

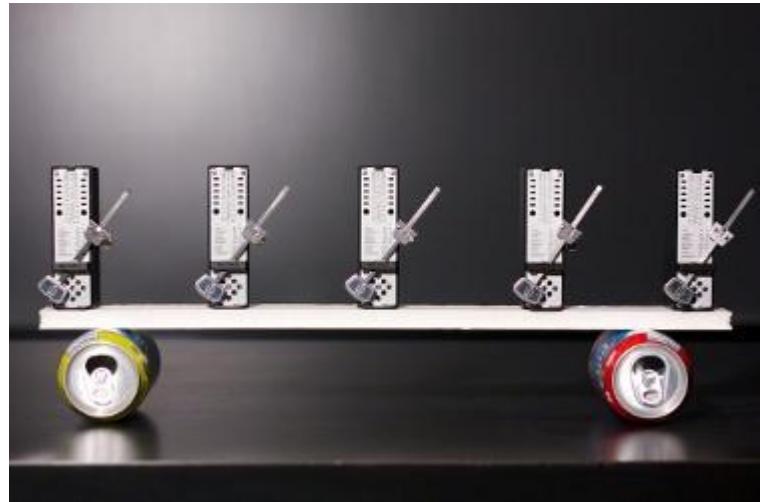


The influence of external drives on the Frustrated Kuramoto model

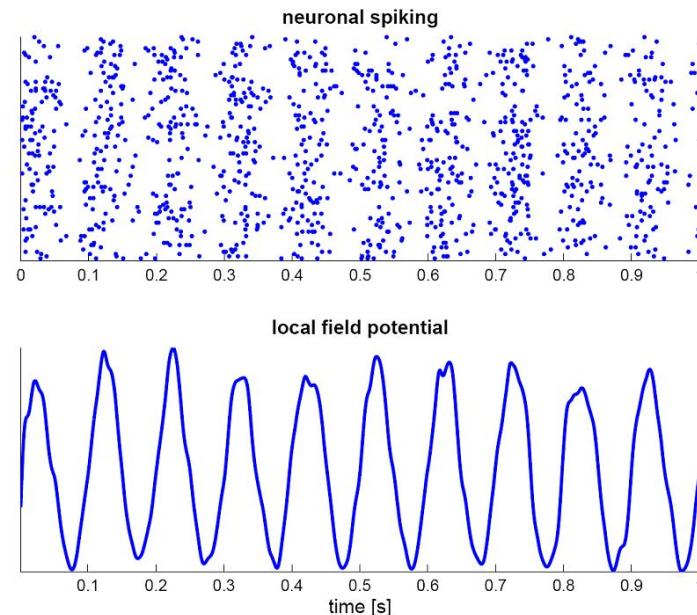
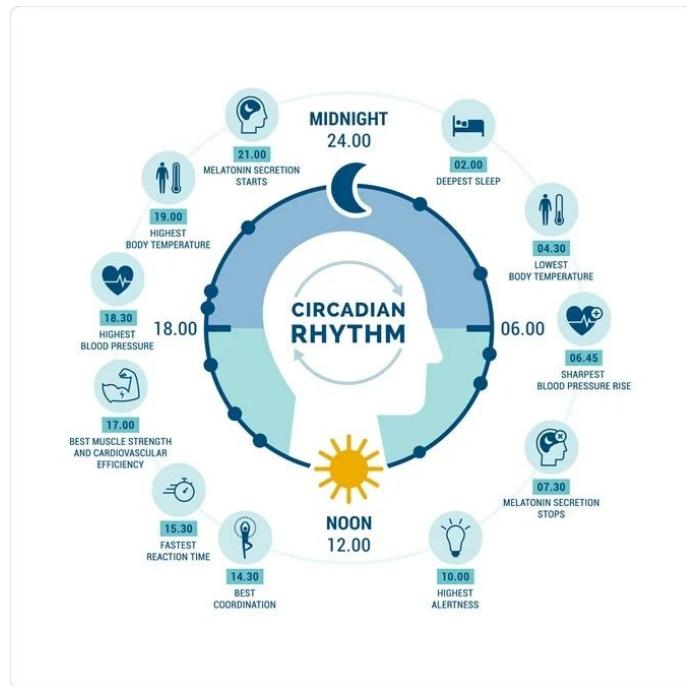
Guilherme S. Costa and Marcus A. M. de Aguiar

