NEUTRINO MASSES AND LEPTOGENESIS

Thomas Hambye
Univ. of Brussels
ICTP-SAIFR Sao Paulo
November 2023

OUTLINE

I. NEUTRINO MASSES

A. Dirac, Weyl and Majorana spinors
B. Neutrino mass matrix
C. Neutrino oscillations
D. Neutrino mass origin

II. LEPTOGENESIS IN TYPE-I SEESAW: BASIC PICTURE

A. The 3 basic ingredients
   1. The CP-asymmetry
   2. The efficiency factor
   3. The L asymmetry to B asymmetry sphaleron conversion factor
B. CP-asymmetry calculation
C. Efficiency factor: Boltzmann equations
D. Two intriguing coinciolences
E. Flavor leptogenesis
F. Quasi-degenerate N_i's: Low scale leptogenesis
G. Very low scale leptogenesis: ARS leptogenesis

III. LEPTOGENESIS IN ALTERNATIVE SCENARIOS

I. NEUTRINO MASSES

A. Dirac, Weyl and Majorana spinors

- A Dirac spinor transforms under Lorentz group as

$$\psi(x) \to \psi'(x') = \exp\left(-\frac{i}{2}\omega_{\alpha\beta} S_{\alpha\beta}\right)\psi(\Lambda^{-1}x')$$

$$x' = \Lambda . x$$

$$S_{\alpha\beta} = \frac{i}{4}[\gamma_\alpha, \gamma_\beta]$$

$\psi(x)$ can be decomposed in L and R components

$$\psi = \underbrace{P_L\psi}_{\psi_L} + \underbrace{P_R\psi}_{\psi_R}$$

In Weyl representation of γ matrices: $\psi_L = \begin{pmatrix}\chi_L\\ 0\end{pmatrix}$ $\psi_R = \begin{pmatrix}0\\ \chi_R\end{pmatrix}$

These are 2 $\neq$ physical objects because under Lorentz each component transforms independently and differently:

$$\chi_{L,R}(x) \to \chi'_{L,R}(x') = \exp\left(-\frac{1}{2}\omega_{0i}\sigma_i - \frac{i}{4}\omega_{ij}\varepsilon^{ijk}\sigma_k\right) . \chi_{L,R}(\Lambda^{-1}x')$$

$\Rightarrow$ Dirac spinor: sum of 2 irreducible representat^{o} of dimension 2 of Lorentz group, one L and one R in the sense of L and R Lorentz transform., each with 2 d.o.f.
↳ degrees of freedom

$$\Rightarrow \mathcal{L} = \bar\psi(i\not\partial - m)\psi = \bar\psi_L i\not\partial\psi_L + \bar\psi_R i\not\partial\psi_R - m(\bar\psi_L\psi_R + \bar\psi_R\psi_L)$$

$\Rightarrow$ the mass term is what makes ψ_L and ψ_R not independent $\Rightarrow$ makes ψ a single physical object

↳ e.o.m.: $i(\partial_t - \sigma.\nabla)\chi_L = m\chi_R$
$i(\partial_t + \sigma.\nabla)\chi_R = m\chi_L$ Weyl equat^{o}s

$\chi_L \xrightarrow{m} \chi_R$

$\chi_R \xrightarrow{m} \chi_L$

but $\psi \xrightarrow{m} \psi$

- if $m=0$: no need for both χ_L and χ_R : one can have for instance just a χ_L and its conjugate state χ_L^c : a weyl spinor

  $\chi_L^c \equiv -i\sigma_2 \chi_L^*$: transforms under Lorentz as a χ_R
  ↳ better to use χ_L^c than χ_L^* which transforms as $i\sigma_2 \chi_R$

- question: can a single χ_L (or ψ_L) have a mass??

  ↳ yes! ← $\exists$ a mass term : → lorentz invar. → bilinear in χ_L and/or χ_L^c → non derivative

  ↓

  $\chi_L^{c\dagger}\chi_L$ is Lorentz invariant

$$\Rightarrow \mathcal{L} \ni -\frac{1}{2} m_M \left(\chi_L^{c\dagger}\chi_L + \chi_L^\dagger \chi_L^c\right)$$

$\Rightarrow$ 2 types of fermion masses: DIRAC: $\chi_L \; m_D \; \chi_R$ $\quad$ $\chi_R \; m_D \; \chi_L$

MAJOR.: $\chi_L \; m_M \; \chi_L^c$ $\quad$ $\chi_L^c \; m_M \; \chi_L$

$\chi_L \; m_M \; \chi_L$ $\quad$ $\chi_L \; m_M \; \chi_L$

χ_L is not a physical state if $m_M \neq 0$: it mixes with χ_L^c

$\Rightarrow$ the physical state is $\psi_M = \chi_L \text{"+"} \chi_L^c = \begin{pmatrix} \chi_L \\ \chi_L^c \end{pmatrix} \begin{matrix} \leftarrow L \\ \leftarrow R \end{matrix}$

↳ $\psi_M \; m_M \; \psi_M$ $\quad$ quantum superposito

(as for Dirac spinor: $\psi = \psi_L \text{"+"} \psi_R$)

