# Dynamics in Many-Body Quantum Systems

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# Outline of Lectures

- Dynamics in Closed Many-Body Systems:

  . thermalization / many-body localisation

- Open Systems: GKSL (Gorini-Kossakowski-Sudarshan-Lindblad) Master equation - “*Lindbladian*”

- Examples (*synthetic* open quantum systems)

  . boundary time-crystals
  . dark states (dissipative state engineering)

- Applications:
  . quantum metrology
  . (topological) quantum computation

# Lecture 1
# -
# dynamics in many-body closed systems

- Introduction

- Thermalization in closed quantum systems

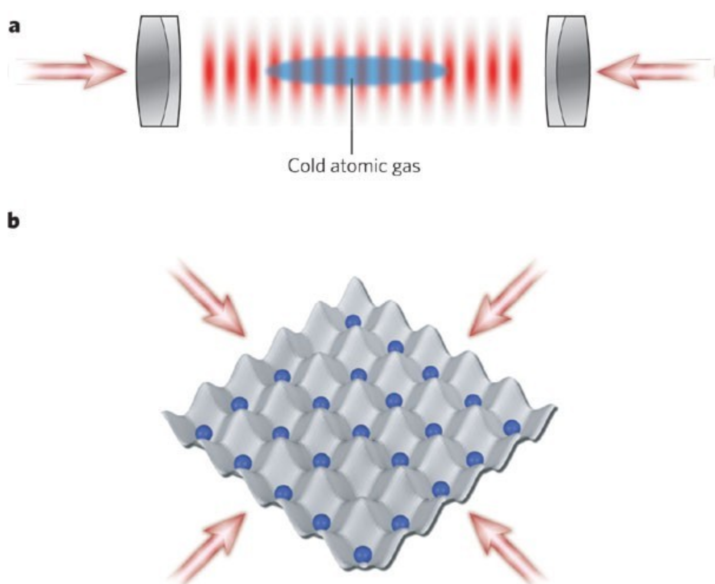
- (Many-body) Localisation

- Conclusions

# Introduction

- Dramatic experimental progress of the last few years:

***Ultracold atoms in optical lattices***
(interference of counter-propagating laser beams)

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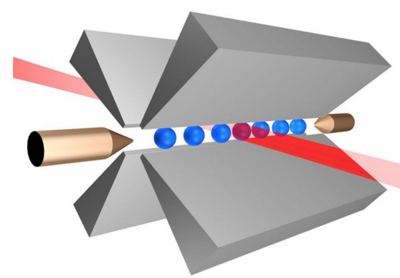
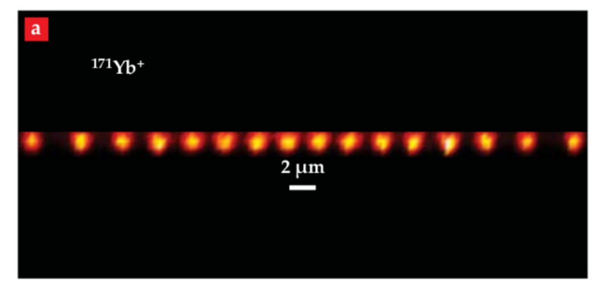
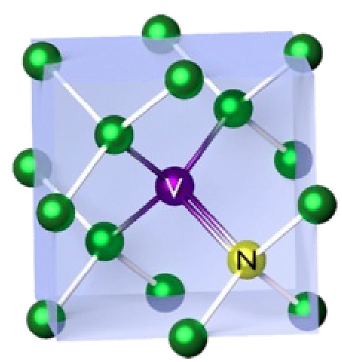
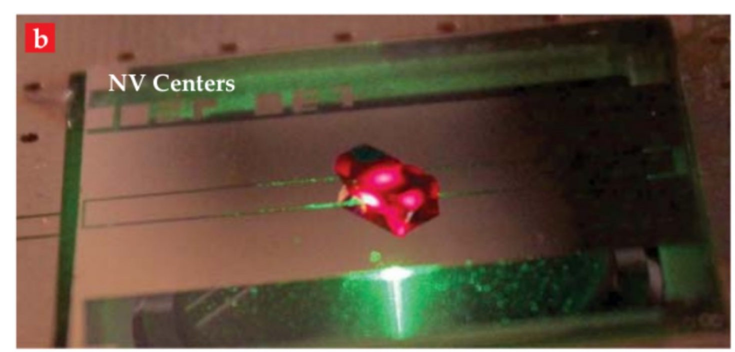
(dynamic eletric fields trap charged particles)






***Nitrogen-Vancay (NV) center***
(point defects in diamonds)



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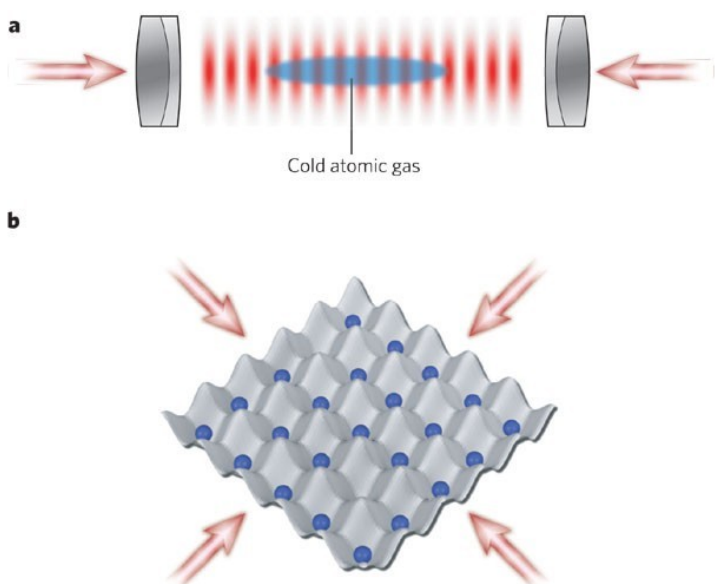


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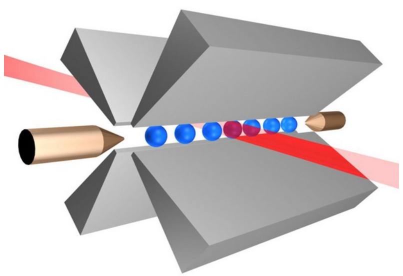

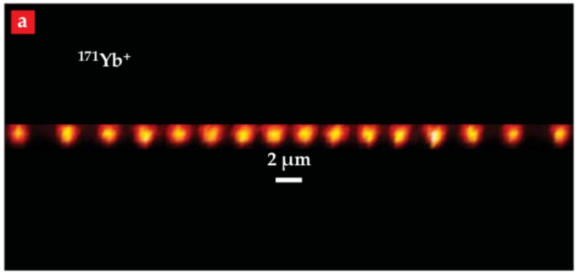


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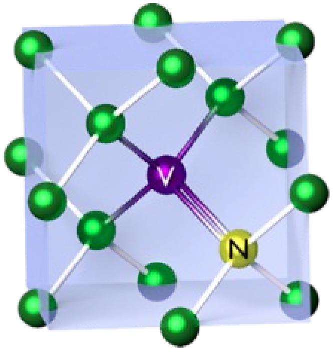


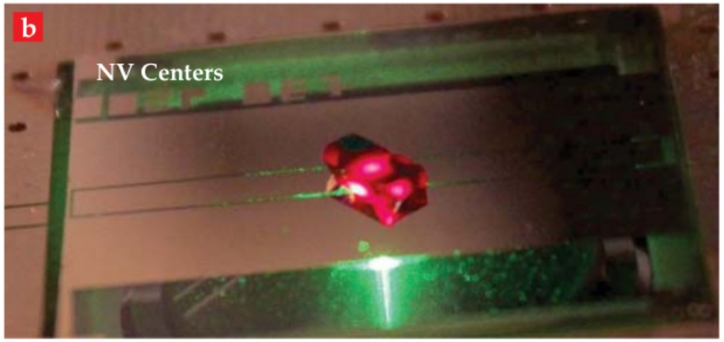

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. can ***quantum effects*** survive at long times in many-body systems?

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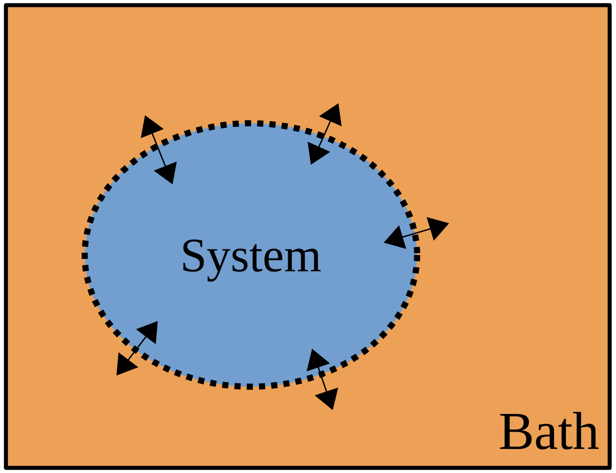

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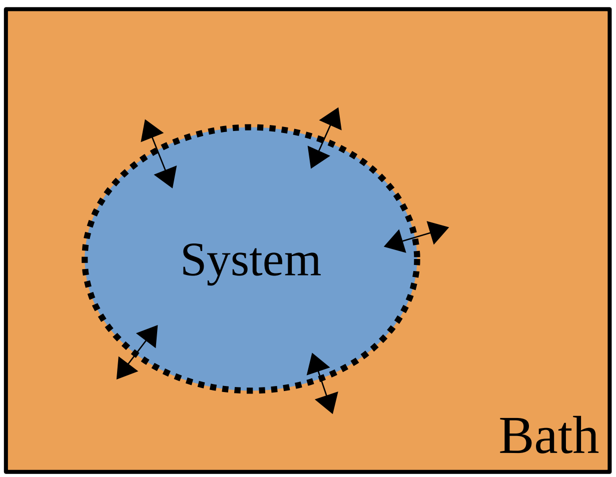


System

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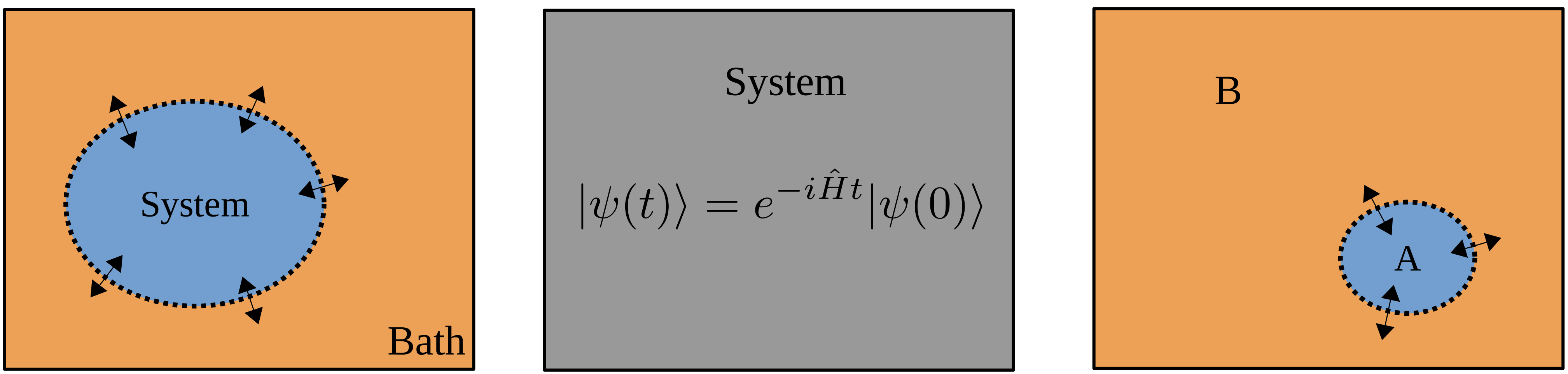


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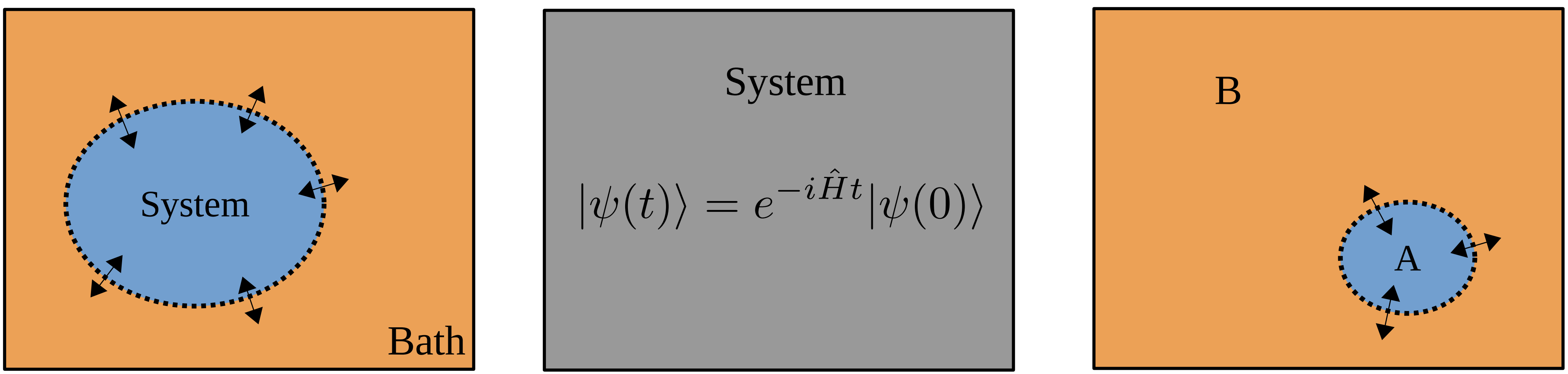


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. if so... stationary states would be described by ***quantum statistical mechanics***

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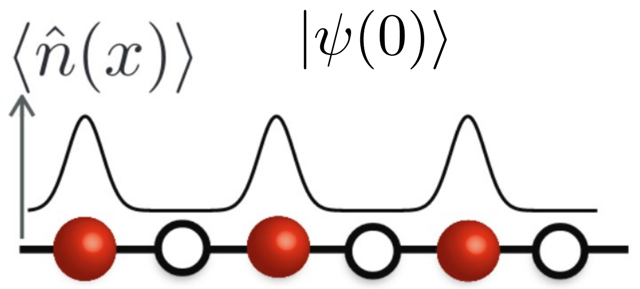


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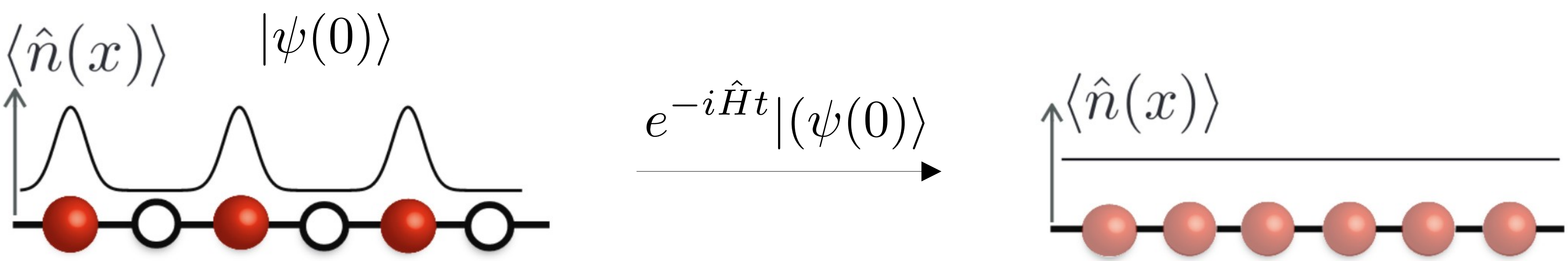


