Harvesting Vacuum Entanglement in Cosmic String Spacetime

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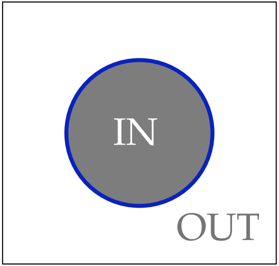
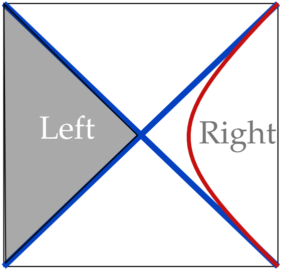
Vacuum Entanglement

The quantum vacuum is a state with mechanical properties (Casimir Polder effect) and thermal characteristics (Unruh Hawking effect).



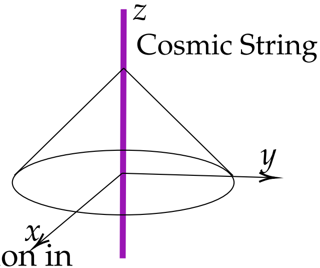


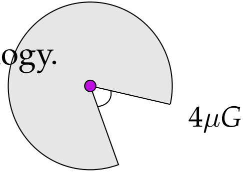
Also there is entanglement associated to the quantum vacuum state.

$$|0_k\rangle \sim \frac{1}{\cosh r} \sum_n \tanh^n r |n_k\rangle^I |n_k\rangle^{II}. \tag{1}$$

Cosmic String

- Topological defect (Kibble).
- Relativistic version of Abrikosov vortices.
- Symmetry breaking phase transition in the early universe.
- Flat spacetime with non-trivial topology.
- Conical space with angular deficit.





The line element of the spacetime generated by the cosmic string is given by

$$ds^2 = dt^2 - dr^2 - dz^2 - r^2 d\varphi^2, \tag{2}$$

where $0 < r < \infty$, $-\infty < z < \infty$, $0 < \varphi < 2\pi b$ with

$$b = 1 - 4\mu G.$$

In this space the field modes are given by

$$\psi_{\mathbf{k}}(r,\varphi,z) = \sqrt{\frac{p}{2\pi b}} J_{|\lambda l|}(pr) e^{i\lambda l \varphi} e^{i\kappa z}, \tag{3}$$

where $\lambda = 1/b$, $l = \{0, \pm 1, \pm 2, ...\}$, $p \in (0,\infty)$ and $\kappa \in (-\infty,\infty)$ and the Green function of the field can be written as

$$G(t,\mathbf{x},t'\mathbf{x}') = -\frac{1}{2\pi} \int d\mu(\mathbf{k}) \int d\omega \frac{1}{\omega^2 - \omega_{\mathbf{k}}^2} e^{i\omega(t-t')} \psi_{\mathbf{k}}(\mathbf{x}) \psi^*_{\mathbf{k}}(\mathbf{x}'). \tag{4}$$

The string quantization condition requires that the parameter $\lambda = b^{-1}$ must be a positive integer. This is the topological charge of the string $\lambda = n$, with $n \in \mathbb{N}$. Therefore the Wightman function

$$G_+(t, \mathbf{x}, t', \mathbf{x}') = \frac{1}{4\pi^2} \sum_{k=0}^{n-1} \frac{1}{(\Delta t + i\epsilon)^2 - d_{kn}^2}, \tag{5}$$

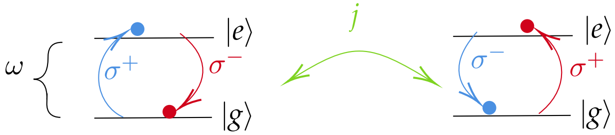
where $\Delta t = t - t'$, $\Delta r = r - r'$, $\Delta z = z - z'$, $\Delta\varphi = \varphi - \varphi'$ and

$$d_{kn} = \sqrt{\Delta r^2 + \Delta z^2 + 4rr' \sin^2(\pi k/n + \Delta\varphi/2)}. \tag{6}$$

