Dynamics in Quantum Systems - Metrology and Time Crystals

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INSTITUTO DE FÍSICA
Universidade Federal Fluminense

Outline

- Metrology;

- Sensing with Time Crystals;

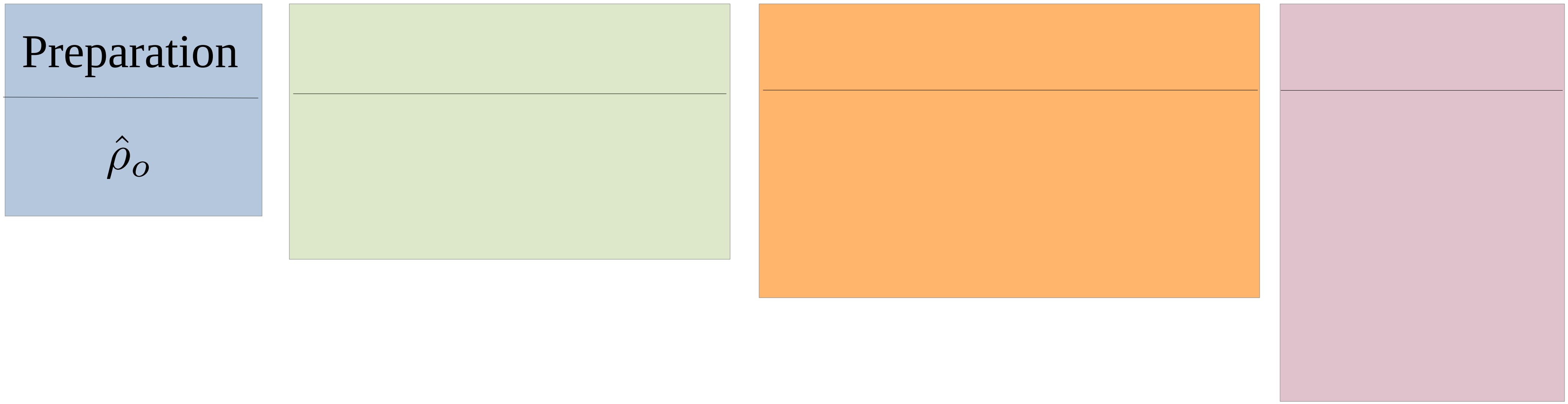
Metrology

Goal: estimate a physical quantity - electric/magnetic fields, frequencies, temperature…

basic steps: use a sensor (quantum system) to interact with the physical signal, and then read out the sensor and estimate it.

C. L. Degen et al., Quantum Sensing,
Reviews of Modern Physics, 89, 035002 (2017).

Metrology



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Metrology

Preparation

$\hat{\rho}_o$

Evolution

$$\hat{\rho}_\theta = \hat{U}_\theta \hat{\rho}_o \hat{U}_\theta^\dagger$$

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Metrology

Preparation	Evolution	Measurements	
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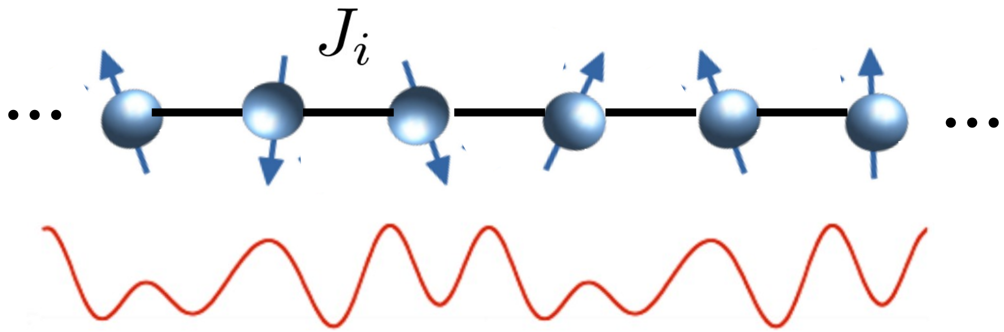
Floquet time-crystals

[Dominic V. Else, Bela Bauer, and Chetan Nayak, PRL 117, 090402 (2016)]

Combining **Floquet dynamics** + **MBL** (many-body localization) can support a **DTC**:

$$\hat{H}(t) = \hat{H}_{MBL} + \hat{X} \sum_{n=0}^{\infty} \delta(t - nT)$$



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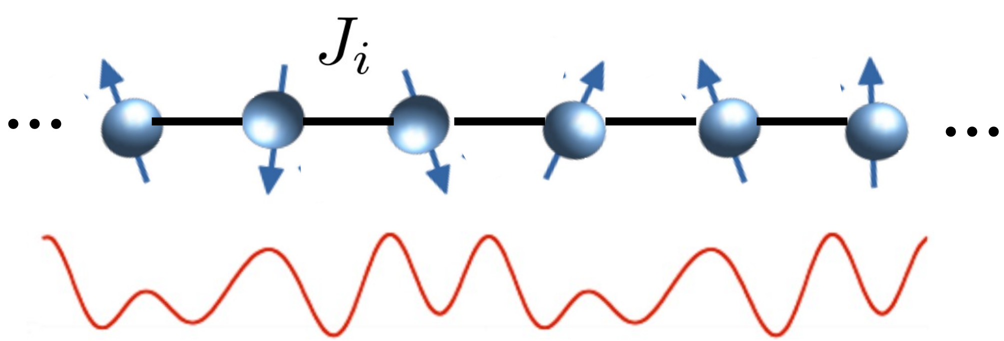
$$\phi \sim \pi$$



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$$|\psi(0)\rangle = |\uparrow\uparrow\downarrow\uparrow\downarrow \ldots\rangle$$

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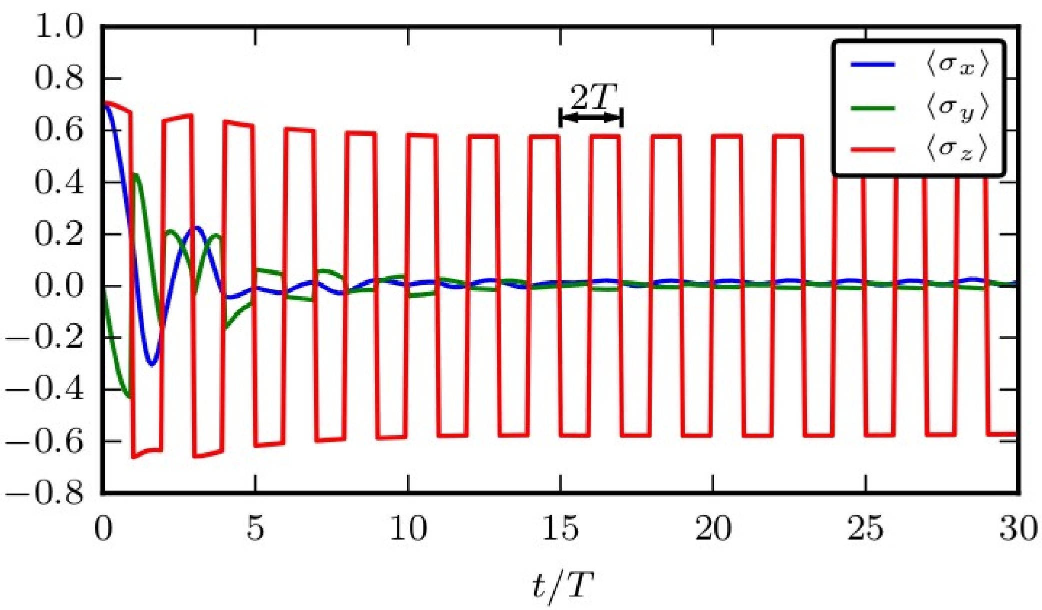
period doubling for magnetization



