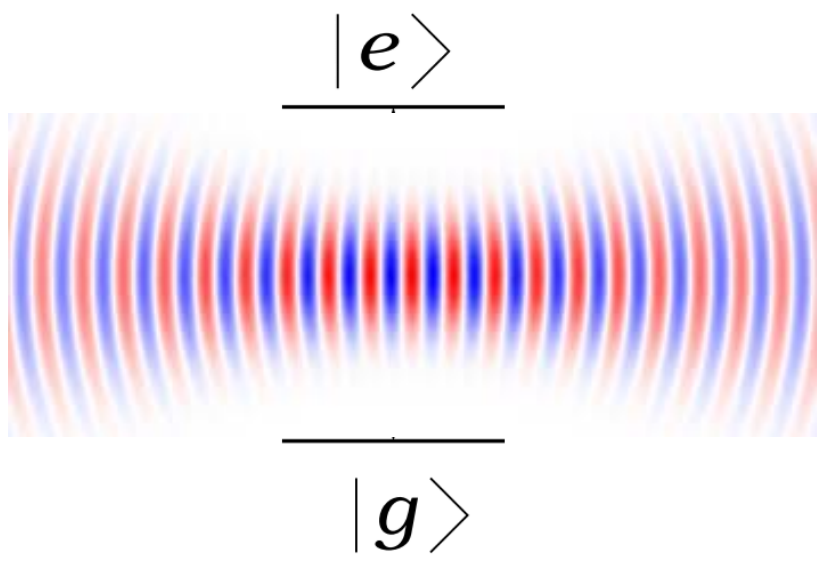
Validity of Born-Markov approximation



atom interacting with quantized radiation field

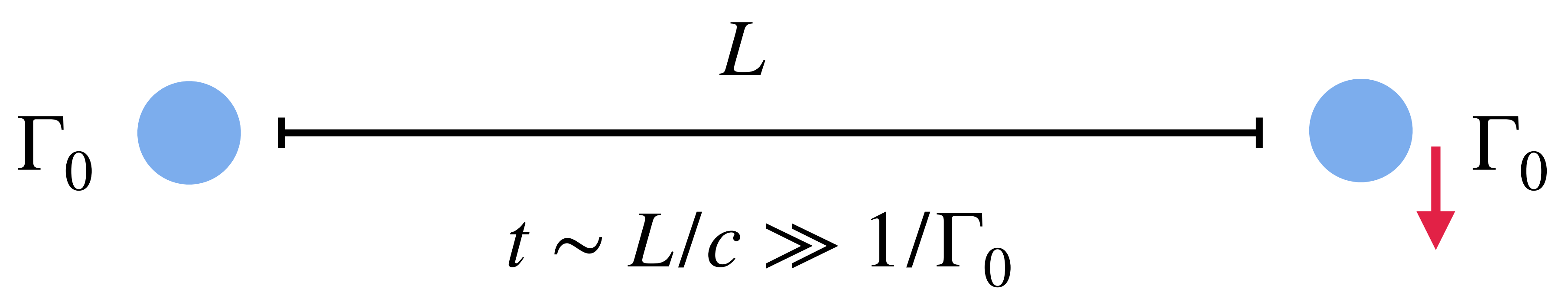
(inverse) atomic radiative lifetimes $\sim 10^{7-9} s^{-1}$ $(\simeq \tau_R^{-1})$

vs optical frequencies of the EM field continuum $\sim 10^{15} s^{-1} (\simeq \tau_B^{-1})$

$\longrightarrow$ Markov approximation

$\tau_B \ll \tau_R$ ✓

When does Markov approximation fail?



atomic decay rate ✗

large propagation time (delay effects!)

