Many-body Open Quantum Systems - Boundary Time Crystals

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Universidade Federal Fluminense

- *What is a Time Crystal?*

Time Crystals

- Landau's symmetry breaking is a cornerstone of modern physics:

. equilibrium ***spontaneuos symmetry breaking (SSB)*** occurs when the ground state or low-temperature states of a system fail to be invariant under symmetries of the Hamiltonian;

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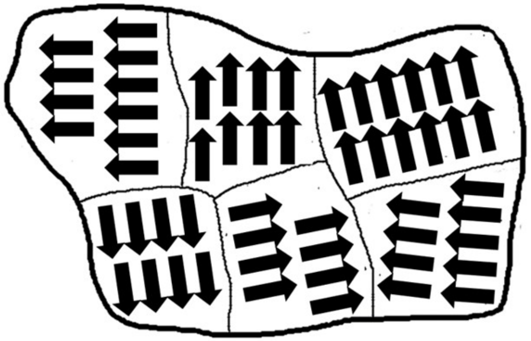

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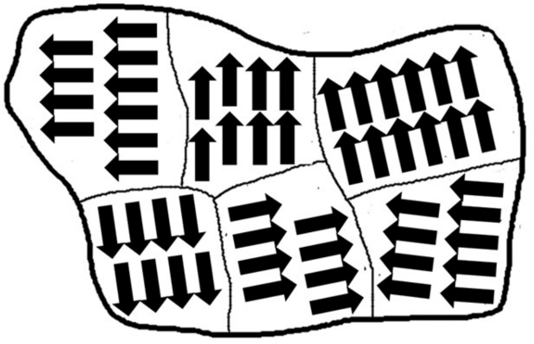

Crystal lattices
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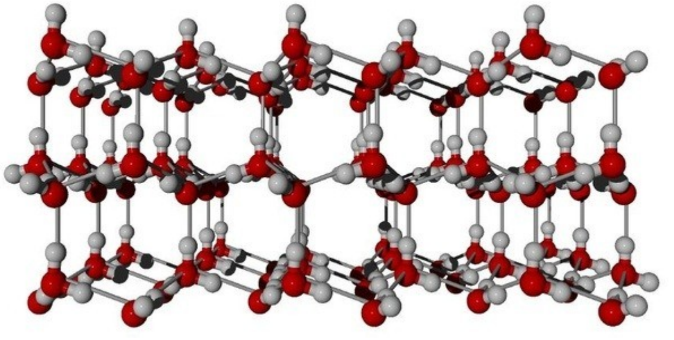

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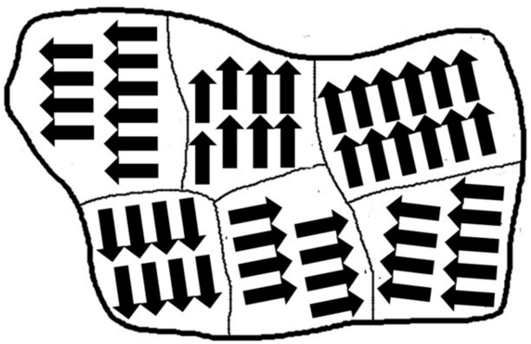

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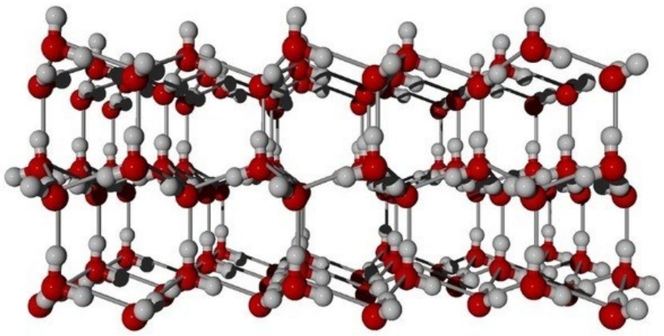

Charge-density-waves
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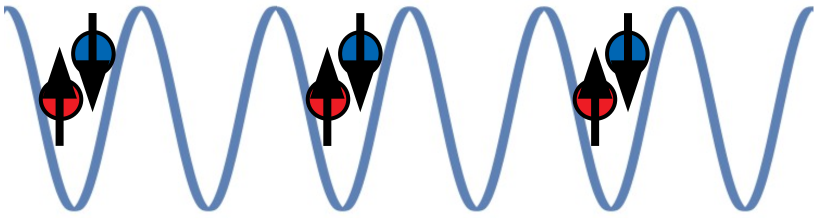

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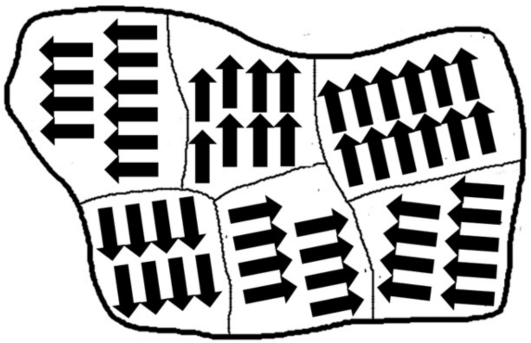

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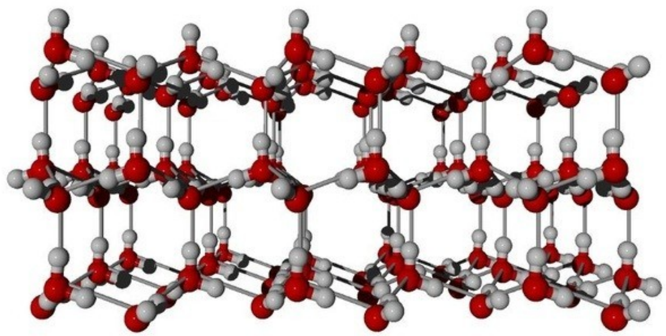

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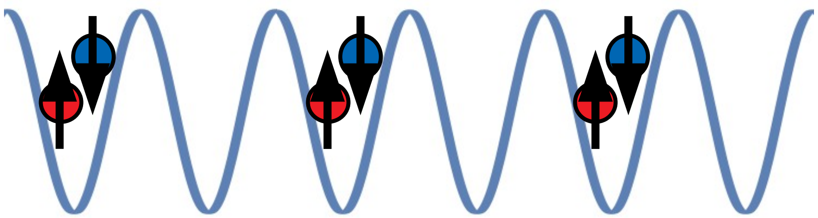

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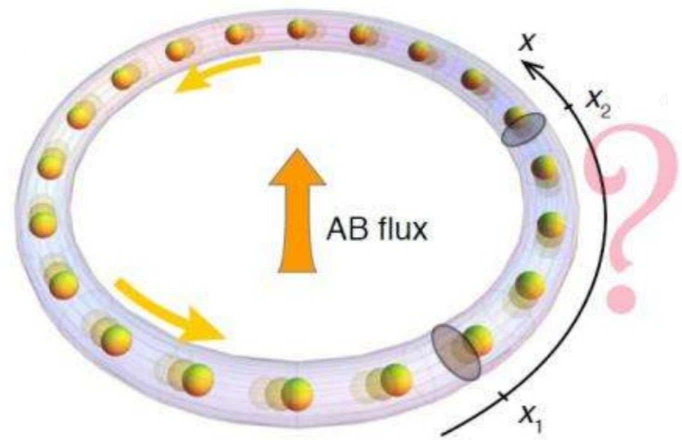
- Can **<u>time-translational invariance</u>** be spontaneously broken?

(F. Wilczek, PRL. 109, 160401 (2012))

ring particle model: **ground states**
with moving "lumps"



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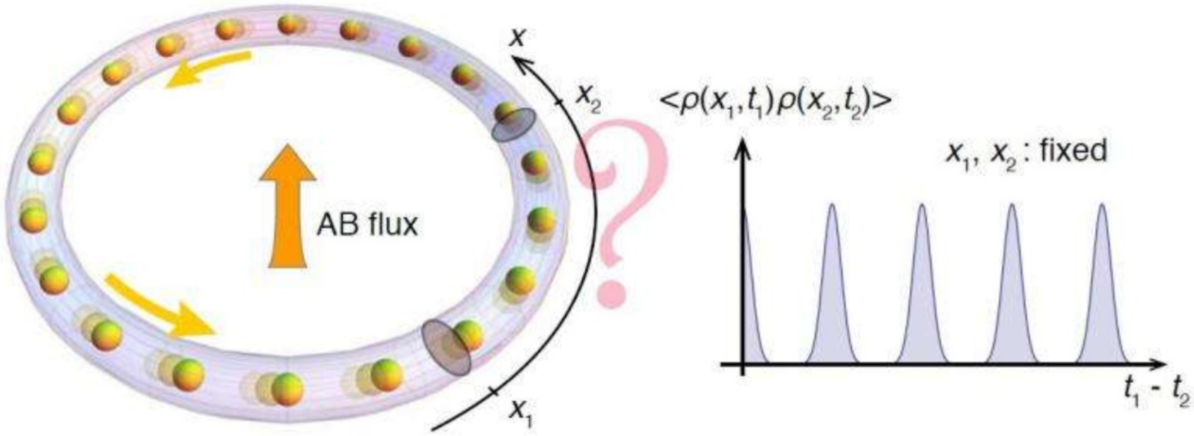

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$$|x-x'|\to\infty$$

$$\rho(x,t) = \text{local order parameter}$$

- intense discussion:
T. Li et al, PRL 109, 163001(2012);
P. Bruno, PRL 110, 118901 (2013);
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- arguments: rotating particles should radiate, incorrect ground state ansatz...

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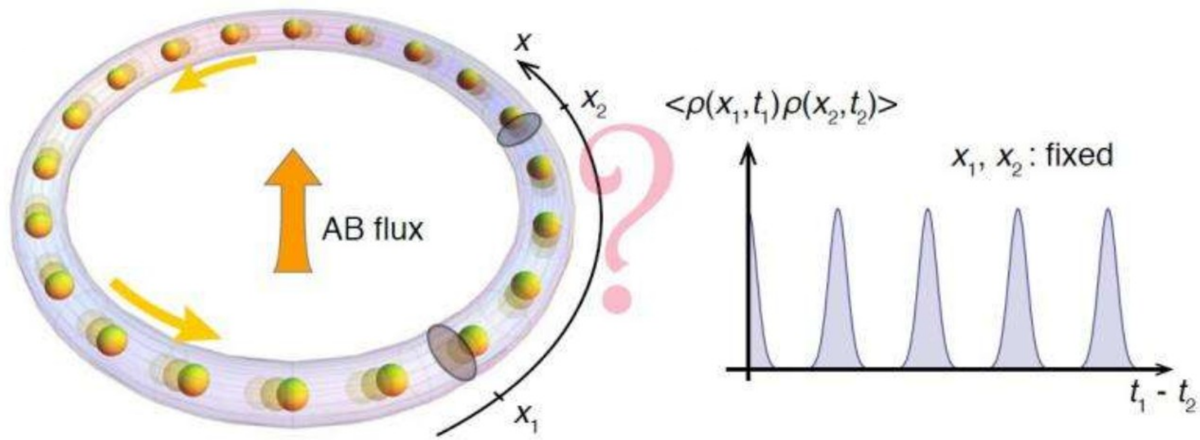

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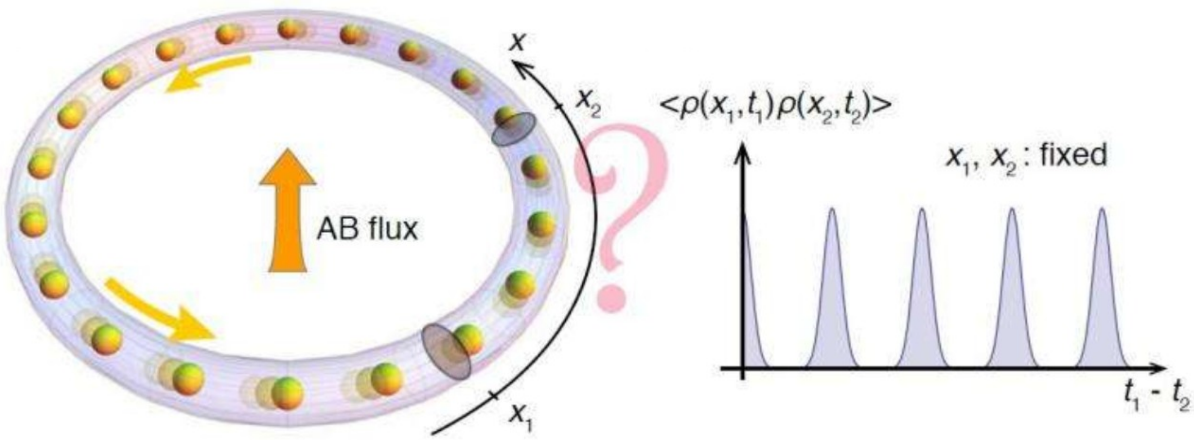

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- No-go theorem: systems in thermal equilibrium cannot manifest any time-crystalline behavior (*short-range Hamiltonians)

(H. Watanabe and M. Oshikawa, PRL 114, 251603 (2015))

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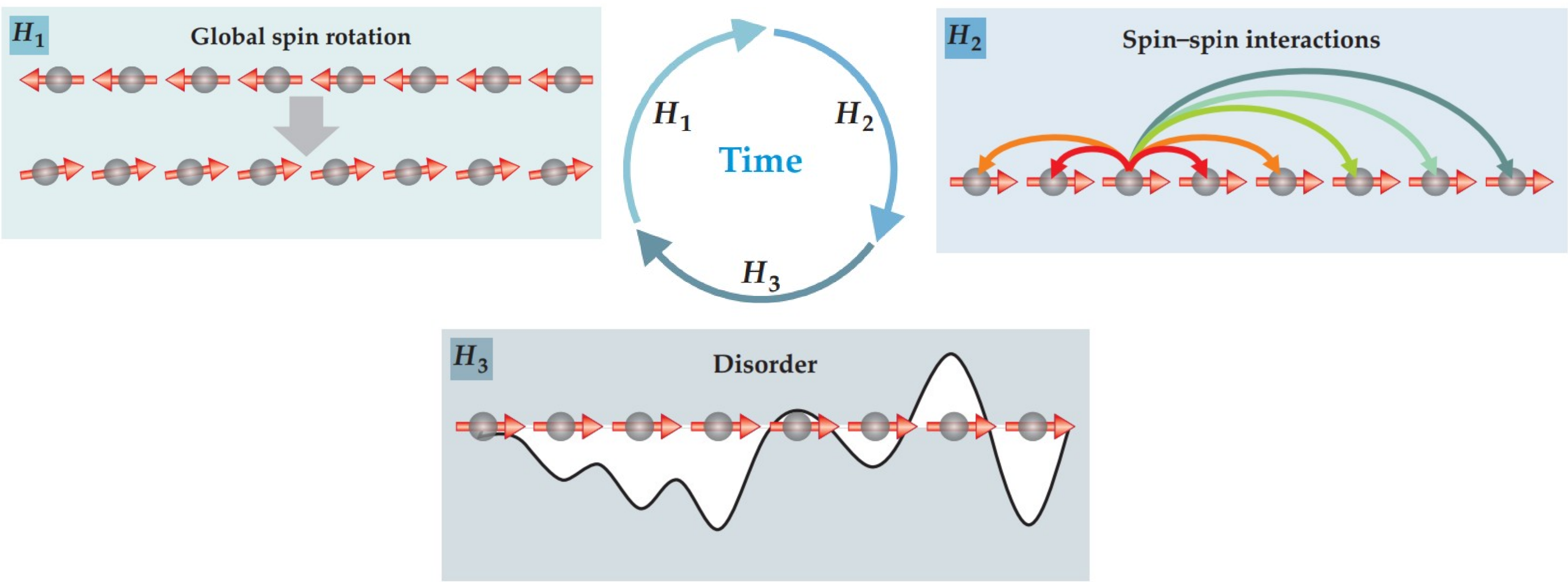
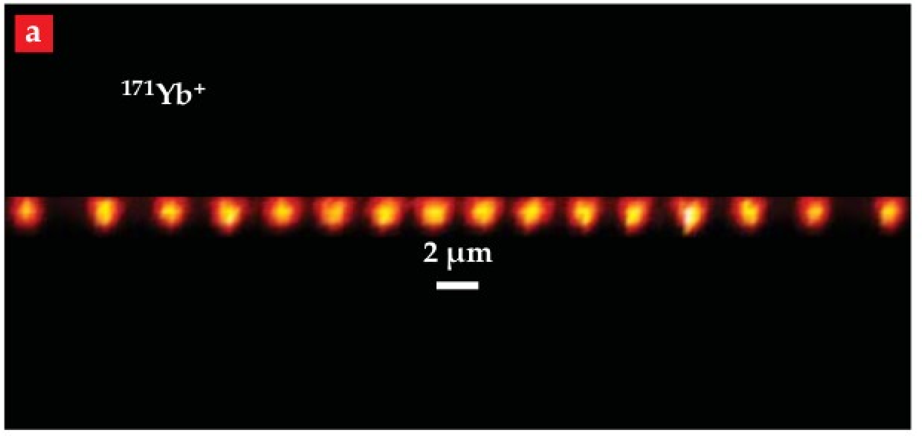


trapped ytterbium ions

J. Zhang et al., Nature 543, 217 (2017).



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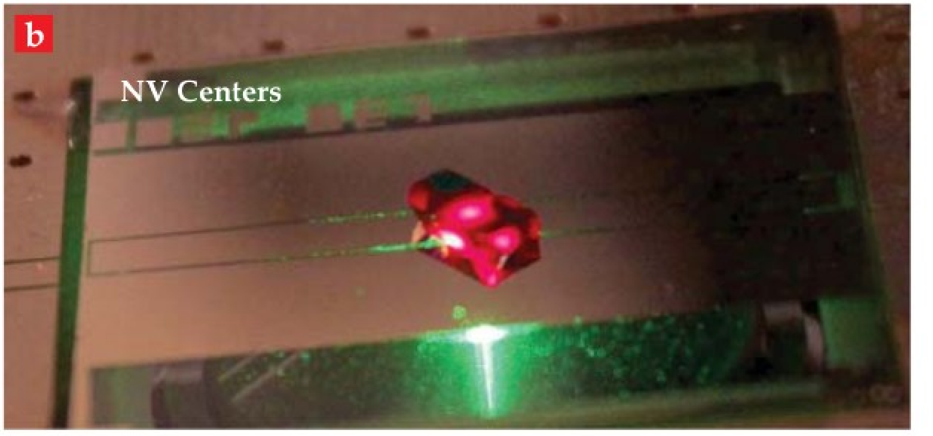