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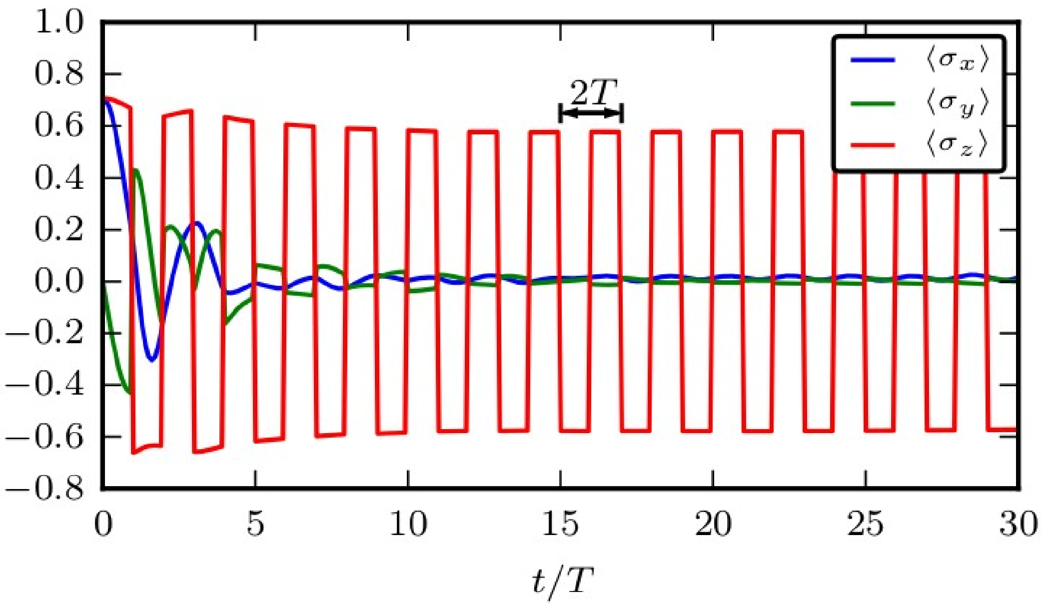
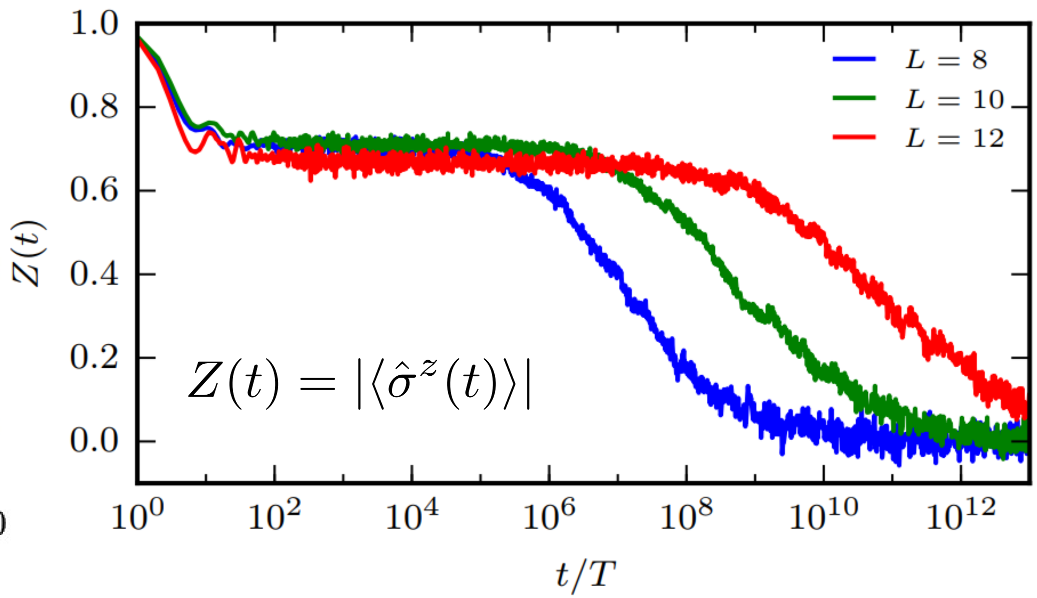
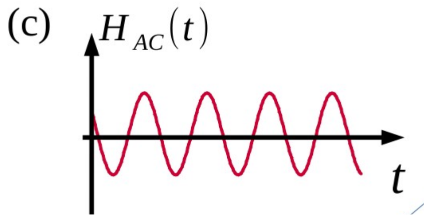


*persistence **only** in the thermodynamic limit*



Sensing AC fields



$$\hat{H}_{\rm AC} = h_{\rm ac} \sin(\omega_{\rm ac} t + \phi_{\rm ac}) \hat{S}^z$$

???

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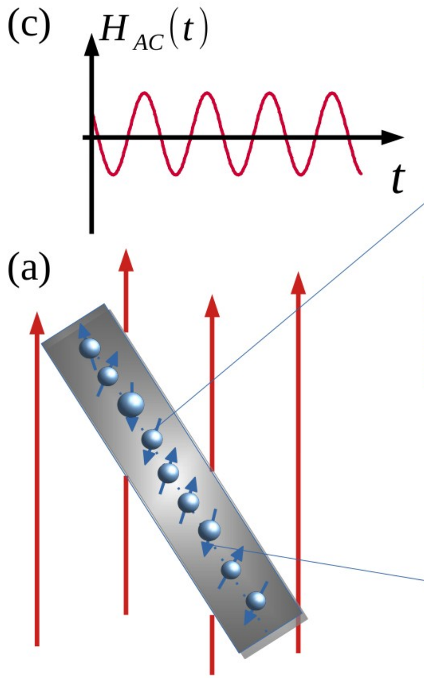

