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Gravitational Waves meets Amplitudes in the Southern Hemisphere

Waveform Modelling for Gravitational Wave Data Analysis II

Geraint Pratten

ICTP-SAIFR, São Paolo, Brazil, 15th August 2023

Introduction

- Aim to build on Part I (Maria Haney)
  - Impossible to cover all work by the community over the past 5+ years
  - Ambitious slides… more material than we can cover but hopefully useful reference point

- Today we will provide a broad introduction to:
  - Numerical Relativity (emphasis on codes/coverage not so much on methods)
  - Surrogates and Reduced Order Models
  - The Effective One Body Framework

- Focus *mainly* on inspiral-merger-ringdown (IMR) models
  - Bulk of compact binaries observed by LIGO-Virgo-KAGRA are heavy(-ish) binary black holes

Anatomy of an Inspiral…

einstein toolkit

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Flux Balance:

$$\frac{dE_{\mathrm{orbital}}}{dt} = \mathcal{L}_{\mathrm{GW}} \approx \frac{32}{5}\frac{c^5}{G}\frac{(m_1 m_2)^2}{M^4}\left(\frac{v}{c}\right)^5$$

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Anatomy of an Inspiral…

post-Newtonian

$$h^{\alpha\beta}(t,r) = -\frac{4G}{c^4 r}\int T^{\alpha\beta}(t - r/c + \mathbf{n}\cdot\mathbf{r}'/c, \mathbf{r}')\, d^3r'$$

scattering amplitudes

$$\mathcal{M}^{\Delta+\nabla} = \frac{2\pi^2 G^2 \epsilon_1\cdot\epsilon_4 \epsilon_2\cdot\epsilon_3}{\sqrt{-q^2}} \sum_{n,i} \alpha^{(n,i)} \mathcal{O}^{(n,i)}$$

numerical relativity

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + D_i\beta_j + D_j\beta_i$$

$$\partial_t K_{ij} = \cdots$$

BH perturbation theory

$$h = \sum C_{[p]\ell mn}\, e^{-i\tilde{\omega}_{[p]\ell mn} t}\, {}_{-2}S_{[p]\ell mn}(\iota,\varphi)$$

EFT

$$S_{\rm eff} = -\frac{1}{16\pi G}\int d^4x\sqrt{\bar{g}}\, R[\bar{g}_{\mu\nu}] + \sum_{i=1}^{\infty} C_i(r_s)\int d\sigma \mathcal{O}_i(\sigma)$$

gravitational self-force

$$g_{\alpha\beta} + \sum_{m=-\infty}^{\infty}\left[q h^{1,m}_{\alpha\beta}(\Omega) + q^2 h^{2,m}_{\alpha\beta}(\Omega)\right] e^{-im\phi} + \mathcal{O}(q^3)$$

post-Minkowski

$$h^{\alpha\beta} = \sum_{n=1} G^{(n)} h^{\alpha\beta}_{(n)}$$

effective one body

$$H_{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\mu} - 1\right)}$$

+ other approaches

Inspiral

Analytical approximations begin to break down

Merger

Ringdown

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we will focus on the basics of EOB + NR + Surrogates

einstein toolkit

Numerical Relativity

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  - Example 1: Post-Newtonian expansion breaks down as $v \sim c$
  - Example 2: BH perturbation theory requires input initial conditions (infer from NR)
- For binaries: 3+1 decomposition and reformulate the EFE as a Cauchy problem + constraints

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Numerical Relativity

- Start from Einstein field equations as set of 10 coupled non-linear 2nd order PDEs

Numerical Relativity

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$$
\begin{aligned}
R_{\mu\nu} \equiv & \frac{1}{2} g^{\sigma\rho} \left(g_{\sigma\nu,\mu\rho} + g_{\mu\rho,\sigma\nu} - g_{\sigma\rho,\mu\nu} - g_{\mu\nu,\sigma\rho} \right) \\
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$$\frac{\partial}{\partial t} u\left(t, x^i\right) = F\left(u\left(t, x^i\right), \partial u\left(t, x^i\right), \partial^2 u\left(t, x^i\right)\right)$$

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- Initial data $u(0,\boldsymbol{x}) = f(\boldsymbol{x})$ evolved forward in time with evolution equations $\partial_t u(t, \boldsymbol{x}) = F(u, \partial u, \partial^2 u)$

Numerical Relativity

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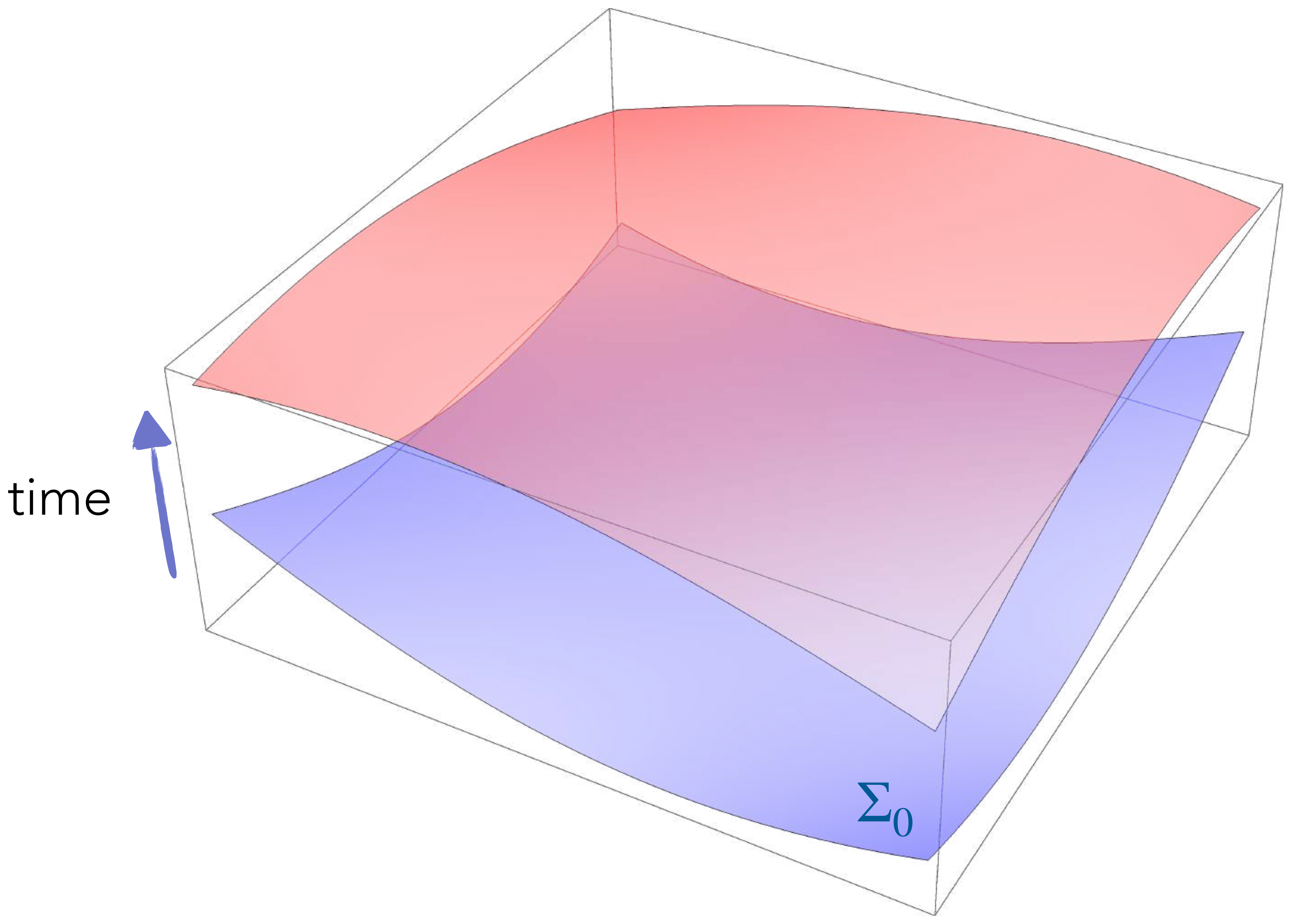

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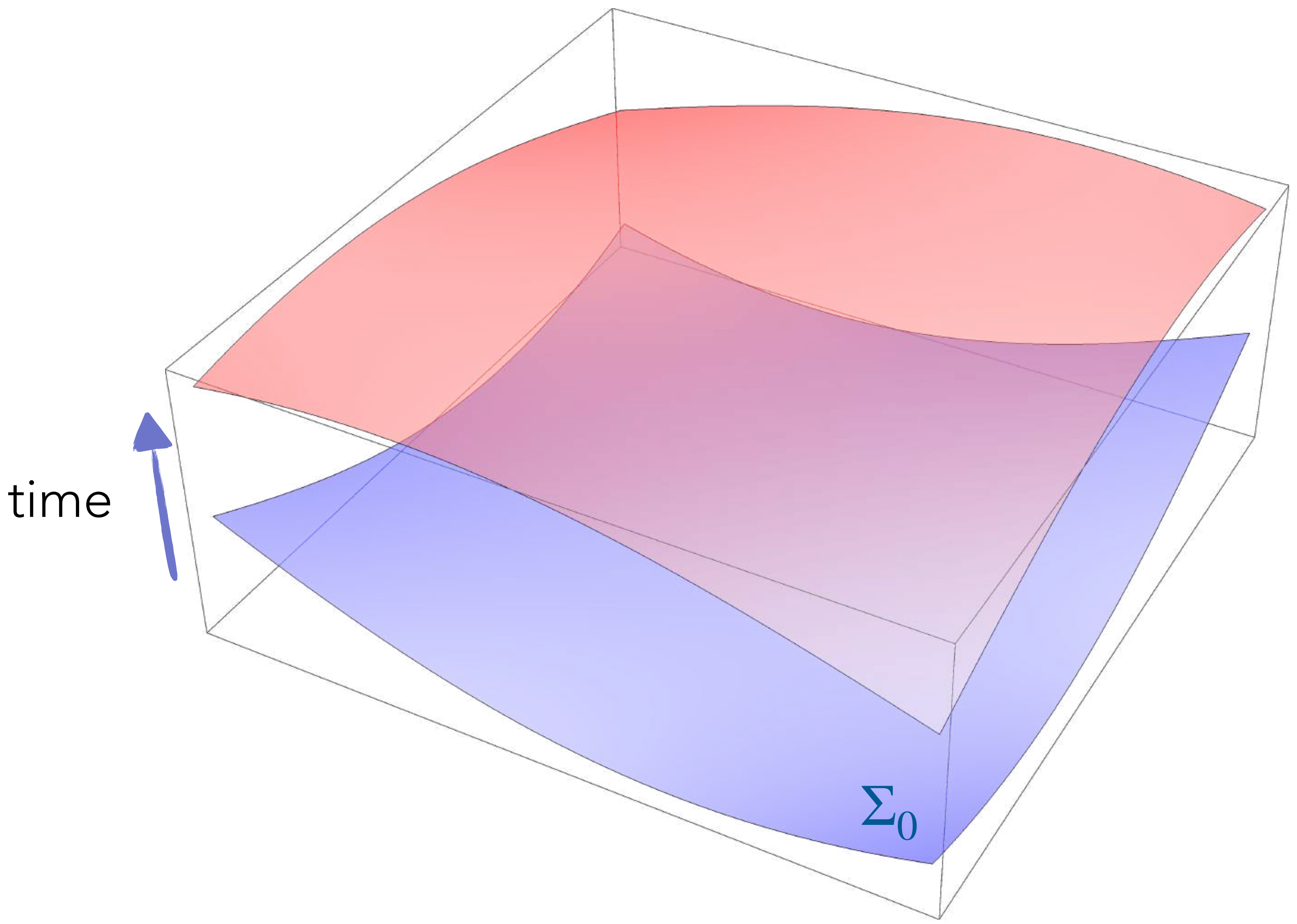

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- Initial data $u(0,\boldsymbol{x}) = f(\boldsymbol{x})$ evolved forward in time with evolution equations $\partial_t u(t,\boldsymbol{x}) = F(u, \partial u, \partial^2 u)$
- Constraint equations defined on initial hyper surfaces of equal time Σ_t [*assuming a 3+1 perspective*]

E.g. textbooks by Alcubierre, Gourgoulhon, or Baumgarte and Shapiro

- 3+1 decomposition → foliation of spacetime by spatial hyper surfaces along timeline vector field
- Normal to slices defined by: $n^\mu = -\alpha g^{\mu\nu} \nabla_\nu t$
- The 3-metric intrinsic to spatial hypersurfaces is $\gamma_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$
- Decompose the spacetime metric as $ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$
- Lapse function α defines a proper time
- Shift vector β^μ determines relative velocity between Eulerian observers and lines of constant spatial coordinates
- Extrinsic curvature: $K_{\mu\nu} = \gamma^\lambda{}_\mu \gamma^\rho{}_\nu \nabla_{(\lambda} n_{\rho)} = -\dfrac{1}{2}\mathscr{L}_n \gamma_{\mu\nu}$
- Extrinsic curvature ~ relative change of spatial metric as it moves along normal vector-field (related to flow of time) ~ time derivative of spatial metric

Numerical Relativity

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Arnowitt, Deser and Misner 1962
York 1979

- Evolution equations

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + D_i\beta_j + D_j\beta_i$$

$$\begin{aligned}\partial_t K_{ij} =& - D_iD_j\alpha + \alpha\left(R_{ij} - 2K_{ik}K^k_j + KK_{ij}\right) \\ & - \alpha 8\pi\left(S_{ij} - \frac{1}{2}\gamma_{ij}(S-\rho)\right) + \beta^k D_k K_{ij} + K_{ik}D_j\beta^k + K_{kj}D_i\beta^k\end{aligned}$$

- Constraint Equations

$$^{(3)}R + K^2 - K_{ij}K^{ij} = 16\pi\rho \qquad \text{Hamiltonian constraint}$$

$$K^i_{j;i} - K_{;j} = 8\pi S_j \qquad \text{Momentum constraint}$$

- 12 evolved variables γ_{ij}, K_{ij} + 12 evolution equations
- 4 free variables α, β^i + 4 constraint equations

Numerical Relativity

- Problem: ADM equations are **not** well posed [strong hyperbolicity]
  - Amplify small errors in the constraints leading to severe instabilities
  - No stable long term evolution
- Instabilities in Ricci tensor → mixed 2nd derivatives of spatial metric prevent hyperbolicity
- Coordinates arbitrary but there are many *bad* choices

$$\partial_{tt}\gamma_{ij} \simeq -2\alpha R_{ij} \simeq \Delta\gamma_{ij} + \gamma_{ik}\partial_j\partial_l\gamma^{kl} + \gamma_{jk}\partial_i\partial_l\gamma^{kl}$$

- Black holes have singularities…
  - Circumvent through gauge choice? Moving punctures!
  - Numerically excise from domain? Popular in conjunction with generalised harmonic gauge

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Multiple widely different methods to circumvent all of these issues

Numerical Relativity

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Finite Difference Codes: BAM, ET, LazEv, Maya, Goddard, GRChombo, …

- Initial Data: **CTT** [York+] + **Bowen-York** (conformally flat, $\chi \leq 0.9$) [Brandt+, Ansorg+]; Beyond Bowen-York [e.g. Ruchlin+] or **CTS** [East+14, Corman+22]
- Formulation: **BSSN** [Shibata+, Nakamura+, Baumgarte+], **or CCZ4** [Weyhausen+, Alic+] **or G**eneralised **H**armonic
- **Moving punctures** gauge conditions [Alcubierre+02, Campanelli+05, Baker+05] → robust merger (esp. at higher mass ratios) or **Excision** for **GH** [e.g. Pretorius+04, East+14, East+21, Corman+22]
- BHs advected on AMR (e.g. **Berger-Oliger**) grids + optionally multipatch grids adapted to spherical topology [Reisswig+]
- **Boundary Conditions**: Sommerfeld/Robin GCs
- Spatial **finite difference** (typically 6th/8th order)
- **Time evolution**: Method of lines w/ 4th order RK
- **GW extrapolation** + **CCE** e.g. PITTNull [Bishop+, Winicour+]

Spectral Codes: SpEC, SpECTRE, BAMPS, …

- Initial Data: **XCTS** [York+,Pfeiffer+] + **SKS** [Lovelace+, Varma+] (conformally curved, $\chi \leq 0.999$)
- Formulation: First order **GH** + constraint damping [Pretorius+, Lindblom+, Friedrich, Garfinkle, Gundlach+] (BSSN/CCZ4 in SpECTRE)
- **Excision** = BH mergers difficult [Szilagyi+, Scheel+, Hemberger+, Ossokine+]
- **Change gauge** from ~ coronating to damped harmonic gauge near merger [e.g. Lindblom+]
- Dynamical grids with multiple subdomains
- **Boundary Conditions**: Constraint preserving/minimally reflective
- Multi-domain **spectral methods** (~ exponentially convergent)
- **Time evolution**: Method of lines w/ 5th order DP
- **GW extrapolation** + **CCE**

See Boyle+19 for details on SXS

* Impossible to list all the contributions to date, provided some starter references

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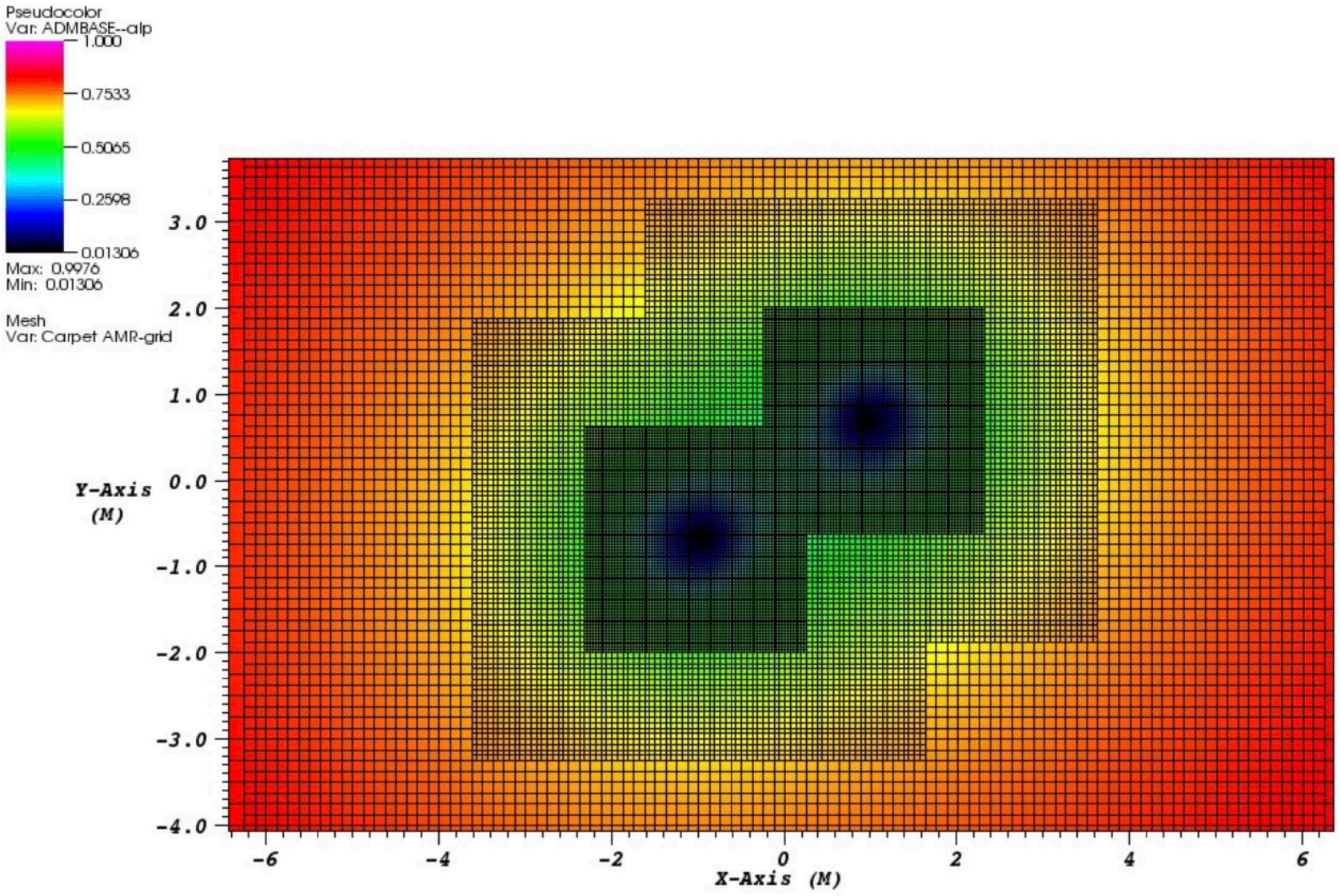

