Towards an experimental test for a quantum extension of Einstein’s Equivalence Principle

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Motivation

Approaches to a quantum equivalence principle

Quantum Aspects of the Equivalence Principle

Y. Aharonov

Department of Physics, University of South Carolina, Columbia, South Carolina[1]

and G. Carmi[2]

Department of Physics, St. John's University, Jamaica, New York

On Gravity's Role in Quantum State Reduction

Roger Penrose[1,2]

On the Equivalence Principle in Quantum Theory *

Claus Lämmerzahl*

Laboratoire de Gravitation et Cosmologie Relativiste, Université Pierre et Marie Curie, CNRS/URA 679, F - 75252 Paris Cedex 05, France

Quantum formulation of the Einstein equivalence principle

Magdalena Zych & Časlav Brukner

Implementation of the Quantum Equivalence Principle

Lucien Hardy

Quantum mechanics and the equivalence principle

P.C.W. Davies

Can quantum probes satisfy the weak equivalence principle?

Luigi Seveso[a], Matteo G. A. Paris[a,b]

Sagnac interferometer and the quantum nature of gravity

Chiara Marletto[1,2,3] and Vlatko Vedral[1,2,3]

Einstein's Equivalence principle for superpositions of gravitational fields and quantum reference frames

Flaminia Giacomini[1,2,*] and Časlav Brukner[3,4]

A Simple Test of the Equivalence Principle(s) for Quantum Superpositions

Patrick J. Orlando,[1] Robert B. Mann,[2,3] Kavan Modi,[1] and Felix A. Pollock[1,*]

Einstein's equivalence principle in quantum mechanics revisited

Michael Nauenberg

Classical equivalence principles

(At least) three different principles

Weak Equivalence Principle (WEP)

Einstein's Equivalence Principle (EEP)

Strong Equivalence Principle (SEP)

E. Di Casola, S. Liberati, S. Sonego; Nonequivalence of equivalence principles. *Am. J. Phys.* 1 January 2015; 83 (1): 39–46.

Weak Equivalence Principle

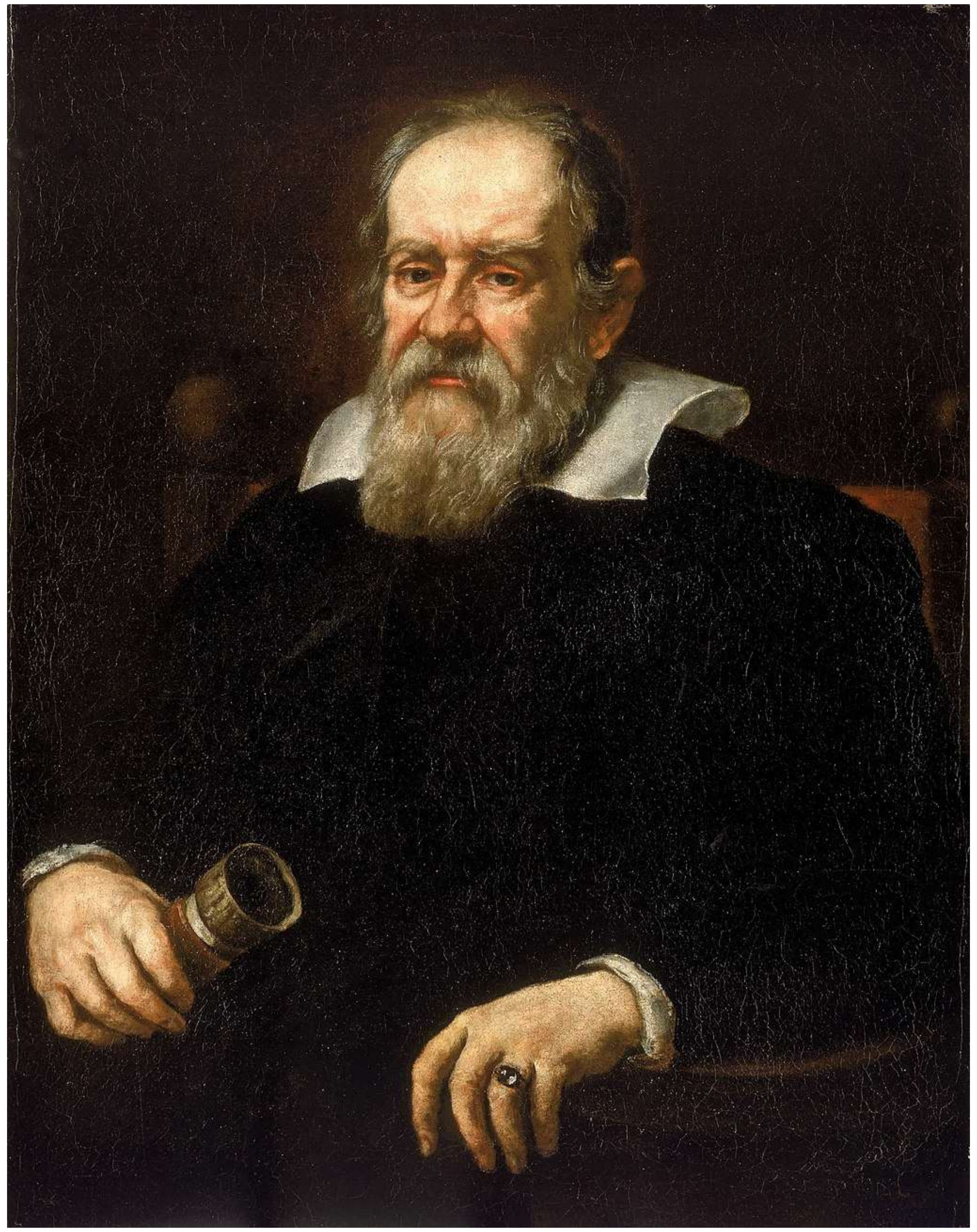
- The weak equivalence principle asserts the equivalence between inertial mass and gravitational mass: it's the universality of free fall.

- Alternatively, it can be formulated saying that a constant acceleration is undistinguishable from a constant gravitational field.

$$m_i a = m_g g$$

$$m_i = m_g$$

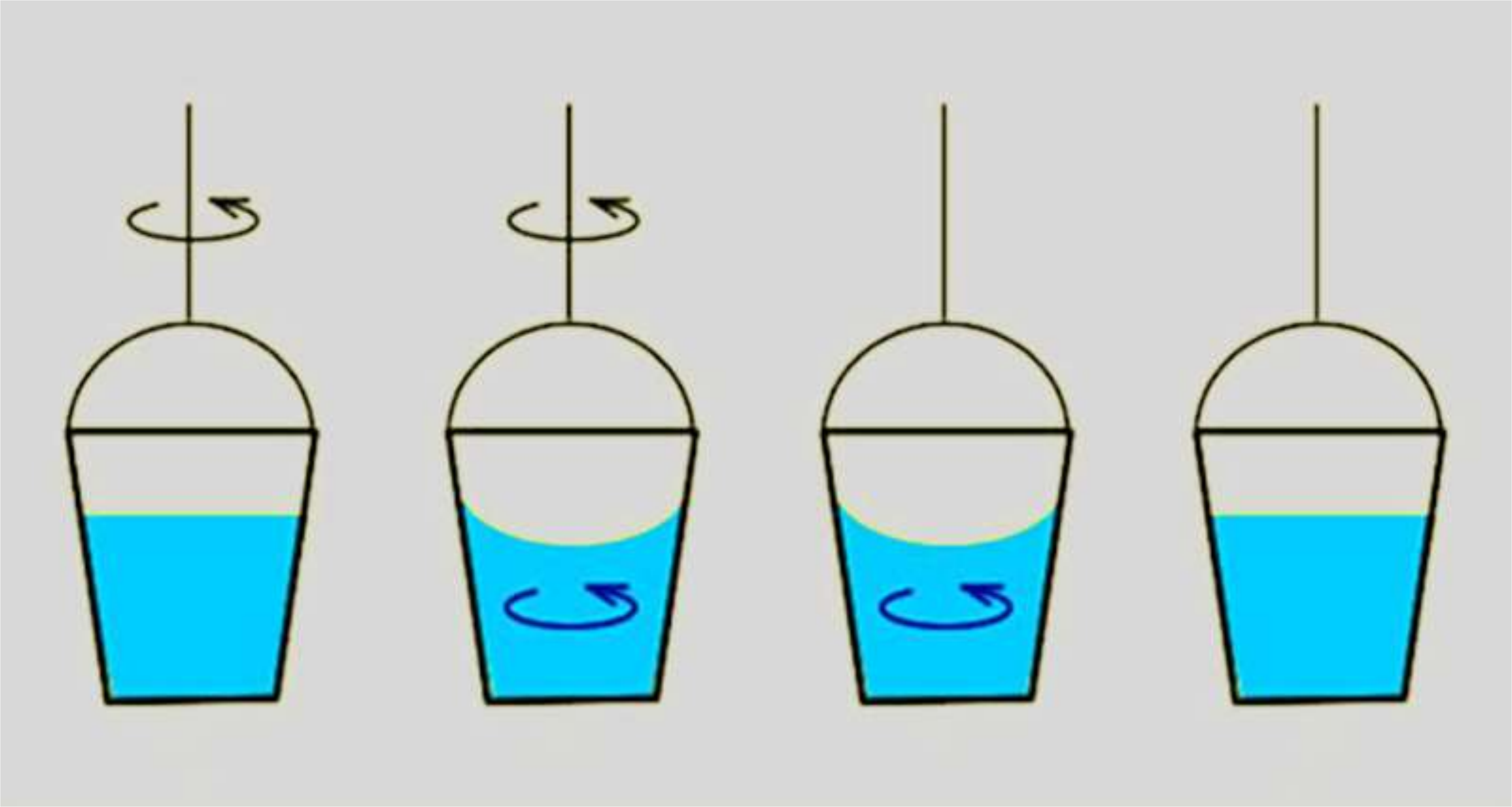
$$a = g$$



Newton’s bucket or the absoluteness of motion

- According to Newton, this experiment shows that motion (in particular, acceleration) is absolute.

- From this idea, he defines his notion of inertial frame: inertial frames are moving with constant velocity with respect to an absolute space



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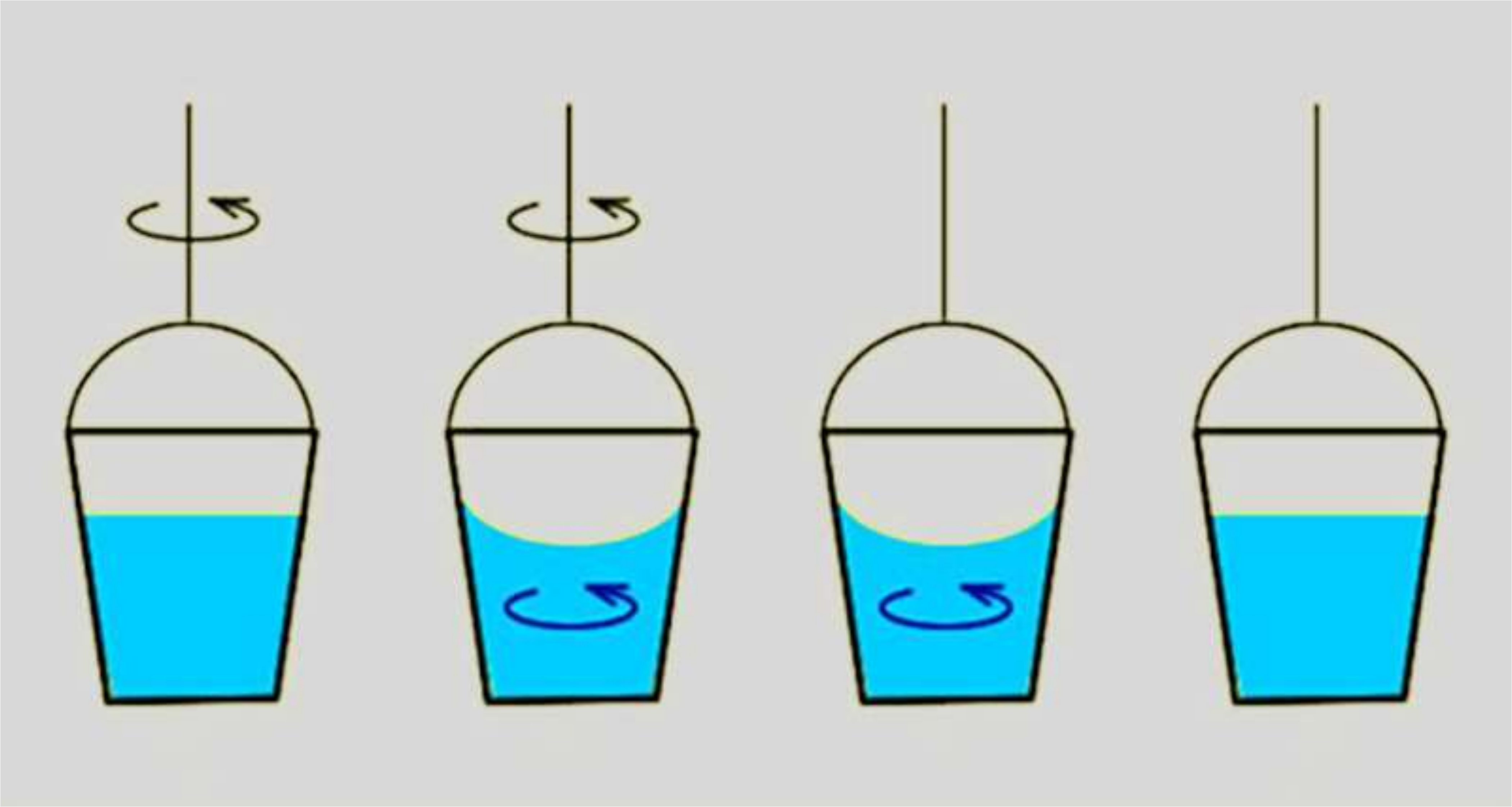

Ernst Mach (1983):
When, accordingly, we say that a body preserves unchanged its direction and velocity in space, our assertion is nothing more or less than an abbreviated reference to the entire universe.

Einstein’s Equivalence Principle

- Einstein understands that inertia and the gravitational field are the same object: it is the gravitational field that defines what frame is inertial, hence the gravitational field is not a force, but provides the structure of spacetime.

- The inertial frames are the freely falling frames. This is the core of Einstein’s equivalence principle, what he called his *happiest thought*.

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In any and every locally inertial frame, anywhere and anytime in the universe, all the (nongravitational) laws of physics must take on their familiar non-relativistic form.

C. W. Misner, K. S. Thorne, J. A. Wheeler, Gravitation (W. H. Freeman, 1973).

Einstein’s Equivalence Principle

- The Einstein’s Equivalence Principle prescribes a metric theory for gravity, and gives a recipe to write equations of motion:

$$\frac{d^2 X^\mu}{d^2 T^2} = 0 \qquad \frac{d^2 x^\lambda}{dt^2} + \tilde{\Gamma}^\lambda_{\nu\alpha} \frac{dx^\nu}{dT} \frac{dx^\alpha}{dT} = 0$$

$$\tilde{g}_{\mu\nu} = \frac{\partial X^\rho}{\partial x^\mu} \frac{\partial X^\sigma}{\partial x^\nu} \eta_{\rho\sigma}$$

Weinberg, Steven. *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (Wiley 1972).

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- Einstein's Equivalence Principle:

$$\tilde{\Gamma}^\lambda_{\nu\alpha} \to \Gamma^\lambda_{\nu\alpha} \qquad\qquad \tilde{g}_{\mu\nu} \to g_{\mu\nu}$$

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Different aspects of Einstein's Equivalence Principle

- Weak Equivalence Principle (WEP): The local effects of motion in a curved spacetime (gravitation) are indistinguishable from those of an accelerated observer in flat spacetime.

C.M. Will, The Confrontation between General Relativity and Experiment, Living Rev. Relativity 17 (2014).

Different aspects of Einstein's Equivalence Principle

- Weak Equivalence Principle (WEP): The local effects of motion in a curved spacetime (gravitation) are indistinguishable from those of an accelerated observer in flat spacetime.

- Local Lorentz Invariance (LLI): The outcome of any local nongravitational experiment is independent of the velocity of the freely-falling reference frame in which it is performed.

C.M. Will, The Confrontation between General Relativity and Experiment, Living Rev. Relativity 17 (2014).

Different aspects of Einstein's Equivalence Principle

- Weak Equivalence Principle (WEP): The local effects of motion in a curved spacetime (gravitation) are indistinguishable from those of an accelerated observer in flat spacetime.

- Local Lorentz Invariance (LLI): The outcome of any local nongravitational experiment is independent of the velocity of the freely-falling reference frame in which it is performed.

- Local Position Invariance (LPI): The outcome of any local nongravitational experiment is independent of where and when in the universe it is performed.

C.M. Will, The Confrontation between General Relativity and Experiment, Living Rev. Relativity 17 (2014).

The Strong Equivalence Principle

- EEP: In any and every locally inertial frame, anywhere and anytime in the universe, all the (nongravitational) laws of physics must take on their familiar non-relativistic form.

> In any and every locally inertial frame, anywhere and anytime in the universe, all the (gravitational or not) laws of physics must take on their familiar non-relativistic form.

- The Strong Equivalence Principle singles out metric-only theories, such as General Relativity.

Will CM. *Theory and Experiment in Gravitational Physics*. 2nd ed. Cambridge University Press; 2018.