Workshop on Dynamical Processes on Complex Networks

Synchronization examples



More “complex” mechanisms



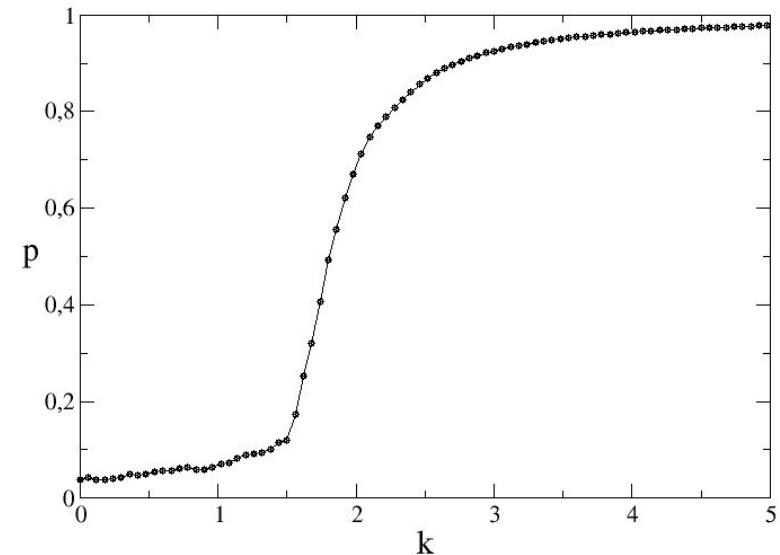
Original Kuramoto model

Population of N oscillators described by phases θ_i with natural frequencies ω_i :

$$\dot{\theta}_i = \omega_i + \frac{k}{N} \sum_{j=1}^N \sin(\theta_j - \theta_i)$$

Complex order parameter:

$$z = pe^{i\psi} \equiv \frac{1}{N} \sum_{j=1}^N e^{i\theta_j}$$

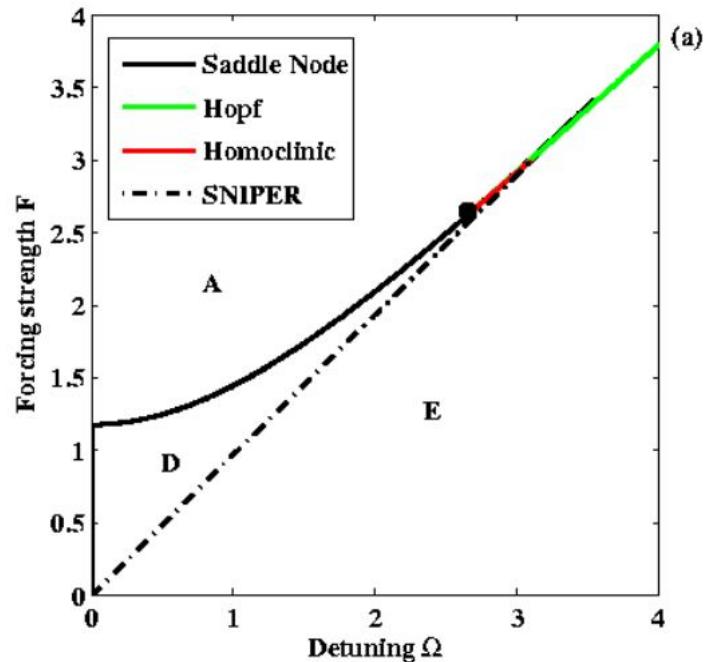


Kuramoto with external drives

$$\dot{\theta}_i = \omega_i + \frac{k}{N} \sum_{j=1}^N \sin(\theta_j - \theta_i) + F \sin(\Omega t)$$

$$\dot{p} = -\Delta p + \frac{kp}{2}(1-p^2) + \frac{F}{2}(1-p^2) \cos(\psi - \Omega t)$$

$$\dot{\psi} = \omega_0 - \Omega - \frac{F}{2} \left(p + \frac{1}{p} \right) \sin(\psi - \Omega t)$$

