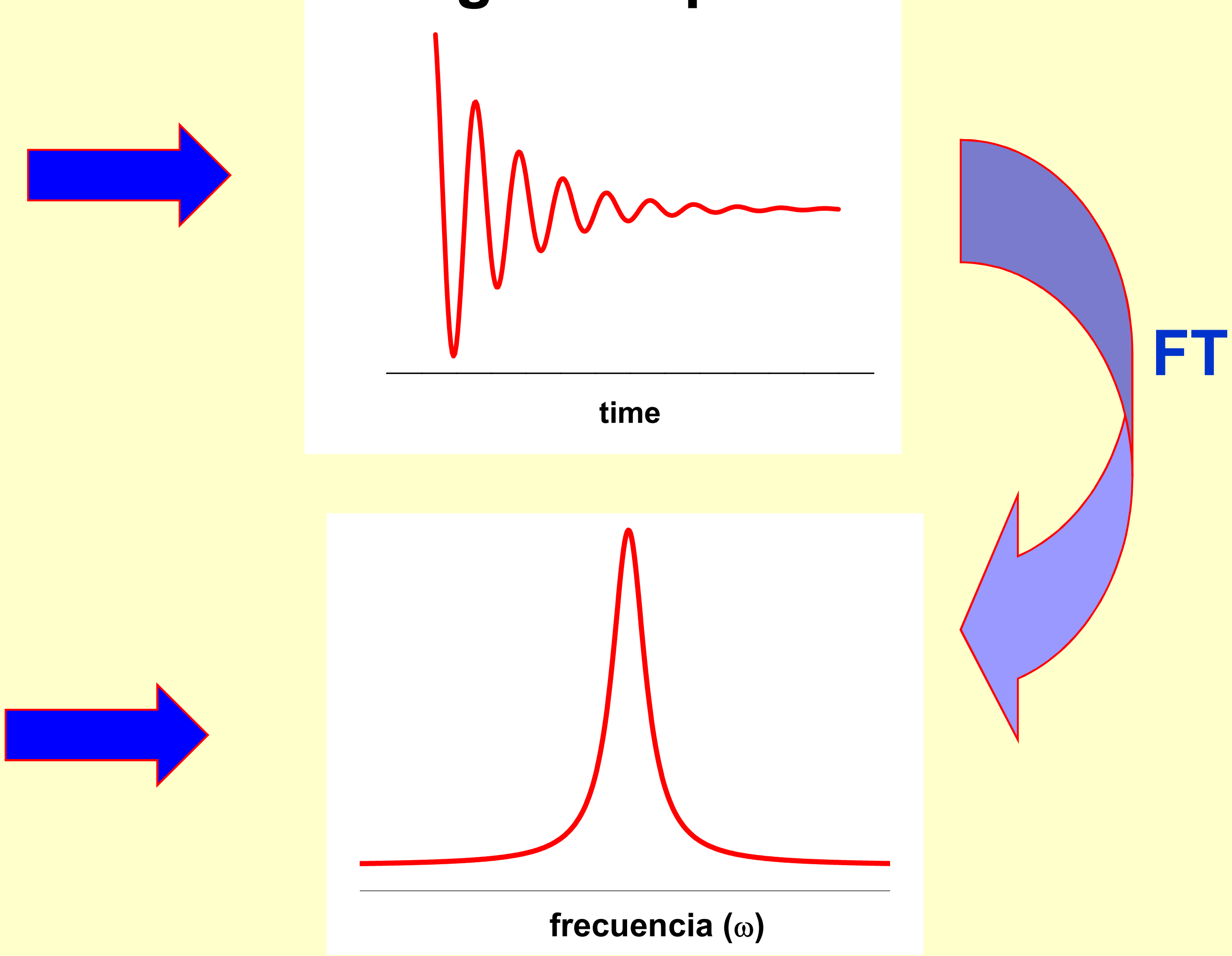
$$\begin{aligned} M(t) &= \frac{\langle\Psi| e^{+\mathrm{i}\hat{H}t/\hbar} \hat{B}^+ e^{-\mathrm{i}\hat{H}t/\hbar} \hat{S}_1^- e^{+\mathrm{i}\hat{H}t/\hbar} \hat{B} e^{-\mathrm{i}\hat{H}t/\hbar} \hat{S}_1^+ |\Psi\rangle}{\langle\Psi| \hat{S}_1^- \hat{S}_1^+ |\Psi\rangle} \\ &= \langle\Psi| \hat{B}^\dagger(t)\hat{A}(0)\hat{B}(t)\hat{A}^\dagger(0) |\Psi\rangle = F(t) \end{aligned}$$

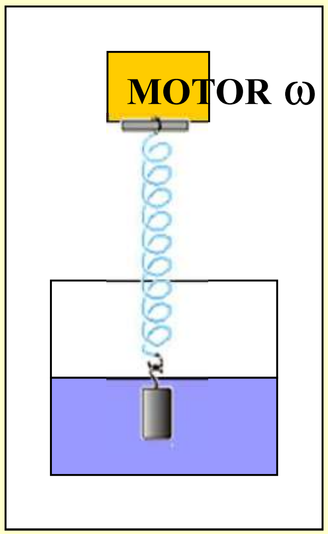
Pulsed NMR (Ernst) vs. continuous wave (CW)

Saves time!!



and allows the design Floquet Hamiltonians



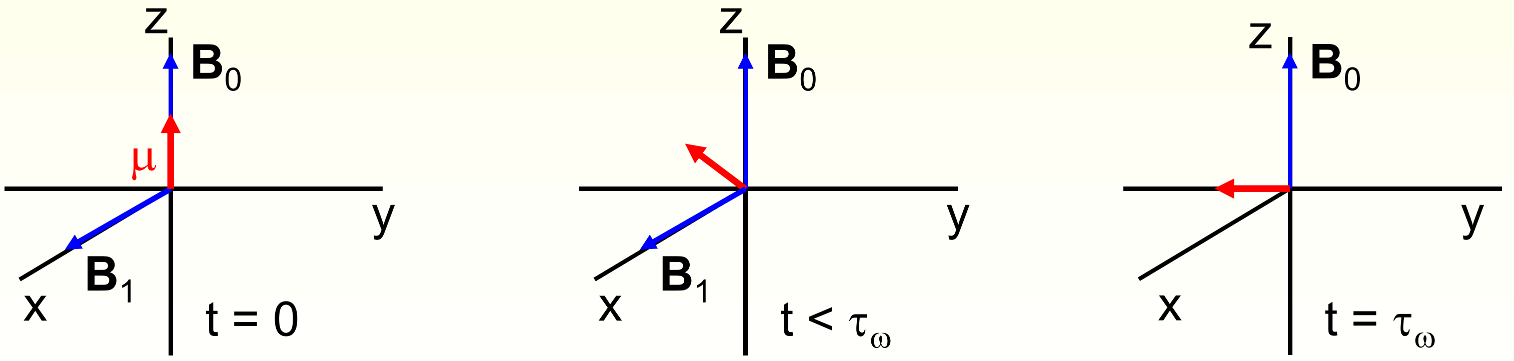


A pulse

$$\omega = \omega_0 \equiv \gamma B_0$$ → Resonance condition

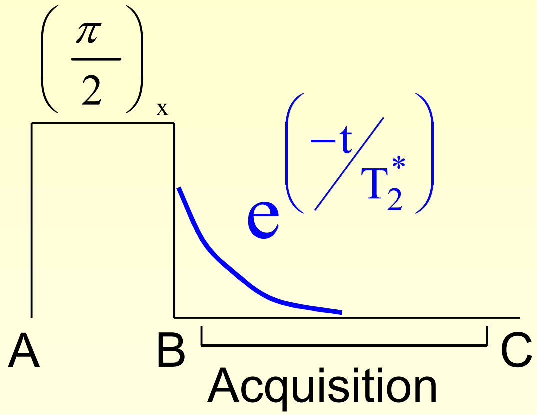
$$B_{ef} = B_1 \Rightarrow \Omega = \gamma\, B_1$$ → The magnetic moment precesses around $\mathbf{B}_1$.

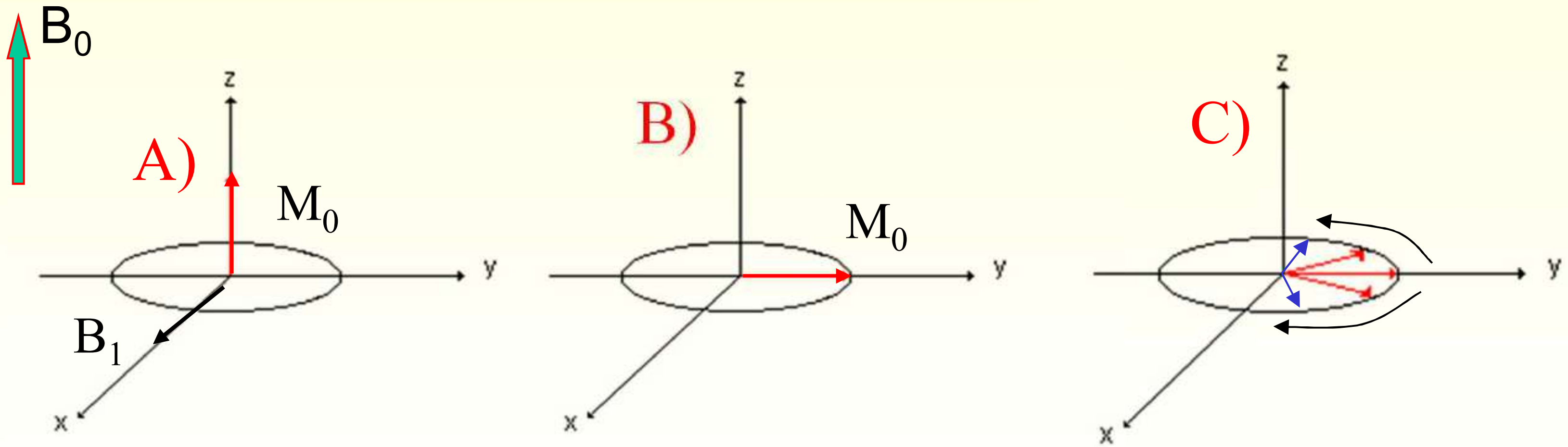
The rf field $\mathbf{B}_1 = B_1\mathbf{i}$ is turned on during a time τ_ω. The magnetic moment precesses in the z-y plane until it forms an angle $\theta = \gamma B_1 \tau_\omega$ with $\mathbf{B}_0$.



$$\theta = \gamma B_1 \tau_\omega = \frac{\pi}{2}$$ → Represents the tilting angle

The FID (Free Induction Decay)





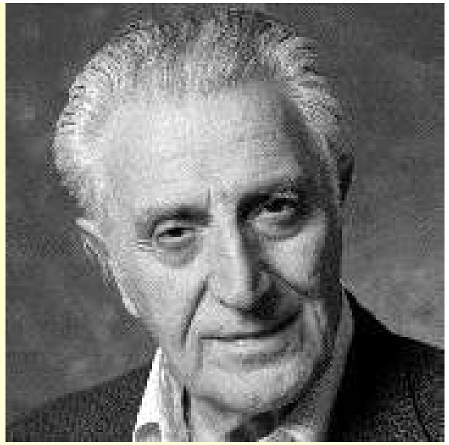
The FID decays because of the spin-spin interactions, but also as a consequence of the dephasing among the spins due to different electronic shieldings, B_0 inhomogeneities, etc. It decays in a characteristic time T_2^* .