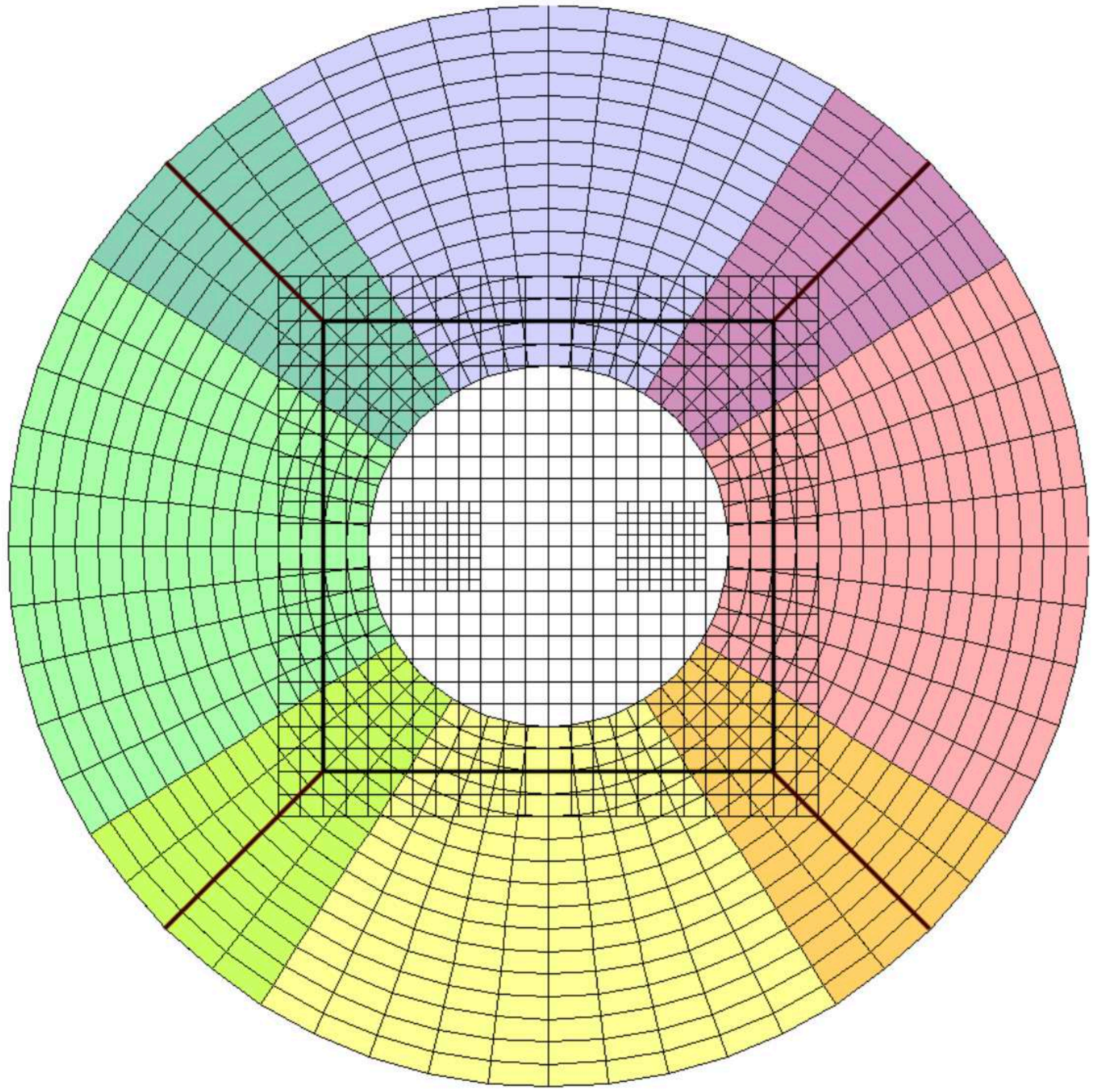

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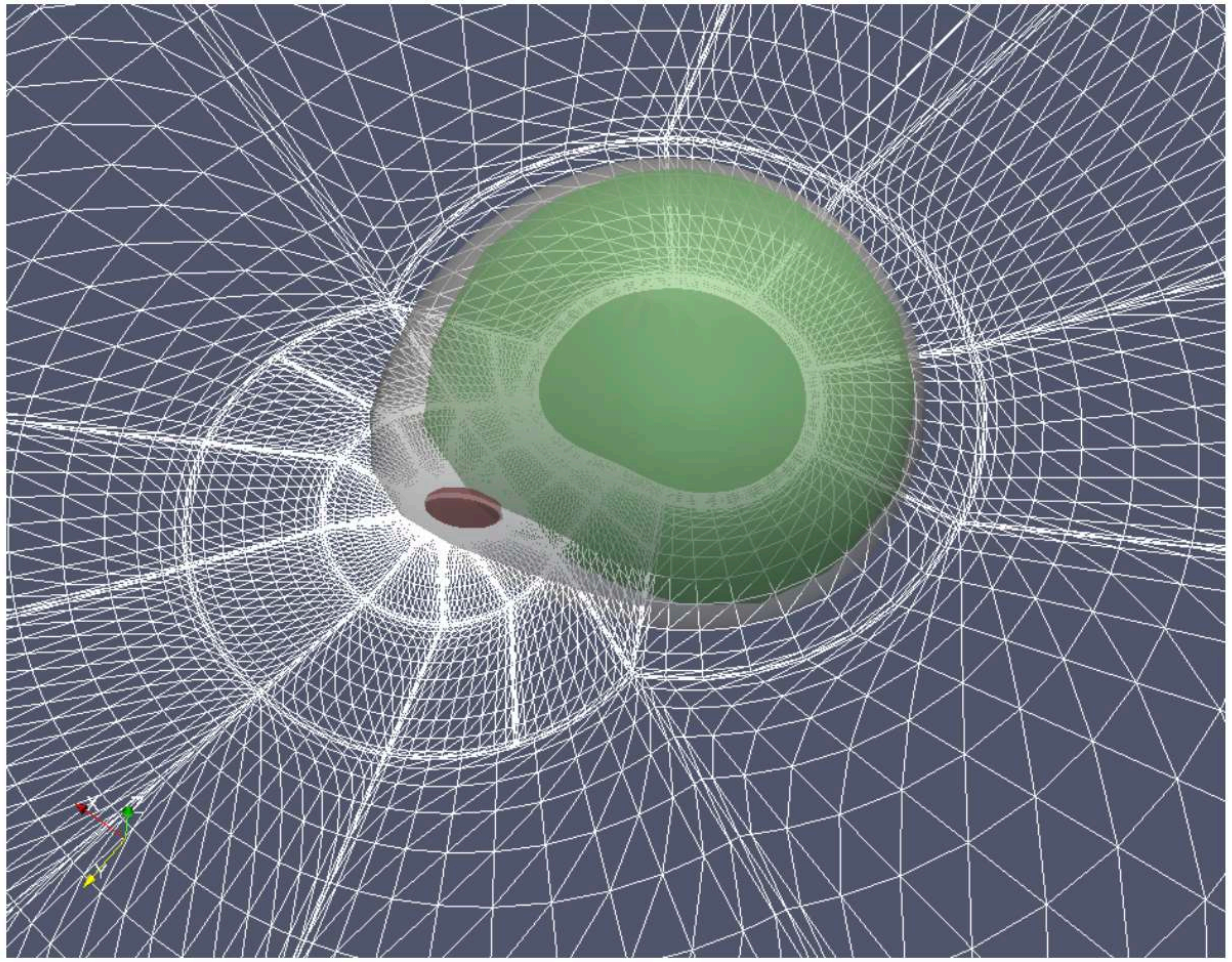

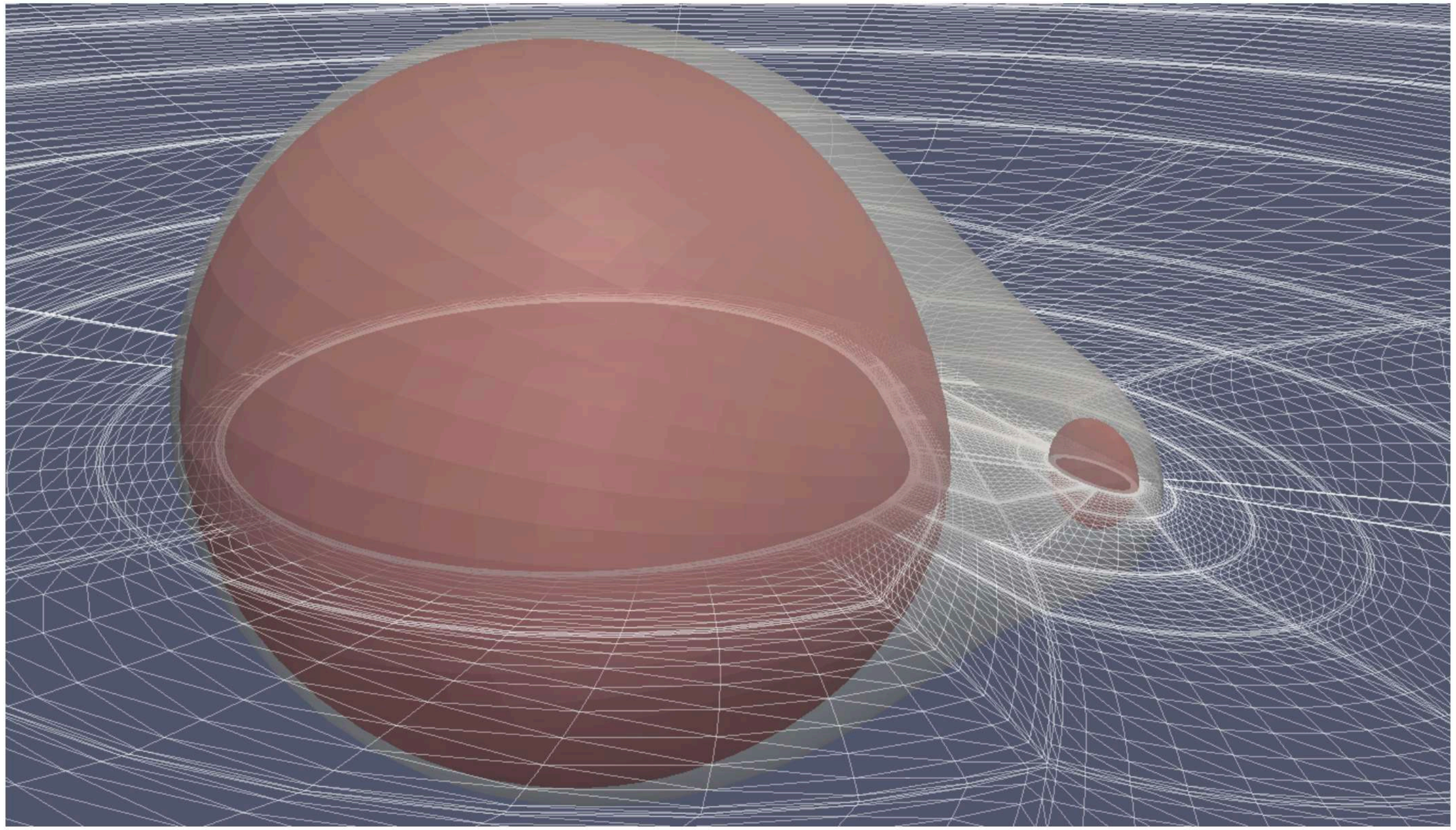

SXS Collaboration: Mass ratio 8:1

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- Interlude on initial data…
- Want to solve the constraint equations

$$\mathcal{C}_0 := R + K^2 - K_{ij}K^{ij} - 16\pi E = 0$$
$$\mathcal{C}_i := D_j K^j_i - D_i K - 8\pi P_i = 0 \;,$$

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4 equations: prescribe 8 fields and solve for 4

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Want to determine K_{ij} and γ_{ij} on $\Sigma_0 \to$ 12 components

What is our **free** data and what is **constrained**?

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Want to determine K_{ij} and γ_{ij} on $\Sigma_0 \rightarrow$ 12 components

What is our **free** data and what is **constrained**?

- Must satisfy constraint equations
- Must be physically meaningful
- Choice of free/constrained data also impacted by mathematical properties (linearity, decoupling, etc)

- Conformal transformation of spatial metric: $\gamma_{ij} = \psi^4 \bar{\gamma}_{ij}$

Lichnerowicz 1944

- Decompose extrinsic curvature: $K_{ij} = \psi^{-2}\bar{A}_{ij} + \frac{1}{3}\gamma_{ij}K$

- Leads to the following system of equations

$$8\bar{D}^2\psi - \psi\bar{R} - \frac{2}{3}\psi^5 K^2 + \psi^{-7}\bar{A}_{ij}\bar{A}^{ij} = 0$$

$$\bar{D}_j\bar{A}^{ij} - \frac{2}{3}\psi^6\bar{\gamma}^{ij}\bar{D}_jK = 0$$

York 79, O Murchadha+ 74

Numerical Relativity

- Moving punctures assumes conformal flatness → decouple the equations $\bar{\gamma}_{ij} = f_{ij}\ K = 0$

$$\bar{D}^2\psi = \frac{1}{8}\psi^{-7}\bar{A}_{ij}\bar{A}^{ij}$$
$$\bar{D}_j\bar{A}^{ij} = 0$$

- Bowen and York derived solution for binary black hole in terms of momenta and spin

$$\bar{A}^{ij} = \frac{3}{2}\sum_I \frac{1}{r_I^2}\left(2P_I^{(i}n_I^{j)} - (\eta^{ij} - n_I^i n_I^j)P_I^k n_I^k + \frac{4}{r_I}n_I^{(i}\epsilon^{j)kl}S_I^k n_I^l\right)$$

Bowen and York 1980

- Decompose conformal factor

$$\psi = 1 + u + \sum_i \frac{M_i}{r_i}$$

Brandt & Brügmann 1997

Numerical Relativity

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Solution to homogenous equation which we call α

Brandt & Brügmann 1997

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$$\bar{A}^{ij} = \frac{3}{2}\sum_I \frac{1}{r_I^2}\left(2P_I^{(i}n_I^{j)} - (\eta^{ij} - n_I^i n_I^j)P_I^k n_I^k + \frac{4}{r_I}n_I^{(i}\epsilon^{j)kl}S_I^k n_I^l\right)$$

Bowen and York 1980

- Decompose conformal factor

$$\psi = 1 + u + \sum_i \frac{M_i}{r_i}$$

Puncture field corrects momentum and spin of BH such that ψ solves inhomogeneous equation

Brandt & Brügmann 1997

Numerical Relativity

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- Moving punctures assumes conformal flatness → decouple the equations $\bar{\gamma}_{ij} = f_{ij} \; K = 0$

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$$\psi = 1 + u + \sum_i \frac{M_i}{r_i} \longrightarrow \bar{D}^2 u = -\frac{1}{8}\alpha^7 \bar{A}_{ij}\bar{A}^{ij}\left[\alpha\left(1+u\right)+1\right]^{-7}$$

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conformal transverse traceless (CTT)

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Bowen and York 1980

- Decompose conformal factor

Free data: $\bar{\gamma}_{ij}, \bar{A}_{ij}^{\mathrm{TT}}, K$ Constrained data: ψ, u

$$\psi = 1 + u + \sum_i \frac{M_i}{r_i} \longrightarrow \bar{D}^2 u = -\frac{1}{8}\alpha^7\bar{A}_{ij}\bar{A}^{ij}\left[\alpha\left(1+u\right)+1\right]^{-7}$$

conformal transverse traceless (CTT)

Brandt & Brügmann 1997

Numerical Relativity

- Alternatively we can include notion of evolution between hyper surfaces $\bar{u}_{ij} = \partial_t \bar{\gamma}_{ij}$
- Introduce auxiliary field for evolution of intrinsic curvature

$$\mathcal{D}^2\alpha = -\partial_t K + \alpha\left(K_{ij}K^{ij} + 4\pi\left(\rho_{\mathrm{H}} + S\right)\right) + \beta^i\mathcal{D}_i K.$$

Numerical Relativity

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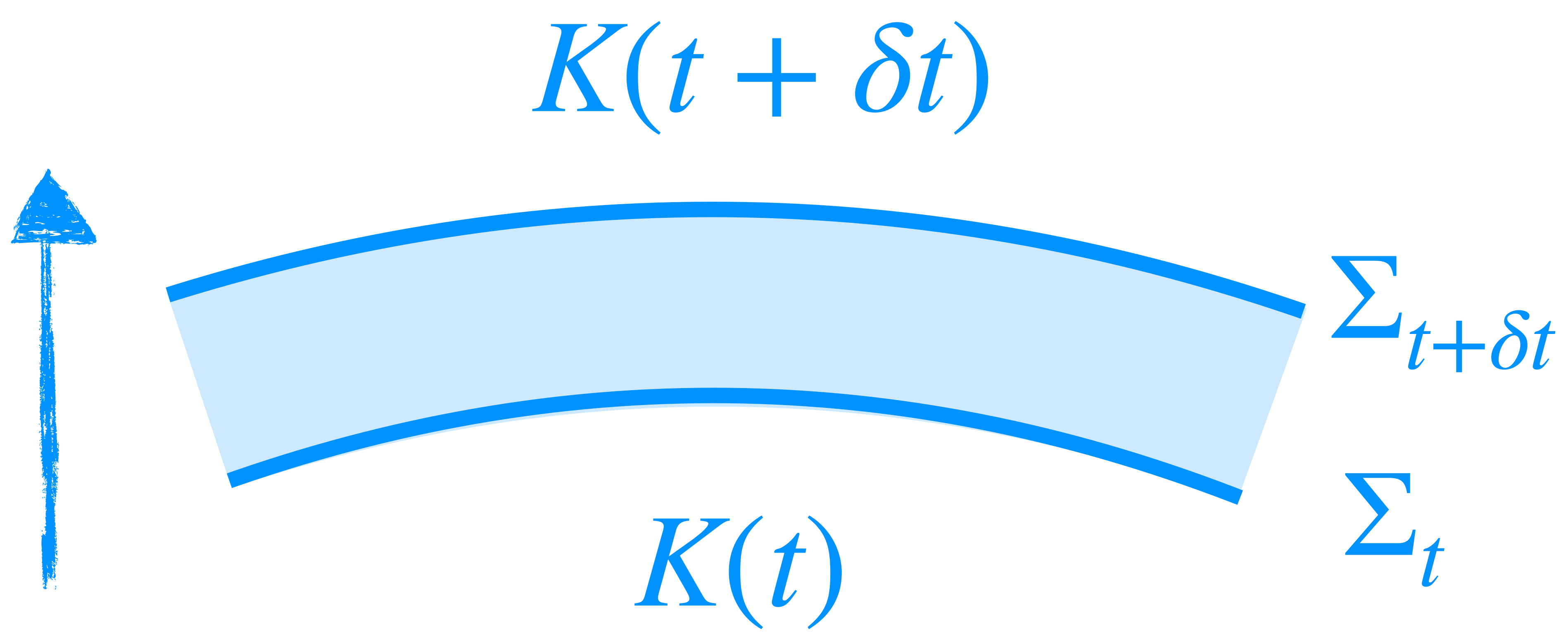
"Thin sandwich" as we specify the evolution between hyper surfaces



York 1999
Pfeiffer and York 2003

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- Leads to a modified system of equations

$$\bar{\mathcal{D}}^2\psi = \frac{1}{8}\psi\bar{R} + \frac{1}{12}\psi^5K^2 - \frac{1}{8}\psi^{-7}\bar{A}_{ij}\bar{A}^{ij}$$

$$\bar{\mathcal{D}}^2(\alpha\psi) = \alpha\psi\left(\frac{7}{8}\psi^{-8}\bar{A}_{ij}\bar{A}^{ij} + \frac{5}{12}\psi^4K^2 + \frac{1}{8}\bar{R}\right) - \psi^5\partial_t K + \psi^5\beta^i\bar{\mathcal{D}}_i K,$$

$$\bar{\mathcal{D}}_i(\bar{L}\beta)^{ij} = (\bar{L}\beta)^{ij}\bar{\mathcal{D}}_i\ln(\bar{\alpha}) + \bar{\alpha}\bar{\mathcal{D}}_i\left(\bar{\alpha}^{-1}\bar{u}^{ij}\right) + \frac{4}{3}\bar{\alpha}\psi^6\bar{\mathcal{D}}^j K$$

Pfeiffer 2005
Pfeiffer and York 2003

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THE ROYAL SOCIETY

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- Leads to a modified system of equations
- Free data: $\bar{\gamma}_{ij}, \bar{u}_{ij}, K, \dot{K}$
- Constrained data: ψ, α, β^i

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- Parameter space coverage for BBH?

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THE ROYAL SOCIETY

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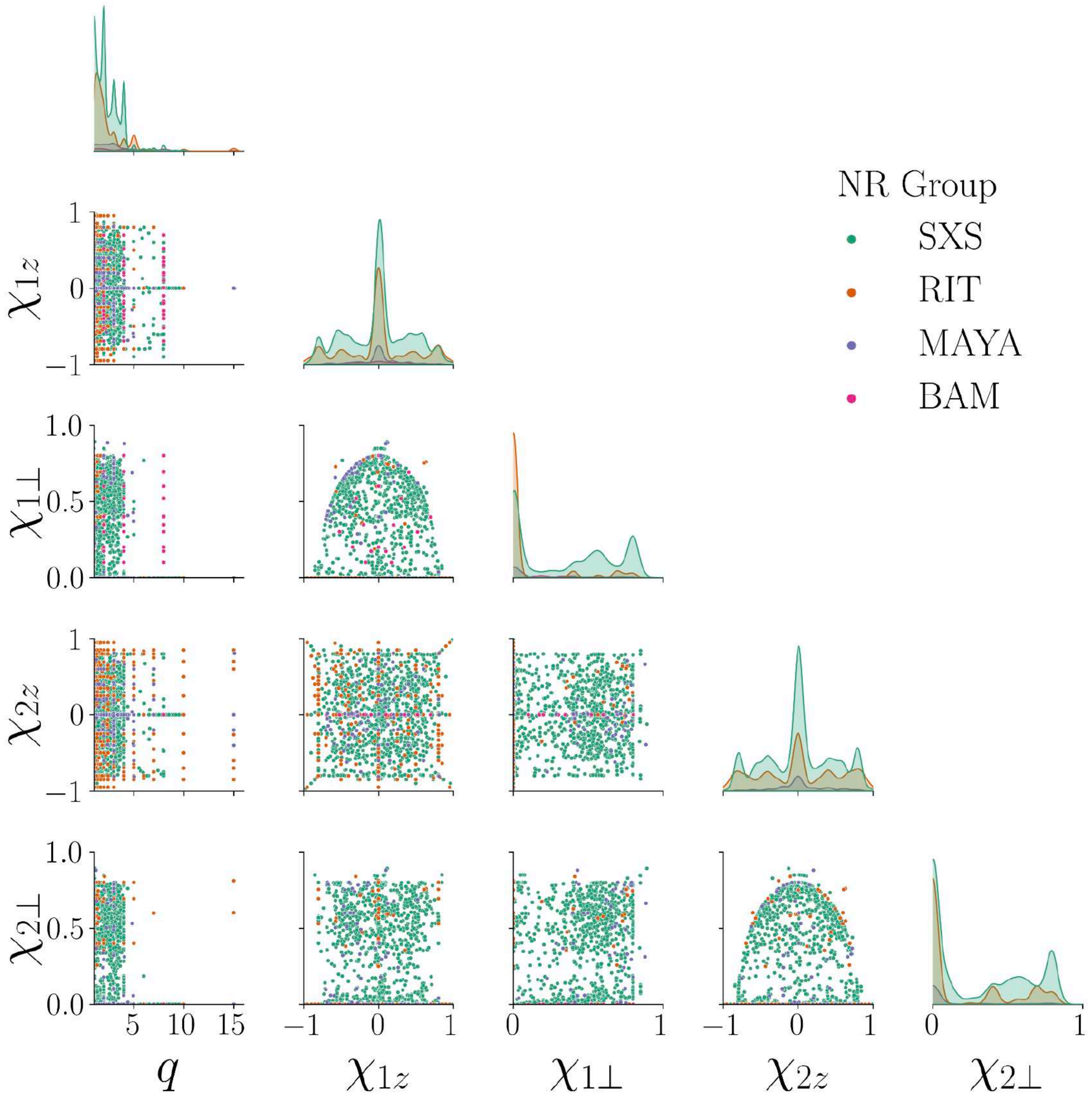

Figure 3. Segment of the public catalogs' coverage of the quasi-circular parameter space as of June 2023, courtesy of Deborah Ferguson.