ψ_M is autoconjugate $\psi_M = \chi_L \text{"+"} \chi_L^c \rightarrow \chi_L^c \text{"+"} \chi_L = \psi_M$

in 4-components: $\quad C \equiv i\gamma^2\gamma^0 = \begin{pmatrix} i\sigma_2 & \\ & -i\sigma_2 \end{pmatrix}$

$$\psi_M^c \equiv C\,\psi_M^T = C\gamma_0^T \begin{pmatrix} \chi_L^* \\ \chi_L^{c*} \end{pmatrix} = \begin{pmatrix} i\sigma_2 \chi_L^{c*} \\ -i\sigma_2 \chi_L^* \end{pmatrix} = \begin{pmatrix} i\sigma_2(-i\sigma_2\chi_L^*)^* \\ \chi_L^c \end{pmatrix} = \begin{pmatrix} \chi_L \\ \chi_L^c \end{pmatrix}$$

instead ψ_D is not autoconjugate : $\psi_D^c = C\bar{\psi}_D^T = \begin{pmatrix} -\chi_R^c \\ \chi_L^c \end{pmatrix}$

for a Majorana spinor:

$$\mathcal{L} = \bar{\psi}_L i\partial\!\!\!/ \psi_L - \frac{m_M}{2}(\overline{\psi_L^c}\psi_L + \bar{\psi}_L \psi_L^c)$$
$$= \frac{1}{2}\bar{\psi}_M i\partial\!\!\!/ \psi_M - \frac{m_M}{2}\bar{\psi}_M \psi_M$$
$$= i\chi_L^\dagger \bar{\sigma}^\mu \partial_\mu \chi_L - \frac{m_M}{2}(\chi_L^{c\dagger}\chi_L + \chi_L^\dagger \chi_L^c)$$

B. NEUTRINO MASS MATRIX

↳ if ν have a mass it could be of the Dirac type or of the Majorana type

- Dirac mass case: requires the existence of right-handed ν : if one assumes on top of the 3 observed neutrinos, $\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$, 3 ν_R (which we call $\nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$)

$$\mathcal{L} \ni -(\bar{\nu}_{eR}\ \bar{\nu}_{\mu R}\ \bar{\nu}_{\tau R}) \underbrace{\begin{pmatrix} m_{\nu_{ee}} & m_{\nu_{e\mu}} & m_{\nu_{e\tau}} \\ m_{\nu_{\mu e}} & \cdot & \cdot \\ \vdots & \cdot & \cdot \end{pmatrix}}_{\mathcal{M}_\nu} \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} + h.c.$$

$$= -(\bar{\nu}_{1R}\ \bar{\nu}_{2R}\ \bar{\nu}_{\tau R}) \underbrace{U_{\nu_R}^\dagger \begin{pmatrix} & & \\ & \cdot\cdot & \\ & & \end{pmatrix} \cdot U_{\nu_L}}_{\mathcal{M}_\nu^{DIAG} = \begin{pmatrix} m_{\nu 1} & & \\ & m_{\nu 2} & \\ & & m_{\nu 3} \end{pmatrix}} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix} + h.c.$$

diagonalisat° by bi-unitary transformat°

↳ $\nu_{1,2,3}$ mass eigenstates

↳ in this case one gets in the charged current interaction a non-diagonal matrix (just in the way one gets the CKM matrix but with now $(\nu_{eL}, \nu_{\mu L}, \nu_{\tau L})$ instead of (u_L, c_L, t_L) and (e_L, μ_L, τ_L) instead of (d_L, s_L, b_L):

$$\mathcal{L} \ni -(\bar{e}_L' \, \bar{\mu}_L' \, \bar{\tau}_L') \gamma^\mu W_\mu^+ \underbrace{U_{e_L}^+ U_{\nu_L}}_{} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix} + h.c.$$

$\hookrightarrow U_{PMNS}$ matrix: same as V_{CKM} but for leptons

$\begin{pmatrix} e_L' \\ \mu_L' \\ \tau_L' \end{pmatrix} = U_{eL} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$: diagonal. of the charged lepton mass matrix $\mathcal{M}_\ell$

$$= -(\bar{e}_L'' \, \bar{\mu}_L'' \, \bar{\tau}_L'') \gamma^\mu W_\mu^+ \underbrace{\begin{pmatrix} e^{-i\phi_e} & & \\ & e^{-i\phi_\mu} & \\ & & e^{-i\phi_\tau} \end{pmatrix} \underbrace{U_{e_L}^+ U_{\nu_L}}_{U_{PMNS}} \begin{pmatrix} e^{i\phi_{\nu_1}} & & \\ & e^{i\phi_{\nu_2}} & \\ & & e^{i\phi_{\nu_3}} \end{pmatrix}}_{U_{PMNS}'} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}$$

one can always redefine L and R e, μ, τ fields with a same phase $\phi_e, \phi_\mu, \phi_\tau$ and L and R ν_1, ν_2, ν_3 fields with a same phase $\phi_{\nu_1}, \phi_{\nu_2}, \phi_{\nu_3}$ without changing anything in the rest of the $\mathcal{L}$: in particular $\mathcal{M}_\ell^{DIAG}$ and $\mathcal{M}_\nu^{DIAG}$ unchanged in this case: $\bar{\psi}_{e_L} \psi_{e_R}, \ldots$ unchanged

$\Rightarrow$ since U_{PMNS} is an arbitrary unitary matrix it has 3 angles + 6 phases $\Rightarrow$ one can choose the phases to eliminate 5 of these 6 phases but not the last one because for $\phi_e = \phi_\mu = \phi_\tau = \phi_{\nu_1} = \phi_{\nu_2} = \phi_{\nu_3}$, U_{PMNS} is unchanged

$\Rightarrow U_{PMNS}' =$ 3 angles + one phase "δ": "Dirac phase"

$\hookrightarrow$ breaks CP.