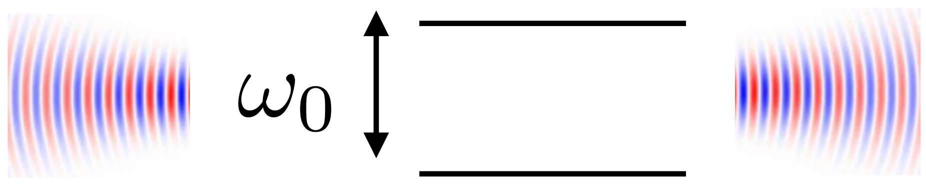
<u>another example</u>: spin coupled to bosonic bath $(H_I = g a^\dagger \sigma^- + h.c.)$

if bosons have long lifetime (spectral function like a delta Dirac) the system performs several coherent oscillations (revivals): breakdown of Markov bath approximation ✗

Examples & Exercises

Decay of a two-level system

$$H_S = \frac{1}{2}\omega_0\sigma_3$$



$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

interacts with continuum of EM modes (several harmonic oscillators)~bath

emission

$$\frac{d}{dt}\rho(t) = \gamma_0(N+1)\left(\sigma_-\rho(t)\sigma_+ - \frac{1}{2}\sigma_+\sigma_-\rho(t) - \frac{1}{2}\rho(t)\sigma_+\sigma_-\right)$$

absorption

$$+\gamma_0 N\left(\sigma_+\rho(t)\sigma_- - \frac{1}{2}\sigma_-\sigma_+\rho(t) - \frac{1}{2}\rho(t)\sigma_-\sigma_+\right),$$

$$H_I \sim \sigma^- \sum_i b_i^\dagger + h.c$$

→ thermal occupation of the boson (bath)/ temperature T

HOW TO SOLVE

write generic ansatz: $\rho = \begin{bmatrix} p & a \\ a^* & 1-p \end{bmatrix}$, and plug into Lindblad equation

$a(t) \sim \exp(-\gamma t/2) \to 0 \quad p(t \to \infty) \sim 1/2(1 - 1/(2N+1))$

when $T = 0 \to N = 0$ and $p = 0$ (no population in the excited state, all lost via emission || look also at the rates in Lindblad equation)

Your exercise

$H_S = \frac{1}{2}\omega_0\sigma_3$ with jump operator $L = \sigma_2$

this problem is equivalent to $H_S + h(t)\sigma_2$

where $h(t)$ is Gaussian delta correlated noise ||

explore literature to find proof of this statement

and convince yourself that in this case the only possible steady state is $\rho \propto 1$

Examples & Exercises

Damped harmonic oscillator

$$\frac{d}{dt}\rho_S(t) = -i\omega_0\left[a^\dagger a, \rho_S(t)\right]$$
$$+\gamma_0(N+1)\left\{a\rho_S(t)a^\dagger - \frac{1}{2}a^\dagger a\rho_S(t) - \frac{1}{2}\rho_S(t)a^\dagger a\right\}$$
$$+\gamma_0 N\left\{a^\dagger\rho_S(t)a - \frac{1}{2}aa^\dagger\rho_S(t) - \frac{1}{2}\rho_S(t)aa^\dagger\right\}$$

there exists an Heisenberg evolution for operators in Lindblad dynamics ($\rightarrow$ **derivation in (3.84 of Breuer)**):

$$\frac{d}{dt}A_H(t) = +i\omega_0\left[a^\dagger a, A_H(t)\right]$$
$$+\gamma_0(N+1)\left\{a^\dagger A_H(t)a - \frac{1}{2}a^\dagger a A_H(t) - \frac{1}{2}A_H(t)a^\dagger a\right\}$$
$$+\gamma_0 N\left\{aA_H(t)a^\dagger - \frac{1}{2}aa^\dagger A_H(t) - \frac{1}{2}A_H(t)aa^\dagger\right\},$$

using bosonic commutation relations:

$$a_H(t) = e^{(-i\omega_0-\gamma_0/2)t}a,$$
$$a_H^\dagger(t) = e^{(+i\omega_0-\gamma_0/2)t}a^\dagger,$$
$$\left(a^\dagger a\right)_H(t) = e^{-\gamma_0 t}a^\dagger a + N\left(1-e^{-\gamma_0 t}\right)$$

$\longrightarrow$

$$\langle a(t)\rangle = \mathrm{tr}\left\{a_H(t)\rho_S(0)\right\} = \langle a(0)\rangle e^{(-i\omega_0-\gamma_0/2)t}$$

$$\langle a^\dagger a(t)\rangle = \mathrm{tr}\left\{\left(a^\dagger a\right)_H(t)\rho_S(0)\right\} = \langle a^\dagger a(0)\rangle e^{-\gamma_0 t} + N\left(1-e^{-\gamma_0 t}\right)$$