Numerical Relativity

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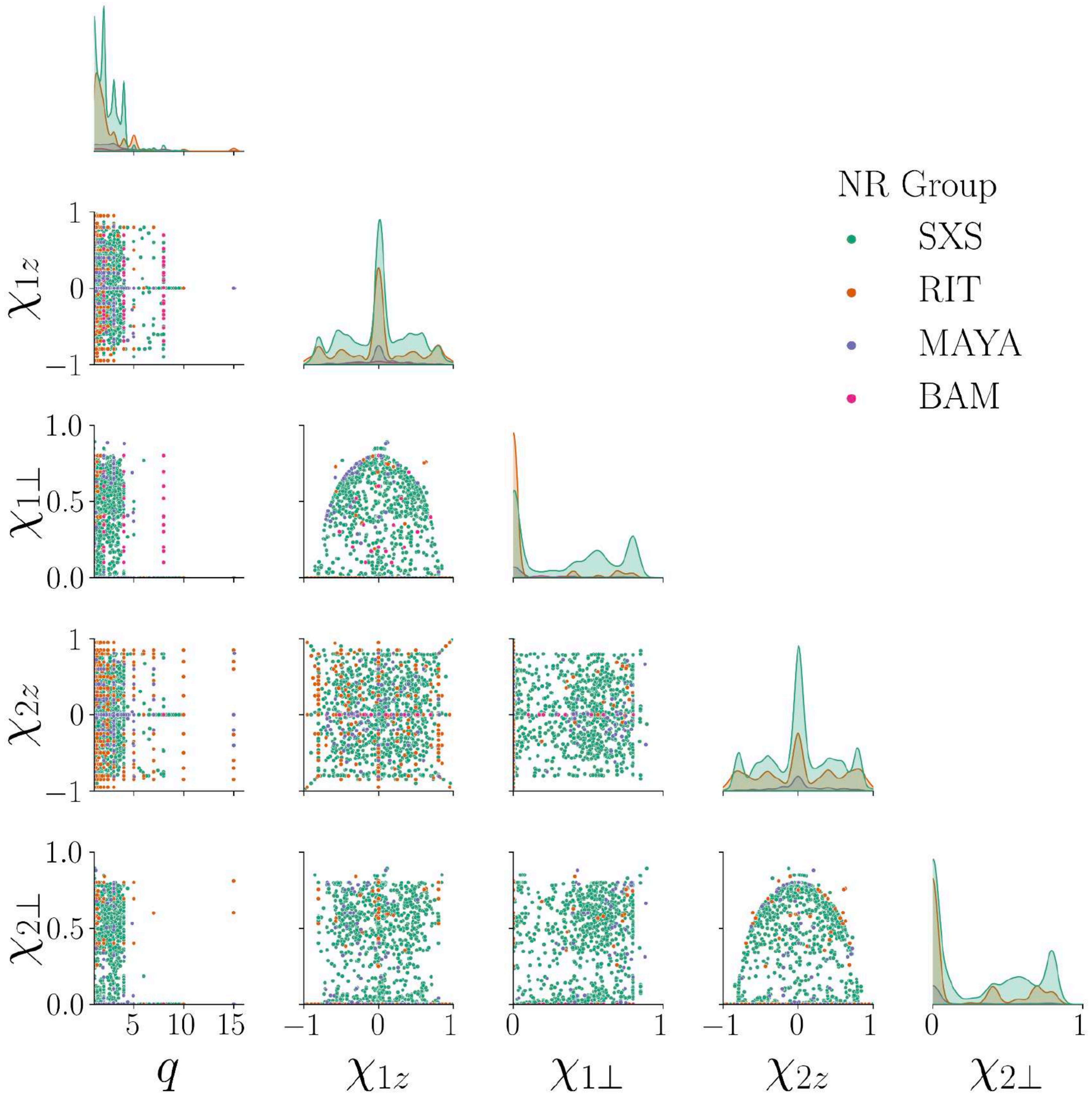
- Parameter space coverage for BBH?
- ~ 5700 NR waveforms publicly available to date
- Broad range of mass ratios, spins and eccentricities



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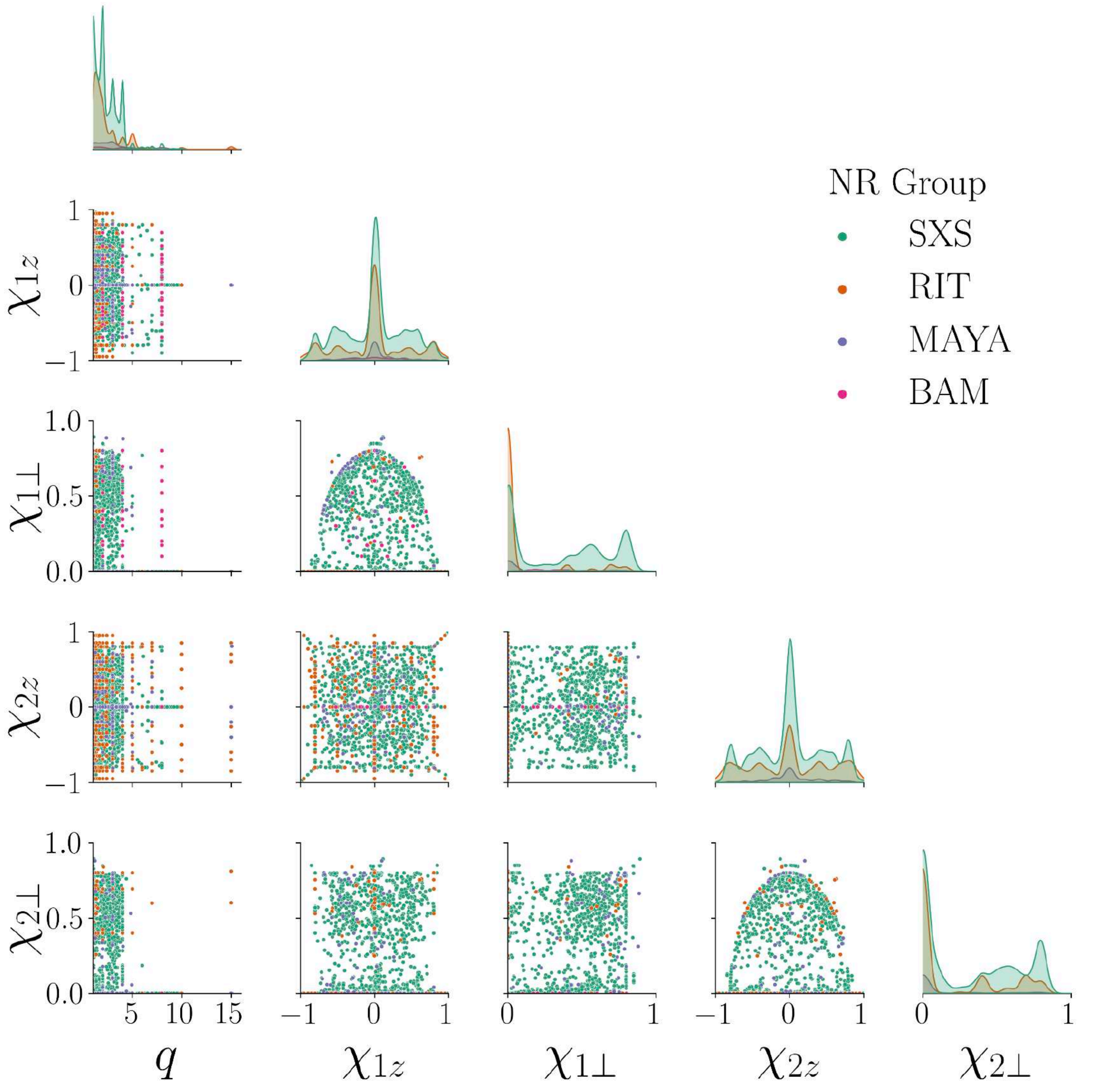
Numerical Relativity

- Parameter space coverage for BBH?
- ~ 5700 NR waveforms publicly available to date
- Broad range of mass ratios, spins and eccentricities
- Non-spinning: $q \leq 18$
- Moderate spins: $q \leq 8$
- Aligned-spins up to $\chi \leq 0.85$ for $q \leq 18$
- Eccentric BBHs for $q \leq 10$



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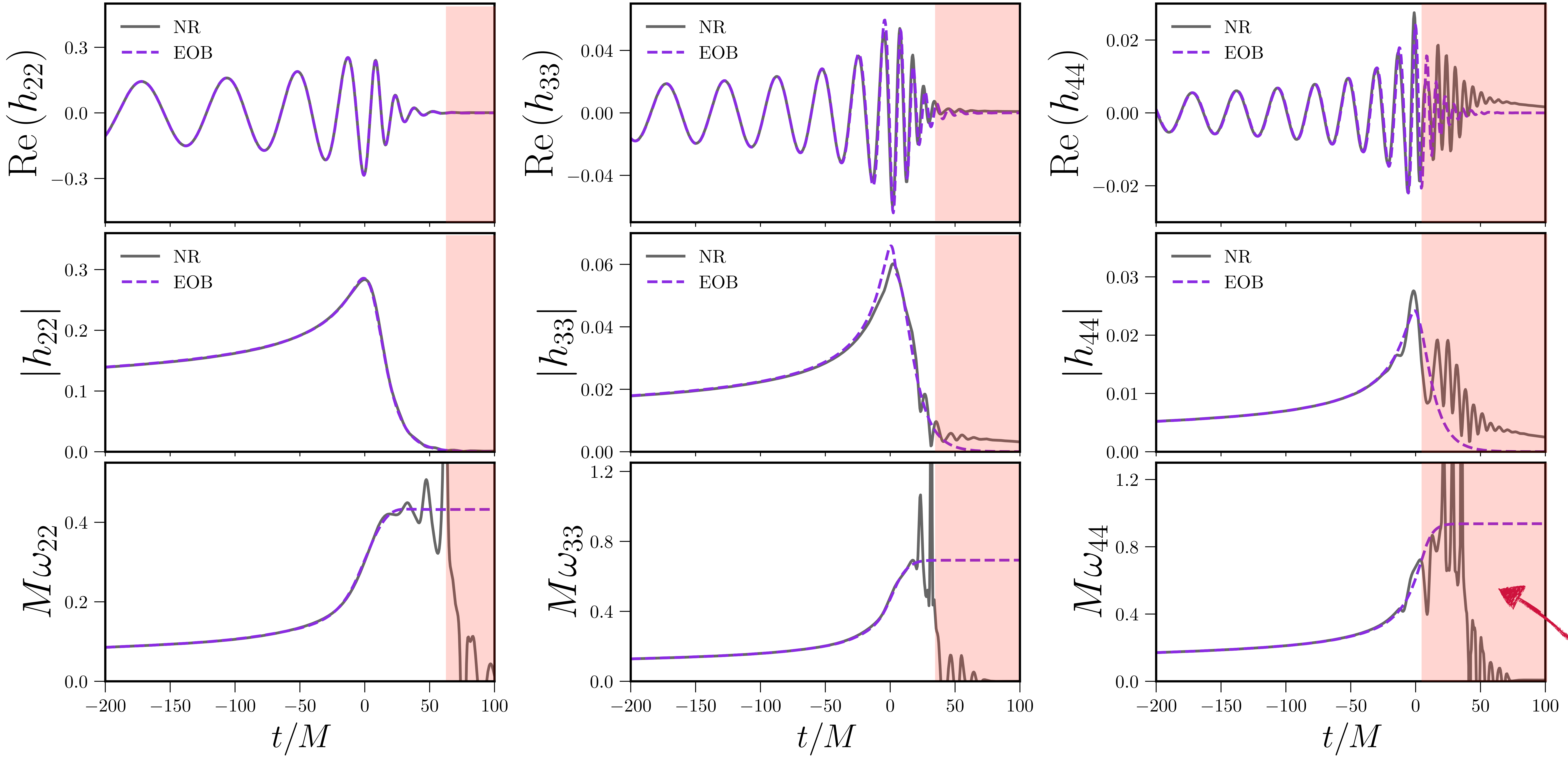
Numerical Relativity

- Numerical relativity allows us to incorporate full non-perturbative information in strong-field regime
- Not free from systematics and couples to how models are informed and calibrated

Nagar+ inc GP



Unphysical behaviour in amplitude and phase

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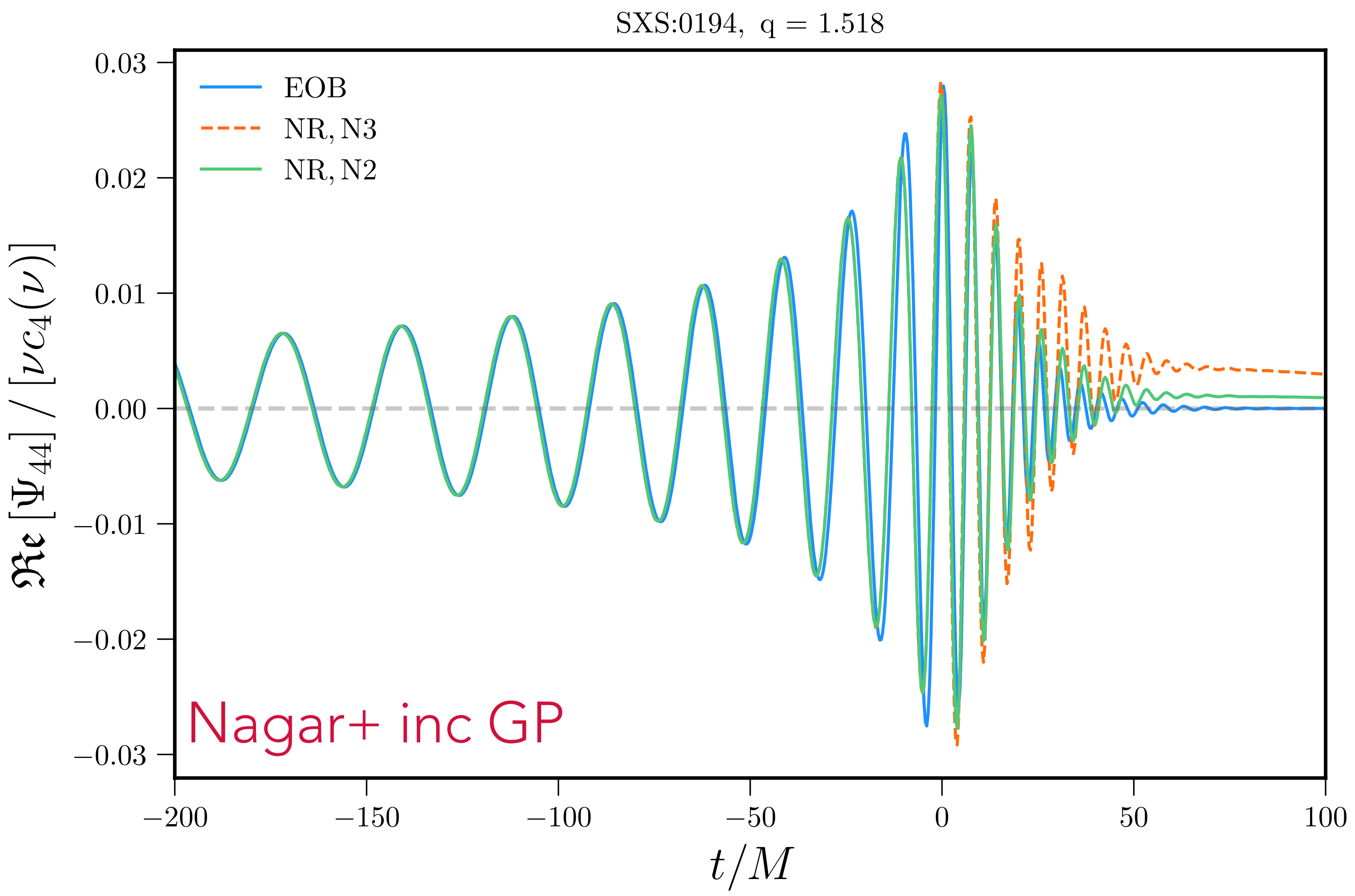
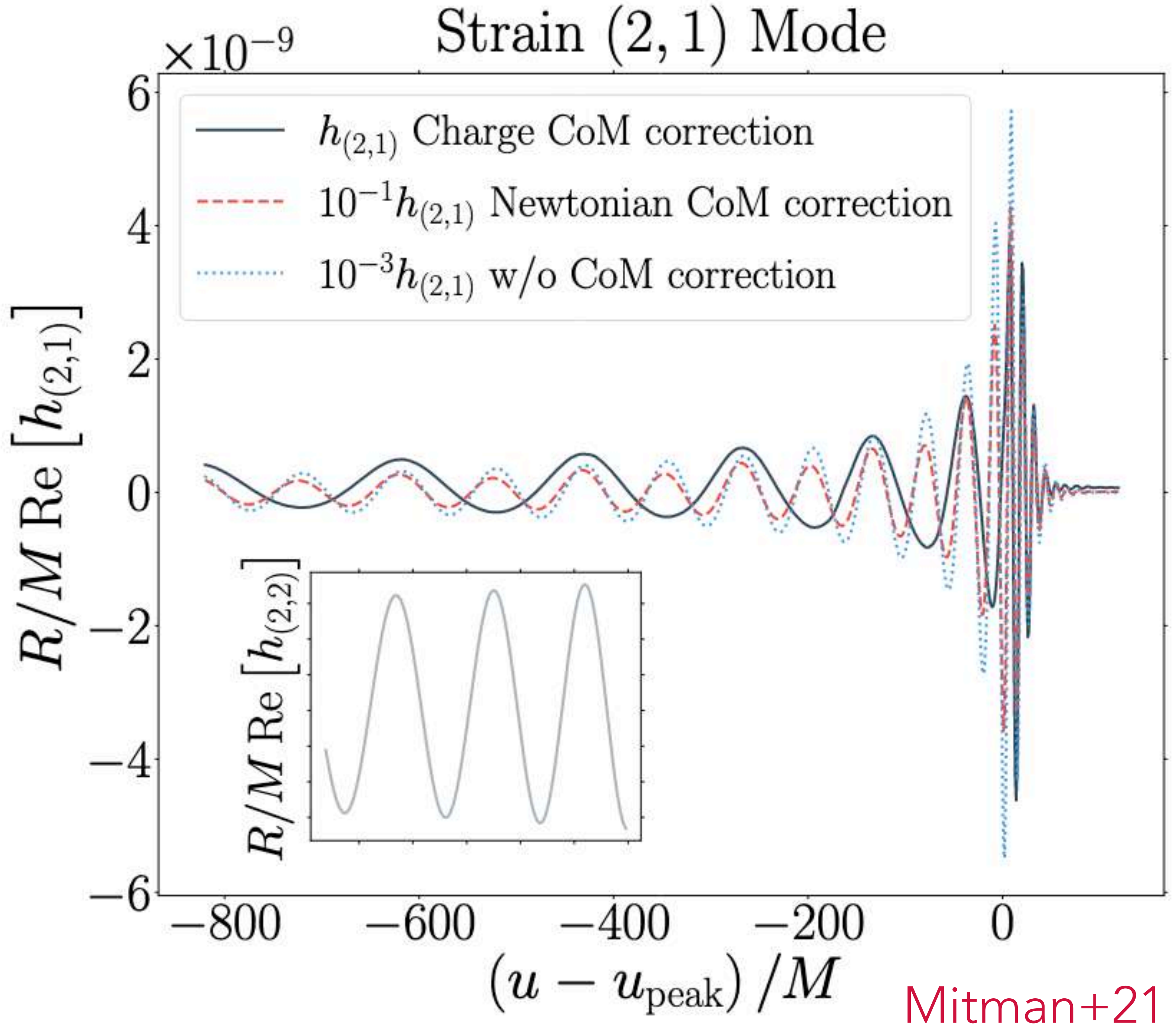
- Extrapolation of waveforms to $\mathscr{I}^+$ can introduce unphysical features [Chu+, Boyle+, Nagar (inc GP)+]
- Mitigate with cauchy characteristic extrapolation (CCE) [Bishop+, Reisswig+, Taylor+, Barkett+, Moxon+]
    - Help reduce near-zone and gauge-effects on waveform



- Recent work to understand impact of frame choice on waveform
    - Fix Poincaré (by mapping to center-of-mass) frame [Boyle+, Woodford+]
    - Use Poincaré charges and super translation charges to fix BMS frame [Mitman+]
    - Methodology increasingly important to meet accuracy requirements



Reduced Order Modelling and Surrogates

Reduced Order Models and Surrogates

- Reduced order models ~ compressing the number of degrees of freedom [e.g. Field+, Antil+, Tiglio+, Canizares+, Pürrer+, Blackman+, Varma+, …]

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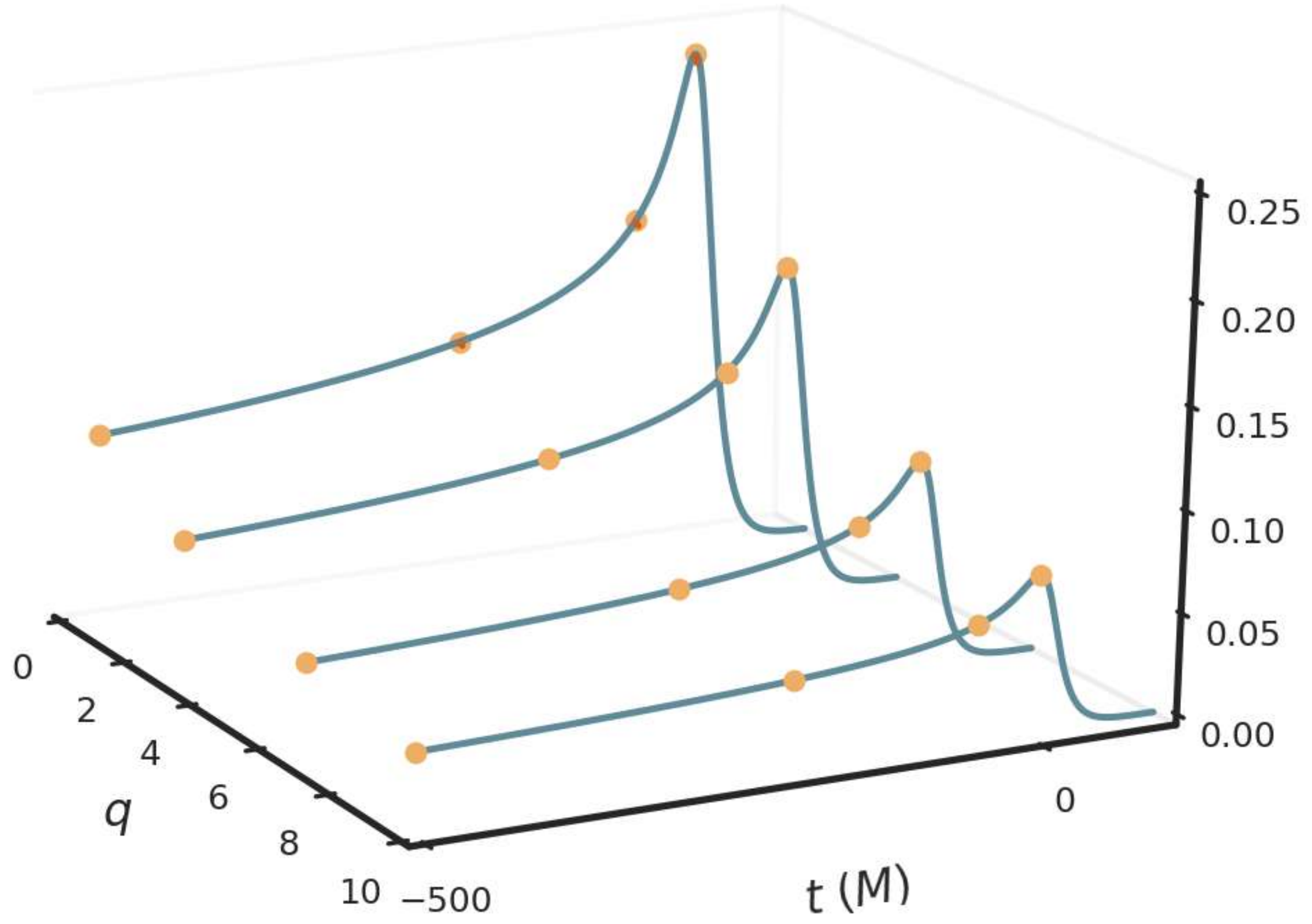
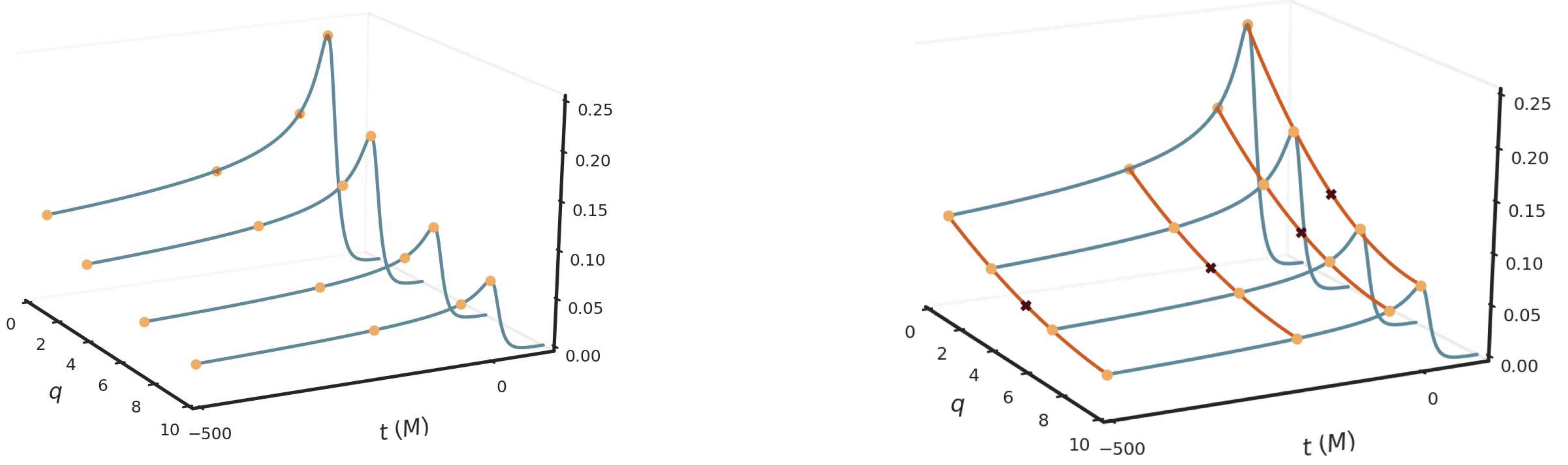