Quantum equivalence principles

Quantum Interpretations of WEP

The trajectories of freely falling bodies are independent of their composition

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The equations of motion of freely falling bodies do not depend on their mass

Quantum Interpretations of WEP

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The equations of motion of freely falling bodies do not depend on their mass

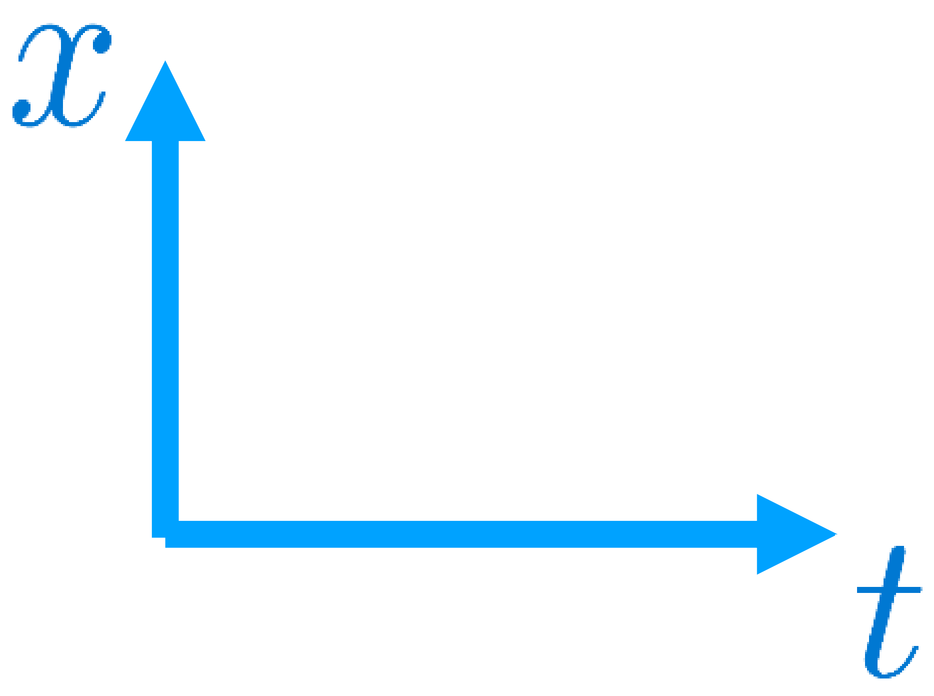
The equations of motion under constant gravitational field are indistiguishable from the ones with constant acceleration

Quantum Reference Frames

Motivation

An operational approach to reference frames

Reference frames are typically treated as an **abstraction** of rods and clocks.



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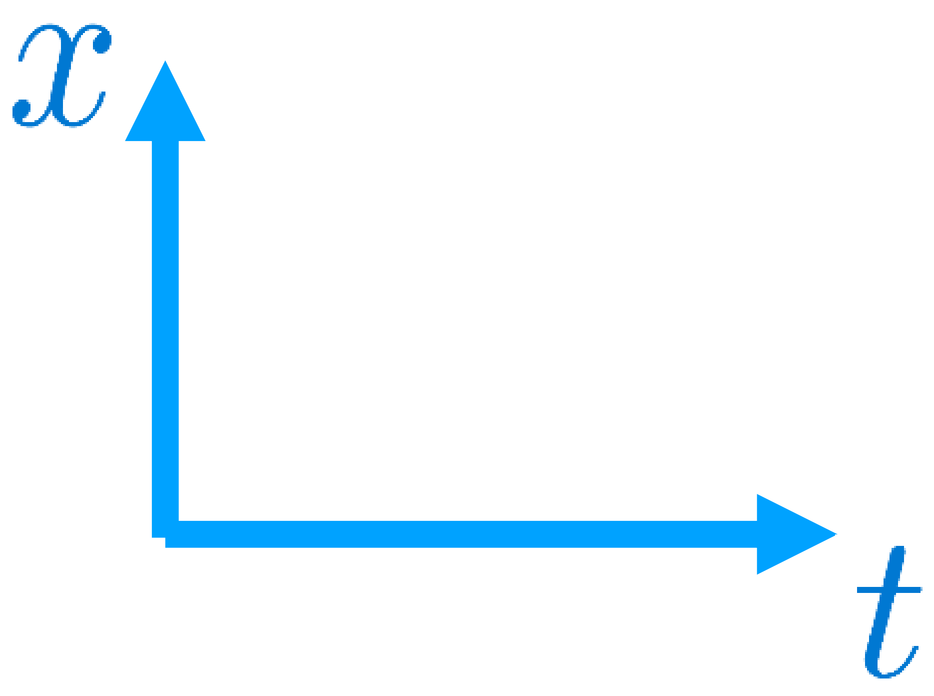
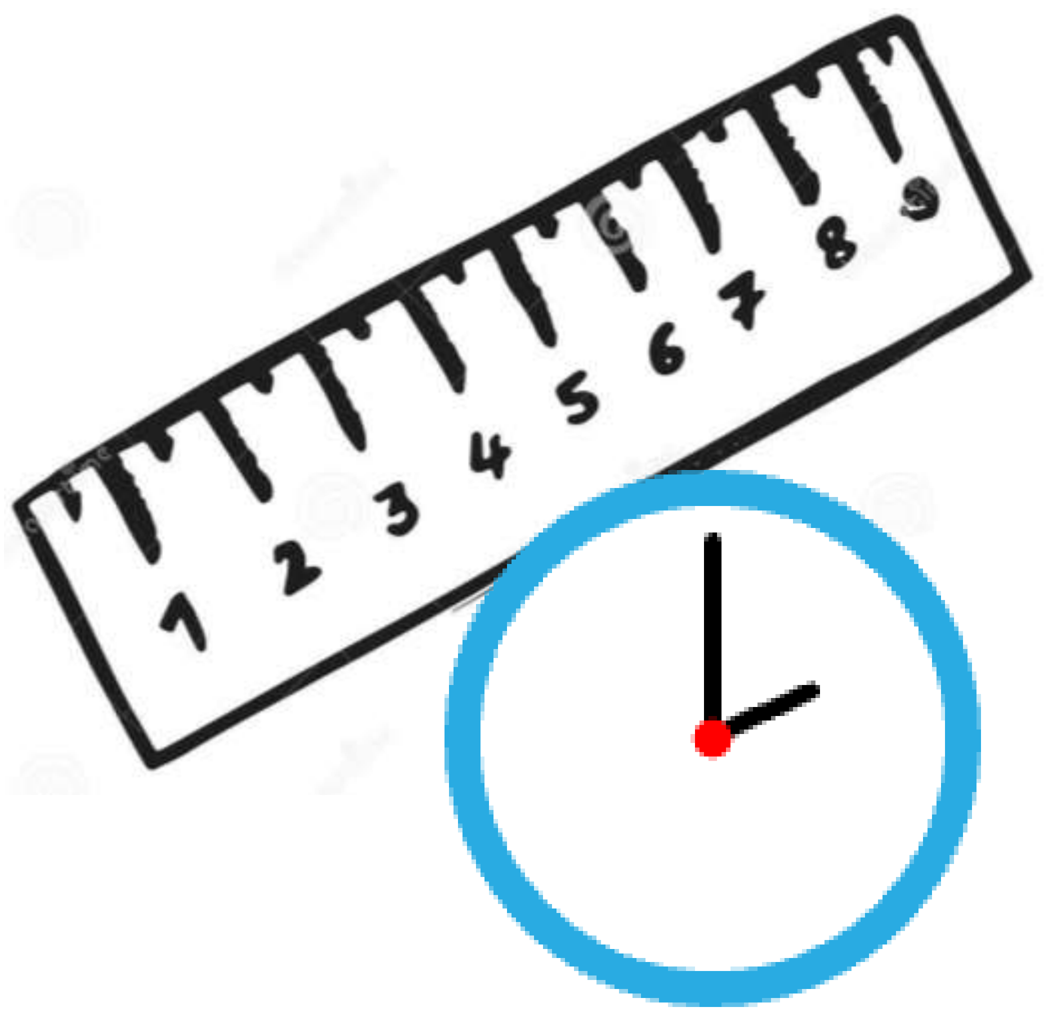
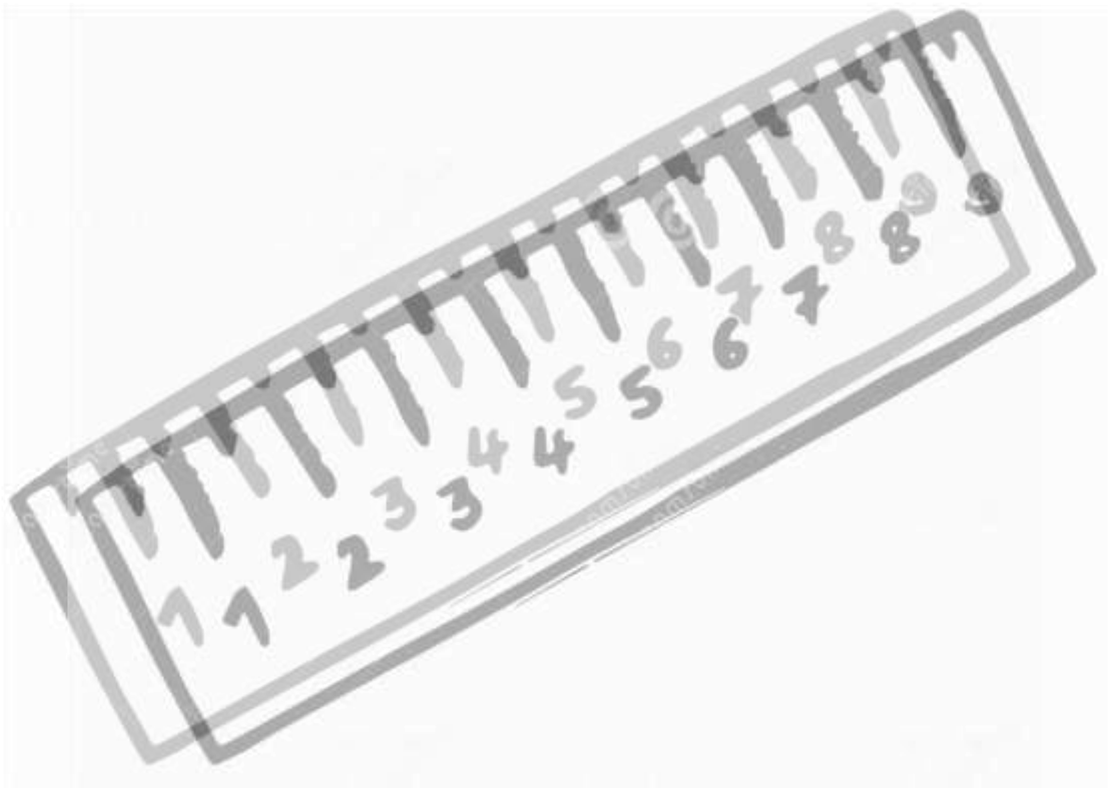


Nevertheless, they are always attached to **actual physical systems**, and as such they must obey dynamical laws.



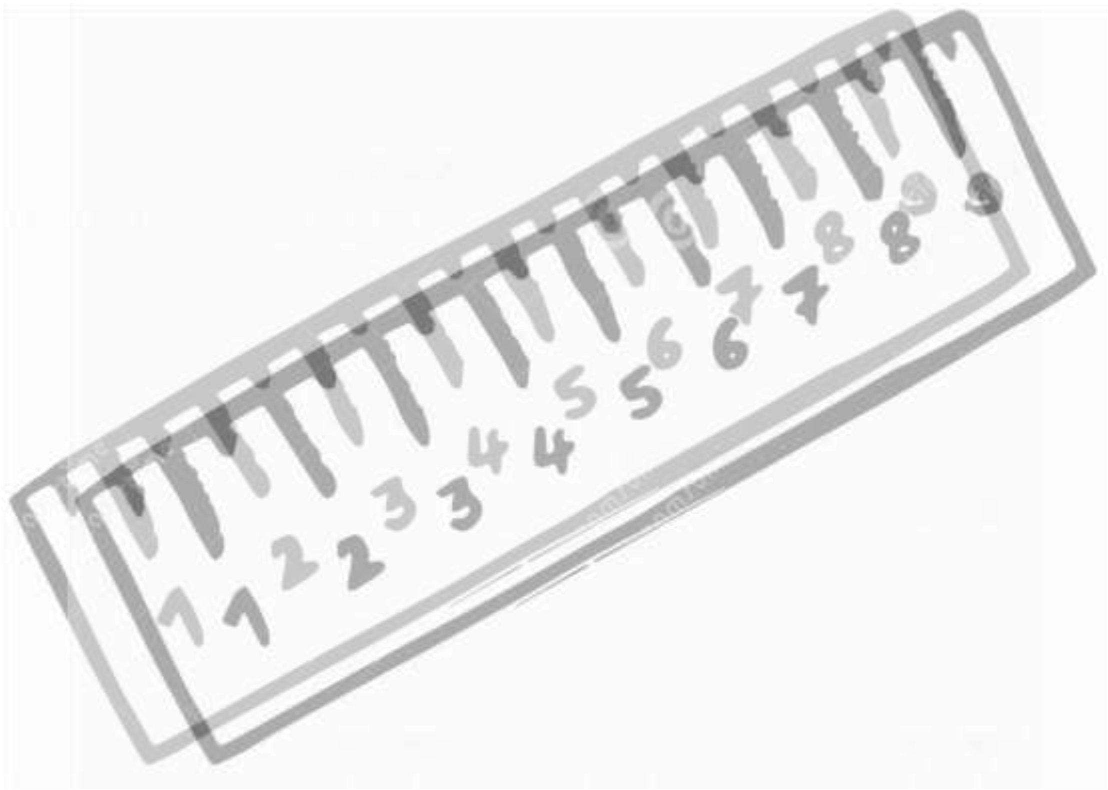
And in particular, they must obey **quantum mechanics**.

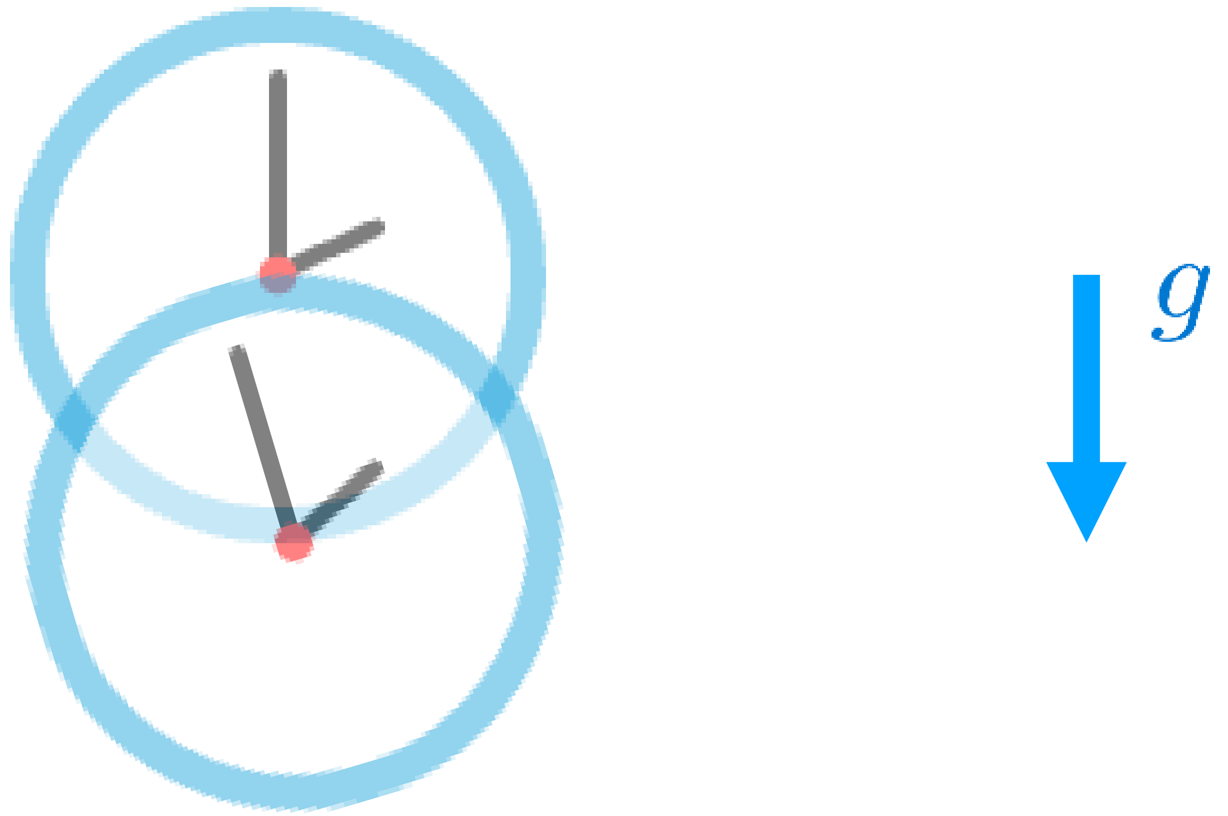
A quantum ruler and a quantum clock



Given a ruler that is in quantum superposition of different locations, can we still use it to describe position?

A quantum ruler and a quantum clock





Given a ruler that is in quantum superposition of different locations, can we still use it to describe position?

Given a clock that is in quantum superposition of different heights in a gravitational field, what time will tell?