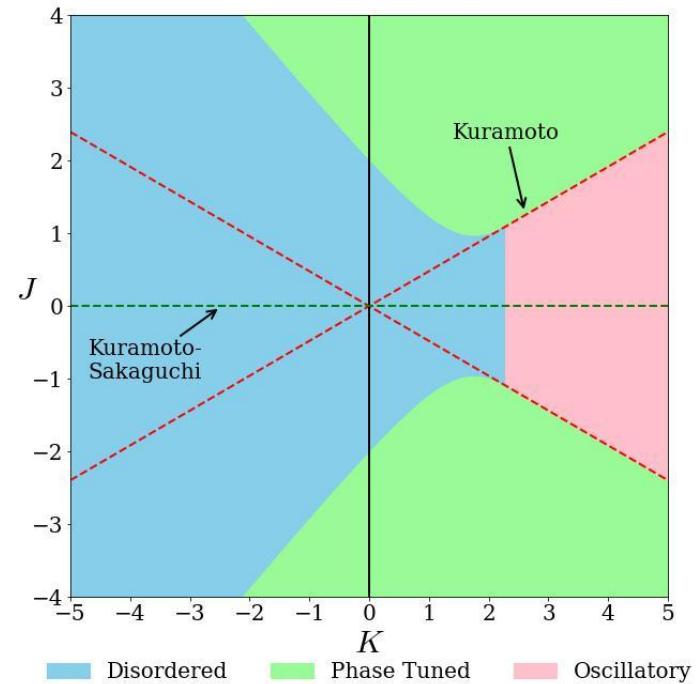


Matrix coupling and generalized frustration

$$\frac{d\vec{\sigma}_i}{dt} = \mathbf{W}_i \vec{\sigma}_i + \frac{1}{N} \sum_{j=1}^N [\mathbf{K} \vec{\sigma}_j - (\vec{\sigma}_i \cdot \mathbf{K} \vec{\sigma}_j) \vec{\sigma}_i]$$

$$\mathbf{K} \equiv \mathbf{K}_R + \mathbf{K}_S = K \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} + J \begin{pmatrix} -\cos \beta & \sin \beta \\ \sin \beta & \cos \beta \end{pmatrix}$$

$$\dot{\theta}_i = \omega_i + \sum_{j=1}^N [K \sin(\theta_i - \theta_j - \alpha) + J \sin(\theta_i + \theta_j + \beta)]$$