Credit to Vijay Varma for images!
https://github.com/vijayvarma392/SurrogateMovie

Reduced Order Models and Surrogates

- Reduced order models ~ compressing the number of degrees of freedom [e.g. Field+, Antil+, Tiglio+, Canizares+, Pürrer+, Blackman+, Varma+, …]
- Build **reduced basis** with SVD of greedy algorithms [expensive offline + fast online evaluation]
- **Interpolate** projection coefficients at **empirical** interpolation nodes (time/frequency) over parameter space



Credit to Vijay Varma for images!
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- Begin by building a *reduced basis*

$$h(t, \boldsymbol{\lambda}) \approx \sum_{i=1}^{n} c_i(\boldsymbol{\lambda}) \hat{e}_i(t)$$

E.g. Field+ 13

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$$\max \epsilon = \max \left| h(t, \boldsymbol{\lambda}) - \sum_{i=1}^{n} c_i(\boldsymbol{\lambda})\, \hat{e}_i(t) \right|^2 \leq \sigma$$

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$$\mathrm{EI}[h](t; \boldsymbol{\lambda}) = \sum_{j=1}^{n} B_j(t) h(T_j; \boldsymbol{\lambda})$$

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Reduced Order Models and Surrogates

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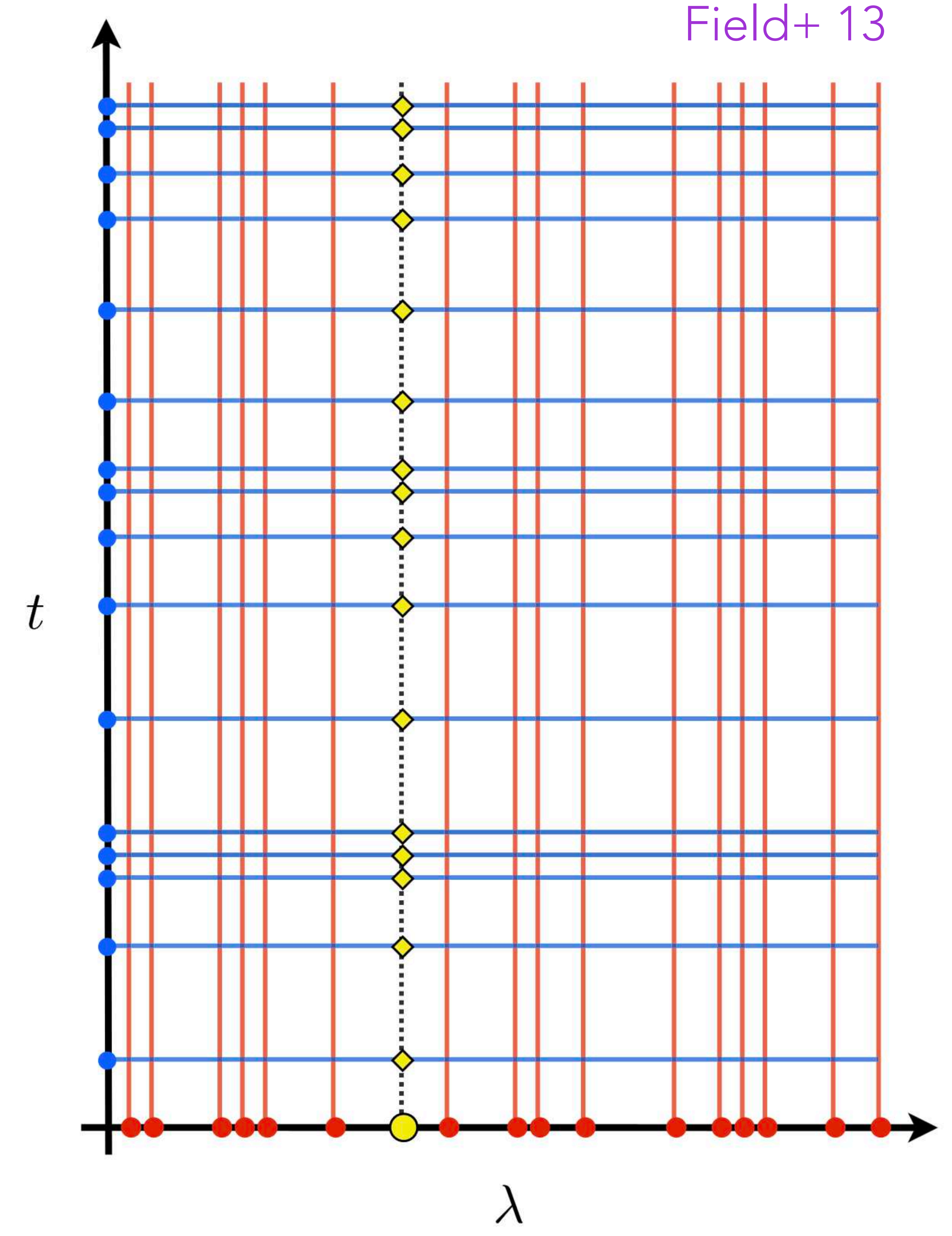
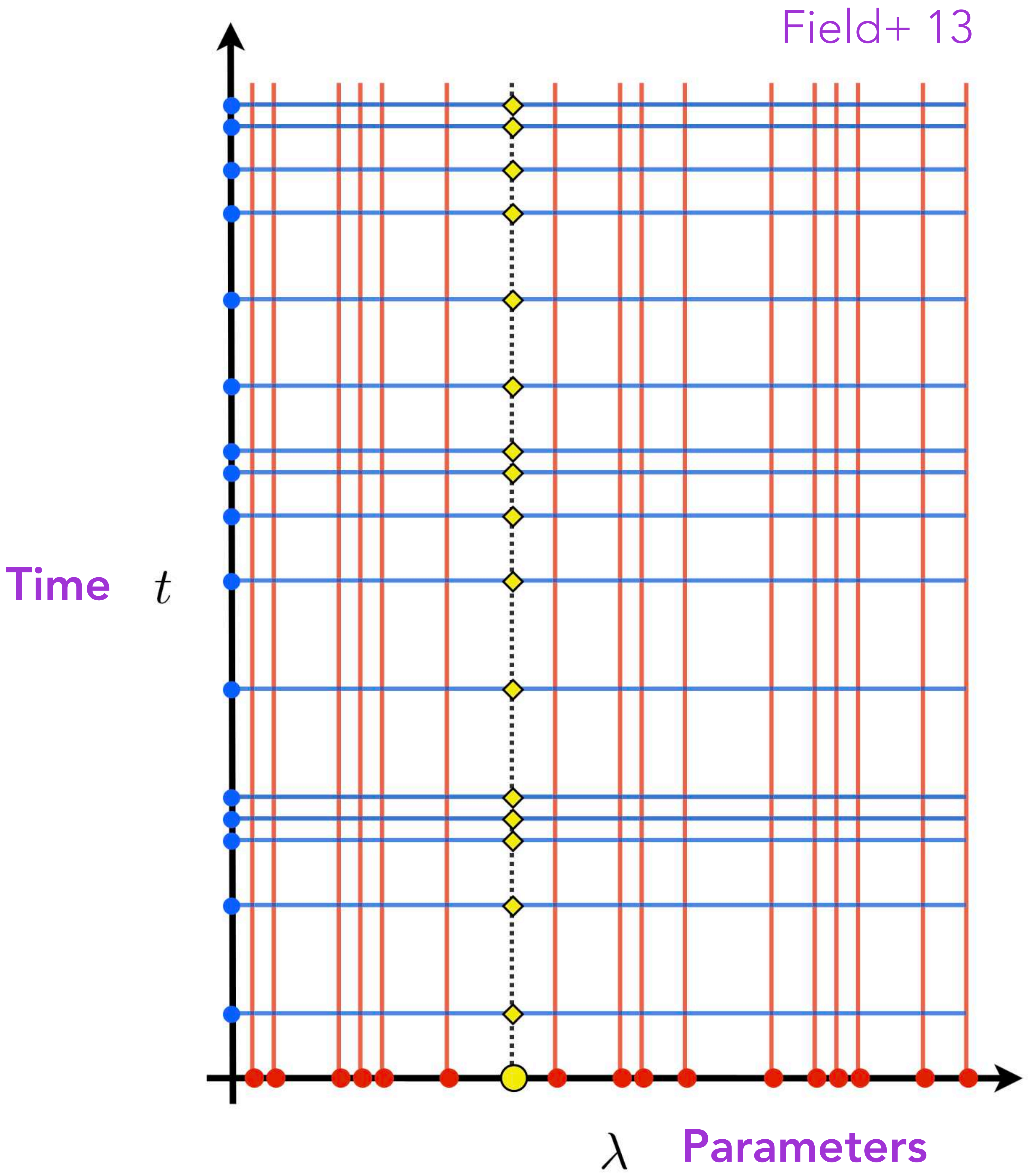
need a global fit across parameter space

E.g. Field+ 13


Field+ 13
t
λ


Field+ 13
Time t
λ Parameters

Reduced Order Models and Surrogates

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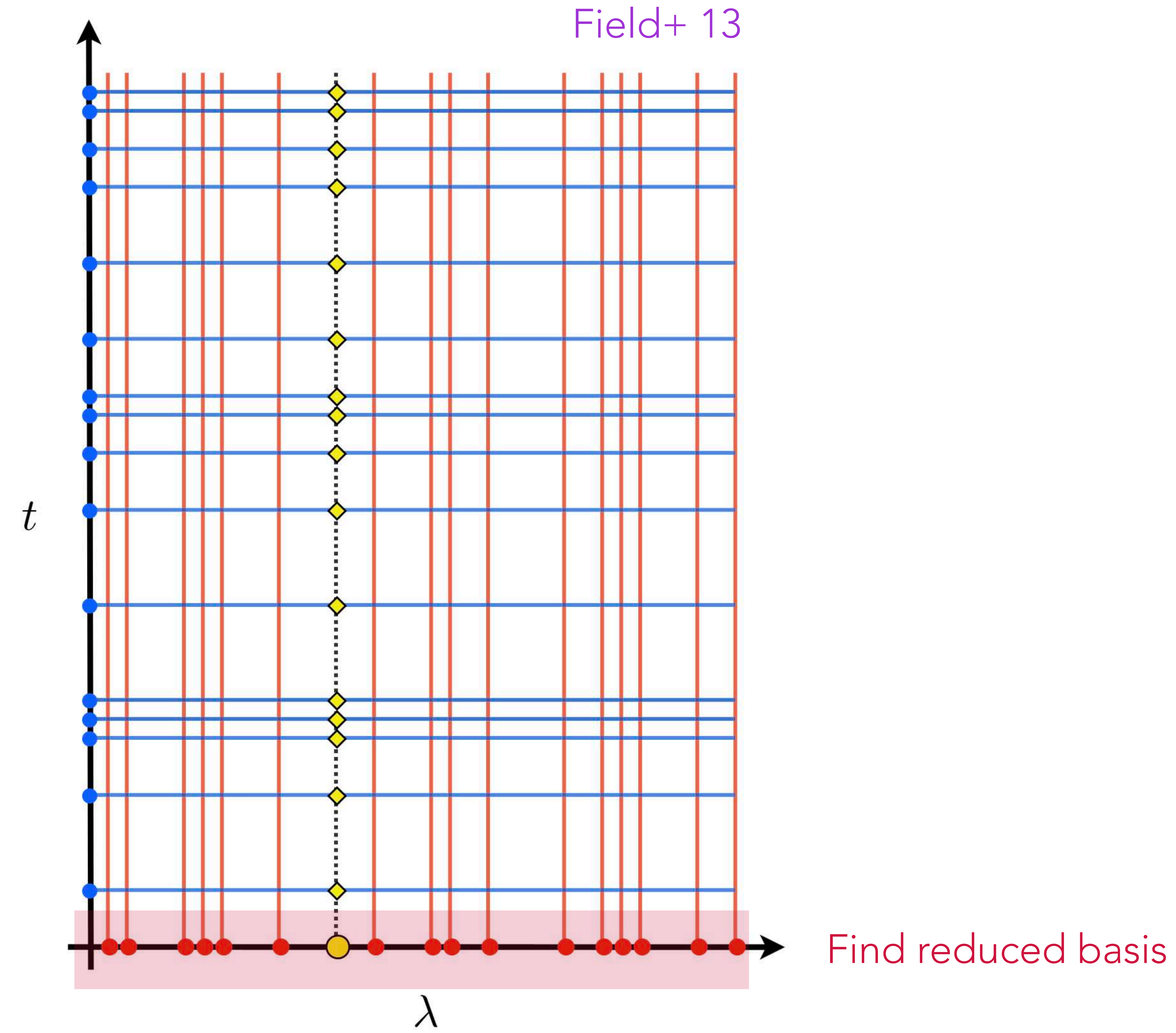
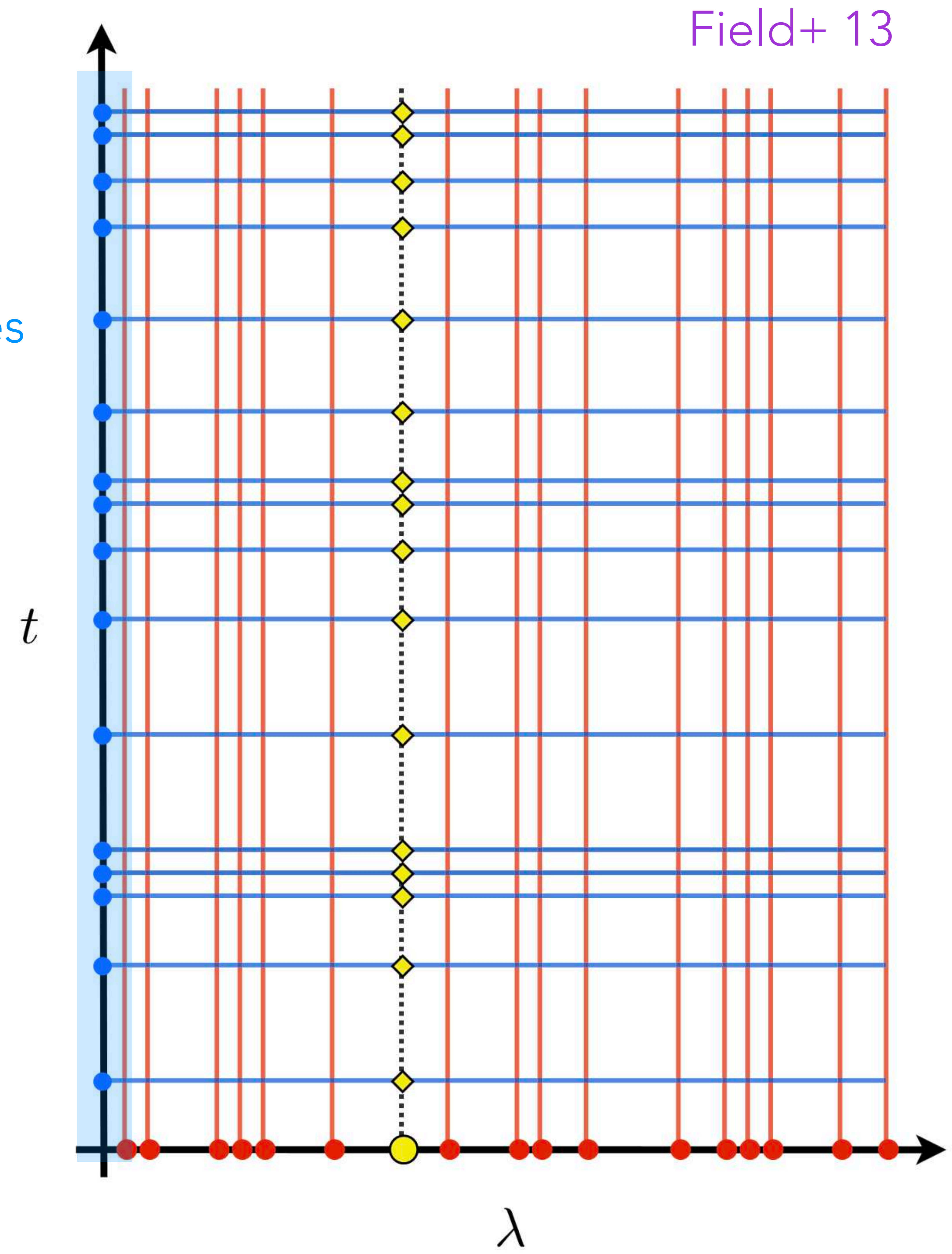
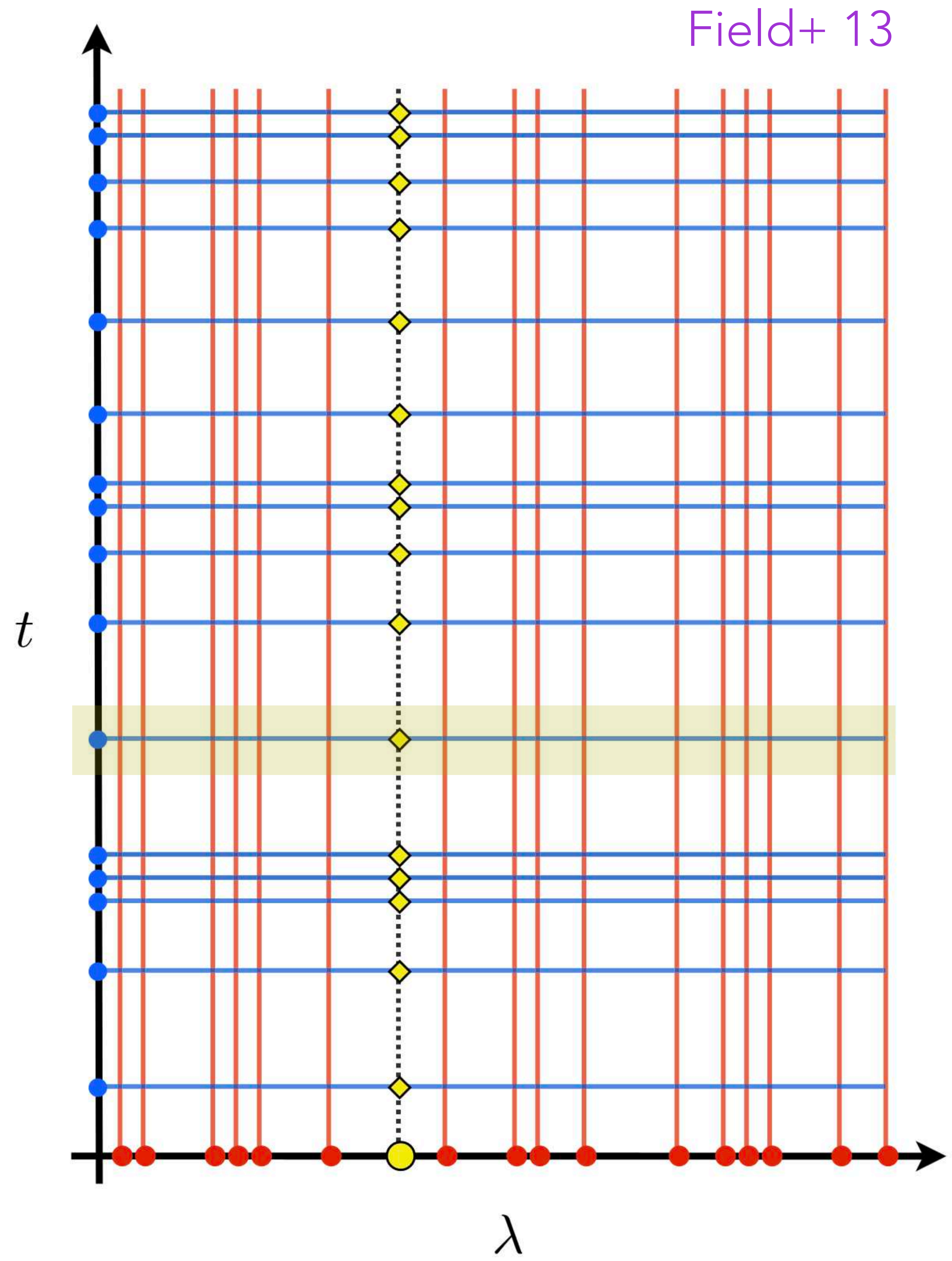


Then find empirical interpolation nodes

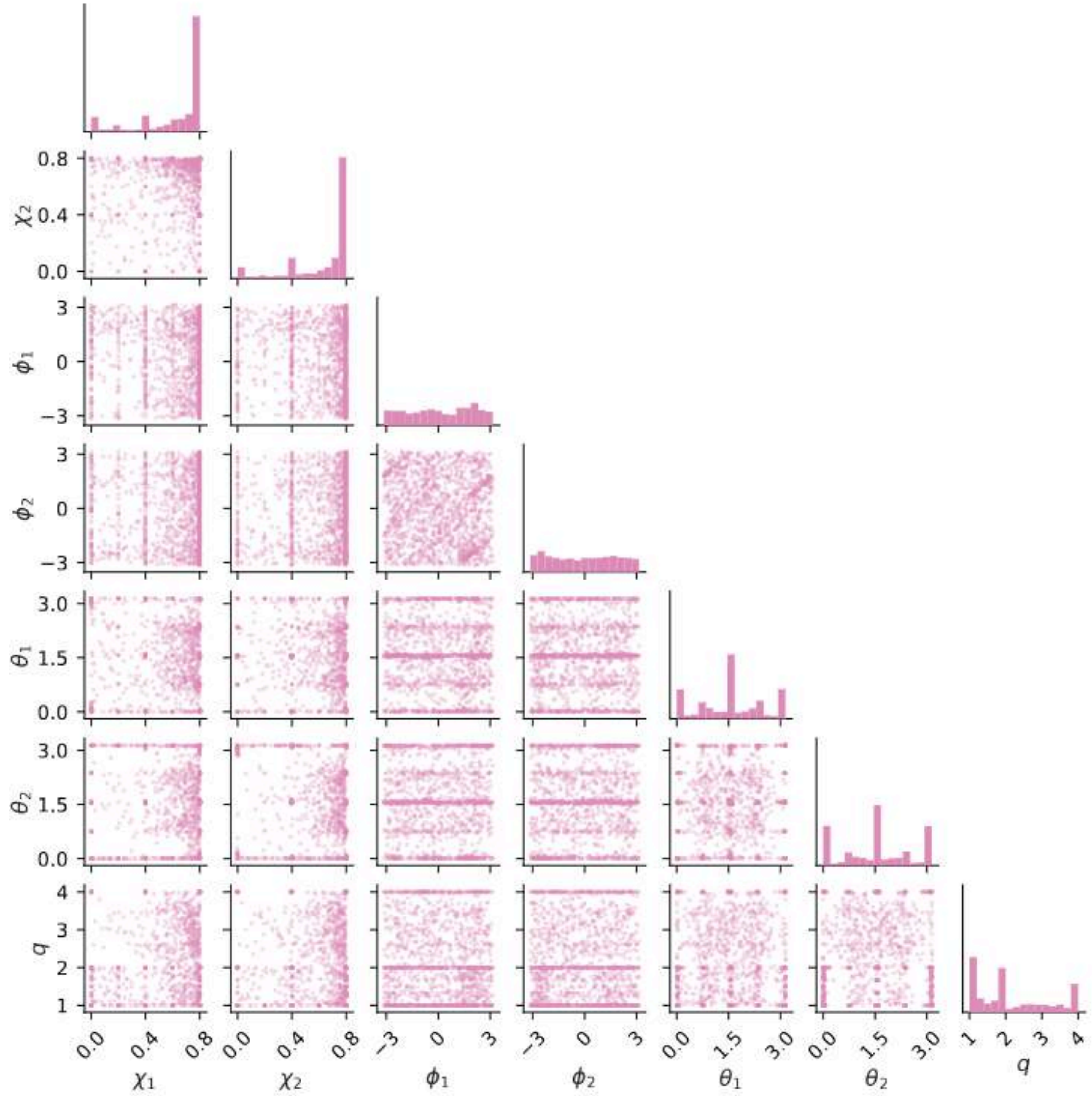




Fit empirical interpolation nodes across parameter space

Reduced Order Models and Surrogates

- Current state-of-the-art is NRSur7dq4
- 1528 precessing NR simulations used to build surrogate
  - Calibrated to $q = 4$ and $|\chi_i| = 0.8$
  - But extrapolation up to $q \sim 6$ and $|\chi_i| \sim 0.99$
- Recent surrogate work includes
  - Aligned-spin NR+PN surrogate with memory [Yoo+23]
  - Extension to extremal BH spins [Walker+22]
  - Surrogate for test particle waveforms [Islam+22]
  - Eccentric aligned-spin surrogate [Islam+22]