• Majorana mass case: ← no right-hol neutrinos in ν mass term.

$$\mathcal{L} \ni -(\overline{\nu_{e_L}^c}\ \overline{\nu_{\mu_L}^c}\ \overline{\nu_{\tau_L}^c}) \underbrace{\begin{pmatrix} m_{\nu_{ee}} & m_{\nu_{e\mu}} & m_{\nu_{e\tau}} \\ m_{\nu_{e\mu}} & m_{\nu_{\mu\mu}} & m_{\nu_{\mu\tau}} \\ m_{\nu_{e\tau}} & m_{\nu_{\tau\mu}} & m_{\nu_{\tau\tau}} \end{pmatrix}}_{\mathcal{M}_\nu^M} \begin{pmatrix} \nu_{e_L} \\ \nu_{\mu_L} \\ \nu_{\tau_L} \end{pmatrix} + h.c.$$

$\mathcal{M}_\nu^M$: symmetric matrix: $\overline{\nu^c}.\nu \sim \nu^T.\nu$

$$= -(\overline{\nu_{1_L}^c}\ \overline{\nu_{2_L}^c}\ \overline{\nu_{3_L}^c})\ \underbrace{U_{\nu_L}^T \begin{pmatrix} & & \\ & '' & \\ & & \end{pmatrix} U_{\nu_L}}_{\mathcal{M}_\nu^{DIAG} = \begin{pmatrix} m_{\nu_1} & & \\ & m_{\nu_2} & \\ & & m_{\nu_3} \end{pmatrix} \leftarrow m_{\nu_i}:\text{ real}} \begin{pmatrix} \nu_{1_L} \\ \nu_{2_L} \\ \nu_{3_L} \end{pmatrix} + h.c.$$

↑ diagonalisat° by single unitary matrix because $\mathcal{M}_\nu$: sym. matrix

⇒ in changed current interact°:

$$\mathcal{L} \ni -(\overline{e_L'}\ \overline{\mu_L'}\ \overline{\tau_L'})\gamma^\mu W_\mu^+ \underbrace{U_{e_L}^+ U_{\nu_L}}_{U_{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} + h.c.$$

$$= -(\overline{e_L''}\ \overline{\mu_L''}\ \overline{\tau_L''})\gamma^\mu W_\mu^+ \begin{pmatrix} e^{-i\phi_e} & & \\ & e^{-i\phi_\mu} & \\ & & e^{-i\phi_\tau} \end{pmatrix} U_{e_L}^+ U_{\nu_L} \begin{pmatrix} \nu_{1_L} \\ \nu_{2_L} \\ \nu_{3_L} \end{pmatrix}$$

⇒ same as Dirac case except that $\nu_{1_L}, \nu_{2_L}, \nu_{3_L}$ cannot be rephased because that would give $m_{\nu_{1,2,3}}$ imaginary: $\overline{\nu^c}.\nu \sim \nu^T.\nu$

⇒ Dirac case: $3\, m_\nu + 3$ angles $+ 1$ phase: 7 params in $\mathcal{M}_\nu$

Majorana case: " + " + 3 phases: 9 " " "

$$U_{PMNS}' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} e^{i\alpha} & & \\ & e^{i\beta} & \\ & & 1 \end{pmatrix}}_{\text{additional } \alpha,\beta \text{ Majorana phases for Majorana case}}$$

3 Euler angles ↗ ↗ ↗ Dirac phase

C. NEUTRINO OSCILLATIONS

↳ if $\mathcal{M}_\nu$ is non diagonal the off diagonal elements of $\mathcal{M}_\nu$ allow a transition of a ν of a given flavor into a ν of a $\neq$ flavor

for instance $\mathcal{M}_{\nu_{e\mu}}$: $\nu_\mu \rightarrow \times \rightarrow \nu_e$

⇒ ν oscillations : their observations allowed to determine the 3 mixing angles of the U_{PMNS} matrix as well as the 2 $\Delta m^2 \equiv m^2_{\nu_3} - m^2_{\nu_2}$ and $m^2_{\nu_2} - m^2_{\nu_1}$

oscillations also depends on δ but CP in oscillat° s not yet observed.

ν oscillat° s do not depend on absolute m_ν scale and on α and β.

global fit : arxiv:2111.03086

Normal ordering : ν_3, ν_2, ν_1

Inverted ordering : ν_2, ν_1, ν_3

		Normal Ordering (Best Fit)		Inverted Ordering ($\Delta\chi^2 = 7.0$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
	$\theta_{12}/^\circ$	$33.45^{+0.77}_{-0.75}$	$31.27 \to 35.87$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
	$\sin^2\theta_{23}$	$0.450^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.570^{+0.016}_{-0.022}$	$0.410 \to 0.613$
	$\theta_{23}/^\circ$	$42.1^{+1.1}_{-0.9}$	$39.7 \to 50.9$	$49.0^{+0.9}_{-1.3}$	$39.8 \to 51.6$
	$\sin^2\theta_{13}$	$0.02246^{+0.00062}_{-0.00062}$	$0.02060 \to 0.02435$	$0.02241^{+0.00074}_{-0.00062}$	$0.02055 \to 0.02457$
	$\theta_{13}/^\circ$	$8.62^{+0.12}_{-0.12}$	$8.25 \to 8.98$	$8.61^{+0.14}_{-0.12}$	$8.24 \to 9.02$
	$\delta_{CP}/^\circ$	230^{+36}_{-25}	$144 \to 350$	278^{+22}_{-30}	$194 \to 345$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.510^{+0.027}_{-0.027}$	$+2.430 \to +2.593$	$-2.490^{+0.026}_{-0.028}$	$-2.574 \to -2.410$

a) Oscillations between 2 ν :

