Neutrino Phenomenology - Practical Activity

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Here is a list of important and (allegedly) fun exercises we could do throughout the week. For the Practical Activities, pick the one exercise you are most interested in and work it out. I suggest you form groups of 3-5 people. It will be helpful to do things in Python (packages: numpy, vegas, scipy, matplotlib).

——— DO NOT HANG OUT ONLY WITH PEOPLE YOU KNOW ———

Exercise 1 - oscillation probability

From

$$P(\nu_\alpha \to \nu_\beta) = |\langle \nu_\alpha | \nu_\beta \rangle|^2 \,, \tag{1}$$

derive the following formula for the neutrino oscillation probability:

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{j>i} \mathrm{Re}(U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}) \sin^2\left(\frac{\Delta m^2_{ji} L}{4E}\right) \\ + 2 \sum_{j>i} \mathrm{Im}(U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j}) \sin\left(\frac{\Delta m^2_{ji} L}{2E}\right). \end{aligned} \tag{2}$$

Here, U is the PMNS matrix defined as $|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle$, $\Delta m^2_{ji} = m^2_j - m^2_i$ is the mass-squared difference, L is the distance traveled by the neutrino, and E is the neutrino energy.

Exercise 2 - matter potential*

Derive the matter potential without cutting corners – *this may be challenging.* First, write down the weak currents that will help you out in the large expressions. Then, you should integrate out the W and Z bosons, dropping out the heavy quarks as they will not matter for this exercise. To do that, derive their equations of motion, and expand on large $M_{W/Z}$, setting the field derivatives to zero. After integrating out the W and Z bosons, trace over the background, that is, integrate over the background, which should give you the density of electrons, ups, and downs (which can be translated into protons and neutrons). Finally, you can use the equations of motion for the neutrino to derive a Dirac equation, and you should identify the integrated out piece of the Lagrangian.

For detailed guidance, see section 4.2 of arXiv:2206.13449.

Exercise 3 - bi-probabilities

Plot the bi-probability plot. That is, you plot the probability of ν_μ oscillating to ν_e versus $\bar{\nu}_\mu$ oscillating to $\bar{\nu}_e$ as you vary δ_{CP}. Do this plot for the NOvA (810 km, 2 GeV) and T2K (295 km, 650 MeV) configurations with and without matter effects to see the difference for both normal and inverted mass orderings. Play a bit with the oscillation parameters to see the dependence of the bi-probability on them (e.g. change one at a time and see how the probabilities change).

This may help you in understanding the recent results from NOvA and T2K on the mass ordering and CP violation (Dan may cover that).

Exercise 4 - CEvNS cross section

Calculate the cross-section for coherent neutrino nucleus scattering. Start from either the effective Lagrangian, integrating out the W and the Z, or the full Lagrangian but only keep leading order terms on the W and Z masses. Calculate the amplitude, as I will set up in the class. You can solve the traces of Dirac matrices by hand or using automated tools (e.g. Mathematica's FeynCalc). If you never did traces of Dirac matrices, do not cut corners, do it at least once. Use the PDG kinematics section to calculate the 2-to-2 neutrino-nucleus scattering.

After this calculation, you'll get the differential cross-section as a function of t, the momentum transfer. Write this differential cross-section as a function of the recoil energy of the nucleus (defined as the kinetic energy of the outgoing nucleus $E_R = E_N - M_N$), and then integrate this differential cross-section to obtain the total cross-section down to some minimum recoil energy. Observe how the cross-section grows as a function of the energy threshold.

CEvNS will eventually become a very hard to surpass background for DM direct detection (see lectures by Graciela and Javier).

Exercise 5 - simple oscillation experiment simulation

Set up a simple simulation of a neutrino experiment. Calculate the number of events as a function of the reconstructed neutrino energy by folding the flux, cross-section, efficiency and energy resolution (a.k.a. migration matrix). The latter is a function of the true neutrino energy and the reconstructed neutrino energy. This can be written as an integral,

$$\frac{dN}{dE_\nu^{\rm rec}} = N_{\rm tgt}T_{\rm exp}\int dE_\nu^{\rm true}\frac{d\phi}{dE_\nu^{\rm true}}\sigma\left(E_\nu^{\rm true}\right)M\left(E_\nu^{\rm true},E_\nu^{\rm rec}\right)\epsilon\left(E_\nu^{\rm rec}\right) \tag{3}$$

where

- $\frac{d\phi}{dE_\nu^{\rm true}}$ is the neutrino flux as a function of the true neutrino energy,
- $\sigma\left(E_\nu^{\rm true}\right)$ is the total cross-section,
- $M\left(E_\nu^{\rm true},E_\nu^{\rm rec}\right)$ is the migration matrix, and
- $\epsilon\left(E_\nu^{\rm rec}\right)$ is the efficiency as a function of the reconstructed neutrino energy.

If this is too easy, you can also write a simple χ^2 with systematic uncertainties to get closer to an experimental simulation,

$$\chi^2 = \sum_{\rm bins}\left\{\frac{\left[(1+x)N_i^{\rm th} - N_i^{\rm true}\right]^2}{(1+x)N_i^{\rm th}} + \left(\frac{x}{\sigma_x}\right)^2\right\}, \tag{4}$$

where σ_x is an overall normalization uncertainty, and x is a pull parameter (you should minimize on that). This is an example of how to implement simplified systematic uncertainties in a χ^2.

Note that this is very simplified, Dario will cover statistics better than I could do.

Exercise 6 - generating events

Generate events using the Vegas Python package for a given differential cross-section. Take a differential cross-section, for example, the quasi-elastic cross-section, or the neutrino-electron scattering cross-section. To do this, you need to:

- Set up Vegas to integrate the differential cross-section.
- Use Vegas to generate weighted events given the integration.
- Figure out the relations in kinematics to generate different electron recoils and translate this into the full momentum for the electrons.

You can start by setting up the differential cross-section in the Vegas package, perform the integration, and then use the results to generate events. Make sure to take into account the kinematics to properly simulate the electron recoil and momentum.

Several BSM searches in neutrino experiment will rely on event generation to improve signal-to-noise ratio (see talk by Matheus).

Exercise 7 - flavor triangles

Plot a flavor composition for ultra-high-energy neutrinos at detection for different source compositions. For example, $\left(\frac{1}{3} : \frac{2}{3} : 0\right)$, $(1 : 0 : 0)$, $(0 : 1 : 0)$, or a more general source $(x : 1 - x : 0)$. If you feel brave enough, make a triangle plot. You can Google for Mathematica or Python code to find it easily. If you're even braver, you can add the current IceCube constraints on that triangle plot to see if we can actually distinguish the sources or not using ultra-high-energy neutrinos.