Varma+ arXiv:1905.09300

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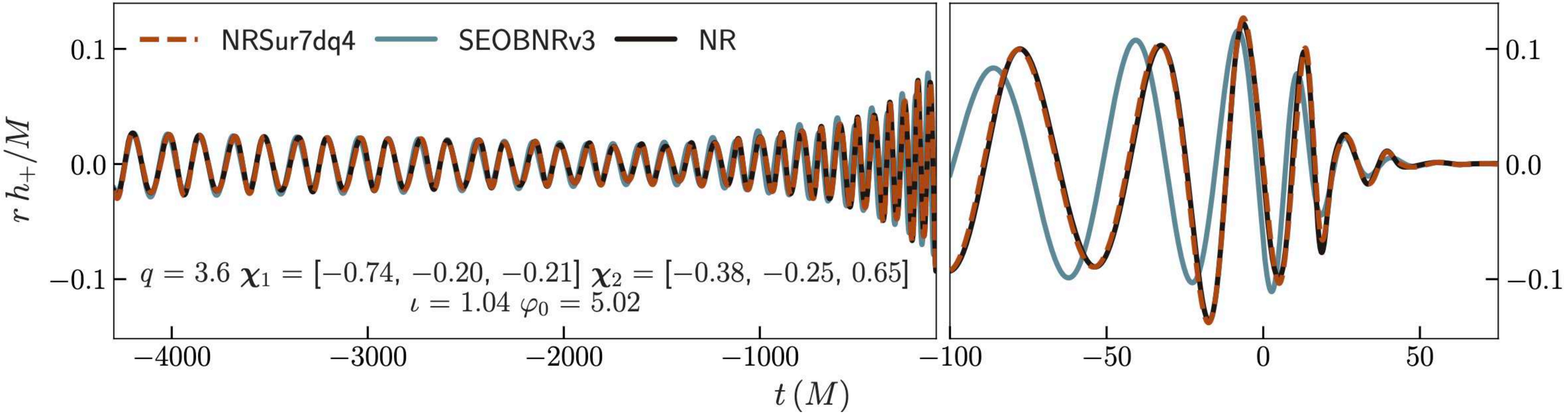
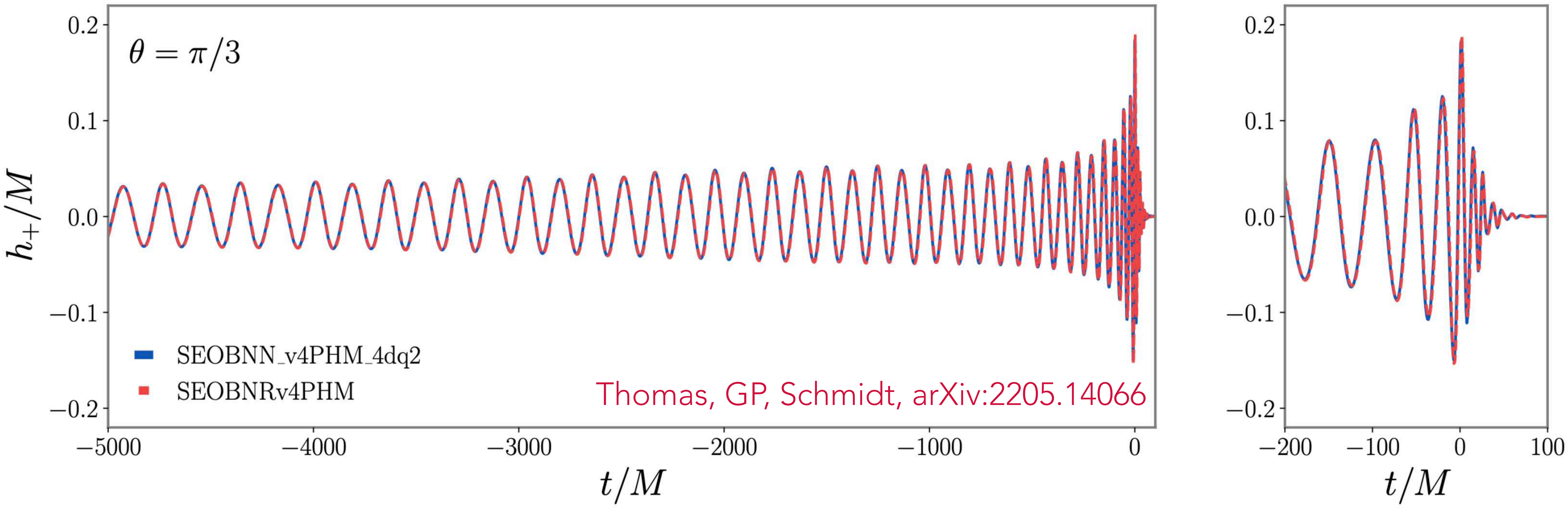


Reduced Order Models and Surrogates

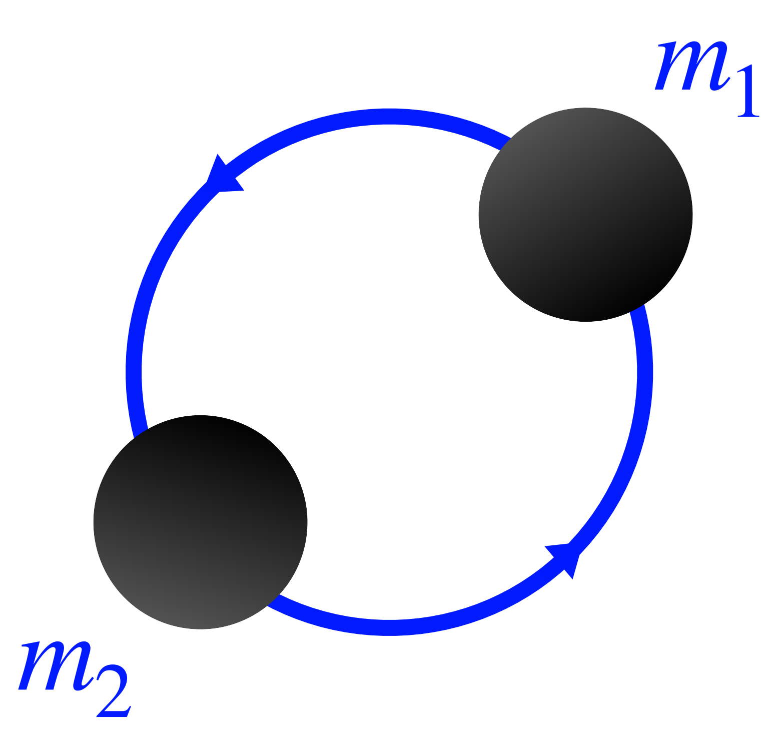
- Procedure can be applied to any semi-analytical waveform models [Fields+, Pürrer+, Lackey+, Thomas+, Varma+]

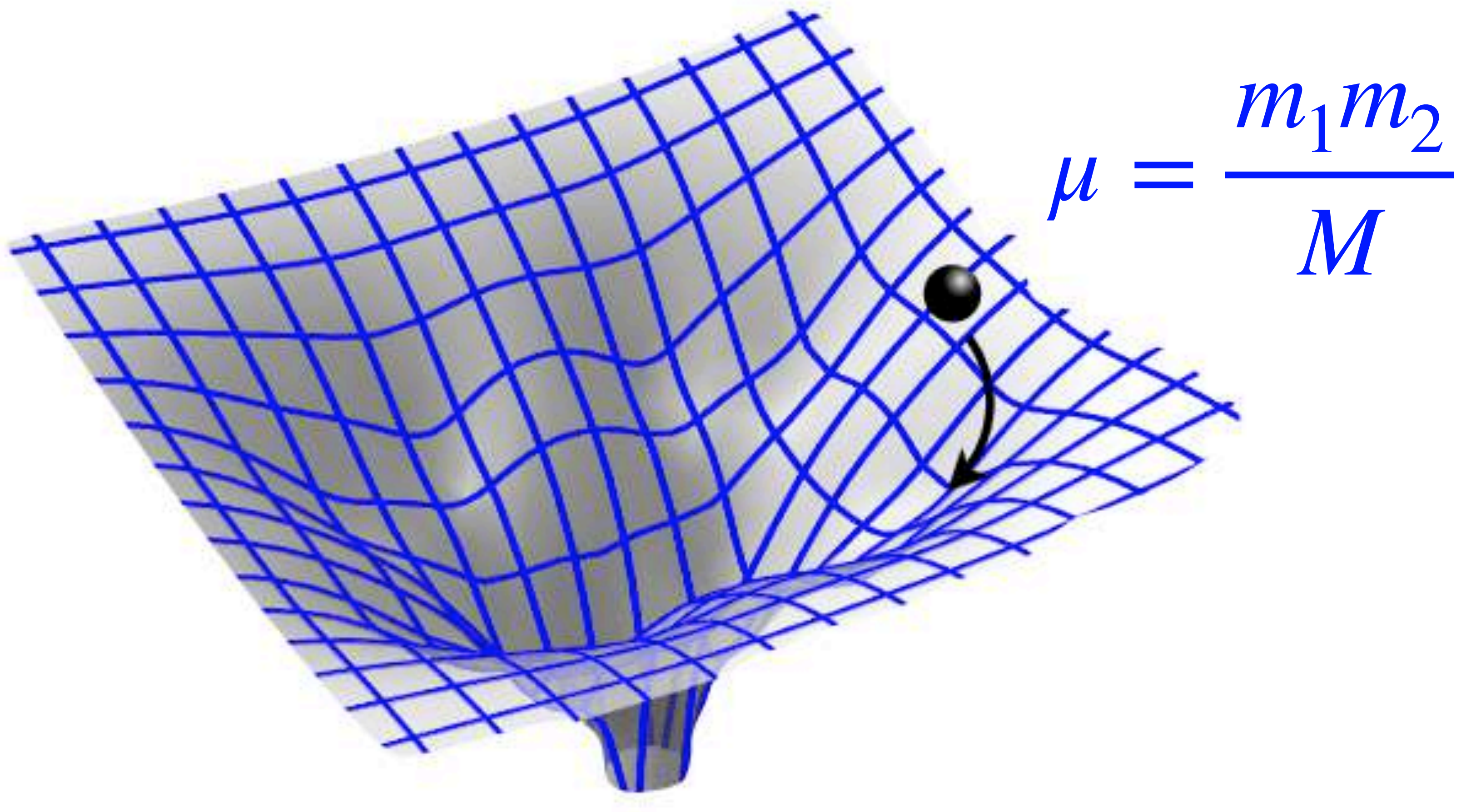


The Effective One Body Framework

Effective One Body

- Novel approach introduced by Buonanno and Damour in 1999
- Inspired by approach to EM interacting quantum two body problem [Brézin+ 1970]
- Basic idea is to map two-body problem onto an effective one-body problem via a canonical transformation





- Calculate motion of a test-particle in a deformed/effective external metric ↔ equations of motion

Buonanno and Damour 1999: arXiv:gr-qc/9811091
Buonanno and Damour 2000: arXiv:gr-qc/0001013

Effective One Body

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- Three main analytical components to an EOB model

Effective One Body

- Three main analytical components to an EOB model

1. Hamiltonian to describe the conservative binary dynamics

2. Radiation reaction (RR) force to account for loss of energy and angular momentum via emission of GWs

3. Gravitational waveform for inspiral, merger, and ringdown

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- Consider a test-particle orbiting a non-spinning BH of mass M

$$ds^2 = -\left(1 - \frac{2M}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2$$

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- We can write down the Hamiltonian for a test-particle of mass μ orbiting the BH

$$H_{\rm Schw}(\boldsymbol{r}, \boldsymbol{p}) = \sqrt{\left(1 - \frac{2M}{r}\right)\left[\mu^2 + \left(1 - \frac{2M}{r}\right) p_r^2 + \frac{p_\varphi^2}{r^2}\right]}$$

Effective One Body

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- And the effective radial potential

$$\frac{V_{\rm eff}^2(r)}{\mu^2} = \left(1 - \frac{2M}{r}\right)\left(1 + \frac{L^2}{\mu^2 r^2}\right)$$

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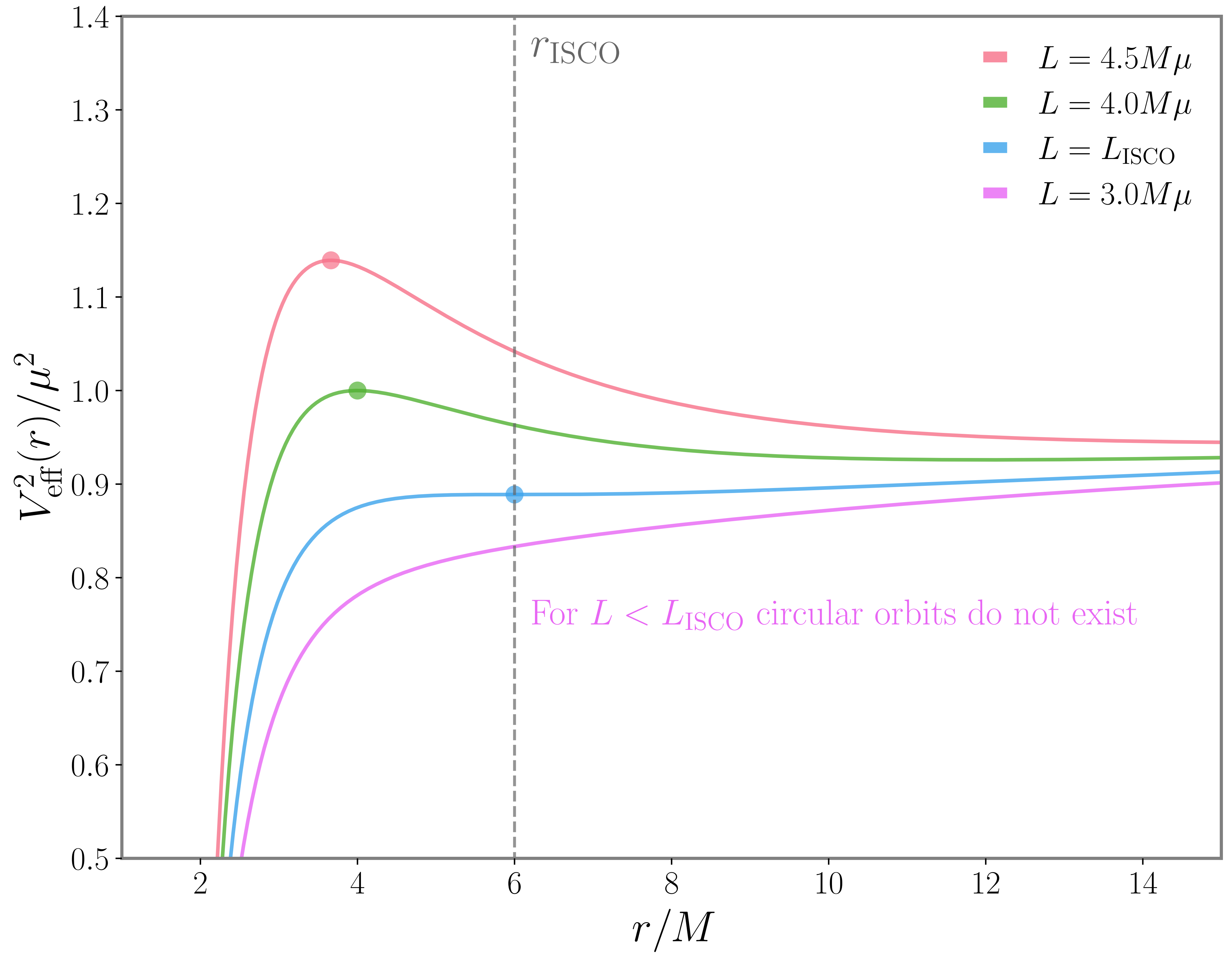

Effective One Body

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- In EOB *map* the two-body dynamics to geodesic motion of a single particle in effective metric

$$S_{\text{tot}}\left[z_1^\mu, z_2^\mu, g_{\mu\nu}\right] = -\int m_1 ds_1 - \int m_2 ds_2 + S_{\text{field}}\left[g_{\mu\nu}(x)\right]$$

See Buonanno+99

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gauge-fixed Einstein-Hilbert action

$$ds_i = \sqrt{-g_{\mu\nu}(z_i^\lambda)dz_i^\mu dz_i^\nu}$$

Line element along trajectory of particle

See Buonanno+99

Effective One Body

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THE ROYAL SOCIETY

- In EOB *map* the two-body dynamics to geodesic motion of a single particle in effective metric

$$S_{\text{tot}}\left[z_1^\mu, z_2^\mu, g_{\mu\nu}\right] = -\int m_1 ds_1 - \int m_2 ds_2 + S_{\text{field}}\left[g_{\mu\nu}(x)\right]$$

gauge-fixed Einstein-Hilbert action

$$ds_i = \sqrt{-g_{\mu\nu}(z_i^\lambda) dz_i^\mu dz_i^\nu}$$

Line element along trajectory of particle

- Associate real two-body dynamics to effective one-body dynamics in external spacetime

$$S_{\text{eff}}\left[z_0^\mu\right] = -\int m_0\, ds_0 \qquad ds_0 = \sqrt{-g_{\mu\nu}^{\text{eff}}(z_0^\lambda) dz_0^\mu dz_0^\nu}$$

Line element along trajectory of *effective* particle

deformed mass-shell condition
E.g. Damour 2001 arXiv:gr-qc/0103018

$$g_{\text{eff}}^{\mu\nu} p_\mu p_\nu + \mu^2 + Q = 0 \quad \rightarrow \quad H_{\text{eff}} = E_{\text{eff}} = -p_0$$

See Buonanno+99

Effective One Body

- Starting from the real nPN Hamiltonian

$$H_{\rm real}^{\rm PN} = H_{\rm Newt} + H_{\rm 1PN} + H_{\rm 2PN} + \cdots$$

- Write down ansatz for relation between EOB Hamiltonian and effective Hamiltonian

$$H_{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\nu} - 1\right)}$$

Not magical! Arises from inverting energy mapping. See Buonanno and Damour 1999

- $H_{\rm EOB}$ related to $H_{\rm PN}$ in a different gauge via canonical transformation

$$H_{\rm EOB} = H_{\rm PN} + \{\mathcal{G}, H_{\rm PN}\} + \frac{1}{2!}\{\mathcal{G}, \{\mathcal{G}, H_{\rm PN}\}\} + \cdots$$

Each bracket introduces factor of c^{-2}

- Find unknown coefficients in $H_{\rm eff}$ by matching LHS to RHS

Effective One Body

- Effective Hamiltonian describes geodesic motion of test-particle in deformed external spacetime

$$H_{\text{eff}}^{\nu} = \mu \sqrt{A_{\nu}(r) \left[\mu^2 + A_{\nu}(r) \bar{D}_{\nu}(r) p_r^2 + \frac{p_{\varphi}^2}{r^2} + Q_{\nu}(r, p_r) \right]}$$

Effective One Body

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Effective One Body

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- The dynamics is encoded in the potentials A_ν and D_ν

$u = 1/r$

$$A^{\rm Taylor}_{\rm non-spin}(u) = 1 - 2u + 2\nu u^3 + \nu\left(\frac{94}{3} - \frac{41\pi^2}{32}\right)u^4 + \left[\nu\left(\cdots\right) + \nu^2\left(\cdots\right) + \frac{64}{5}\nu\ln u\right]u^5 + \left[\nu a_6 + \cdots\right]u^6$$

E.g. Pompili+23 for SEOBNRv5

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a_6 is as-of-yet unknown PN coefficient that gets calibrated to NR

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Effective One Body

- Significant freedom in structure of Hamiltonian, gauge, PN information included, resummation etc

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We will come back to this later…

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THE ROYAL SOCIETY

- Significant freedom in structure of Hamiltonian, gauge, PN information included, resummation etc
- Resummation of analytical information critical in reducing difference with NR information

Effective One Body

- Significant freedom in structure of Hamiltonian, gauge, PN information included, resummation etc
- Resummation of analytical information critical in reducing difference with NR information
- Padé resummation is an effective strategy

$$F(u) = P^1_n \left[F_{\rm PN} \right] = \frac{1 + n_1 u}{1 + d_1 u + \cdots + d_n u^n}$$

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Damour & Nagar 09

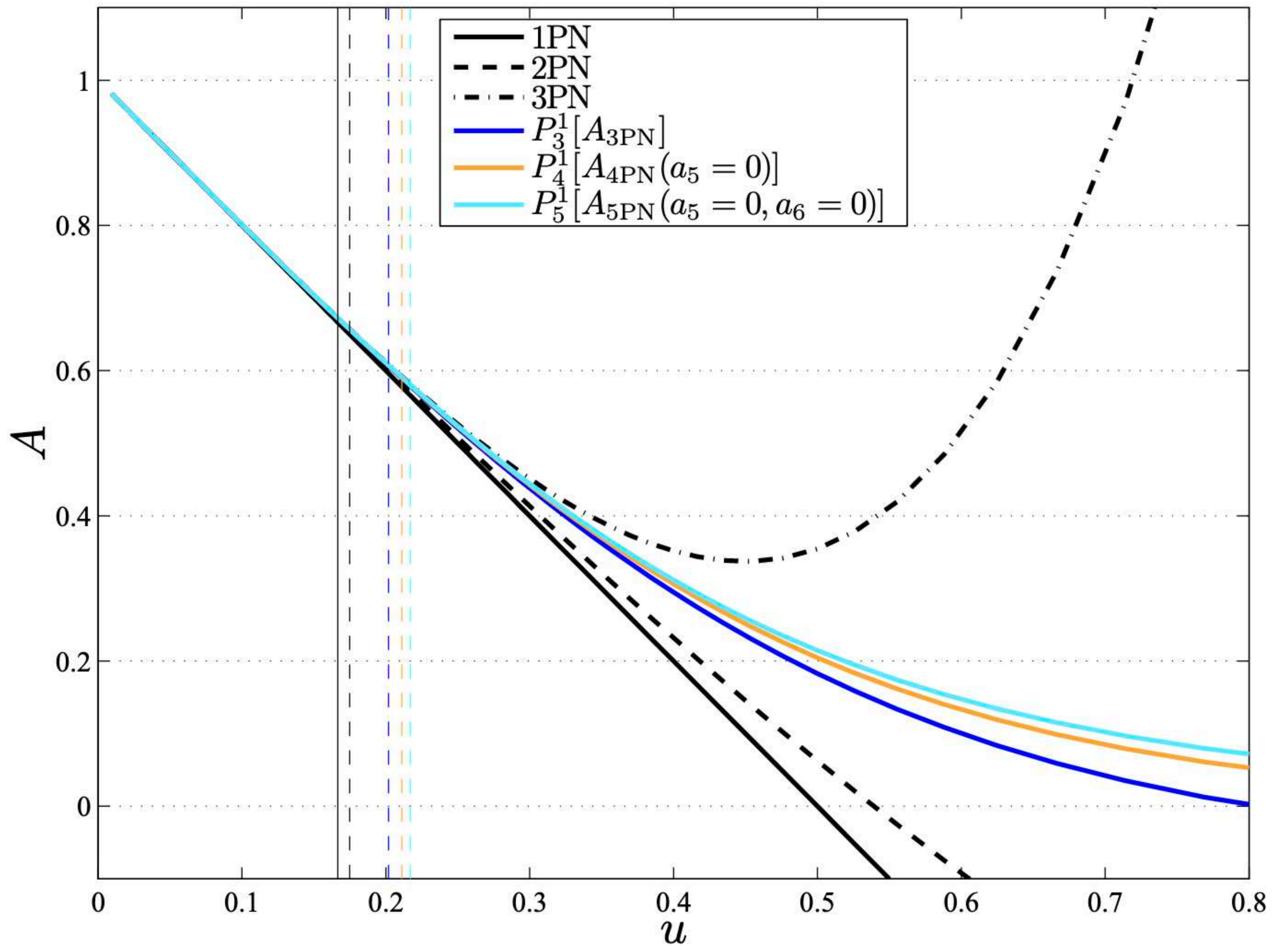

Fig. 2 Various approximations and Padé resummation of the EOB radial potential $A(u)$, where $u = GM/(c^2R)$, for the equal-mass case $\nu = 1/4$. The vertical dashed lines indicate the corresponding (adiabatic) LSO location [2] defined by the condition $d^2\mathscr{E}^0_{\rm eff}/dR^2 = d\mathscr{E}^0_{\rm eff}/dR = 0$, where $\mathscr{E}^0_{\rm eff}$ is the effective energy along the sequence of circular orbits (*i.e.*, when $P^{\rm eff}_R = 0$).