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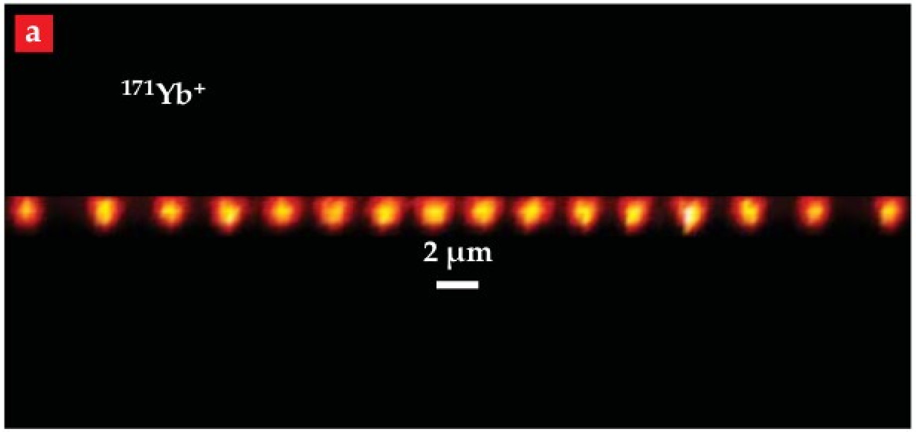

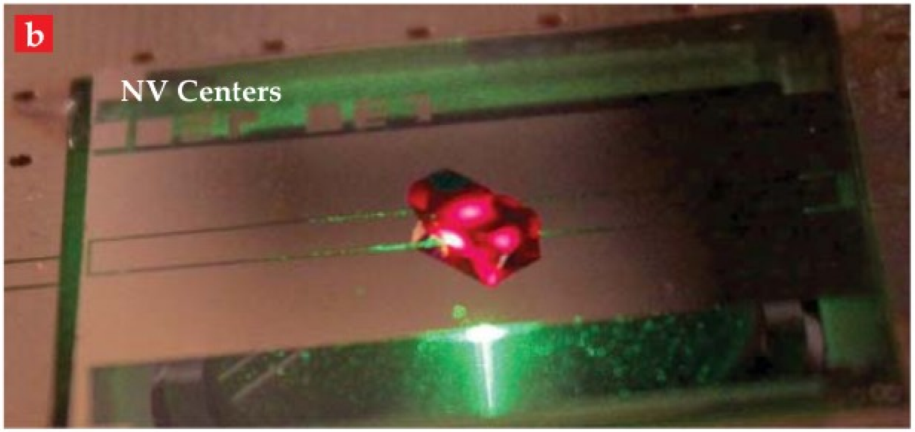

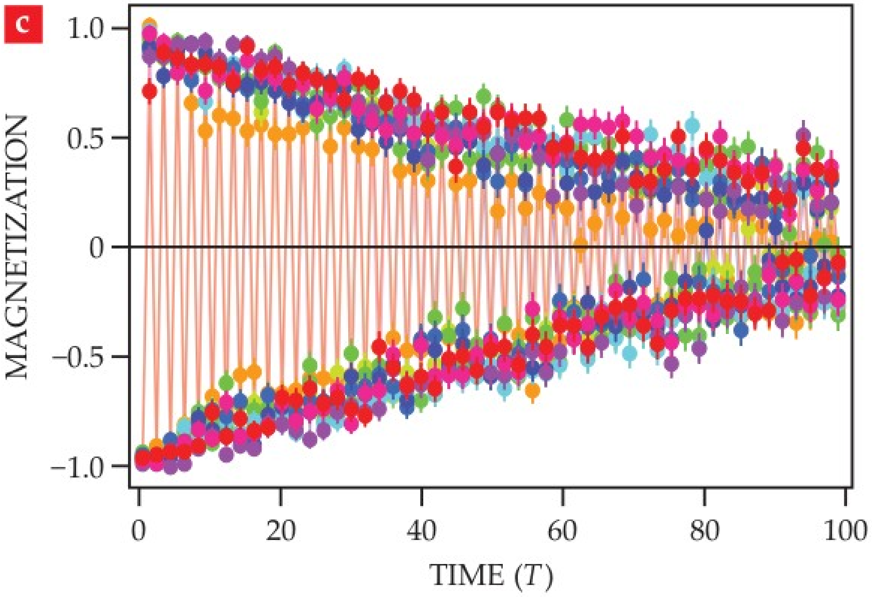

period-doubling dynamics

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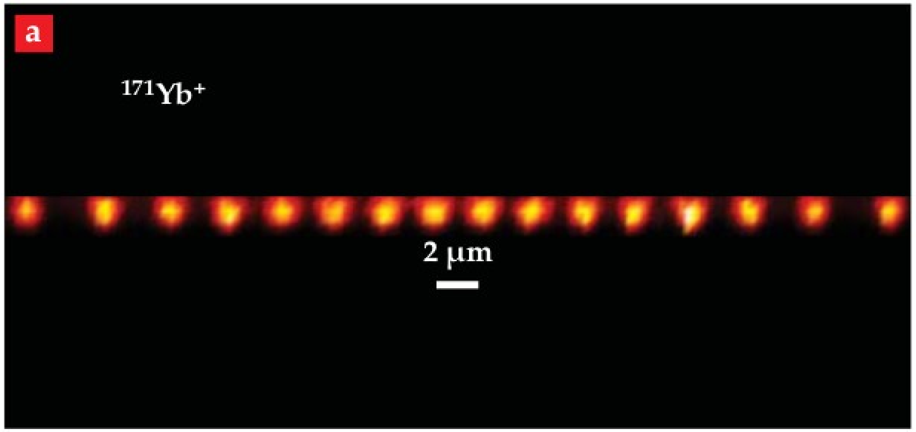

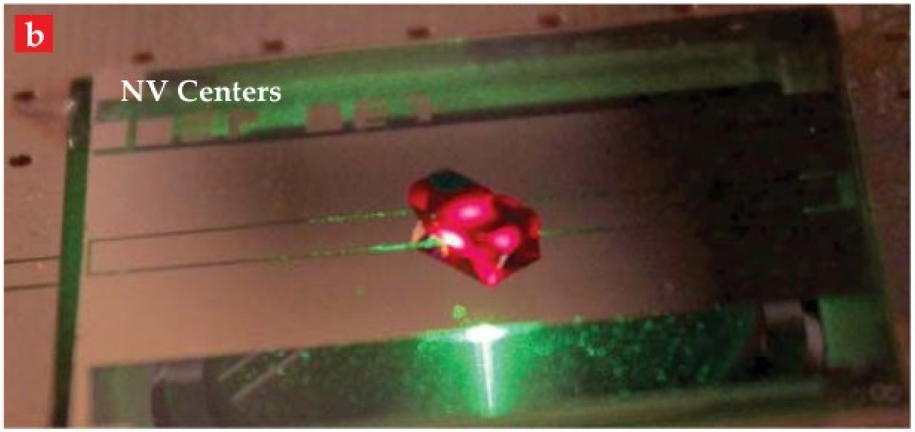

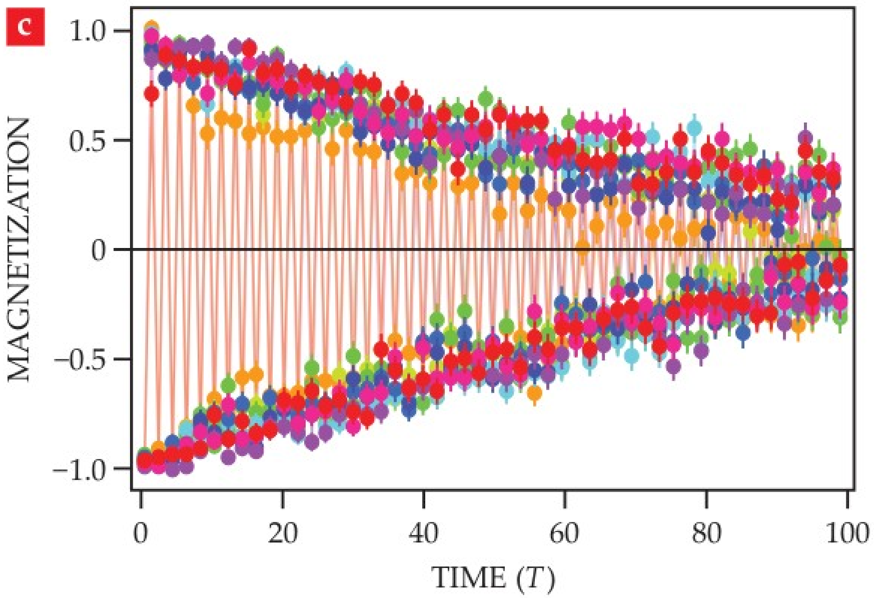

period-doubling dynamics

Discrete Time Crystals:
(variety of platforms)

- MBL (short-range);
- Quantum Scars;
- Stark localisation;
- Long-range spins;
- Clock models (n-tuplings)
- ...

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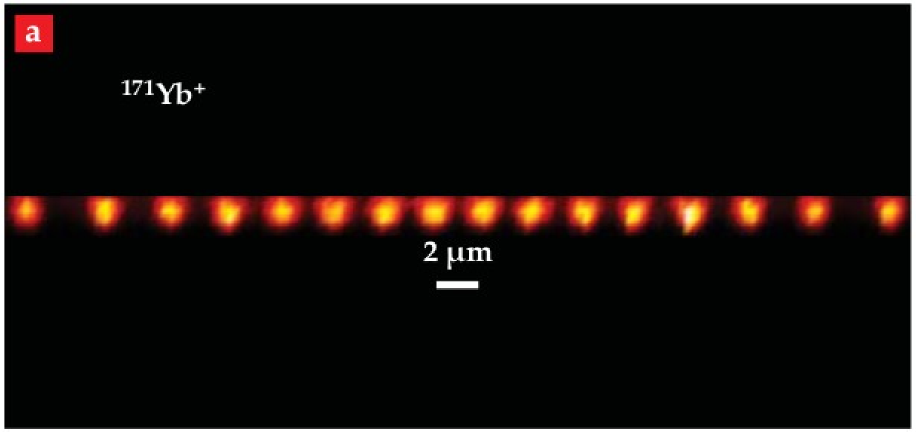

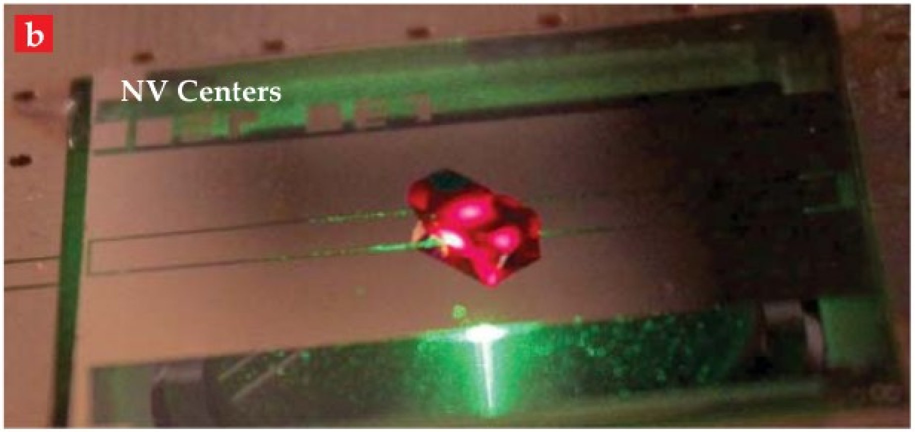

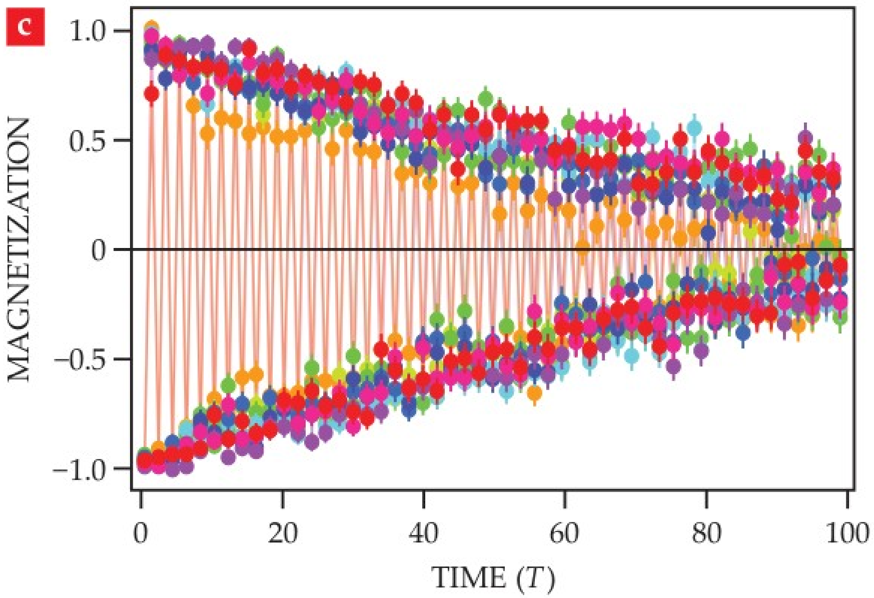

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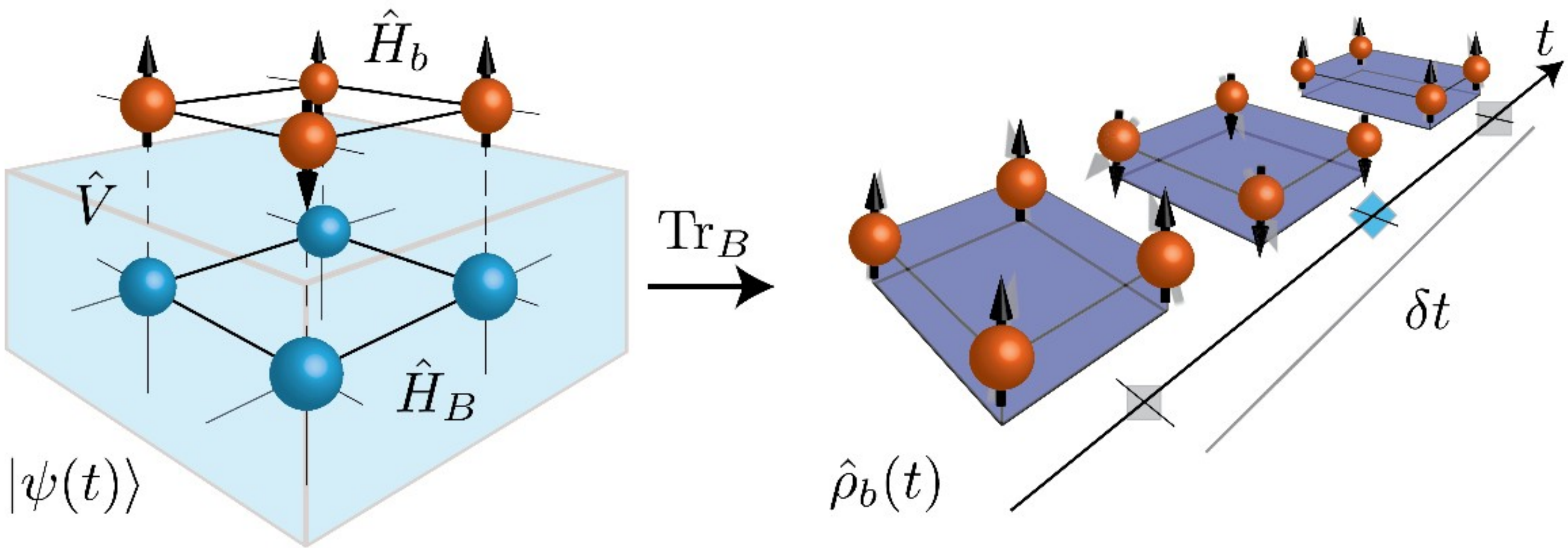
What about ***continuous time crystals***?