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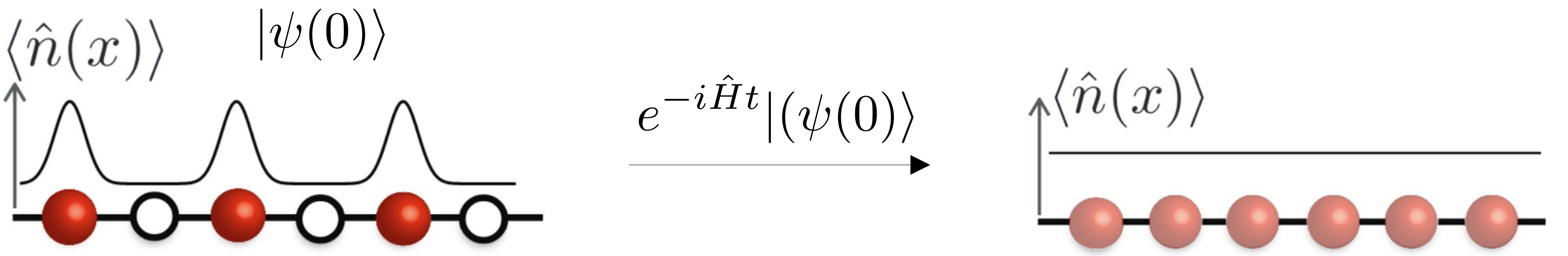


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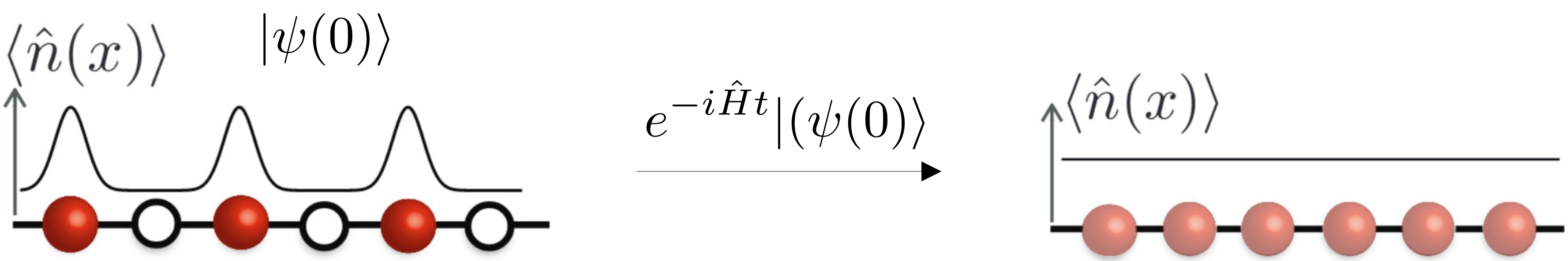


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. in fact, ***all local physical observables*** determined by few global conserved quantities, total energy, particle number etc.

$$\langle \hat{O} \rangle = Tr(\hat{\rho}_{\mathrm{th}} \hat{O}) \qquad \hat{\rho}_{\mathrm{th}}(T, E, \mu, \ldots)$$

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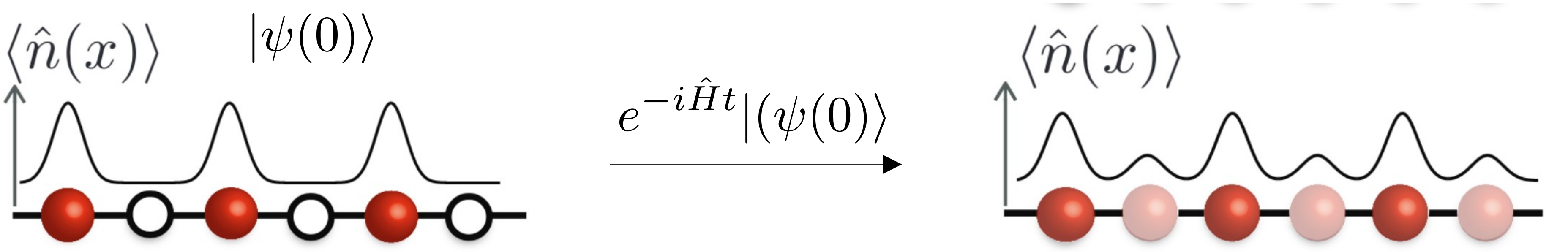

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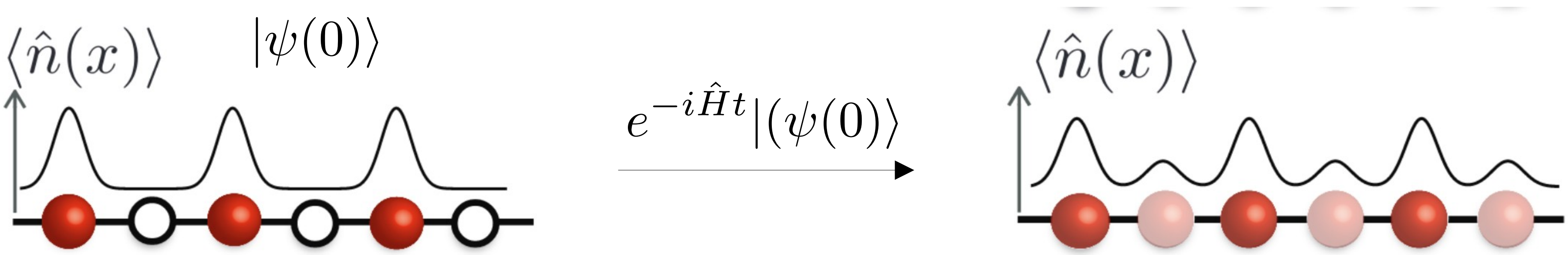


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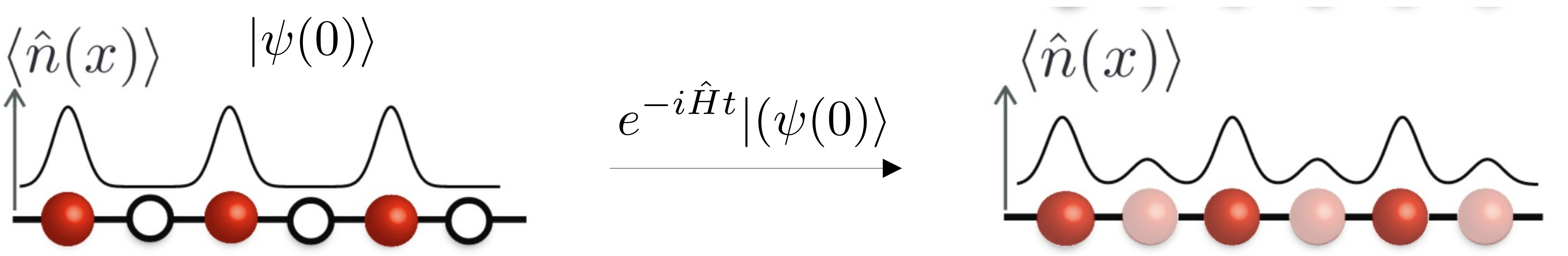


- ***ergodicity-breaking systems:*** new forms of ***stable*** quantum phases and phase transitions that are unique to the non-equilibrium settings.

- Different instances where it occurs:

. (main focus) *Anderson localisation / Many-body localisation;*
. (but also) Time Crystals / Many-body scars / Stark localisation / ...

# Outline

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# Thermalization in quantum systems

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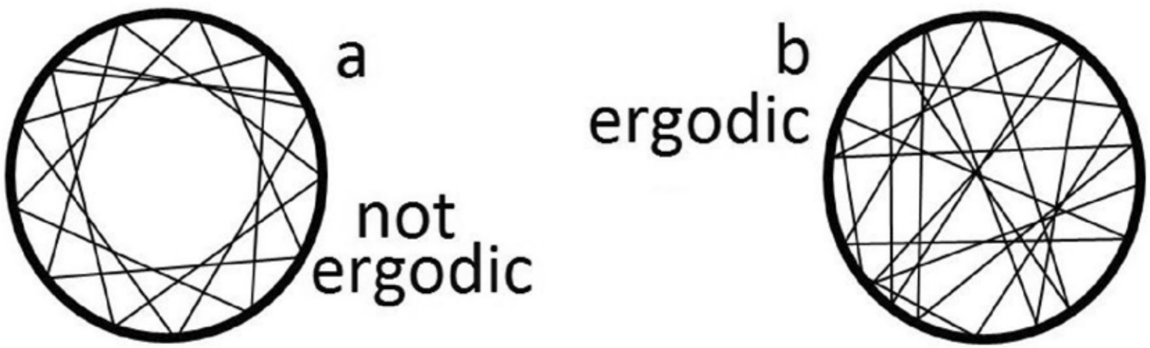

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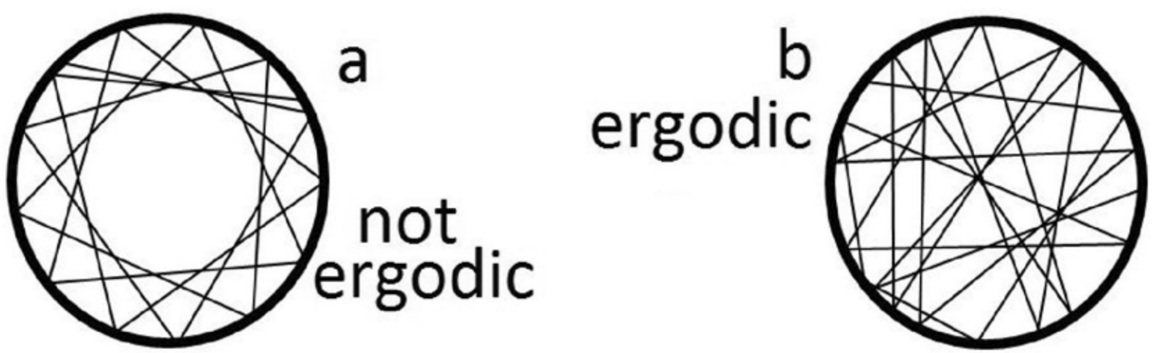


- direct translation ***classical*** → ***quantum*** is problematic!

  . quantum mechanics operates in **Hilbert space**, one cannot track a trajectory in the ***phase space*** (e.g. (q,p))

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- Thus, we need to modify the notion of ergodicity in the quantum case!

. How do we do it?

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i.e.,

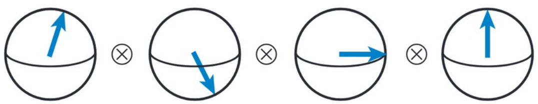
$$\langle \hat{O} \rangle_\infty = \sum_\alpha p_\alpha \langle \alpha|\hat{O}|\alpha\rangle$$

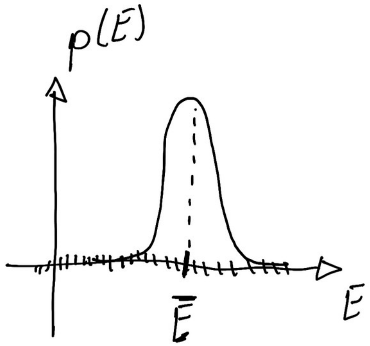
as expected, ***depends on initial state***.

**physical initial states** have support in narrow energy window:

e.g. product states

low-entangled/fast prep.





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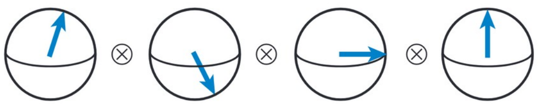

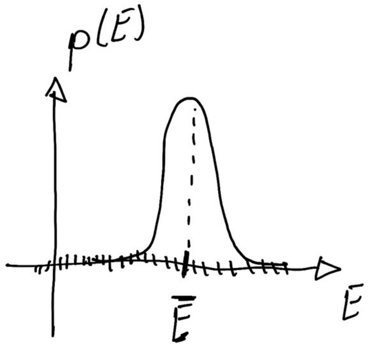


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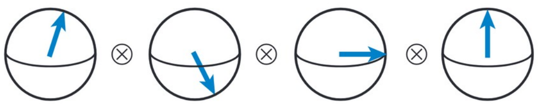

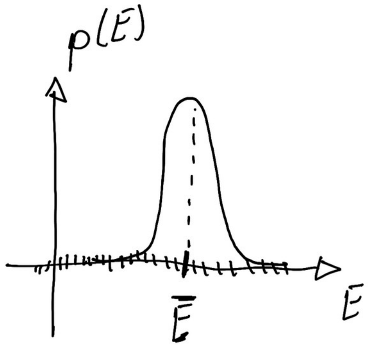


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i.e., ***all individual eigenstates*** have thermal observables, identical to a mc ensemble at $E_\alpha$

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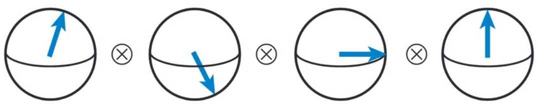

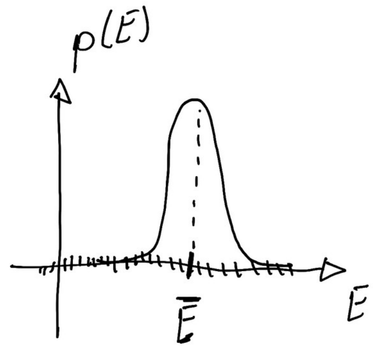


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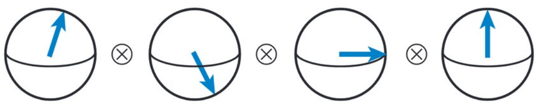

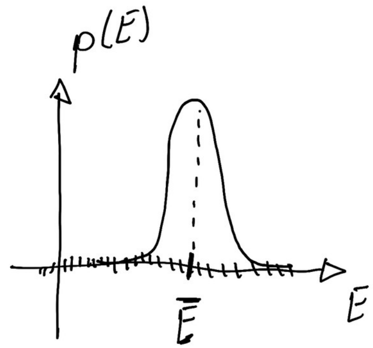


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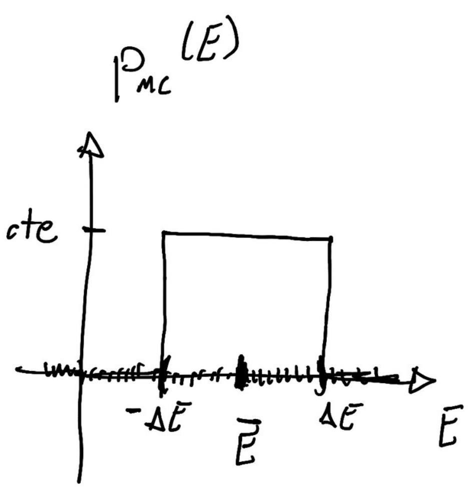


$\hat{\rho}_{mc}(\bar{E}) =$ ***equal probability*** of eigenstates within "$\bar{E} \pm \Delta E$ "

$$= \frac{1}{\mathcal{N}} \sum_{E_\alpha = \bar{E} - \Delta E}^{\bar{E} + \Delta E} |\alpha\rangle\langle\alpha|$$

