

II LEPTOGENESIS IN TYPE-I SEESAW: BASIC PICTURE

$$h = 0.67$$

PLANCK data: $\Omega_B h^2 = 0.0224 \pm 0.0001$

BBN: $0.019 < \Omega_B h^2 < 0.024$

impressive agreement from physics at 2 completely \neq early universe epochs!

$$\hookrightarrow \frac{m_B}{m_Y} \Big|_{\text{today}} \sim 6 \cdot 10^{-10}$$

$$\frac{m_B}{s} \Big|_{\text{today}} \sim 9 \cdot 10^{-11}$$

\hookrightarrow this value of Ω_B is:

- large: if $m_b = m_{\bar{b}} \Rightarrow$ annihilat. catastrophe at $T \sim m_p$ ($p\bar{p} \rightarrow \pi\pi, \gamma\gamma, \dots$) very efficient:

$$\frac{m_b}{m_{\bar{b}}} = \frac{m_b}{m_{\bar{b}}} \sim 10^{-18} \gg 10^{-10}$$

- small: at $T \gg m_p$: $m_q \sim m_{\bar{q}} \sim m_{\gamma}$ ($q\bar{q} \leftrightarrow \gamma\gamma, \dots$) \Rightarrow if during annih. catastrophe B number is conserved

$$\text{(as expected): } \frac{m_q - m_{\bar{q}}}{m_q} \Big|_{T \gg m_p} \sim \frac{m_q - m_{\bar{q}}}{m_{\gamma}} \Big|_{T \gg m_p} \\ \sim \frac{m_q - m_{\bar{q}}}{m_{\gamma}} \Big|_{T \ll m_p} \sim 10^{-10}$$

\hookrightarrow need to create a very small asym. at $T > m_p$ (N.B: at end of inflation (before reheating) one expects $m_q = m_{\bar{q}} = 0$).

\Rightarrow Leptogenesis motivation:

- baryogenesis requires BSM physics

- leptogenesis provides a straightforward explanation for baryogenesis from same BSM physics as the one one invokes to account for

\vee masses

A] TYPE-I SEESAW LEPTOGENESIS: THE 3 BASIC INGREDIENTS

1) The CP-asymmetry:

$$\text{type-I seesaw: } \mathcal{L} \ni -(\sum_{ij} Y_{\nu ij} \bar{N}_{Ri} \tilde{H}^+ L_j + \text{h.c.}) - \frac{1}{2} (m_{N_i} \bar{N}_{Ri} N_{Ri} + \text{h.c.})$$

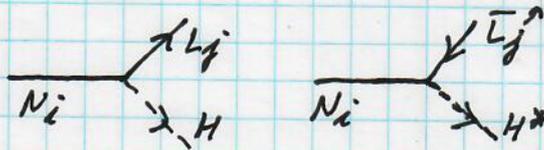
early universe thermal bath:

- at $T \gg m_{N_i}$: N_{Ri} in thermal equilibrium

$$m_{N_i}(T) = \underbrace{g_{N_i}}_{\frac{1}{2}} \cdot \frac{3}{4} \frac{\zeta(3)}{\pi^2} T^3 \sim m_{\gamma} = \underbrace{g_{\gamma}}_{\frac{1}{2}} \frac{\zeta(3)}{\pi^2} T^3$$

↳ thermalization from Y_{ν} interactions or other interactions

- at $T \leq m_{N_i}$: N_i disappears by decaying:



↳ we work in physical N_i state basis: $N_i = N_{Ri} + N_{Ri}^c$

⇒ the averaged ΔL produced each time $\propto N_i$ decays:

$$E_{N_i} = \frac{\Gamma(N_i \rightarrow L_j H) - \Gamma(N_i \rightarrow \bar{L}_j H^*)}{\Gamma_{N_i}^{\text{TOT}}}$$

↳ "CP asymmetry":

1st Sakharov cond. → requires L violation: $\begin{matrix} m_{N_i} \\ N_{Ri} \downarrow \quad N_{Ri}^c \rightarrow \\ \rightarrow \quad \times \quad \leftarrow \end{matrix} : \Delta L = 2$

- " CP " : if not: $\Gamma(i \rightarrow f) = \Gamma(i_{CP} \rightarrow f_{CP})$

2nd Sakharov cond. → $\begin{matrix} \downarrow \\ N_i \rightarrow L_j H \quad N_i \rightarrow \bar{L}_j H^* \\ \downarrow \end{matrix}$