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- Example: resummation used in SEOBNRv5 (Pompili+23)

$$A_{\mathrm{non-spin}}(u) = P^1_5\left[A^{\mathrm{Taylor}}_{\mathrm{non-spin}}(u)\right]$$

$$\bar{D}_{\mathrm{non-spin}}(u) = P^2_3\left[\bar{D}^{\mathrm{Taylor}}_{\mathrm{non-spin}}(u)\right]$$

Effective One Body

- Can generalise the Hamiltonian to include spin

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THE ROYAL SOCIETY

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$$H_{\text{eff}} = \sqrt{A(r)\left[\mu^2 c^2 + A(r)\bar{D}(r)\frac{p_r^2}{c^2} + \frac{L^2}{c^2 r^2} + Q(r, p_r)\right]}$$
$$+ \frac{1}{c^3 r^3}\boldsymbol{L} \cdot \left[g_S(r, p_r)\boldsymbol{S} + g_{S^*}(r, p_r)\boldsymbol{S}^*\right].$$

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inclusion of spin effects is not unique
(e.g. gauge choices)

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inclusion of spin effects is not unique (e.g. gauge choices)

- Include PN SO information through gyro-gravitomagnetic terms

$$g_S = 2 - \frac{1}{c^2}\left[\frac{27\nu}{16}\frac{p_r^2}{\mu^2} + \frac{5\nu M}{16 r}\right] + \cdots + \frac{1}{c^8}\left(g_S^{5.5\text{PN,loc}} + g_S^{5.5\text{PN,nonloc}}\right),$$
$$g_{S^*} = \frac{3}{2} - \frac{1}{c^2}\left[\left(\frac{3\nu}{2} + \frac{5}{4}\right)\frac{p_r^2}{\mu^2} + \left(\frac{3}{4} + \frac{\nu}{2}\right)\frac{M}{r}\right] + \cdots + \frac{1}{c^8}\left(g_{S^*}^{5.5\text{PN,loc}} + g_{S^*}^{5.5\text{PN,nonloc}}\right)$$

Damour 00, Barausse 11, Nagar 11, Khalil 23 + many others

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THE ROYAL SOCIETY

- Can also generalise the Hamiltonian to arbitrary spins

E.g. Balmelli+15, Khalil+20, Khalil+23

Effective One Body

- Can also generalise the Hamiltonian to arbitrary spins

$$ds^2 = g^{\mu\nu}_{\mathrm{Kerr}} \partial_\mu \partial_\nu$$

$$ds^2 = -\frac{\Lambda}{\Delta\Sigma}\partial_t^2 + \frac{\Delta}{\Sigma}\partial_r^2 + \frac{1}{\Sigma}\partial_\theta^2 + \frac{\Sigma - 2Mr}{\Sigma\Delta\sin^2\theta}\partial_\phi^2 - \frac{4Mra}{\Sigma\Delta}\partial_t\partial_\phi$$

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↓

$$H^{\rm Kerr} = \frac{2Mr}{\Lambda}\boldsymbol{L}\cdot\boldsymbol{a} + \left[A^{\rm Kerr}\left(\mu^2 + B^{\rm Kerr}_{\rm np}(\boldsymbol{n}\cdot\boldsymbol{p})^2 + B^{\rm Kerr}_{p}\boldsymbol{p}^2 + B^{\rm Kerr}_{\rm npa}(\boldsymbol{n}\times\boldsymbol{p}\cdot\boldsymbol{a})^2\right)\right]^{1/2}$$

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$$H^{\text{Kerr}} = \frac{2Mr}{\Lambda}\boldsymbol{L}\cdot\boldsymbol{a} + \left[A^{\text{Kerr}}\left(\mu^2 + B^{\text{Kerr}}_{\text{np}}(\boldsymbol{n}\cdot\boldsymbol{p})^2 + B^{\text{Kerr}}_{p}\boldsymbol{p}^2 + B^{\text{Kerr}}_{\text{npa}}(\boldsymbol{n}\times\boldsymbol{p}\cdot\boldsymbol{a})^2\right)\right]^{1/2}$$

$$H^{\text{prec}}_{\text{eff}} = \frac{Mr}{\Lambda}\left[\boldsymbol{L}\cdot\left(g_{a_+}\boldsymbol{a}_+ + g_{a_-}\delta\boldsymbol{a}_-\right) + \text{SO}_{\text{calib}} + G^{\text{prec}}_{a^3}\right] + \left[A^{\text{prec}}\left(\mu^2 + B^{\text{prec}}_{p}\boldsymbol{p}^2 + B^{\text{prec}}_{np}(\boldsymbol{n}\cdot\boldsymbol{p})^2 + B^{\text{Kerr}}_{npa}(\boldsymbol{n}\times\boldsymbol{p}\cdot\boldsymbol{a}_+)^2 + Q^{\text{prec}}\right)\right]^{1/2}$$

Map to effective metric of deformed Kerr background with $\boldsymbol{a} = \boldsymbol{a}_1 + \boldsymbol{a}_2$

E.g. Balmelli+15, Khalil+20, Khalil+23

Effective One Body

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↓

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Now things are getting messy…

E.g. Balmelli+15, Khalil+20, Khalil+23

- Great, so we have a Hamiltonian…

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- Great, so we have a Hamiltonian… now what?

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- Great, so we have a Hamiltonian… now what?
- The Hamilton-Jacobi equations provide the equations of motion…

$$\frac{d\boldsymbol{r}}{dt} = \frac{\partial H_{\mathrm{EOB}}}{\partial \boldsymbol{p}}\,, \qquad \frac{d\boldsymbol{p}}{dt} = -\frac{\partial H_{\mathrm{EOB}}}{\partial \boldsymbol{r}} + \boldsymbol{\mathcal{F}}\,, \qquad \frac{d\boldsymbol{S}_{1,2}}{dt} = \frac{\partial H_{\mathrm{EOB}}}{\partial \boldsymbol{S}_{1,2}} \times \boldsymbol{S}_{1,2},$$

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radiation reaction force

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- The RR force ↔ flux of angular momentum expressed as sum of factorised and resummed multipoles

$$\mathcal{F}_\varphi = -\frac{M\Omega}{8\pi} \sum_{\ell=2}^{\ell_{\rm max}} \sum_{m=1}^{\ell} m^2 |d_L\, h^F_{\ell m}|^2$$

Effective One Body

- The inspiral-plunge EOB modes can be written as

factorised modes

$$h_{\ell m}^{\mathrm{IM}} = h_{\ell m}^{F} \, N_{\ell m}$$

non quasi-circular (NQC) correction

- Factorisation of each multipole, e.g. Damour+07, Damour+08, Pan+10

$$h_{\ell m}^{F}(t) = h_{\ell m}^{(N,\epsilon)} \, S_{\mathrm{eff}}^{(\epsilon)} \, T_{\ell m} \, e^{i\delta_{\ell m}} \, (\rho_{\ell m})^{\ell} \, h_{\ell m}^{\mathrm{NQC}}$$

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Newtonian contribution, ϵ denotes parity of mode

$$h_{\ell m}^{F}(t) = h_{\ell m}^{(N,\epsilon)}\, S_{\rm eff}^{(\epsilon)}\, T_{\ell m}\, e^{i\delta_{\ell m}}\, (\rho_{\ell m})^{\ell}\, h_{\ell m}^{\rm NQC}$$

Effective One Body

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Effective source term: $$\hat{S}_{\mathrm{eff}} = \begin{cases} \dfrac{E_{\mathrm{eff}}(v_\Omega)}{\mu}, & \ell + m \text{ even}, \\ v_\Omega \dfrac{p_\phi(v_\Omega)}{M\mu}, & \ell + m \text{ odd}, \end{cases}$$

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Resums infinite number of leading logarithms: $T_{\ell m} = \frac{\Gamma(\ell + 1 - 2i\hat{k})}{\Gamma(\ell + 1)} e^{\pi \hat{k}} e^{2i\hat{k} \ln(2m\Omega r_0)}$

$$h_{\ell m}^{F}(t) = h_{\ell m}^{(N,\epsilon)} \, S_{\mathrm{eff}}^{(\epsilon)} \, T_{\ell m} \, e^{i\delta_{\ell m}} \, (\rho_{\ell m})^{\ell} \, h_{\ell m}^{\mathrm{NQC}}$$

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Residual phase correction due to sub-leading logarithms

- The inspiral-plunge EOB modes can be written as

factorised modes

$$h_{\ell m}^{\mathrm{IM}} = h_{\ell m}^{F} \, N_{\ell m}$$

non quasi-circular (NQC) correction

- Factorisation of each multipole, e.g. Damour+07, Damour+08, Pan+10

Residual amplitude correction such that expansion agrees with PN-expanded modes

$$h_{\ell m}^{F}(t) = h_{\ell m}^{(N,\epsilon)} \, S_{\mathrm{eff}}^{(\epsilon)} \, T_{\ell m} \, e^{i\delta_{\ell m}} \, (\rho_{\ell m})^{\ell} \, h_{\ell m}^{\mathrm{NQC}}$$

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Phenomenological NQC corrections to shape waveform during late-plunge up to merger

Effective One Body

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- Complete EOB waveform by attaching a merger-ringdown

$$h_{\ell m}(t) = h^{\mathrm{IM}}_{\ell m}(t) \, \Theta\left(t^{\ell m}_{\mathrm{match}} - t\right) + h^{\mathrm{R}}_{\ell m}(t) \, \Theta\left(t - t^{\ell m}_{\mathrm{match}}\right) \, ,$$

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$$h_{\ell m}^{\mathrm{IM}} = h_{\ell m}^{F} \, N_{\ell m}$$ non quasi-circular (NQC) correction

- Factorisation of each multipole, e.g. Damour+07, Damour+08, Pan+10

$$h_{\ell m}^{F}(t) = h_{\ell m}^{(N,\epsilon)} \, S_{\mathrm{eff}}^{(\epsilon)} \, T_{\ell m} \, e^{i\delta_{\ell m}} \, (\rho_{\ell m})^{\ell} \, h_{\ell m}^{\mathrm{NQC}}$$

- Complete EOB waveform by attaching a merger-ringdown

$$h_{\ell m}^{\mathrm{R}}(t) \sim \sum_{n=0}^{\infty} A_{\ell m n} e^{-i\sigma_{\ell m n} t}$$

$$h_{\ell m}(t) = h_{\ell m}^{\mathrm{IM}}(t)\,\Theta\left(t_{\mathrm{match}}^{\ell m} - t\right) + h_{\ell m}^{\mathrm{R}}(t)\,\Theta\left(t - t_{\mathrm{match}}^{\ell m}\right),$$

Ringdown well described via BH perturbation theory as superposition of QNM

Effective One Body

- Two main flavours of EOB are SEOBNR and TEOBResumS

Effective One Body

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  - Pompili+23 https://arxiv.org/abs/2303.18039
  - Khalil+23 https://arxiv.org/abs/2303.18143
  - Ramos-Buades+23 https://arxiv.org/abs/2303.18046
  - Maarten+23 https://arxiv.org/abs/2303.18026
  - Damour+14 https://arxiv.org/abs/1406.6913
  - Nagar+18 https://arxiv.org/abs/1806.01772
  - Nagar+21 https://arxiv.org/abs/2108.02043
  - Nagar+23 https://arxiv.org/abs/2304.09662

Some starter references (but highly incomplete)

Effective One Body

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higher-order spin information	NNLO SS (4PN), LO S^3 (3.5PN), LO S^4 (4PN)	LO SS (2PN)	NNLO SS (4PN) for circular orbits
precessing-spin Hamiltonian	yes	yes	no
spin-multipole constants included	yes	no	yes (in the SS contributions for circular orbits)

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precessing-spin Hamiltonian	yes	yes	no
spin-multipole constants included	yes	no	yes (in the SS contributions for circular orbits)

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Different analytical information included in the various potentials

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precessing-spin Hamiltonian	yes	yes	no
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Same resummation of A-potential

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precessing-spin Hamiltonian	yes	yes	no
spin-multipole constants included	yes	no	yes (in the SS contributions for circular orbits)

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Different resummation of D-potential

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Different limits of Hamiltonian in test-particle limit

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Different gauge choices and resummation for spin-orbit interactions

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Differing analytical spin information

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SEOBNRv5 has incorporates precession into the Hamiltonian

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Again, differing analytical information in multipole moments

- SEOBNRv5 is the current fifth generation of SEOBNR model

Laying the foundation of the effective-one-body waveform models SEOBNRv5: improved accuracy and efficiency for spinning non-precessing binary black holes

Lorenzo Pompili,[1,*] Alessandra Buonanno,[1,2] Héctor Estellés,[1] Mohammed Khalil,[3,1,2] Maarten van de Meent,[1,4] Deyan P. Mihaylov,[1] Serguei Ossokine,[1] Michael Pürrer,[5,6,1] Antoni Ramos-Buades,[1] Ajit Kumar Mehta,[7,1] Roberto Cotesta,[8] Sylvain Marsat,[9] Michael Boyle,[10] Lawrence E. Kidder,[10] Harald P. Pfeiffer,[1] Mark A. Scheel,[11] Hannes R. Rüter,[12] Nils Vu,[11] Reetika Dudi,[1] Sizheng Ma,[11] Keefe Mitman,[11] Denyz Melchor,[13] Sierra Thomas,[13,14] and Jennifer Sanchez[15]

SEOBNRv5PHM: Next generation of accurate and efficient multipolar precessing-spin effective-one-body waveforms for binary black holes

Antoni Ramos-Buades,[1,*] Alessandra Buonanno,[1,2] Héctor Estellés,[1] Mohammed Khalil,[3,1,2] Deyan P. Mihaylov,[1] Serguei Ossokine,[1] Lorenzo Pompili,[1] and Mahlet Shiferaw[1,4]

Theoretical groundwork supporting the precessing-spin two-body dynamics of the effective-one-body waveform models SEOBNRv5

Mohammed Khalil,[1,2,3,*] Alessandra Buonanno,[2,3] Héctor Estellés,[2] Deyan Mihaylov,[2] Serguei Ossokine,[2] Lorenzo Pompili,[2] and Antoni Ramos-Buades[2]

Enhancing the SEOBNRv5 effective-one-body waveform model with second-order gravitational self-force fluxes

Maarten van de Meent,[1,2] Alessandra Buonanno,[2] Deyan P. Mihaylov,[2] Serguei Ossokine,[2] Lorenzo Pompili,[2] Niels Warburton,[3] Adam Pound,[4] Barry Wardell,[3] Leanne Durkan,[3,5] and Jeremy Miller[6]

Effective One Body

- Key highlights from SEOBNRv5
- Most of 5PN NS terms and all generic-spin terms up to 4PN [SO + SS at NNLO, S^3 + S^4 terms at LO]
- Improved NR calibration [442 simulations + 13 test particle]
- PN information from orbit-averaged generic Hamiltonian
- In-plane spin-effects via partially precessing Hamiltonian
- Calibration of NS modes and radiation-reaction up to 2-GSF
- Precession equations PN-expanded with self-consistent SO and SS couplings at NNLO (e.g. SSC)
- Mismatch against 1543 precessing-spin NR simulations have 99.8% of cases with $1 - \mathcal{M} < 3\,\%$
- Calibration to 2GSF flux

Effective One Body: Highlights from SEOBNRv5

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- Partial-precession effects in Hamiltonian

$$H_{\rm eff}^{\rm pprec} = \frac{M p_\phi \boldsymbol{l} \cdot (g_{a_+} \boldsymbol{a}_+ + g_{a_-} \delta \boldsymbol{a}_-) + {\rm SO}_{\rm calib} + \left\langle G_{a^3}^{\rm pprec} \right\rangle}{r^3 + a_+^2 (r + 2M)} + \left[A^{\rm pprec} \left(\mu^2 + B_p^{\rm pprec} \frac{p_\phi^2}{r^2} + \left(1 + B_{np}^{\rm pprec}\right) (\boldsymbol{n} \cdot \boldsymbol{p})^2 \right.\right.$$

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(Khalil+2023)

full-precessing-spin (prec) Hamiltonian

$$H_{\text{EOB}}^{\text{prec}}(\mathbf{r}, \mathbf{p}, \mathbf{a}_+, \mathbf{a}_-)$$

$\mathbf{r}, \mathbf{p} \rightarrow$ canonical variables in center of mass

$\mathbf{a}_{1,2} \rightarrow$ body's spins $\qquad \mathbf{a}_\pm = \mathbf{a}_1 \pm \mathbf{a}_2$

$\nu \rightarrow$ symmetric mass ratio

→ orbit-averaging in-plane spins for circular orbits

simplifying PN resummation, requiring that $H_{\text{EOB}}^{\text{pprec}}$ reduces to $H_{\text{EOB}}^{\text{prec}}$ when PN expanded through $\mathcal{O}(a_\pm^3)$

(Khalil+2023)

partial-precessing-spin (pprec) Hamiltonian

$$H_{\text{EOB}}^{\text{pprec}}(r, p_r, \mathbf{l}^2, \mathbf{a}_\pm \cdot \mathbf{l}_\text{N}, \mathbf{a}_\pm \cdot \mathbf{l}, \mathbf{a}_\pm^2, \mathbf{a}_+ \cdot \mathbf{a}_-)$$

$\mathbf{l} \rightarrow$ orbital angular momentum

$\mathbf{l}_\text{N} \rightarrow$ Newtonian angular momentum

PN-expanded EOB equations for spins, angular momentum, velocity parameter $\text{v} = (M\omega)^{1/3}$

$$\dot{\mathbf{a}}_\pm = \boldsymbol{\Omega}_{\mathbf{a}_\pm} \times \mathbf{a}_\pm$$

$$\mathbf{l} = \mathbf{l}(\mathbf{l}_\text{N}, \text{v}, \mathbf{a}_\pm)$$

$$\dot{\mathbf{l}}_\text{N} = \dot{\mathbf{l}}_\text{N}(\mathbf{l}_\text{N}, \text{v}, \mathbf{a}_\pm)$$

$$\dot{\text{v}} = \left[\frac{\dot{E}(\text{v})}{dE_{\text{EOB}}(\text{v})/d\text{v}} \right]_{\text{PN expanded}}$$

→

EOB orbital equations in co-precessing (P) frame

$$\dot{r} = \frac{\partial H_{\text{EOB}}^{\text{pprec}}}{\partial p_r} \qquad \omega \equiv \dot{\phi} = \frac{\partial H_{\text{EOB}}^{\text{pprec}}}{\partial p_\phi}$$

$$\dot{p}_r = -\frac{\partial H_{\text{EOB}}^{\text{pprec}}}{\partial r} + \mathcal{F}_r \qquad \dot{p}_\phi = \mathcal{F}_\phi$$

↓

Co-precessing frame EOB waveform multipoles

$$h_{\ell m}^{\text{P}}(t) = h_{\ell m}^{\text{P}}(r, \text{v}, \mathbf{a}_\pm \cdot \mathbf{l}_\text{N})$$

←

Inertial-frame EOB waveform multipoles

$$h_{\ell m}^{\text{I}}(t) = \sum_{m', m''} \mathbf{R}_{mm'}^{\text{JI}} \mathbf{R}_{m'm''}^{\text{PJ}} h_{\ell m''}^{\text{P}}(t)$$

Effective One Body: Highlights from SEOBNRv5

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- Calibration to 2nd order gravitational-self-force (GSF) flux

Effective One Body: Highlights from SEOBNRv5

- Calibration to 2nd order gravitational-self-force (GSF) flux
- GSF models metric of binary as expansion in $\epsilon = m_2/m_1$

$$g_{\alpha\beta} + \sum_{m=\infty}^{\infty} \left[\epsilon h^{1,m}_{\alpha\beta}(\Omega) + \epsilon^2 h^{2,m}_{\alpha\beta}(\Omega)\right] e^{-im\varphi} + \mathcal{O}(\epsilon^3)$$

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Schwarzschild metric

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metric amplitudes

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$$\mathcal{F}^{\mathrm{GSF}\epsilon}_{\ell m}(\epsilon, y) = \epsilon^2 \mathcal{F}^{\mathrm{GSF}1\epsilon}_{\ell m}(y) + \epsilon^3 \mathcal{F}^{\mathrm{GSF}2\epsilon}_{\ell m}(y) + \mathcal{O}(\epsilon^4)$$

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$$\mathcal{F}^{\mathrm{EOB}}_{\ell m} = d_L^2 \frac{(mM\Omega)^2}{8\pi} \left|h^{\mathrm{N}}_{\ell m}\right|^2 \left|\hat{S}_{\ell m}\right|^2 \left|T_{\ell m}\right|^2 \left|\rho_{\ell m}\right|^{2\ell}$$

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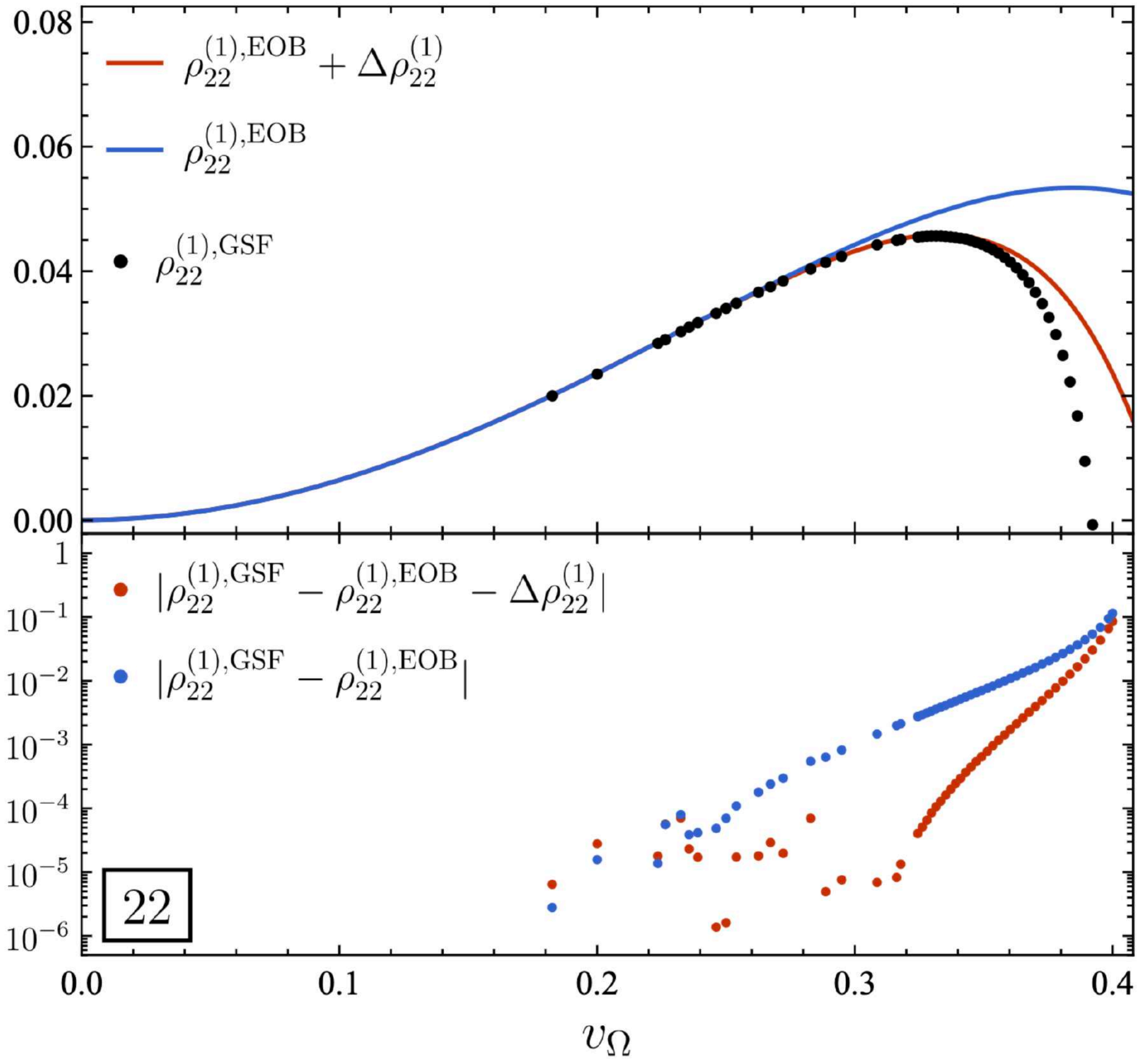