PHYSICAL REVIEW VOLUME 80, NUMBER 4 NOVEMBER 15, 1950

Spin Echoes*†

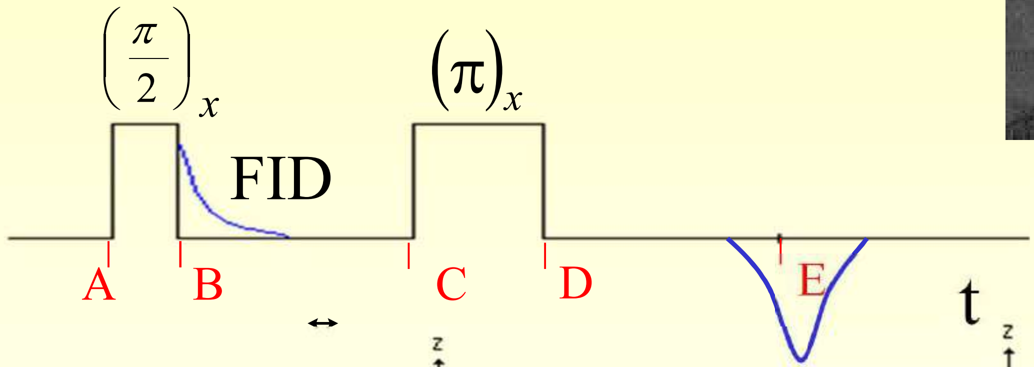
E. L. HAHN‡
Physics Department, University of Illinois, Urbana, Illinois

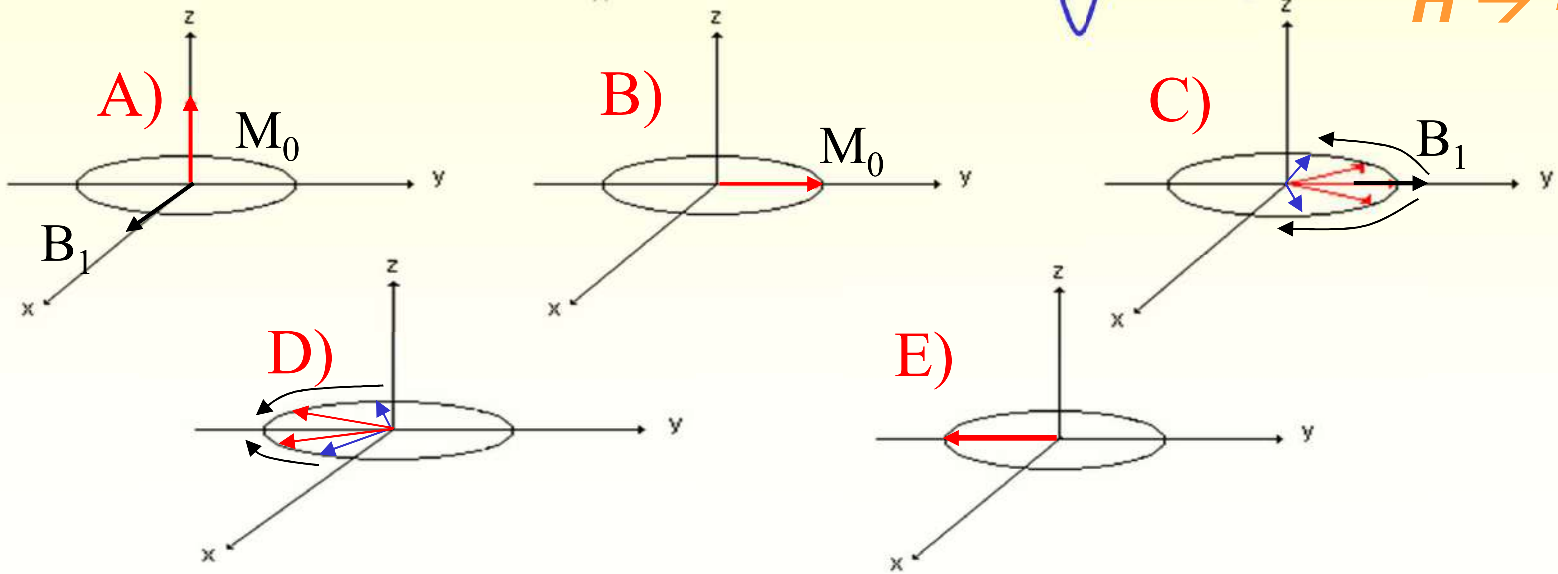
(Received May 22, 1950)



"The Wizard"

H → -H





This sequence reverses the field inhomogeneities. The magnetization decays because of dipolar interactions in a characteristic time "T_2".

Understanding "scrambling"

$$\rho(0) \propto \sum_i I_i^z$$

$$H_{\rm DQ} = -\frac{1}{2}\sum_{i<j} d_{ij}(I_i^+ I_j^+ + I_i^- I_j^-)$$

$$\rho(t+\delta t) = \exp(-i\delta t\, H/\hbar)\rho(t)\exp(i\delta t\, H/\hbar)$$
$$\cong \rho(t) - i\frac{\delta t}{\hbar}[H,\rho(t)]\,.$$

$$[I_j^+ I_0^+, I_0^z] = -I_j^+ I_0^+$$

$$[I_k^- I_j^-, -I_j^+ I_0^+] = 2 I_k^- I_j^z I_0^+$$

$$[I_l^+ I_j^+, [I_k^- I_j^-, -I_j^+ I_0^+]] = [I_l^+ I_j^+, 2 I_k^- I_j^z I_0^+]$$
$$= -2 I_l^+ I_k^- I_j^+ I_0^+\,.$$

$$\cdots I_m^+ I_l^+ I_k^z I_j^+ I_0^+$$

$$\rho(t) = \begin{array}{c|cccc} & |{++}\rangle & |{+-}\rangle & |{-+}\rangle & |{--}\rangle \\ \hline \langle{++}| & \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\ \langle{+-}| & \rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} \\ \langle{-+}| & \rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} \\ \langle{--}| & \rho_{41} & \rho_{42} & \rho_{43} & \rho_{44} \end{array}$$

The non-diagonal elements

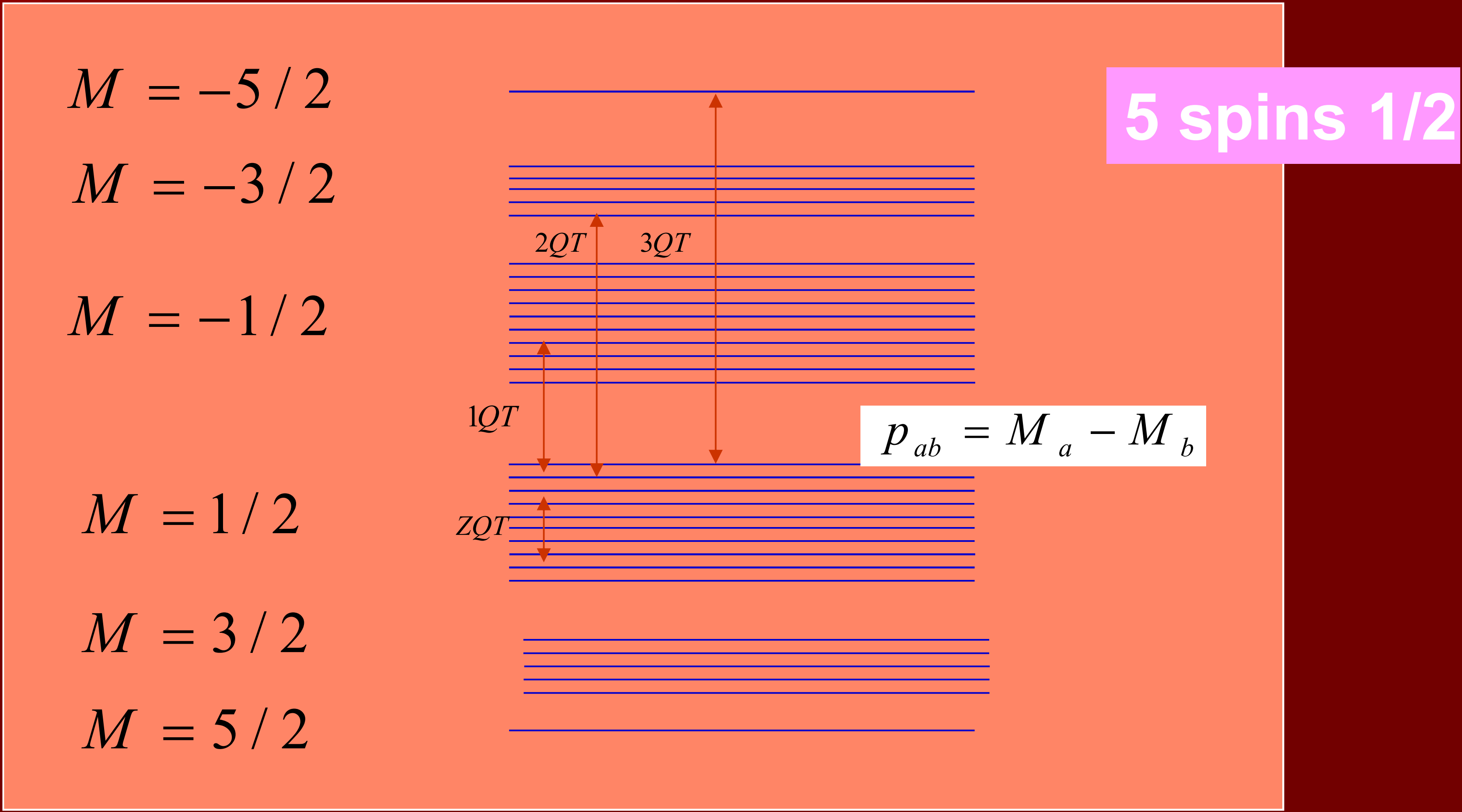
Populations

$$\boxed{\rho_{rs} = \langle r|\rho(t)|s\rangle = \overline{c_r(t)c_s^*(t)}}$$

represent coherent superpositions of eigenstates in the wave function

$$\boxed{c_r(t)|r\rangle + c_s(t)|s\rangle}$$

Can we see in more detail how the coherence is lost?
Can we measure highly correlated many spin states?