↳ suppose only 2 ν : ν_μ and ν_τ with non-diag. $\mathcal{M}_\nu$:

Flavour states : $|\nu_\mu\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$

$|\nu_\tau\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$

consider a ν_μ state produced at $t=x=0$ (from a W^+ decay for instance) with energy E: how this ν_μ evolves in time??

$$|\nu_\mu(t=0,x=0)\rangle = |\nu_\mu\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle \qquad (*)$$

$|\nu_{1,2}\rangle$: plane waves : $|\nu_{1,2}(t,x)\rangle = e^{-i(E_{1,2}t - p_{1,2}x)}|\nu_{1,2}\rangle$ $(**)$

mass eigenstates

$$\Rightarrow |\nu_\mu(t,x)\rangle = \cos\theta\,|\nu_1(t,x)\rangle + \sin\theta\,|\nu_2(t,x)\rangle$$

$$(*)+(**) \quad = \left(\cos^2\theta\, e^{-i(E_1t-p_1x)} + \sin^2\theta\, e^{-i(E_2t-p_2x)}\right)|\nu_\mu\rangle$$

$$+\left(-\sin\theta\cos\theta\, e^{-i(E_1t-p_1x)} + \sin\theta\cos\theta\, e^{-i(E_2t-p_2x)}\right)|\nu_\tau\rangle$$

$\Rightarrow$ probability that a ν_μ produced at $t=x=0$ is seen as a ν_τ at t,x:

$$P(\nu_\mu(t=0,x=0)\to\nu_\tau(t,x)) = |\langle\nu_\tau|\nu_\mu(t,x)\rangle|^2$$

$$= \left|\sin\theta\cos\theta\left(-e^{-i(E_1t-p_1x)} + e^{-i(E_2t-p_2x)}\right)\right|$$

$$E_it - p_ix \simeq \frac{m_{\nu_i}^2 L}{2E}$$

relativistic ν: $\begin{cases} t=x=L \\ E_i=p_i=E \end{cases}$

$$= \sin^2 2\theta\,\sin^2\frac{\Delta m_\nu^2 L}{4E}$$

$\Delta m_\nu^2 = m_{\nu_2}^2 - m_{\nu_1}^2$

$$P(\nu_\mu(t=0,x=0)\to\nu_\mu(t,x)) = 1-\sin^2 2\theta\,\sin^2\frac{\Delta m_\nu^2 L}{4E}$$

$\Rightarrow$ example: maximum mixing: $\theta=\pi/4$: $\quad L_{osc} \equiv \frac{4E}{\Delta m_\nu^2}\cdot\pi$

b) Oscillations for the 3 ν case:

↳ we proceed in same way as for the 2 ν case but now writing: $|\nu_\mu(t,x)\rangle = U_{\mu i}\, |\nu_i(t,x)\rangle$ $\qquad U \equiv U_{\nu_L}$

$$= U_{\mu i}\, e^{-i(E_i t - p_i x)} |\nu_i\rangle$$

$$= U_{\mu i}\, e^{-i\,\text{"}}\ \text{"}\ U^\dagger_{i\alpha} |\nu_\alpha\rangle \qquad \alpha = e, \mu, \tau$$

$$\Rightarrow P(\nu_\mu(t=0, x=0) \to \nu_\tau(t,x)) = |\langle \nu_\tau | \nu_\mu(t,x)\rangle|^2$$

$$= \sum_{i,j} U_{\mu i} U^*_{\tau i} U^*_{\mu j} U_{\tau j}\, e^{-i\frac{\Delta m^2_{ij} L}{2E}}$$

$$= -4 \sum_{i<j} \mathrm{Re}\, |J^{\mu\tau}_{ij}| \sin^2 \frac{\Delta m^2_{\nu_i \nu_j} L}{4E}$$

$$-2 \sum_{i<j} \underbrace{\mathrm{Im}\, |J^{\mu\tau}_{ij}|}_{\text{↳ CP part}} \sin^2 \frac{\text{"}}{\text{"}}$$

$$\Delta m^2_{ij} \equiv m^2_{\nu_i} - m^2_{\nu_j}$$

$$J^{\mu\tau}_{ij} = U_{\mu i} U_{\tau j} U^*_{\tau i} U^*_{\mu j}$$

Now in practice one splitting is much larger than the other one, say $\Delta m^2_{32} \gg \Delta m^2_{21} \Rightarrow$ 2 possibilities

"normal hierarchy" "inverted hierarchy"

In both case we can make the approximat°: $\Delta m^2_{31} = \Delta m^2_{32} \gg \Delta m^2_{21}$ and writing

Dirac phase $\qquad$ ← 3 EULER angles

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} e^{i\alpha} & & \\ & e^{i\beta} & \\ & & 1 \end{pmatrix}}_{\text{addition. phases for Major. case}}$$

we get:

$$P(\nu_e \to \nu_\mu) = s_{23}^2 \sin^2 2\theta_{13} S_{23} + c_{23}^2 \sin^2 2\theta_{12} S_{12} - P_{CP}$$

$$P(\nu_e \to \nu_\tau) = c_{23}^2 \sin^2 2\theta_{13} S_{23} + s_{23}^2 \sin^2 2\theta_{12} S_{12} + P_{CP}$$

$$S_{ij} = \sin^2 \frac{\Delta m_{ij}^2 L}{4E}$$

$$P(\nu_\mu \to \nu_\tau) = c_{13}^4 \sin^2 2\theta_{23} S_{23} - s_{23}^2 c_{23}^2 \sin^2 2\theta_{12} S_{12} - P_{CP}$$