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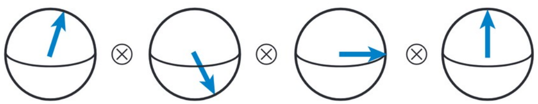
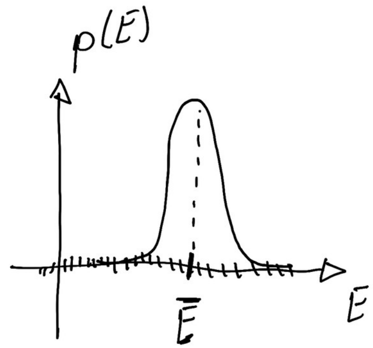





$$\langle \hat{O} \rangle_\infty = \sum_{E_\alpha \approx \bar{E}} p_\alpha \langle \alpha | \hat{O} | \alpha \rangle$$

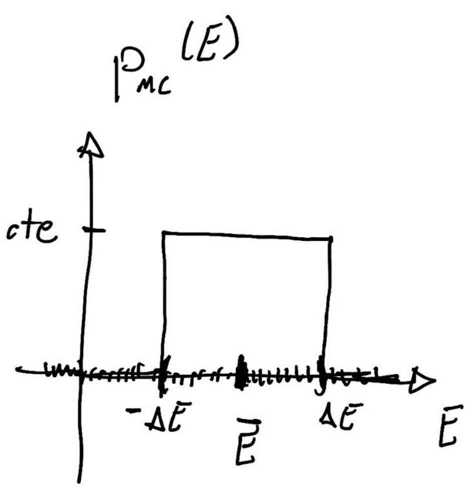
"physical states" $\qquad \approx O_{\mathrm{mc}}(\bar{E})$

- to ensure that $\langle \hat{O} \rangle_\infty$ is thermal for ***generic initial states*** is to assume that,

$$\langle \alpha | \hat{O} | \alpha \rangle = O_{\mathrm{mc}}(\bar{E}), \qquad \forall \alpha$$

***ETH (Eigenstate Thermalization Hyphothesis)***

i.e., ***all individual eigenstates*** have thermal observables, identical to a mc ensemble at $\bar{E}$



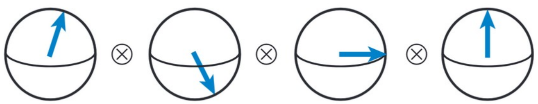

$\hat{\rho}_{mc}(\bar{E}) =$ ***equal probability*** of eigenstates within "$\bar{E} \pm \Delta E$ "

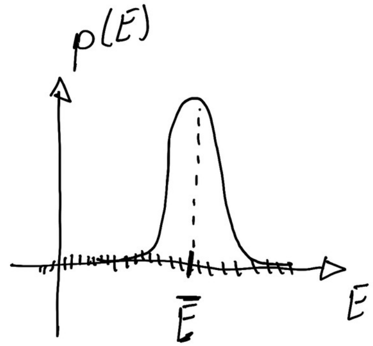
$$= \frac{1}{\mathcal{N}} \sum_{E_\alpha = \bar{E} - \Delta E}^{\bar{E} + \Delta E} |\alpha\rangle\langle\alpha|$$

****physical initial states*** have support in narrow energy window:

e.g. product states

low-entangled/fast prep.



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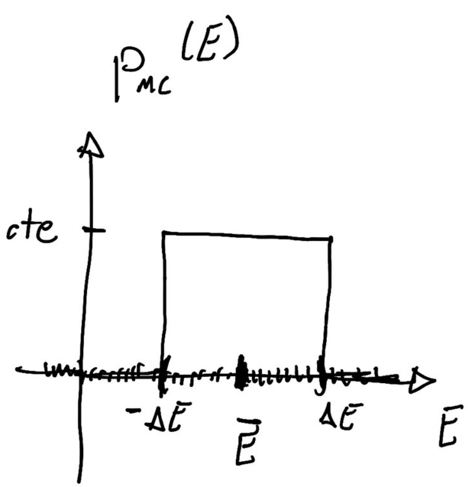


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      lattice systems: spins, bosons, fermions...

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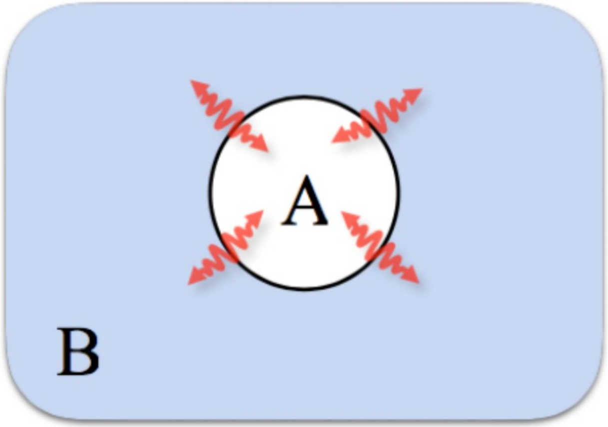

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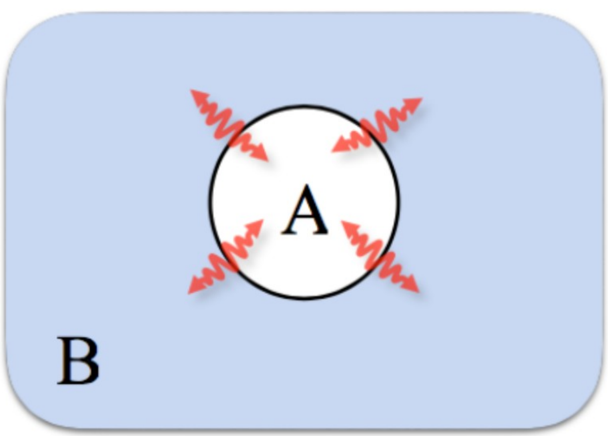



i.e,

- ***volume law-entanglement*** (S ~ vol(A)) between A/B,

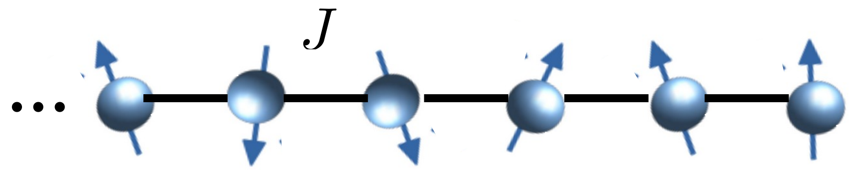
$$S_A = -Tr(\hat{\rho}_A \log(\hat{\rho}_A))$$

- correlations must spread "***fast***" in thermalizing systems

# Thermalization in quantum systems

- Ising model:

$$\hat{H} = \sum_{i=1}^{L-1} J \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z + \sum_{i=1}^{L} g \hat{\sigma}_i^x + h \hat{\sigma}_i^z$$



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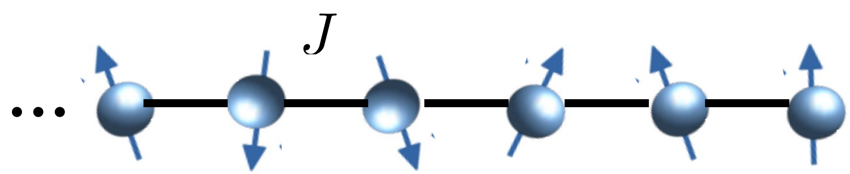
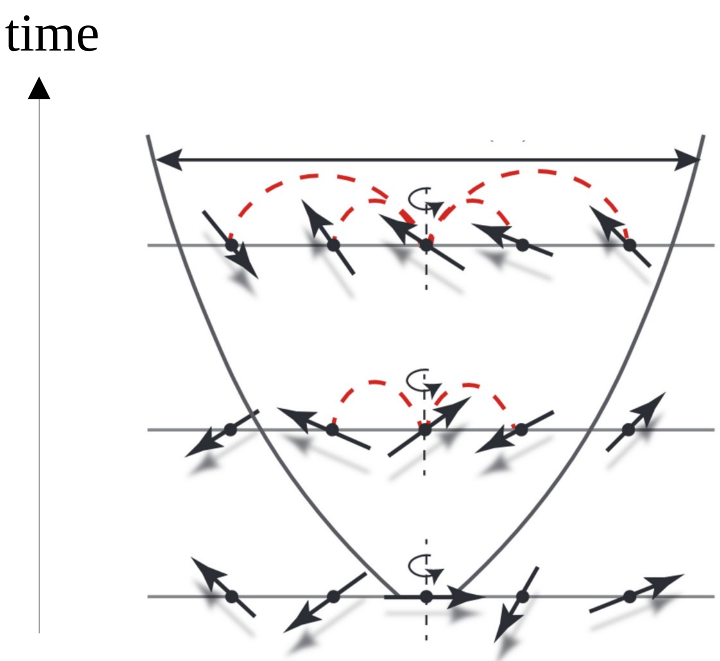



$$|\psi(0)\rangle = |\uparrow\downarrow \dots \downarrow\rangle$$

“quench”
(non eigenstate)



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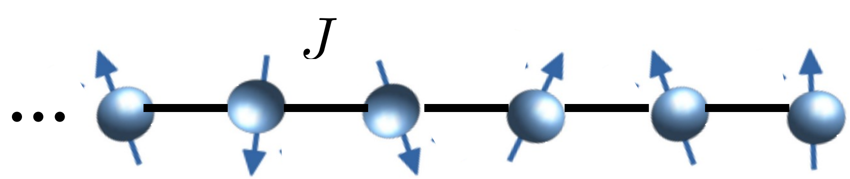


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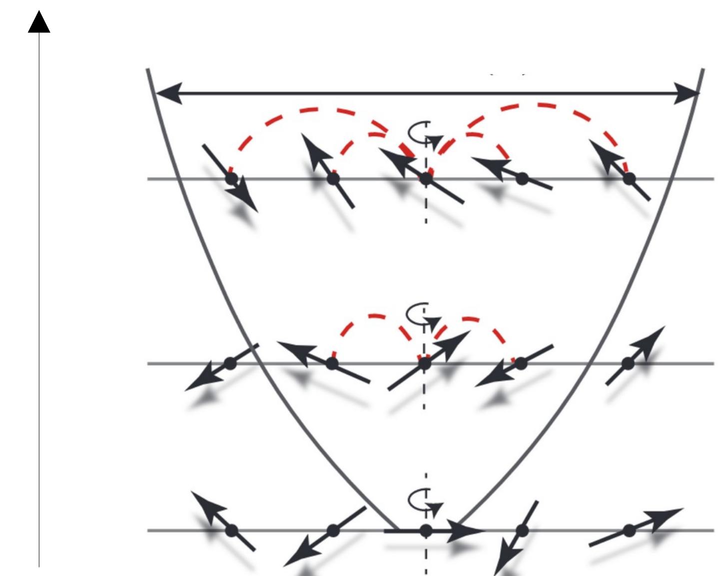


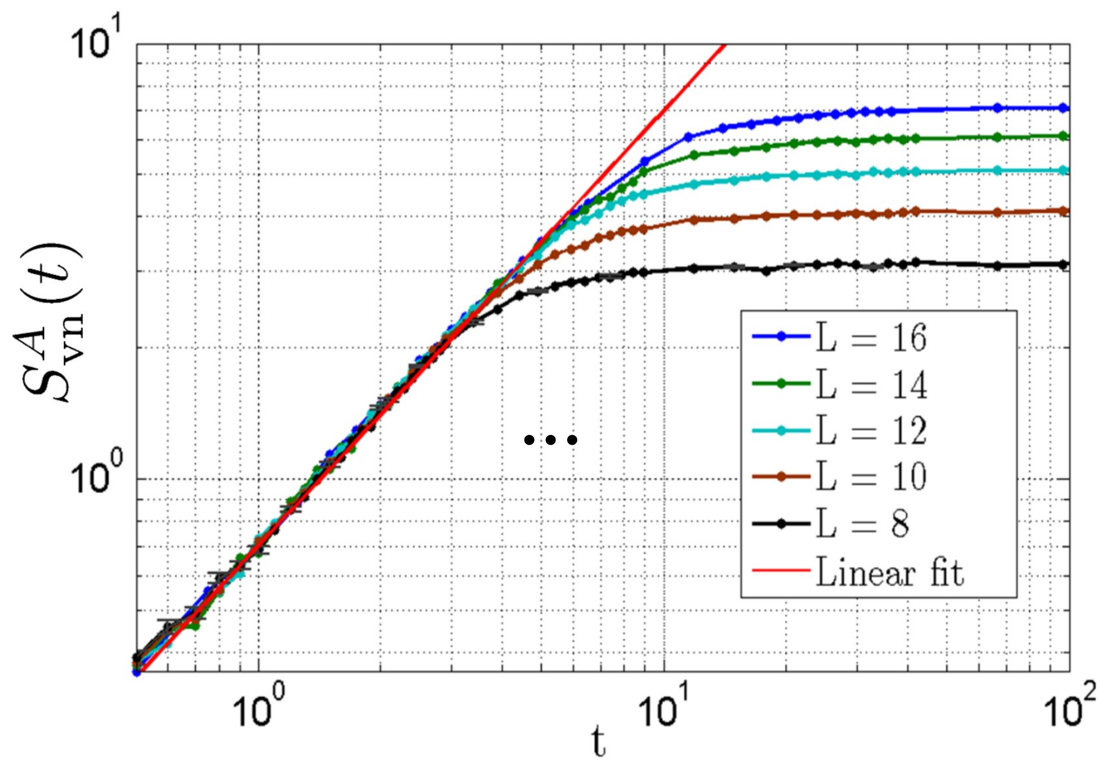


$$S^A_{\mathrm{vn}}(t) \xrightarrow{t\to\infty} L\ln 2$$

PRL 111, 127205 (2013)

# Outline

- Introduction
- Thermalization in quantum systems
- (Many-body) Localisation
- Conclusions

# Localisation

- What are the possible routes of escaping thermalization?

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- What are the possible routes of escaping thermalization?

. a possibility is looking for system with absence of transport (energy, spin excitations, …)

. disordered systems / Anderson localisation are natural starting points

# Localisation

In fact, Anderson himself pointed out this possibility in 1957, stating that a localized system provides:

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

## Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
*Bell Telephone Laboratories, Murray Hill, New Jersey*
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

### I. INTRODUCTION

A NUMBER of physical phenomena seem to involve quantum-mechanical motion, without any particular thermal activation, among sites at which the mobile entities (spins or electrons, for example) may be localized. The clearest case is that of spin diffusion[1,2]; another might be the so-called impurity band conduction at low concentrations of impurities. In such situations we suspect that transport occurs not by motion of free carriers (or spin waves), scattered as they move through a medium, but in some sense by quantum-mechanical jumps of the mobile entities from site to site. A second common feature of these phenomena is randomness: random spacings of impurities, random interactions with the "atmosphere" of other impurities, random arrangements of electronic or nuclear spins, etc.

Our eventual purpose in this work will be to lay the foundation for a quantum-mechanical theory of trans- reasonably well, and to prove a theorem about the model. The theorem is that at sufficiently low densities, transport does not take place; the exact wave functions are localized in a small region of space. We also obtain a fairly good estimate of the critical density at which the theorem fails. An additional criterion is that the forces be of sufficiently short range—actually, falling off as $r \to \infty$ faster than $1/r^3$—and we derive a rough estimate of the rate of transport in the $V \propto 1/r^3$ case.