- " C " : if not $\Gamma(i \rightarrow f) = \Gamma(i_c \rightarrow f_c)$

↳ V-A structure of Y_{ν} : no R doublet

\Rightarrow expectation: $\frac{n_L}{n} = \underbrace{\epsilon_{N_i}}_{\Delta L} \underbrace{\left(\frac{n_{N_i}}{n}\right)}_{\# \text{ of decays.}} \Big|_{\text{before decay } (T \gg m_{N_i})} \quad (*)$

after the N_i decays

$\hookrightarrow n_L = n_L - n_{\bar{L}}$ number density

$n_{N_i} = N_i$ number density

$n =$ entropy density $= \frac{2\pi^2}{45} g_{\text{bos}} T^3$

$\hookrightarrow g_{\text{bos}} = \sum_{i=\text{bos}} g_i \left(\frac{T_i}{T}\right)^3 + \sum_{i=\text{ferm}} g_i \frac{7}{8} \left(\frac{T_i}{T}\right)^3$

\hookrightarrow comoving quantity: $\propto 1/\text{Volume}$

2) The efficiency factor: (*) true only if N_i decays occur perfectly out-of-equilibrium.
3rd Sakharov condition

\hookrightarrow if not: $\frac{n_L}{n} = \underbrace{\eta_i}_{\uparrow} \epsilon_{N_i} \left(\frac{n_{N_i}}{n}\right) \Big|_{T \gg m_{N_i}} \quad (**)$

efficiency suppression factor: $\eta_i \leq 1$

3) The L asym. to B asym. sphaleron conversion factor:

\hookrightarrow breaks B+L but not B-L

\hookrightarrow are in thermal equilib.

for $T \gtrsim 150 \text{ GeV}$

$\Rightarrow \frac{n_B}{n} = \underbrace{\frac{n_L}{n} \Big|_{\text{init}}}_{= (**)} \cdot \frac{28}{79} \Rightarrow$ baryogenesis!

$\hookrightarrow \Rightarrow$ if $\eta_i \sim 1$ one needs $\epsilon_{N_i} \sim 10^{-7}$ to

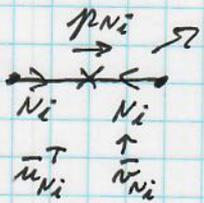
get $\frac{n_B}{n} \sim 10^{-10} \leftarrow \left(\frac{n_{N_i}}{n}\right) \Big|_{T \gg m_{N_i}} \sim 10^{-3}$

$\Rightarrow l_l \bar{l}_l$: usual propag. of a fermion $l_l: i \sum_{\vec{p}} \frac{u_{\vec{p}} \bar{u}_{\vec{p}}}{\vec{p}^2 - m_l^2} = \frac{i}{\vec{p}^2 - m_l^2}$

$\bar{l}_j^T l_j^T: (l_j \bar{l}_j)^T =$ transpose of usual external fermion line: $(\sum_{\vec{p}} u_{\vec{p}} \bar{u}_{\vec{p}})^T = (\vec{p}^2 + m_l^2)^T$

$\bar{N}_i^T N_i$: external N_i line: $\sum_{\vec{p}} \bar{u}_{\vec{p}}^T u_{\vec{p}} = C^{-1} (M_{N_i} - M_{N_i})$

$N_b N_b^T$: propag. of $N_b: i \sum_{\vec{p}} \frac{v_{\vec{p}} u_{\vec{p}}}{\vec{p}^2 - M_{N_b}^2} = -i \frac{(M_{N_b} - M_{N_b}) C^T}{\vec{p}^2 - M_{N_b}^2}$



\Rightarrow commuting C and γ matrices and performing the trace

$$M^{(1)} M^{(0)*} = i \int \frac{d^4 b}{(2\pi)^4} \frac{1}{(b+p_j)^2} \frac{1}{(b-p_H)^2} \frac{1}{(b^2 - M_{N_b}^2)} (M_{N_i}^2 + 2b p_j)$$