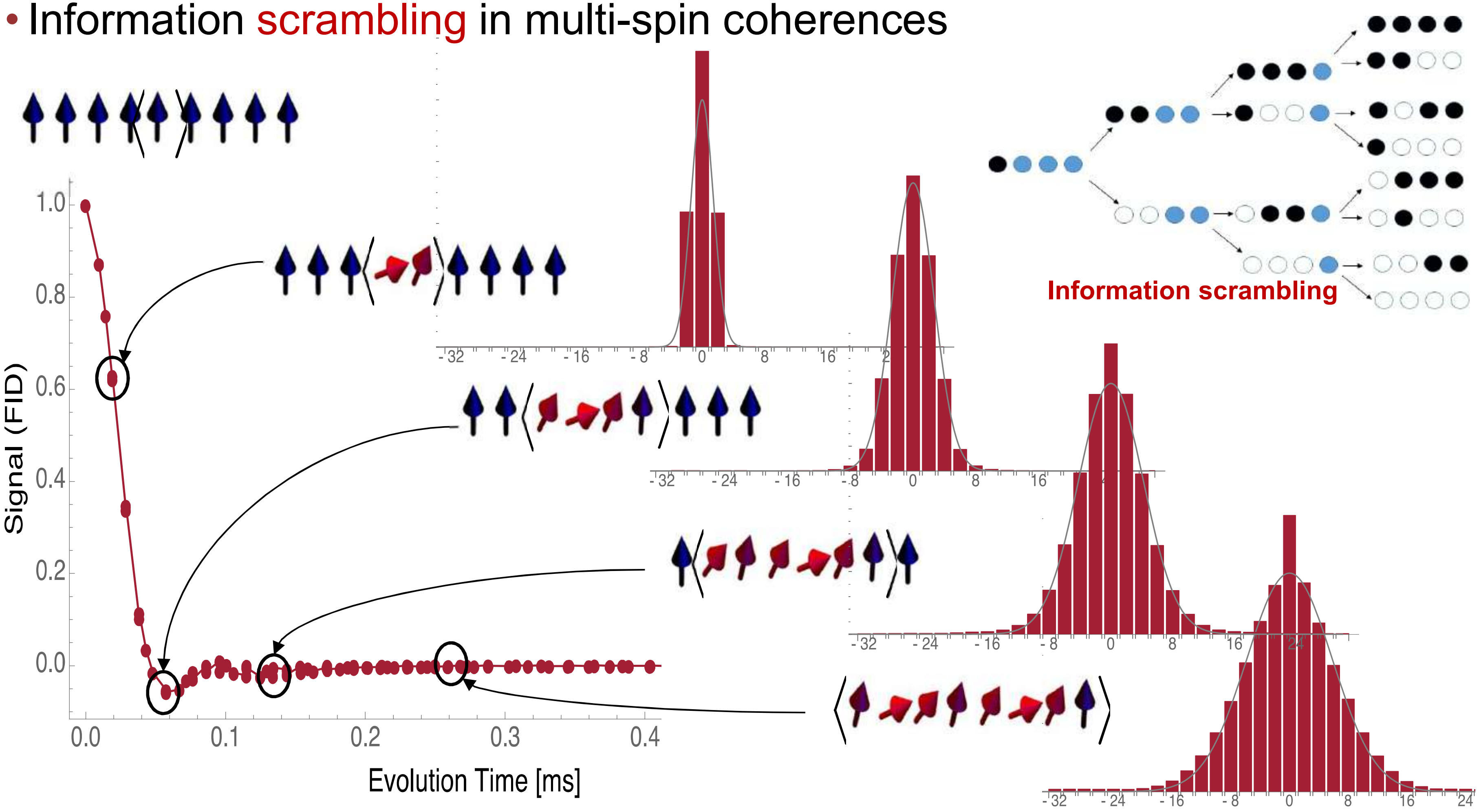


Energy levels in a multi-spin system at high magnetic field. Coherence is an extension of the concept of transversal magnetization. The coherences can be associated to transitions between two states.

$$\hat{H}_d^Z = \sum_{i<j} d_{ij}(2I_i^Z I_j^Z - \tfrac{1}{2}[I_i^+ I_j^- + I_i^- I_j^+] - \tfrac{3}{2}[I_i^+ I_j^+ + I_i^- I_j^-])$$

"Decay" of a closed system

- Information scrambling in multi-spin coherences



Multiple Quantum Coherences are Out of Time Order Commutators

$$\hat{A} = \hat{I}_1^z$$
$$\hat{B} = \exp[\mathrm{i}\varphi\, \hat{I}^z]$$
$$\hat{I}^z = \sum_{i(\text{spins})} \hat{I}_i^z$$

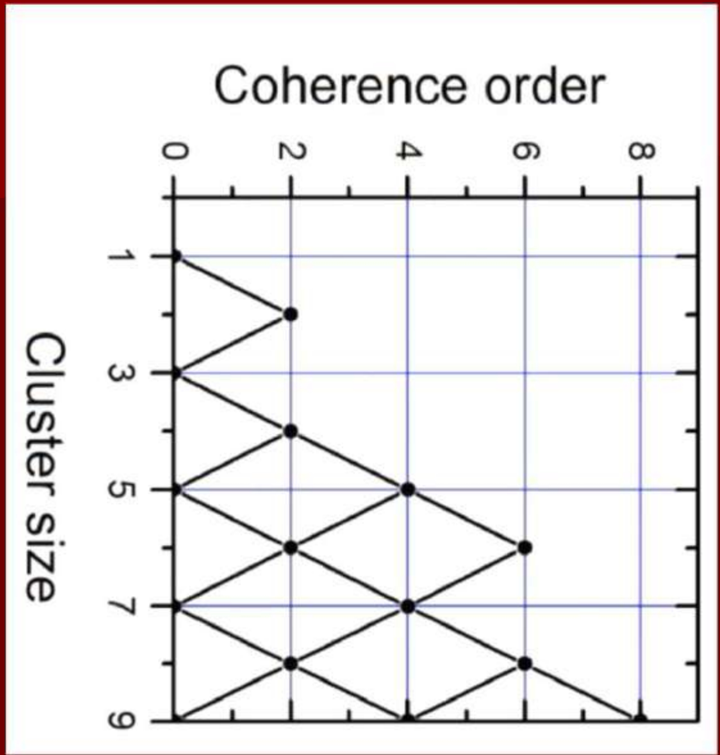
$$S_\varphi(t) = G(t,\varphi) = \frac{\langle\Psi|\exp\left[-\mathrm{i}\hat{S}^z(t)\right]\hat{S}^z\exp\left[\mathrm{i}\varphi\hat{S}^z(t)\right]\hat{S}_1^z|\Psi\rangle}{} \simeq 1 - \frac{1}{2}\varphi^2\frac{\langle\Psi|\left(\mathrm{i}\left[\hat{S}^z(t),\hat{S}_1^z(0)\right]\right)^2|\Psi\rangle}{\langle\Psi|\hat{S}_1^z\hat{S}_1^z|\Psi\rangle}$$

$$\varphi_n = n2\pi/Q$$
$$Q = 2^s$$
$$S_{\varphi_n}^\delta(t)$$

$$S_q(t) = \sum_{n=1}^{Q} e^{-\mathrm{i}q\varphi_n} S_{\varphi_n}(t)$$

$$m_2(t) = \sum_q q^2 S_q(t) = \frac{\langle\Psi|\left(\mathrm{i}\left[\hat{S}^z(t),\hat{S}_1^z(0)\right]\right)^2|\Psi\rangle}{\langle\Psi|\hat{S}_1^z\hat{S}_1^z|\Psi\rangle} \simeq t^2 M_2 + \ldots$$