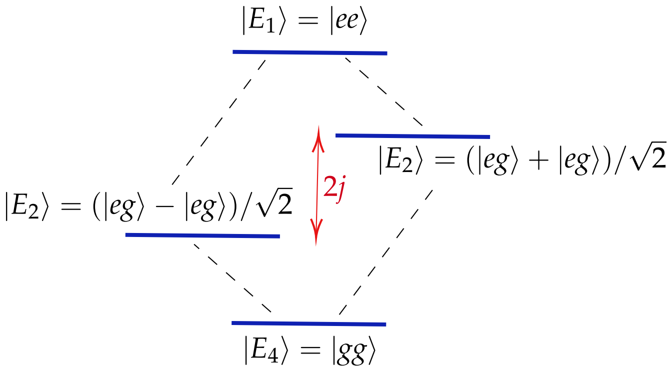
We model a qubit as a two-level system and consider a pair of identical qubits with a Hamiltonian given by

$$\mathcal{H}_0 = \omega\left(\tilde{\sigma}\otimes\mathbb{I} + \mathbb{I}\otimes\tilde{\sigma}\right) + j\left(\sigma^+\otimes\sigma^- + \sigma^-\otimes\sigma^+\right), \qquad (7)$$

where ω is the energy gap and the projector operator has been defined $\tilde{\sigma} = |e\rangle\langle e|$. In addition, we consider a direct Heisenberg XY-interaction between the qubits with coupling constant j



The energy levels structure of the two qubits system is as follows



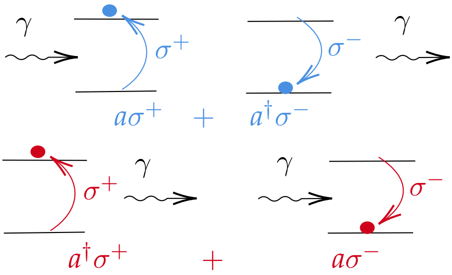
with their corresponding energy levels $E_1 = 2\omega$, $E_2 = \omega + j$, $E_3 = \omega - j$ and $E_4 = 0$.

Unruh-De Witt Detector

The interaction between the detectors and the scalar field is given by

$$\mathcal{H}_{int} = g\left(\mathcal{M}_1\varphi(\chi_1(\tau)) + \mathcal{M}_2\varphi(\chi_2(\tau))\right), \tag{8}$$

where the field is evaluated at the spacetime locations of the qubits $\chi_n(\tau)$, with $n = 1, 2$.



The operators $\mathcal{M}_1 = m \otimes \mathbb{I}$ with $\mathcal{M}_2 = \mathbb{I} \otimes m$ are the monopole operators of the two-qubit system.

Effective Qubits Dynamics

The total Hamiltonian of the qubits-field system is given by

$$\mathbb{H} = \mathbb{H}_0 + \mathcal{H}_{int}, \tag{9}$$

where the free Hamiltonian is $\mathbb{H}_0 = \mathcal{H}_0 + \mathcal{H}_0^{field}$. The dynamic of the system is given by the Sturn-Liouville equation for the total density operator in the interaction picture

$$i\frac{\partial \varrho_I(\tau)}{\partial \tau} = [\mathcal{H}^I_{int}(\tau), \varrho_I(\tau)], \tag{10}$$

with the initial condition $\varrho_I(\tau_0) = \rho_{in} \otimes |0\rangle\langle 0|$, where $|0\rangle$ is the vacuum of quantum field on the cosmic string spacetime. We solve perturbatively this equation with the Dyson series

$$\begin{aligned}\varrho_I(\tau) =& \varrho_I(\tau_0) - i\int_{\tau_0}^{\tau} d\tau \left[\mathcal{H}^I_{int}(\tau), \varrho_I(\tau_0)\right] \\ & - \int_{\tau_0}^{\tau} d\tau \int_{\tau_0}^{\tau} d\tau' \left[\mathcal{H}^I_{int}(\tau), \left[\mathcal{H}^I_{int}(\tau'), \varrho_I(\tau_0)\right]\right] + \cdots \end{aligned} \tag{11}$$

Effective Qubits Dynamics

Since we are interested only on the qubits final state, realize a trace over the field DOF $\rho_I(\tau) = \mathrm{Tr}_{field}\, \varrho_I(\tau)$.

$$\rho_I(\tau) = \rho_{in} + g^2 \sum_{n,p=1}^{2} \int_{\tau_0}^{\tau} d\tau \int_{\tau_0}^{\tau} d\tau' \delta\rho_{np}(\tau, \tau') G^{+}_{np}(\tau, \tau'), \qquad (12)$$

where

$$\delta\rho_{np}(\tau, \tau') = \mathcal{M}^{I}_{p}(\tau')\rho_{in}\mathcal{M}^{I}_{n}(\tau) - \theta(\Delta\tau)\mathcal{M}^{I}_{n}(\tau)\mathcal{M}^{I}_{p}(\tau')\rho_{in} - \theta(-\Delta\tau)\rho_{in}\mathcal{M}^{I}_{n}(\tau)\mathcal{M}^{I}_{p}(\tau'), \qquad (13)$$

and $\Delta\tau = \tau - \tau'$, the Heaviside function is θ and the Wightman function of the field evaluated at the qubits trajectory points is denoted by

$$G^{+}_{np}(\tau, \tau') = G^{+}(\chi_n(\tau), \chi_p(\tau')), \qquad (14)$$

where $n, p = \{1, 2\}$.