(avoid somehow no-go theorem...)

Boundary Time Crystals



- We focus on the dynamics (and possible SSB) of the ***boundary*** of the system;

. equivalently, in open systems (which are the *boundary* of an environment)

. idea ~ ***surface*** critical phenomena: only the surface, representing a (macroscopic) portion of the system is ordered.

PRL 121, 035301 (2018)

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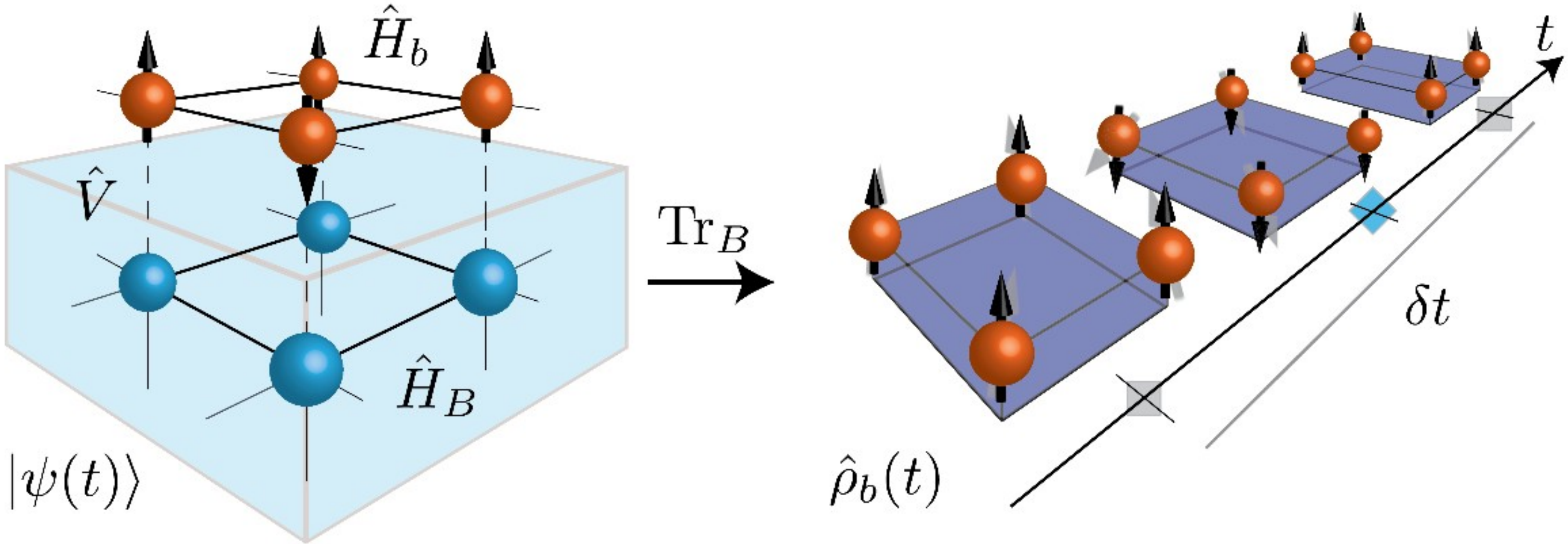

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- automatically ***out of "no-go theorem"***
. intrinsically out of equilibrium situation;
. while no-go prevents SSB in the ***whole*** system, the boundary is just a ***fraction*** of it, thus do not violate the theorem

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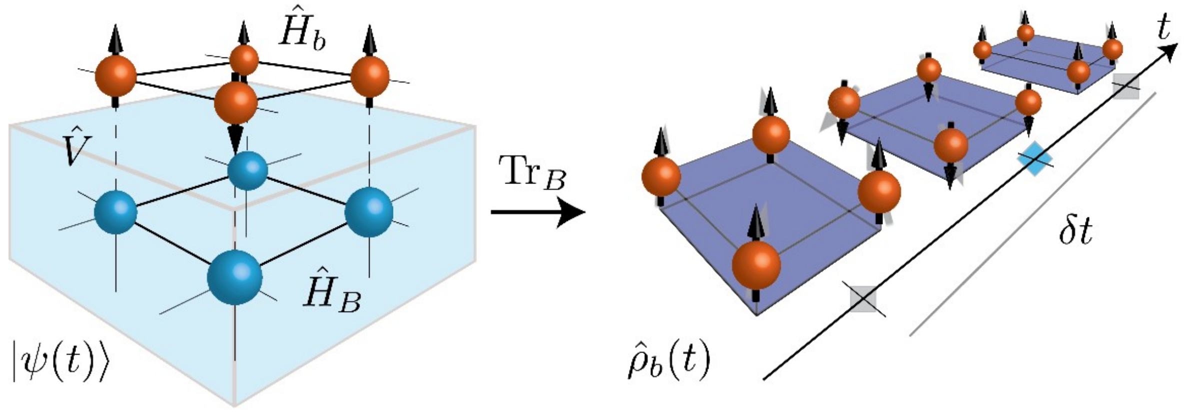

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$$\langle \hat{O}_S \rangle = f(t)$$

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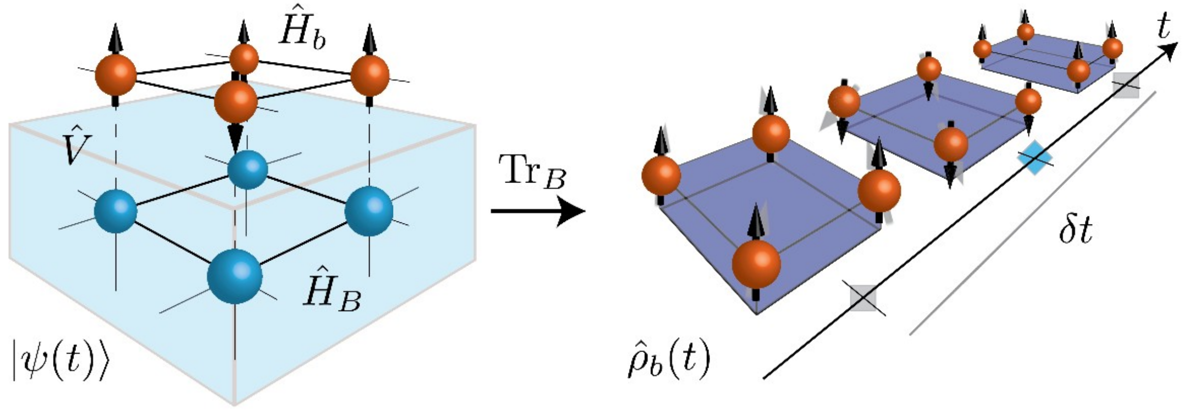

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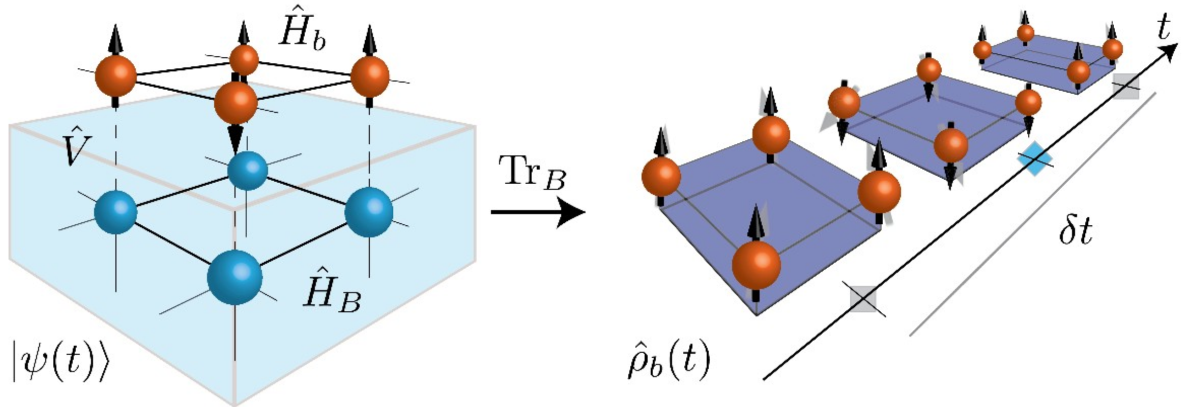

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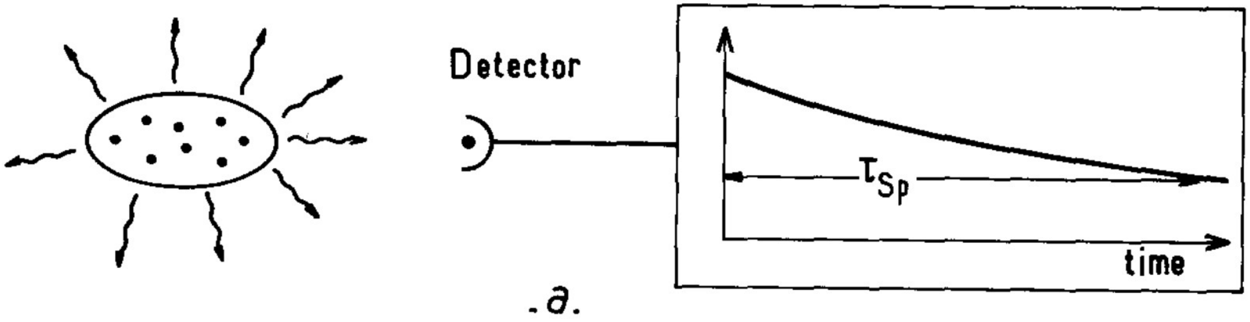

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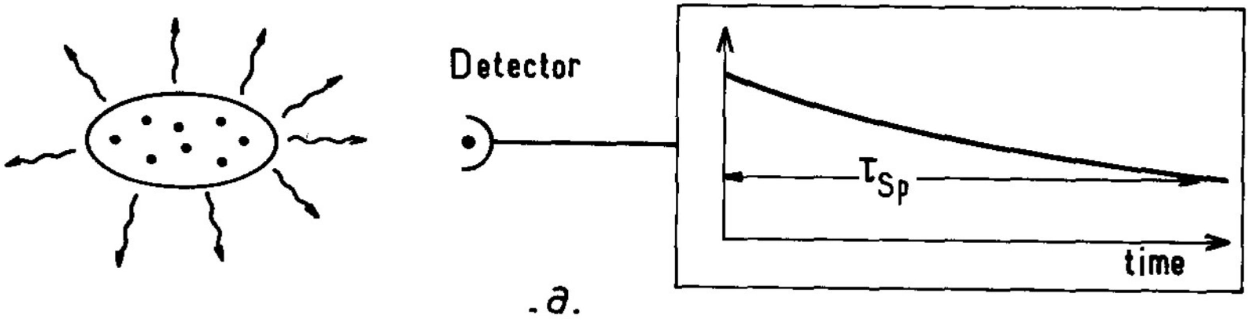

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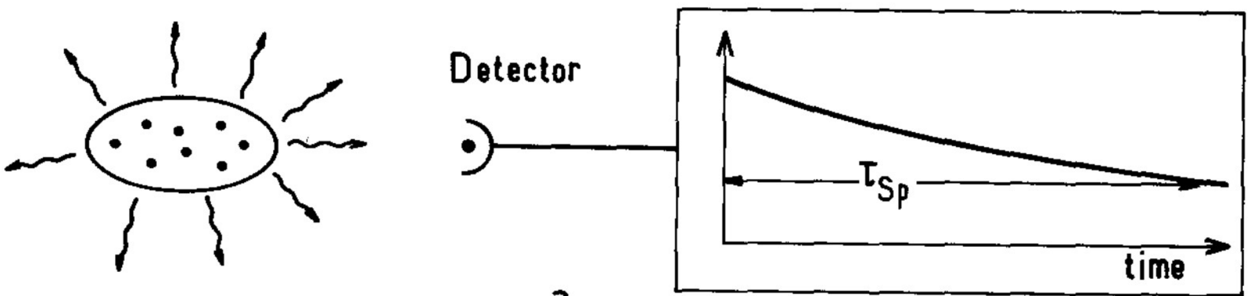

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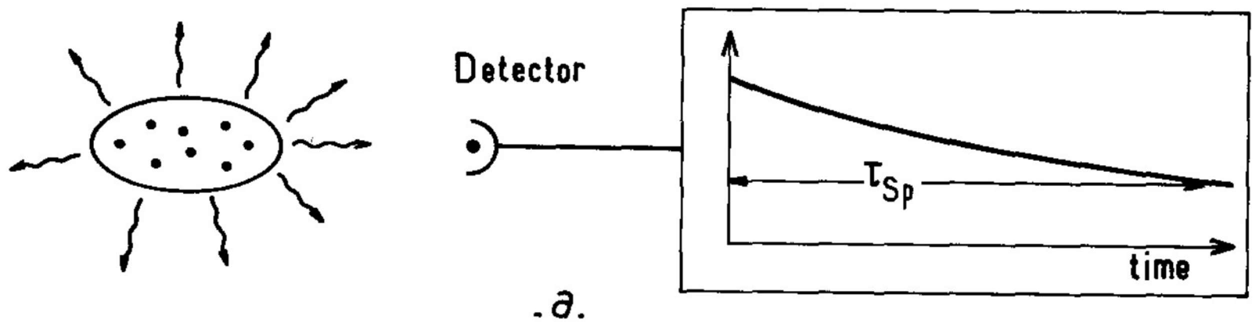

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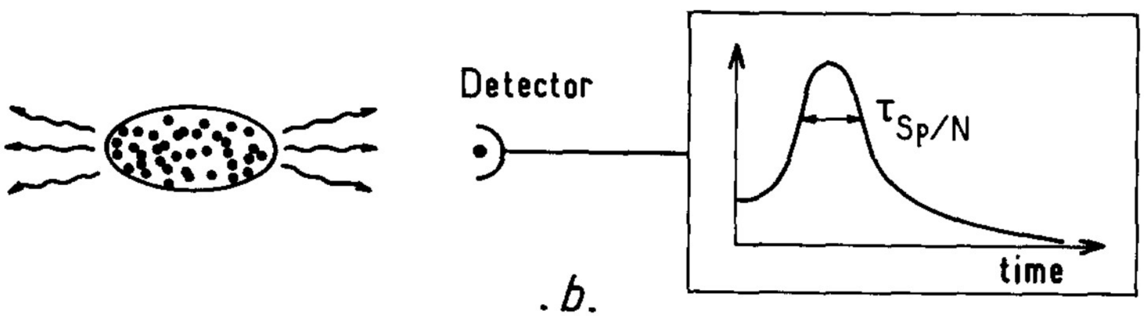

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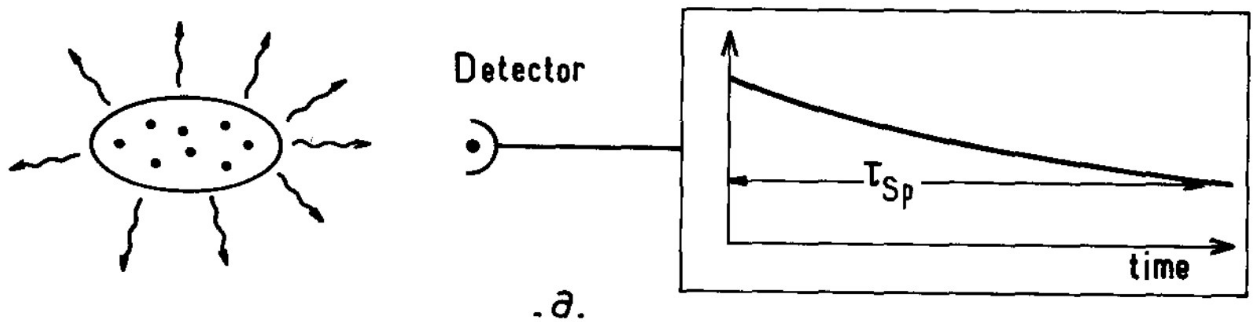

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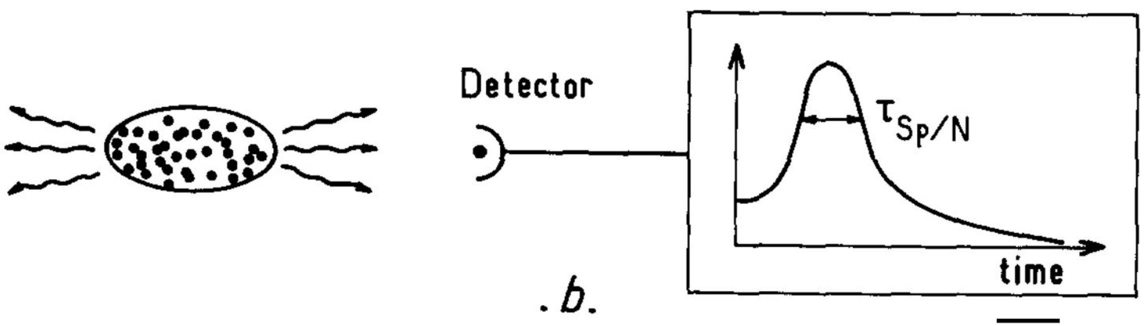

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Superradiance

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~ ***identical*** coupling of all the atoms to the electromagnetic field.