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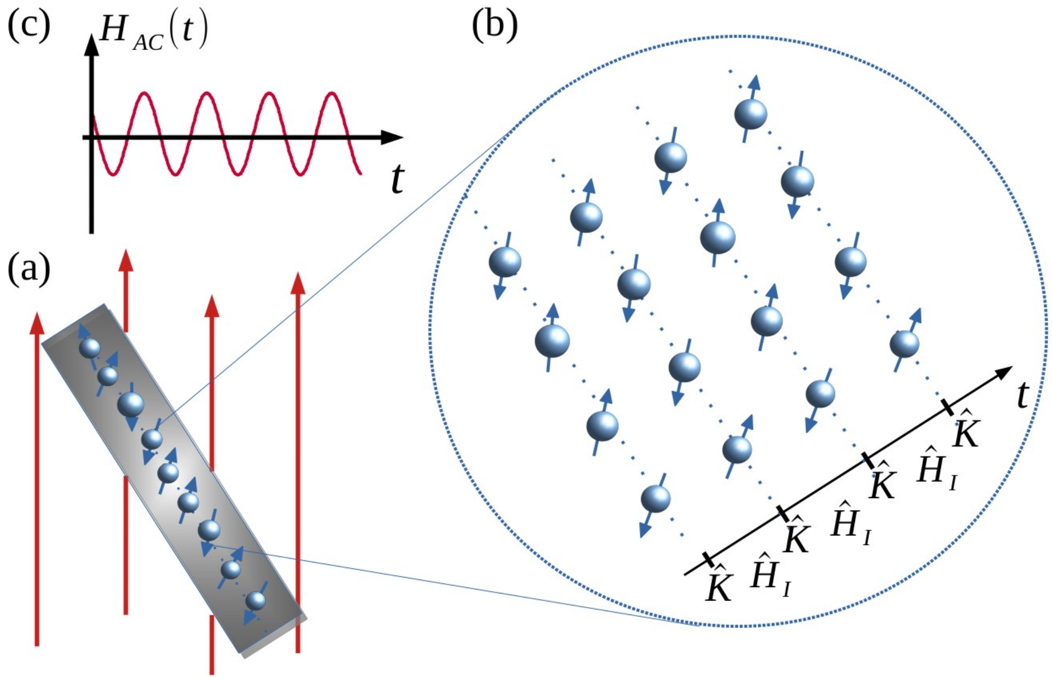

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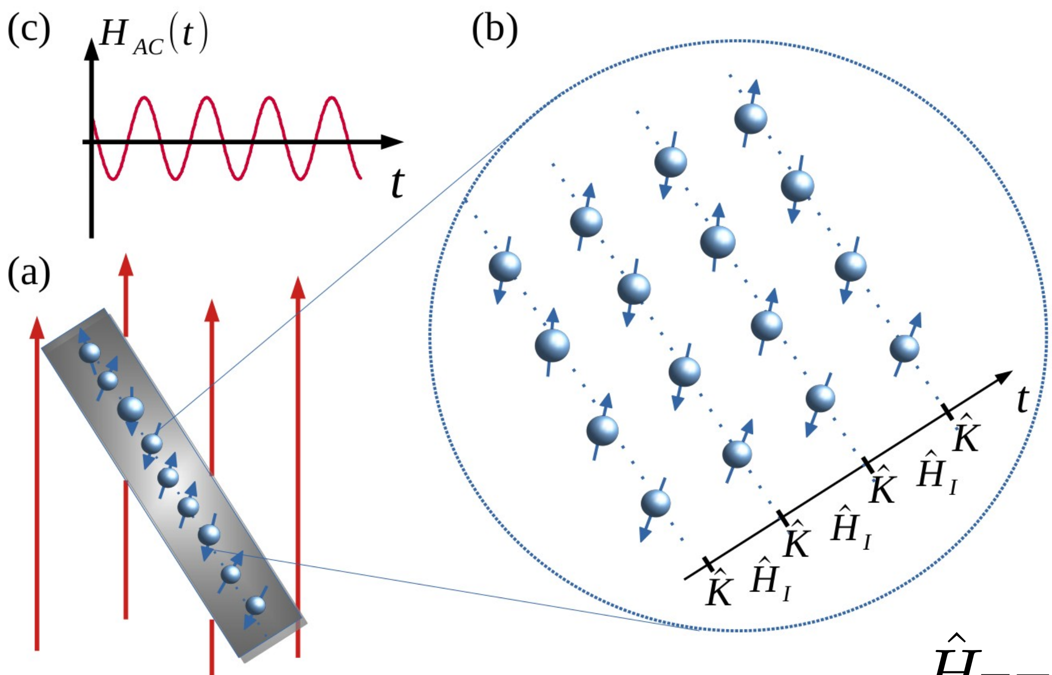

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???

$$\hat{H}_{\mathrm{FTC}}(t) = \hat{H}_{MBL} + \hat{X} \sum_{n=0}^{\infty} \delta(t - nT)$$

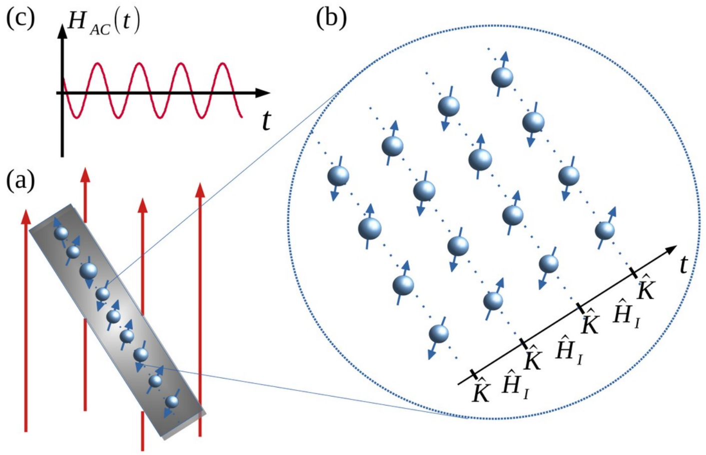
$$\hat{H}(t) = \hat{H}_{\mathrm{FTC}}(t) + \hat{H}_{\mathrm{AC}}(t)$$

FTC sensor

exploit long-range spatial and time ordering for enhanced metrological protocols.

F. Iemini, Rosario Fazio, Anna Sanpera, arXiv:2306.03927 (2023)

Sensing AC fields



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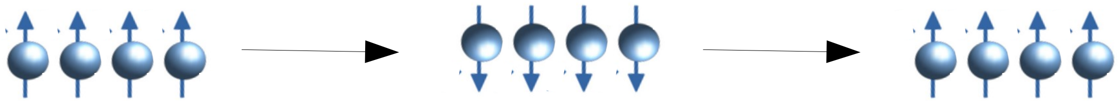
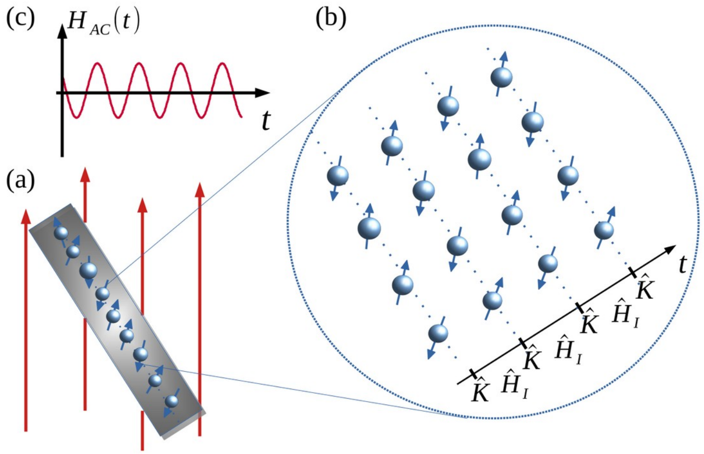
Physical picture