Effective Qubits Dynamics

$$i\frac{\partial \varrho_I(\tau)}{\partial \tau} = \left[\mathcal{H}^I_{int}, \varrho_I(\tau)\right]$$

$$\varrho_I(\tau) = U(\tau,\tau_0)\varrho_I(\tau_0)U^\dagger(\tau,\tau_0)$$

Unitary evolution,
Closed system,
Perturbation theory,
Small times regime.

Trace over field's DOF

Trace over field's DOF

$$\rho_{qubits}(\tau) = \rho_{qubits}(\tau_0) + \Delta\rho(\tau)$$

(Lindblad master equation)

$$i\frac{\partial \rho(\tau)}{\partial \tau} = [H_0, \varrho(\tau)] + \{\mathcal{L}_0, \rho\} + \sum_k \mathcal{L}_k \rho \mathcal{L}_k^\dagger$$

Weak coupling limit,
Born approximation,
Numerical approach,
Large times regime.

Considering the qubits initial state, at time $\tau = \tau_0$, with zero entanglement

$$\rho_{in} = \begin{pmatrix} p & 0 & 0 & 0 \\ 0 & 1/2 - p & 0 & 0 \\ 0 & 0 & 1/2 - p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}, \tag{15}$$

where the single probability that defines the initial state has to be $0 \leq p \leq 1/2$. The qubits density state after the interaction with the vacuum becomes

$$\rho(\tau) = \begin{pmatrix} p + \delta p_1 & 0 & 0 & \delta\alpha \\ 0 & 1/2 - p + \delta p_2 & \delta\beta & 0 \\ 0 & \delta\beta^* & 1/2 - p + \delta p_3 & 0 \\ \delta\alpha^* & 0 & 0 & p + \delta p_4 \end{pmatrix}. \tag{16}$$

Entanglement measurement

To measure the entanglement of a two qubit system we use the **Negativity**. We define the negativity of a quantum state described by the density operator $\rho(\tau)$ as

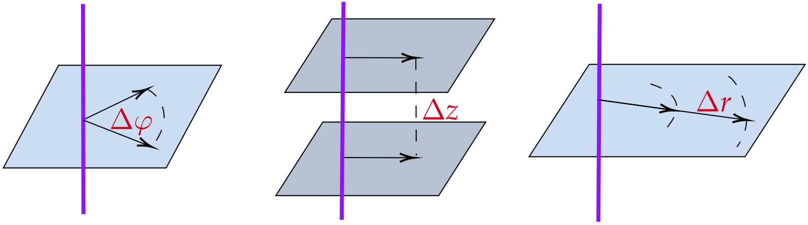
$$\mathcal{N}(\tau) = 2\left|\sum_{\lambda_i<0} \lambda_i\right| = \left(\sum_i |\lambda_i| - \lambda_i\right), \tag{17}$$

where λ_i are all the negative eigenvalues of the partial transpose $\rho^{PT}(\tau)$ of the density matrix $\rho(\tau)$. For our case the Negativity depends on various parameter that define the initial qubits state, the topologoical charge of the string, and the spatial lcoations of the qubits

$$\mathcal{N} = \mathcal{N}(\tau, \omega, j, p, n, \mathbf{x}_1, \mathbf{x}_2). \tag{18}$$

Qubits spatial configurations

We can consider stationary qubits that only have spatial separations on the angular, axial or radial variables.



After some interaction time there is a sudden birth of entanglement

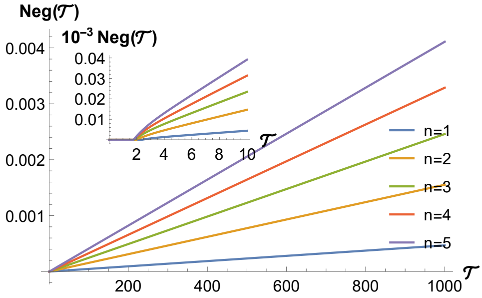


Figure: Negativity as a function of interaction time for the axial case. We use $\mathcal{T}$, evaluated with $\omega = 1$, $j = -0.5$, $\Delta z = 1$, and $r = 1$.