(field mediates a cooperative coupling among the atoms → collective dynamics)

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use cavities* … (*cavity QED***):

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. *tuning resonance*
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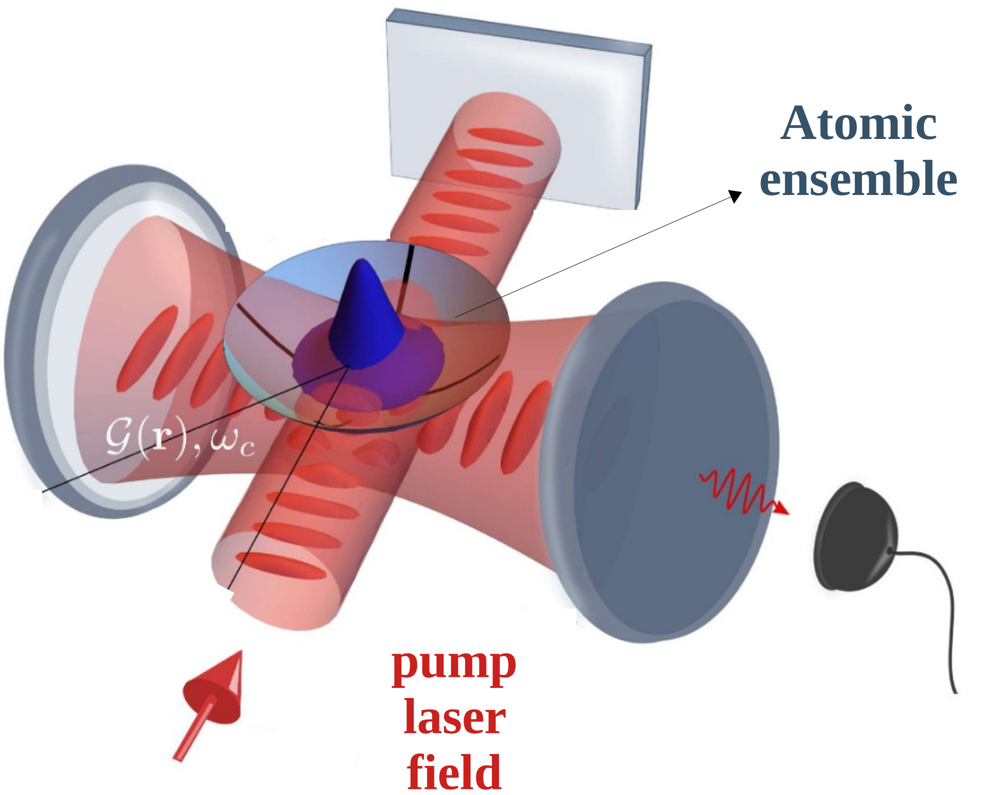

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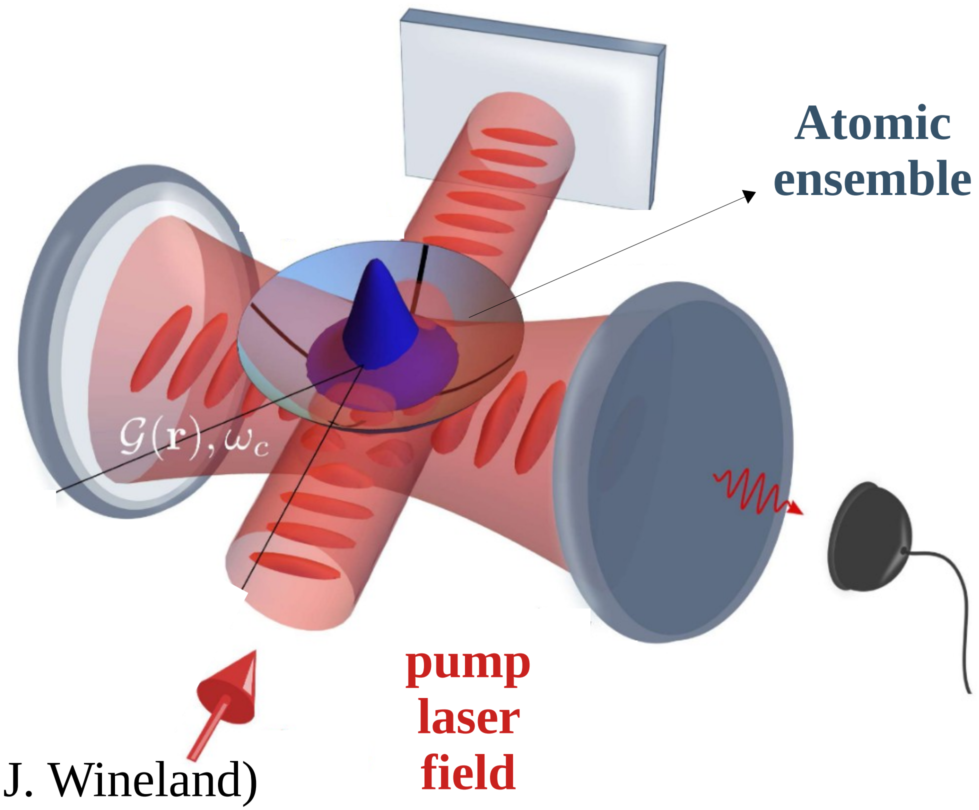

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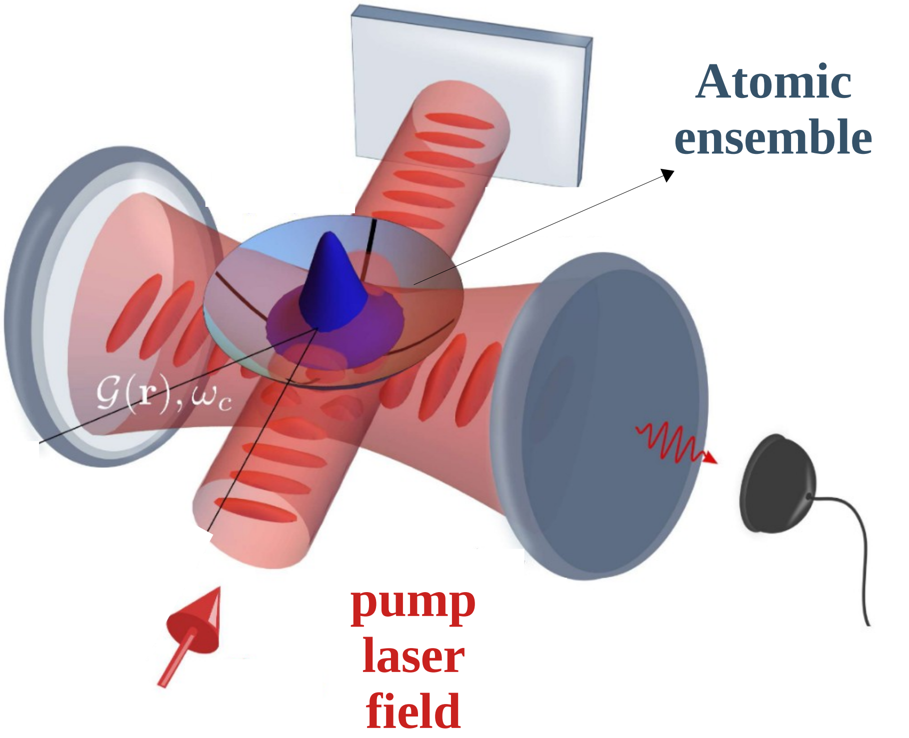
. *high level of controlability*:
. Nobel Prize 2012 (Serge Haroche and David J. Wineland)

for "ground-breaking experimental methods that enable measuring and manipulation of individual quantum systems"

. applications in quantum information/computation
. quantum simulation long-range interacting systems / ...

. ***A single photon mode coupled to N spin-1/2 atoms:***



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$$\hat{H} = \sum_{j=1}^{N} g(\hat{a}\hat{\sigma}_i^- + \hat{a}^\dagger\hat{\sigma}_i^+) + \Omega\hat{\sigma}_j^x$$

$$\hat{L}_i = \sqrt{\Gamma}\hat{a}$$

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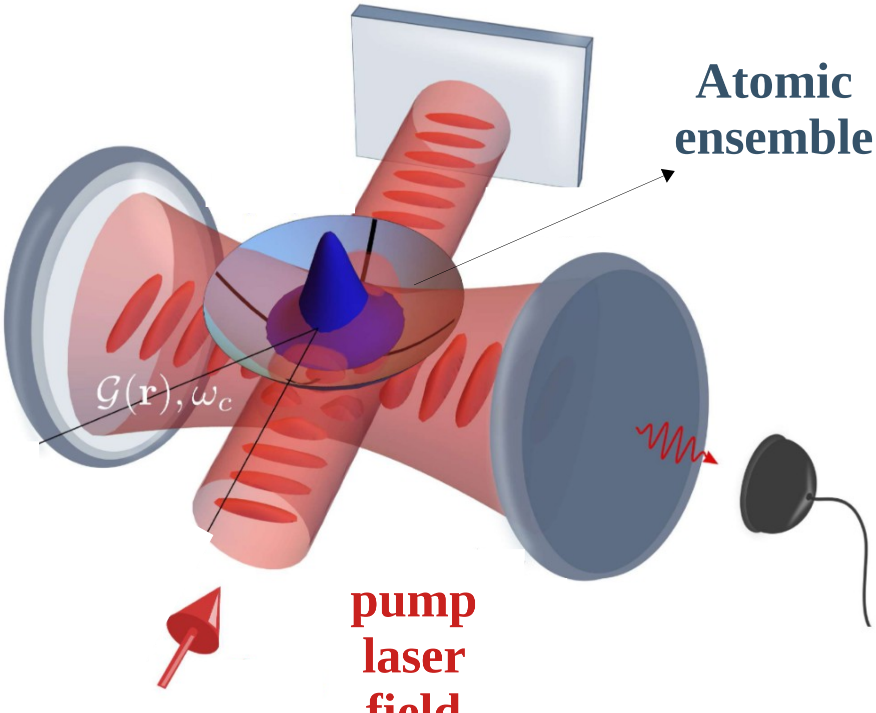
$\eta =$ atom's energy splitting

$\Gamma =$ photon losses in the cavity



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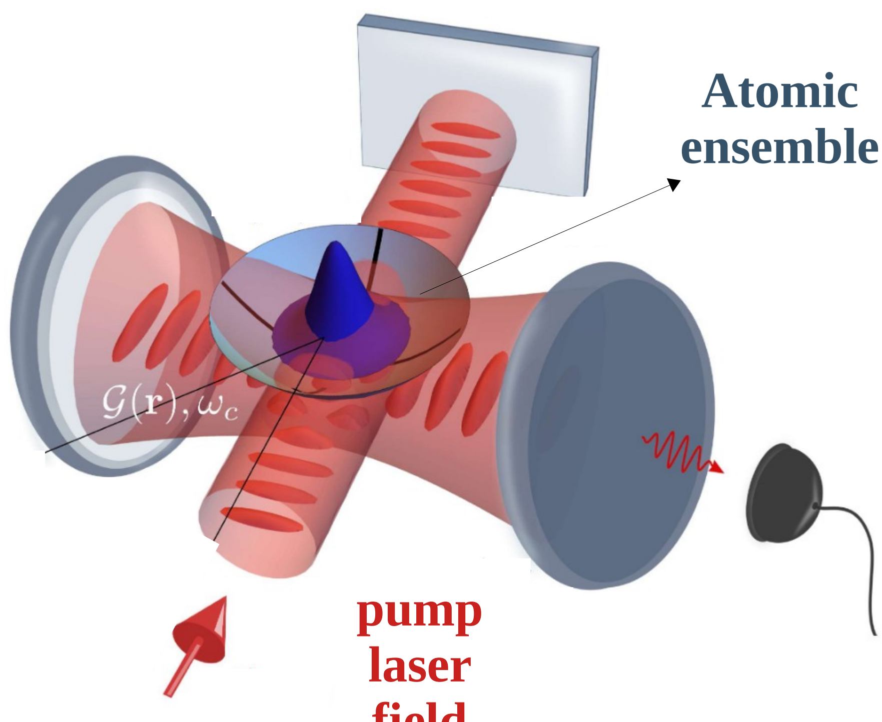

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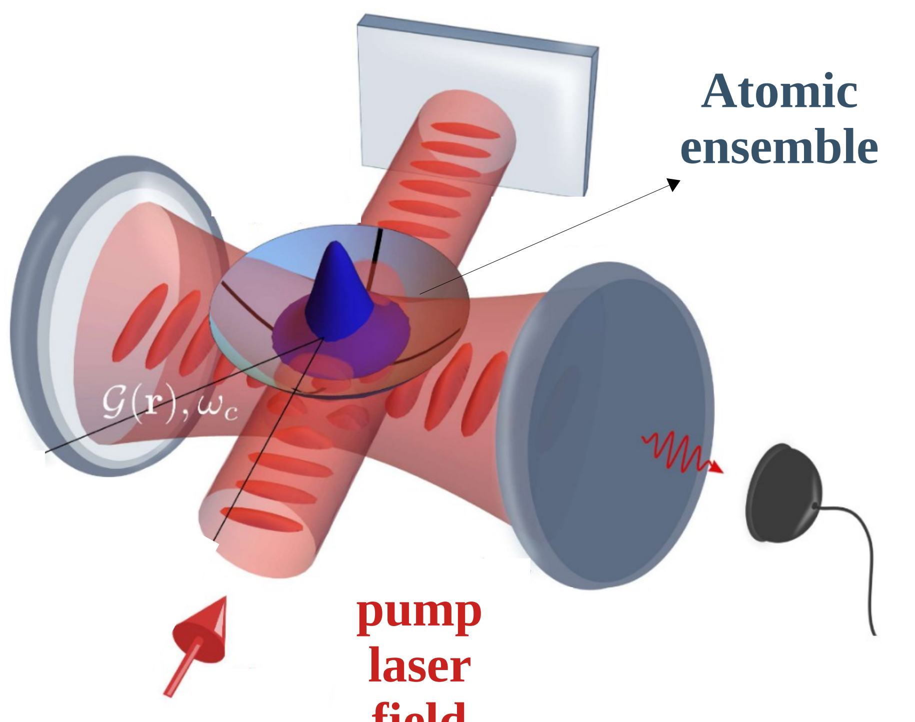
$$\frac{d\hat{\rho}_{\text{spins}}}{dt} = \mathcal{L}[\hat{\rho}_{\text{spins}}]$$

cooperative dissipation
among spins



- ***Dissipative collective spin Lindbladian*** *(open Dicke model, diven Dicke model, ...)****:***

$$\hat{\mathcal{L}}[\hat{\rho}] = -i\omega_0 \left[\hat{S}^x, \hat{\rho}\right] + \frac{2\kappa}{N}\left(\hat{S}_-\hat{\rho}\hat{S}_+ - \frac{1}{2}\left\{\hat{S}_+\hat{S}_-, \hat{\rho}\right\}\right)$$

coherent dynamics

effective decay rate

$$\hat{S}^x = \sum_{j=1}^{N} \hat{\sigma}_j^x$$

$$\hat{S}^- = \sum_{j=1}^{N} \hat{\sigma}_j^-$$

- ***Dissipative collective spin Lindbladian*** *(open Dicke model, diven Dicke model, ...)***:**

$$\hat{\mathcal{L}}[\hat{\rho}] = -i\omega_0 \left[\hat{S}^x, \hat{\rho}\right] + \frac{2\kappa}{N} \left(\hat{S}_- \hat{\rho} \hat{S}_+ - \frac{1}{2}\left\{\hat{S}_+ \hat{S}_-, \hat{\rho}\right\}\right)$$