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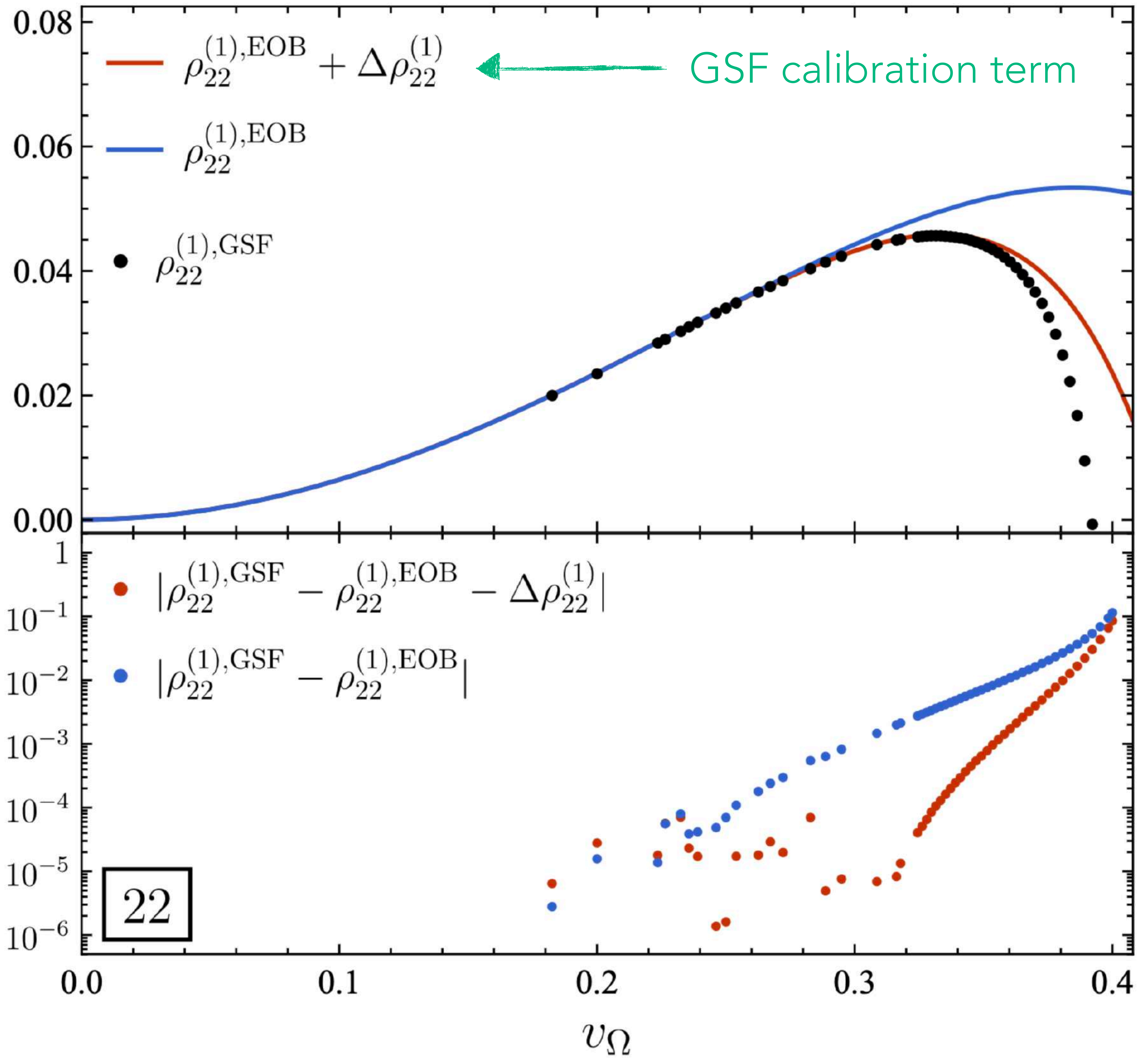

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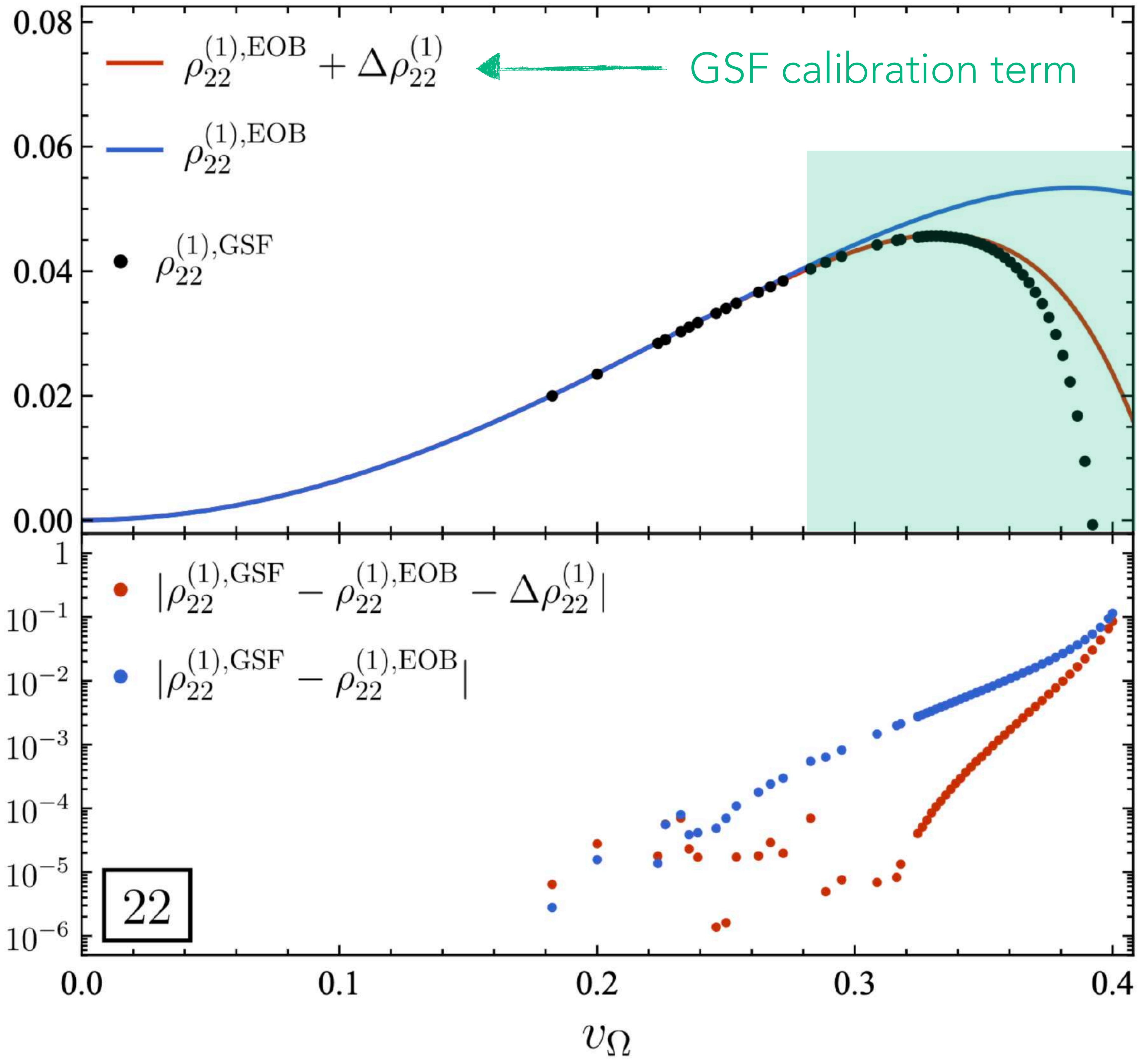

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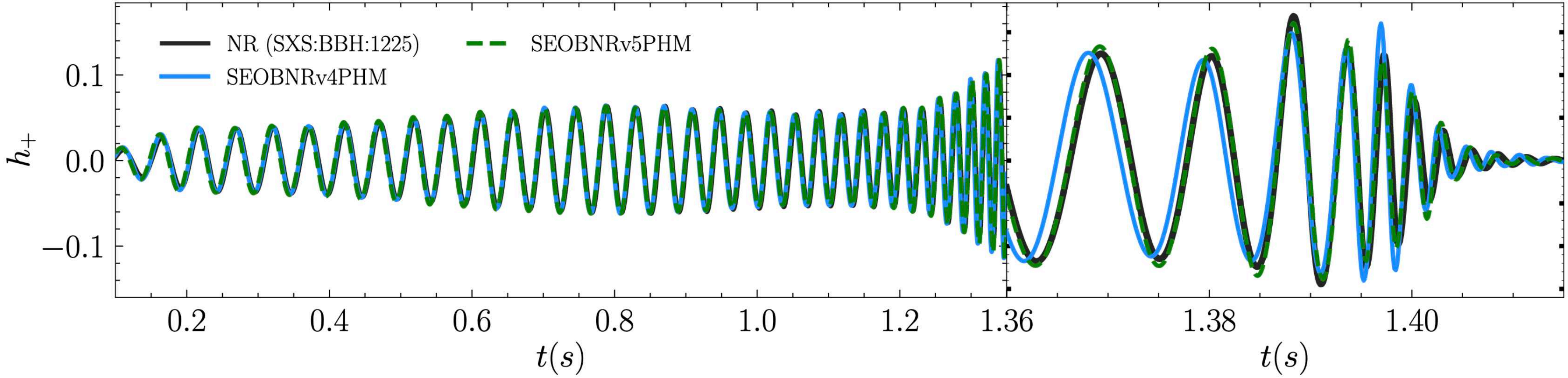

Approximant	SEOBNRv4PHM	SEOBNRv5PHM	IMRPhenomXPHM	TEOBResumS-GIOTTO
median $\max_M \overline{\mathcal{M}}_{\rm SNR}$	$7.49 \cdot 10^{-3}$	$4.75 \cdot 10^{-3}$	$14.35 \cdot 10^{-3}$	$11.47 \cdot 10^{-3}$
% cases with $\max_M \overline{\mathcal{M}}_{\rm SNR} < 1\%$	60.8%	84.4%	38.3%	44.9%
% cases with $\max_M \overline{\mathcal{M}}_{\rm SNR} < 3\%$	95.3%	99.8%	78.3%	83.3%

Effective One Body: Highlights from SEOBNRv5

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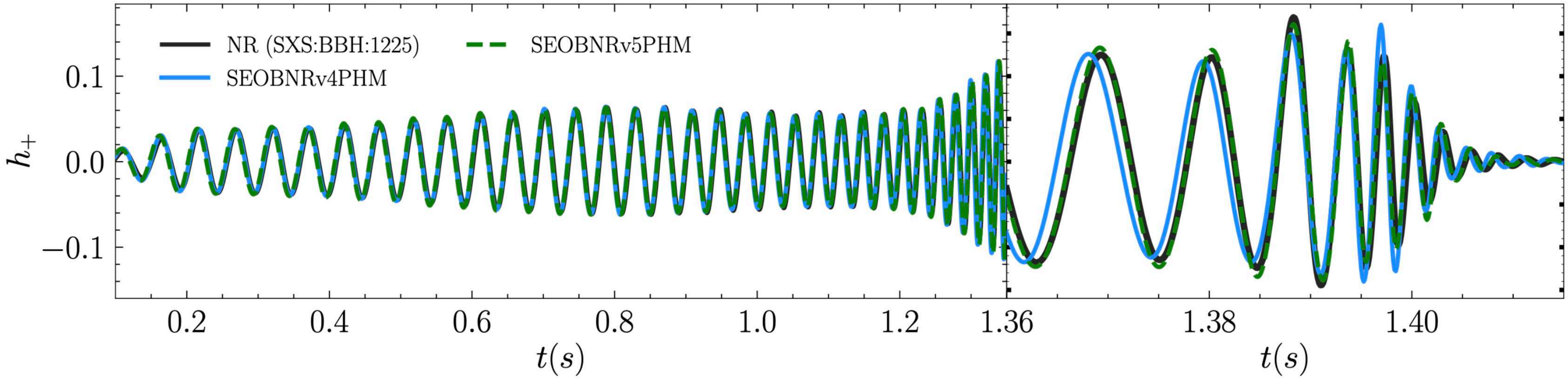

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Model Accuracy?

Metrics for Model Accuracy?

- Mismatch as one way to gauge (point-wise) level of agreement between models (and/or NR)
- Overlap is the noise-weighted inner product - weighted by PSD of detector

$$\langle h_1, h_2 \rangle = 4\,\mathrm{Re} \int_{\mathrm{f_{low}}}^{f_{\mathrm{high}}} df\, \frac{\tilde{h}_1(f)\, \tilde{h}_2^*(f)}{S_n(f)}$$

- Interested in the mismatch optimised over polarisation angle as well as time and phase (gauge)

$$\mathcal{M} \approx 1 - \max_{t_c, \varphi_0, \psi} \left[\frac{\langle h_1, h_2 \rangle}{\sqrt{\langle h_1, h_1 \rangle \langle h_2, h_2 \rangle}} \right]$$

- Lot's of different variations [e.g. sky-and-polarization averaged]
- Treat as a measure of agreement between two waveforms at point in parameter space

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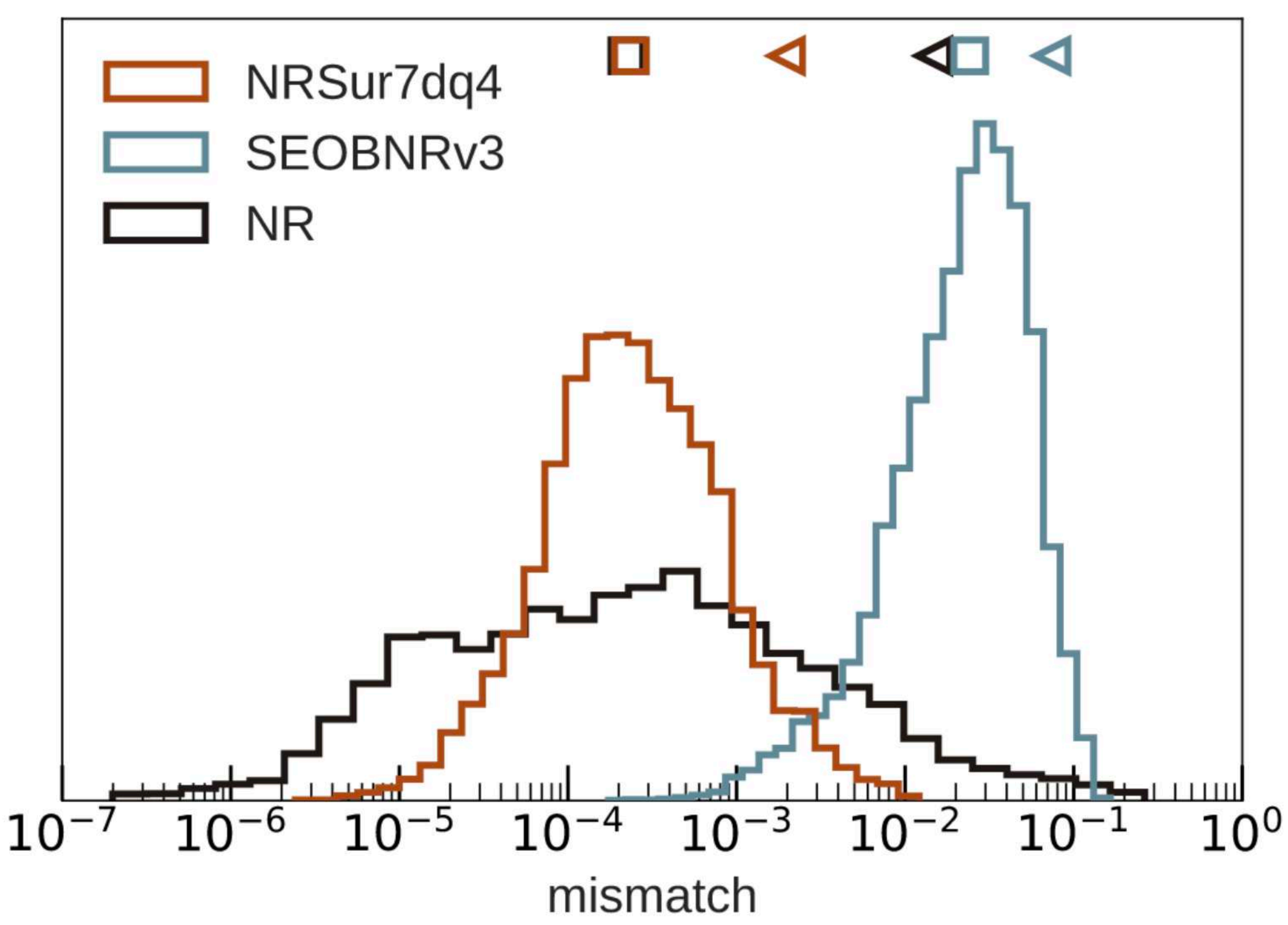
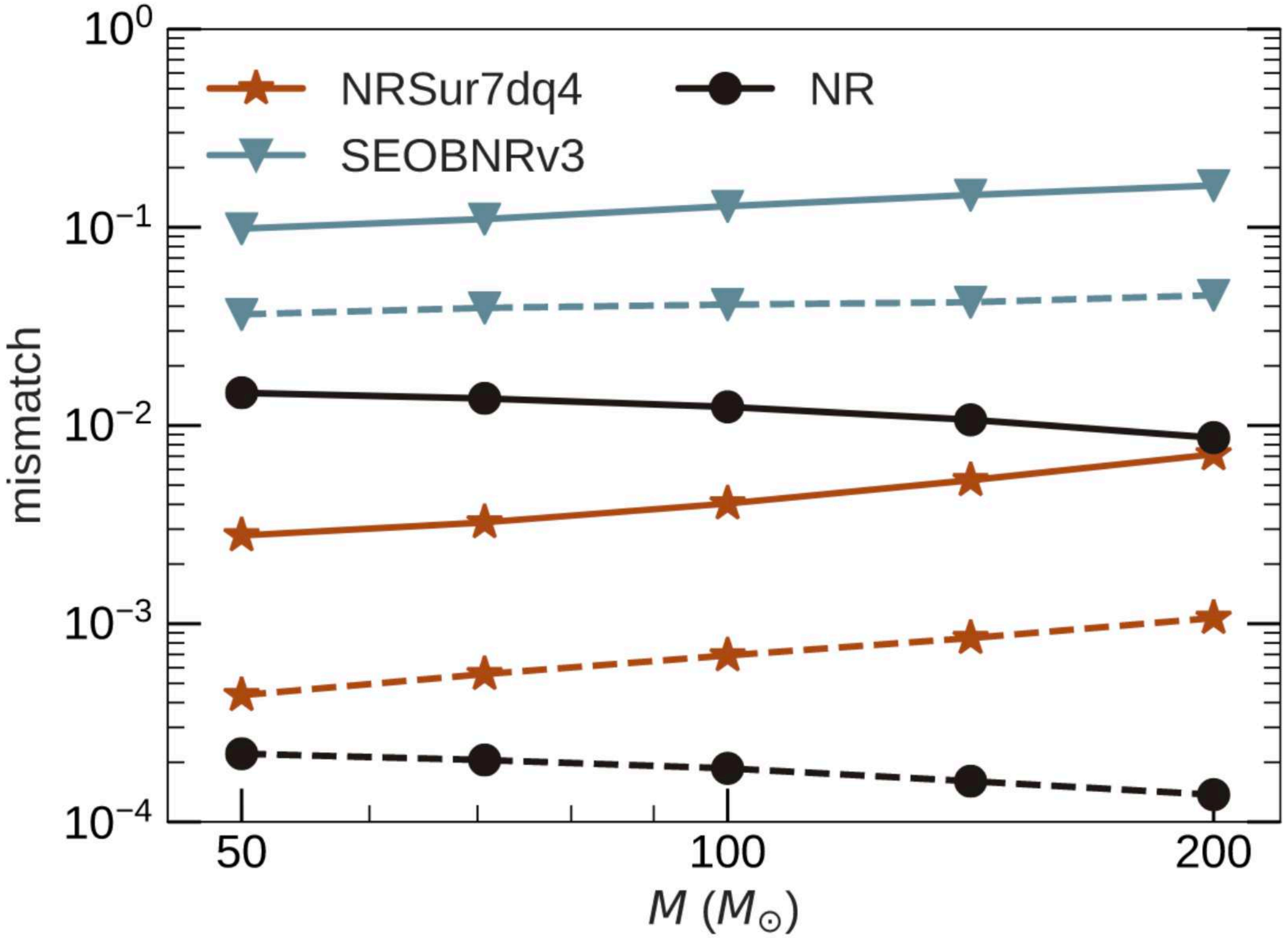
- NR surrogates amongst most accurate models available ~ at level of NR errors

Varma+



(a) Flat noise mismatches



(b) LIGO noise mismatches

Mismatches Across the Parameter Space?