Experiments

arXiv > gr-qc > arXiv:2009.09546

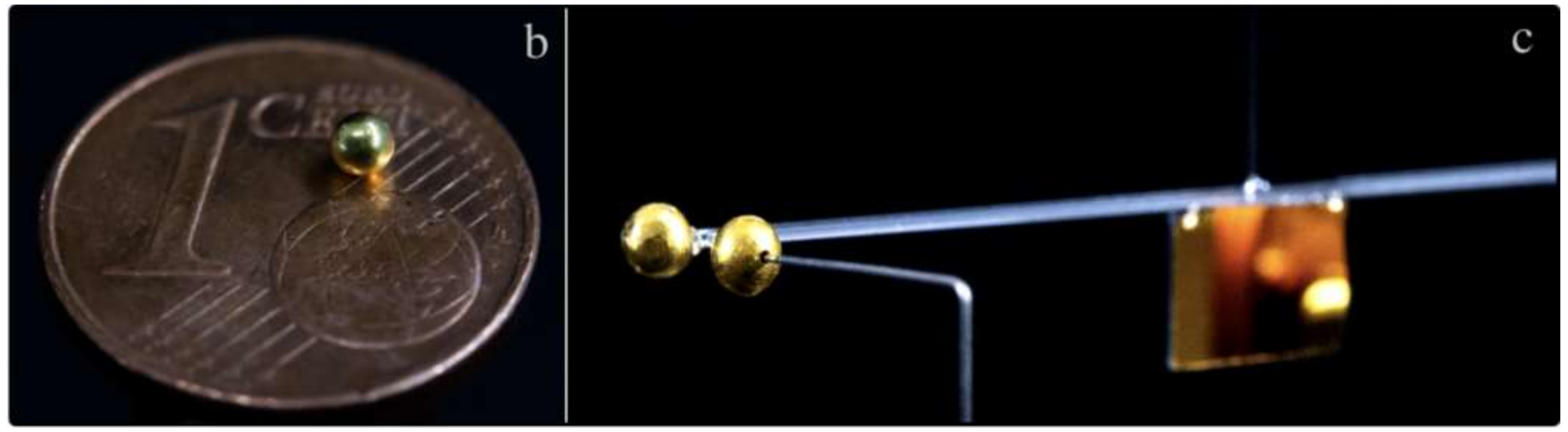
General Relativity and Quantum Cosmology

[Submitted on 21 Sep 2020 (v1), last revised 2 Mar 2021 (this version, v2)]

Measurement of Gravitational Coupling between Millimeter-Sized Masses

Tobias Westphal, Hans Hepach, Jeremias Pfaff, Markus Aspelmeyer

$90\ \mathrm{mg}$



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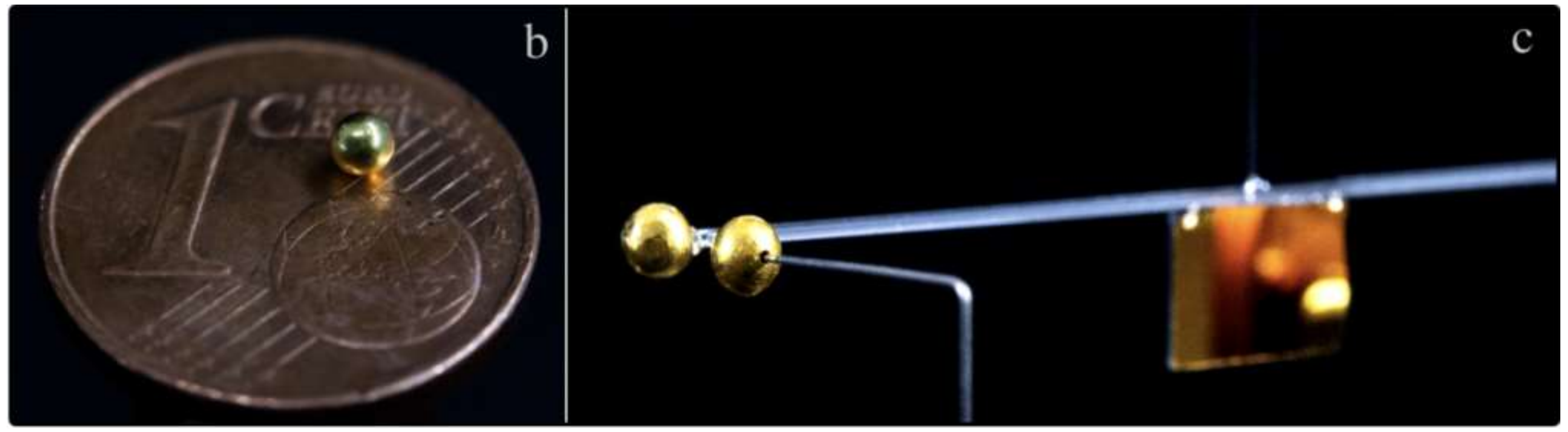


Published: 23 December 2015

Quantum superposition at the half-metre scale

T. Kovachy, P. Asenbaum, C. Overstreet, C. A. Donnelly, S. M. Dickerson, A. Sugarbaker, J. M. Hogan & M. A. Kasevich

Nature **528**, 530–533 (2015) | Cite this article

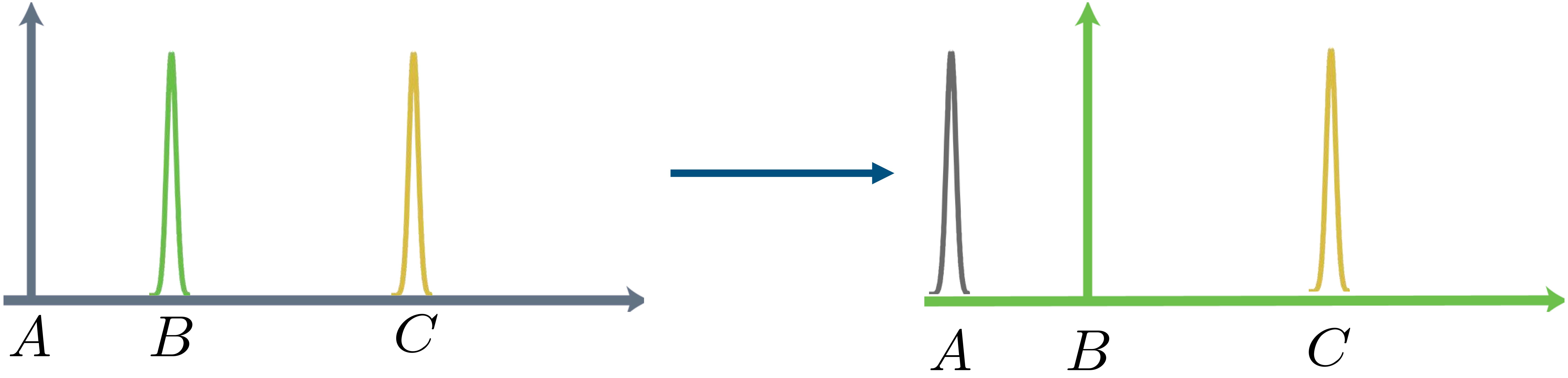
(Some) literature on QRFs

- **1712.07207**
- 1809.00556
- 1809.05093
- 1811.08228
- **1908.10165**
- 1911.04903
- 2004.14292
- 2006.06364
- 2006.07298
- 2008.03317
- 2011.01951
- **2012.13754**
- 2012.15769
- **2101.11628**
- 2103.01232
- 2103.05022
- 2107.07545
- **2109.01405**
- 2109.06184
- 2110.13199
- 2110.13824
- 2112.00046
- **2112.03303**
- **2112.11473**
- **2201.03120**
- 2203.06755
- **2207.00021**
- **2209.02214**
- **2211.01657**
- 2212.14081
- **2302.11629**
- 2308.09131
- **2308.10967**
- **2308.12912**
- **2402.10267**
- **2405.00114**
- 2406.15838
- 2406.19448

An introduction to QRFs

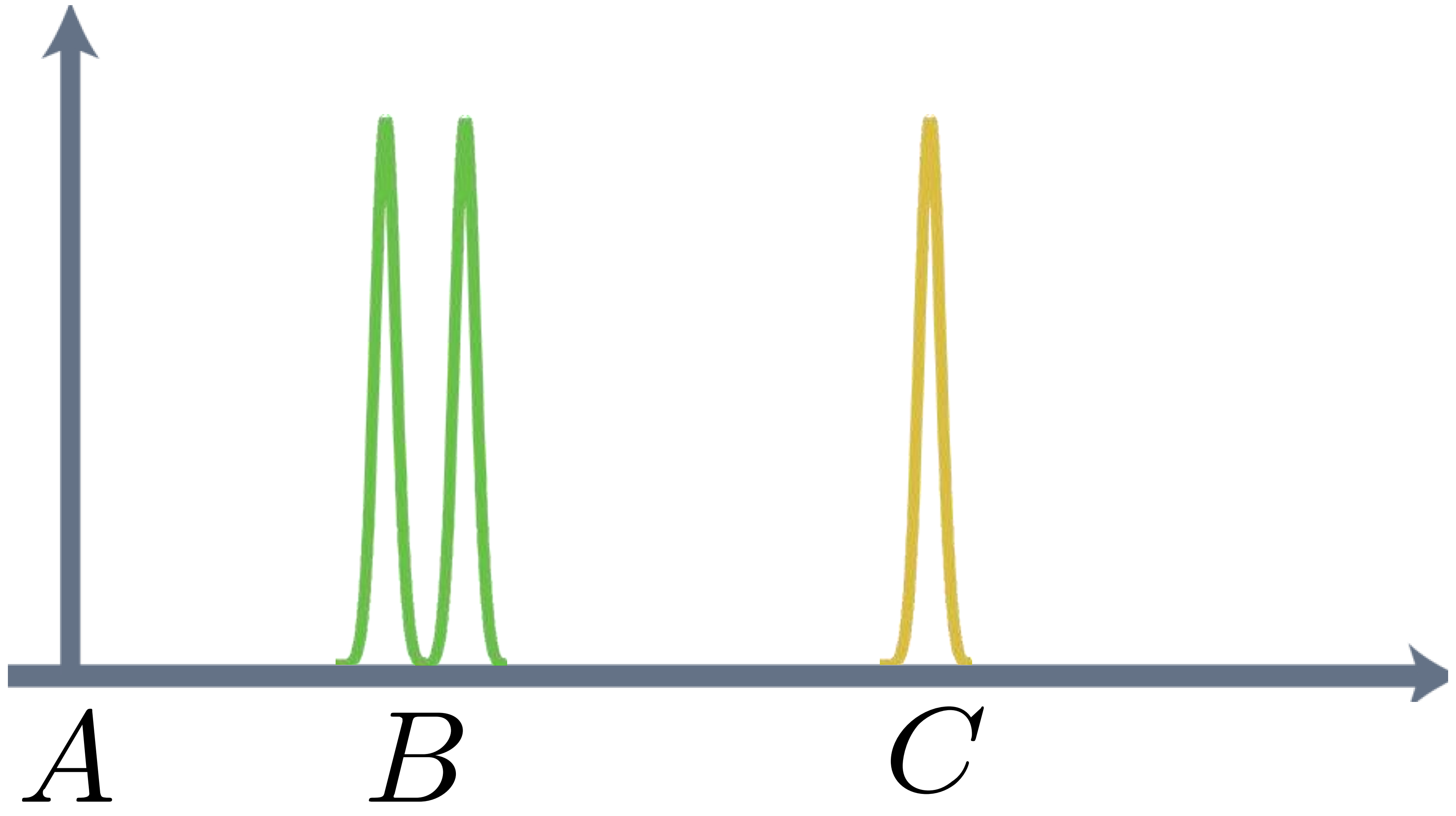
F. Giacomini, E. Castro-Ruiz, C. Brukner, *Quantum mechanics and the covariance of physical laws in quantum reference frames*. Nat Commun 10, 494 (2019).

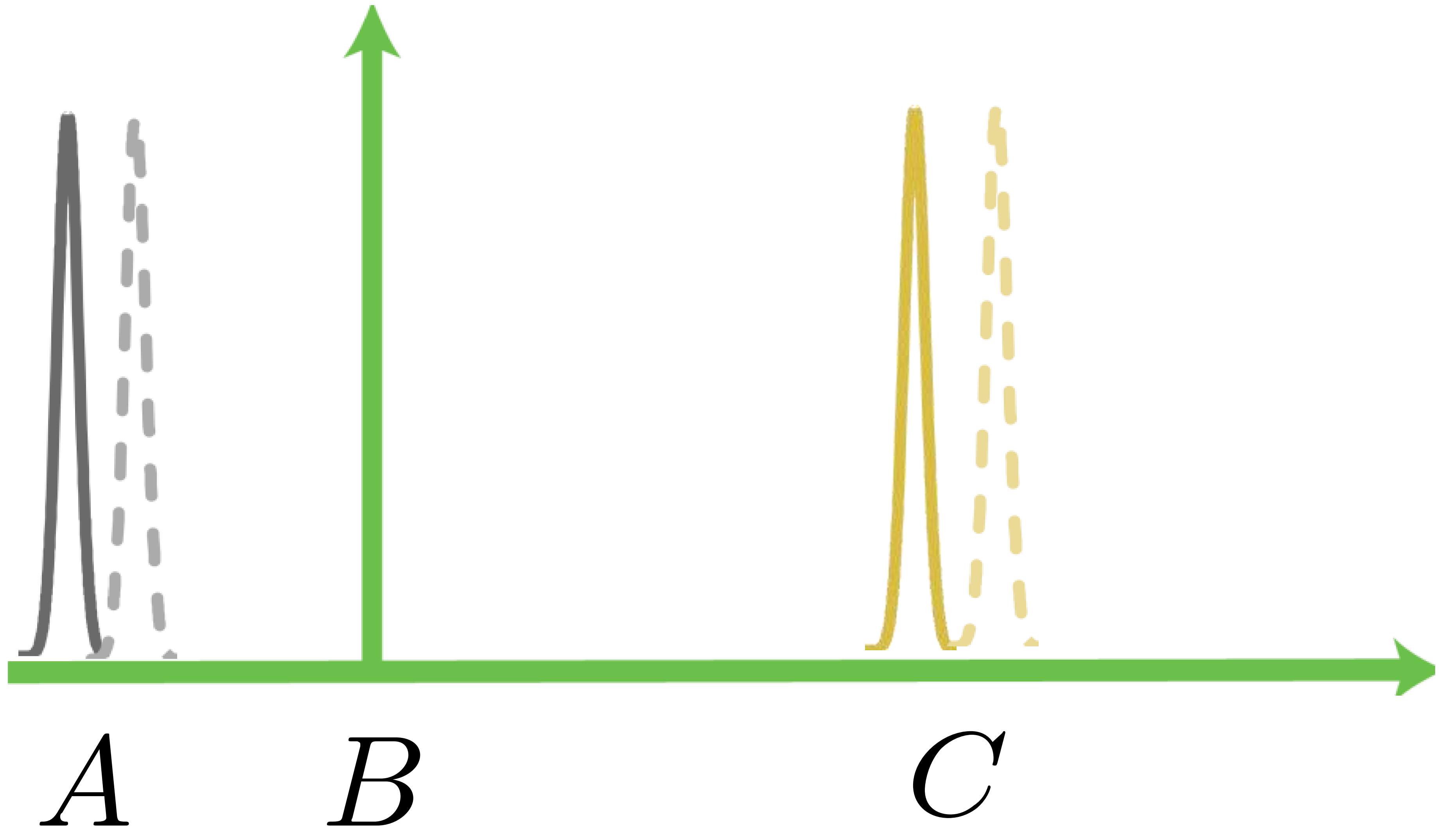
Classical RF transformations



$$|\psi\rangle^{(A)}_{ABC} = |0\rangle_A \, |x_1\rangle_B \, |x_2\rangle_C \qquad |\psi\rangle^{(B)}_{ABC} = |-x_1\rangle_A \, |0\rangle_B \, |x_2 - x_1\rangle_C$$

Quantum superposition of translations





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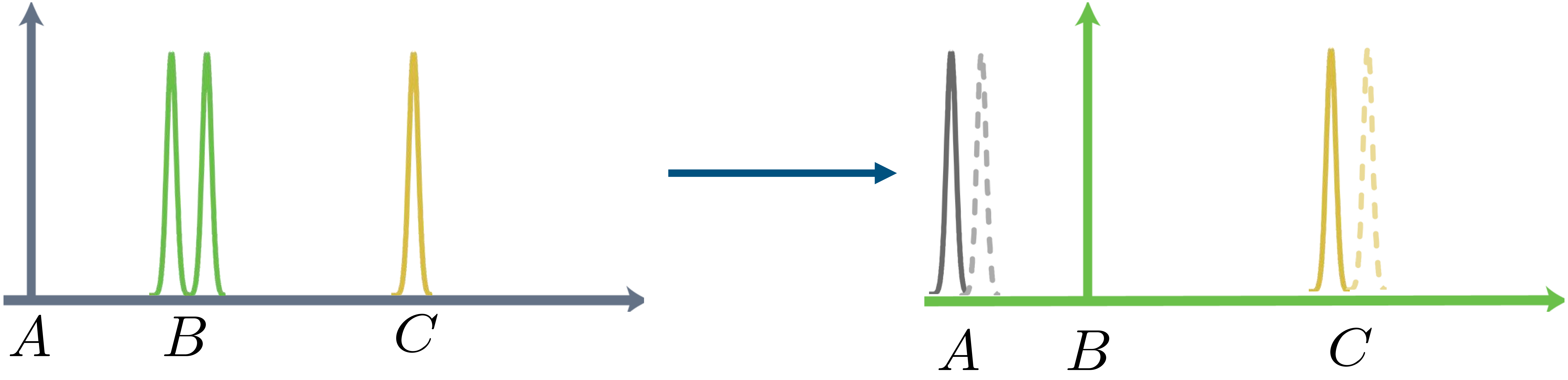
Quantum superposition of translations



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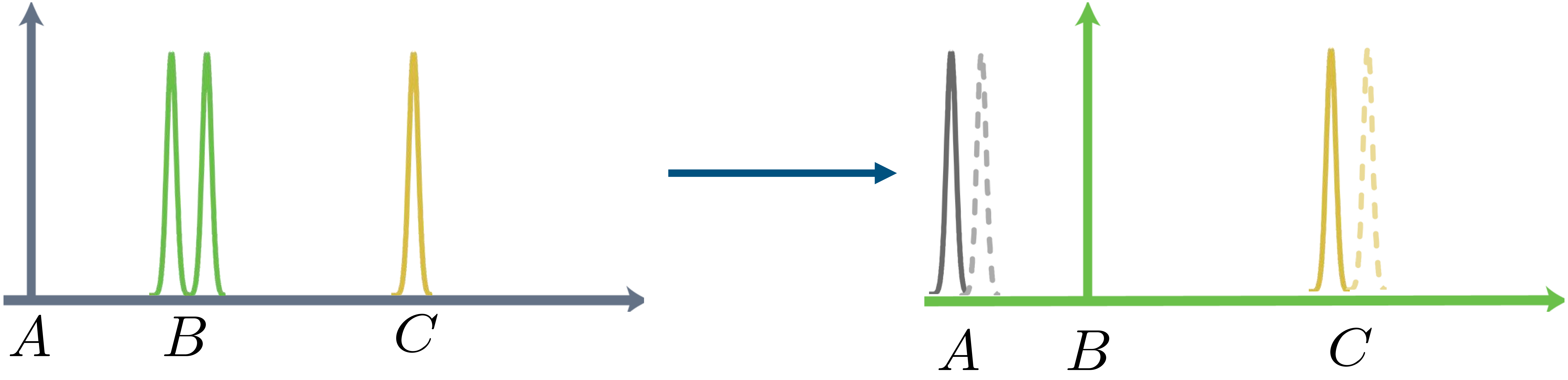