$\cdot M_{N_i} M_{N_b} \cdot C_{ib}$

$C_{ib} = Y_{\nu i l} Y_{\nu l b}^* Y_{\nu b j} Y_{\nu j i}$

$\Rightarrow \Gamma(N_i \rightarrow L_j H^+) = \frac{1}{16\pi} \frac{1}{M_{N_i}} \frac{1}{2} (M^{(1)} M^{(0)*} + M^{(0)} M^{(1)*}) \cdot 2$
 $I_{ib} =$ loop integral $I_{ib} C_{ib} + I_{ib}^* C_{ib}^*$ \uparrow $l_j H^+ + \nu_j H^0$

$\Gamma(N_i \rightarrow L_j H^+) = \dots (I_{ib} C_{ib}^* + I_{ib}^* C_{ib}) \cdot 2$

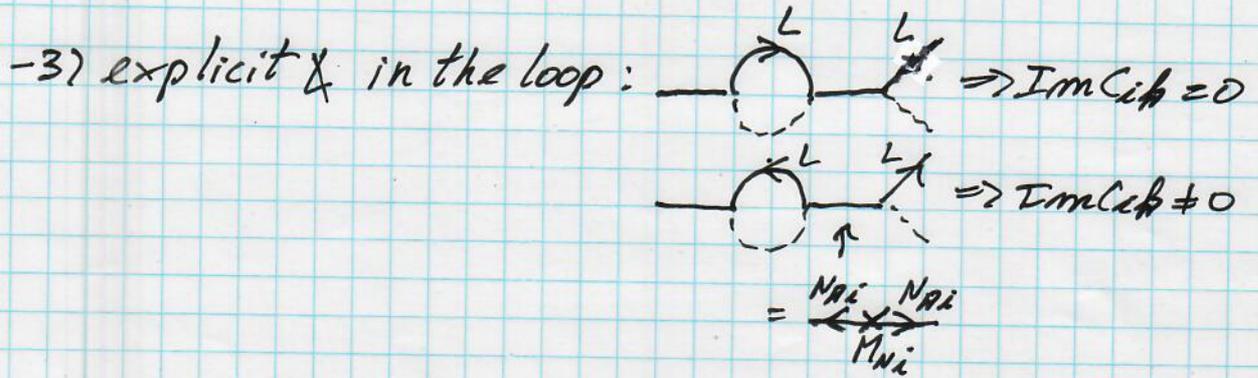
$$\Rightarrow \Sigma_{N_i} = \frac{1}{16\pi} \frac{1}{M_{N_i}} \frac{(I_{ib} C_{ib} + I_{ib}^* C_{ib}^* - I_{ib} C_{ib}^* - I_{ib}^* C_{ib})}{\frac{1}{8\pi} |Y_{\nu i l}|^2 M_{N_i}}$$

$$= -\frac{1}{2} \frac{1}{M_{N_i}^2} \cdot \frac{4 \text{Im} I_{ib} \text{Im} C_{ib}}{|Y_{\nu i l}|^2}$$

\Rightarrow non trivial requirements for α CP asym.: :

- 1) $\text{Im} C_{ib} \neq 0 \Rightarrow$ need for δP phases in C_{ib}
- 2) $\text{Im} I_{ib} \neq 0 \Rightarrow$ need for an absorptive part

$\Rightarrow M_{N_i}^2 > (m_H + m_e)^2$



-4) if $i = b \Rightarrow \text{Im} C_{ii} = 0 \Rightarrow$ need for at least 2 heavy states.

$$\Rightarrow \epsilon_{Ni} = \frac{1}{8\pi} \frac{M_{Nb}}{M_{Ni}} \frac{\text{Im} [Y_{vi1} Y_{vbe}^* Y_{vbj}^* Y_{vij}]}{|Y_{vi1}|^2}$$

$$\cdot \left\{ \left[1 - \left(1 + \frac{M_{Nb}^2}{M_{Ni}^2} \right) \log \left(1 + \frac{M_{Ni}^2}{M_{Nb}^2} \right) \right] + \left[\frac{M_{Ni}^2}{M_{Ni}^2 - M_{Nb}^2} \right] \right\}$$

↑
vertex

↑
self-energy

□ Efficiency factor: Boltzmann equations

↳ requires to compare reaction rate with Hubble expansion rate H

of reacts a single particle i undergoes per second
 $= (1/\Delta t)$

↳ time between 2 reacts.