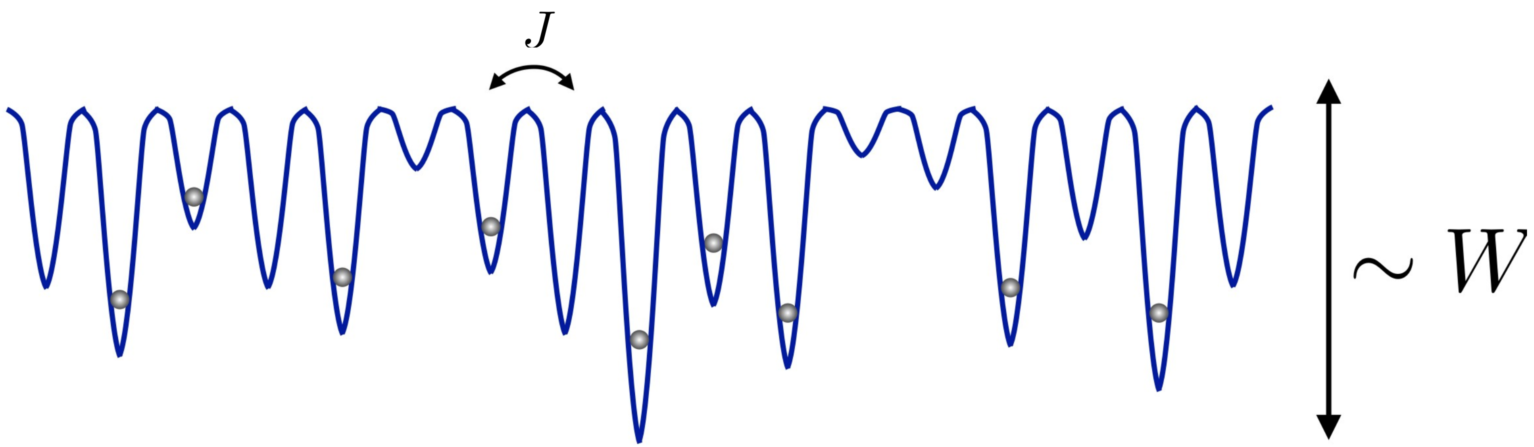
Such a theorem is of interest for a number of reasons: first, because it may apply directly to spin diffusion among donor electrons in Si, a situation in which Feher[3] has shown experimentally that spin diffusion is negligible; second, and probably more important, as an example of a real physical system with an infinite number of degrees of freedom, having no obvious oversimplification, in which the approach to equilibrium is simply impossible; and third, as the irreducible minimum from which a theory of this kind of transport,

- A simple model with AL: 1d tight-binding fermions in a disordered potential

$$\hat{H} = J \sum_{j=1}^{L-1} \hat{c}_j^\dagger \hat{c}_{j+1} + W \sum_{j=1}^{L} \underline{h_j} \hat{c}_j^\dagger \hat{c}_j$$

on-site disordered potential (random/ quasiperiodic/...)

$\{\hat{c}_i, \hat{c}_i^\dagger\}$ = fermions



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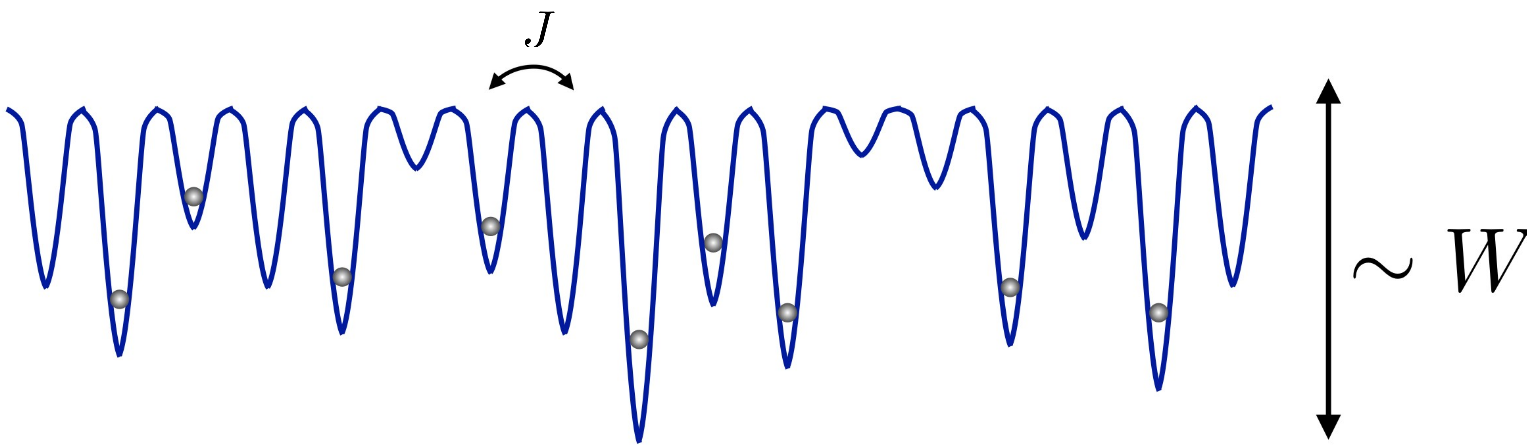


- “***classical***” perspective:

. if kinetic energy is larger than potential barrier, J>W: particle escaper and spreads balistically

. otherwise, J<W: particle is trapped, and therefore localised in the potential.

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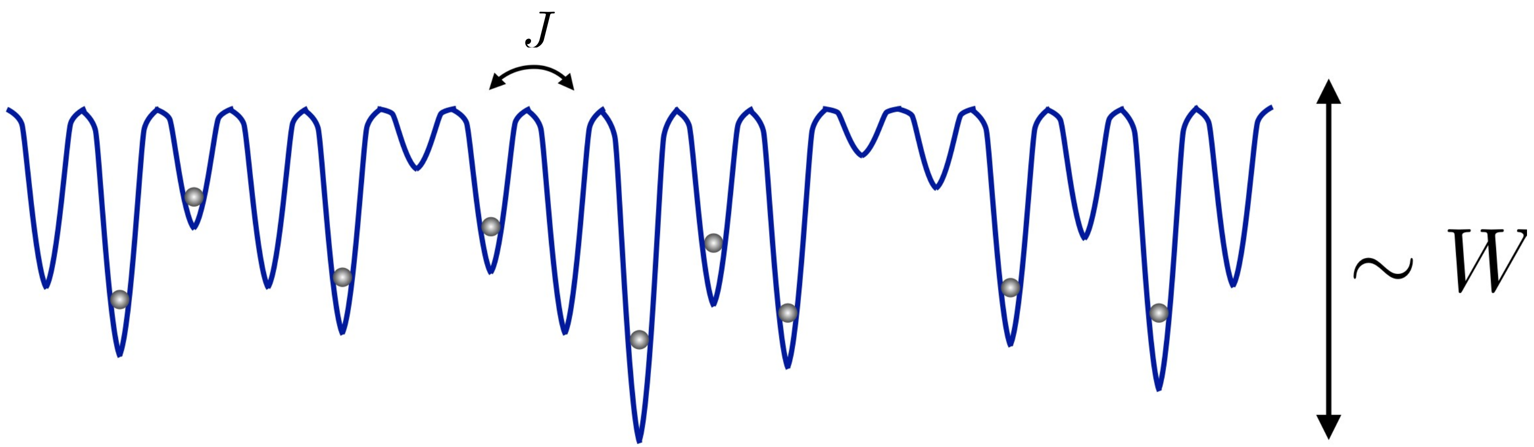


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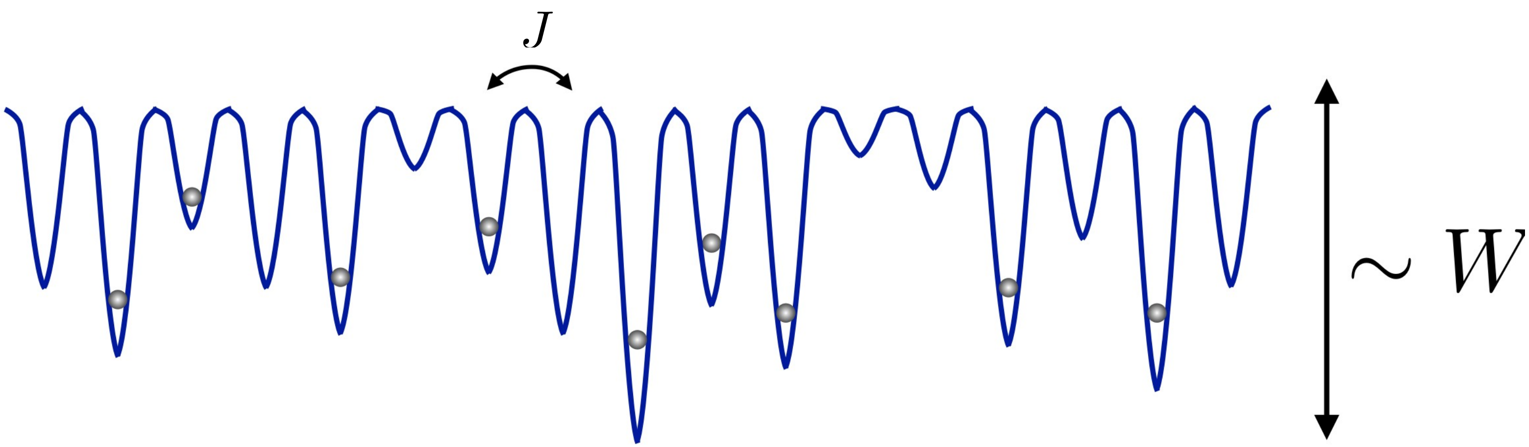


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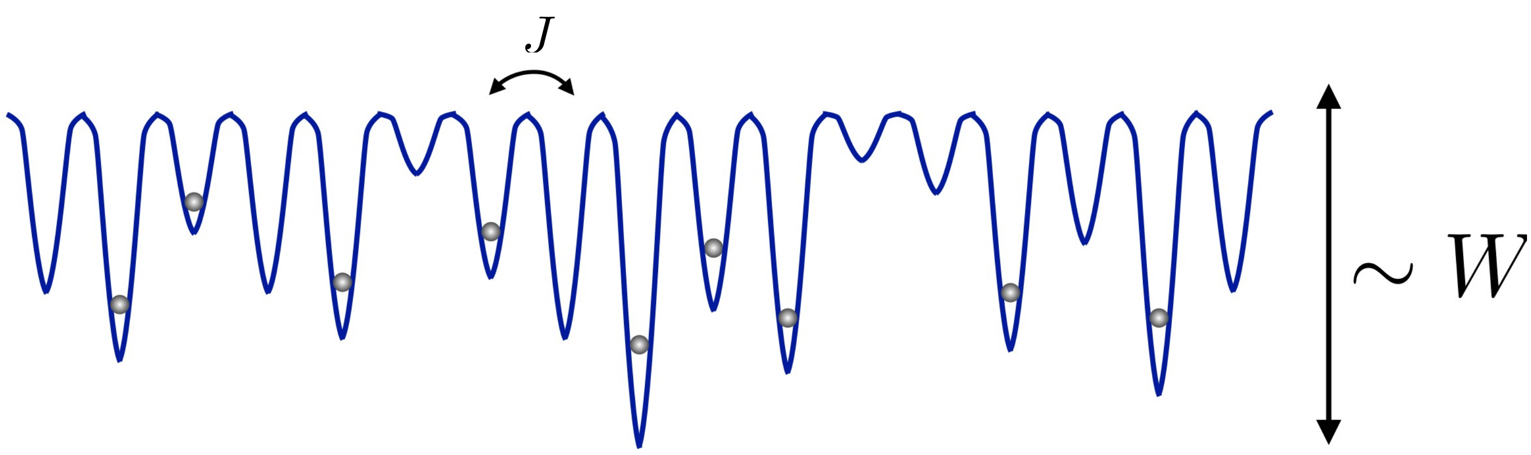


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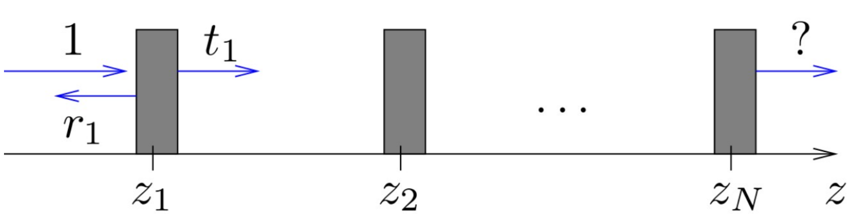

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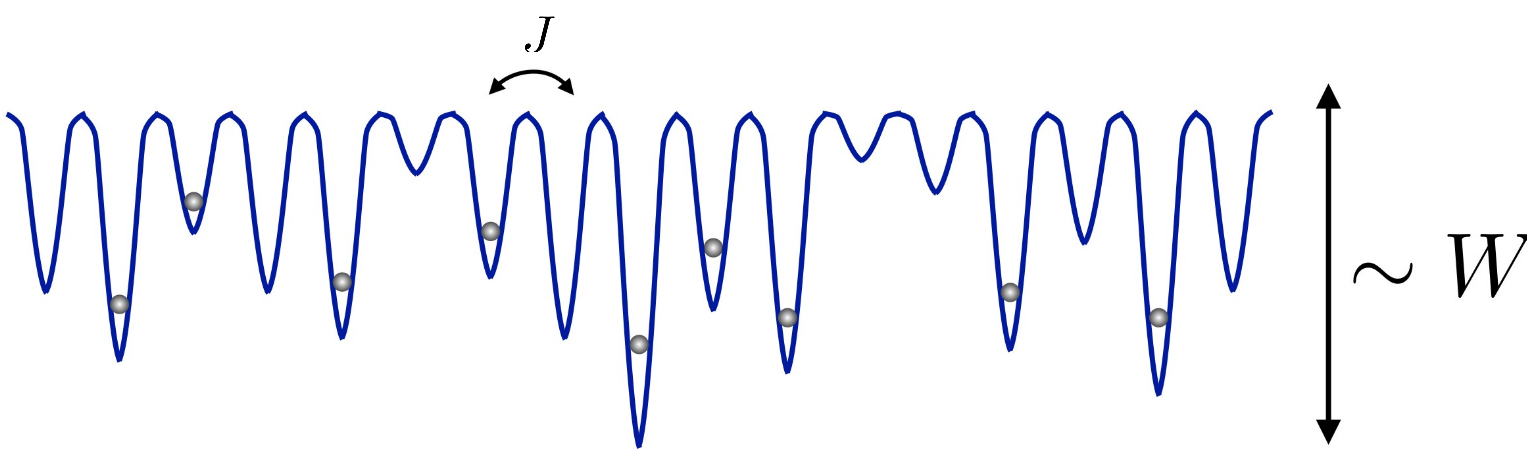


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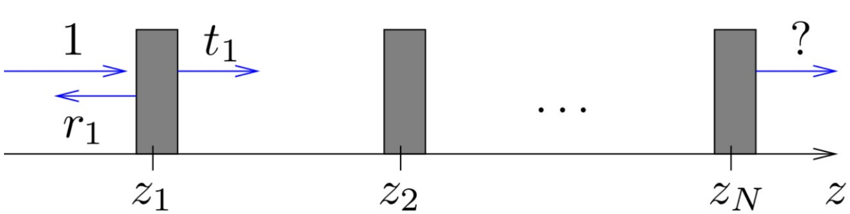


. since potential is disordered, waves ***interfere off-resonantly (descrtuctively)***
and wavepacket does not spread along the chain (localised)