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_{13} S_{23} - c_{13}^4 \sin^2 2\theta_{12} S_{12}$$

$$P(\nu_\mu \to \nu_\mu) = 1 - 4 c_{13}^2 s_{23}^2 (1 - c_{13}^2 s_{23}^2) S_{23} - c_{23}^4 \sin^2 2\theta_{12} S_{12}$$

$$P(\nu_\tau \to \nu_\tau) = 1 - 4 c_{13}^2 c_{23}^2 (1 - c_{13}^2 c_{23}^2) S_{23} - s_{23}^4 \sin^2 2\theta_{12} S_{12}$$

↳ replacing ν by $\bar{\nu}$ gives the same except that $P_{CP} \to -P_{CP}$ (see below)

⇒ very brief survey on oscillation parameter determinat°

e) "Atmospheric parameters": $\theta_{23}, \Delta m_{23}^2$

↳ a useful formula: $\sin^2 \frac{\Delta m^2 L}{4E} = \sin^2 1{,}27 \frac{\Delta m^2}{eV^2} \frac{L}{km} \frac{GeV}{E}$

⇒ atmospheric ν_μ: $E \sim GeV$, $\Delta m^2 \sim 2{,}5 \cdot 10^{-3} eV$

↳ ⇒ $L_{osc} \sim 250 km$ ← $\frac{\Delta m^2 L_{osc}}{4E} \sim \pi$

← $P(\nu_\mu \to \nu_\mu)$

$$S_{ij} = \sin^2 \frac{\Delta m_{ij}^2 L}{4E}$$

$$P(\nu_\mu \to \nu_\mu) = 1 - 4 \underbrace{c_{13}^2}_{\sim 1} s_{23}^2 (1 - \underbrace{c_{13}^2}_{\sim 1} s_{23}^2) S_{23} - c_{23}^4 \underbrace{\sin^2 2\theta_{12}}_{\text{small}} S_{12}$$

$$\simeq 1 - 4 s_{23}^2 (1 - s_{23}^2) S_{23}$$

⇒ fitting the observed oscillat° pattern of $P(\nu_\mu \to \nu_\mu)$ as a function of L (⟺ zenithal angle distribut°)

SK got: $|\Delta m_{23}^2| = 2.5 \cdot 10^{-3} eV^2$, $\theta_{23} = \pi/4$

← The Sapenkamiokonde 1998 breakthrough

↳ Nobel prize 2015

↳ note that to determine $\sin\theta_{23}$ it is enough to observe ν_μ with $L \gg L_{osc} \Leftrightarrow s_{23} = \sin^2 \frac{\Delta m^2_{23} L}{4E} \simeq \frac{1}{2}$ (average)

in case $P(\nu_\mu \to \nu_\mu) \simeq 1 - 2 s^2_{23}(1 - s^2_{23}) \simeq \frac{1}{2}$

↳ $\Rightarrow \theta_{23} \sim \frac{\pi}{4}$: maximal mixing observed

↳ note also that $P(\nu_\mu \to \nu_\mu)$ deficit has been also observed from a beam of ν_μ : K2K (~2000) : first laboratory evidence for BSM physics!

d) "Solar parameters": θ_{12}, Δm^2_{12}

↳ solar ν_e neutrinos

↳ $P(\nu_e \to \nu_\mu) + P(\nu_e \to \nu_\tau) = \underbrace{\sin^2 2\theta_{13}\, s_{23}}_{\text{very small: see below}} + \sin^2 2\theta_{12}\, s_{12}$

$\Rightarrow P(\nu_e \to \nu_e) \simeq 1 - \sin^2 2\theta_{12}\, s_{12}$

↳ solar ν_e experiments : ν_e deficits

already in 1970's (Nobel prize 2002)

Homestake, Kamiokande, SNO, ...

$\Rightarrow \Delta m^2_{21} = (7{,}5 \pm 0{,}2) \cdot 10^{-5}\,\text{eV}^2$

$\sin^2\theta_{12} = 0.304 \pm 0.01$

matter effect crucial for this determination

↳ (skipped here)

e) θ_{13}: "reactor ν parameter": $L \sim 1$ km, $E \sim$ MeV

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2\theta_{13} \underbrace{\sin^2 \frac{\Delta m^2_{23} L}{4E}}_{\sim 1} - c_{13}^4 \sin^2 2\theta_{12} \underbrace{\sin^2 \frac{\Delta m^2_{12} L}{4E}}_{\ll 1}$$

$\leftarrow$ for $\frac{\Delta m^2_{23} L}{4E} \sim \frac{\pi}{2}$

$\Rightarrow \sin^2 2\theta_{13} = 0.09 \pm 0.001$ DOUBLE CHOOZ, DAYA BAY } 2012

f) δ: $A_{CP} \equiv \frac{P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)}{\text{"} \quad + \quad \text{"}} \propto P_{CP}$

$$P_{CP} \equiv \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin\delta_{13} \sin \frac{\Delta m^2_{12} L}{4E} \cdot \sin \frac{\Delta^2_{23} L}{4E}$$

... (yet to be clearly observed)

g) absolute ν mass scale: • Tritium decay (KATRIN, ...)