⇒ no thermal equilibr. if: $\Delta t > \frac{1}{H} \Leftrightarrow \Gamma < H$

↳ age of the universe

↳ with $H = \left(\frac{8\pi\rho_{TOT}}{3M_p^2}\right)^{1/2} \sim 17 \frac{T^2}{Mpl}$

$$\rho_{TOT} = \frac{T^2}{30} T^4 \left[\sum_{bos} g_i \left(\frac{T_i^4}{T^4}\right) + \frac{7}{8} \sum_{ferm.} g_i \left(\frac{T_i^4}{T^4}\right) \right]$$

$\equiv g_{eff}$: # of relativistic degrees of freedom in universe thermal bath

⇒ Boltzmann equations: allow to compute the # density of each particle specie as a function of T .

for DM

of inverse scattering = γ^{EQ} because xx are assumed to be in thermal equil.

$$\Delta Z H(Z) \frac{dY_{DM}}{dZ} = 2\gamma^{EQ} \left(1 - \frac{Y_{DM}^2}{\gamma^{EQ2}}\right)$$

$$= \underbrace{-2\gamma^{EQ} \frac{Y_{DM}^2}{\gamma^{EQ2}}}_{\Delta n_{DM} \text{ per scattering}} + \underbrace{2\gamma^{EQ}}_{\Delta n_{DM} \text{ per inverse scatt}}$$

↳ takes into account that Y_{DM} is not necessarily in thermal equil.

↳ = # of scattering if DM is in thermal equil.

⇒ for Y_N : same logic:

$$\Delta Z H(Z) \frac{dY_N}{dZ} = \gamma_D^{EQ} \left(1 - \frac{Y_N}{\gamma_D^{EQ}}\right)$$

= # of inverse decays = γ_D^{EQ} because L and H are in thermal equil.

$$= \underbrace{-\gamma_D^{EQ} \frac{Y_N}{\gamma_D^{EQ}}}_{\Delta n_N \text{ per } N \rightarrow LH \text{ decay}} + \underbrace{\gamma_D^{EQ}}_{\Delta n_N \text{ per } LH \rightarrow N \text{ inverse decay}}$$

$$\gamma_D^{EQ} = m_N^{EQ} \langle \Gamma_N^{TOT} \rangle = \Gamma_N^{TOT} \langle \frac{E}{m_N} \rangle$$

↑ boost of the lifetime

↳ = # of decays if N is in thermal equilibrium

(6) $m_N \rightarrow Y_N \equiv \frac{m_N}{s}$: comoving # density \Rightarrow no more $3Hm_N$

(7) $t \rightarrow z \equiv m_N / T$ with $t = \frac{1}{2} \frac{1}{H(z)} = \frac{1}{2} \frac{z^2}{H(M_N)}$

$$\hookrightarrow H(M_N) dz = z dz$$

$$\Rightarrow \frac{dY_N}{dz} = -z \underbrace{\frac{\Gamma_N^{\text{TOT}}}{H(M_N)}}_{\equiv K(M_N)} \underbrace{\langle \frac{M}{E} \rangle}_{\substack{\uparrow \\ \text{boost factor of } \Gamma}} (Y_N - Y_N^{\text{EQ}})$$

\uparrow decay \uparrow inverse decay

defining $\gamma_D^{\text{EQ}} \equiv m_N^{\text{EQ}} \Gamma_N^{\text{TOT}} \cdot \langle \frac{M}{E} \rangle =$ "reaction density"

= # of N decay per unit time per unit volume if N is in thermal equil.

= # of inverse decays because Λ and M are in thermal equilibrium.

$$\Rightarrow \Lambda z H(z) \frac{dY_N}{dz} = - \underbrace{1}_{\substack{\Delta m_N \text{ per} \\ \text{decay}}} \cdot \underbrace{\gamma_D^{\text{EQ}} \frac{Y_N}{Y_N^{\text{EQ}}}}_{\substack{\downarrow \\ \text{# of decays per time per volume}}} + \gamma_D^{\text{EQ}} \cdot \underbrace{1}_{\substack{\uparrow \\ \text{# of inverse decays} \\ \text{per time per volume}}} \quad \leftarrow \Delta m_N \text{ per inv. decay}$$

of decays per time per volume taking into account that N is not necessarily in thermal equil.: Y_N / Y_N^{EQ} factor