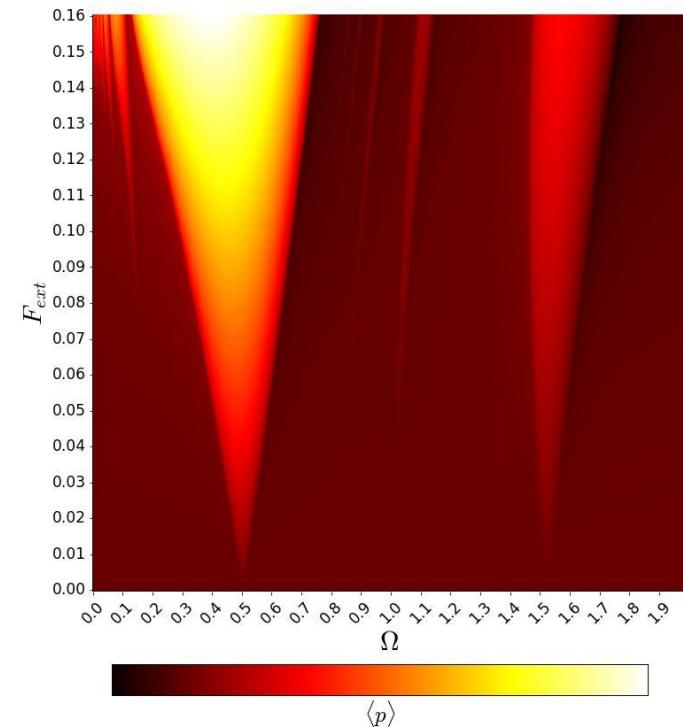
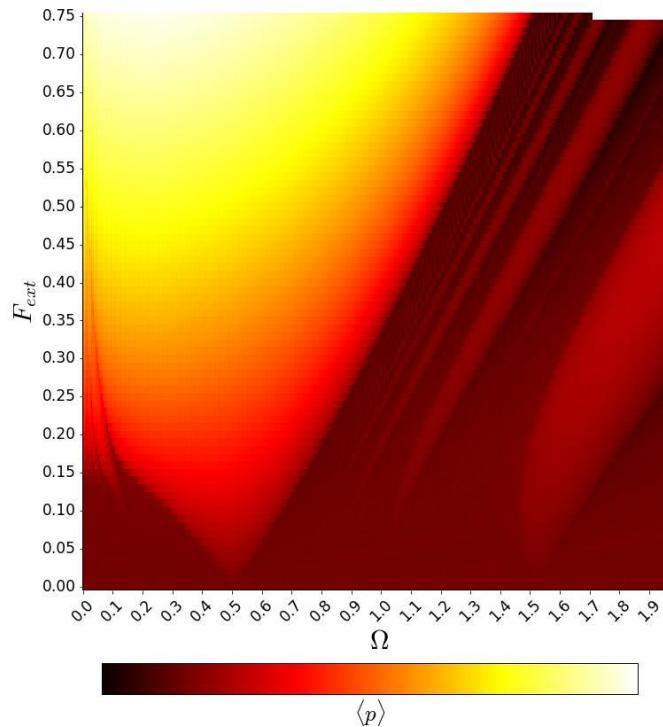
External drives on matrix coupled oscillators

$$\dot{p} = -\Delta p + \frac{(1-p^2)}{2} [pK \cos \alpha - pJ \cos(2\phi + 2\Omega t + \beta) + F \cos \phi]$$

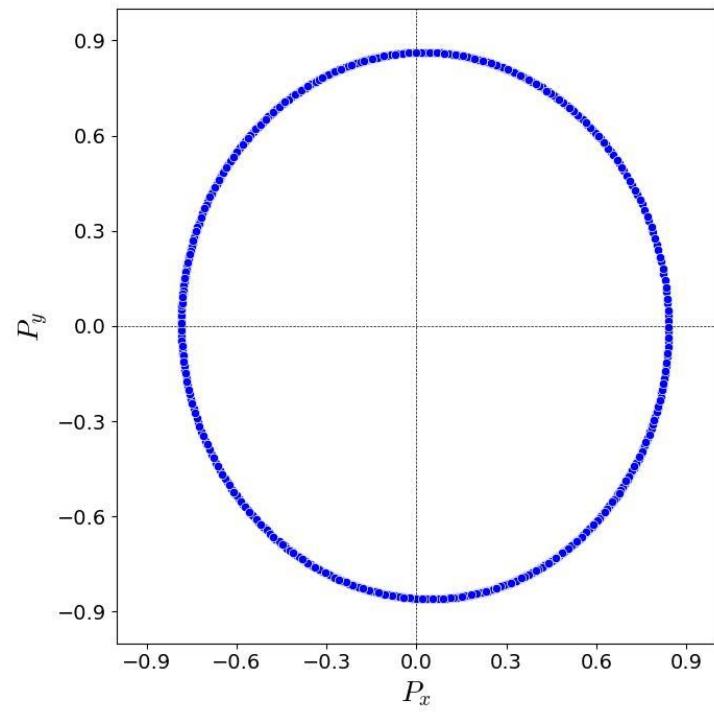
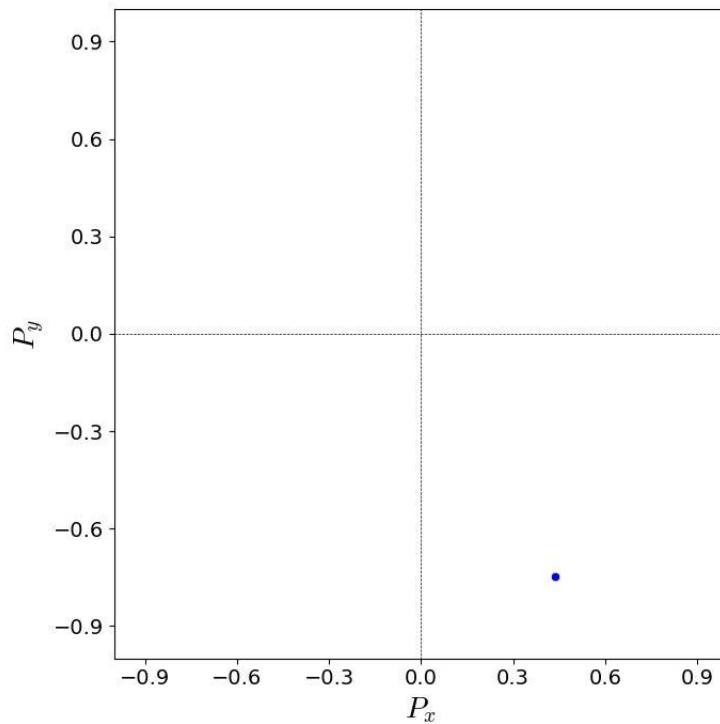
$$\dot{\phi} = (\omega_0 - \Omega) - \frac{(1+p^2)}{2} \left[K \sin \alpha - J \sin(2\phi + 2\Omega t + \beta) + \frac{F}{p} \sin(\phi) \right]$$