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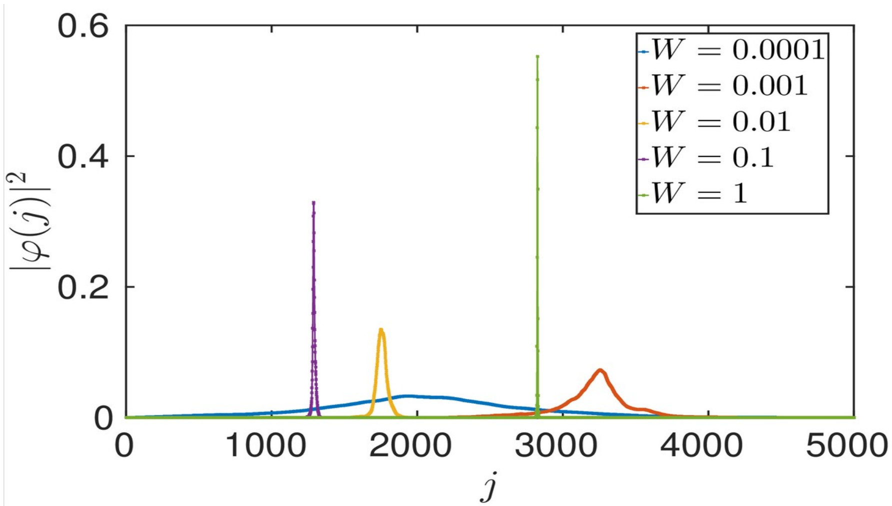
$.W \neq 0 : \varphi_E(j) \sim e^{-\frac{|j-c|}{\eta}},$ (exponentially localised wavefunctions)

$L = 5000$




- stronger disorder, higher localisation

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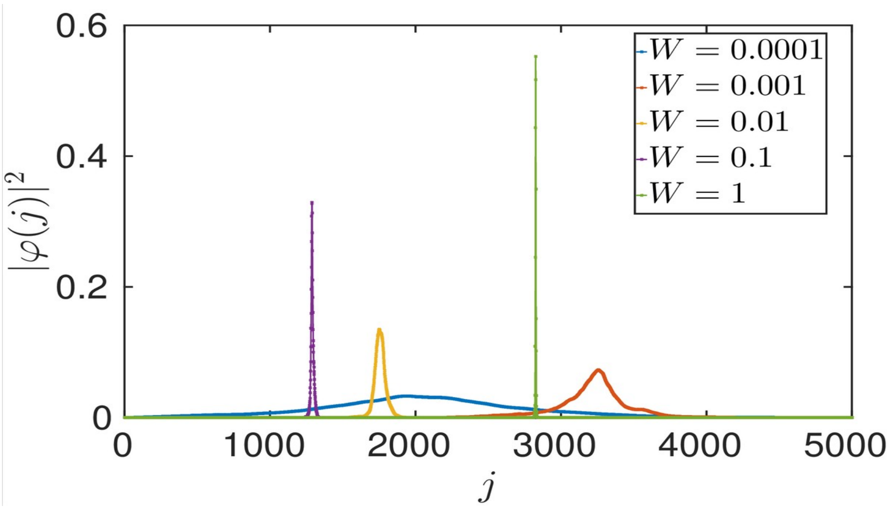


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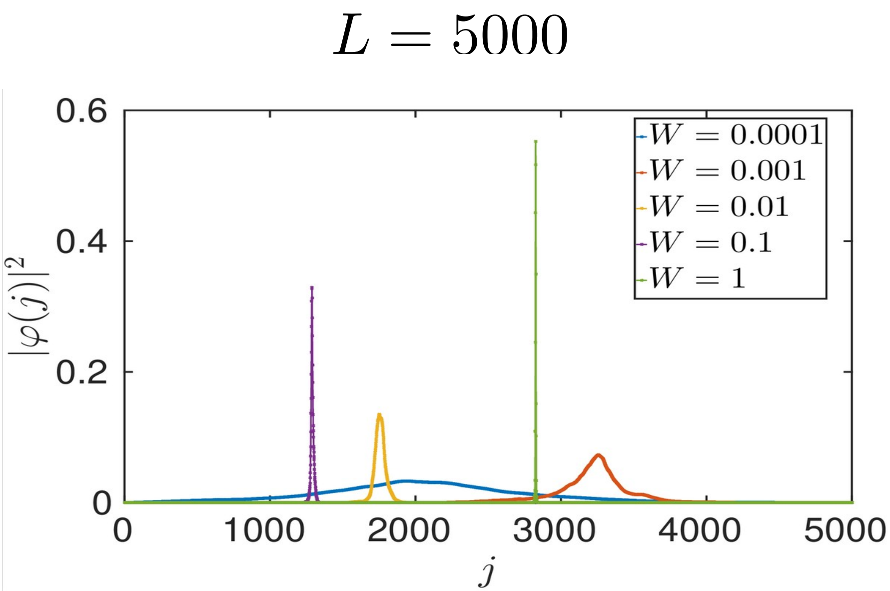
- dynamics is *frozen*…

. full spectrum is localised

”initial localised wavepackets overlap with close localised eigenstates”

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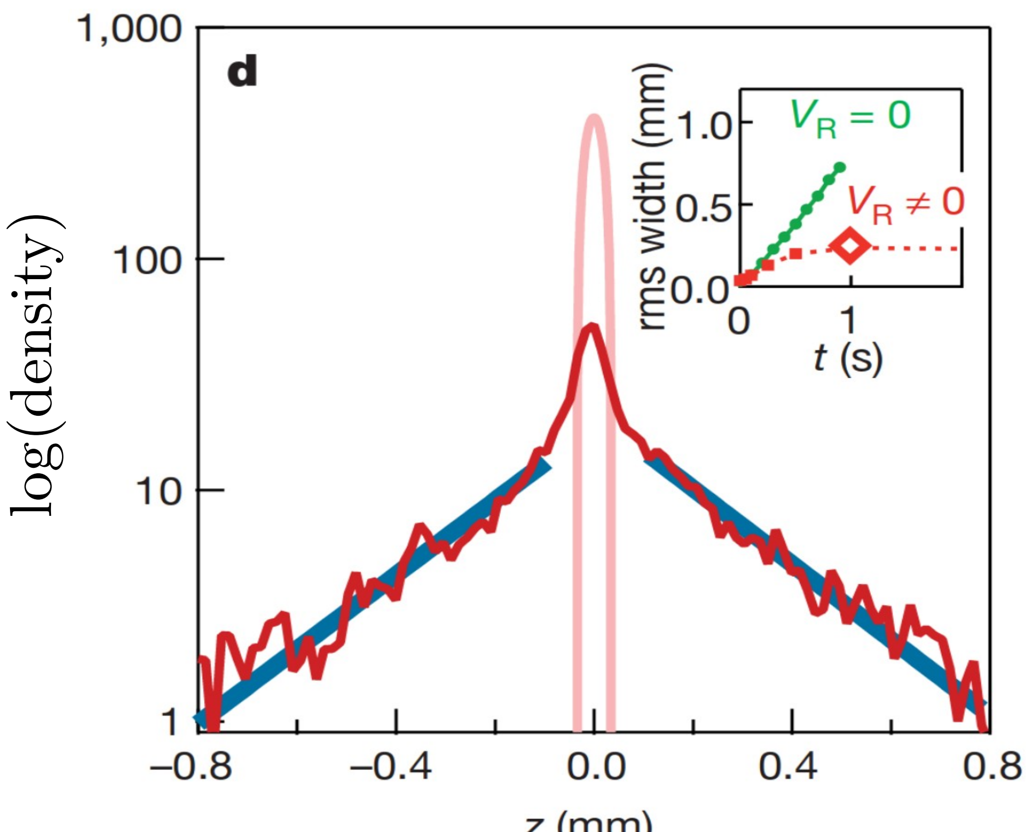
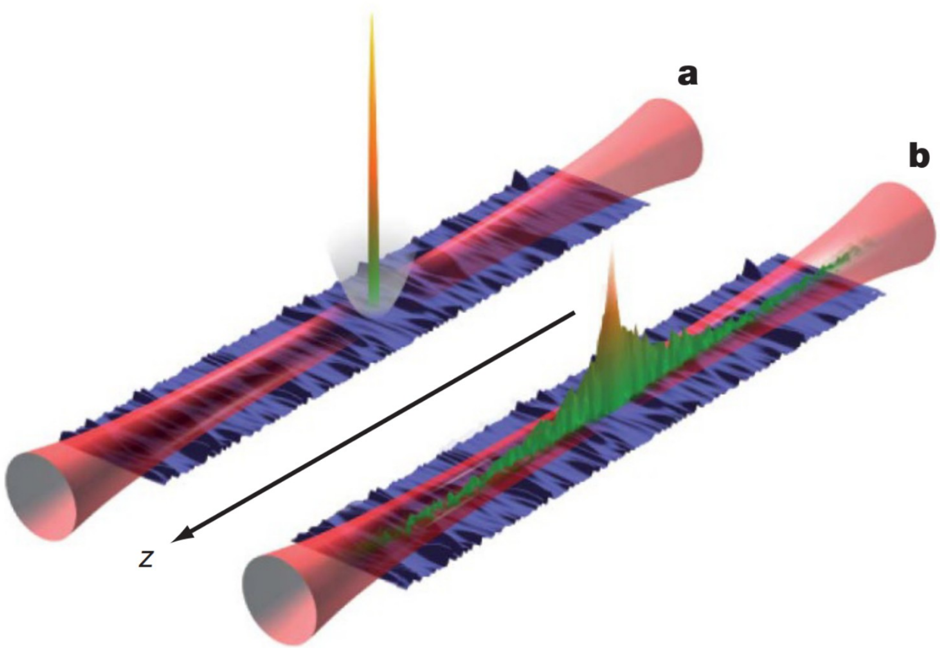
- low-density BEC (1,7x10$^4$ atoms) in a disordered optical trap;




Nature 453, 891–894 (2008)

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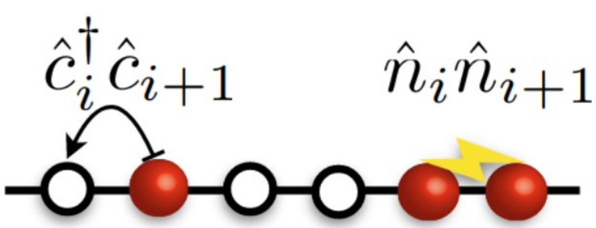

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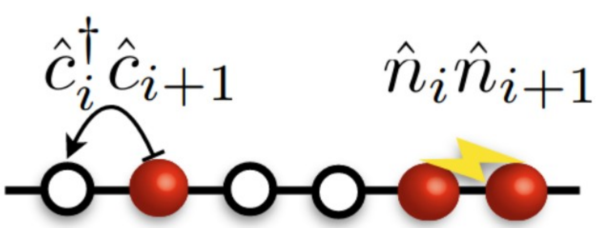


Jordan-Wigner

$$\hat{c}_j^\dagger = \left(\prod_{k<j} \sigma_k^z\right)\hat{\sigma}_j^+$$

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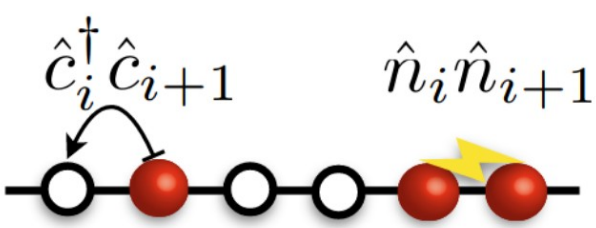
Jordan-Wigner




$$\hat{c}_j^\dagger = (\prod_{k<j} \sigma_k^z)\hat{\sigma}_j^+$$

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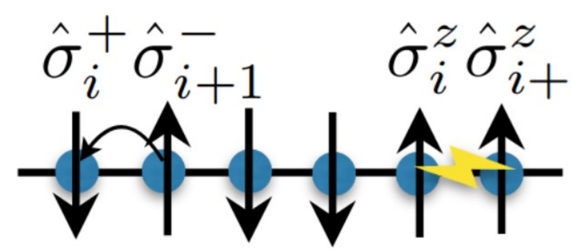

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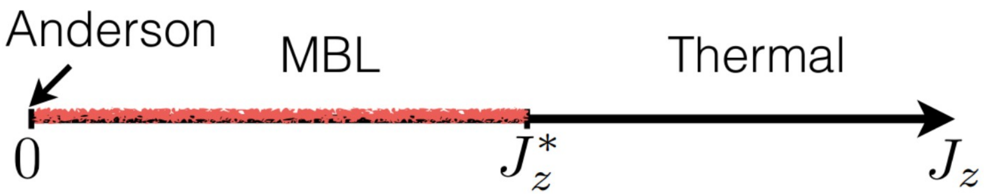


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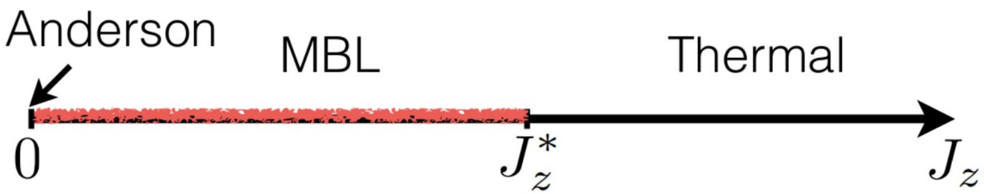


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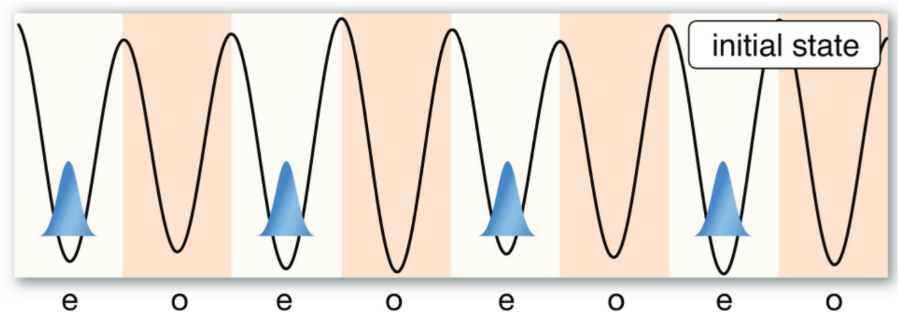

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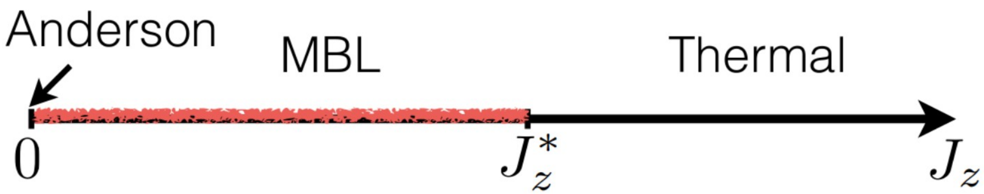


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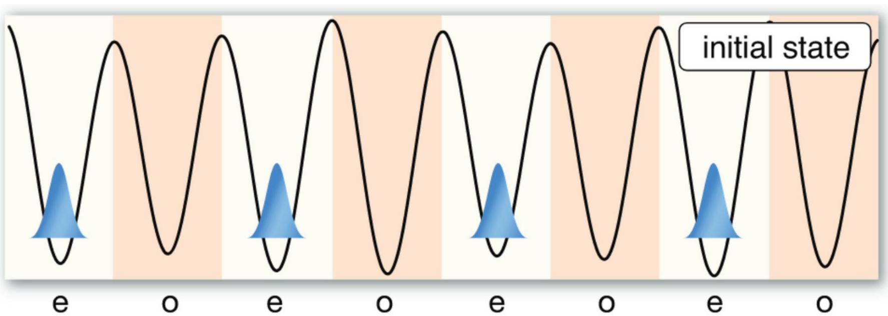