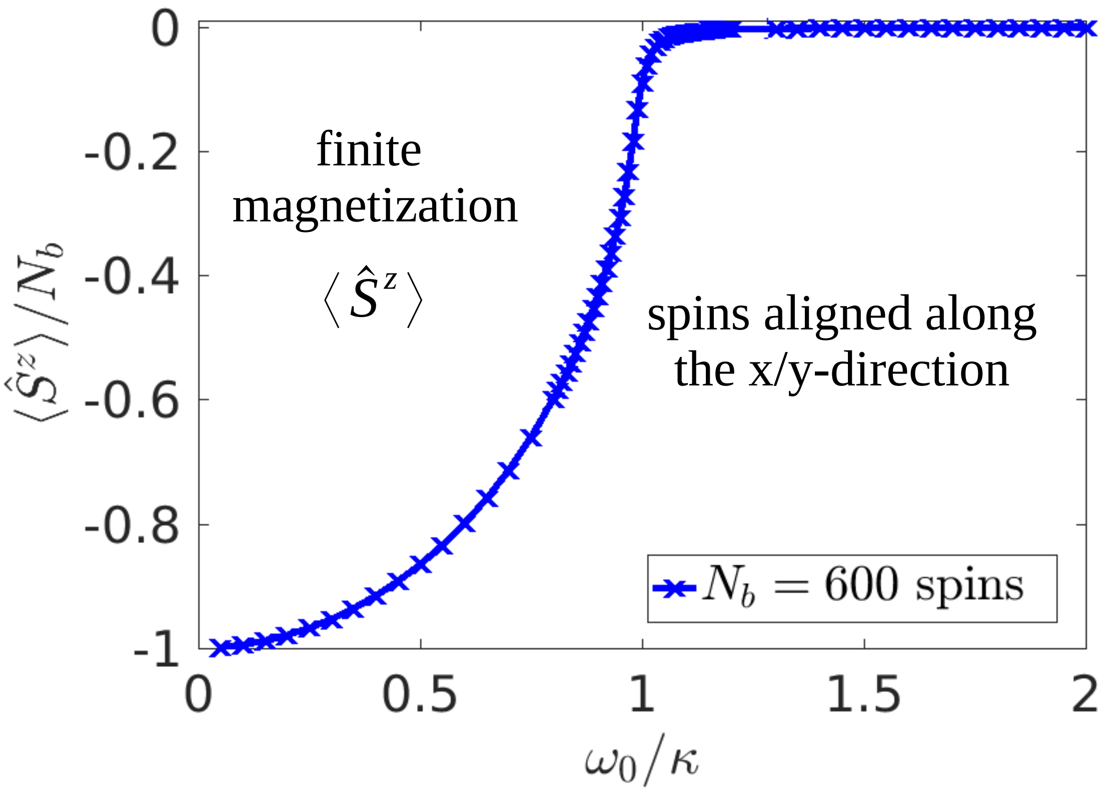
coherent dynamics

effective decay rate

$$\hat{S}^x = \sum_{j=1}^{N} \hat{\sigma}_j^x$$

$$\hat{S}^- = \sum_{j=1}^{N} \hat{\sigma}_j^-$$

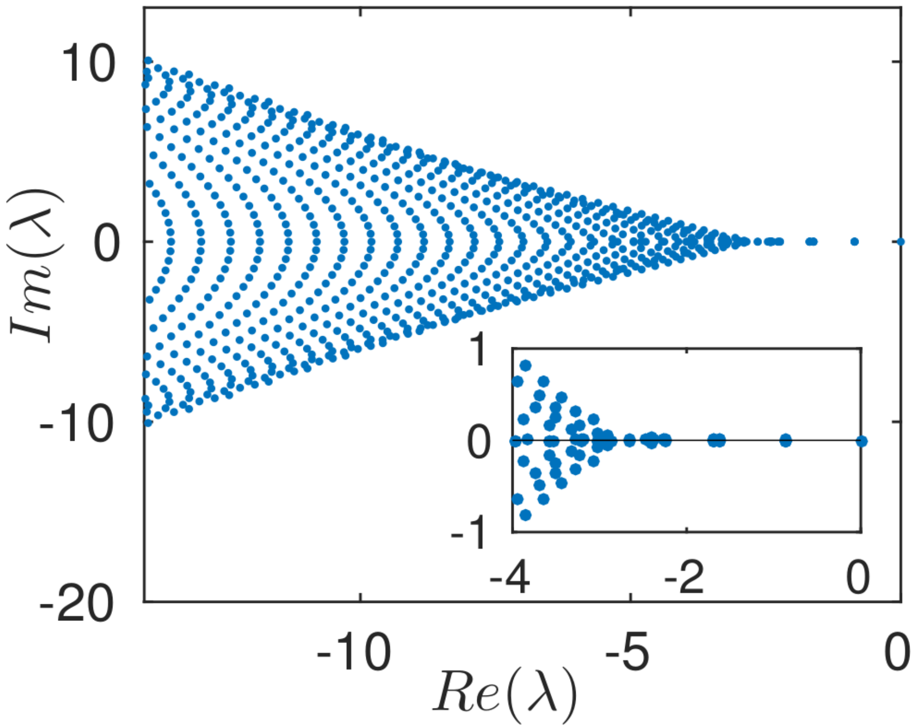
Steady state properties, $t \to \infty$



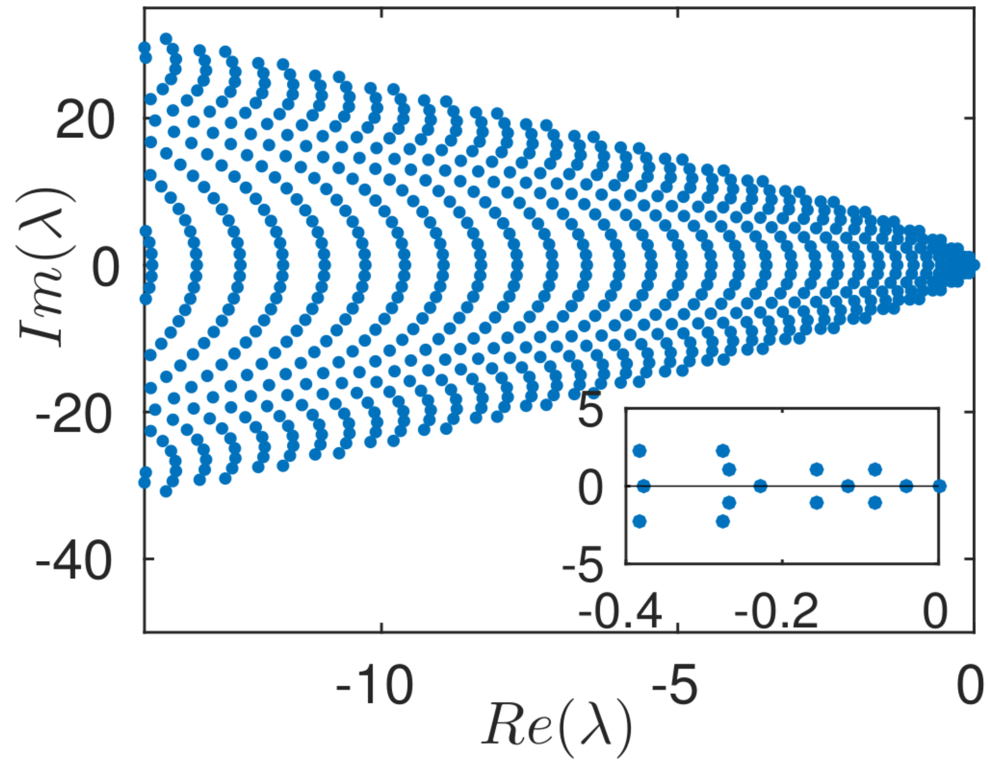
Prog. Theo. Phys. 64, 307 (1978);
Opt. Commun. 27, 160 (1978);
Phys. Lett. A 72, 200 (1979);
PRA 65, 042107 (2002).

- *What about dynamics?* $N = 36$ spins

$\boxed{\kappa > \omega_0}$

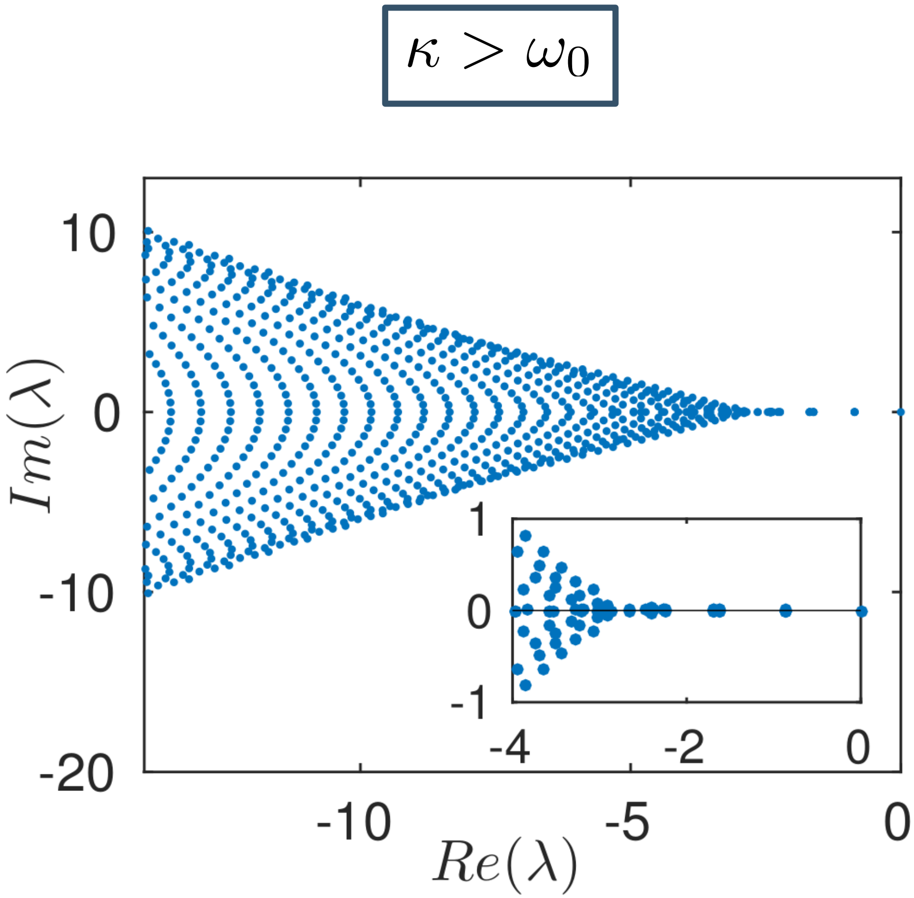


$\boxed{\kappa < \omega_0}$



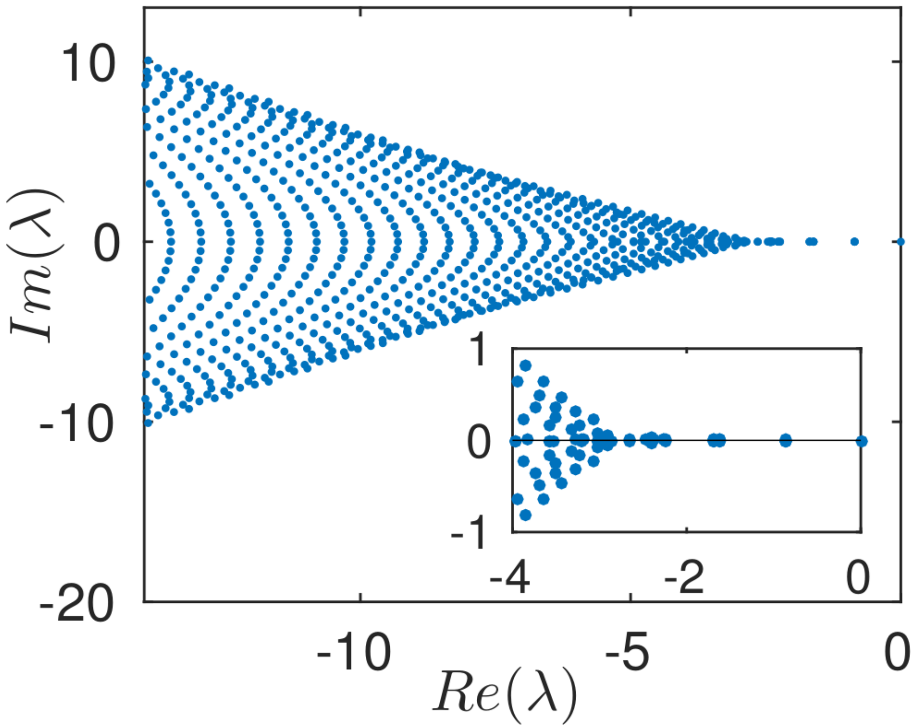
Spectral properties:

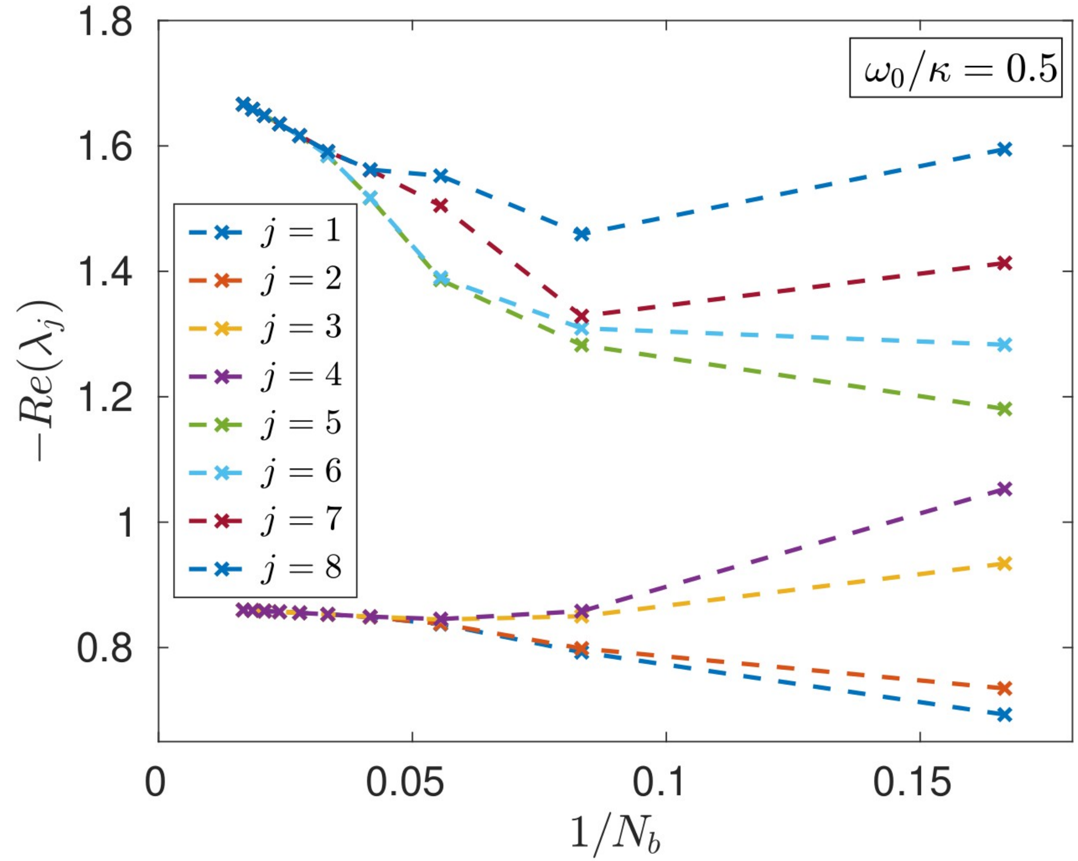
any difference?