Your exercises

- solve the same exercise focusing on the dynamics of $\rho(t)$ and not of observables
- solve the same exercise with:
  $H = \epsilon(a + a^\dagger)$ (coherent pump)
  OR with $L = a^\dagger$(incoherent pump)
  and comment on the results

Structure and symmetries of Lindblad equation

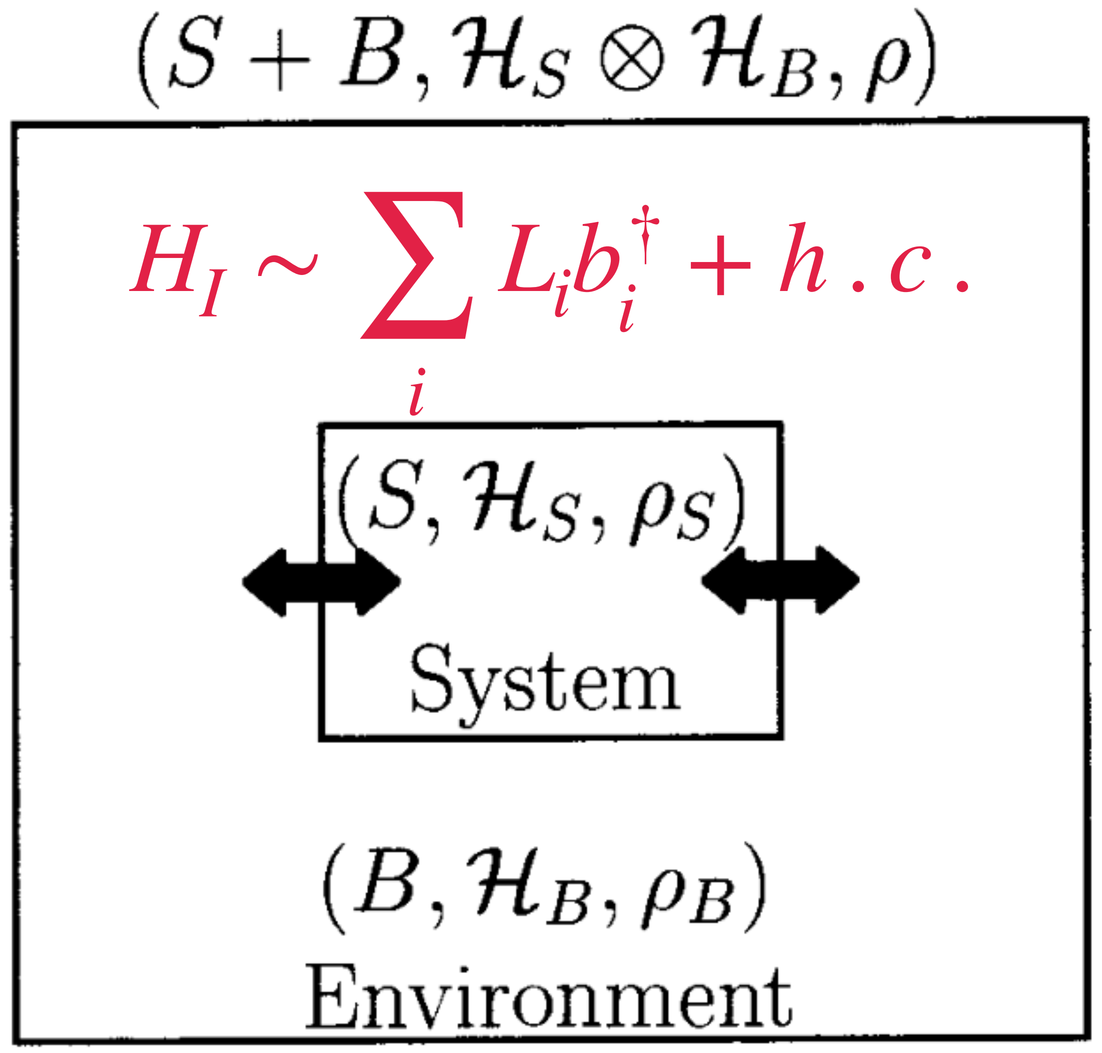
$$\partial_t \rho = -i[H,\rho] + \kappa \sum_i \left(L_i \rho L_i^\dagger - \frac{1}{2}\{L_i^\dagger L_i, \rho\}\right)$$