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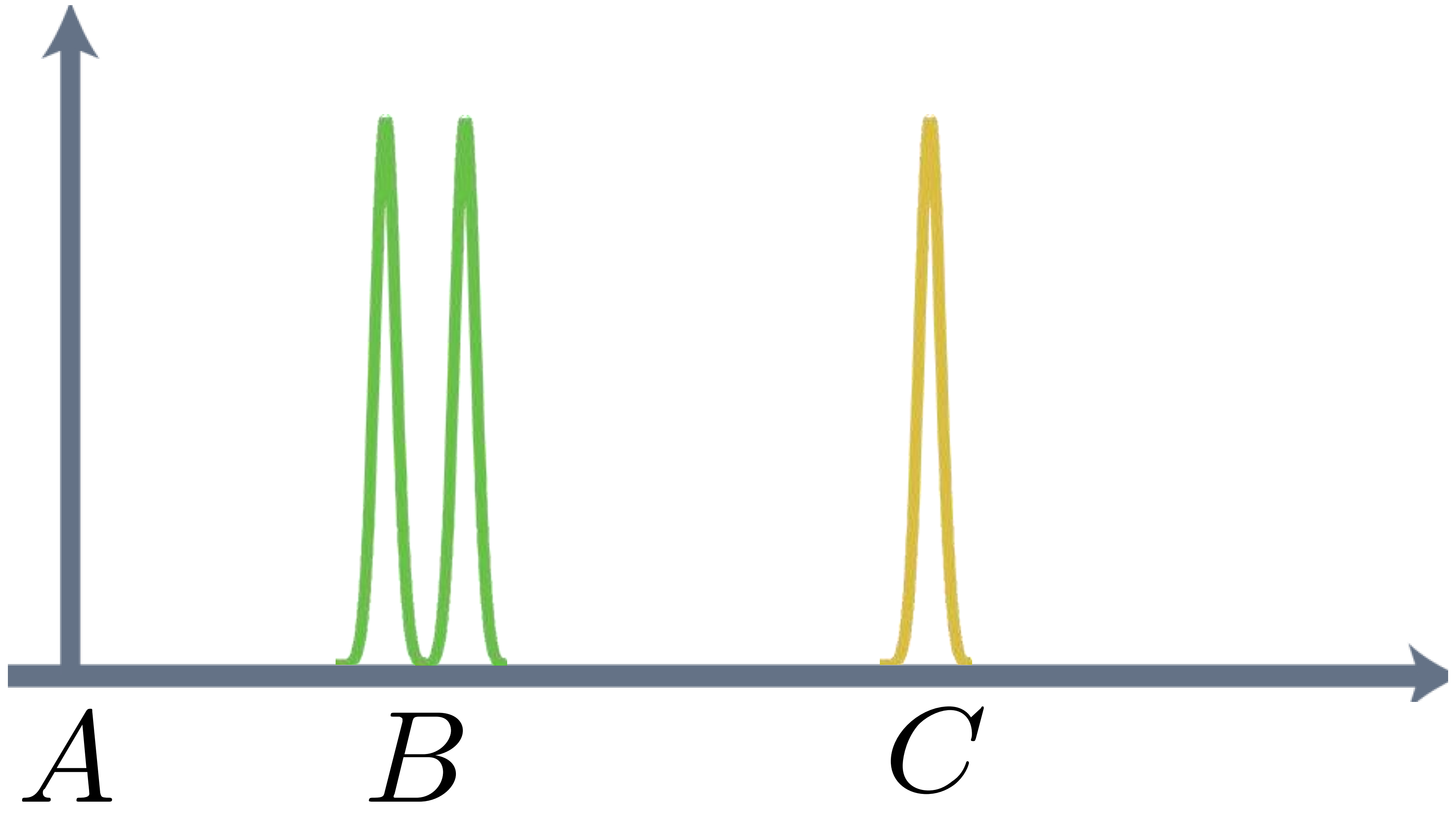
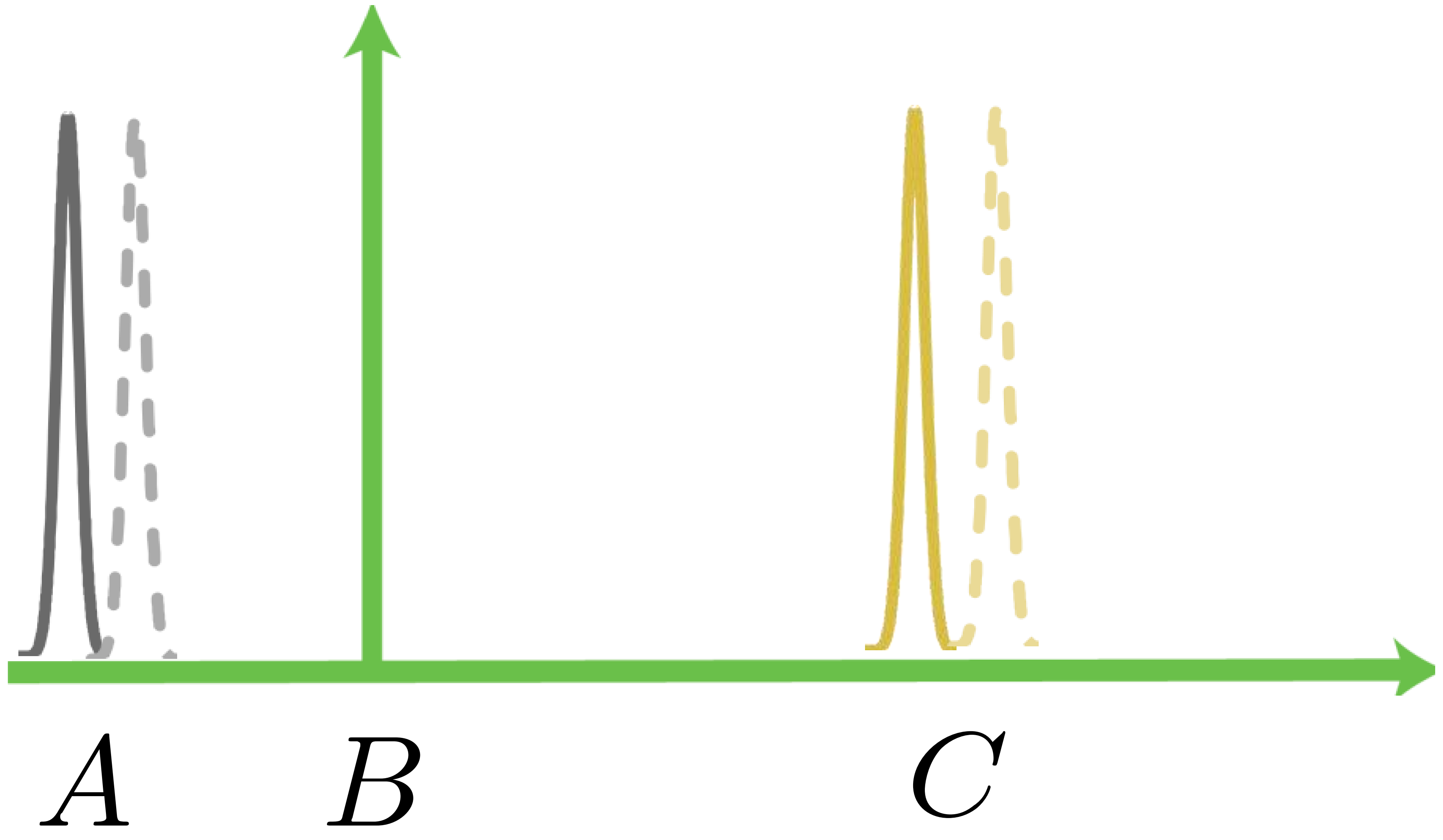
In its frame, B is localized

Quantum superposition of translations





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Quantum Reference Frames

(Ideal) QRF tranformations are
coherent superposition of CRF transformations

Given:

1. A locally compact group G
2. A Hilbert space $\mathcal{H} = L^2(G)$ with orthonormal basis $\{|g\rangle\}_{g\in G}$, i.e. $\langle g|g'\rangle = \delta_{g^{-1}\circ g'}$
3. A unitary regular representation of G on $\mathcal{H}$, $U(g)$, that is $U(g)\,|g'\rangle = |g \circ g'\rangle$

Quantum Reference Frames

(Ideal) QRF tranformations are
coherent superposition of CRF transformations

$$S^{(C)\to(A)} : \mathcal{H}_A \otimes \mathcal{H}_B \to \mathcal{H}_B \otimes \mathcal{H}_C$$

$$|\psi\rangle^{(C)}_{AB} \mapsto |\psi\rangle^{(A)}_{BC}$$

$$S^{(C)\to(A)} = \int_G dg \; |g^{-1}\rangle_{CA} \langle g| \otimes U^\dagger_B(g)$$

Quantum-controlled classical transformation

Frame-dependency of resources

$$|\psi\rangle^{(A)}_{BC} = \frac{|x_1\rangle_B + |x_2\rangle_B}{\sqrt{2}} |x_3\rangle_C \quad \rightarrow \quad |\psi\rangle^{(B)}_{AC} = \frac{|-x_1\rangle_A |x_3 - x_1\rangle_C + |-x_2\rangle_A |x_3 - x_2\rangle_C}{\sqrt{2}}$$

Coherence and entanglement are **frame-dependent** features

$$\boxed{\mathcal{C}_e^{(A)} + \mathcal{E}_e^{(A)} = \mathcal{C}_e^{(B)} + \mathcal{E}_e^{(B)}}$$

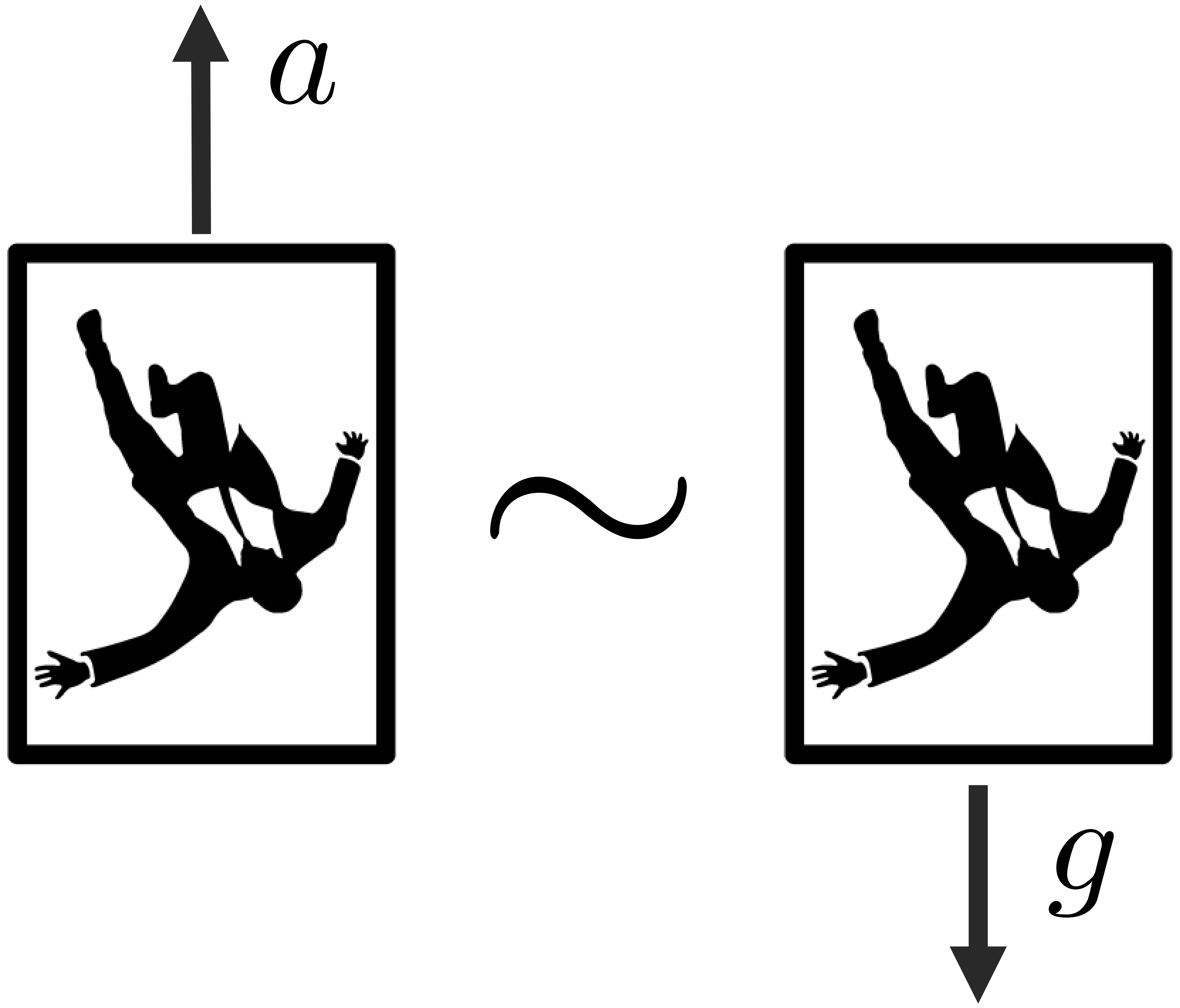
C. Cepollaro et al, The sum of entanglement and subsystem coherence is invariant under quantum reference frame transformations, arXiv:2406.19448

The quantum weak equivalence principle

F. Giacomini, E. Castro-Ruiz, C. Brukner, *Quantum mechanics and the covariance of physical laws in quantum reference frames*. Nat Commun 10, 494 (2019).

The classical weak equivalence principle

Weak equivalence principle:
The physical effects seen in a uniform gravitational field are indistinguishable from those seen from a system in constant acceleration.



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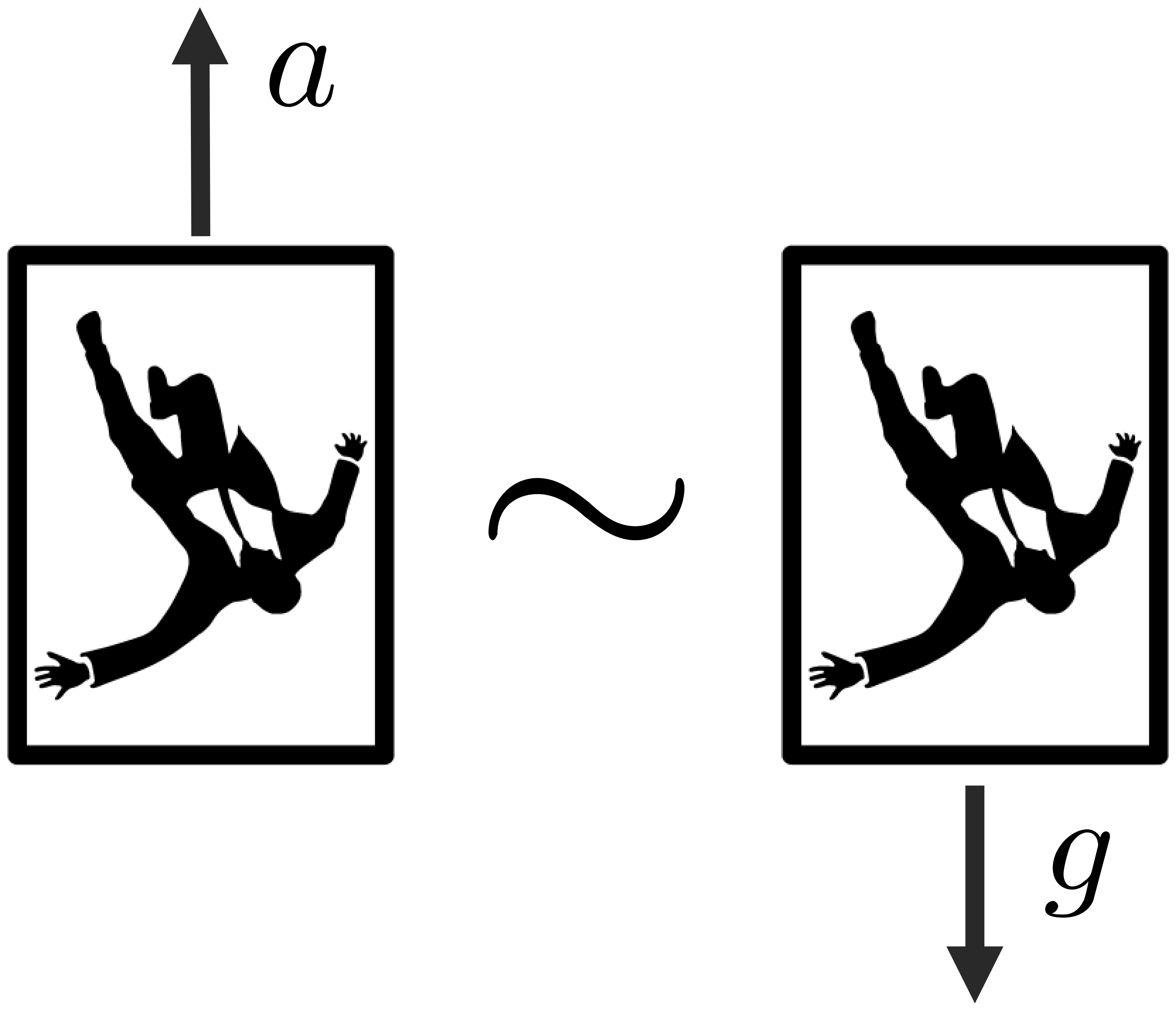
Given two particles A and B, if A freely falls with constant g and B is free,
in the RF of A, B is moving with a constant acceleration a =-g