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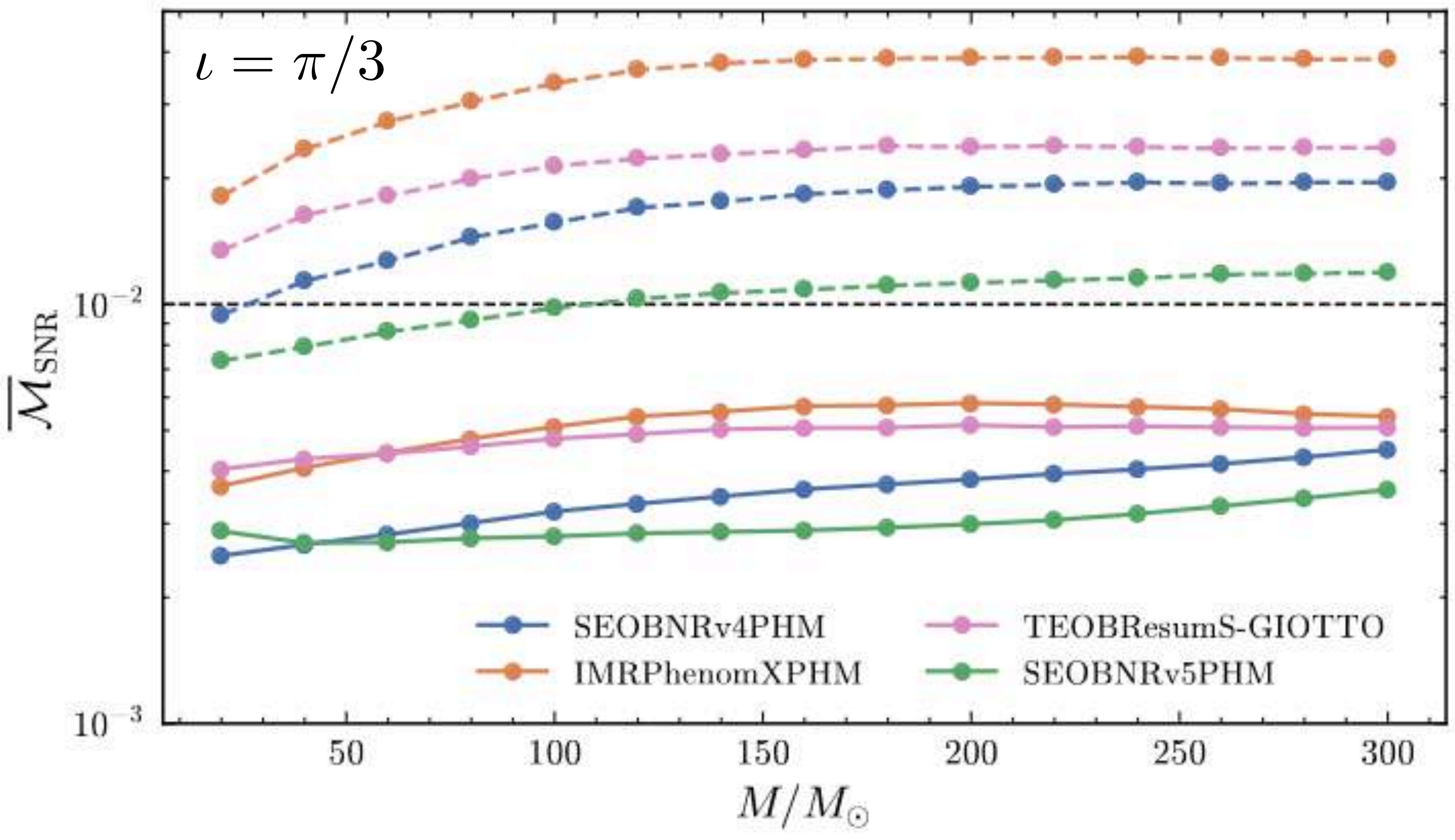
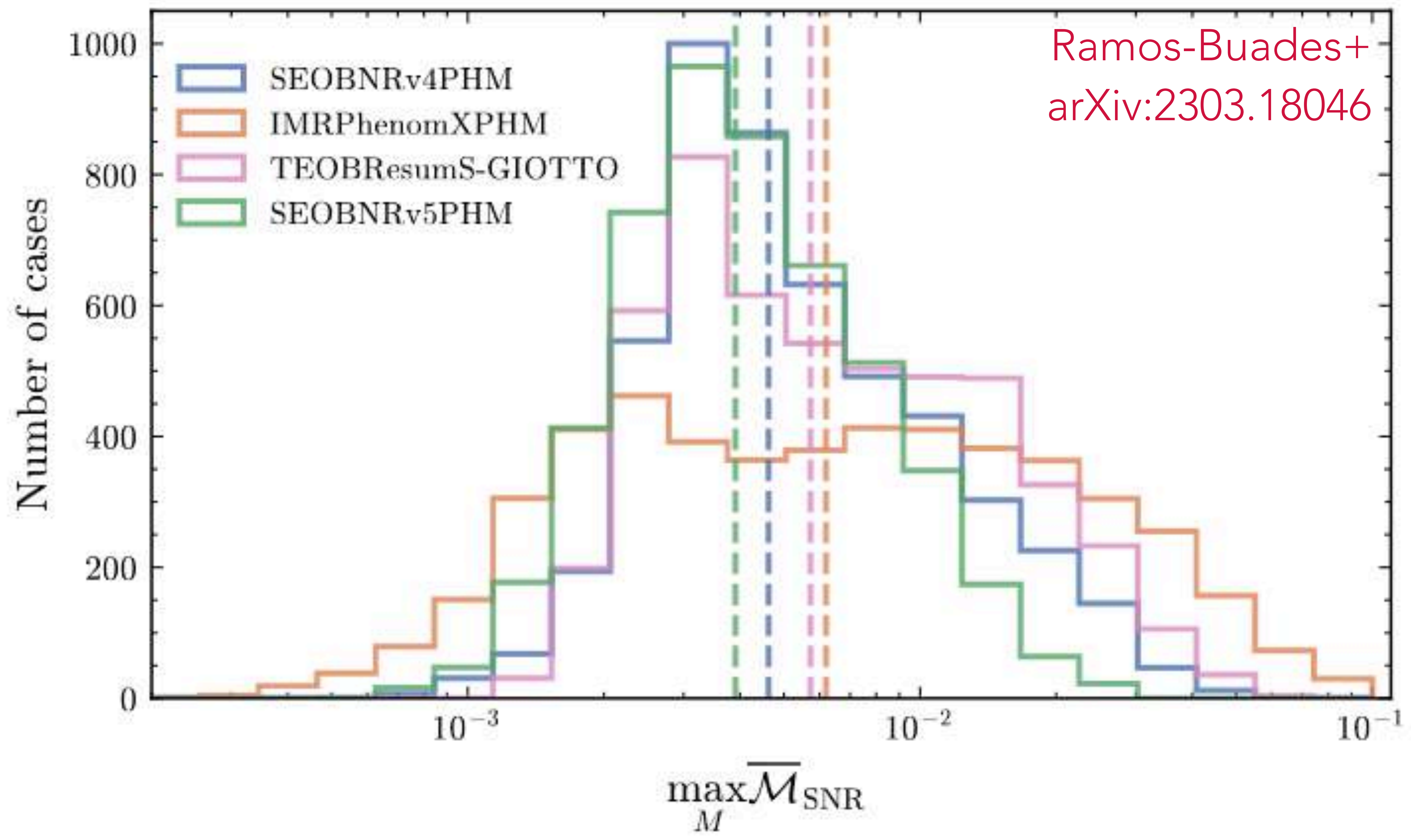
- Can use NR surrogate to gauge performance of semi-analytical models

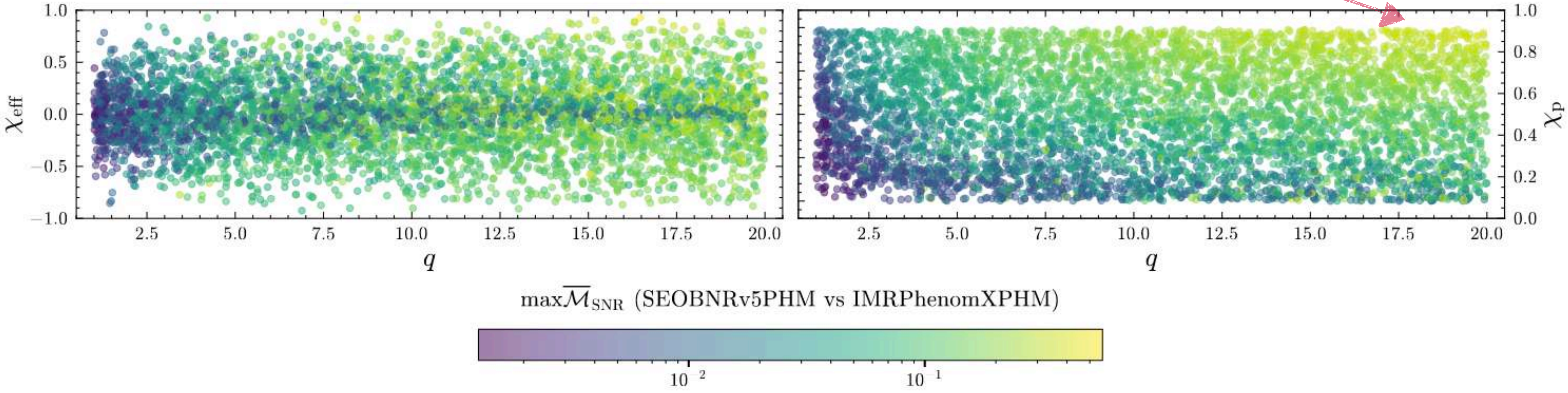




Mismatches Across the Parameter Space?

Largest differences at high(er) mass ratios and precession

Ramos-Buades+ arXiv:2303.18046



Mismatches Across the Parameter Space?

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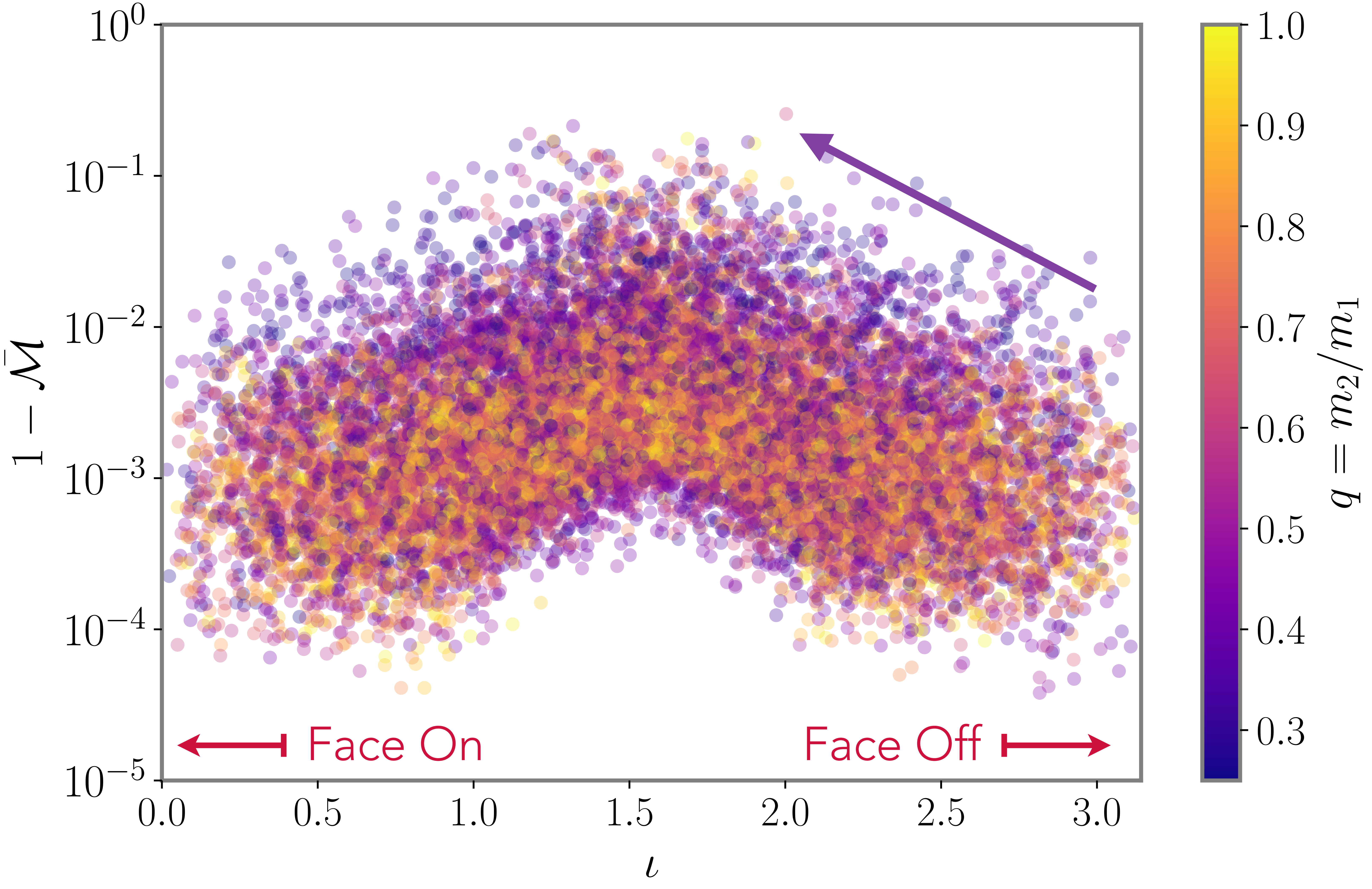
- Accuracy of models highly dependent on binary geometry, mass ratio, spins, etc

$\bar{\mathcal{M}} = \mathcal{M}_{t_0,\varphi_0,\psi_0}$



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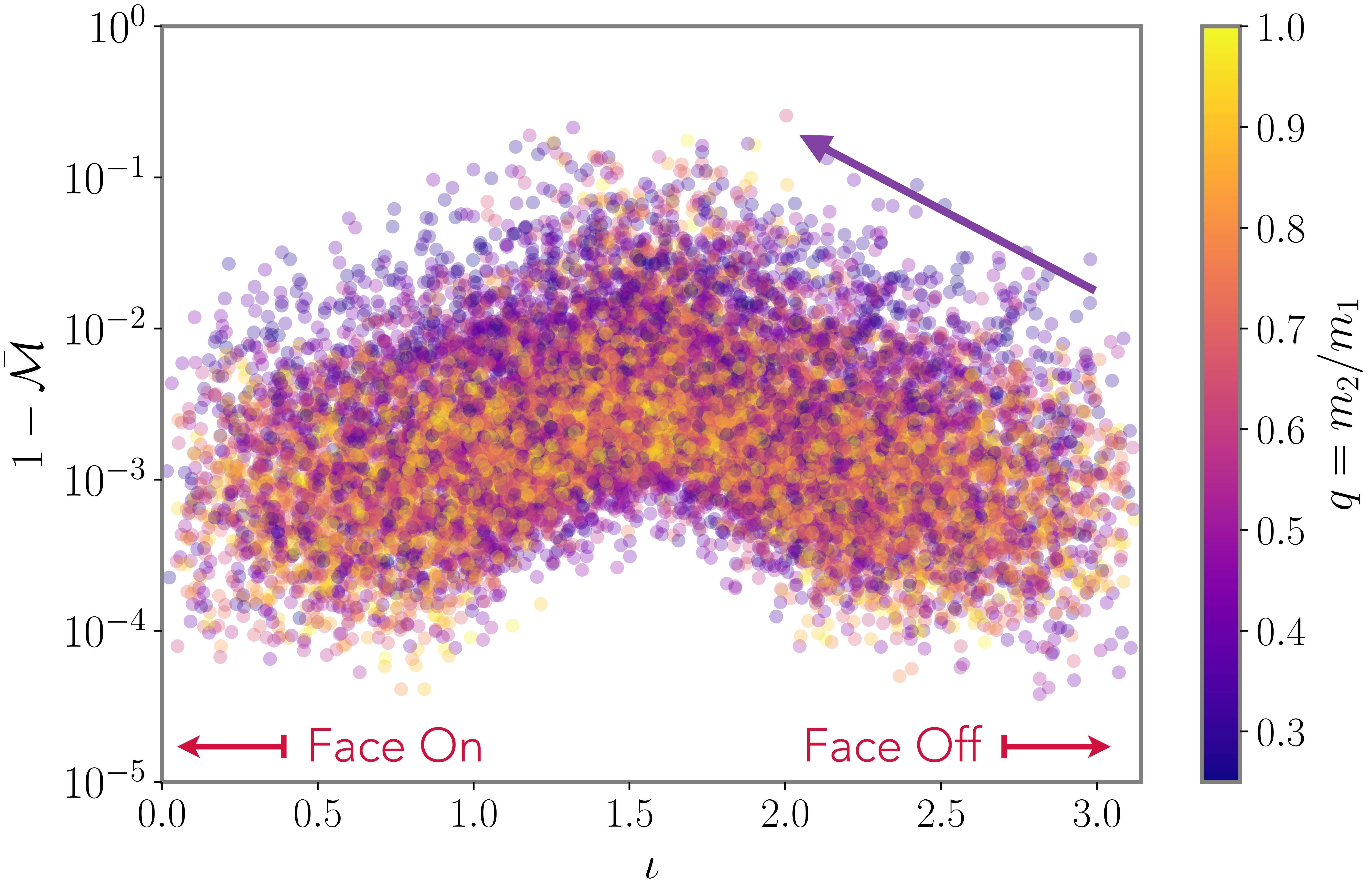

Detailed modelling of higher multipoles

Mismatches Across the Parameter Space?

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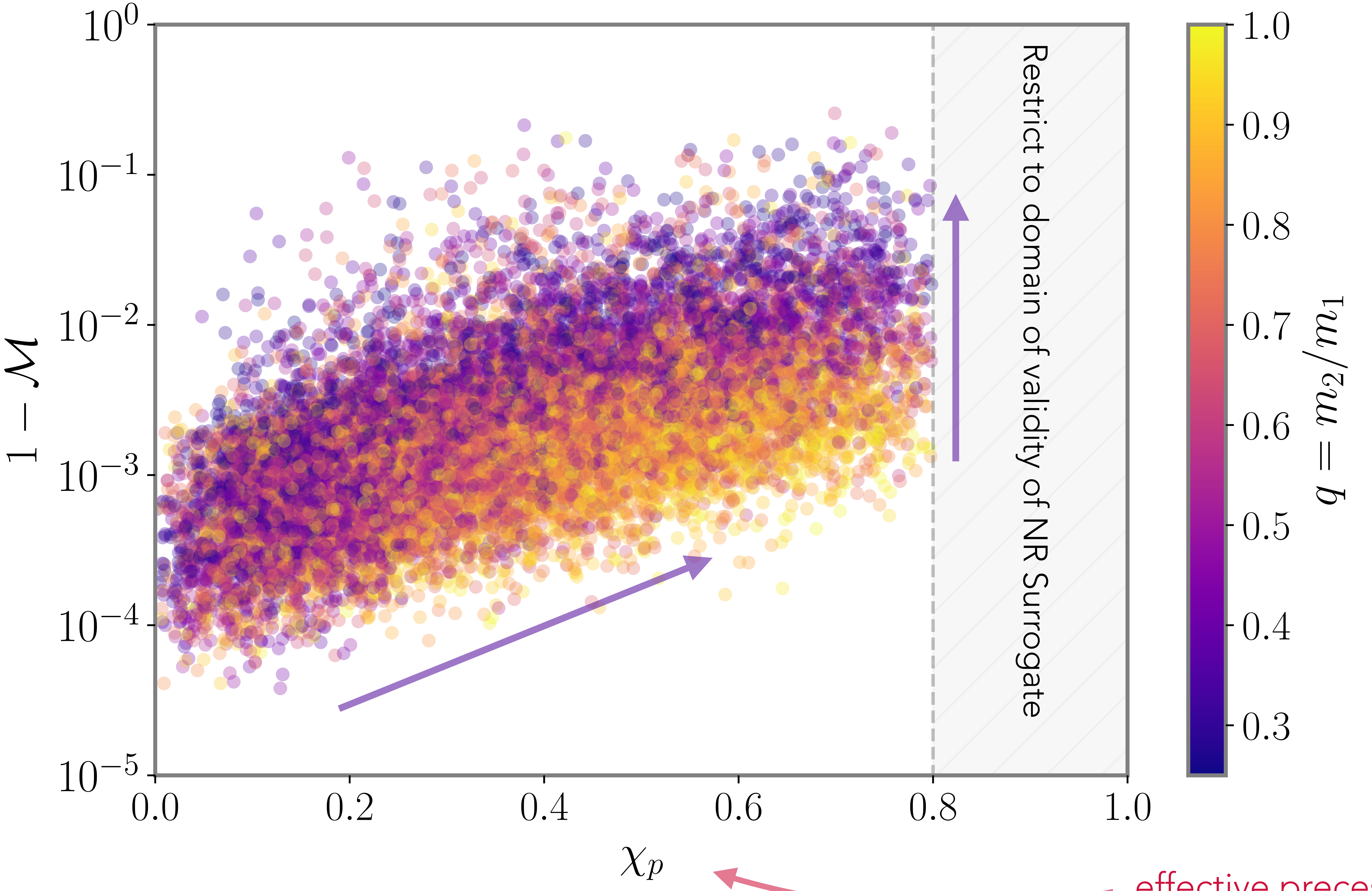

effective precessing spin [Schmidt+]

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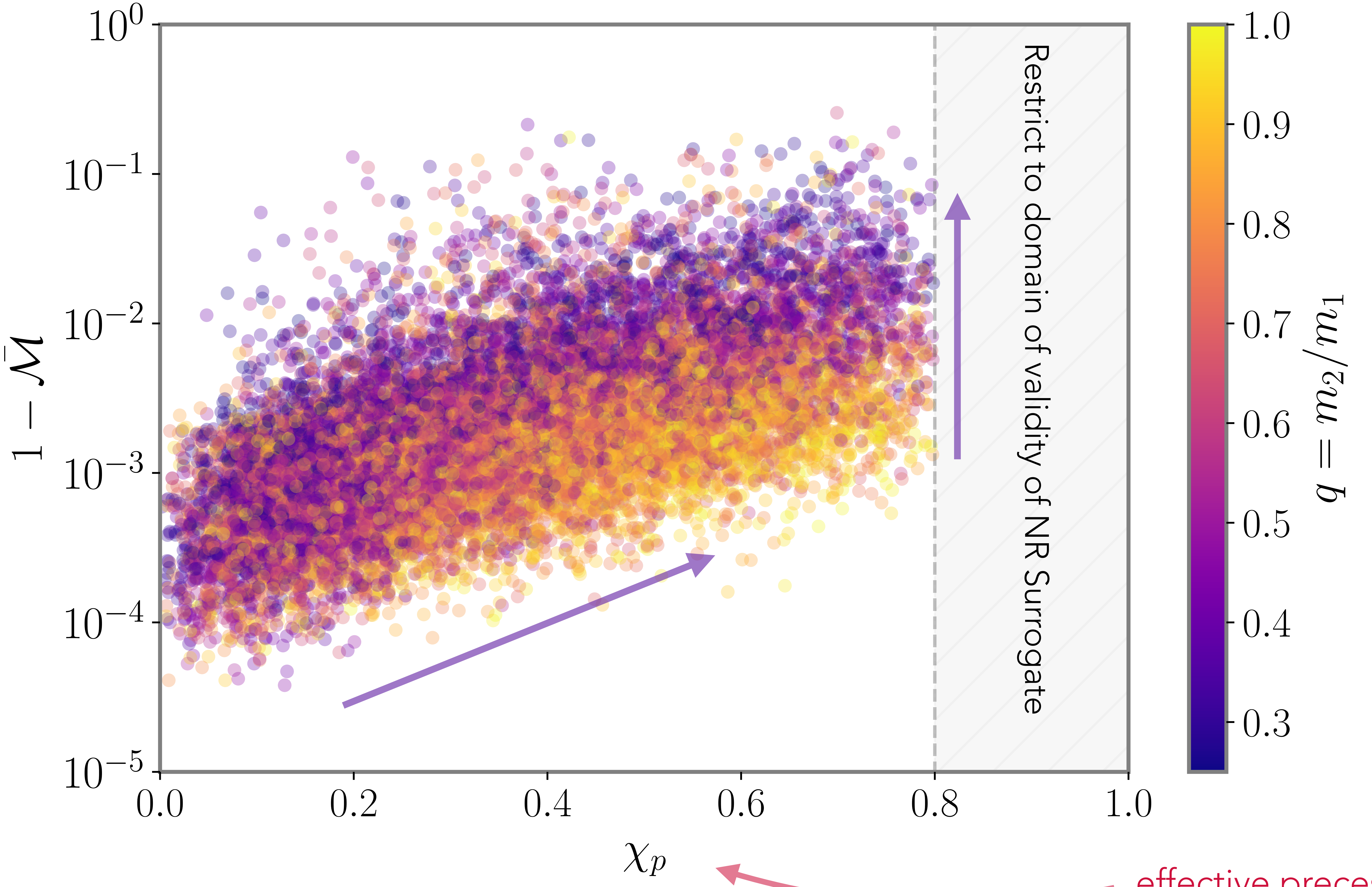

Detailed modelling of precession: mode asymmetries, co-precessing frame, NR tuning, higher multipoles, remnant, …

effective precessing spin [Schmidt+]

Thank You!