Lindblad operators encode dissipative channels

~noise/jump

~anti-hermitian hamiltonian controls the decay rate

$$-i\left(H - i\frac{\kappa}{2}\sum_i L_i^\dagger L_i\right)\rho + H.c.$$



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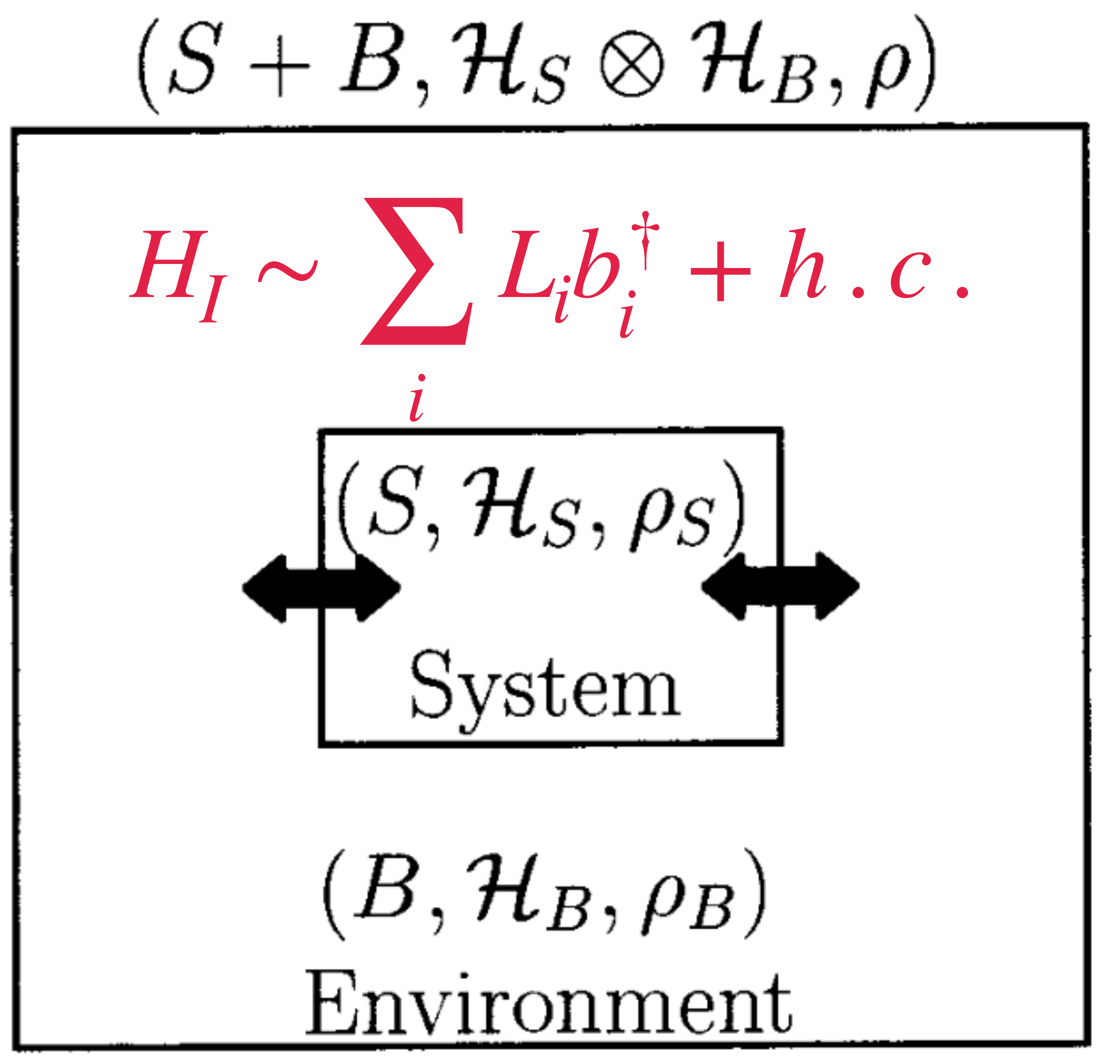
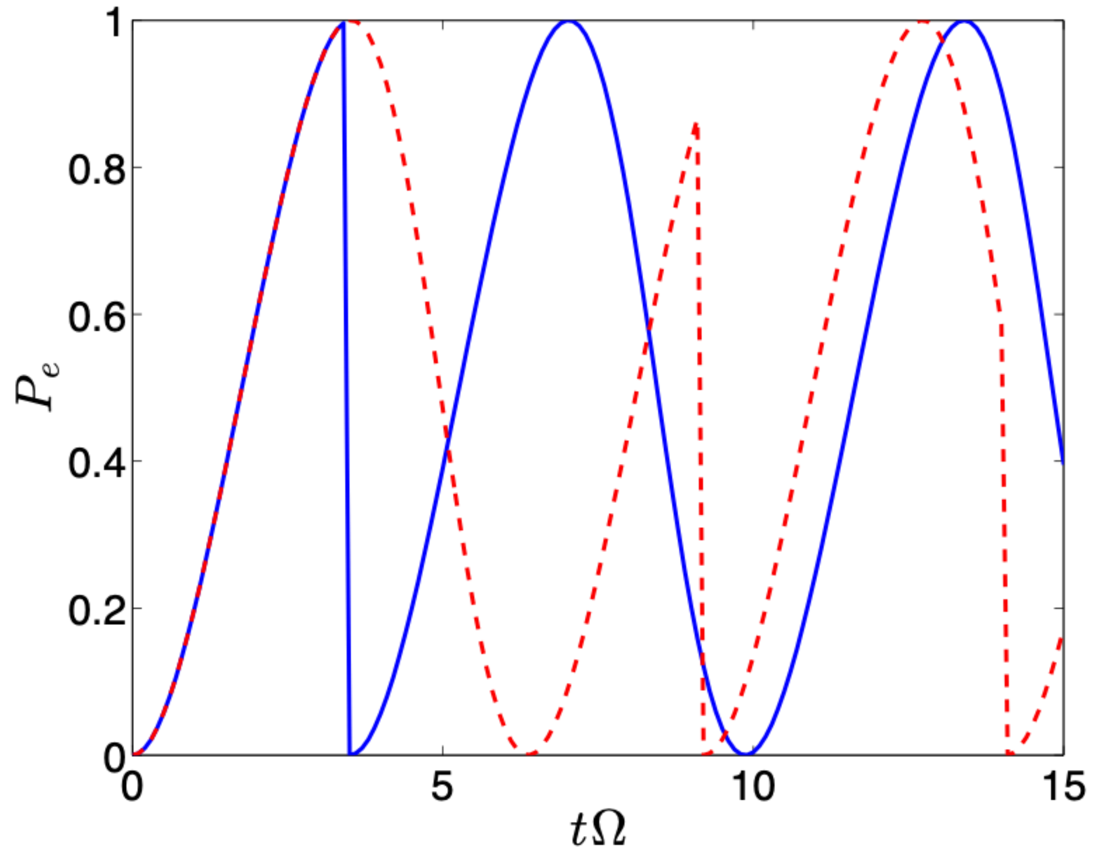