Sensing AC fields



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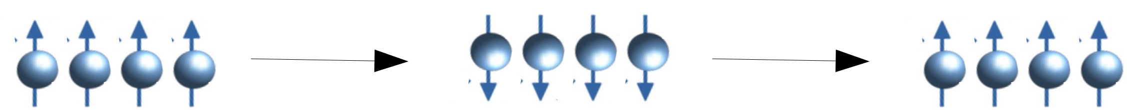
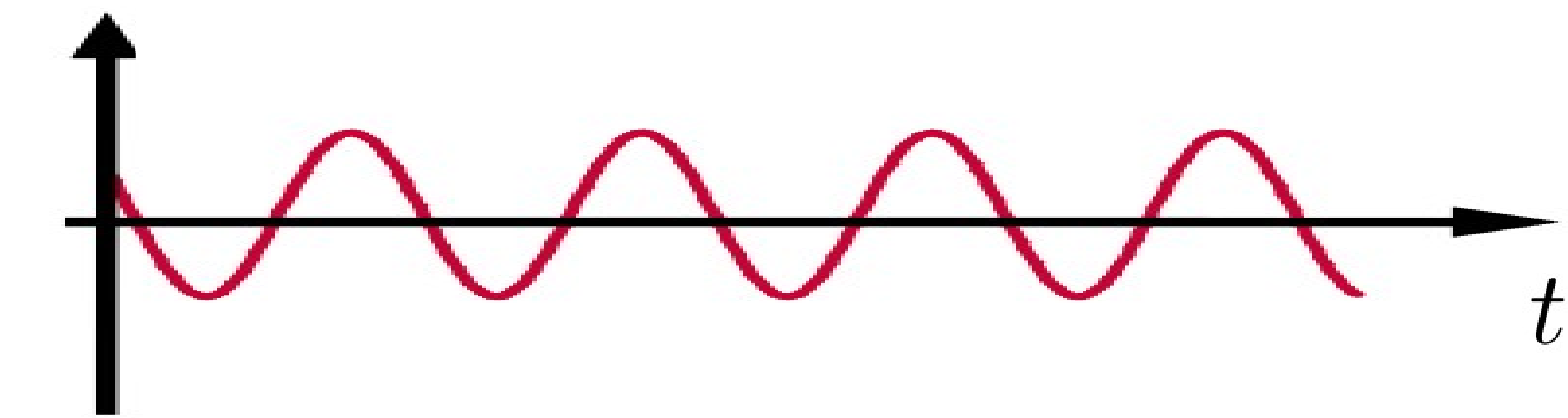
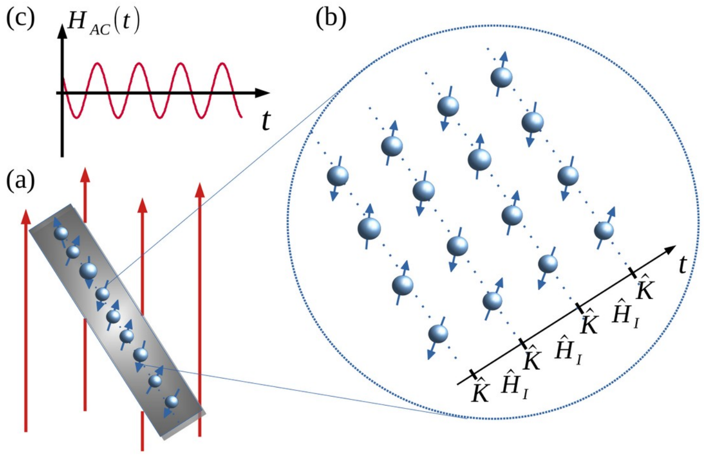