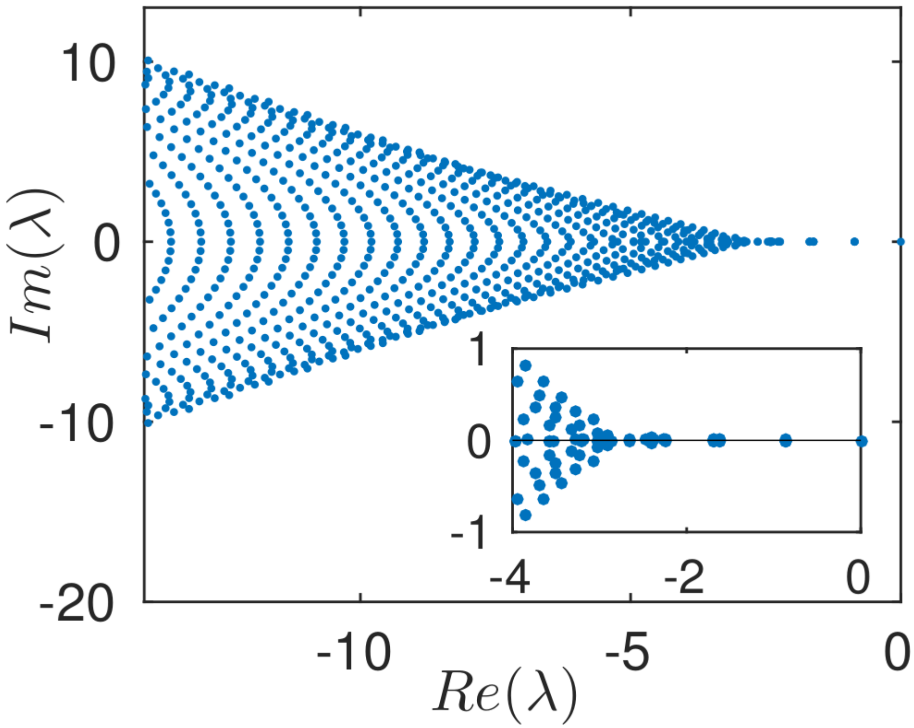
$\kappa > \omega_0$
10
0
-10
-20
$Im(\lambda)$
-10
-5
0
$Re(\lambda)$
1
0
-1
-4
-2
0

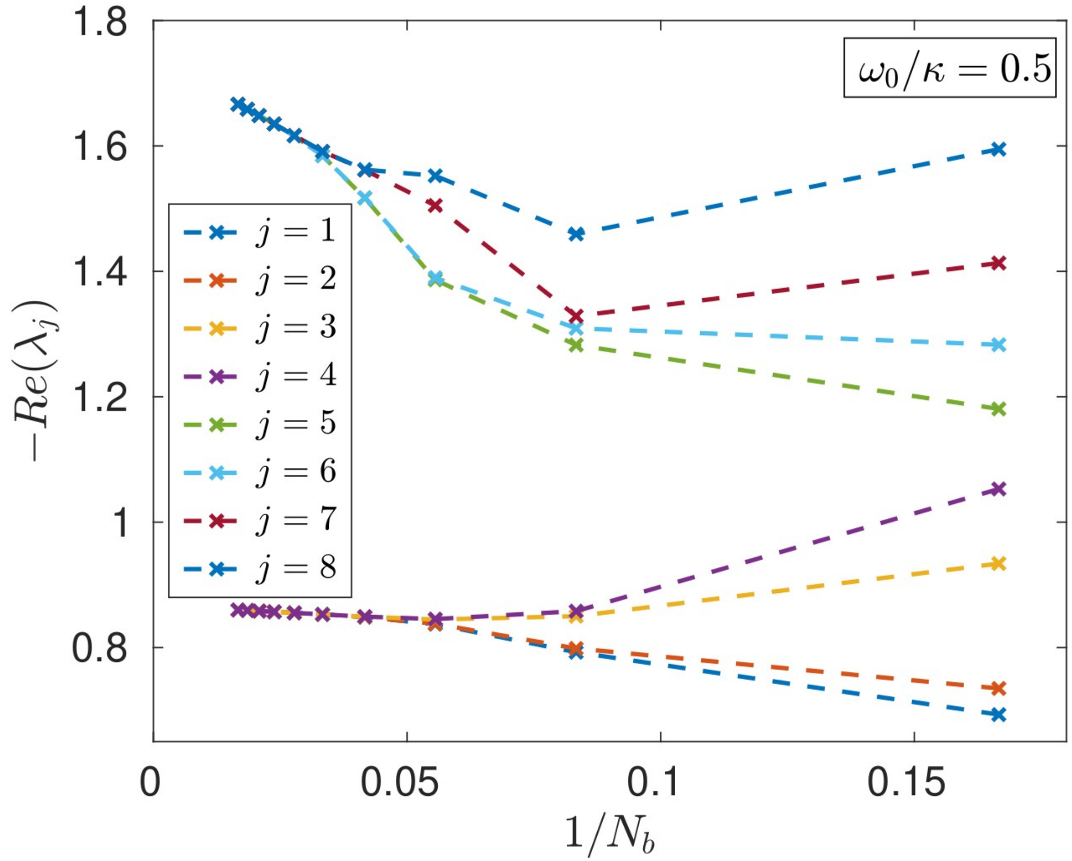
$\kappa > \omega_0$





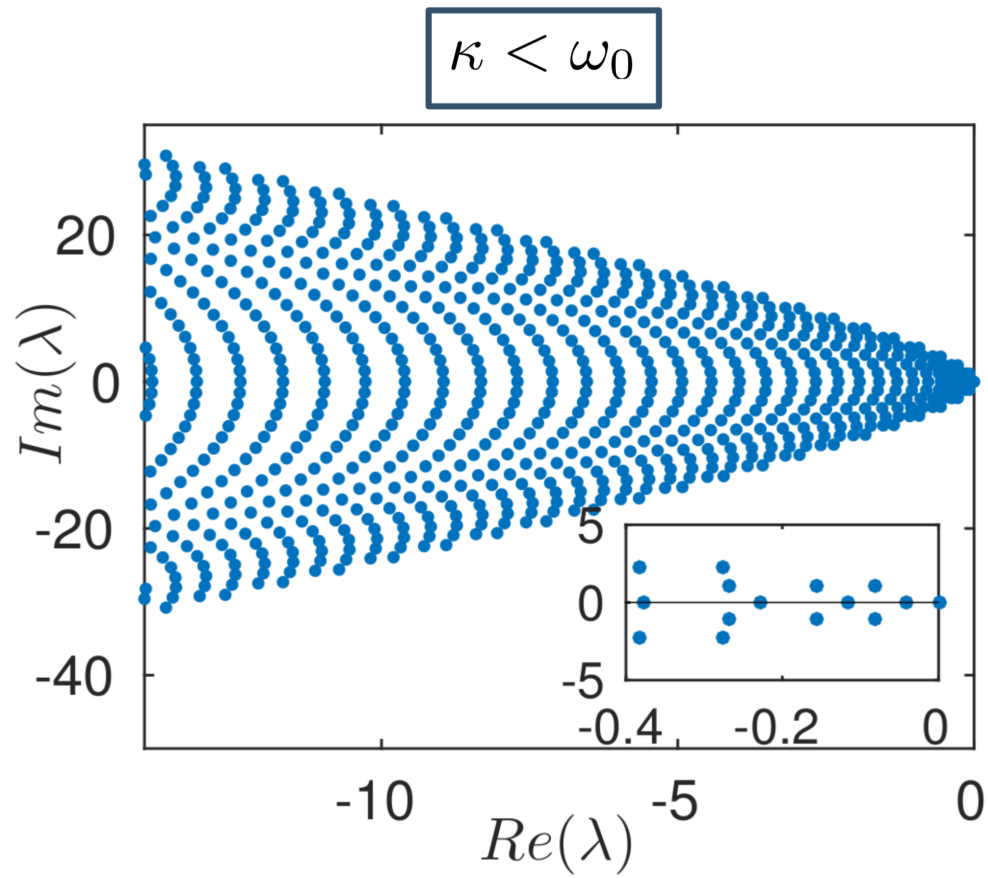
$\boxed{\kappa > \omega_0}$

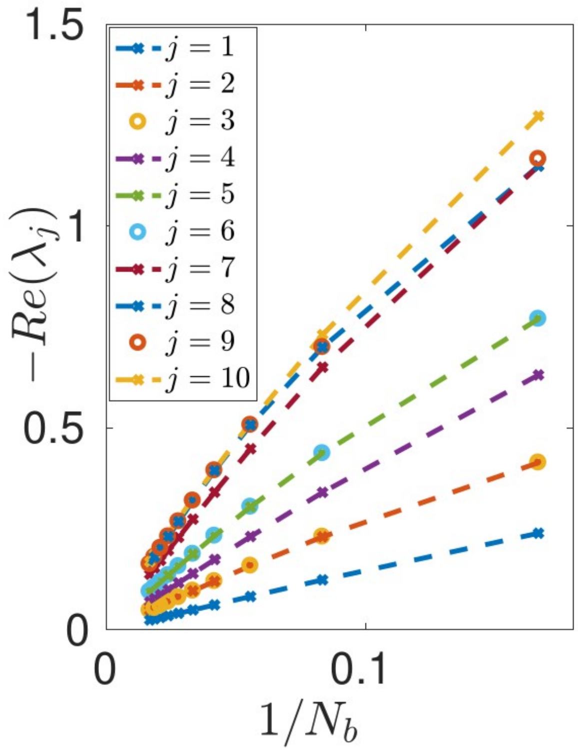




- ***gapped*** Lindbladian:
  exponential decay towards ss

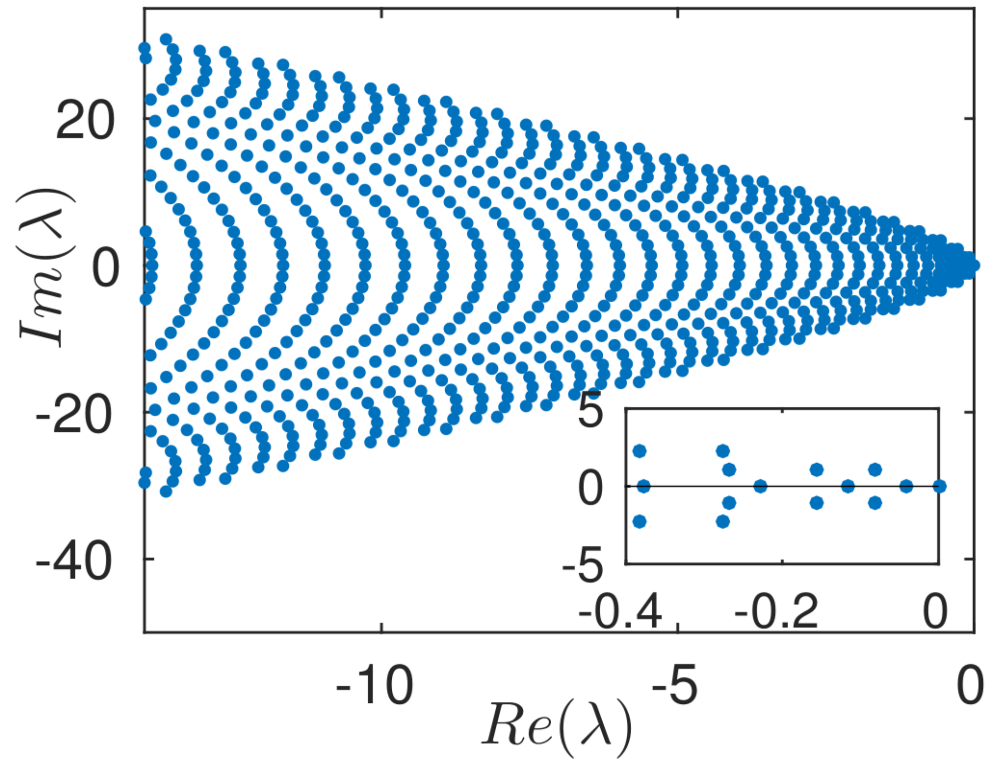
- no imaginary eigenvalues
  no coherent eigenspace oscillations

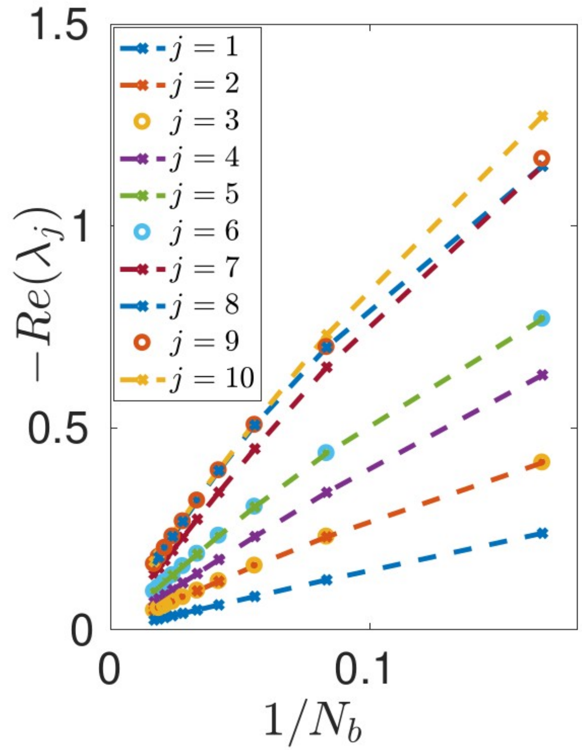

$\kappa < \omega_0$
20
0
-20
-40
$Im(\lambda)$
-10
-5
0
$Re(\lambda)$
5
0
-5
-0.4
-0.2
0



- ***gapless*** Lindbladian:
  slow decay modes diverge for a macroscopic system $N \to \infty$

$\kappa < \omega_0$





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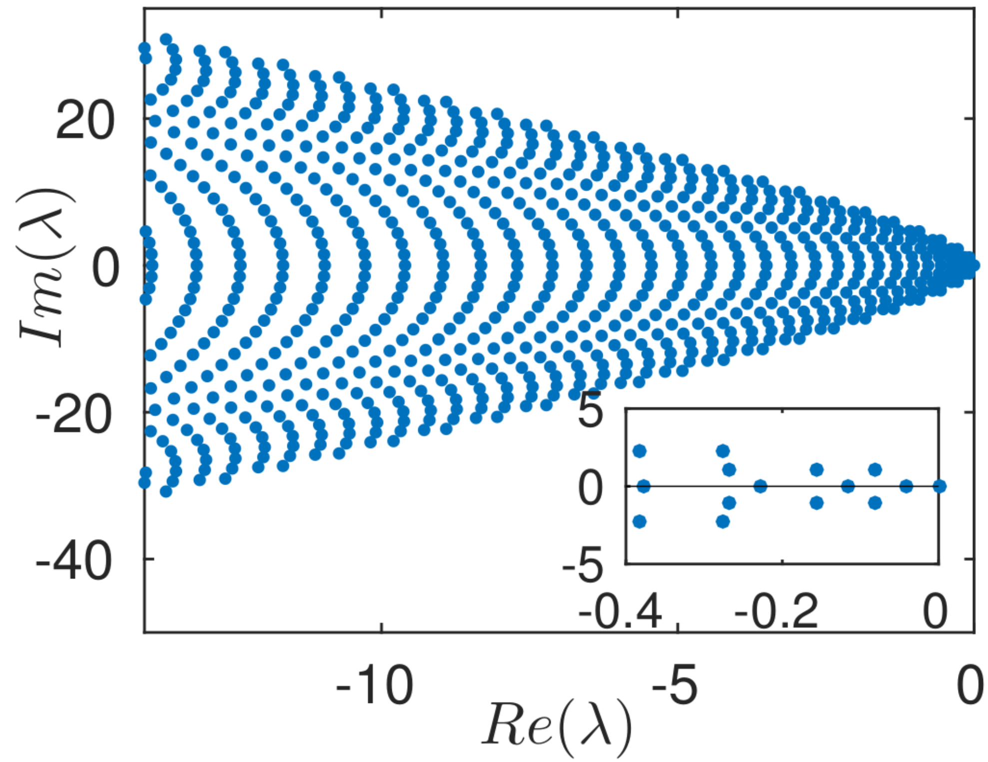
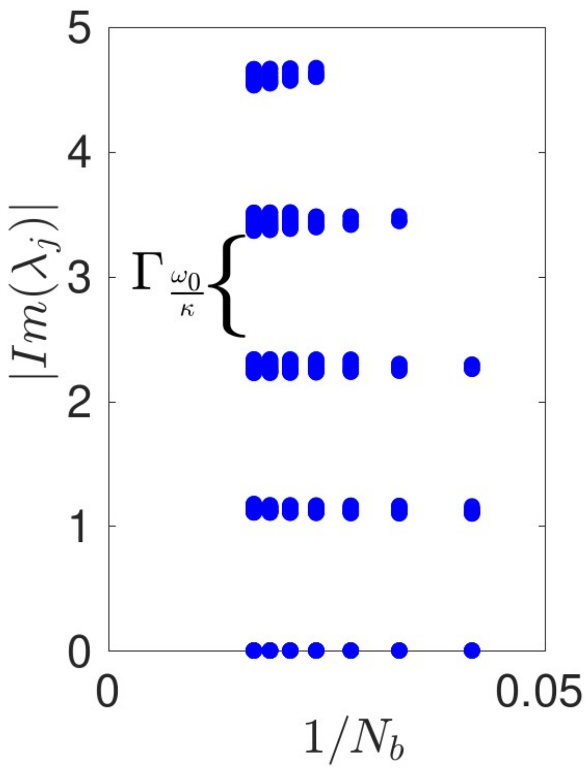