$\Rightarrow m_\nu < 1{,}1$ eV $\qquad {}^3H \to {}^3He + e^- + \bar{\nu}_e$ ($\bar{\nu}_e$: not seen)

• cosmology: $\sum_{i=1,..,3} m_{\nu_i} < 0.4$ eV

$\hookrightarrow$ affects large scale structure formation

h) Dirac or Majorana? neutrinoless double beta decay:

$\leftarrow$ does not exist if ν masses are of the Dirac type.

$\Rightarrow$ rate proportional to $\sum U_{ie}^2 m_{\nu_i} \equiv m_{\nu ee}$ (depends on Majorana phases)

$$= c_{13}^2 (m_{\nu_1} c_{12}^2 + m_{\nu_2} s_{12}^2 e^{2i\alpha}) + m_{\nu_3} s_{13}^2 e^{2i(\beta-\delta)}$$

$\Rightarrow m_{\nu ee}^{exp} \lesssim 0.17$ eV $\leftarrow$ KamLAND-Zen exp.

D] NEUTRINO MASS ORIGIN?

- why neutrinos are massless in the SM:
  - ↳ no Dirac masses in SM because no ν_R in SM
  - ↳ no Majorana masses because:

electric charge not conserved

- this breaks $SU(2)_L \times U(1)_Y$:

- in principle could be generated through SSB of $SU(2)_L \times U(1)_Y$ from interaction involving the SM scalar doublet once replaced by its vev but no such interact in SM:

L = lepton doublet: $\begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$

H = SM doublet: $\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$

$SU(2)$

$\downarrow$

$$\left.\begin{array}{l} L \to UL \\ \tilde{H} \to U\tilde{H} \\ \tilde{H}^+ \to \tilde{H}^+ U^+ \\ \bar{L}^c \sim L^T \to \bar{L}^c U^T \\ \tilde{H}^* \to U^* \tilde{H}^* \end{array}\right\} SU(2)_L \text{ invariant}$$

$\mathcal{L} \ni \bar{L}^c \cdot L H$: not $SU(2)_L \times U(1)_Y$ invar: 3 of doublets

$\mathcal{L} \ni \frac{1}{\Lambda} \underbrace{(\bar{L}^c \tilde{H}^*)}_{Y=0} \underbrace{(\tilde{H}^+ L)}_{Y=0}$ $SU(2)_L \times U(1)_Y$ invar. but not in SM because dim-5 interact.

$\Rightarrow$ non-renormal. $\Rightarrow$ forbidden in SM

- Weinberg operator:

the interact° $\frac{1}{\Lambda}(\bar{L}^c \tilde{H}^*)(\tilde{H}^+ L)$ could nevertheless be induced by dim-4 interactions involving or BSM heavy field (see the 3 tree level possibilities below): if this is the case ν_L Majorana masses are induced:

$$\mathcal{L} \ni \frac{c_{\alpha\beta}}{\Lambda}(\overline{L^c_\alpha}\cdot\tilde{H}^*)(\tilde{H}^\dagger L_\beta)$$

$\alpha,\beta = e,\mu,\tau$

Λ: number with dimen of mass to make $\mathcal{L}$ of dim-4 as must be

$\Downarrow$

$\Downarrow$

$\leftarrow$ Majorana mass matrix for $\nu_{\alpha,\beta}$:

$\hookrightarrow \mathcal{L} \ni \frac{1}{2}\underbrace{\frac{c_{\alpha\beta}}{\Lambda}v^2}_{=m_{\nu_{\alpha\beta}}}\overline{\nu^c_\alpha}\nu_\beta$ ...

$\Rightarrow$ seesaw mechanism: if $\Lambda \gg v$ then $m_\nu \ll v$ (as observed)

$\hookrightarrow$ ~ mass of heavy exchanged particle: see below

$\Rightarrow$ possibility of clear explanat° for small ν masses: because it is induced by exchange of heavy particle $\Leftrightarrow$ because new physics scale is much larger than $v \simeq 246$ GeV

$\hookrightarrow$ for instance for $c_{\alpha\beta} \sim 1$ (couplings): $m_\nu \sim 0.1$ eV $\Leftrightarrow$ $\underline{\underline{\Lambda \sim 10^{15}\text{ GeV}}}$

- Tree level ways to generate the Weinberg operator: $\exists$ three ways: the 3 seesaw models

any combinat° of L and H in weinberg operat. is either a singlet or a triplet of $SU(2)_L$.

$\Rightarrow$ LH = singlet: type-I seesaw

LH = triplet: type-III seesaw

LL and HH are triplets: type-II seesaw

(LL and HH singlets: doesn't work: $\tilde{H}^+\tilde{H}^{*} = H^+H^0 - H^0H^+ = 0$)

- Type-I seesaw: HL couples to a $SU(2)_L$ singlet of $Y=0$

$$\mathcal{L} \ni -Y_{\nu_{i\alpha}} \underbrace{\overline{\nu_{R_i}}}\, \tilde{H}^+ L_\alpha + h.c.$$

↳ a lepton with e-m charg $=0$ is called a neutrino

$\Rightarrow \nu_R$ is a right-handed ν

↳ with a right-handed ν one can expect Dirac ν masses just as with the up quarks in SM:

$$\mathcal{L} \ni -Y_{\nu_{i\alpha}} \overline{\nu_{R_i}} \nu_{L_\alpha} H^0 + h.c. \qquad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$$

$$\ni -\underbrace{Y_{\nu_{i\alpha}} \frac{v}{\sqrt{2}}}_{= m^{DIRAC}_{\nu_{i\alpha}}} \overline{\nu_{R_i}} \nu_{L_\alpha} + h.c$$

↳ if there is only this interact. then $m_\nu \sim 0.1\,eV$ requires $Y_\nu \sim 10^{-11}$: much smaller than Yukawa couplings of other fermions in SM: looks weird but nothing forbids that.