$\hat{H}_{\mathrm{AC}}(t)$



Sensing AC fields



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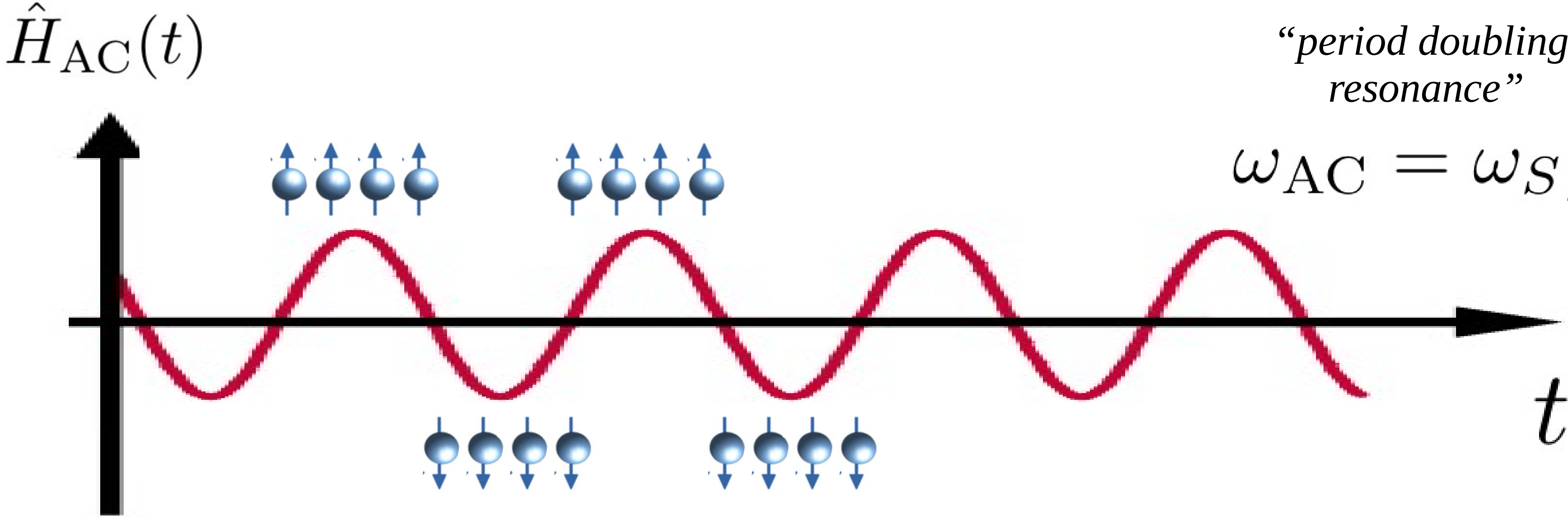
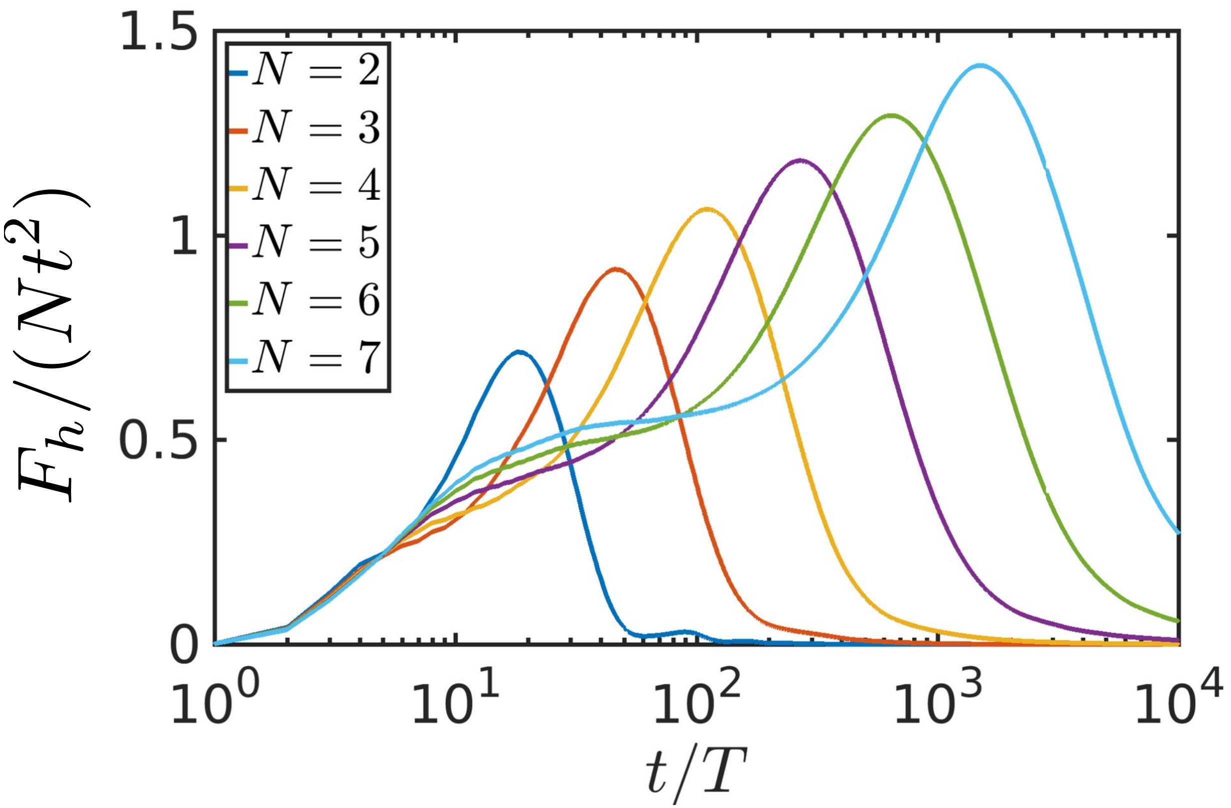
Physical picture

"period doubling resonance"

$$\omega_{\mathrm{AC}} = \omega_S/2$$



Sensor performance: $\phi \neq \pi$, $h_i^x \neq 0$, (linear response $h_{\mathrm{AC}} \to 0$), $\omega_{AC} = \omega_S/2$



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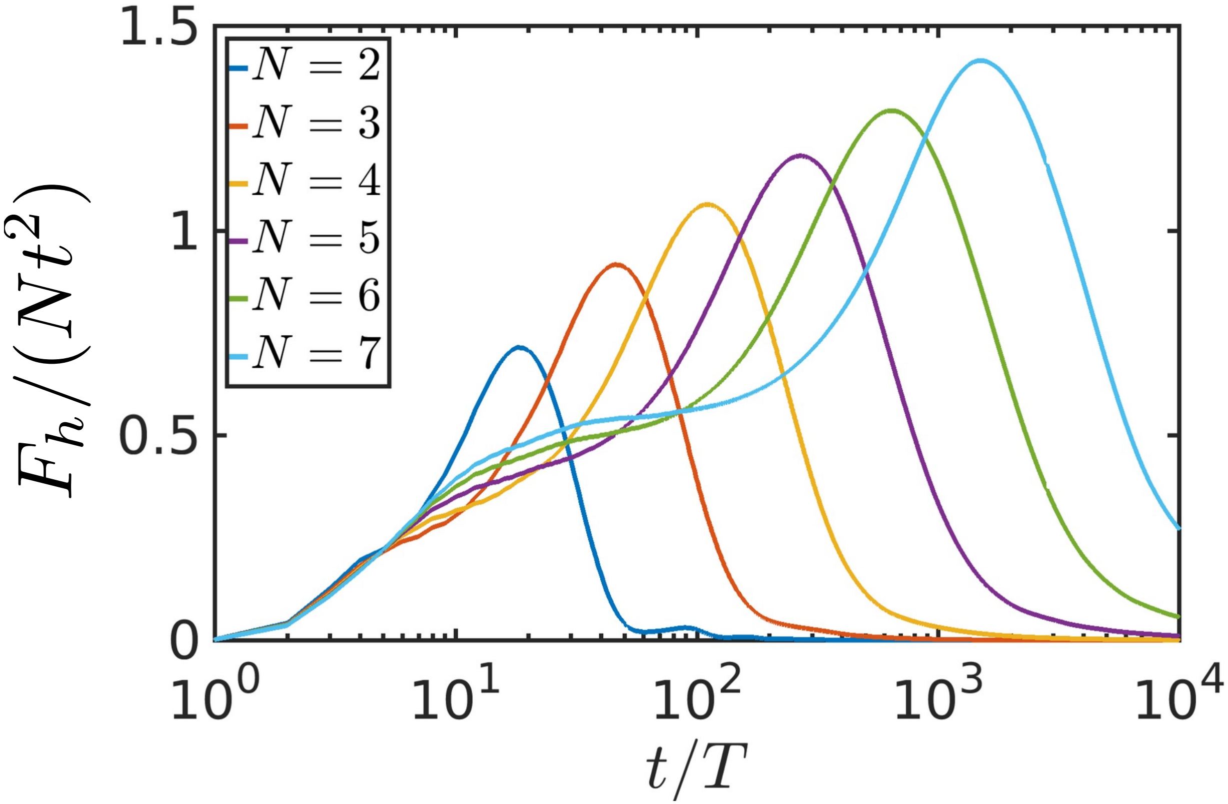

. saturates SQL for a plateau time;

. beyond SQL for exponentially long time;