\Rightarrow to see if N is in thermal equilibrium one has

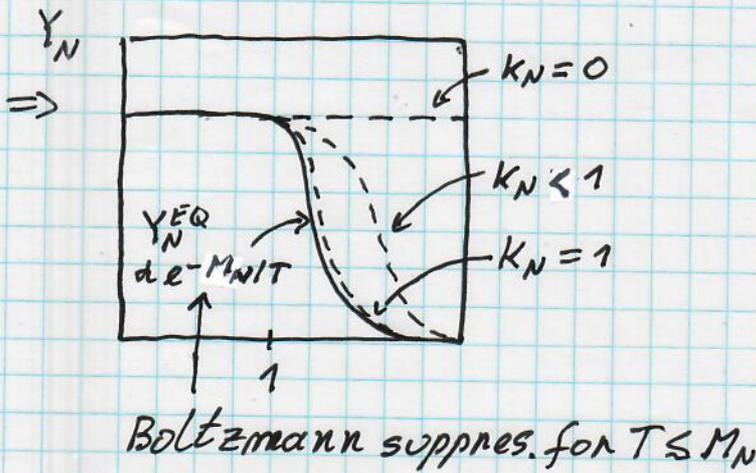
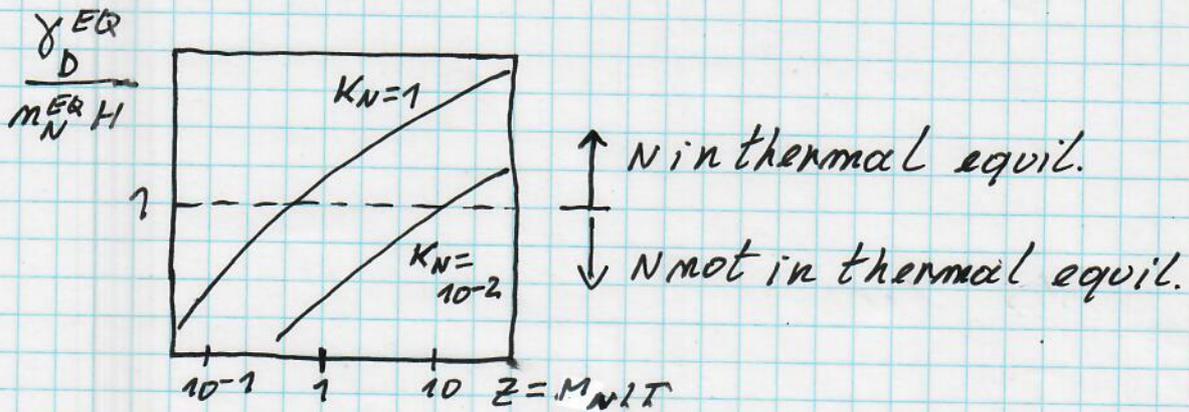
to compare $\frac{\gamma_D^{\text{EQ}}}{m_N^{\text{EQ}}} = \underbrace{\Gamma_N^{\text{TOT}} \langle \frac{M}{E} \rangle}_{\text{reaction rate}} =$ # of decay α

single N undergo per unit time, with Hubble rate

⇒ in summary:

↳ 1st Boltz. eq.: if $\frac{\gamma_D^{EQ}}{m_N^{EQ} H} (z) > 1 \Rightarrow N$ in thermal equilibrium: many decays and inverse decays

↳ ⇒ $Y_N - Y_N^{EQ}$ suppressed



↳ in 2nd Boltz. eq.:

- if no wash-out term: no efficiency suppression even if $Y_N - Y_N^{EQ}$ suppressed from 1st Boltz. eq.:

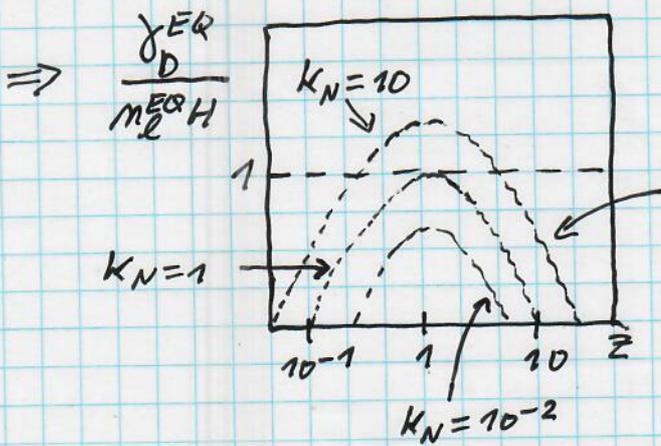
γ_D^{EQ} large in this case $\Rightarrow \gamma_D^{EQ} (Y_N - Y_N^{EQ})$ not suppressed