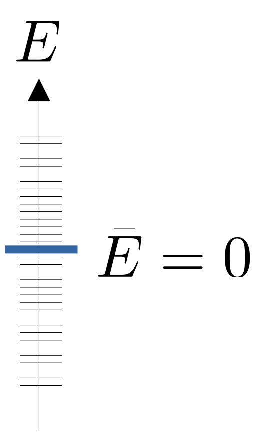


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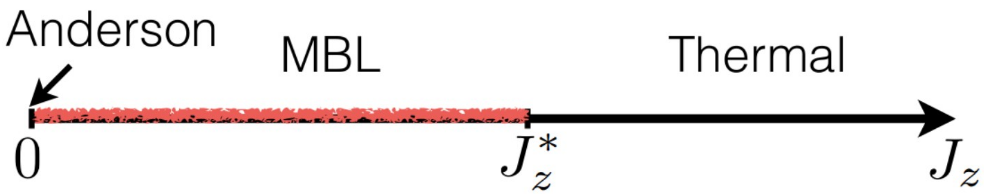


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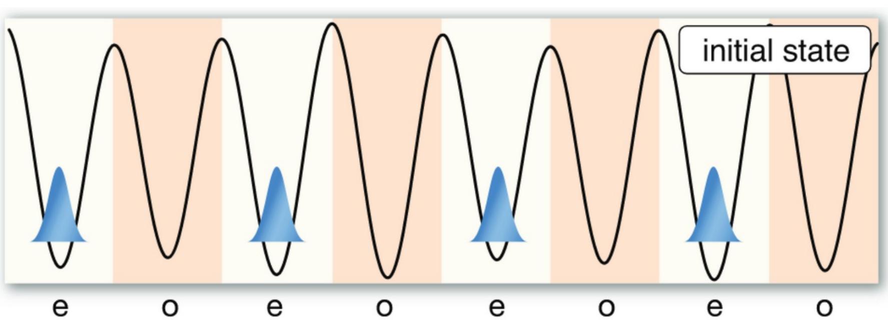


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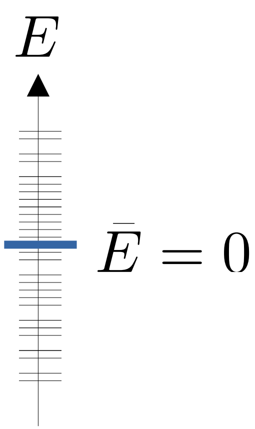


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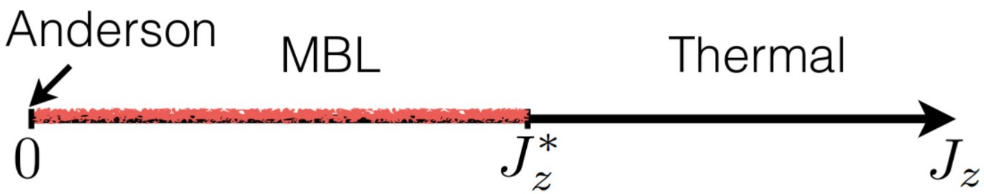


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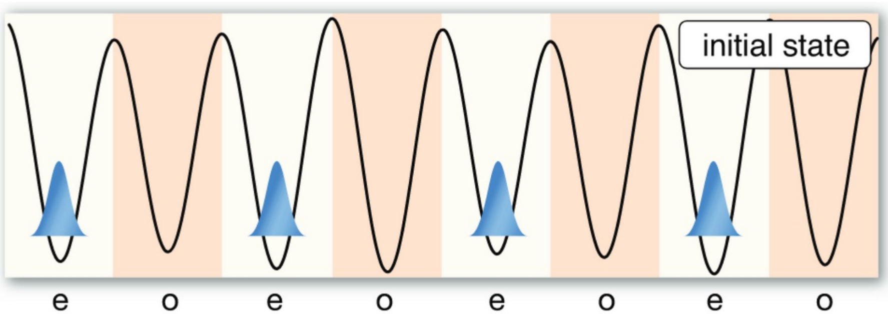


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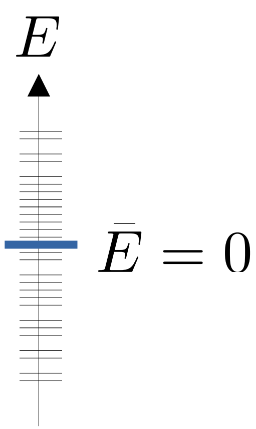


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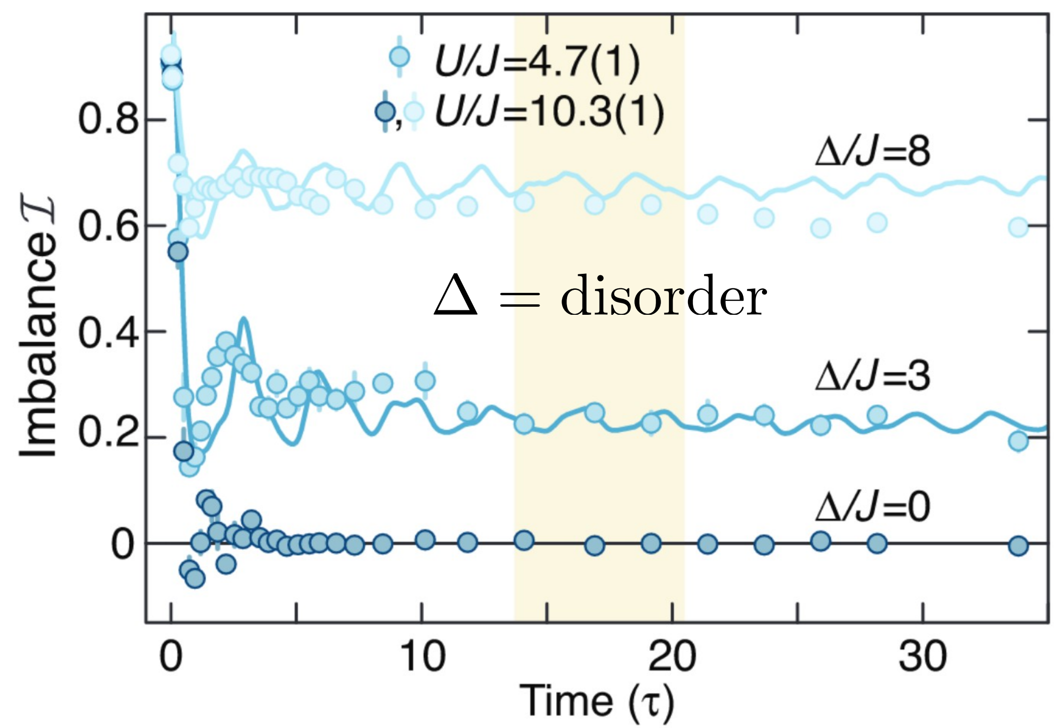

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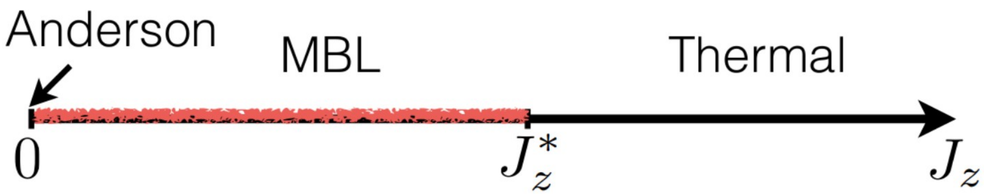


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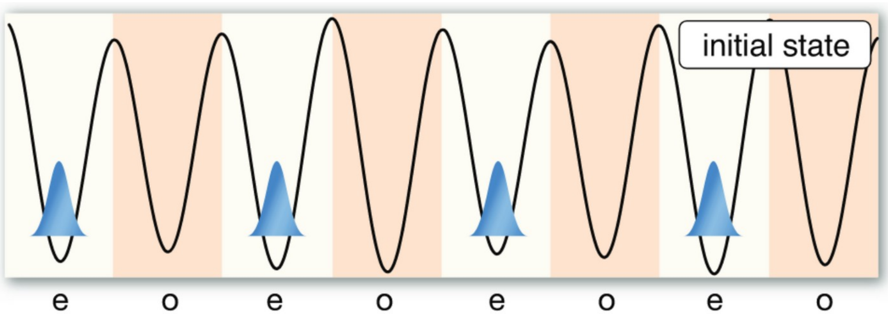


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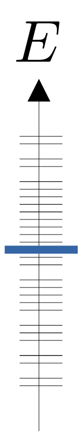


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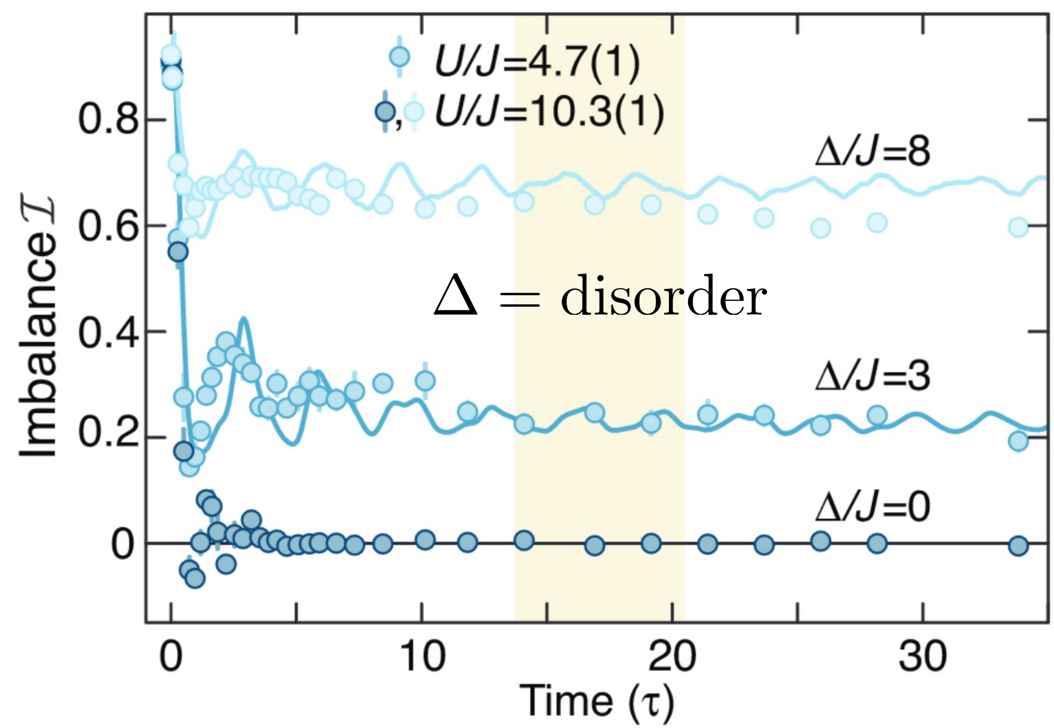
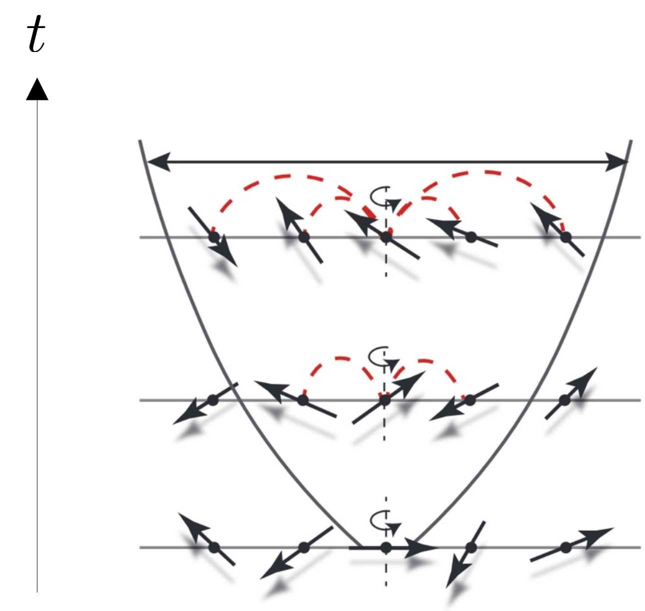



. *frozen* spin transport
(***local memory of initial state***)

# MBL (correlation spreading)

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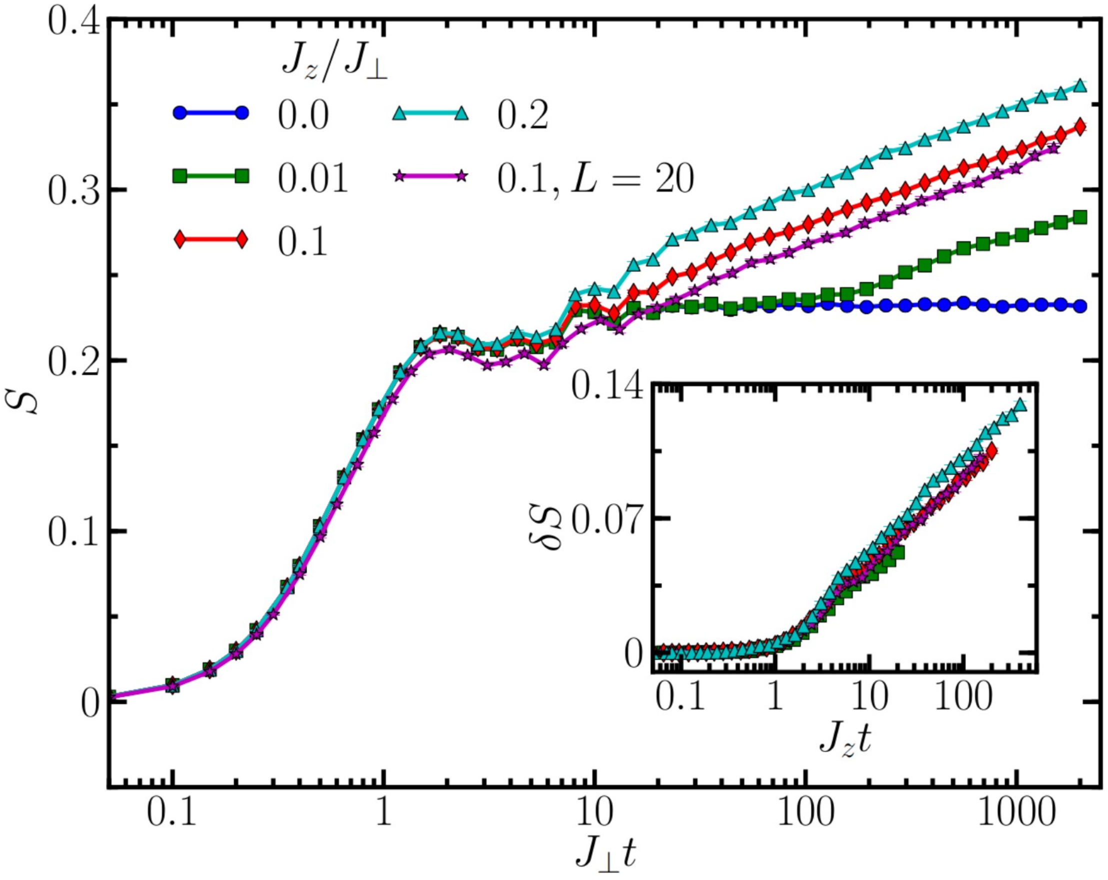