$$H = \frac{p_A^2}{2m_A} + \frac{p_B^2}{2m_B} + m_A g x_A$$

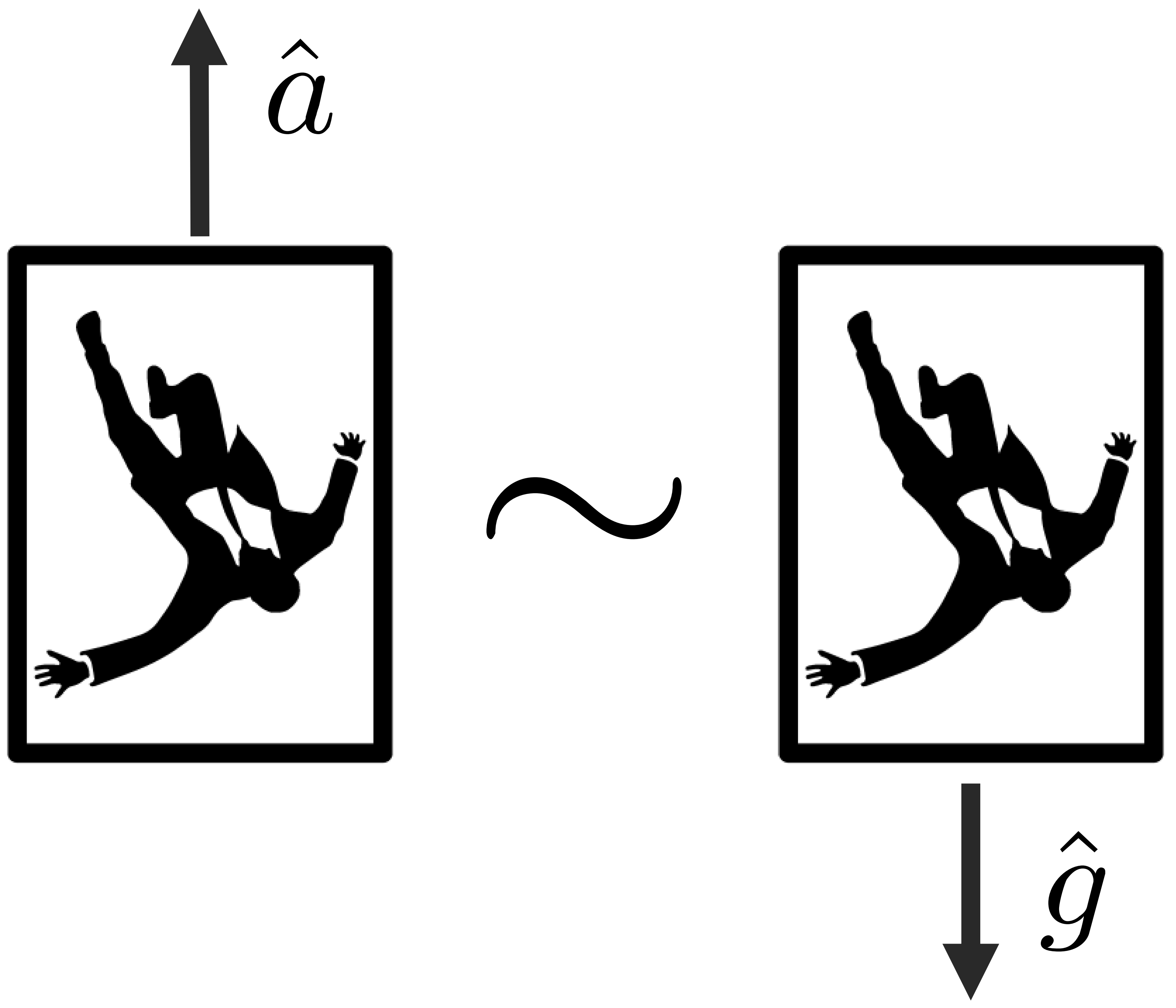
$$x'_{A,B}(t) = x_{A,B} - x_A(t)$$

$$H = \frac{p_A'^2}{2m_A} + \frac{p_B'^2}{2m_B} + m_B a x'_B$$

$$a = -g$$

The quantum weak equivalence principle

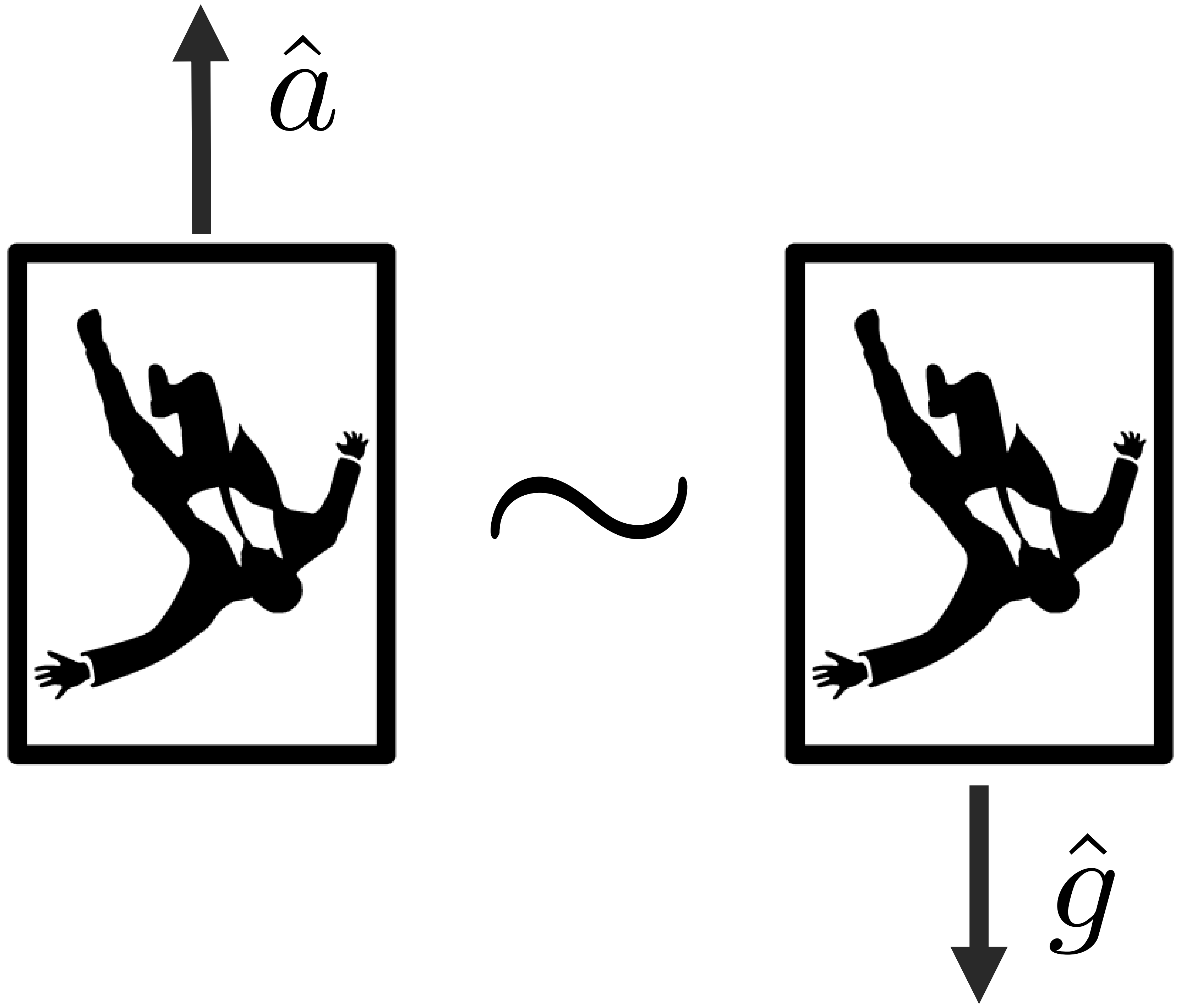
Quantum weak equivalence principle:
The physical effects seen in a superposition of uniform gravitational fields are indistinguishable from those as seen from a system in superposition of accelerations.



The quantum weak equivalence principle

Quantum weak equivalence principle:
The physical effects seen in a superposition of uniform gravitational fields are indistinguishable from those as seen from a system in superposition of accelerations.

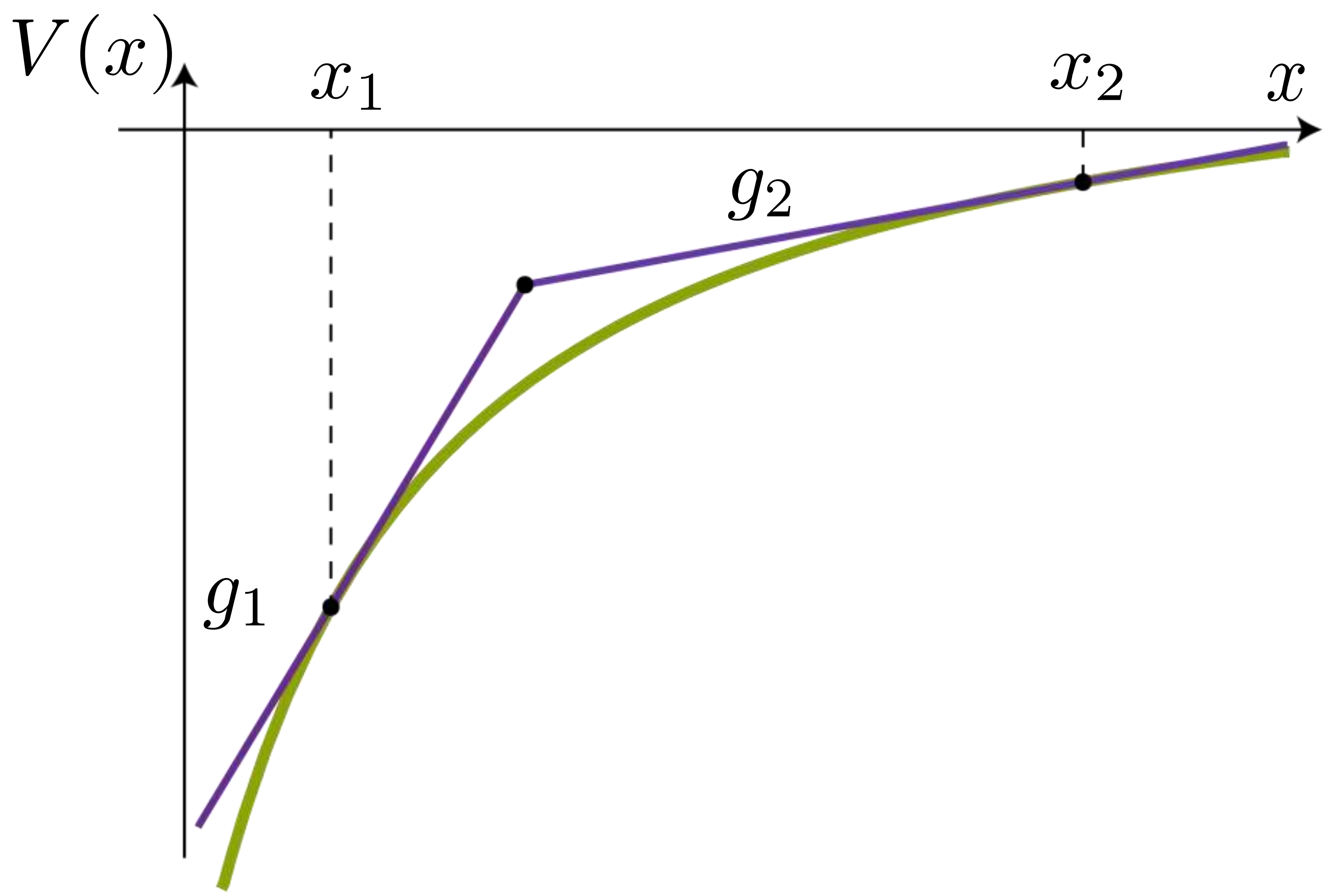
Given two particles A and B, if A freely falls in superposition of different g1, g2 and B is free, can we make a claim similar to the classical case?



QRFs!

The quantum weak equivalence principle

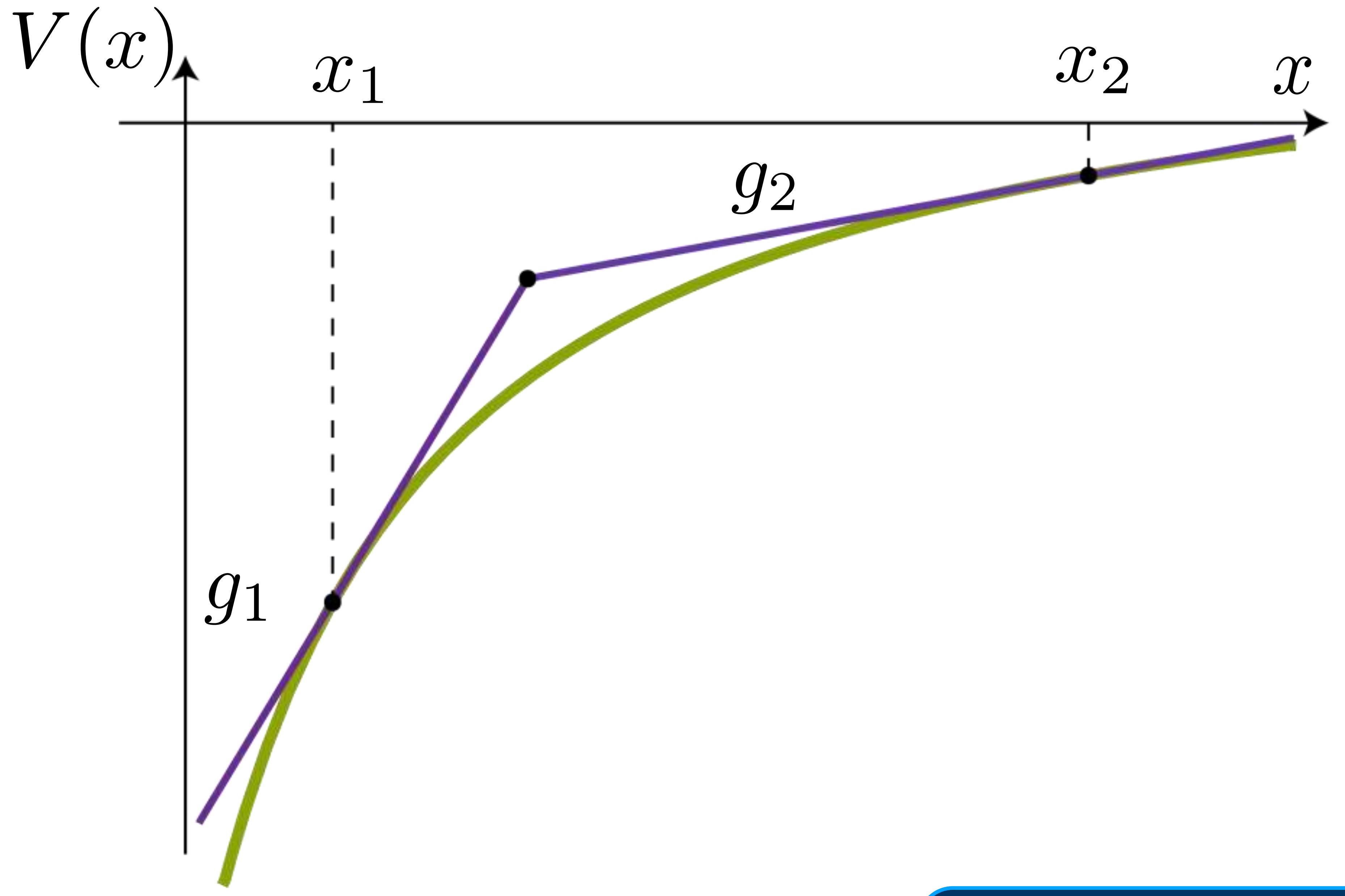
$$\hat{H}_{AB}^{(C)} = \frac{p_A^2}{2m_A} + \frac{p_B^2}{2m_B} + V(\hat{x}_A)$$



The quantum weak equivalence principle

$$\hat{H}^{(C)}_{AB} = \frac{p_A^2}{2m_A} + \frac{p_B^2}{2m_B} + V(\hat{x}_A)$$

$$|\psi\rangle^{(C)}_{AB} = \frac{|x_1\rangle_A + |x_2\rangle_A}{\sqrt{2}} |0\rangle_B$$



$$\hat{g} = \frac{1}{m_A} \frac{dV(\hat{x}_A)}{d\hat{x}_A}$$

$$\hat{g}\, \frac{|x_1\rangle_A + |x_2\rangle_A}{\sqrt{2}} = \frac{g_1 |x_1\rangle_A + g_2 |x_2\rangle_A}{\sqrt{2}}$$

A evolves in a **superposition**
of two gravitational accelerations

The quantum weak equivalence principle

$$\hat{H}^{(C)}_{AB} = \frac{p_A^2}{2m_A} + \frac{p_B^2}{2m_B} + V(\hat{x}_A)$$

$$|\psi\rangle^{(C)}_{AB} = \frac{|x_1\rangle_A + |x_2\rangle_A}{\sqrt{2}} |0\rangle_B$$

$\longrightarrow$

$$|\psi\rangle^{(A)}_{BC} = \frac{|-x_1\rangle_B |-x_1\rangle_C + |-x_2\rangle_B |-x_2\rangle_C}{\sqrt{2}}$$

$$\hat{H}^{(A)}_{BC} = \frac{\pi_B^2}{2m_B} + \frac{\pi_C^2}{2m_C} + \frac{m_C}{m_A} V(-\hat{q}_C) - \frac{m_B}{m_A} \frac{dV}{d\hat{x}_A}\Big|_{-\hat{q}_C} \hat{q}_B$$

The quantum weak equivalence principle

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$$\rightarrow$$

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$$\hat{a}_B = -\hat{g}$$

When A falls in a superposition of gravitational accelerations,
in the QRF of A, B evolves in a superposition of accelerations

This is the Quantum Weak Equivalence Principle

The Einstein Equivalence Principle for Quantum Reference Frames

F. Giacomini, Č. Brukner, Einstein's Equivalence principle for superpositions of gravitational fields and quantum reference frames, arXiv:2012.137544

F. Giacomini, Č. Brukner, Quantum superposition of spacetimes obeys Einstein's equivalence principle. AVS Quantum Sci. 1 March 2022; 4 (1): 015601.

C. Cepollaro, F.Giacomini, Quantum generalisation of Einstein's equivalence principle can be verified with entangled clocks as quantum reference frames, 2024 Class. Quantum Grav. 41 185009

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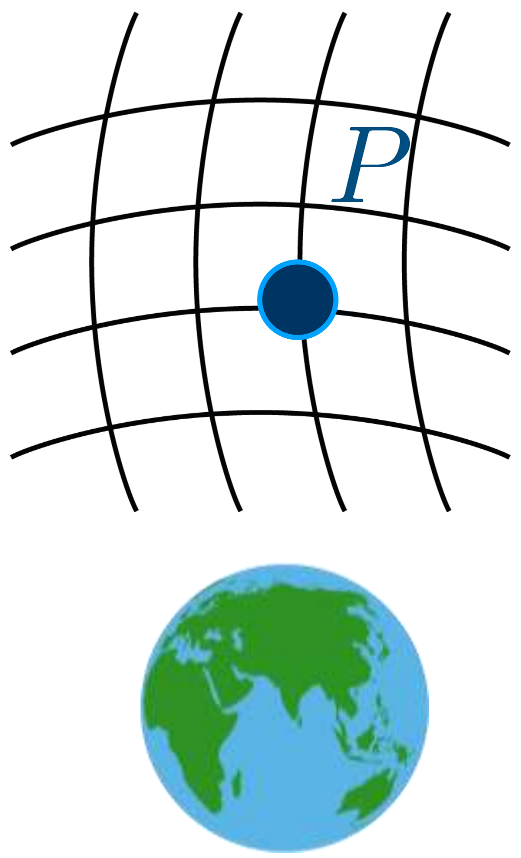
In any and every **locally inertial frame**, anywhere and anytime in the universe, all the (nongravitational) laws of physics must take on their familiar non-relativistic form.