Example

2 possible 'trajectories' (single realization of noisy dynamics)



$$H = \Delta\sigma^z + \Omega\sigma^x$$

$$L = \sqrt{\Gamma}\sigma^-$$

→ A. Daley Sec. III of arXiv:1405.6694

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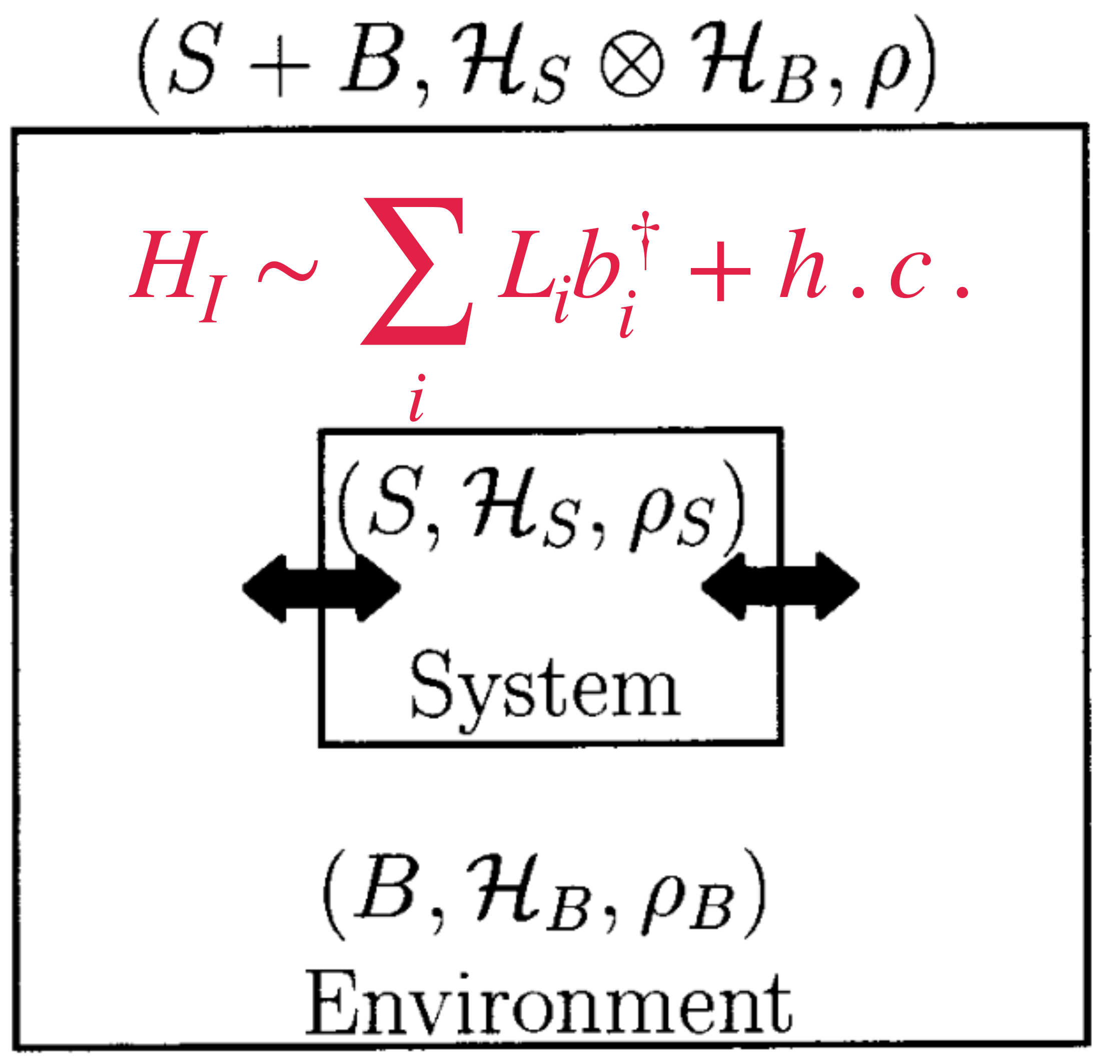

→ conservation of probability: in general we have $\mathrm{Tr}(\rho) = 1$, and Lindblad dynamics preserve it: $\partial_t \mathrm{Tr}(\rho) = 0$

question: is purity preserved?

→ symmetries: consider the specific case of $H_I = \sum_\mu g_\mu a^\dagger b_\mu$ + h.c. with $H_S = a^\dagger a$ and $H_E = \sum_\mu \omega_\mu b_\mu^\dagger b_\mu$

the total number of particles $N = N_s + N_E$ is conserved, but individually N_s and N_E aren't, and, when the environment is traced out in order to derive the Lindblad equation, no symmetry is left (no individual number particle conservation)

→ nevertheless, notice that the jump operator $L = a$ has the interesting property that the whole Liouvillian is symmetric under $a \to -a$ this is called a weak symmetry of the Liouvillian and it does not entail any conservation law