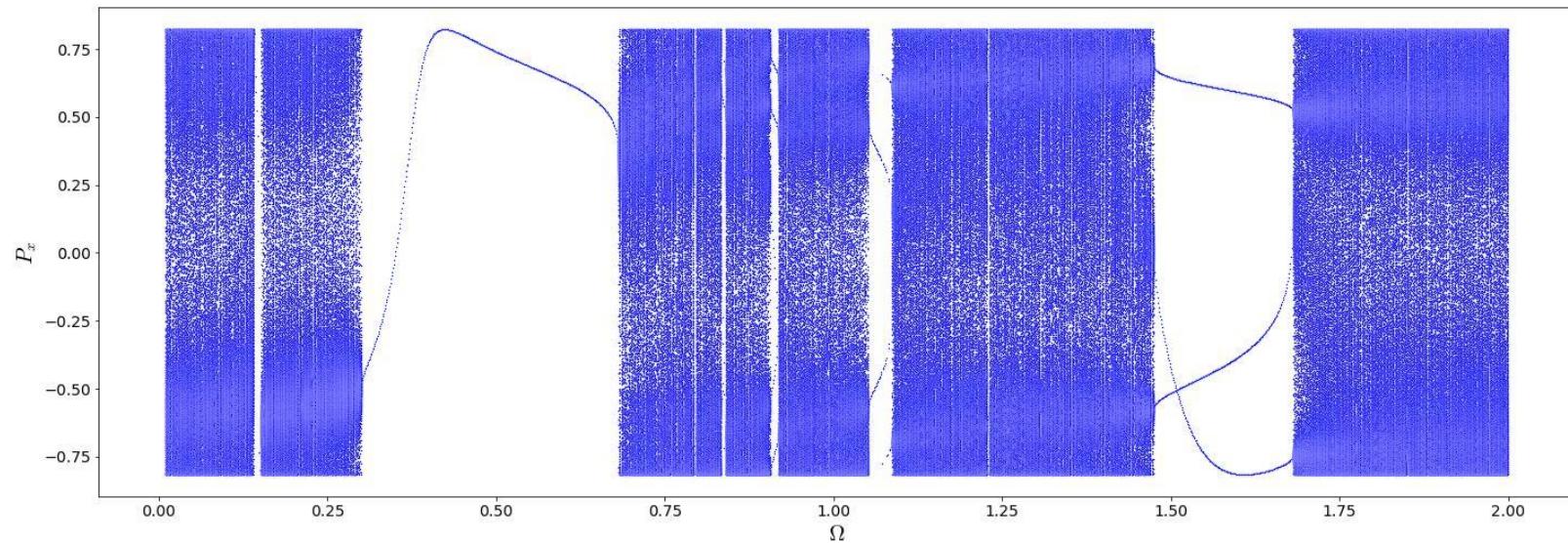
Equations are non-autonomous!