C. W. Misner, K. S. Thorne, J. A. Wheeler, Gravitation (W. H. Freeman, 1973).

Locally inertial frames



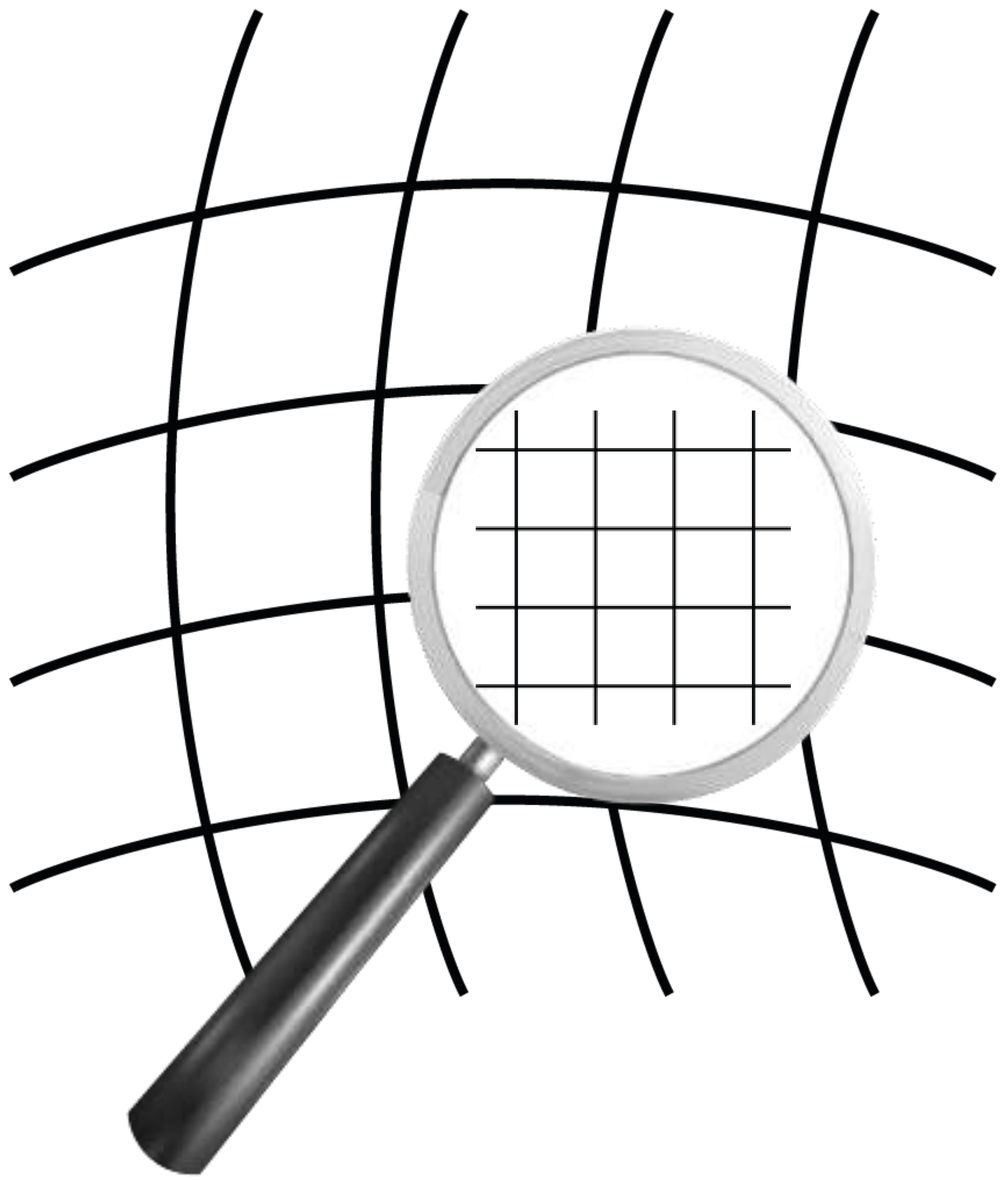
$g_{\mu\nu}(x)$

Change of coordinates:
1. Translation
2. Stretching

$$x^\mu = x^\mu(\xi^\mu) = x^\mu_P + \frac{\partial x^\mu}{\partial \xi^\alpha}\xi^\alpha + O(\xi^2)$$

$$\tilde{g}^{\mu\nu}(\xi) = \frac{\partial x^\mu}{\partial \xi^\alpha}\frac{\partial x^\nu}{\partial \xi^\beta} g_{\alpha\beta}(x(\xi))$$

$$g_{\mu\nu}(x) \to \tilde{g}_{\mu\nu}(\xi) = \eta_{\mu\nu} + O(\xi^2)$$

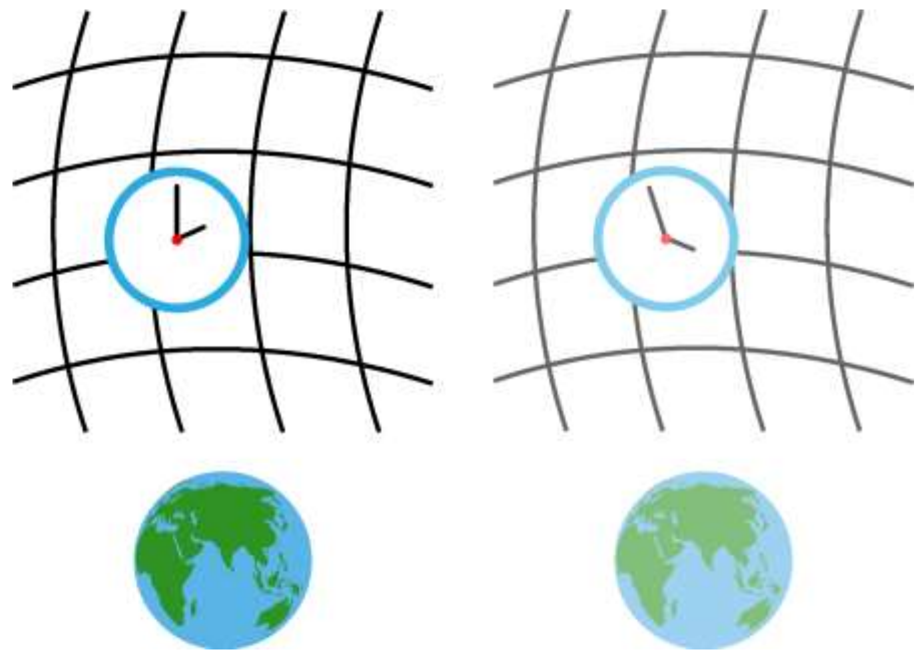


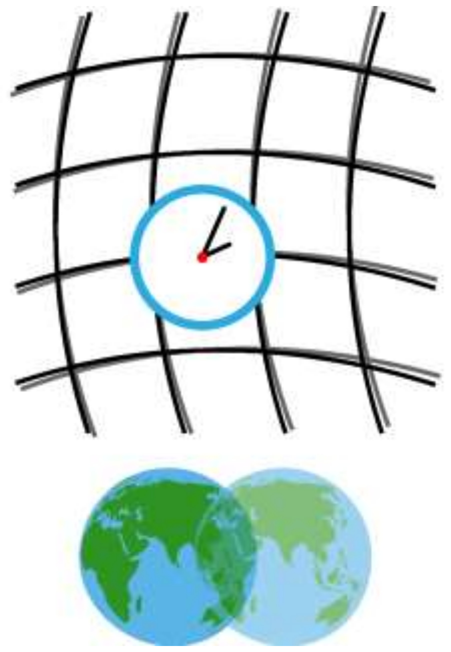
C. W. Misner, K. S. Thorne, J. A. Wheeler, Gravitation (W. H. Freeman, 1973).

Quantum locally inertial frame

A mass in spatial superposition generates a superposition of gravitational fields.

A clock that evolves in this scenario will get entangled with the gravitational field.





F. Giacomini, Č. Brukner, Einstein's Equivalence principle for superpositions of gravitational fields and quantum reference frames, arXiv:2012.137544

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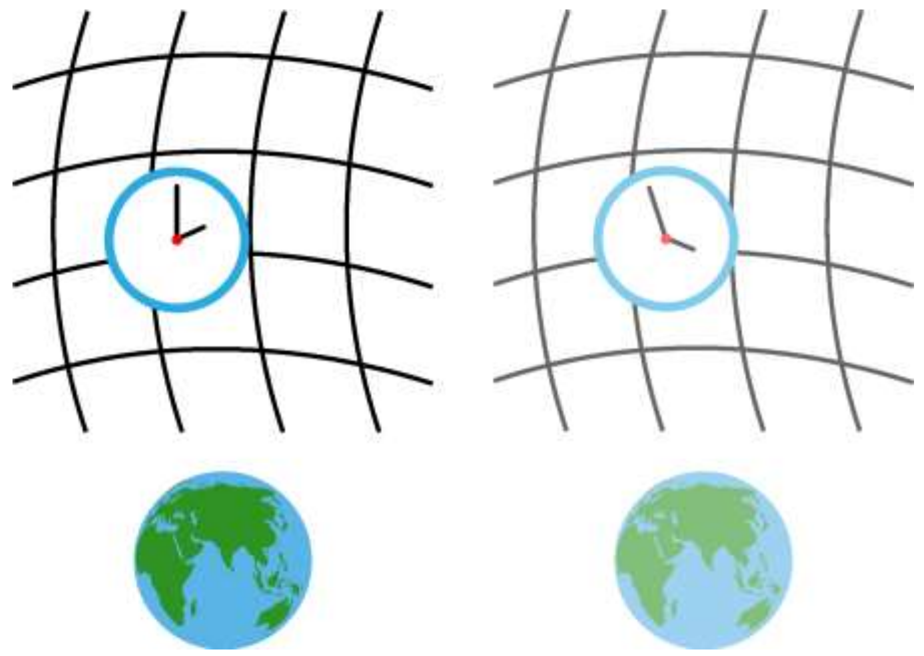
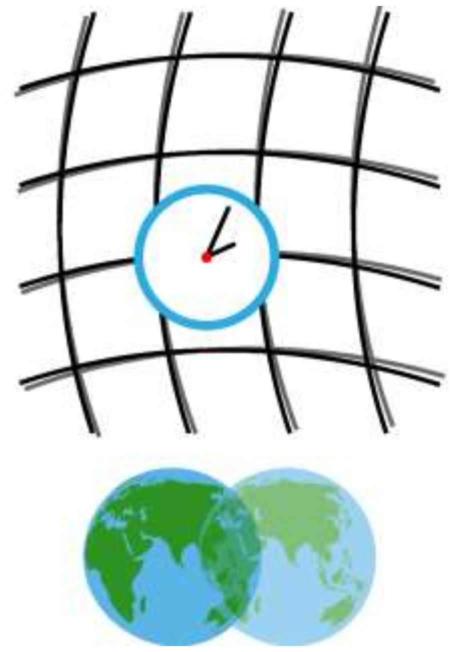


The **Quantum** Locally Inertial Frame (QLIF) of the clock is a frame associated to a quantum particle where the metric is locally flat and well defined.



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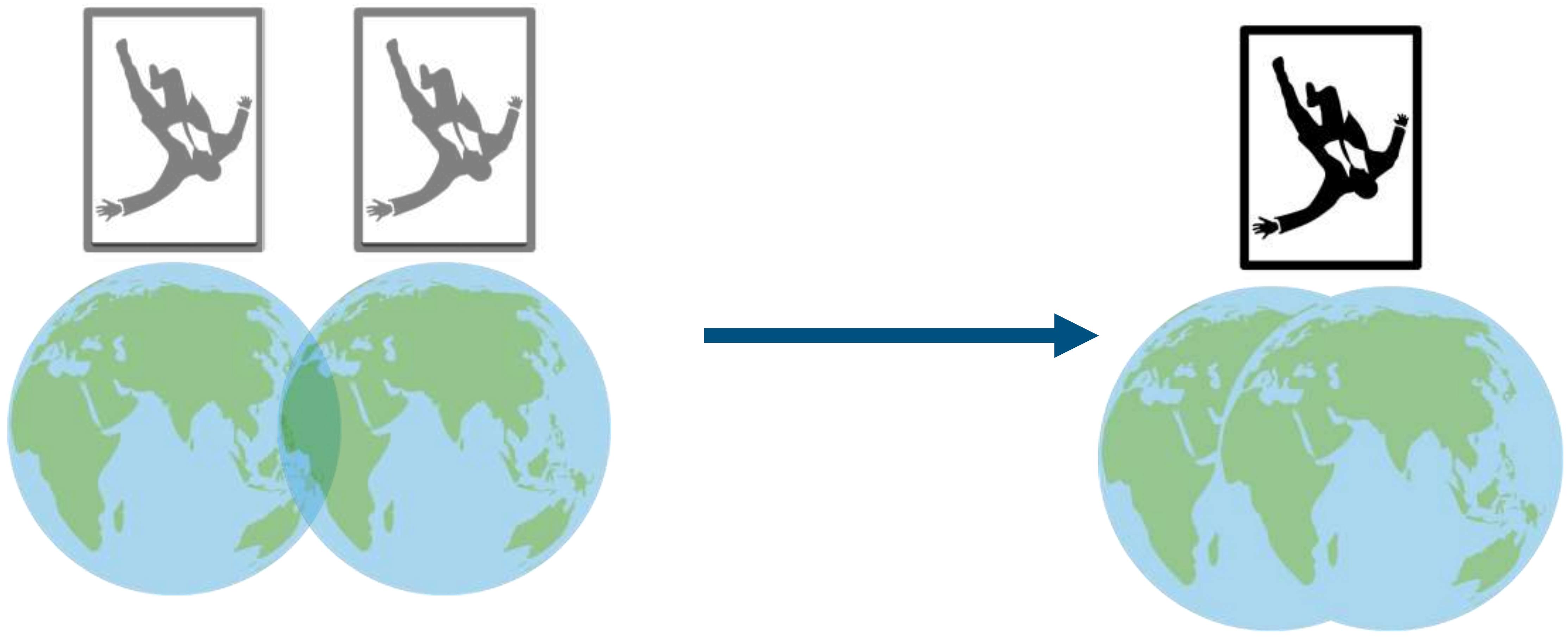
The Einstein Equivalence Principle for QRFs

In any and every **Quantum** Locally Inertial Frame (**QLIF**), anywhere and anytime in the universe, all the (nongravitational) laws of physics must take on their familiar non-relativistic form.



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The three aspects of the EEP for QRFs

Q-WEP

The local effects of **(quantum)** motion in a **superposition** of uniform gravitational fields are indistinguishable from those of an observer in flat spacetime that undergoes a quantum **superposition** of accelerations.

Q-LLI

The outcome of any local nongravitational experiment is independent of the velocity of the freely falling **quantum** reference frame in which it is performed.

Q-LPI

The outcome of any local nongravitational experiment is independent of the position of the **quantum** reference frame in which it is performed.

C. Cepollaro, F. Giacomini, 2024 Class. Quantum Grav. 41 185009

Towards a test for the principle

C. Cepollaro, F. Giacomini, 2024 Class. Quantum Grav. 41 185009

Classical tests of EEP

- WEP tests: Comparison between accelerations of two bodies of different composition in an external gravitational field.
- LLI tests: Tests of special relativity (e.g., Michelson-Morley experiment).
- LPI tests: Gravitational redshift experiment.

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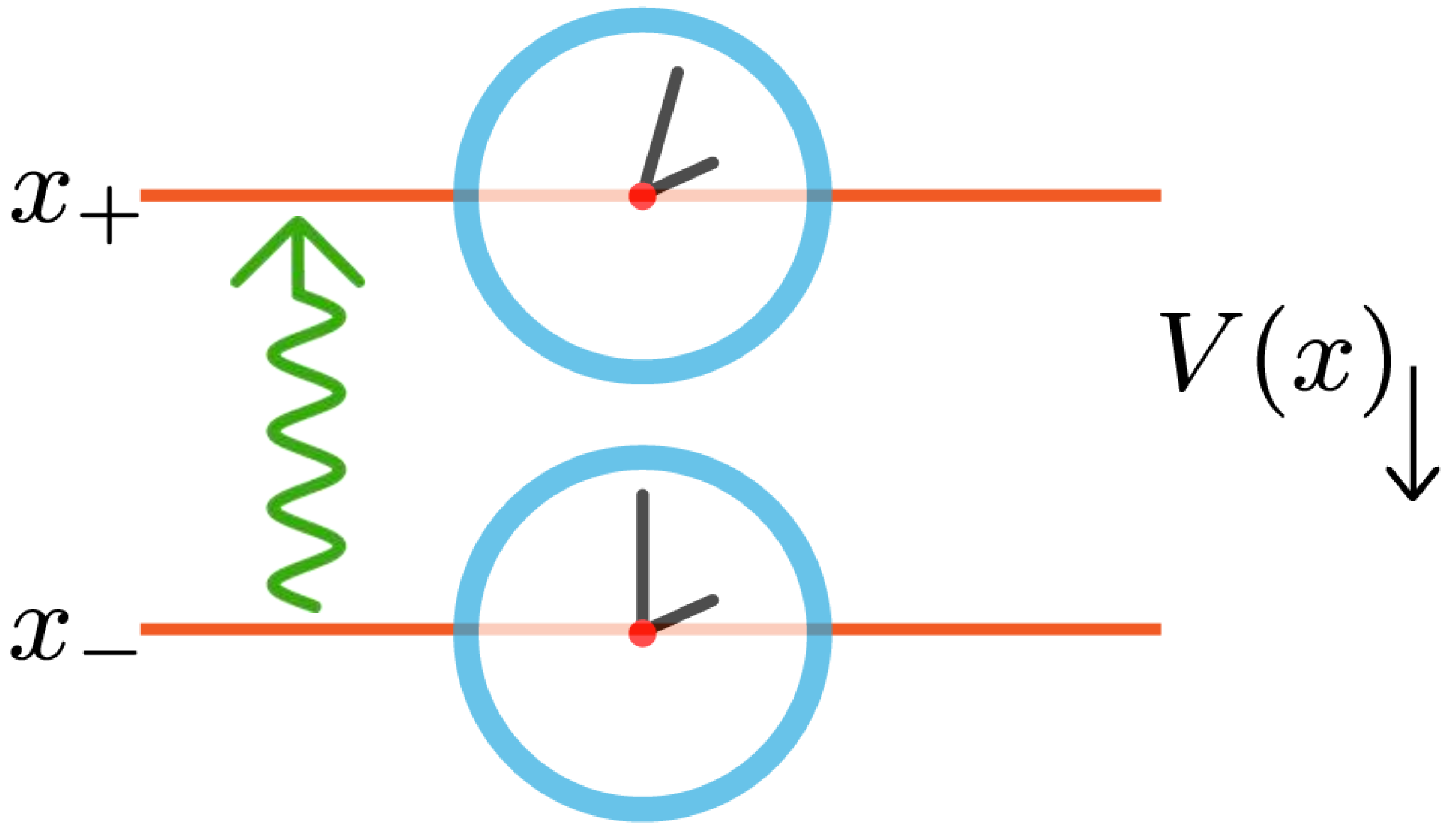