measuring scrambling from the OTOC

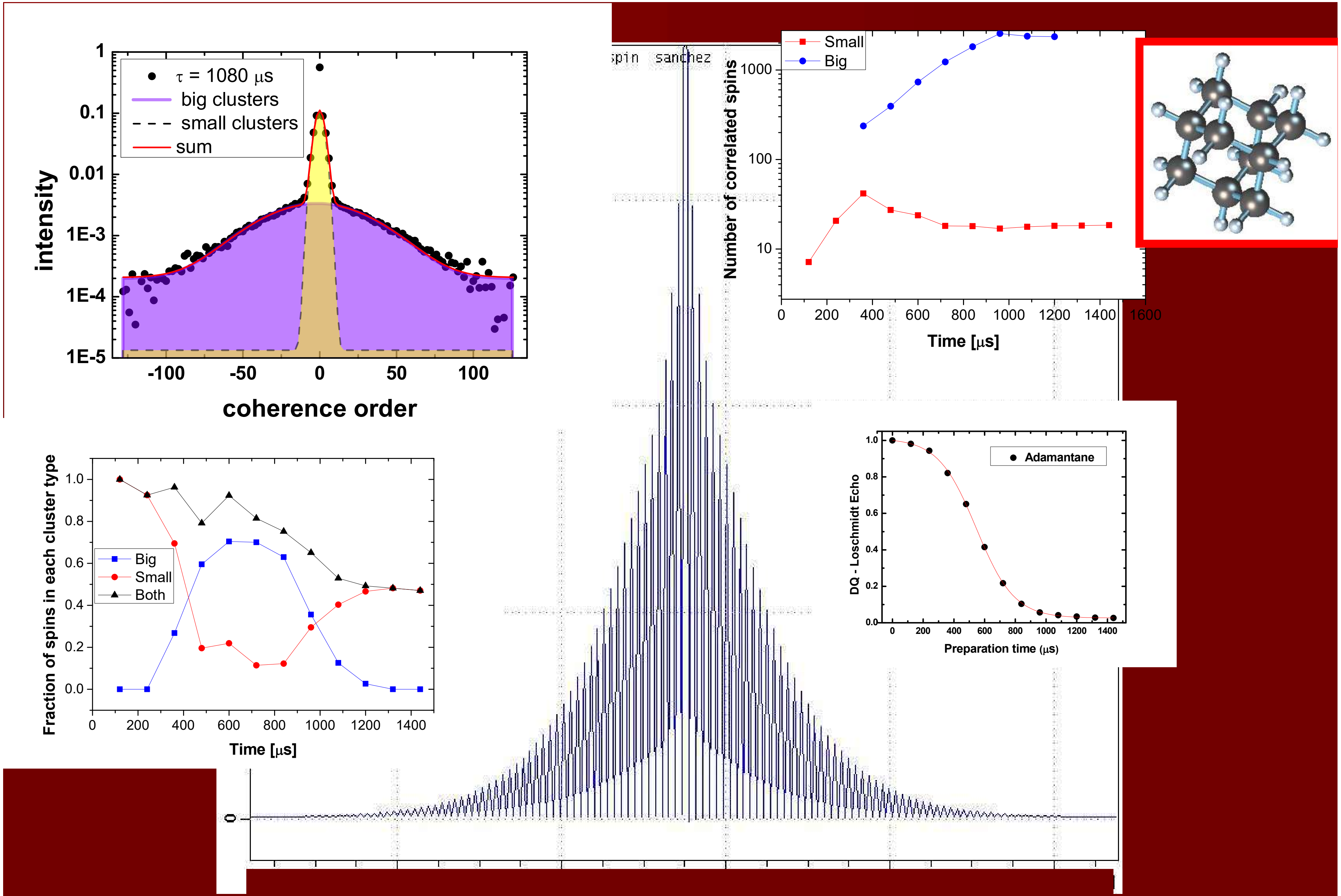


$$S_\varphi = \langle I_z \Phi(t) I_z \Phi(t) \rangle$$

$$\Phi = e^{-i\varphi I_z}$$

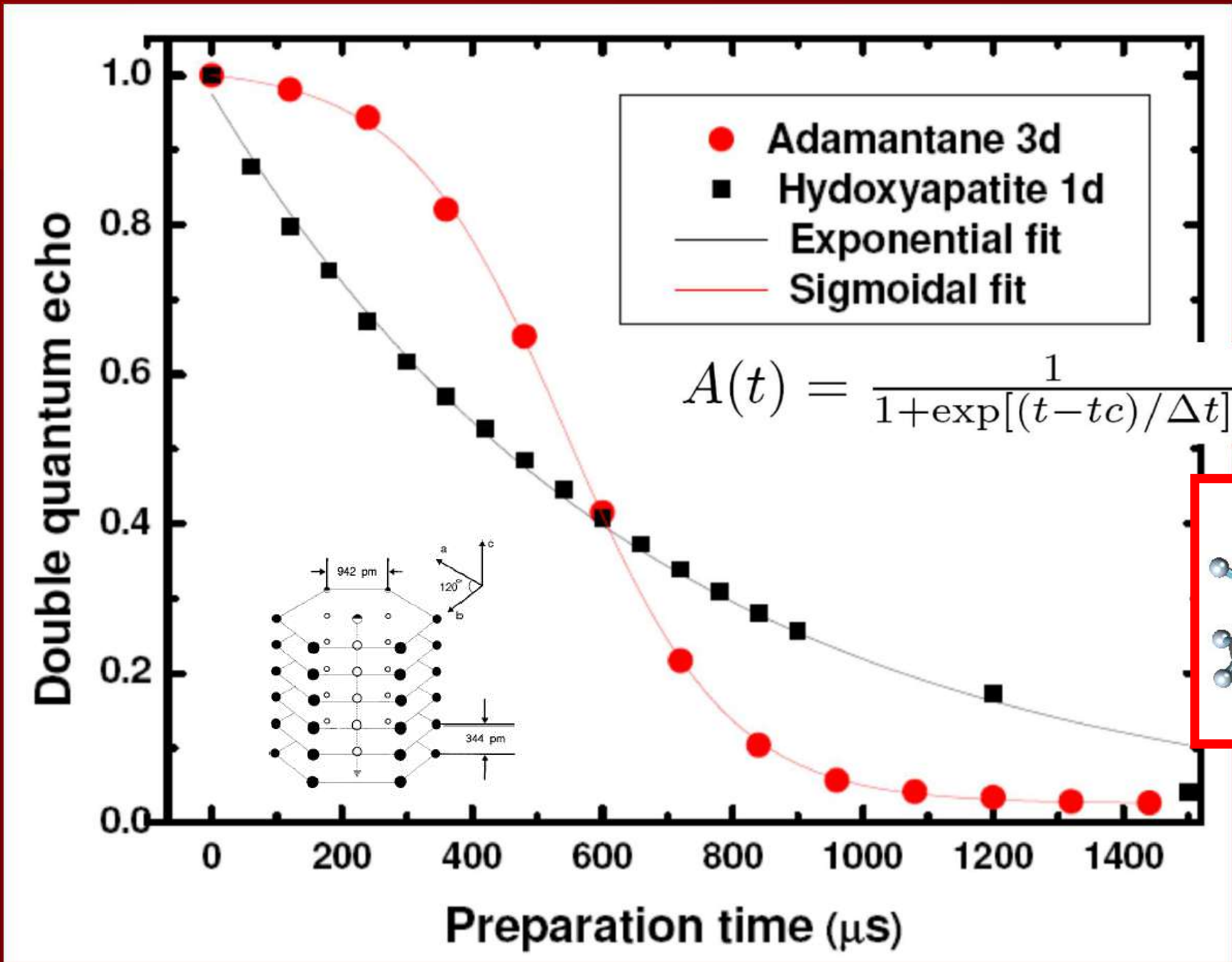
$$S(q, N(t)) \propto e^{-q^2/N^2(t)}$$

$$\binom{2K}{K-n} = \frac{(2K)!}{(K-n)!(K+n)!}$$

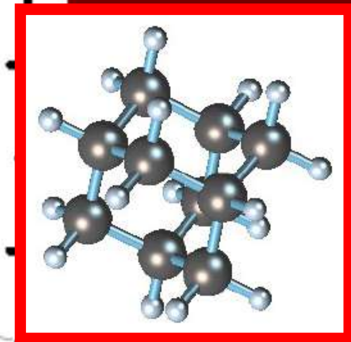


Coherence order

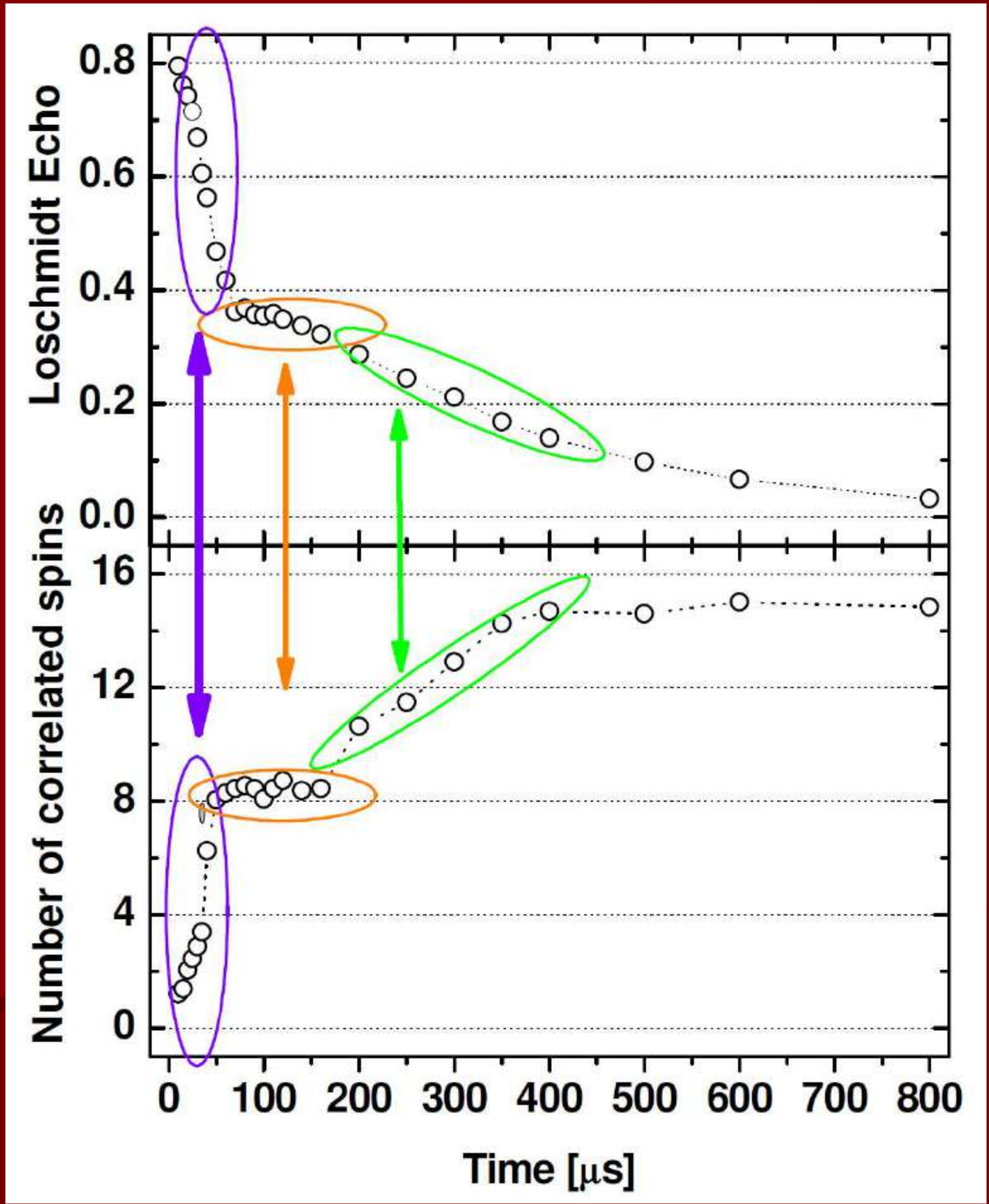
HOW MANY SPINS CAN BECOME CORRELATED?



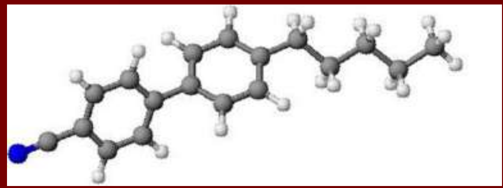
$$A(t) = \frac{1}{1+\exp[(t-tc)/\Delta t]}$$



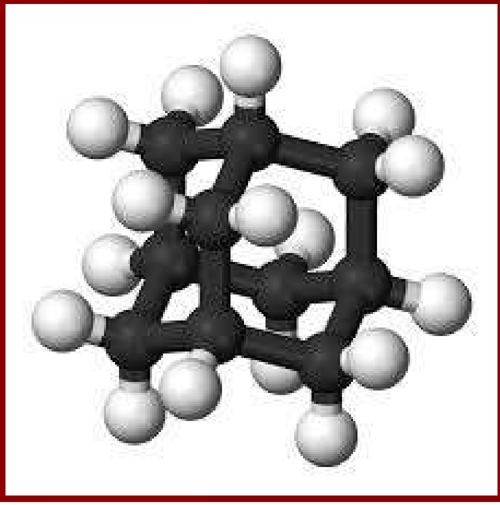
Last findings!



The Loschmidt Echo decays proportionally to the rate of CHANGE in the cluster formation



adamantane: scaled dipolar Hamiltonian



Journal of Magnetic Resonance 281 (2017) 75–81

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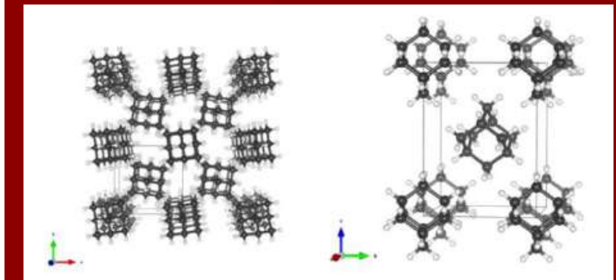
Evolution of multiple quantum coherences with scaled dipolar Hamiltonian

Claudia M. Sánchez [a], Lisandro Buljubasich [a,b], Horacio M. Pastawski [a,b], Ana K. Chattah [a,b,*]



THE JOURNAL OF CHEMICAL PHYSICS 143, 164308 (2015)

Experimental quantification of decoherence via the Loschmidt echo in a many spin system with scaled dipolar Hamiltonians

Lisandro Buljubasich,[1,2] Claudia M. Sánchez,[2] Axel D. Dente,[1,2] Patricia R. Levstein,[1,2,a)] Ana K. Chattah,[1,2,b)] and Horacio M. Pastawski[1,2]

PHYSICAL REVIEW LETTERS 124, 030601 (2020)

Perturbation Independent Decay of the Loschmidt Echo in a Many-Body System

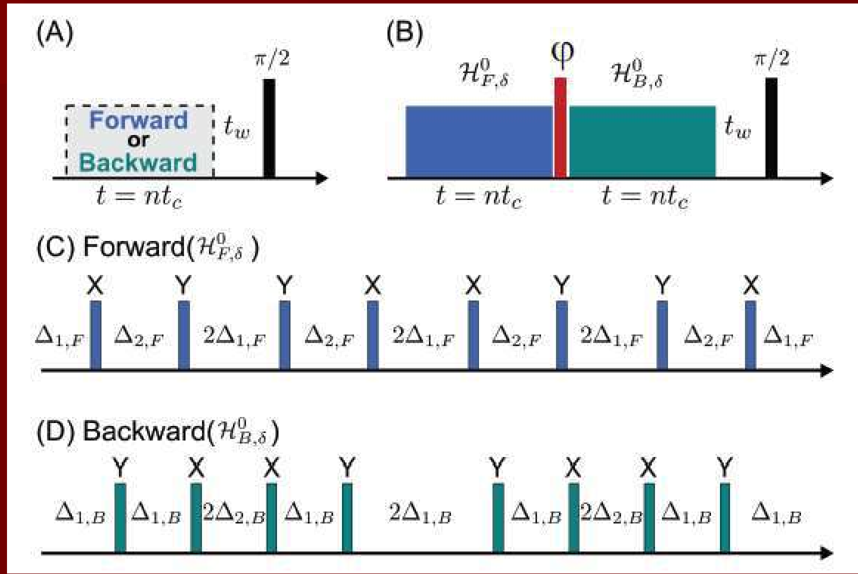
C. M. Sánchez,[1] A. K. Chattah,[1,2*] K. X. Wei,[3,4] L. Buljubasich,[1,2] P. Cappellaro,[5,4] and H. M. Pastawski[1,2]