. exponentially large sensing protocol,

$$t^* \sim e^{\gamma N}$$

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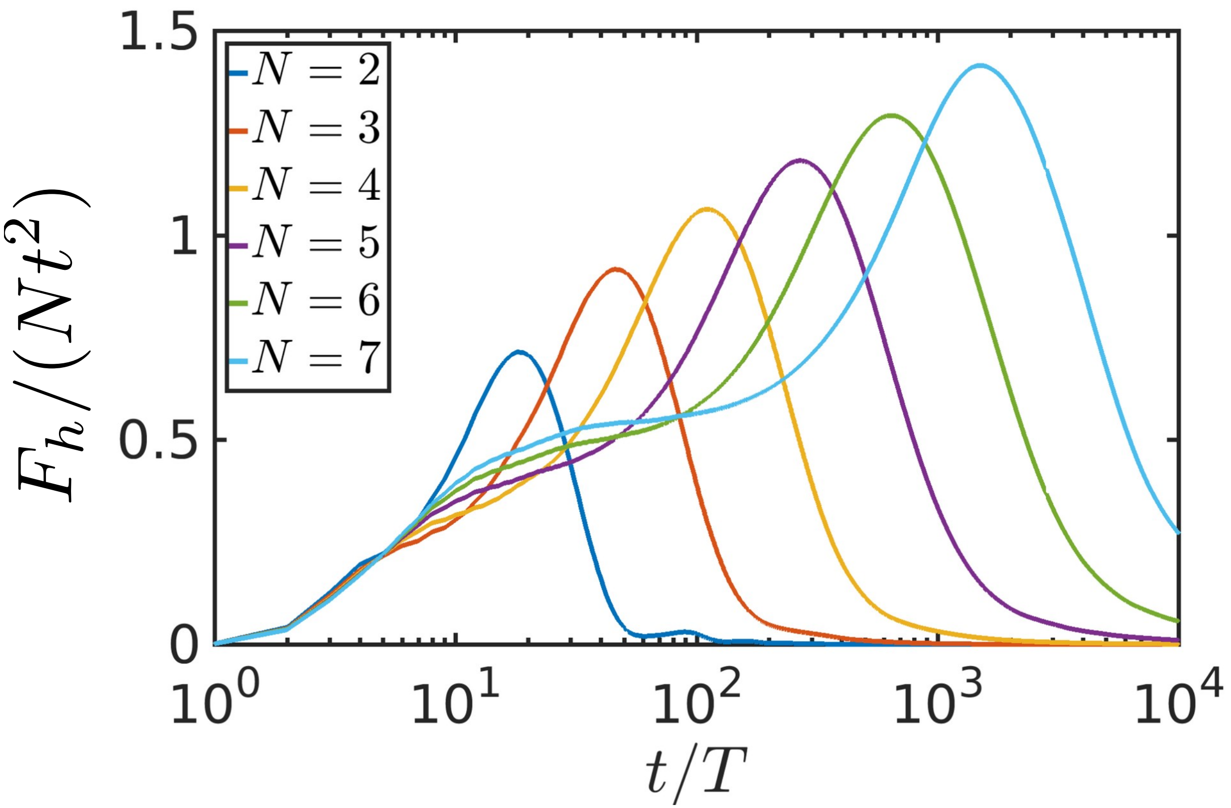