$$\frac{\Delta\nu}{\nu} = \frac{\Delta V}{c^2}$$

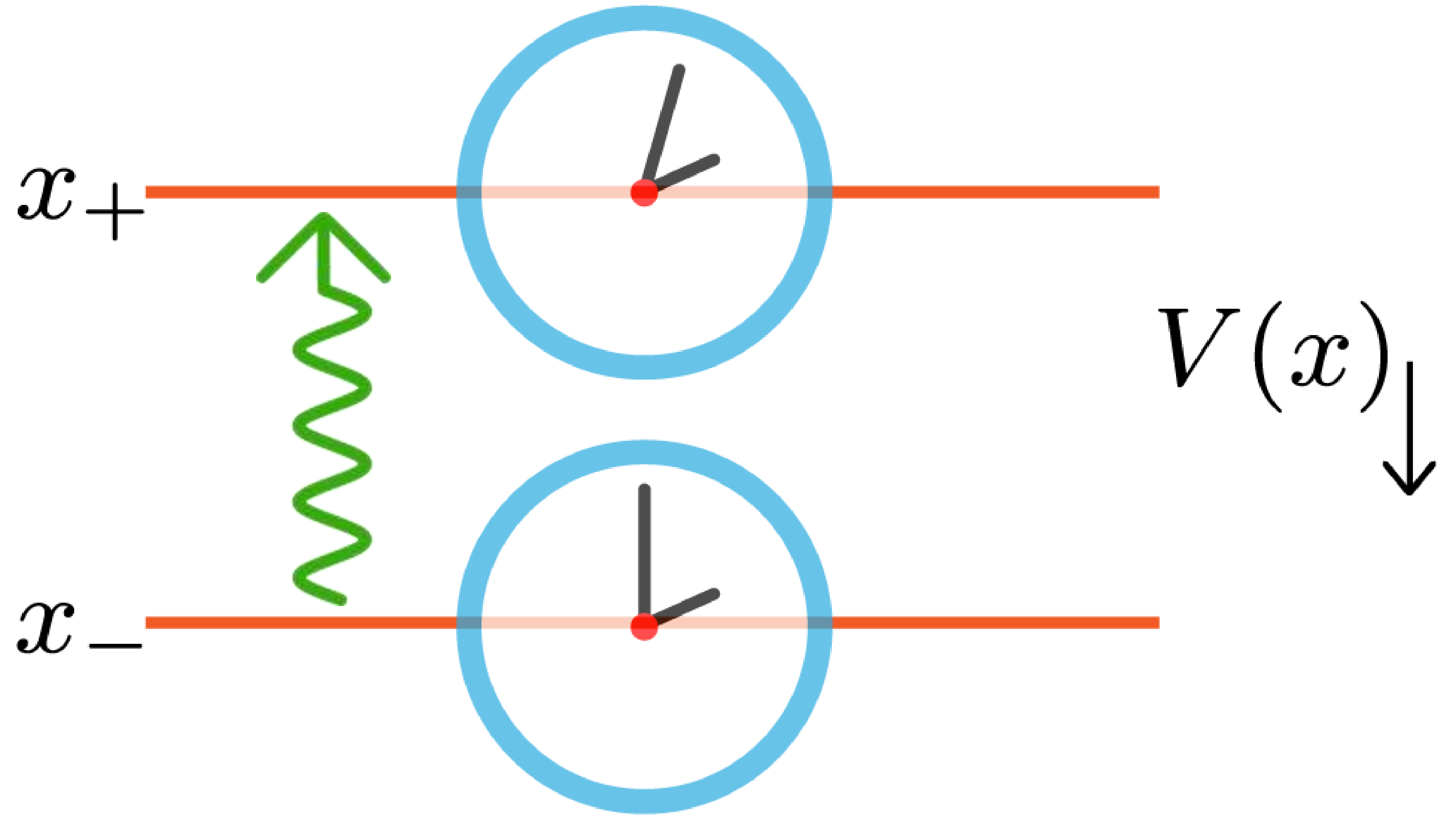
The locally inertial frames momentarily at rest with each clock are related through Lorentz boosts

The doppler effect gives the time dilation factor

A classical test for LPI

LPI

The outcome of any local nongravitational experiment is independent of the position of the reference frame in which it is performed.



IF NOT LPI

$$\tau = \tau(x)$$

$$\frac{\Delta \nu}{\nu} = (1 + \alpha(x)) \frac{\Delta V}{c^2}$$

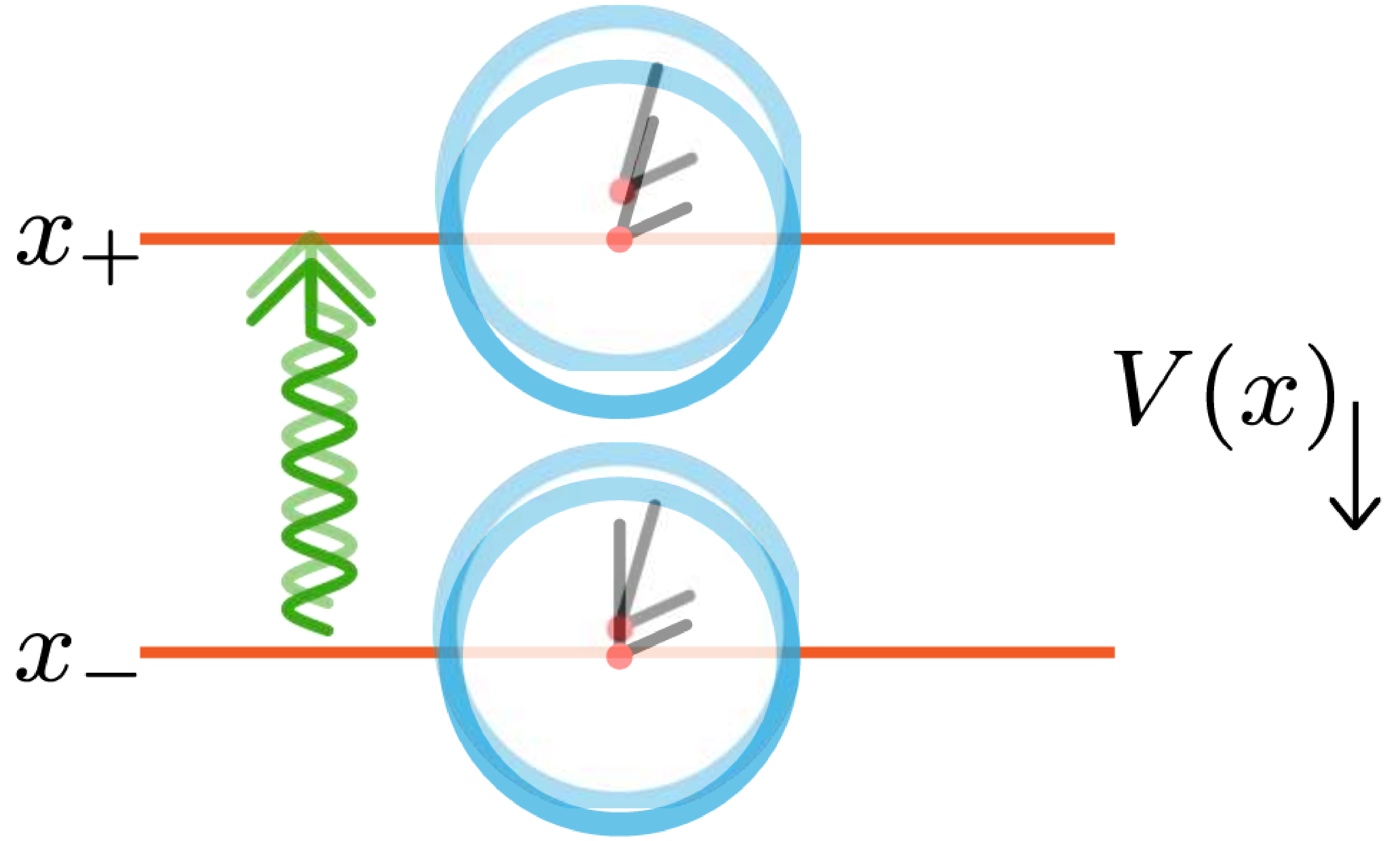
LPI can be verified by measuring $\alpha(x)$

Will CM. *Theory and Experiment in Gravitational Physics*. 2nd ed. Cambridge University Press; 2018.

A quantum test for Q-LPI

Q-LPI

The outcome of any local nongravitational experiment is independent of the position of the **quantum** reference frame in which it is performed.



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Atomic clocks

External + Internal DOF

$$\hat{H} = mc^2 + \frac{\hat{\mathbf{p}}^2}{2m} + mV(\hat{\mathbf{x}})$$

$$m \to \hat{M} = m + \frac{\hat{H}_I}{c^2}$$

Newtonian QM +
mass-energy equivalence

$$\hat{H} = mc^2 + \frac{\hat{\mathbf{p}}^2}{2m} + mV(\hat{\mathbf{x}}) + \hat{H}_I\left(1 + \frac{V(\hat{\mathbf{x}})}{c^2} - \frac{\hat{\mathbf{p}}^2}{2m^2c^2}\right)$$

1. C. Lämmerzahl, On the equivalence principle in quantum theory. Gen Relat Gravit 28, 1043–1070 (1996).
2. M. Zych, F. Costa, I. Pikovski, Č. Brukner, Quantum interferometric visibility as a witness of general relativistic proper time. Nat Commun 2, 505 (2011).
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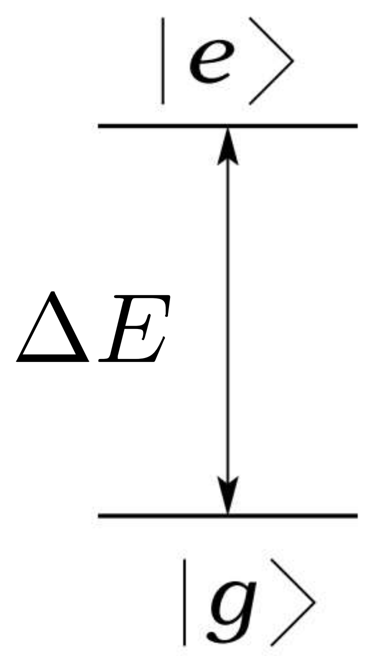
$$\omega' = \omega\left(1 + \frac{V(x)}{c^2} - \frac{p^2}{2m^2c^2}\right) = \omega\frac{d\tau}{dt}$$

$$\omega = \frac{\Delta E}{\hbar}$$



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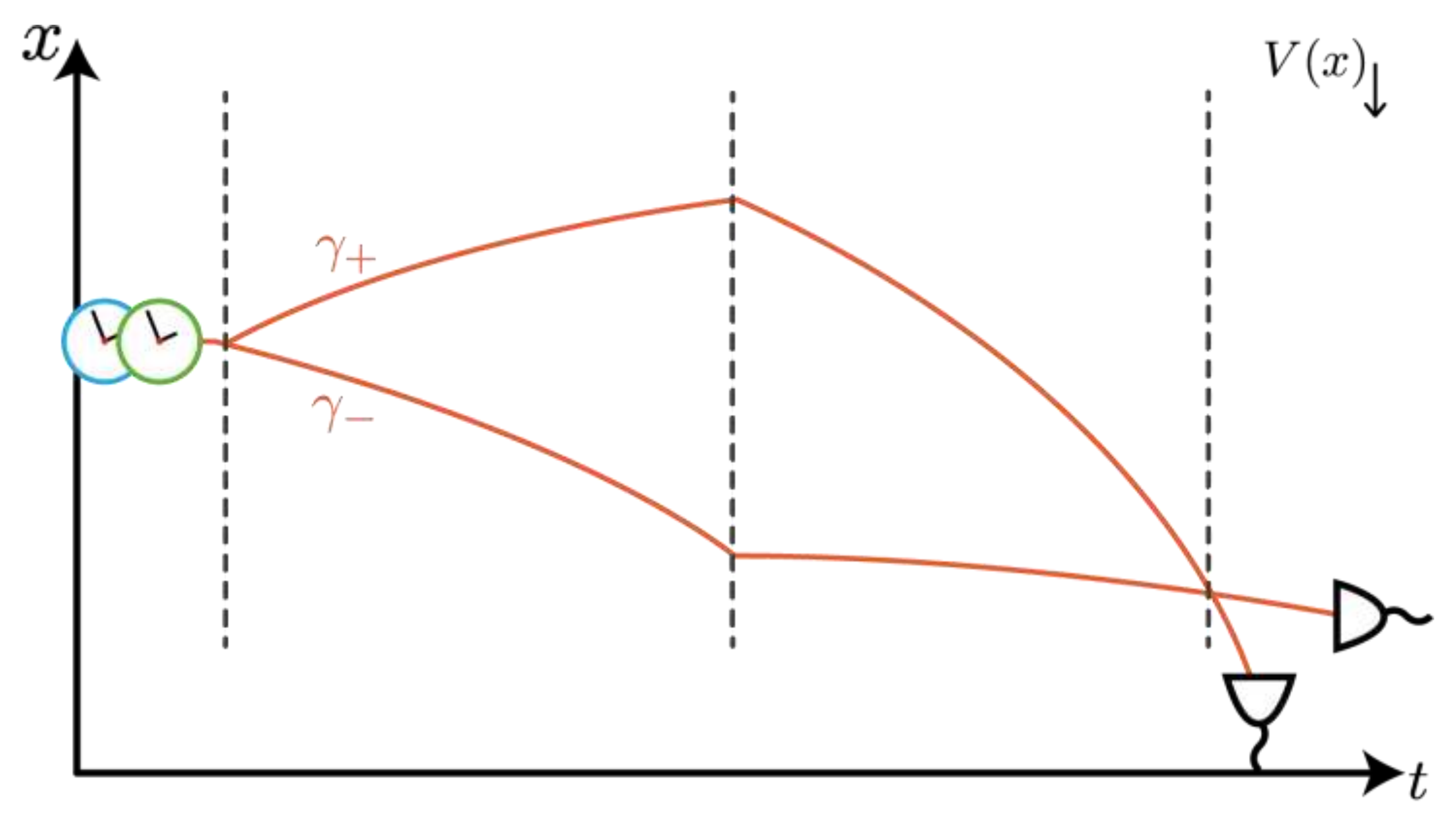
Atomic clock interferometry

$$|\psi_0\rangle_{AB} = \frac{|x_+\rangle_{\mathbf{A}} |x_+\rangle_{\mathbf{B}} + |x_-\rangle_{\mathbf{A}} |x_-\rangle_{\mathbf{B}}}{\sqrt{2}} |\tau_{in}\rangle_{C_A} |\tau_{in}\rangle_{C_B}$$



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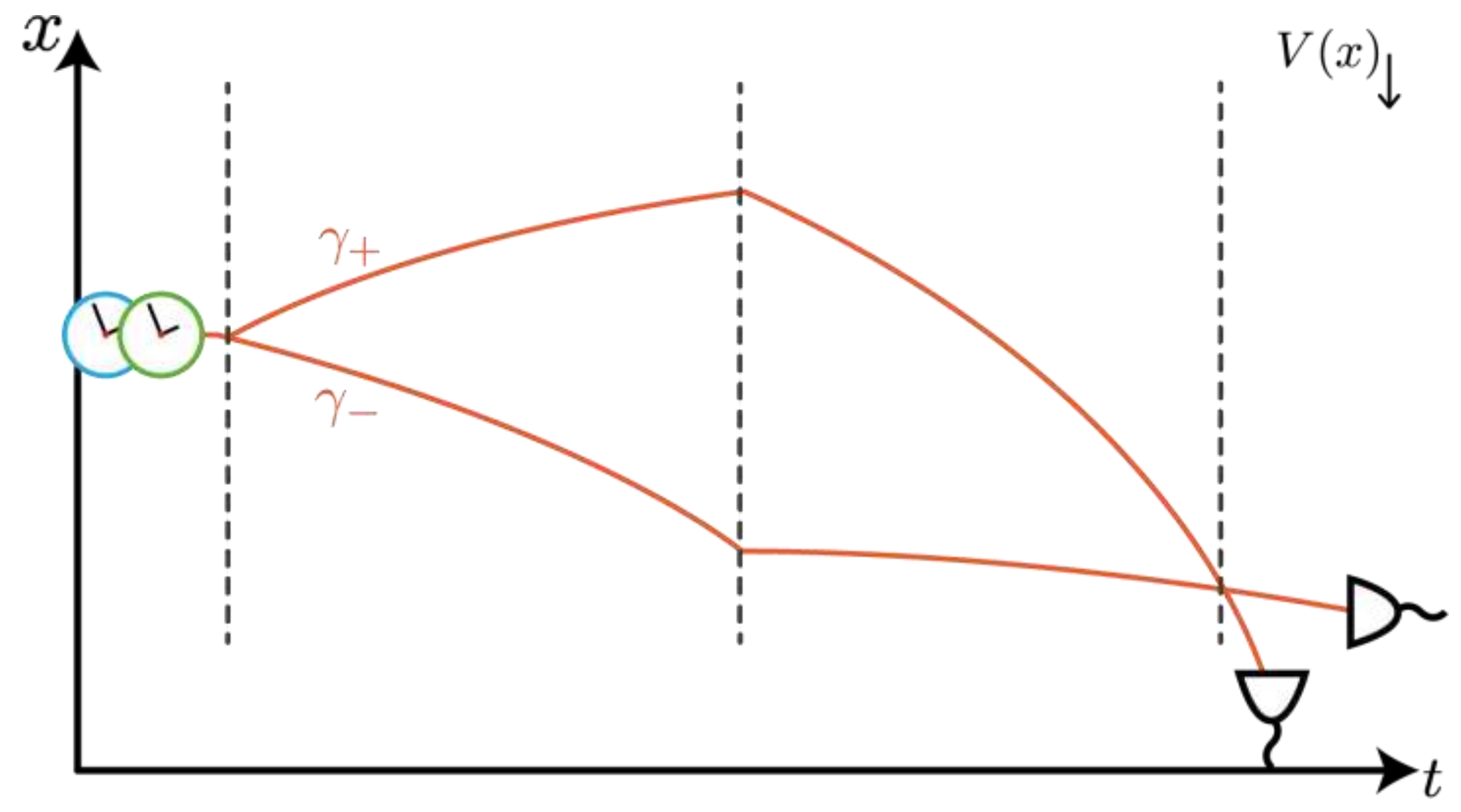