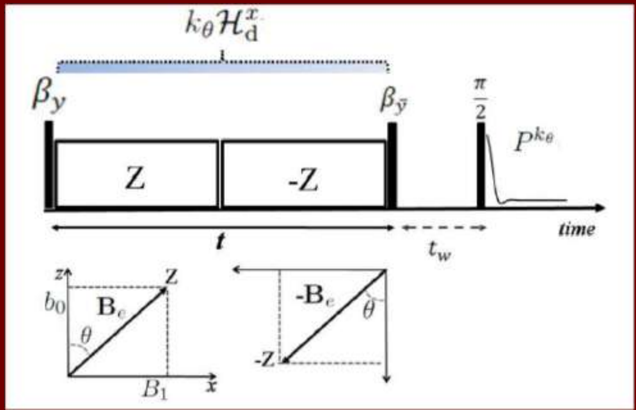
Emergent decoherence induced by quantum chaos in a many-body system. A Loschmidt echo observation through NMR

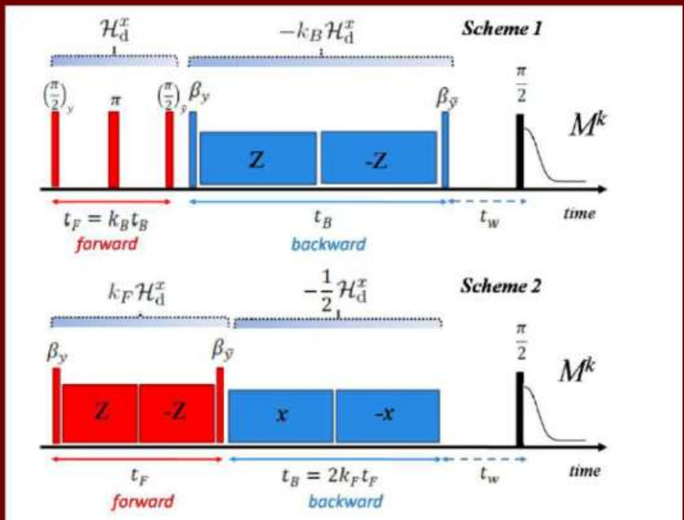
Claudia M. Sánchez, Ana K. Chattah,* and Horacio M. Pastawski*

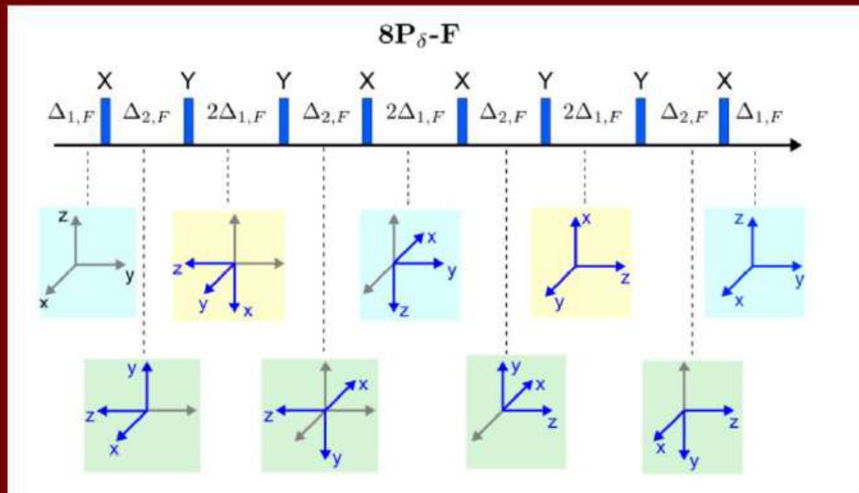
Synthetic Floquet Hamiltonians: rf solid irradiation OR pulsed sequences.

$$\hat{H}_d^Z = \sum_{i<j} d_{ij}\left(2I_i^Z I_j^Z - \frac{1}{2}[I_i^+ I_j^- + I_i^- I_j^+] - \frac{3}{2}[I_i^+ I_j^+ + I_i^- I_j^-]\right)$$





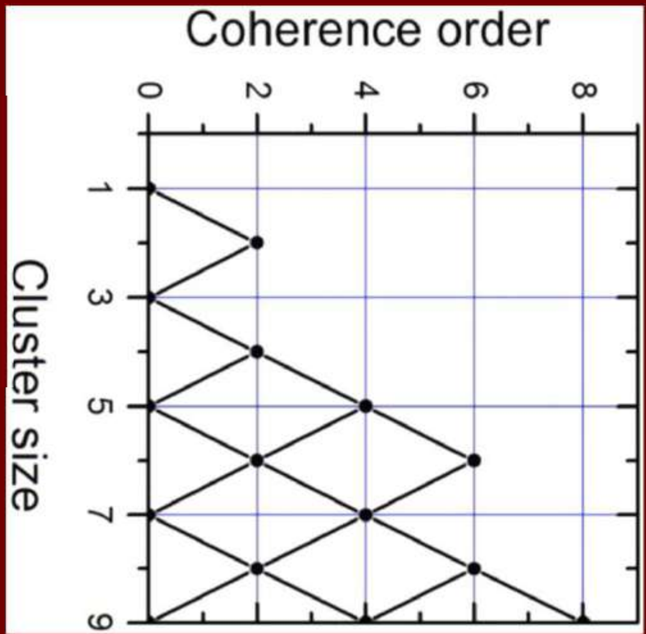




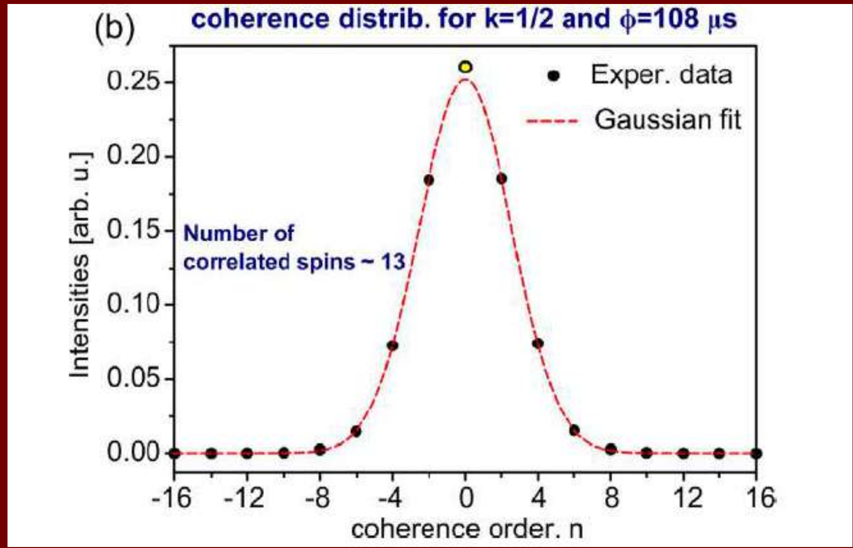
Multiple Quantum Coherences or OTOC's

Hilbert space index →

←time



Galton's board



$$S_{\varphi}^{\delta}(t) = \sum_{n} \exp(in\varphi) S_{n}^{\delta}(\tau).$$

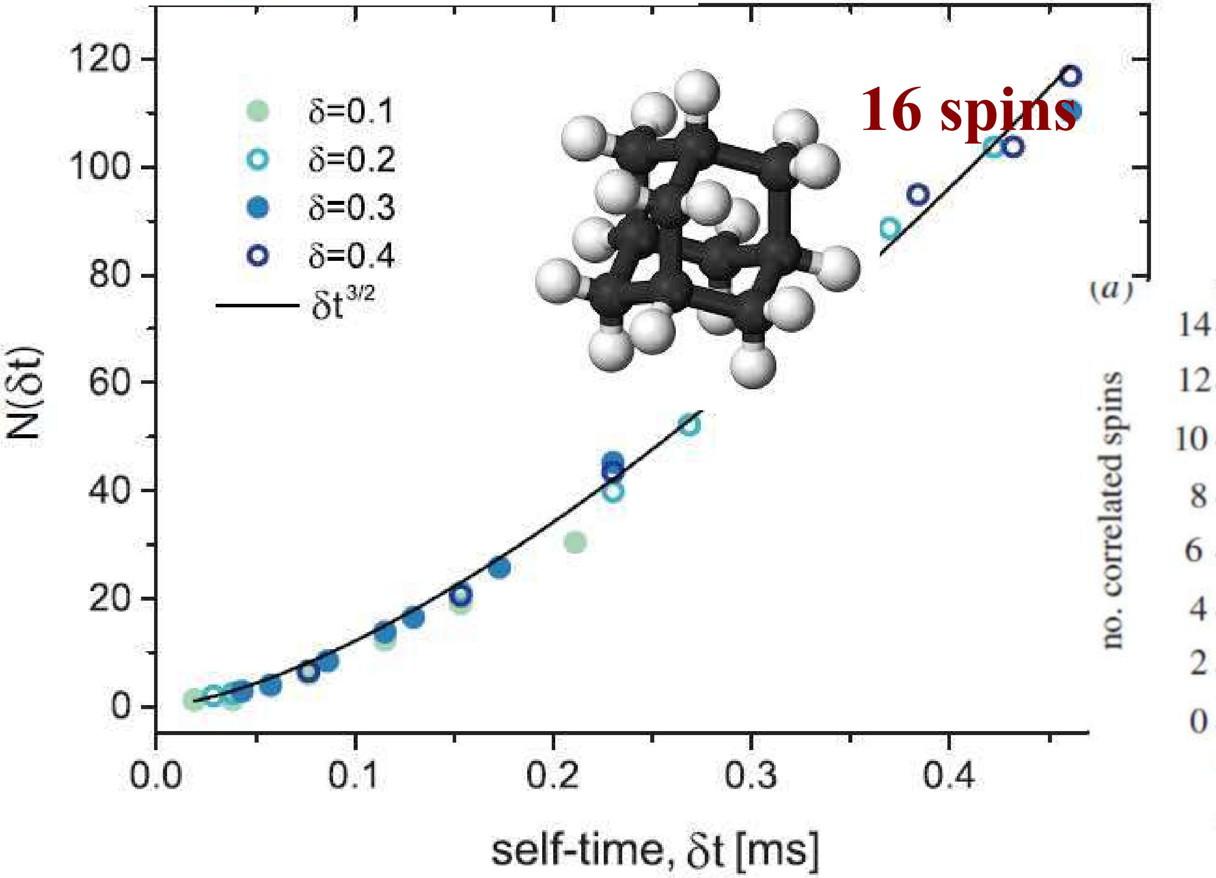
$$LE^{\delta}(t) = \sum_{n} S_{n}^{\delta}(t) = S_{\varphi=0}^{\delta}(t)$$