↳ however nothing forbids also (except if one assumes an extra BSM symmetry and nothing tells us this sym. exists) Majorana masses for the ν_{R_i}:

$$\mathcal{L} \ni -\frac{1}{2} m_{\nu_{R_{ij}}} \overline{\nu^c_{R_i}} \nu_{R_j} + h.c.$$

$\Rightarrow$ let us assume $m_{\nu_{R_{ij}}}$ diagonal here ($\Leftrightarrow$ go to the ν_{R_i} basis where $m_{\nu_{R_{ij}}}$ is diagonal)

$$\Rightarrow \mathcal{L} \ni -Y_{\nu i\alpha}\, \overline{\nu_{Ri}}\, \tilde{H}^{+} L_\alpha - \frac{1}{2} m_{\nu_{Ri}} \overline{\nu_{Ri}^c}\, \nu_{Ri} + h.c.$$

↳ these 2 interactions generates nothing but the weinberg operator interaction

$$\Rightarrow \frac{\mathcal{L}_{\alpha\beta}}{\Lambda} = \frac{-1}{2} \sum_i \frac{Y_{\nu i\alpha} Y_{\nu i\beta}}{m_{\nu_{Ri}}} \leftarrow \text{from propag of } \nu_{Ri}$$

$$\Downarrow$$

$$m_{\nu_{\alpha\beta}} = -\frac{1}{2} \sum_i \frac{Y_{\nu i\alpha} Y_{\nu i\beta}}{m_{\nu_{Ri}}} v^2$$

Another way to see that is to write the total ν mass matrix and to diagonalize it:

$$\mathcal{L} \ni -\underbrace{Y_{\nu i\alpha} \frac{v}{\sqrt{2}}}_{m_{D i\alpha}} \underbrace{\overline{\nu_{Ri}}\, \nu_{L\alpha}}_{\overline{\nu_{L\alpha}^c}\, \nu_{Ri}^c} - \frac{1}{2} m_{\nu_{Ri}} \overline{\nu_{Ri}^c}\, \nu_{Ri}$$

$$- Y_{\nu i\alpha}^{*} \frac{v}{\sqrt{2}} \underbrace{\overline{\nu_{L\alpha}}\, \nu_{Ri}}_{\overline{\nu_{Ri}^c}\, \nu_L^c} - \frac{1}{2} m_{\nu_{Ri}} \overline{\nu_{Ri}}\, \nu_{Ri}^c$$

$$= -\frac{1}{2} \begin{pmatrix} \overline{\nu_L^c} & \overline{\nu_R} \end{pmatrix} \begin{pmatrix} 0 & (m_D^T) \\ (m_D) & (m_{\nu_R}) \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + h.c.$$

$$= -\frac{1}{2} \begin{pmatrix} \overline{\nu_L^c} & \overline{\nu_R} \end{pmatrix} U_\nu^{*} U_\nu^{T} \underbrace{\begin{pmatrix} 0 & (m_D^T) \\ (m_D) & (m_{\nu_R}) \end{pmatrix}}_{\begin{pmatrix} -m_D^T m_{\nu_R}^{-1} m_D & \\ & m_{\nu_R} + \underbrace{m_D^T m_{\nu_R}^{-1} m_D}_{\text{negligible}} \end{pmatrix}} U_\nu U_\nu^{+} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + h.c.$$

$\Rightarrow$ 3 mass eigenstates which have Major. masses $\sim m_{\nu_{Ri}}$ which are essentially the ν_{Ri}: $\nu_{Ri}' = \nu_{Ri} + \mathcal{O}\left(\frac{m_D}{m_{\nu_R}}\right) \cdot \nu_L$ and 3 mass eigenstates which are essentially the ν_L: $\nu_{L\alpha}' = \nu_{L\alpha} + \mathcal{O}\left(\frac{m_D}{m_{\nu_R}}\right) \cdot \nu_R$ with ν mass matrix which is

the one we got from the diagram:

$$m_{\nu_{\alpha\beta}} = -m_D^T m_{\nu_R}^{-1} m_D = -\frac{1}{2} \frac{Y_{\nu i\alpha} Y_{\nu i\beta}}{m_{\nu_{Ri}}} v^2$$

$\Rightarrow$ in summary unless there are no Major. masses ($\Leftrightarrow$ there exists a symmetry forbidding them: total lepton # conservation: automatic in SM but now to be assumed "by hand" if we want to forbid the $m_{\nu_{Ri}}$) the SM left-handed $\nu_{L\alpha}$ have Majorana masses (resulting from Dirac masses and $m_{\nu_{Ri}}$ masses) and not Dirac masses!

$\hookrightarrow$ these Majorana masses are naturally small (as observed) if the scale of new physics ($\Leftrightarrow$ the $m_{\nu_{Ri}}$) is much larger than the EW scale: seesaw

$\hookrightarrow$ and in general we expect $m_{\nu_{Ri}} \gg v$ since the $m_{\nu_{Ri}}$ are not protected by EW scale (i.e. $m_{\nu_{Ri}} \neq 0$ even if $v=0$ unlike all other SM fermion masses)

$\hookrightarrow$ point towards a new physics scale $\sim 10^{15}$ GeV if Yukawa of order unity: GUT scale (see below).

- How many parameters in seesow type-I with 3 right-hd neutrinos?