$\Rightarrow \gamma_D^{EQ} (Y_N / Y_N^{EQ} - 1) \epsilon_N$ term unsuppressed in 2nd Boltz. eq

$\Rightarrow \eta = 1$

- however if γ_D^{EQ} big, wash-out term in 2nd Boltz. eq. big \Rightarrow washout whenever:

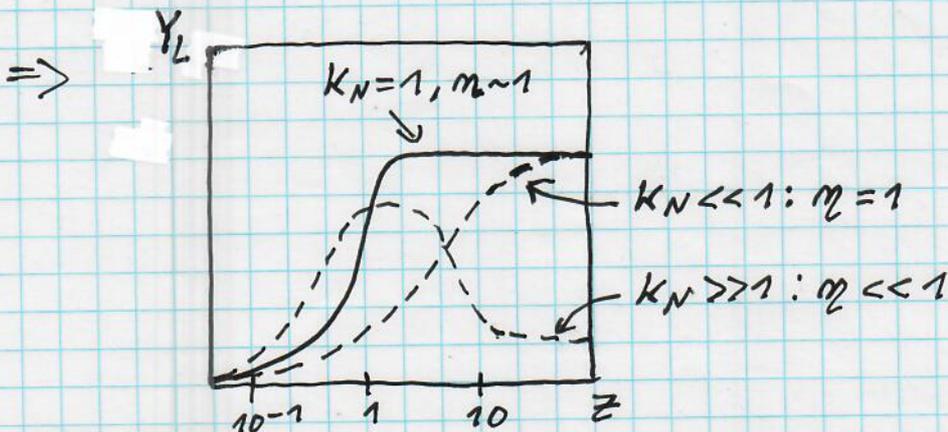
3rd Sakharov cond.: is the decay of ℓ , not of N !
 $\frac{\gamma_D^{EQ}}{m_L^{EQ} H} > 1 \Rightarrow$ inverse decay in equil. from the Γ of ℓ , not of N !
 \leftarrow # of inverse decays of ℓ will undergo per unit time \propto Hubble constant.



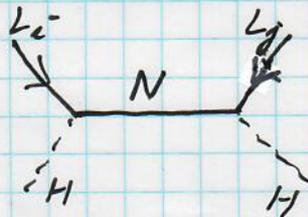
$\frac{\gamma_D^{EQ}}{m_L^{EQ} H} \propto \frac{m_N^{EQ} \cdot \Gamma}{m_L^{EQ}}$
 $\rightarrow m_N^{EQ} \propto e^{-M_N/T}$ for $M_N < T$
 \hookrightarrow Boltz. suppress.
 $\leftarrow m_L^{EQ}$ not Boltz. suppress. for $M_N < T$

\hookrightarrow this is Boltz. suppress which makes leptogenesis possible, so that we are here to talk about.

$\hookrightarrow \frac{\gamma_D^{EQ}}{m_N^{EQ} H} \propto \frac{m_N^{EQ} \cdot \Gamma}{m_N^{EQ}}$ is instead not Boltz suppress. \Rightarrow but that's no problem.



• $\Delta L = 2$ scatterings:



\Rightarrow extra $-\frac{Y_L}{Y_L^{EQ}} \gamma_{scatt}^{EQ} \cdot 2$ term in 2nd Boltz. eq.

\nwarrow # of scatter per unit time and vol.
 $\gamma_{scatt}^{EQ} = \langle \sigma v \rangle n_L^{EQ} n_H^{EQ}$

\hookrightarrow gives washout if $\frac{\gamma_{scatt}}{m_H^{EQ} H} > 1$

\hookrightarrow N.B.: the on-shell N part of $\Delta L = 2$ scatter is nothing but the inverse decays \Rightarrow need to subtract it to avoid double counting.

• erasure of asym. produced by heavier $N: N_2, N_3$

\hookrightarrow if $K_{N_1} > 1$: inverse decay washout term will put back to ~ 0 L asym produced by N_2, N_3 decays, but less L asym produced by N_1 because for the later the creation/washout are occurring at same time while for the former creation occurs before N_1 washout.