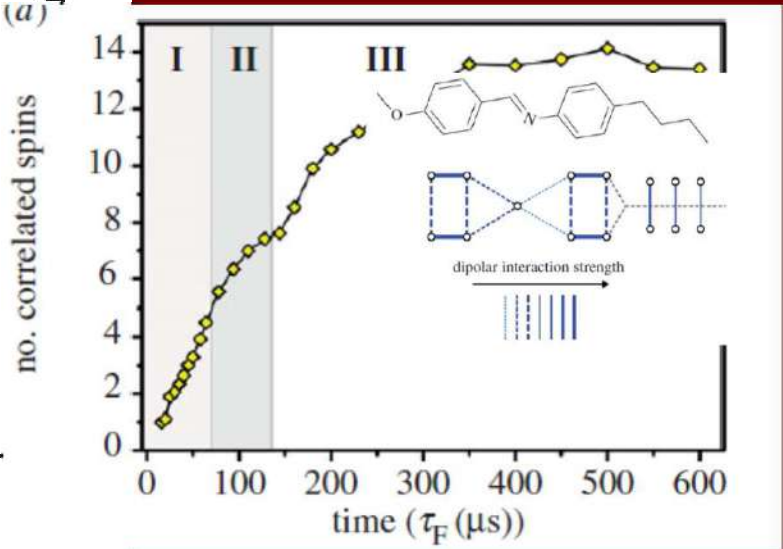
$$S(n, N(t)) \propto e^{-n^2/N(t)}.$$

scrambling through OTOCs

$$N(\delta t) \approx \delta t^{3/2}$$

2^{120} states





Sánchez et al. PTRSA2016

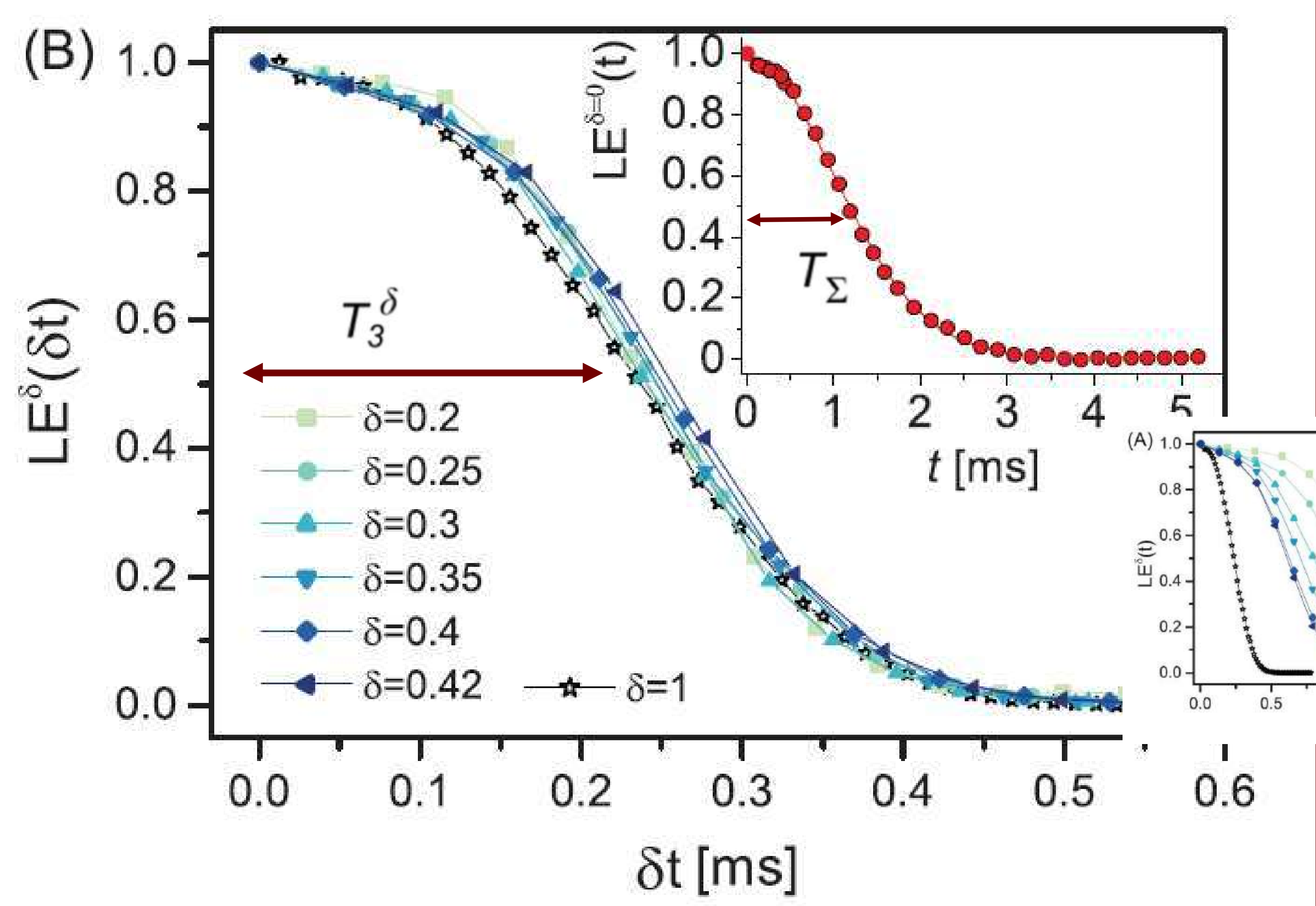
PHYSICAL REVIEW LETTERS **124**, 030601 (2020)

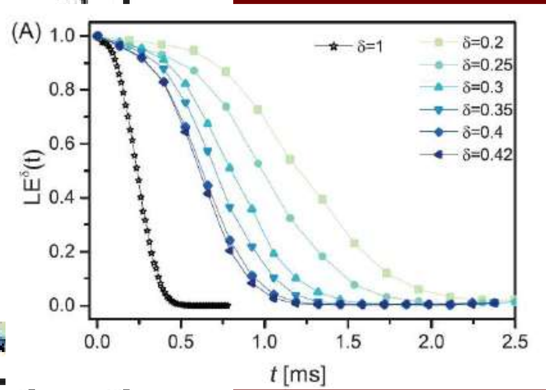
Perturbation Independent Decay of the Loschmidt Echo in a Many-Body System

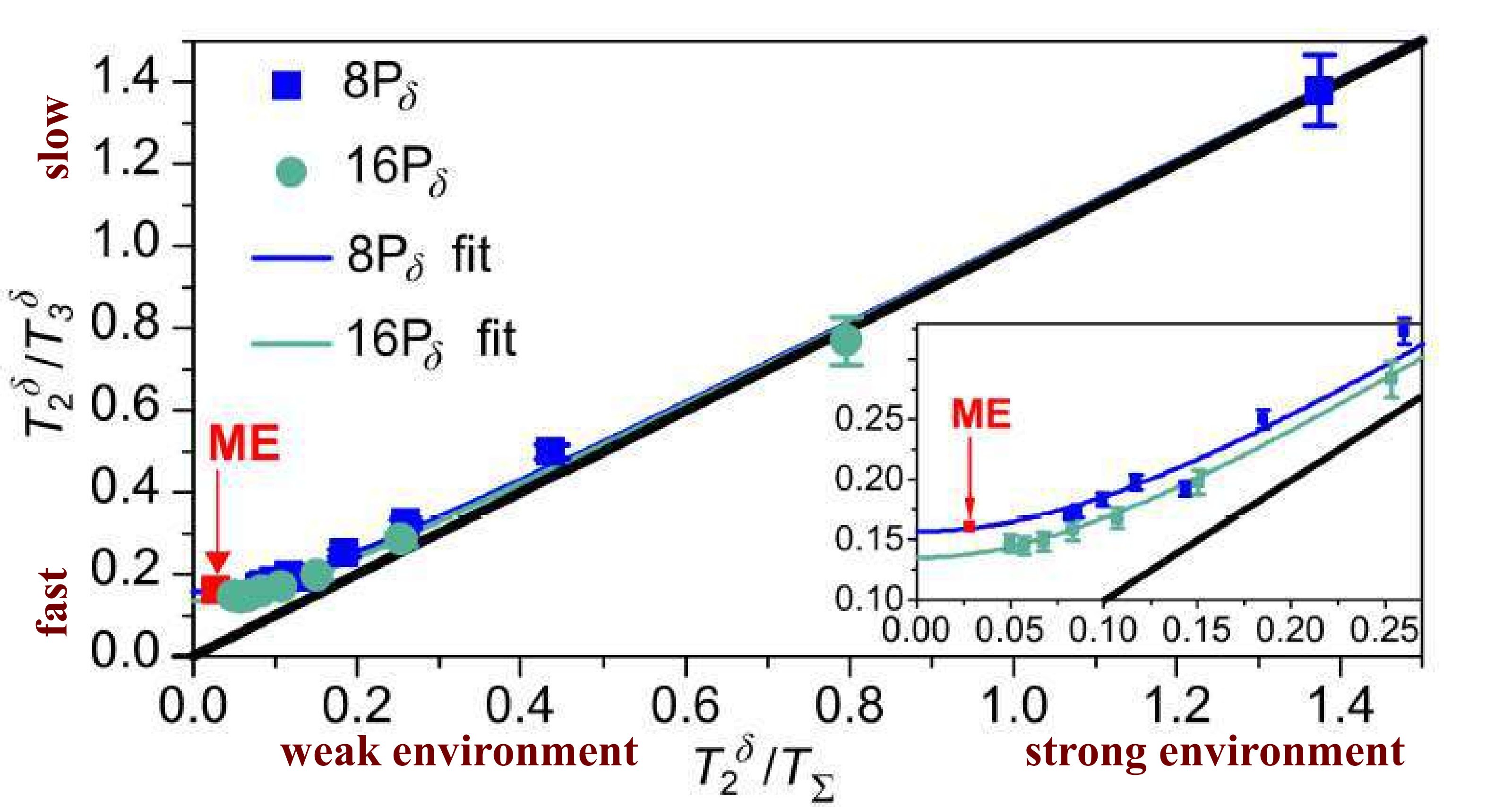
C. M. Sánchez,[1] A. K. Chattah,[1,2*] K. X. Wei,[3,4] L. Buljubasich,[1,2] P. Cappellaro,[3,4] and H. M. Pastawski[1,2]

Loschmidt echo decay

$$P(t) = \exp\left(2\frac{\Gamma^2}{\sigma^2} - 2\sqrt{\left(\frac{\Gamma^4}{\sigma^4} + \Gamma^2 t^2\right)}\right)$$