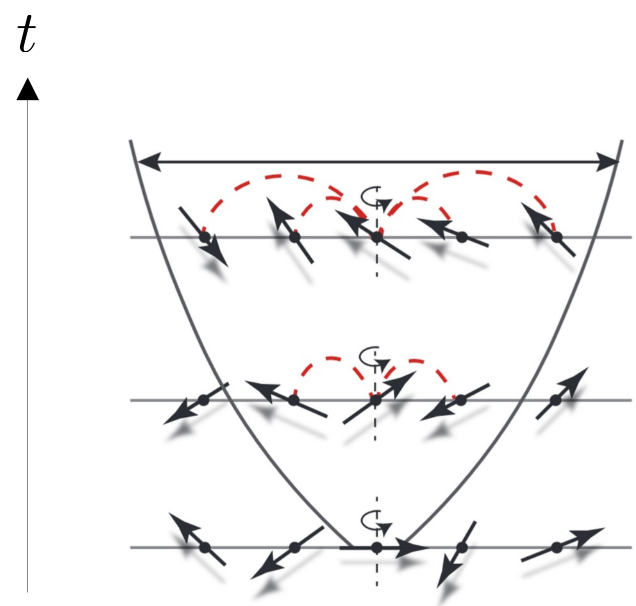


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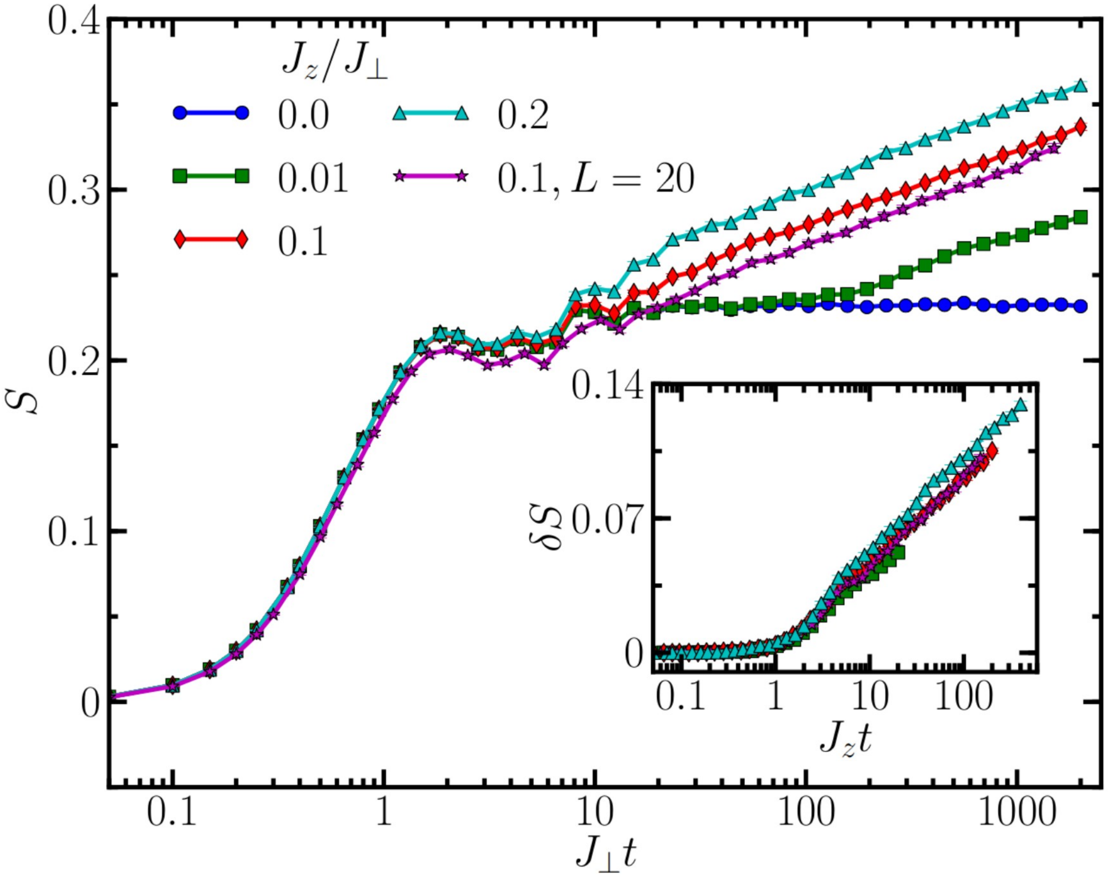


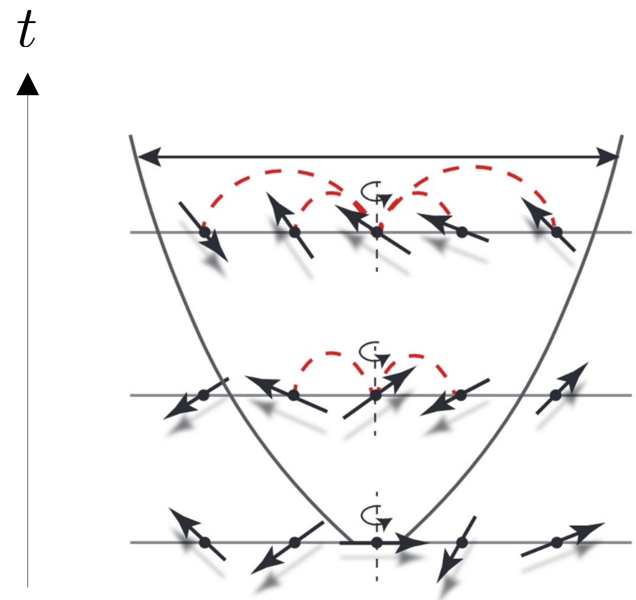


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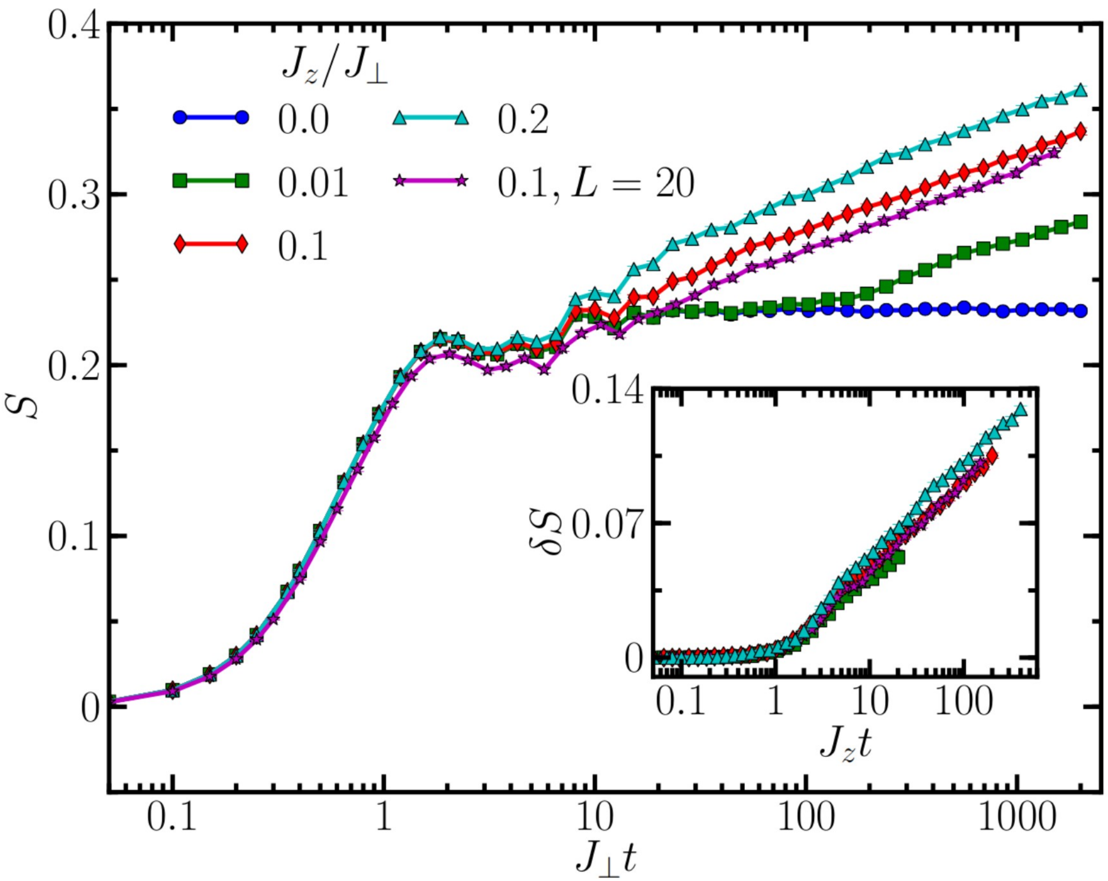


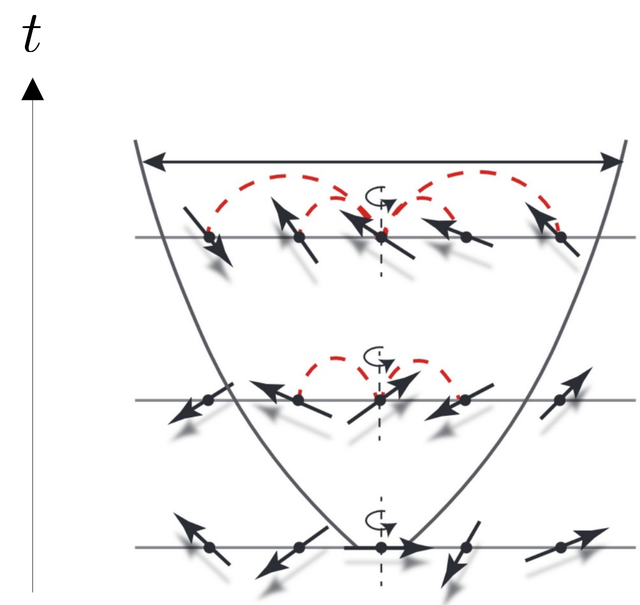


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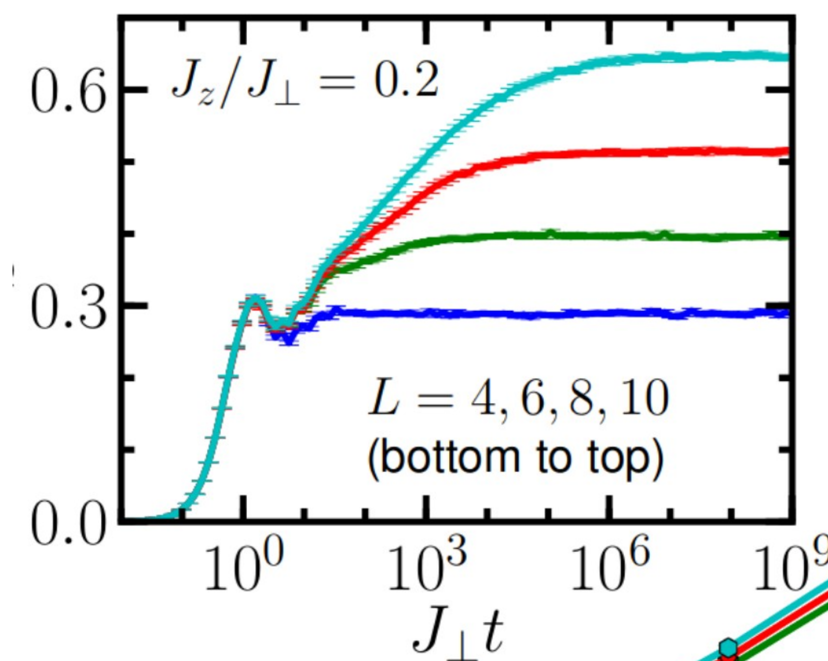

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- ***alternative approach***

  . local correlation spreading ↔ global behavior:

  . ***Monogamy of Entanglement*** (PRB 94, 214206 (2016))

  . experimentally feasible (local observables)

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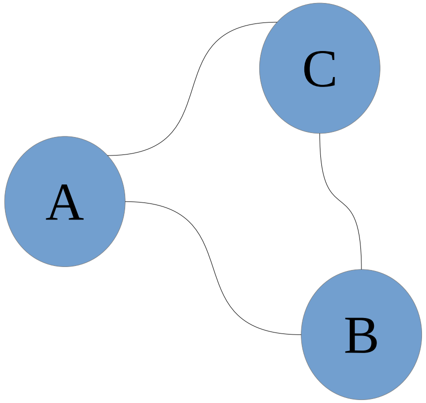


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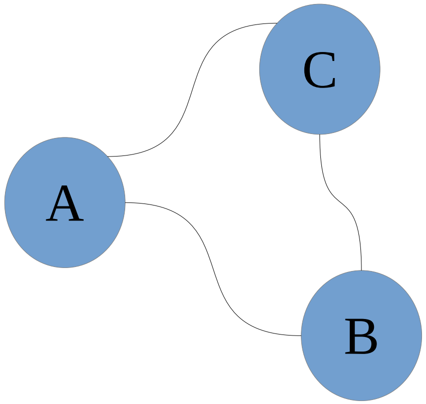


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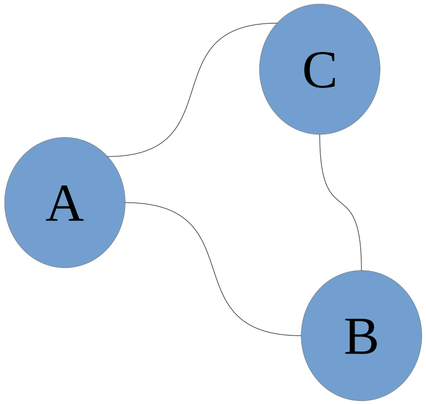


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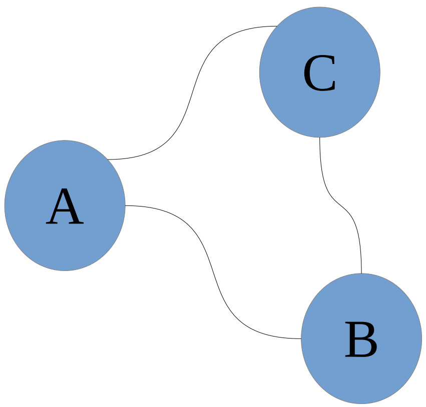


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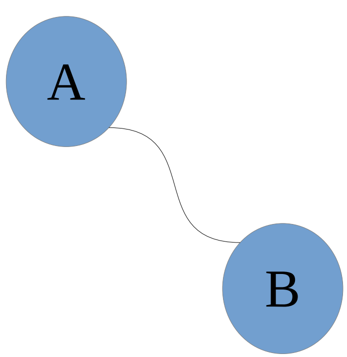

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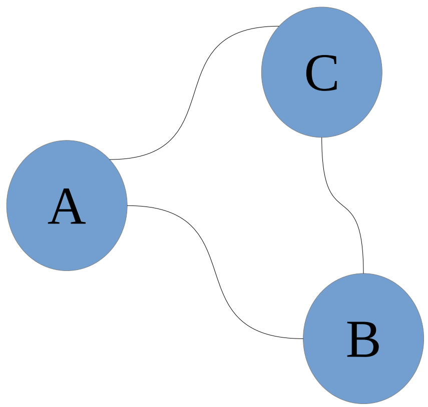