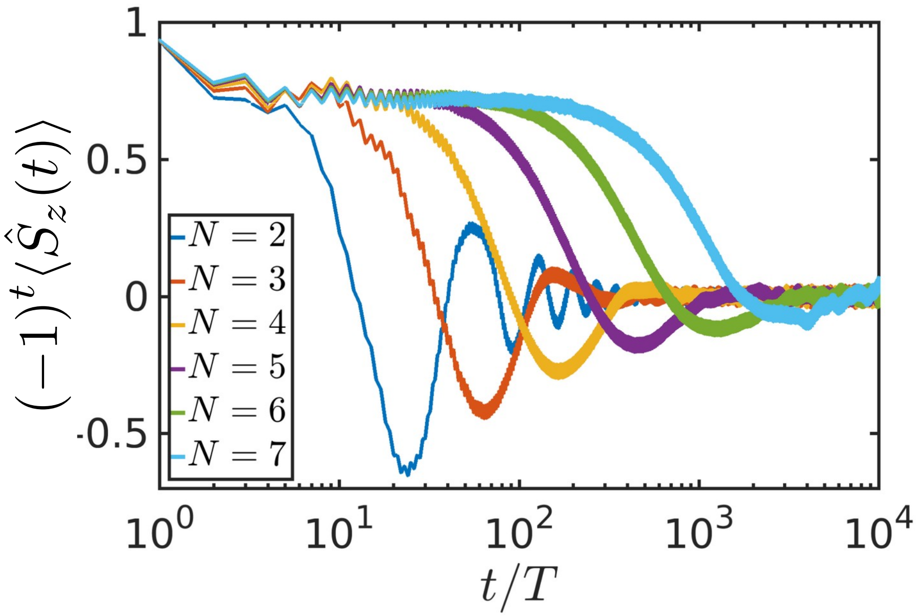

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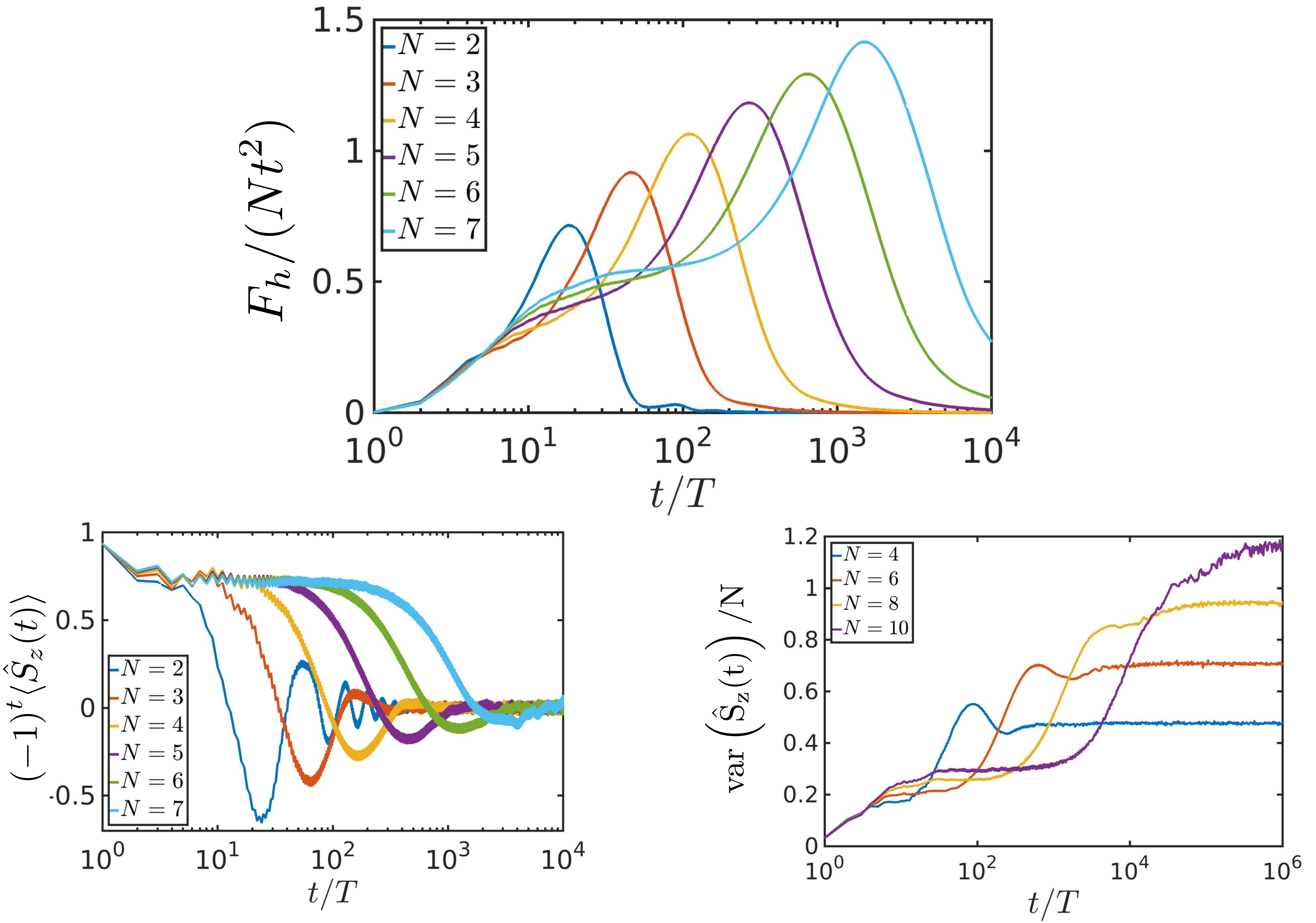
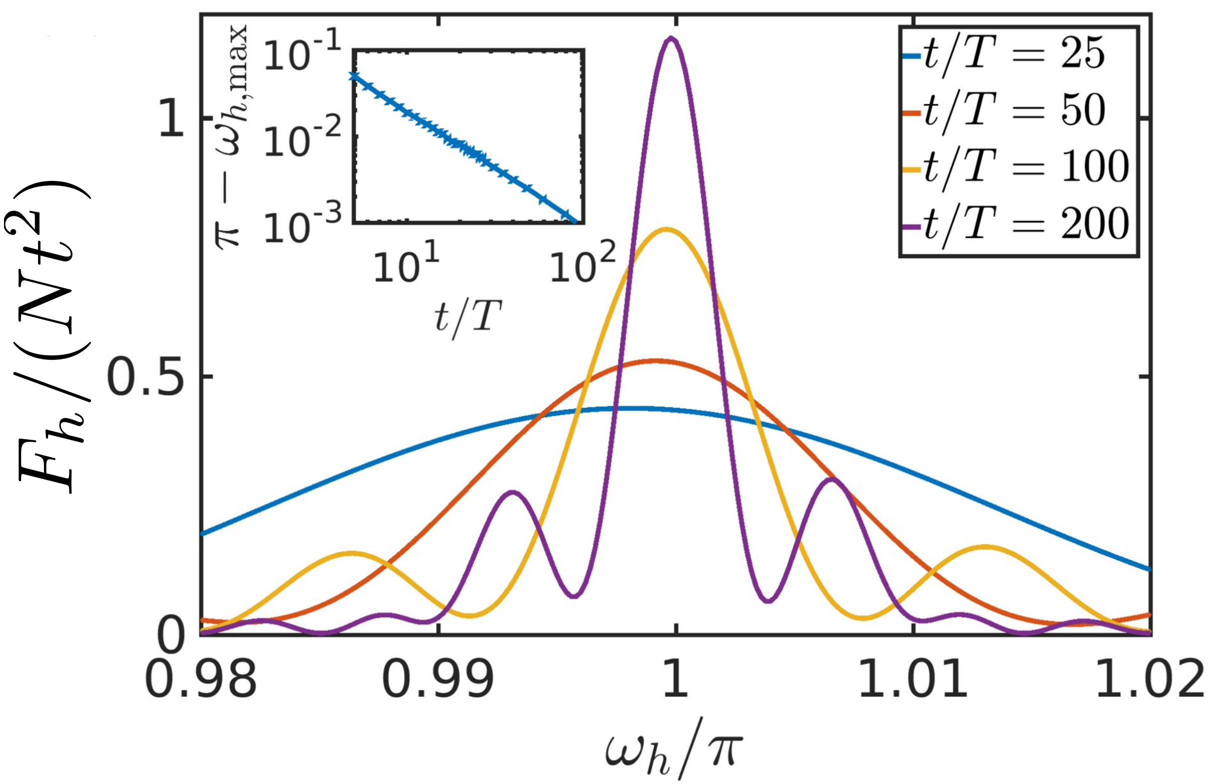


. scanning frequency:



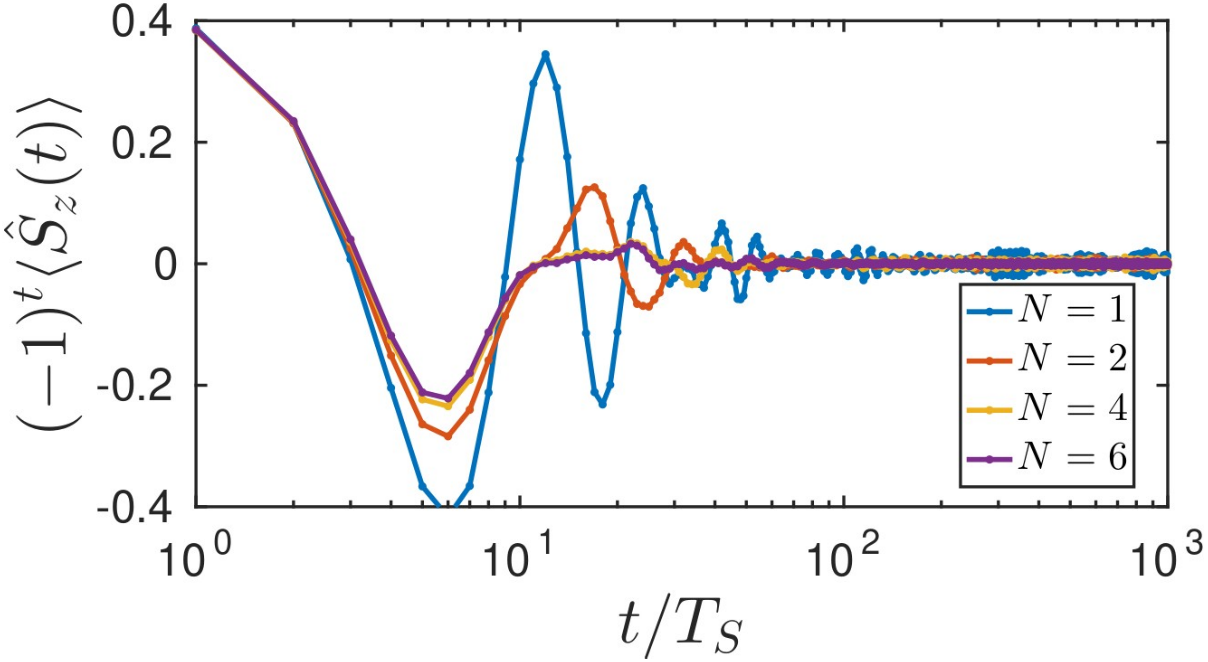
. structured response, robust to decoherence/noise;

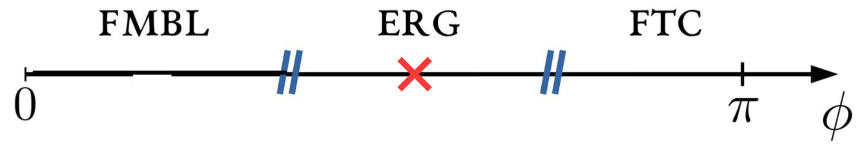
. characteristic time scale:

$$t^* \sim |\omega - \pi|^{-1}$$

. dual role for the sensor.