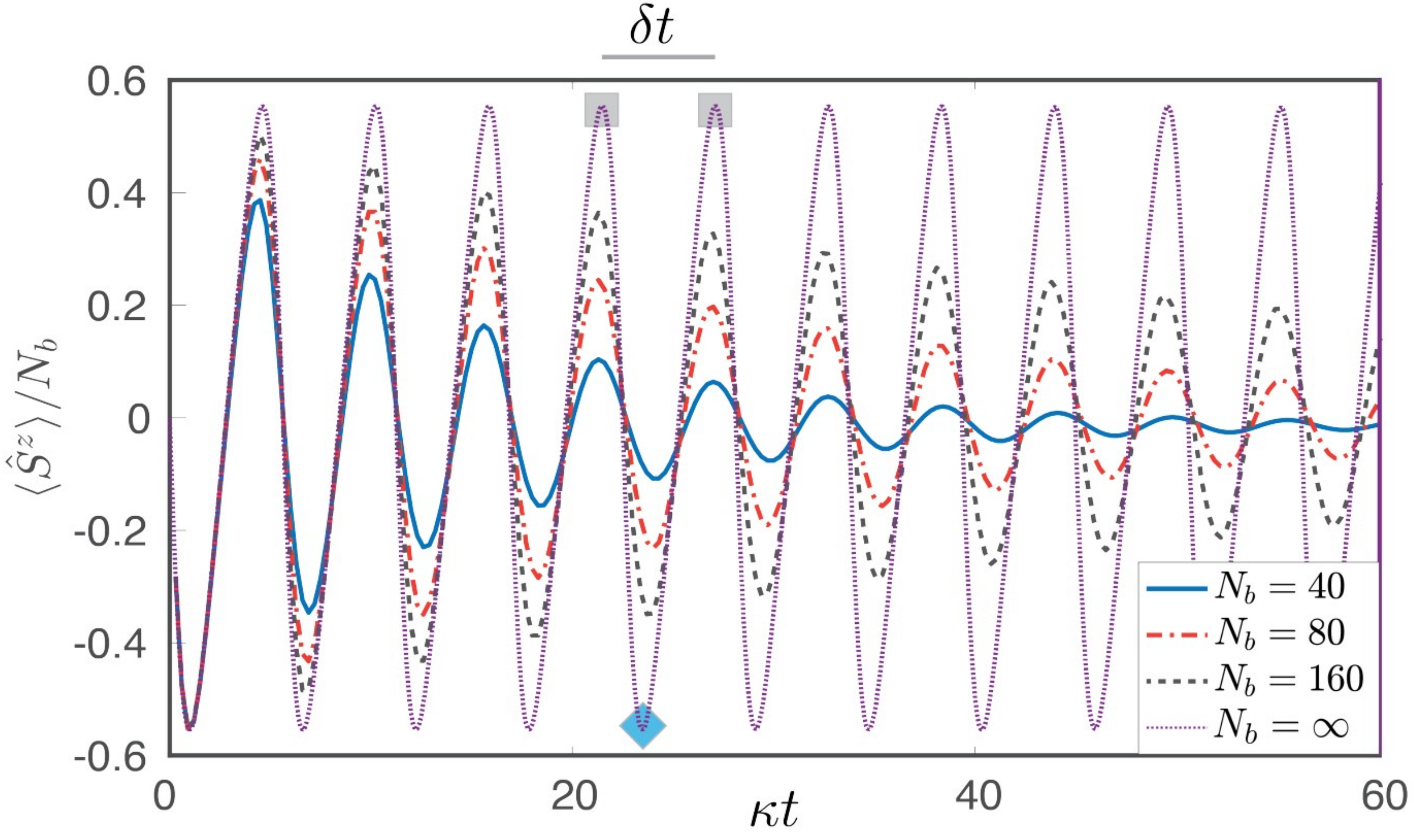
- imaginary eigenvalues:
  coherent oscillations in slow decaying modes

initial state:

$$|\psi(0)\rangle = |+\cdots+\rangle$$

Time Evolution

$\boxed{\kappa < \omega_0}$



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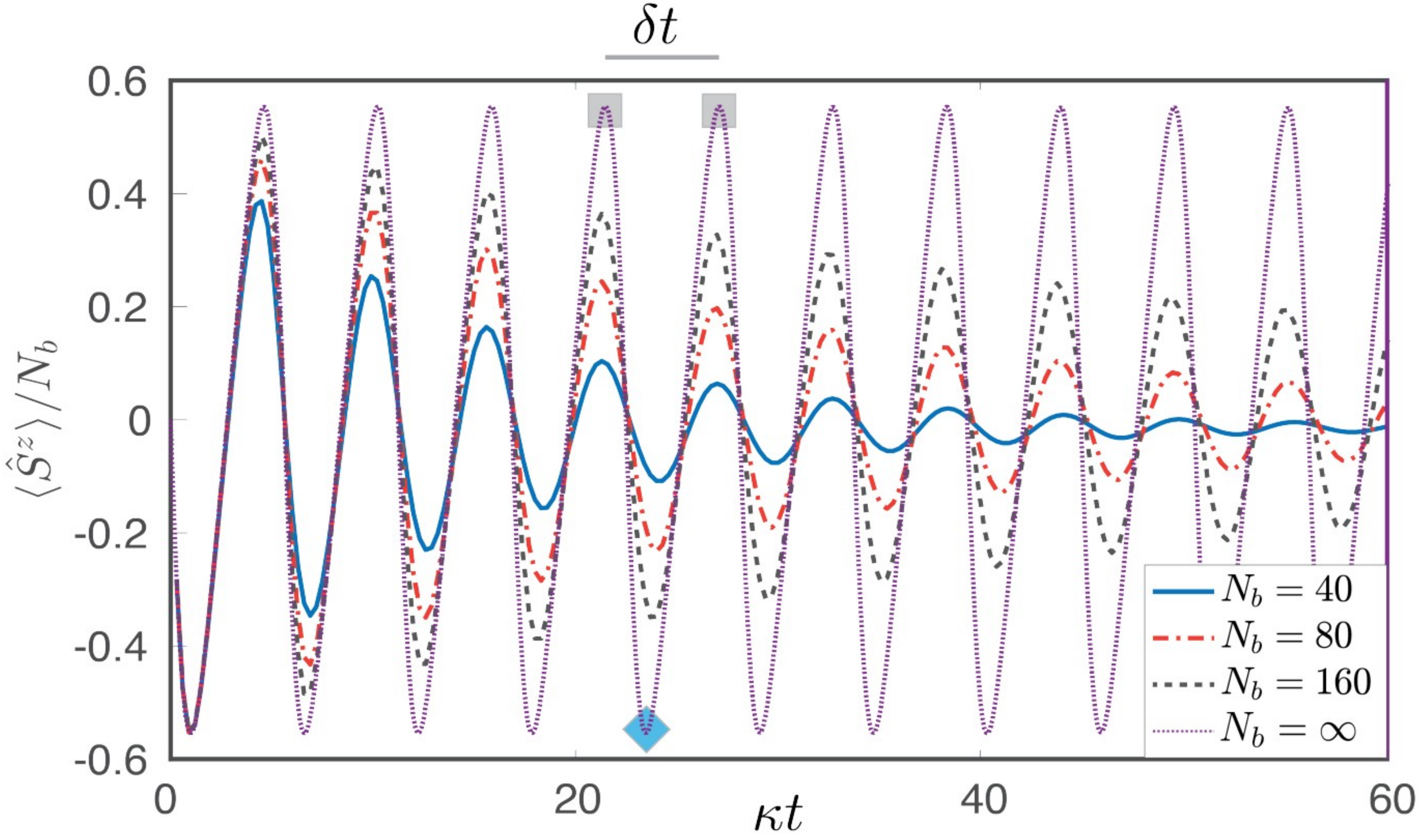
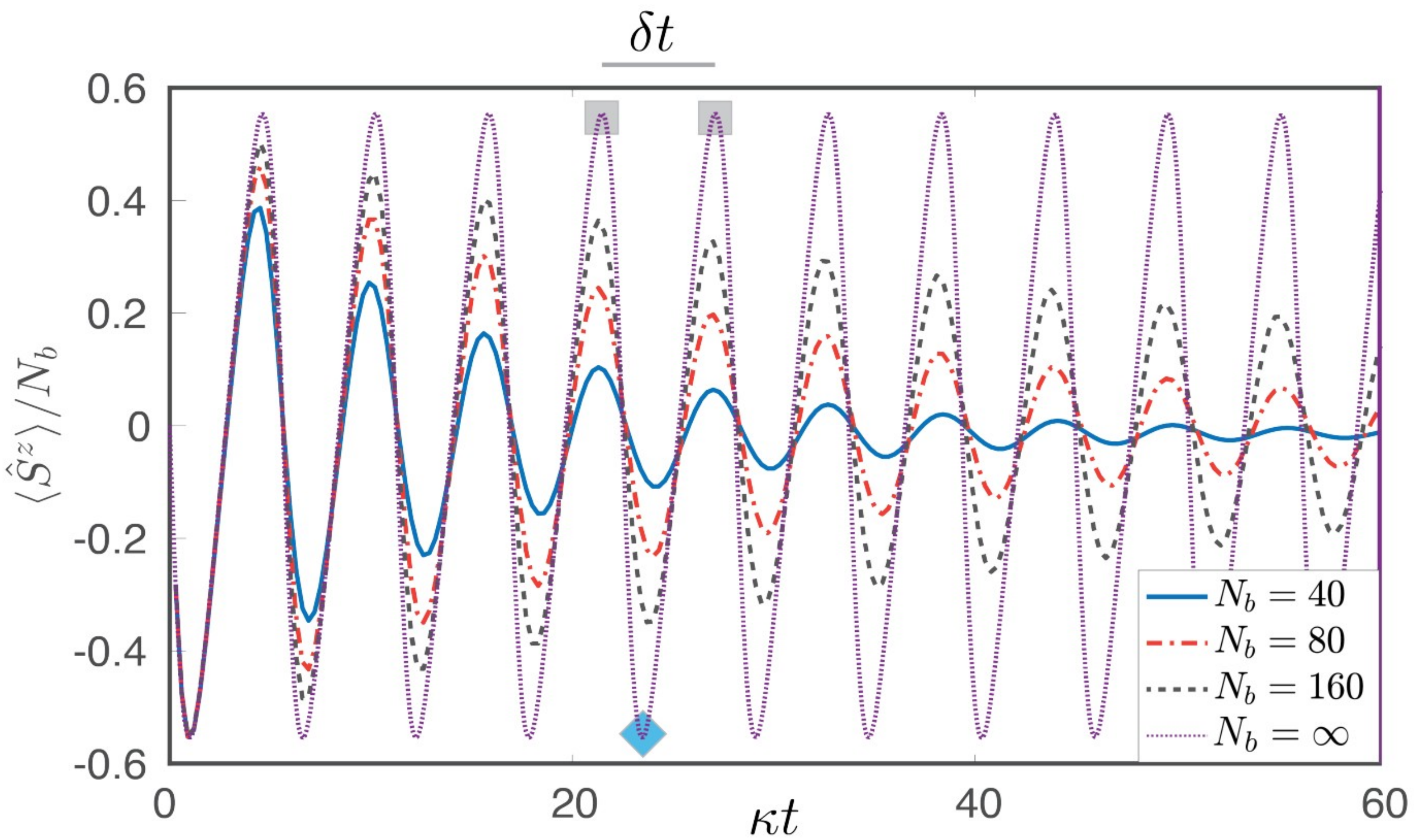


- nontrivial dynamics decays for finite system sizes initial state:

$$|\psi(0)\rangle = |+\cdots+\rangle$$

Time Evolution

$\boxed{\kappa < \omega_0}$



- nontrivial dynamics decays for finite system sizes

- time scale diverges in the macroscopic limit $N \to \infty$

. ***breaking TTSB*** and stabilizing a ***boundary time crystal phase***.

Thermodynamic limit / Mean-field

One can understand what happens in the thermodynamic limit by using a semiclassical approximation:

Heisenberg picture

$$\frac{d\langle\hat{O}\rangle}{dt} = i\langle[\hat{H},\hat{O}]\rangle + \frac{\kappa}{2S}\langle[\hat{L}^\dagger,\hat{O}]\hat{L} + \hat{L}^\dagger[\hat{O},\hat{L}]\rangle$$

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$$\frac{\mathrm{d}}{\mathrm{d}t}\left\langle\hat{S}^y\right\rangle = -\,\omega_0\left\langle\hat{S}^z\right\rangle + \frac{(\omega_z-\omega_x)}{S}\left(\left\langle\hat{S}^x\hat{S}^z\right\rangle + \left\langle\hat{S}^z\hat{S}^x\right\rangle\right) + \frac{\kappa}{2S}\left(\left\langle\hat{S}^y\hat{S}^z\right\rangle + \left\langle\hat{S}^z\hat{S}^y\right\rangle - \left\langle\hat{S}^y\right\rangle\right)$$

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$$d\langle \hat{S}^\alpha \rangle / dt = \sum_{\alpha'=x,y,z} c_{\alpha'} \langle \hat{S}^{\alpha'} \rangle + \sum_{\alpha',\beta'=x,y,z} c_{\alpha',\beta'} \langle \hat{S}^{\alpha'} \hat{S}^{\beta'} \rangle$$

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$$\begin{aligned} d\langle\hat{S}^\alpha\hat{S}^\beta\rangle/dt \quad &= \quad \sum_{\alpha'=x,y,z} c_{\alpha'}\langle\hat{S}^{\alpha'}\rangle + \sum_{\alpha',\beta'=x,y,z} c_{\alpha',\beta'}\langle\hat{S}^{\alpha'}\hat{S}^{\beta'}\rangle + \\ &\quad + \sum_{\alpha',\beta',\gamma=x,y,z} c_{\alpha',\beta',\gamma}\langle\hat{S}^{\alpha'}\hat{S}^{\beta'}\hat{S}^{\gamma}\rangle \end{aligned}$$

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$$\begin{aligned} d\langle\hat{S}^\alpha\hat{S}^\beta\rangle/dt \quad &= \quad \sum_{\alpha'=x,y,z} c_{\alpha'}\langle\hat{S}^{\alpha'}\rangle + \sum_{\alpha',\beta'=x,y,z} c_{\alpha',\beta'}\langle\hat{S}^{\alpha'}\hat{S}^{\beta'}\rangle + \\ &\quad + \sum_{\alpha',\beta',\gamma=x,y,z} c_{\alpha',\beta',\gamma}\langle\hat{S}^{\alpha'}\hat{S}^{\beta'}\hat{S}^{\gamma}\rangle \end{aligned}$$

$d\langle\hat{S}^\alpha\hat{S}^\beta\hat{S}^\gamma\rangle/dt \quad = \quad \cdots$ hierarchy keeps growing ...