Harvesting entanglement

We see that at the resonance points $j = \pm\omega$ there is maximum amount of entanglement. This imply that the Heisenberg XY-interaction enhanced the process of entanglement harvesting.

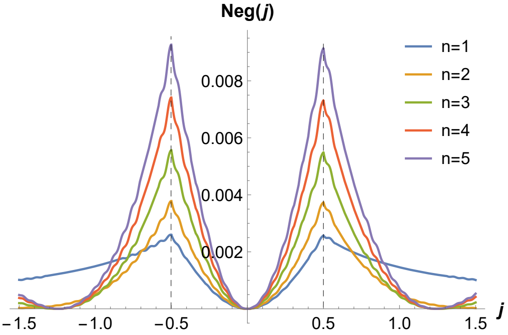


Figure: Negativity as a function of the interaction coupling constant j. These figures are given for $\mathcal{T} = 1000$, $\omega = 0.5$, $r = 1$, $\Delta z = 100$.

The Negativity oscillates as the distance between the qubits increases. For the case $j = 0$, at the limit of large qubits separations there is no entanglement left.

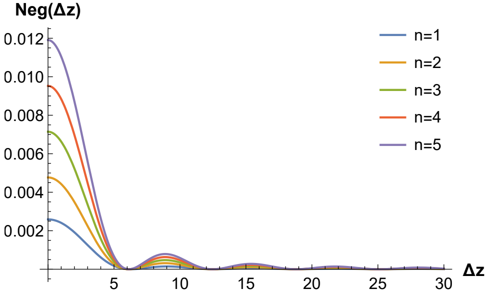


Figure: Negativity as a function of the distance between the qubits Δz for the case without XY-interaction. Here we evaluated with $\omega = 0.5$, $j = 0$, $\mathcal{T} = 1000$, $r = 1$.

The Negativity oscillates as the distance between the qubits to the cosmic string increases. For the case $j \neq 0$, at the limit of large qubits separations there is some remaining entanglement.

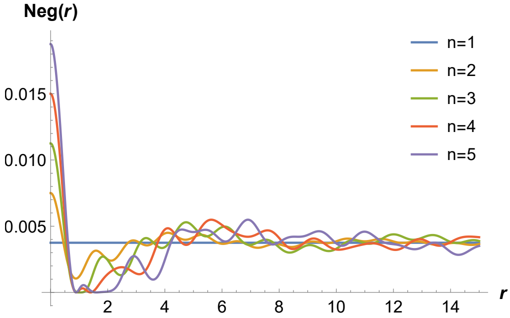


Figure: Negativity as a function of the distance between the qubits to the cosmic string, r. Here it is use $\mathcal{T} = 1000$, $\omega = 1.5$, $j = 1$, $\Delta z = 100$.

Harvesting entanglement

The generation of entanglement from the vacuum fluctuations is a non-local process.

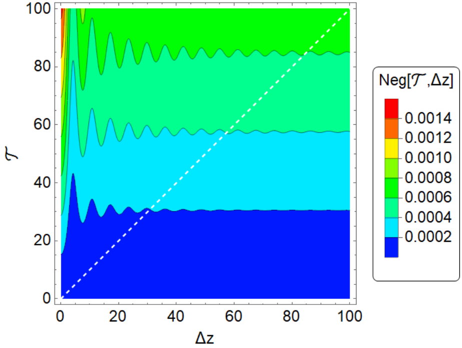


Figure: Contour plot $\mathcal{T}$ vs Δz of the negativity with $n = 4$. this graphic are evaluated with $\omega = 0.5$, $j = 0.5$, $r = 1$, $\Delta\varphi = \Delta r = p = 0$, and $g = 0.01$.

Conclusions

- The quantum vacuum can be a natural source of (small) entanglement.
- The direct qubits coupling via the Heisenberg XY-interaction can benefit the harvesting process.
- The maximum entanglement is obtain at the resonance points $j = \pm\omega$.
- In the limit of large separation between the qubits there is some remaining amount of entanglement for $j \neq 0$.
- The higher the topological charge n the more entanglement we can obtain in the cosmic string spacetime.

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THANKS!