Intrinsic Decoherence

→Central Hypothesis of Irrevesibility

we observe the **thermodynamic limit**

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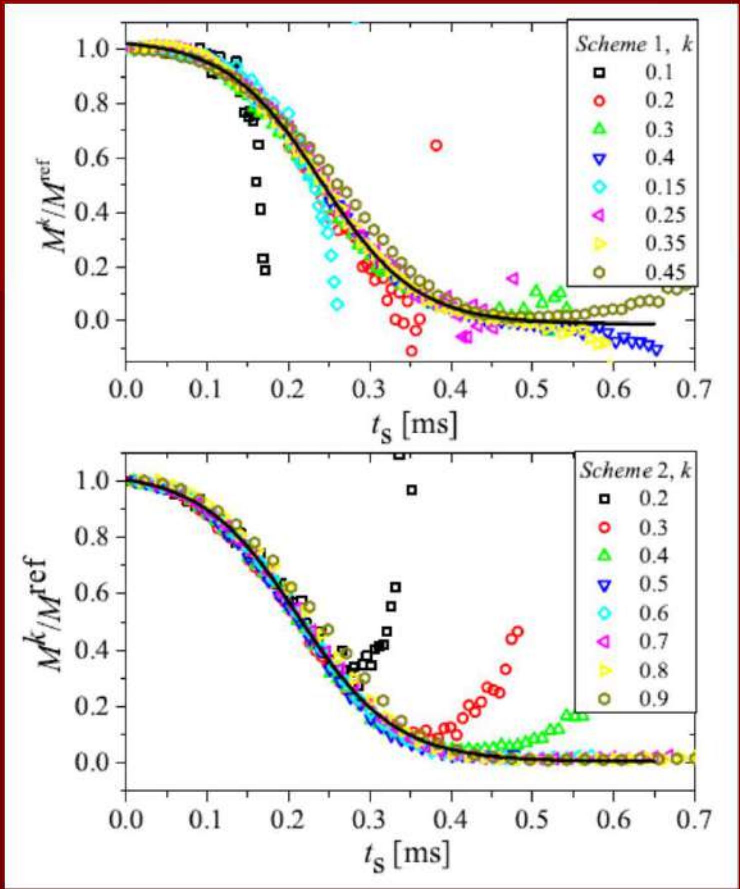
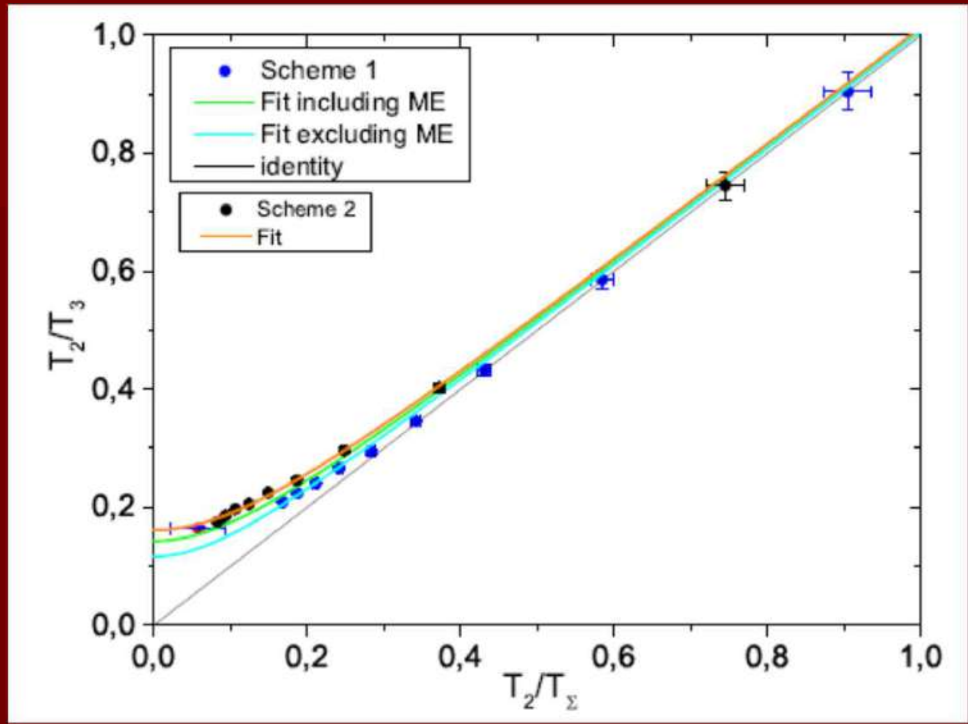
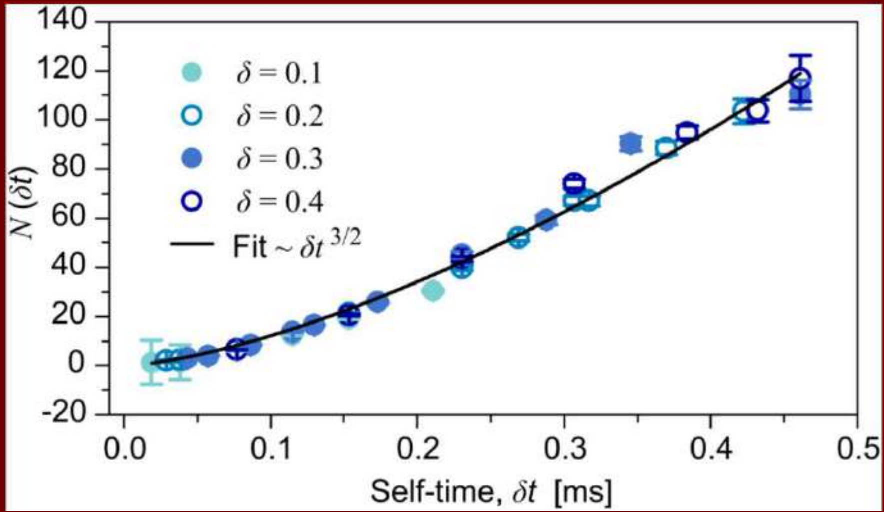




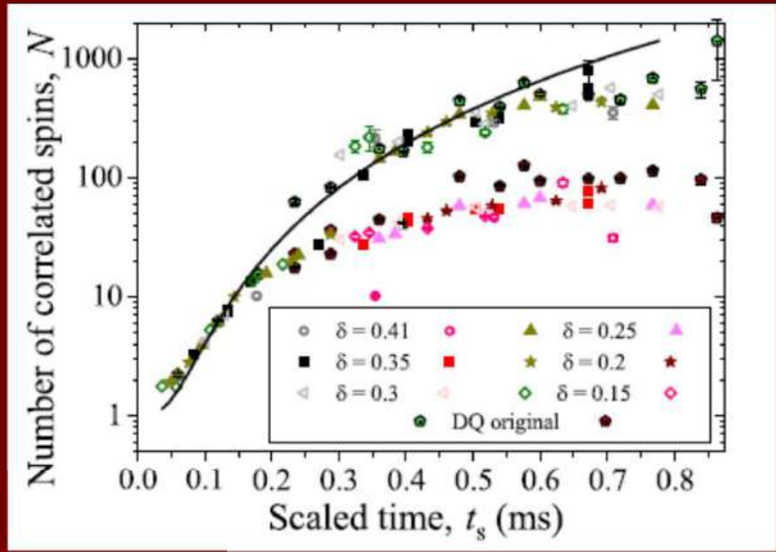
Sigmoid/logistic decay: signature of chaos
Gross-Pitaewsky and hard disk gas

R. Pinto, E. Medina, and H. M. Pastawski, BAPS March Meeting **2004**, J22.001 (2004), URL `http://flux.aps.org/meetings/YR04/MAR04/baps/abs/S3420001.html`.
G. Manfredi and P.-A. Hervieux, Phys. Rev. Lett. **100**, 050405 (2008).

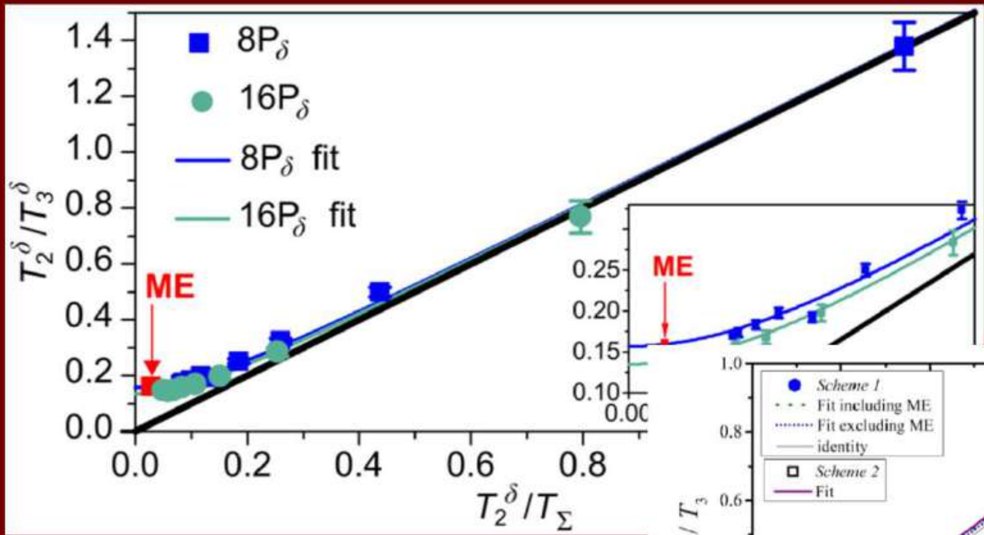
XYX→diffusive spreading

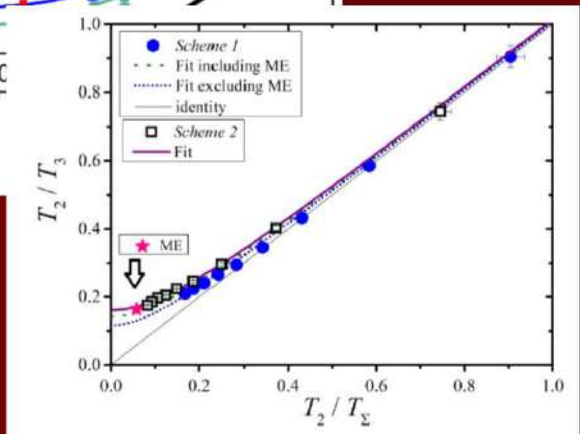


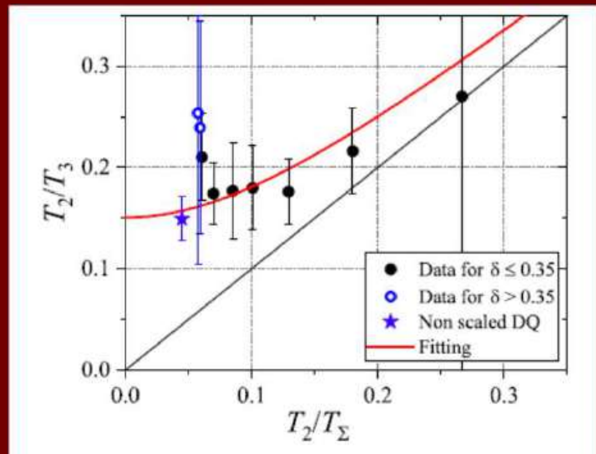
DQ→ballistic spreading



$$N_2(t_s) \approxeq 1 + (\sigma - 1)(t_s/\tau)^3$$







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Article

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Operator growth from global out-of-time-order correlators

Received: 15 June 2022

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Tianci Zhou[1,2] & Brian Swingle[3]

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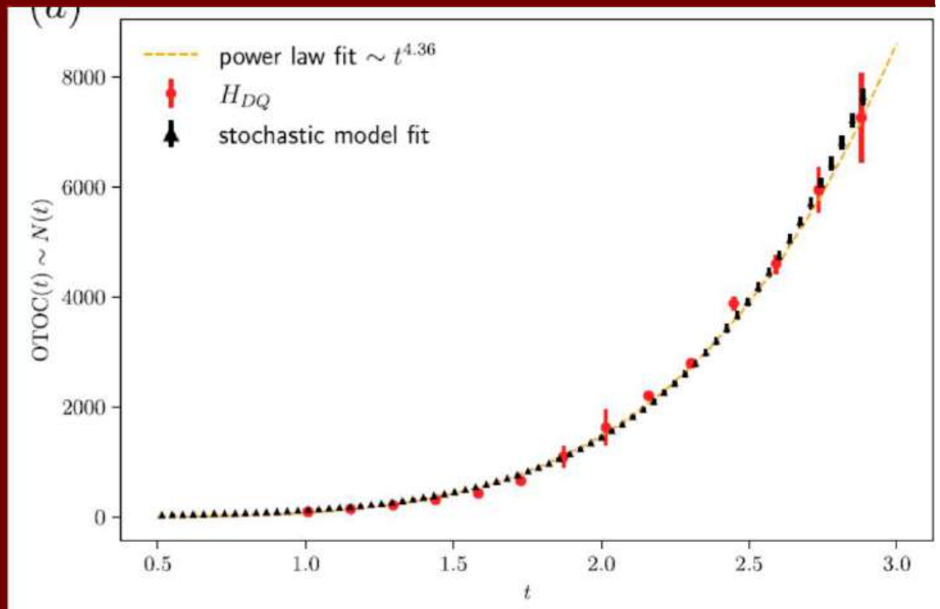
We acknowledge discussions and comments from Horacio Pastawski,