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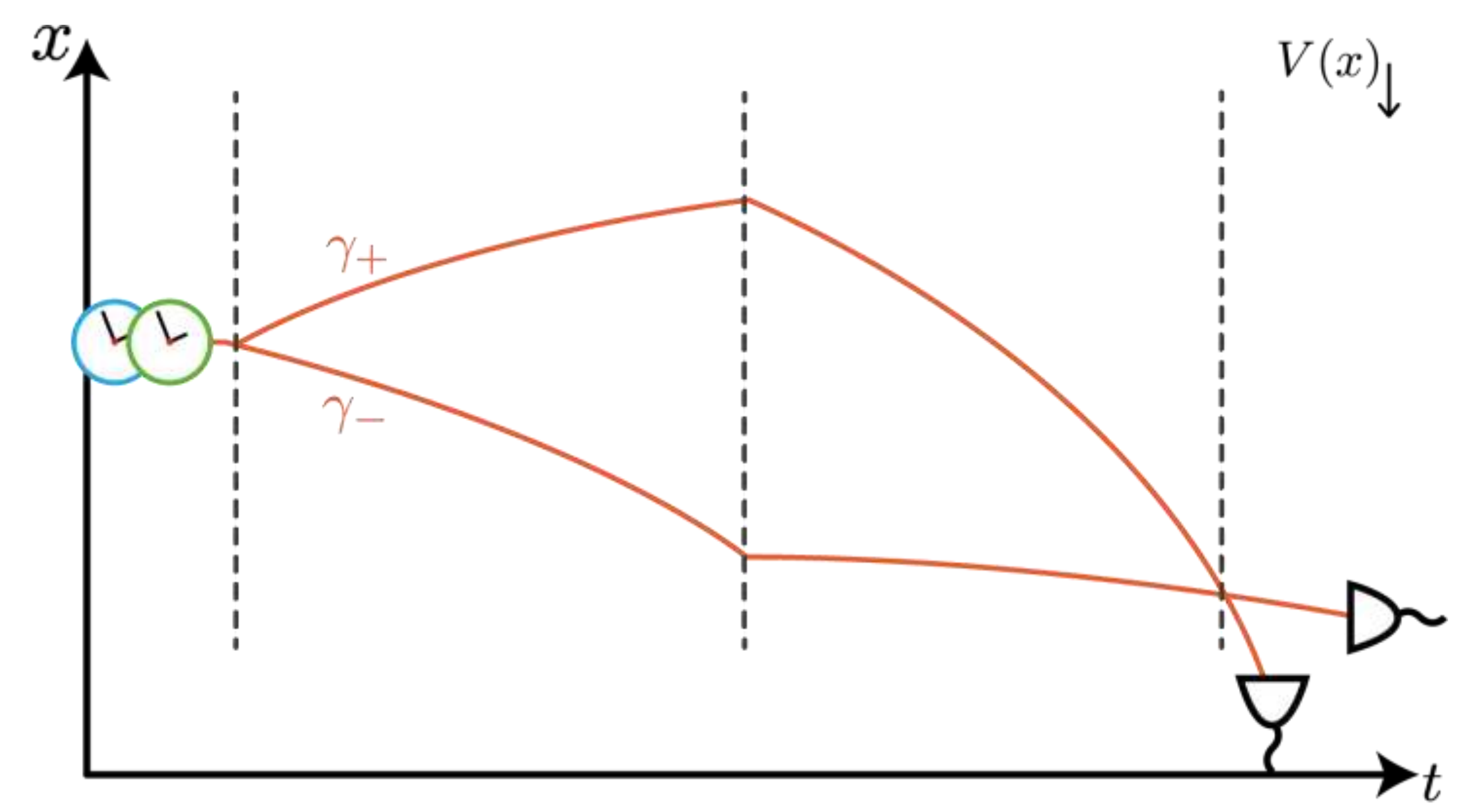
$$\Delta\phi_j = \frac{1}{\hbar}\int_{\Delta\gamma} dt \left(m_j c^2 + \frac{\mathbf{p}_j^2}{2m_j} + m_j V(\mathbf{x}_j) \right)$$

$$|\tau_\pm\rangle_{C_j} = e^{-\frac{i}{\hbar}\int_{\gamma_\pm} dt \left(1 + \frac{V(\mathbf{x}_j)}{c^2} - \frac{\mathbf{p}_j^2}{2m_j^2 c^2}\right)\hat{H}^j_{int}} |\tau_{in}\rangle_{C_j}$$



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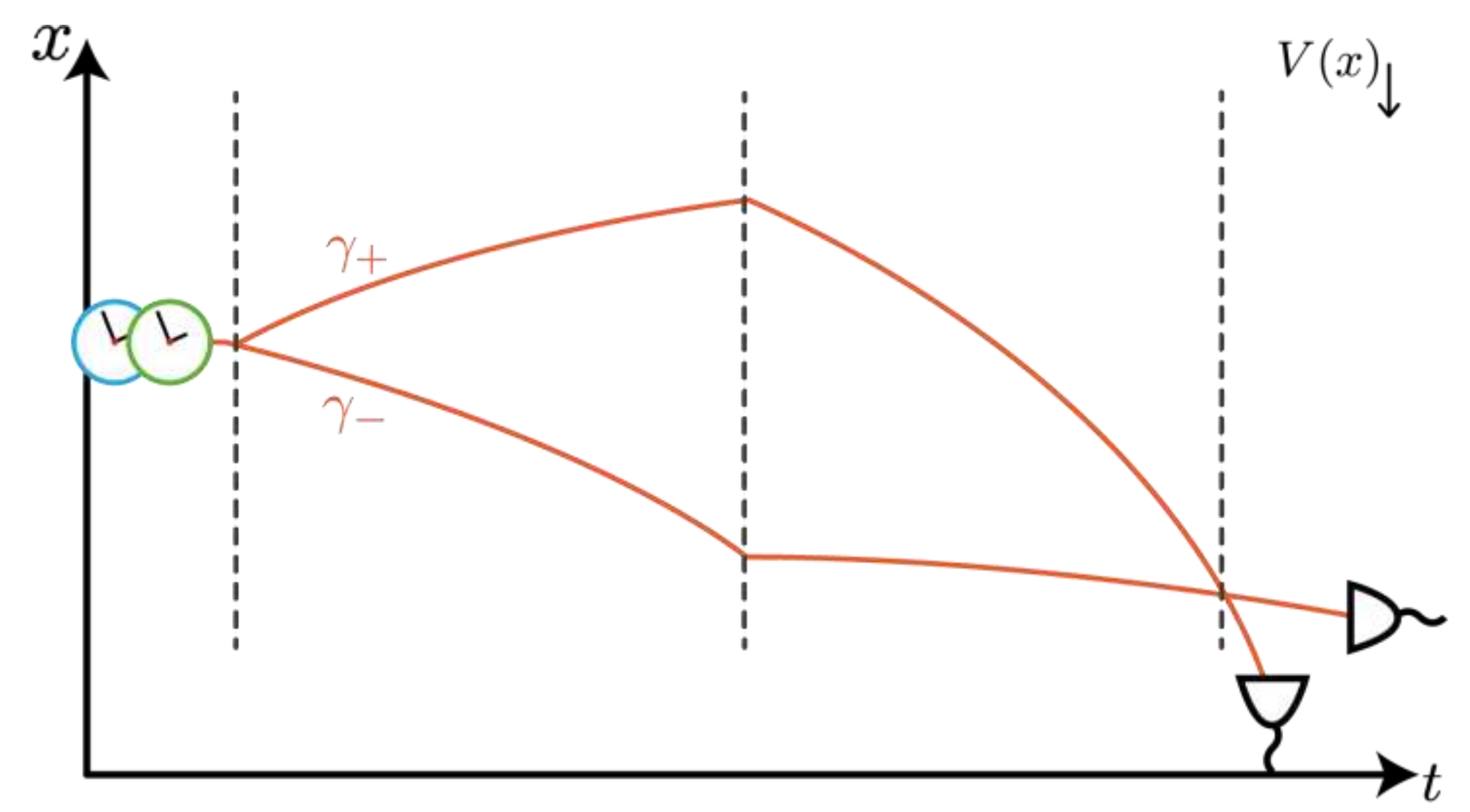

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$$|D_\pm\rangle_{\mathbf{AB}} = \frac{|x_+\rangle_{\mathbf{A}} |x_+\rangle_{\mathbf{B}} \pm |x_-\rangle_{\mathbf{A}} |x_-\rangle_{\mathbf{B}}}{\sqrt{2}}$$

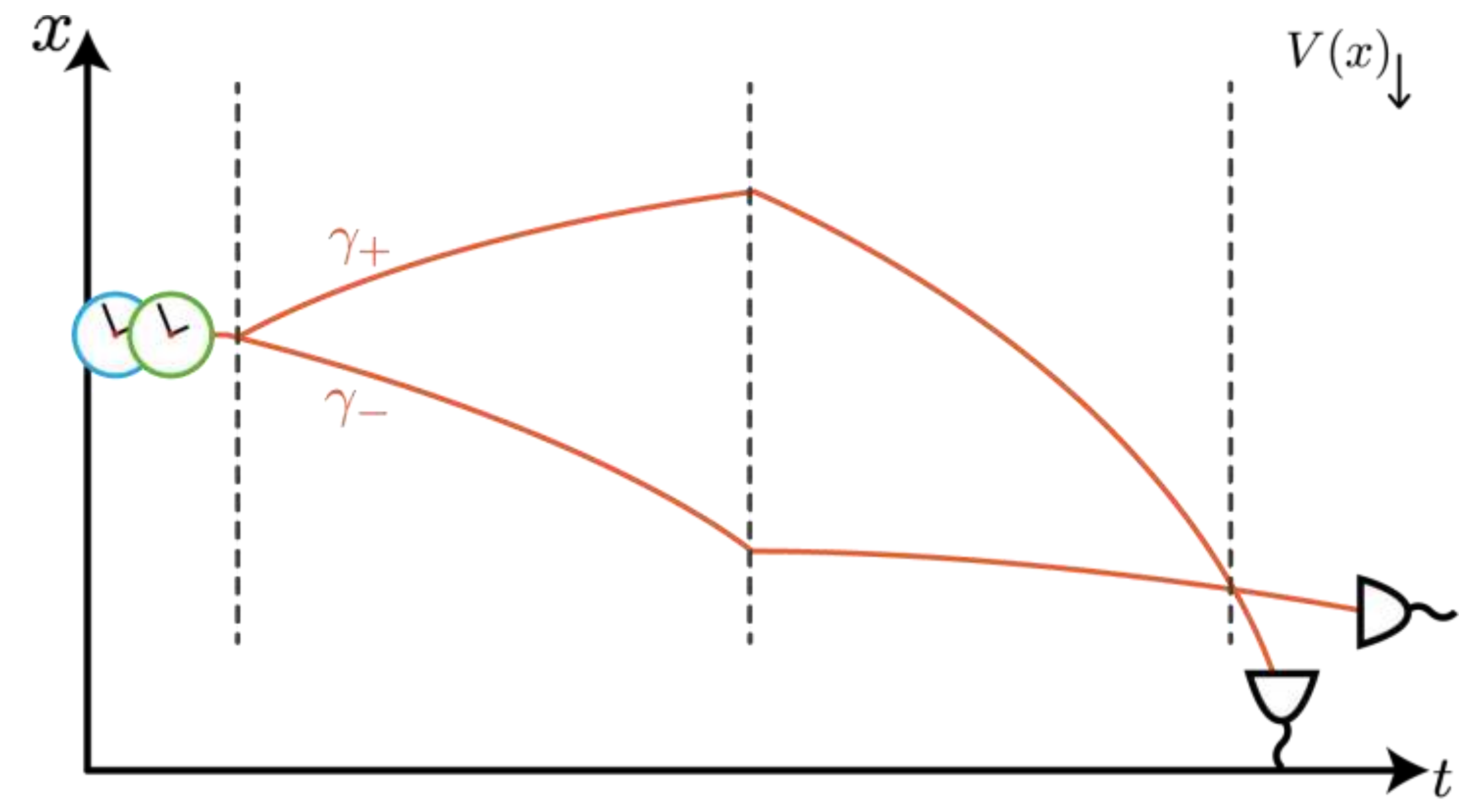
$$|\tau_\pm\rangle_{C_j} = e^{-\frac{i}{\hbar}\int_{\gamma_\pm} dt \left(1+\frac{V(\mathbf{x}_j)}{c^2} - \frac{\mathbf{p}_j^2}{2m_j^2 c^2}\right)\hat{H}^j_{int}} |\tau_{in}\rangle_{C_j}$$

$$P_\pm = \frac{1}{2}\left(1 \pm |\langle\tau_+|\tau_-\rangle|^2 \cos\left(2(\Delta\phi + \varphi')\right)\right)$$

The test for EEP for QRFs

IF NOT Q-LPI

$$\hat{H} = mc^2 + \frac{\hat{\mathbf{p}}^2}{2m} + mV(\hat{\mathbf{x}}) + \hat{H}_I\left(1 + (1+\alpha(\hat{x}))\frac{V(\hat{\mathbf{x}})}{c^2} - \frac{\hat{\mathbf{p}}^2}{2m^2c^2}\right)$$



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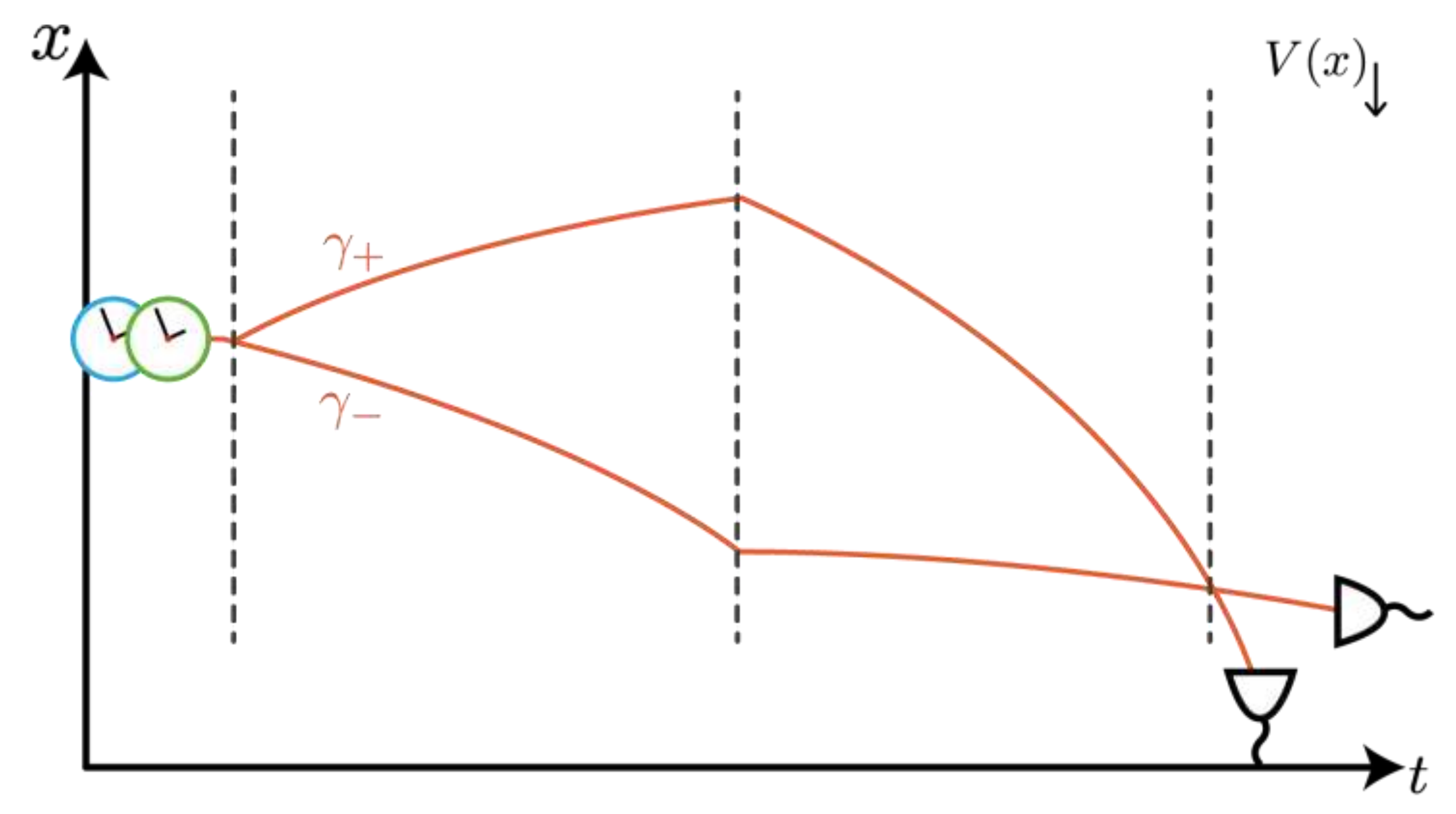
$$\langle\tau_+|\tau_-\rangle' = \langle\tau_+|\tau_-\rangle - \frac{i}{\hbar}\int_{\Delta\gamma} dx\, \frac{V(\mathbf{x}_\gamma)}{c^2}\alpha(\mathbf{x}_\gamma)\langle\tau_+|\,\hat{H}\,|\tau_-\rangle$$



$$P'_\pm = \frac{1}{2}\left(1 \pm \left|\langle\tau_+|\tau_-\rangle'\right|^2 \cos\left(2(\Delta\phi + \varphi')\right)\right) \neq P_\pm$$

Atomic clock interferometers can be used to measure vioaltions of the EEP for QRFs

Summary

Take-home messages

- There are different definitions of equivalence principle, depending on their interpretation one ends up with different compatibility with QM

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- There are different definitions of equivalence principle, depending on their interpretation one ends up with different compatibility with QM
- Quantum Reference Frames (QRFs) generalize classical RFs for particles that can be in quantum superposition
- QRFs can be used to define a quantum extension of the Einstein’s Equivalence Principle
- This EEP for QRFs can be tested with atomic clock interferometers

Thank you!