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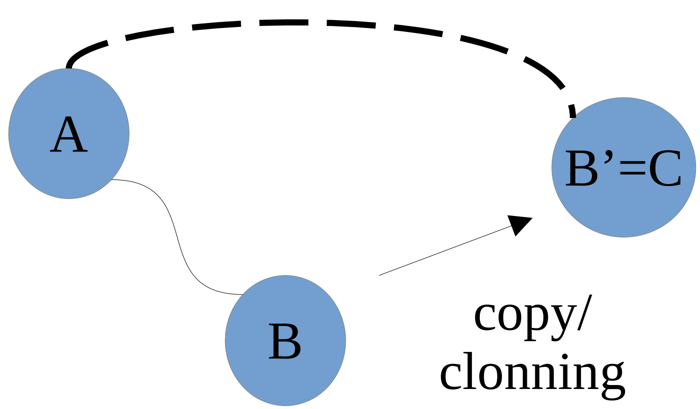


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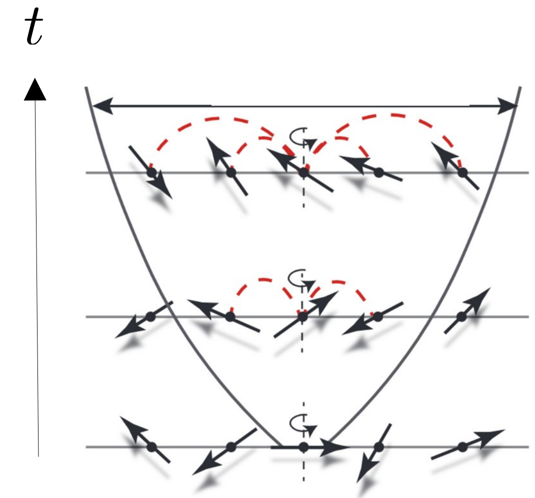

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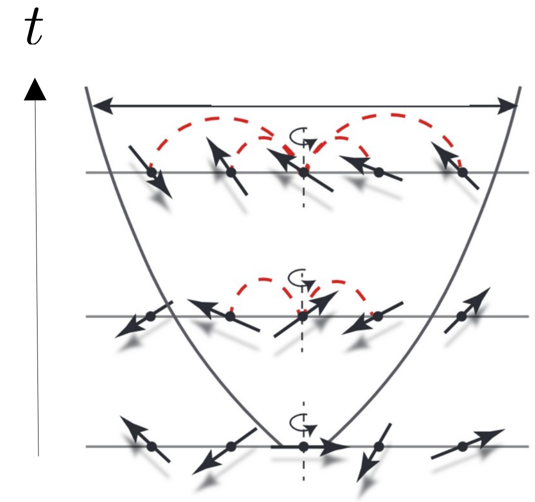

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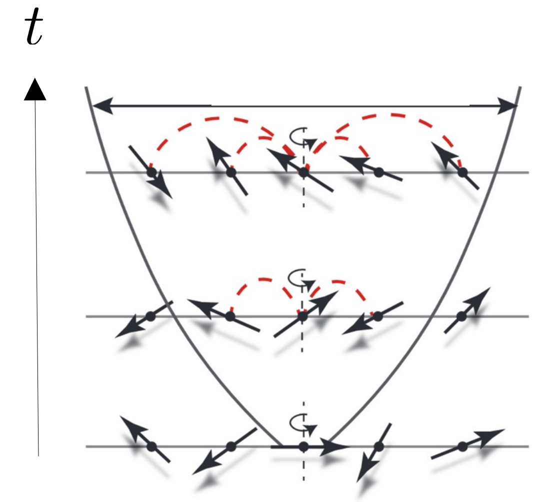

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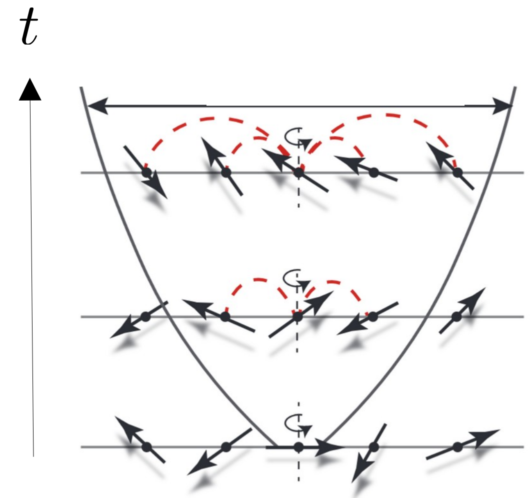


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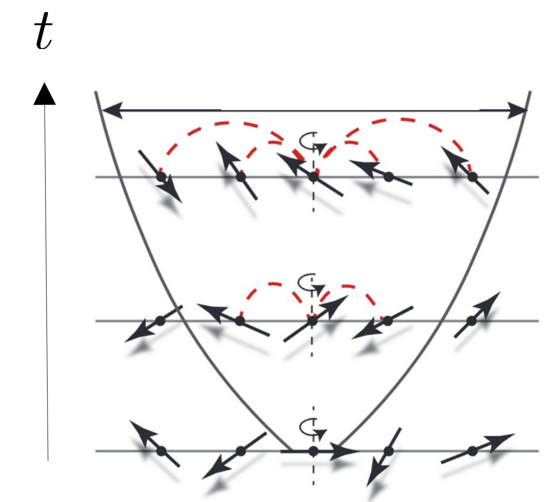


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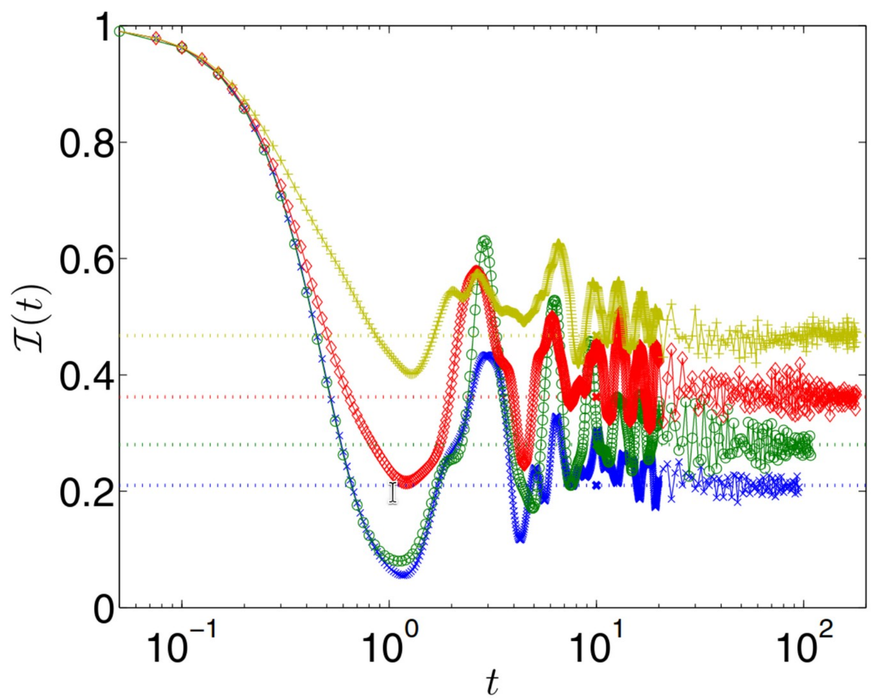


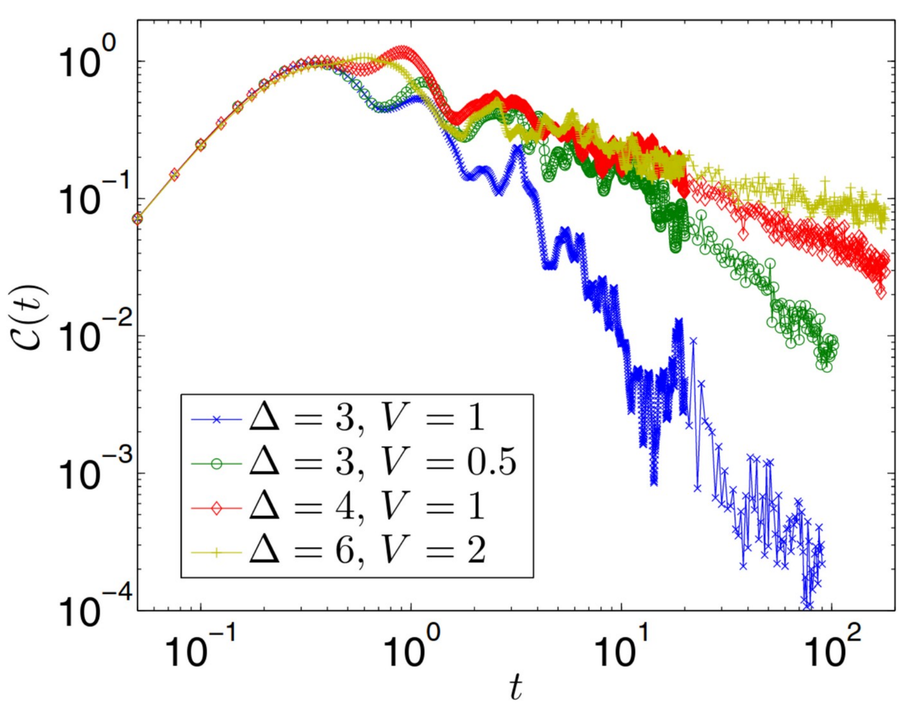

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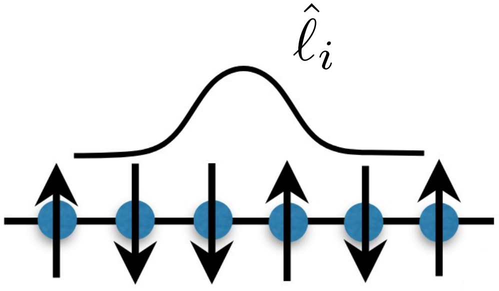

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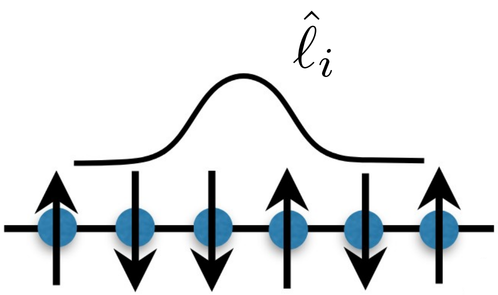

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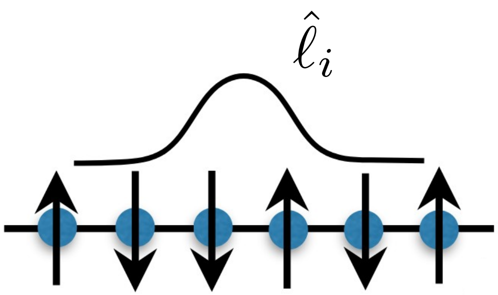


. $J_\perp, J_z \neq 0$ : ***MBL*** – exponentially localised interacting eigenstates,

$$\hat{\ell}_i = \sum_{j=1}^L \sum_{\alpha=x,y,z} c_j^\alpha \hat{\sigma}_j^\alpha + \sum_{j,k=1}^L \sum_{\alpha=x,y,z} \underline{c_{j,k}^{[\alpha,\beta]} \hat{\sigma}_j^\alpha \hat{\sigma}_k^\beta} + ...$$

"localised around i'th site"

. Therefore, a phenomenological ***l-bit model*** is given by,

$$J_{ij} \sim e^{-\frac{|i-j|}{\eta}}$$

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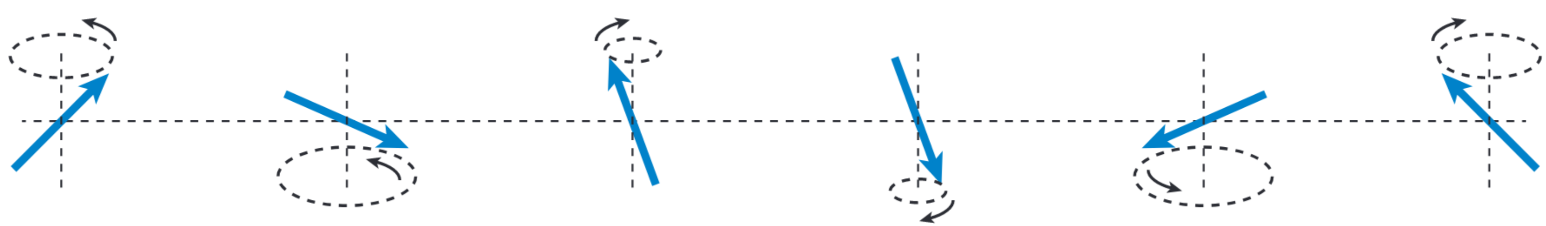

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$$t = \pi/4 \longrightarrow S(\hat{\rho}_1) = 1$$

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. Although significant progress, there still remain many open issues:

. conjectures on thermalization – ETH – necessary?

. other forms of localisation, disorder-free, scars, …?

. different contexts – Floquet?

# Thanks for your attention!

If you are interested…
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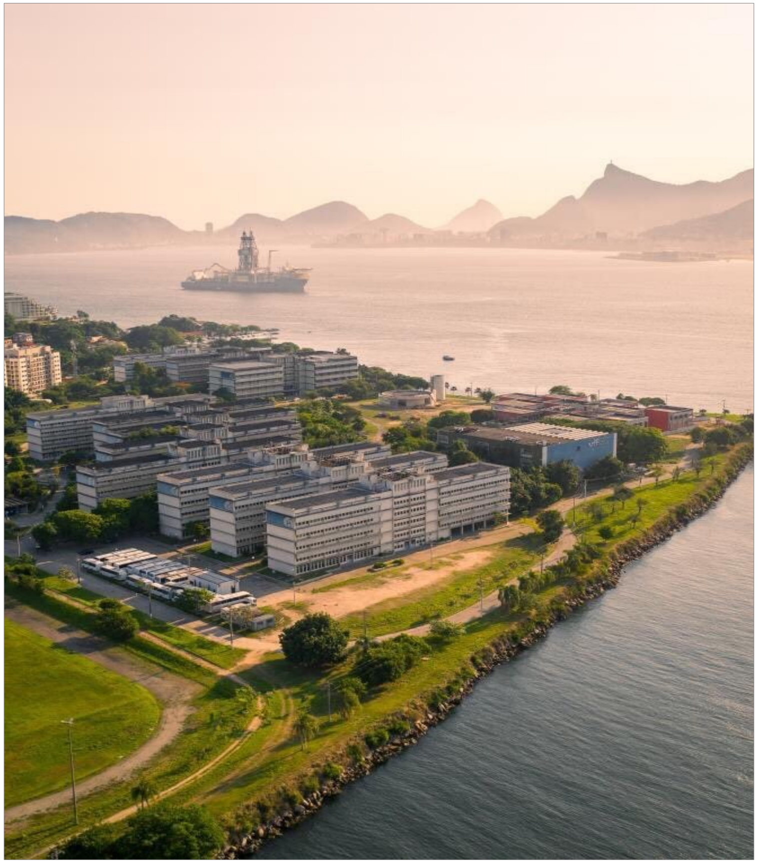
serrapilheira



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