Table 1 | The scalings of the local OTOC predicted by the long range Brownian circuit model, see ref. 17,48,49

α	Light cone	Scaling function	Tail
$[\frac{d}{2}, d)$	$\exp(Bt^{\eta})$	$C\left(\frac{r}{\exp(Bt^{\eta})}\right)$	$\frac{1}{r^{2\alpha}}$ *
d	$\exp\left(\frac{(\ln t)^2}{4d\ln 2}\right)$	$C\left(\frac{r}{t^{\frac{1}{4d}\log_2 t}}\right)$	
$(d, d+\frac{1}{2})$	$t^{\frac{1}{2\alpha-2d}}$	$C\left(\frac{r}{t^{\frac{1}{2\alpha-2d}}}\right)$	
$d+\frac{1}{2}$	$t\ln t$	$C\left(\frac{r}{t\ln t}\right)$	
$(d+\frac{1}{2}, d+1)$	$v_B t$	$C\left(\frac{r-v_B t}{t^{\frac{1}{2\alpha-2d}}}\right)$	$\frac{1}{r^{2\alpha-2d}}$ *
$d+1$	$v_B t$	$C\left(\frac{r-v_B t}{(t\ln t)^{\frac{1}{2}}}\right)$	erf
$[d+1, \infty)$	$v_B t$	$C\left(\frac{r-v_B t}{t^{\frac{1}{2}}}\right)$	erf

Parameters: $B = \frac{d\ln 2}{2(\alpha-d)^2}$, $\eta = \log_2 \frac{d}{\alpha}$. The tail scalings with * only has numerical support for $d=1$ along with a few general scaling conjectures.



---OTO commutator saturates for finite systems

--- OTO commutator keeps growing for unbounded systems. The precise law depends on the system, on the observable and on the Hamiltonian. global~local

---Single-particle systems (theory&numerics): exponential Loschmidt Echo decay with the classical Lyapunov exponent that holds beyond the Eherenfest time.

---puzzling NMR experiments:

→XXZ with transverse (not conserved) polarization
under Dipolar→ Diffusive scrambling, under 2Q→ballistic scrambling
perturbation independent logistic/sigmoid (exponential) Loschmidt Echo decay

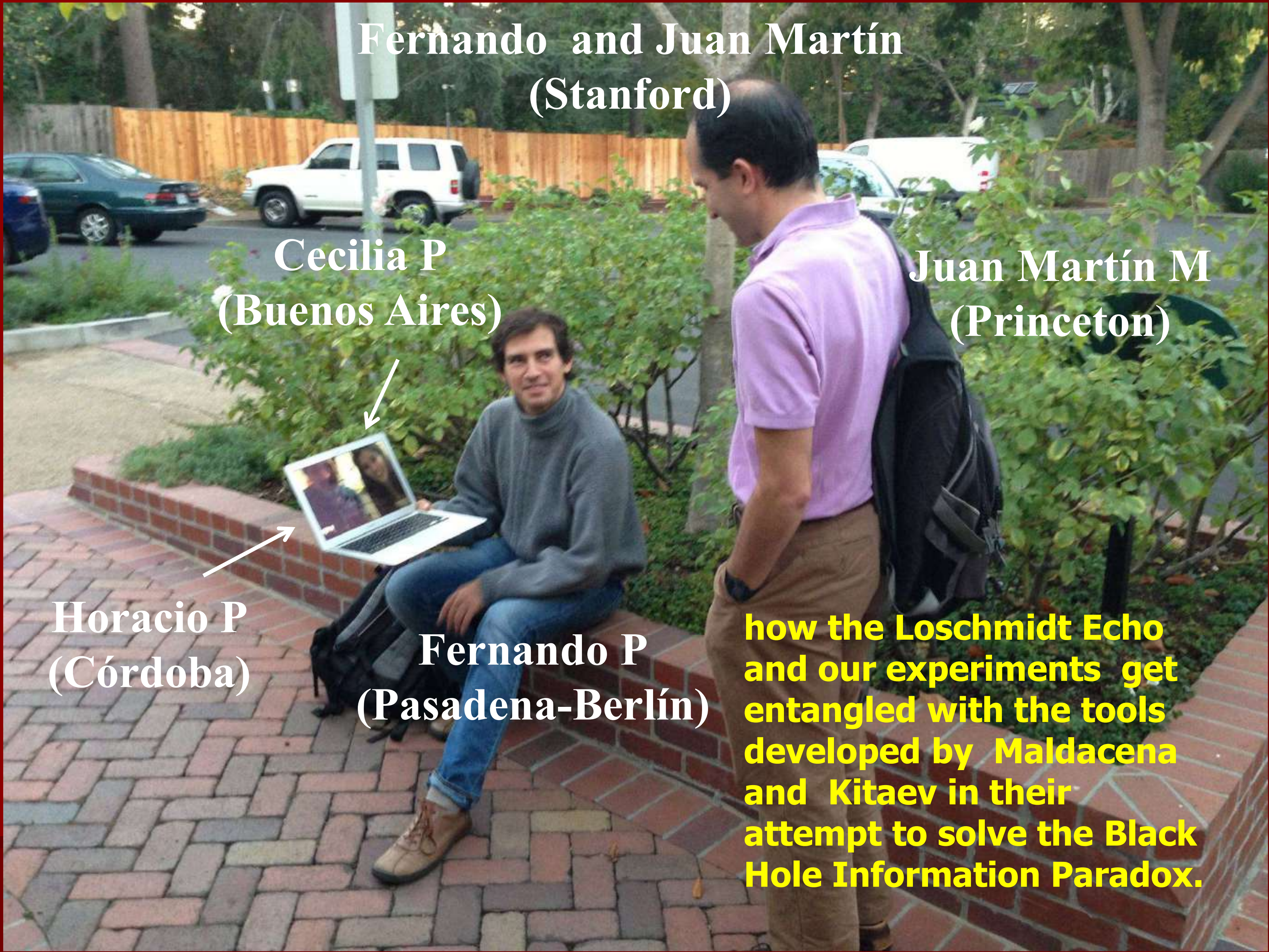
→XXZ with longitudinal conserved polarization, LE is perturbation independent Gaussian

---Many-body spin systems in the thermodynamic limit at high temperature should become irreversible (Keldysh or Lindblad) with intrinsic decoherence/irreversibility even when the coupling with the environment vanishes

$$\lim_{\Sigma\to 0}\lim_{N\to\infty} \neq \lim_{N\to\infty}\lim_{\Sigma\to 0}$$

The class of Lindblad equations contains the Schrödinger equation of ordinary quantum mechanics as a special case, but in general these equations involve a variety of new quantities that represent a departure from quantum mechanics. These are quantities whose details of course we now don't know. Though it has been scarcely noticed outside the theoretical community, there already is a line of interesting papers, going back to an influential 1986 article by Ghirardi, Rimini, and Weber at Trieste, that use the Lindblad equations to generalize quantum mechanics in various ways. Steven Weinberg


CAUTION
EL RESULTADO CLASICO


Fernando and Juan Martín
(Stanford)
Cecilia P
(Buenos Aires)
Juan Martín M
(Princeton)
Horacio P
(Córdoba)
Fernando P
(Pasadena-Berlín)
how the Loschmidt Echo and our experiments get entangled with the tools developed by Maldacena and Kitaev in their attempt to solve the Black Hole Information Paradox.

Bekenstein-Hawking entropy

- The black hole entropy (~ number of degrees of freedom):

$$\boxed{S = \frac{A}{4\ell_P^2}}$$

horizon area: $A = 4\pi a^2, \qquad a = 2GM/c^2$

Planck length: $\ell_P = \sqrt{\hbar G/c^3} \approx 1.6 \cdot 10^{-35}\,\mathrm{m}$

$M \approx 6 \cdot 10^{24}\,\mathrm{kg}$  $\Longrightarrow$

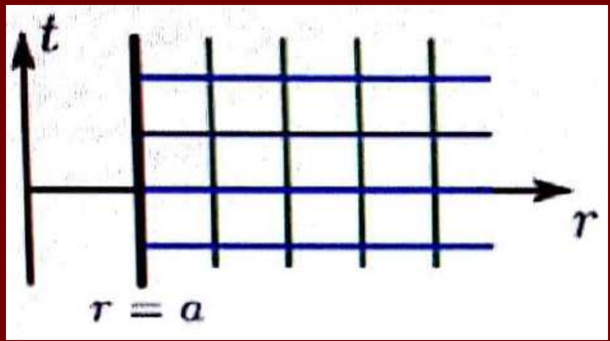
- $a \approx 0.9\,\mathrm{cm}$

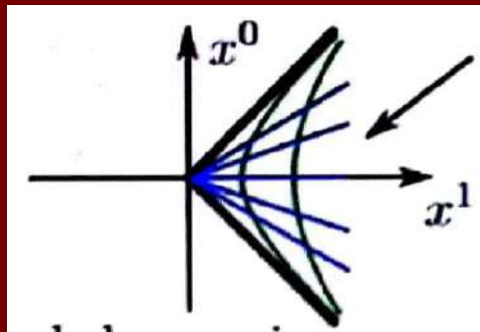
$S \approx 1 \cdot 10^{66}$

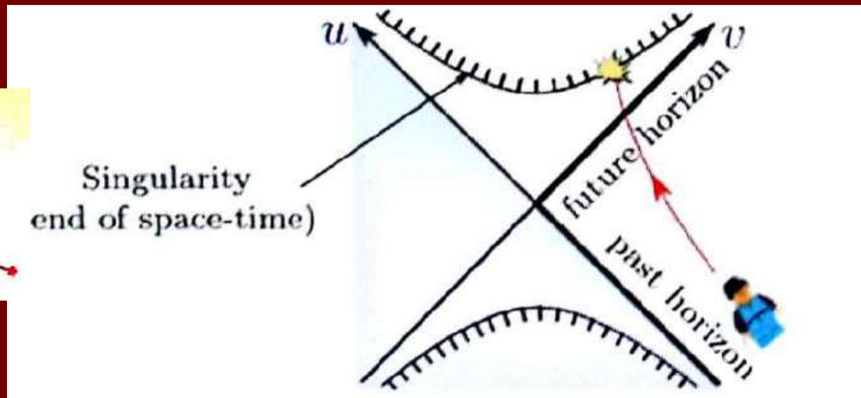
Surface gravity: $\varkappa = 1/(2a)$

$$d\overset{\boxed{\frac{a}{2G}}}{M} = \frac{1}{8\pi G}\,\overset{\boxed{\frac{1}{2a}}}{\varkappa}\,d\overset{\boxed{4\pi a^2}}{A}$$ looks like

$$dE = T\,dS$$







A quantum effect: Hawking radiation

- The black hole horizon is a special type of heat bath. It can be characterized by time-dependent correlation functions.

Hawking temperature: $T = \dfrac{\varkappa}{2\pi}$