Oscillatory states ($\omega = 0.5$; $\Delta=1.0$)



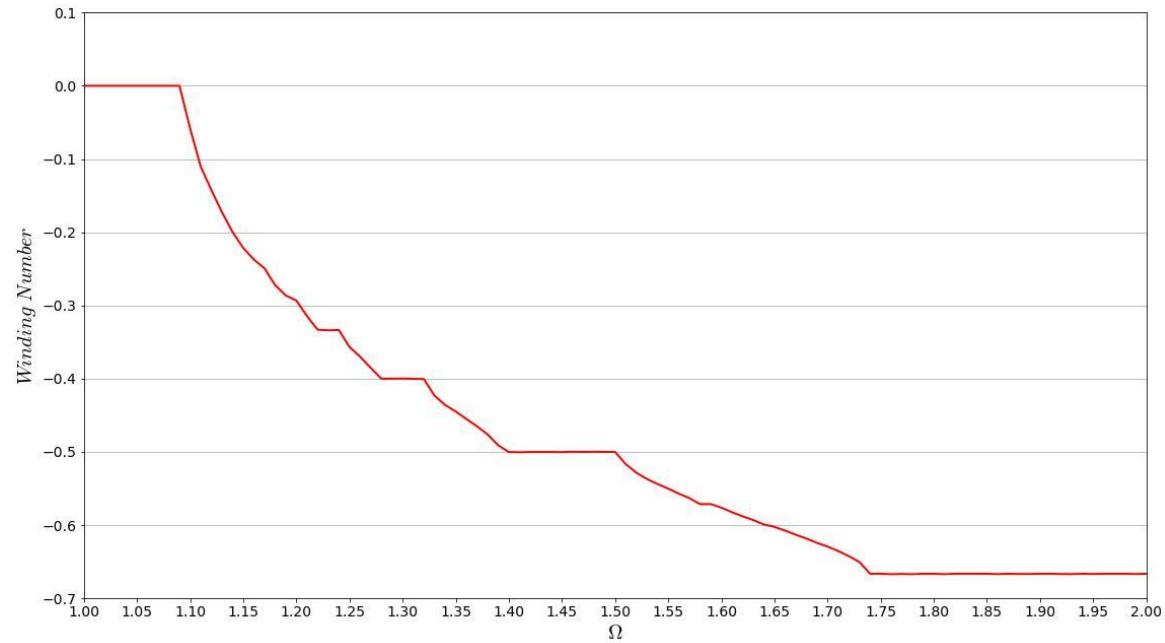
Poincaré maps

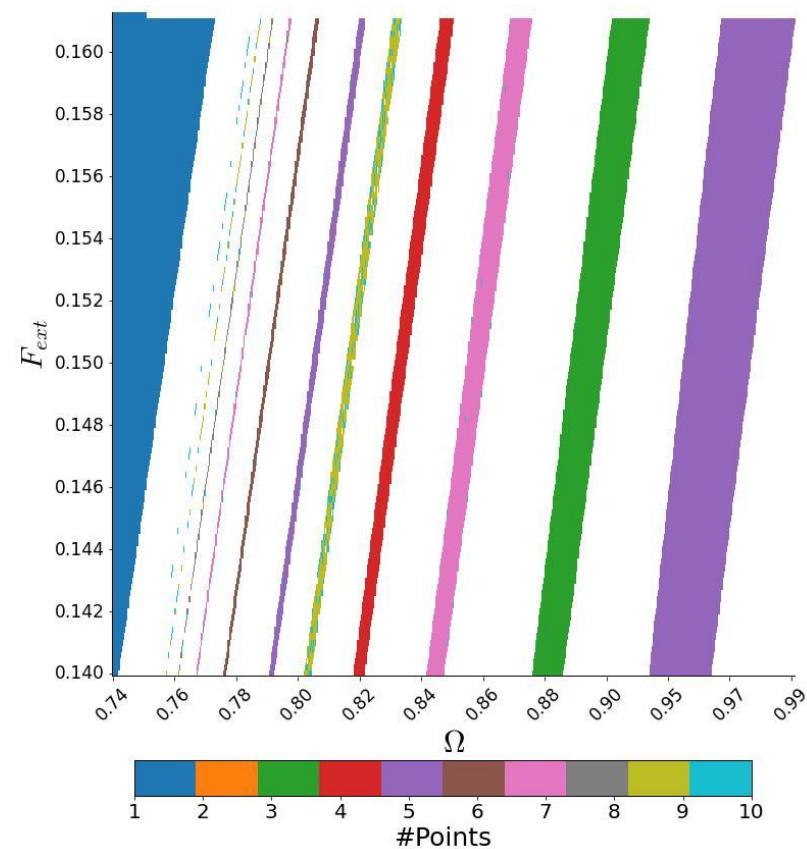
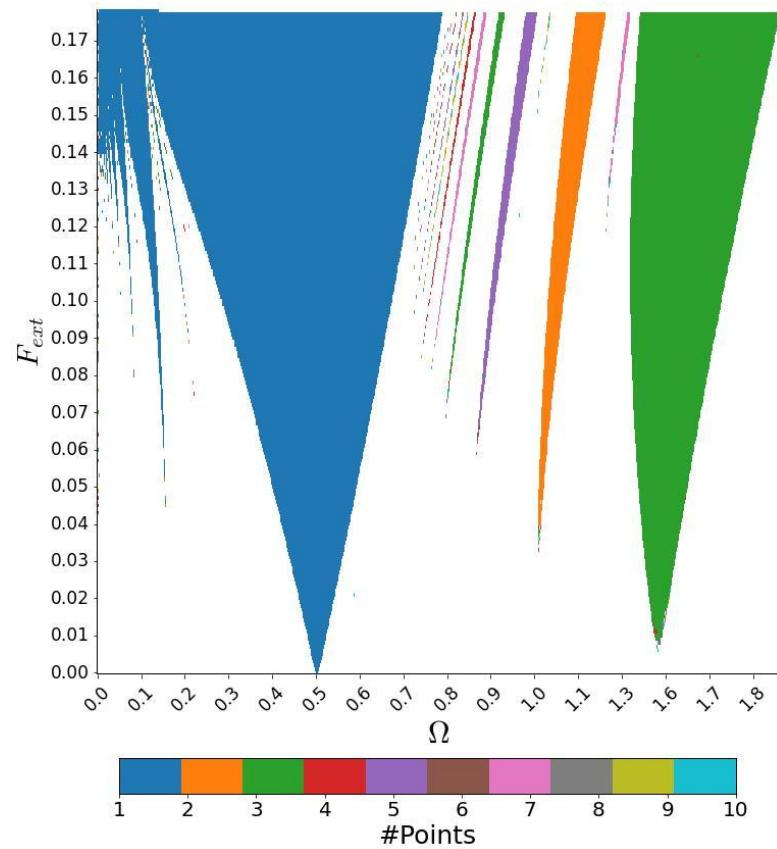




Winding Number

$$w = \lim_{n \rightarrow \infty} \frac{\theta_n}{n}$$





Final remarks and Prospects

- The matrix coupling Kuramoto model exhibits novel and interesting new synchronized states.
- Adding a global external drive to it we can obtain dynamical equations for the order parameter via Ott-Antonsen ansatz.
- The external driving acting on oscillatory synchronized states lead to the formation of Arnold tongues and mode-locking regions.
- What happens in the phase tuned region ?

Thanks for your attention!

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