Thermodynamic limit / Mean-field

Close the hierarchy on second cumulant (mean-field): $\langle \hat{S}^{\alpha} \hat{S}^{\beta} \rangle \simeq \langle \hat{S}^{\alpha} \rangle \langle \hat{S}^{\beta} \rangle$

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$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\left\langle \hat{S}^x \right\rangle &= -\frac{\omega_z}{S}\left(\left\langle \hat{S}^y \hat{S}^z \right\rangle + \left\langle \hat{S}^z \hat{S}^y \right\rangle\right) + \frac{\kappa}{2S}\left(\left\langle \hat{S}^x \hat{S}^z \right\rangle + \left\langle \hat{S}^z \hat{S}^x \right\rangle + \left\langle \hat{S}^x \right\rangle\right) \\
\frac{\mathrm{d}}{\mathrm{d}t}\left\langle \hat{S}^y \right\rangle &= -\omega_0 \left\langle \hat{S}^z \right\rangle + \frac{(\omega_z - \omega_x)}{S}\left(\left\langle \hat{S}^x \hat{S}^z \right\rangle + \left\langle \hat{S}^z \hat{S}^x \right\rangle\right) + \frac{\kappa}{2S}\left(\left\langle \hat{S}^y \hat{S}^z \right\rangle + \left\langle \hat{S}^z \hat{S}^y \right\rangle - \left\langle \hat{S}^y \right\rangle\right) \\
\frac{\mathrm{d}}{\mathrm{d}t}\left\langle \hat{S}^z \right\rangle &= \omega_0 \left\langle \hat{S}^y \right\rangle - \frac{\kappa}{S}\left(\left\langle (\hat{S}^x)^2 \right\rangle + \left\langle (\hat{S}^y)^2 \right\rangle + \left\langle \hat{S}^z \right\rangle\right) + \frac{\omega_x}{S}\left(\left\langle \hat{S}^x \hat{S}^y \right\rangle + \left\langle \hat{S}^y \hat{S}^x \right\rangle\right) .
\end{aligned}
$$

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\end{aligned}
$$

. macroscopic/normalized observables: $\hat{m}^\alpha = \hat{S}^\alpha / S$

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$$
\begin{aligned}
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\end{aligned}
$$

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$$
\begin{aligned}
\frac{d}{dt} m^x &= \kappa m^x m^z \\
\frac{d}{dt} m^y &= -\omega_0 m^z + \kappa m^y m^z \\
\frac{d}{dt} m^z &= \omega_0 m^y - \kappa\left((m^x)^2 + (m^y)^2\right)
\end{aligned}
$$

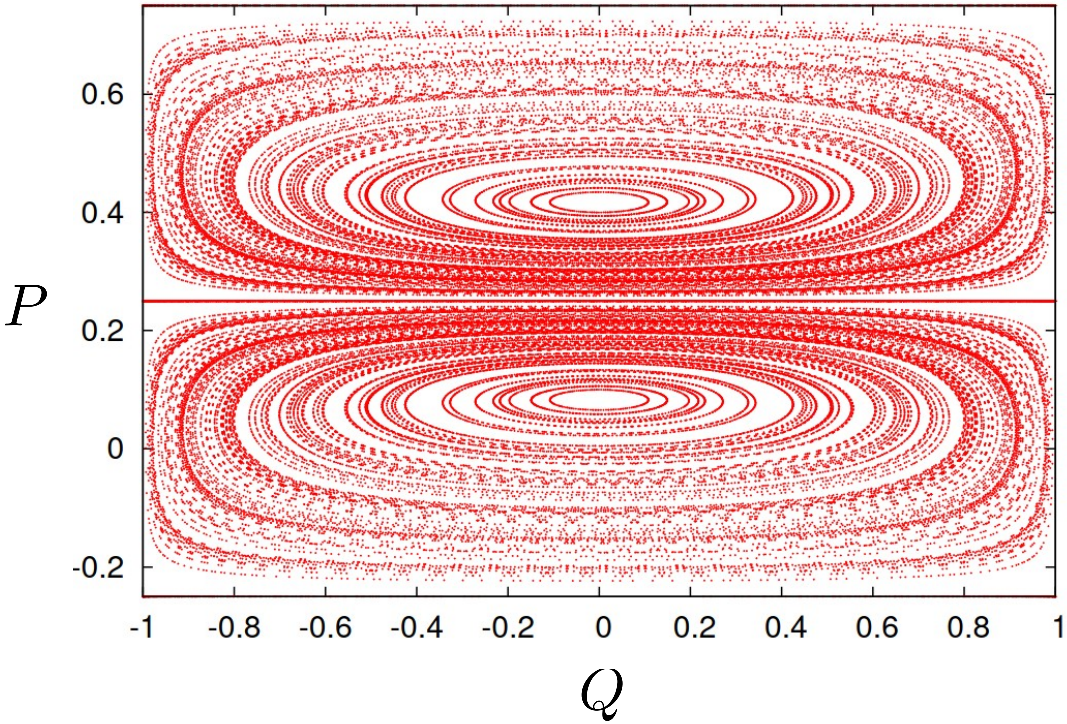
simpler, analytical solutions, stability/Jacobian...

Thermodynamic limit / Mean-field

$\boxed{\kappa < \omega_0}$

Poincaré section: "phase space"

$$
\begin{aligned}
Q &= m^z \\
P &= (m^x)^2 + (m^y)^2
\end{aligned}
$$



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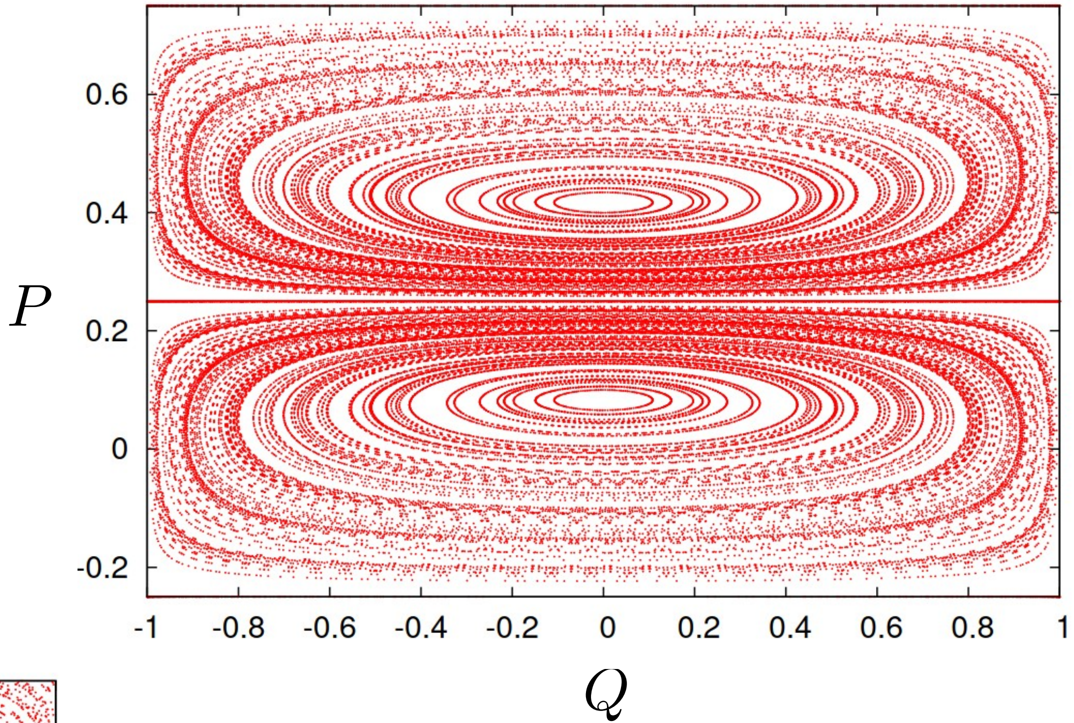
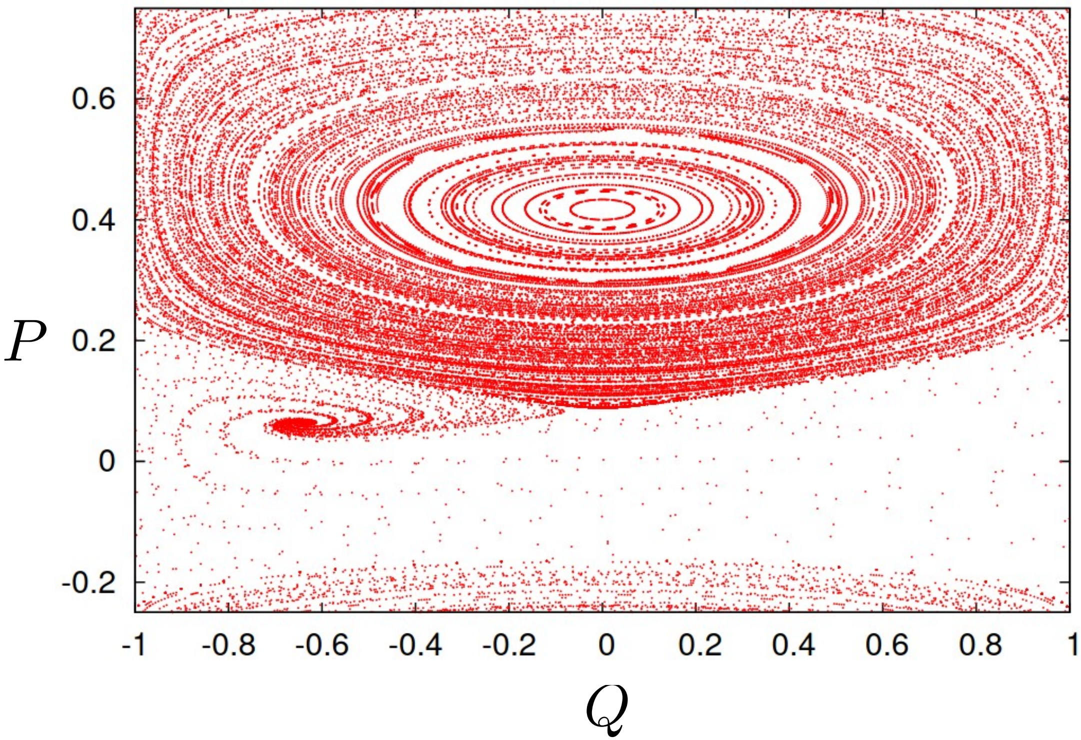


$\omega_0 = 1,\ \kappa = 0.5,\ \omega_z = 0.6$



robust to perturbations

$$\hat{H} = \omega_0 \hat{S}^x + \boxed{\frac{\omega_z}{S}(\hat{S}^z)^2 + \frac{\omega_x}{S}(\hat{S}^x)^2}$$

Conclusions

- we discussed about time crystals, intrisically non-equilibrium phases;

- while Discrete TC arise in closed systems, continuous seems to be "*more likely*" in ***open systems:***

  . ***boundary time crystals*** (macroscopic portion of whole system)

  . collective spin models featuring these phases.

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  . mostly collective spins at the moment… (few exceptions)

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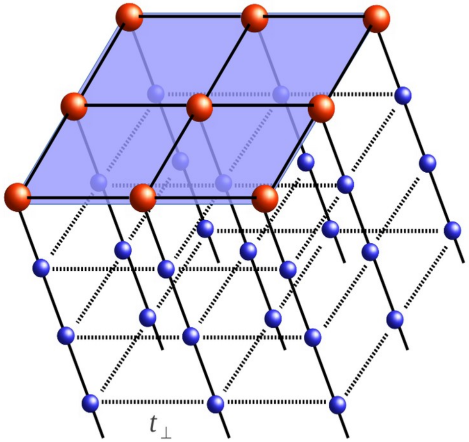

XYZ-Heisenberg, quantum-Ising –>
limit cycles at the mean-field level.

T. E. Lee et al, PRA 84, 031402 (2011);
C.-K. Chan et al, PRA 91, 051601 (2015).

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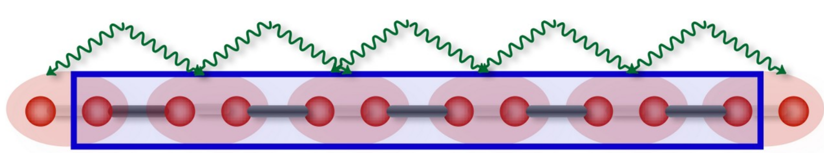
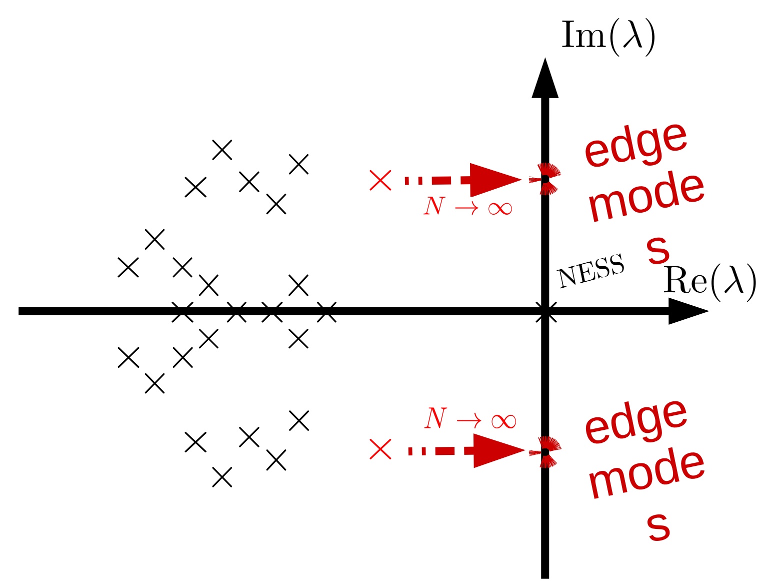


$$\hat{\ell}_j = (\hat{a}_j^\dagger + \hat{a}_j)(\hat{a}_{j+1}^\dagger - \hat{a}_{j+1})$$



S. Diehl et al, Nat. Phys. 7, 971 (2011);
F. Iemini et al, PRB 93, 115113 (2016).

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  . others?

- and... *why should I care*? Applications?

  . Sensors . Clocks
  . Quantum engines . Simulating complex quantum networks
  ….

Thanks for your attention!

If you are interested…
contact: fernandoiemini@id.uff.br

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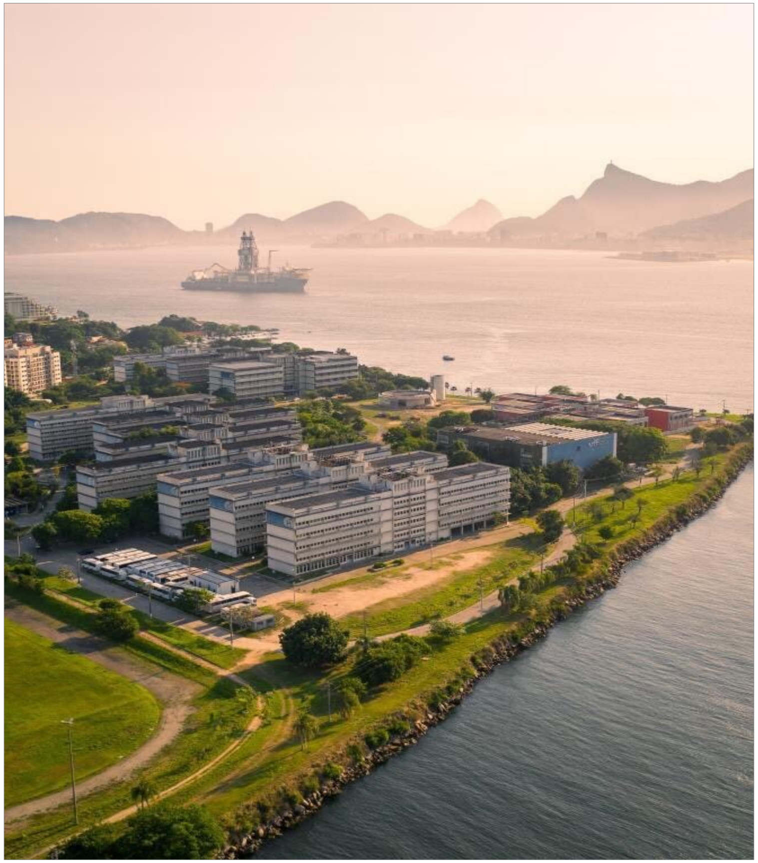
Universidade Federal Fluminense

serrapilheira



UFF,Niterói,Brazil