In charged lepton mass eigenstate basis and basis of ν_{R_i} where $m_{\nu_{R_{ij}}}$ is diagonal and real:

3 real $m_{\ell_i} = m_{e^-}, m_{\mu^-}, m_{\tau^-}$

3 real $m_{\nu_{R_i}}$

Y_ν : 9 real + 9 imaginary

↳ but 3 phases can be rotated away:

from $L_i \to e^{i\alpha_i} L_i$
$\ell_{R_i} \to e^{i\alpha_i} \ell_{R_i}$ } which leaves the $\mathcal{L}$ invar. except Y_ν

$\Rightarrow Y_\nu$: 9 real + 6 imagin.

$\Rightarrow$ TOTAL: 15 real + 6 imagin = 21 param

↳ to be compared with 6 real + 3 phases in ν mass matrix:

$3 m_{\ell_i}, 3 m_{\nu_i}, 3\theta_{ij}, \delta, \alpha, \beta$

$\Rightarrow$ 6 real + 3 phases do not show up at low energy. (except special cases)

↳ "High-energy parameters"

↳ i.e.: decoupling

- Parametrization of decoupling parameters (in $\mathcal{M}_\ell^{DIAG}$ basis):

$$R \equiv \frac{v}{\sqrt{2}} (\mathcal{M}_{\nu_R})^{-1/2} Y_\nu U_{PMNS} (\mathcal{M}_\nu^{DIAG})^{-1/2} \qquad \mathcal{M}_{\nu_R} = \begin{pmatrix} m_{\nu_{R1}} & & \\ & m_{\nu_{R2}} & \\ & & m_{\nu_{R3}} \end{pmatrix}$$

$$\mathcal{M}_\nu^{diag} = \begin{pmatrix} m_{\nu 1} & & \\ & m_{\nu 2} & \\ & & m_{\nu 3} \end{pmatrix}$$

$$\Leftrightarrow Y_\nu = \frac{\sqrt{2}}{v} (\mathcal{M}_{\nu_R})^{1/2} R (\mathcal{M}_\nu^{DIAG})^{1/2} U_{PMNS}^\dagger$$

$$\Rightarrow \mathcal{M}_\nu = -Y_\nu^T (\mathcal{M}_{\nu_R})^{-1} Y_\nu \cdot \frac{v^2}{2} = U_{PMNS}^* \mathcal{M}_\nu^{DIAG} U_{PMNS}^\dagger$$

$\Uparrow$

independent of R by construct°

$\Rightarrow$ R has to do with decoupl. param.

$RR^T = R^T R = 1$ $\Rightarrow$ orthogonal complex 3×3 matrix

$\hookrightarrow$ 3 angles + 3 phases

together with the $3\, m_{\nu_{Ri}}$ it parametrizes fully the high energy parameter space.

- Type-III seesaw: very same as type-I but taking L and H in a triplet combination:

$$\Sigma_{R_i} = \begin{pmatrix} \Sigma_{R_i}^+ \\ \Sigma_{R_i}^0 \\ \Sigma_{R_i}^- \end{pmatrix} \Rightarrow m_\nu \propto \frac{Y_\Sigma^2 v^2}{m_\Sigma}$$

$$\mathcal{L} \ni -Y_{\Sigma_{i\alpha}} \overline{\Sigma_{R_i}} \overbrace{\tilde{H}^+ L_\alpha}^{\text{triplet combination}} + h.c.$$

$$-\frac{1}{2} m_{\Sigma_i} \overline{\Sigma_{R_i}^c} \Sigma_{R_i} + h.c.$$

- Type-II seesaw: LL and HH couple to a heavy scalar triplet Δ of mass m_Δ and hypercharge = 2:

$$\Delta = \begin{pmatrix} \delta^{++} \\ \delta^+ \\ \delta^0 \end{pmatrix}$$

$$\mathcal{L} \ni -Y_{L_{\alpha\beta}} \underbrace{\overline{L_\alpha^c} L_\beta}_{\text{3-plet combin.}} \Delta - \mu_\Delta \underbrace{H^\dagger \tilde{H}}_{\text{3-plet combin.}} \Delta + h.c$$

Majorana masses here too

$$\Rightarrow \frac{c_{\alpha\beta}}{\Lambda} = -\frac{\mu_\Delta Y_{L_{\alpha\beta}}}{m_\Delta^2} \Rightarrow m_{\nu_{\alpha\beta}} = -\frac{\mu_\Delta Y_{L_{\alpha\beta}}}{m_\Delta^2} v^2 = 2 Y_{L_{\alpha\beta}} \cdot v_\Delta$$

$v_\Delta = \langle \delta^0 \rangle$

↳ $m_\nu \ll v$ as soon as $m_\Delta \gg v \Rightarrow$ seesaw mechanism operative too

$\Rightarrow$ also points towards a new physics BSM scale much larger than the E-W scale

P.S.: more precisely for type-II: writing $\Delta = \Sigma T_a \Delta_a \frac{1}{\sqrt{2}} = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}$

$$\mathcal{L} = -m_\Delta^2 \, tr(\Delta^\dagger \Delta) - \mu_\Delta H^\dagger \Delta \tilde{H} - Y_L \overline{L^c} i\tau_2 \Delta L + h.c.$$

↳ $SU(2)_L$: $\Delta \to U \Delta U^\dagger$, $L \to UL$, $H \to UL$, $\tilde{H} \to U\tilde{H}$, $\tilde{H}^\dagger \to \tilde{H}^\dagger U^\dagger$, $\overline{L^c} i\tau_2 \to \overline{L^c} i\tau_2 U^\dagger$