Ergodic Sensor





- fast thermalization time ~ O(T);

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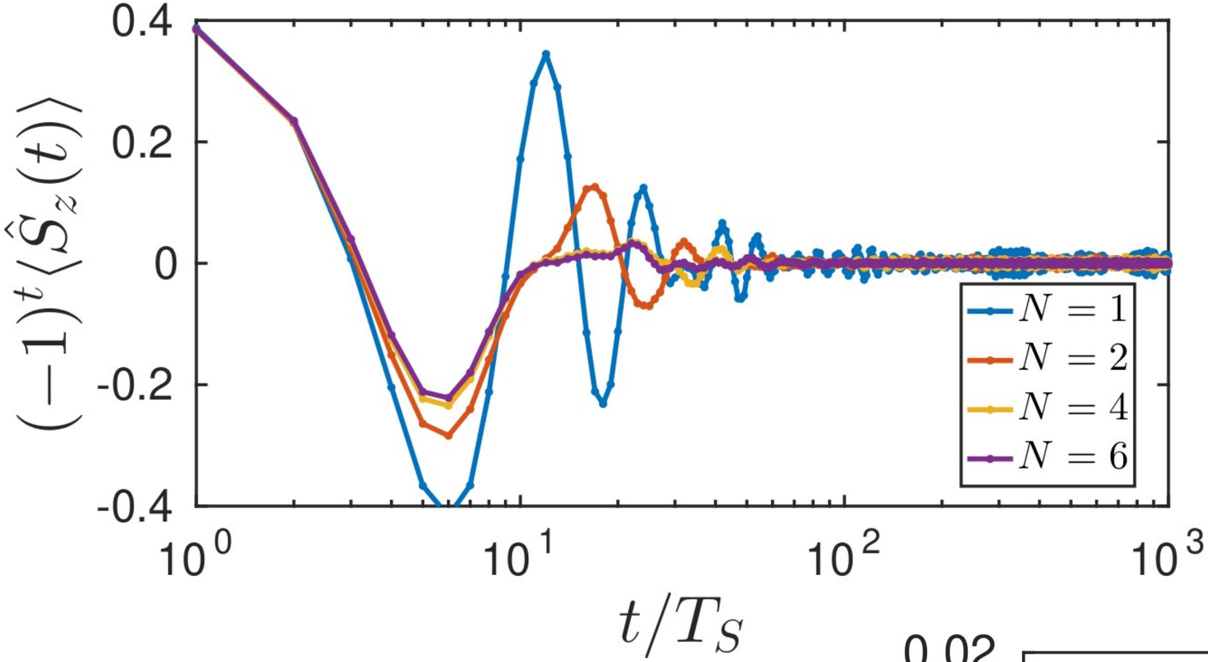

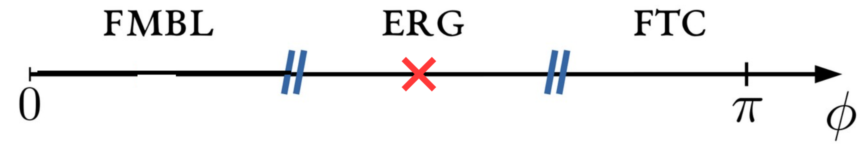

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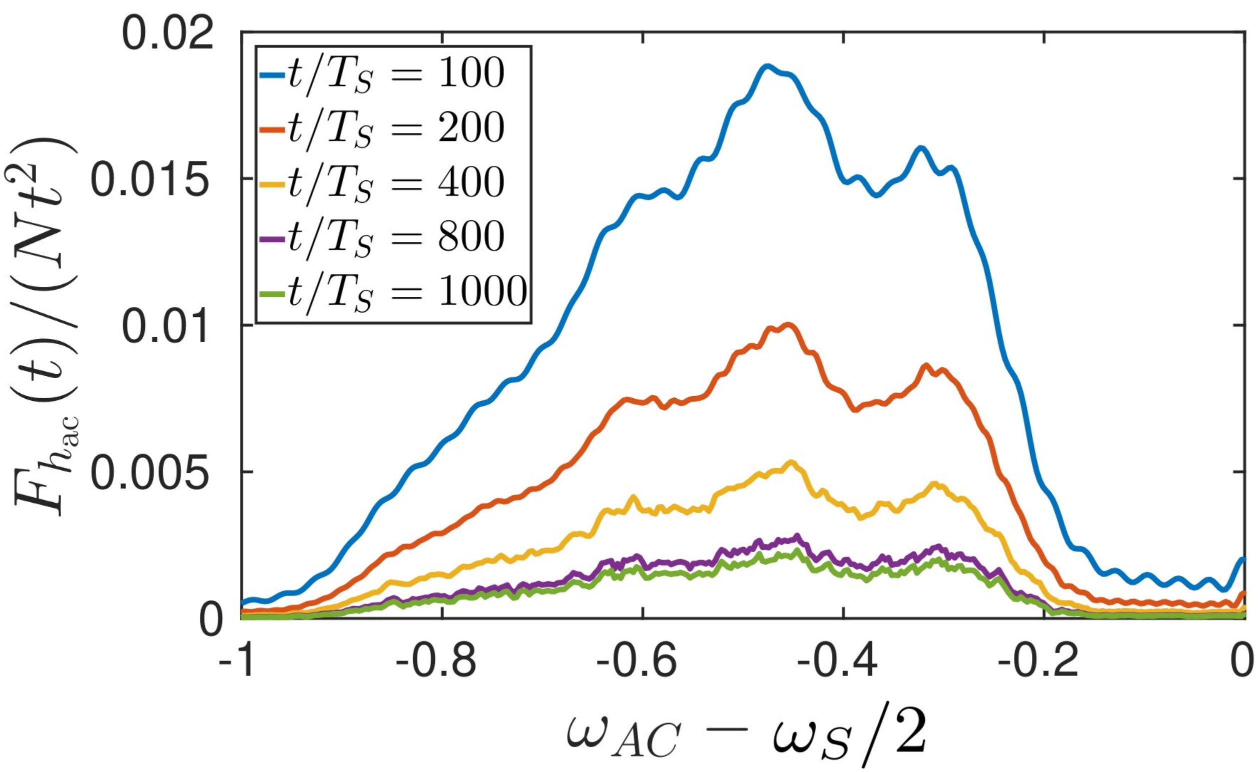
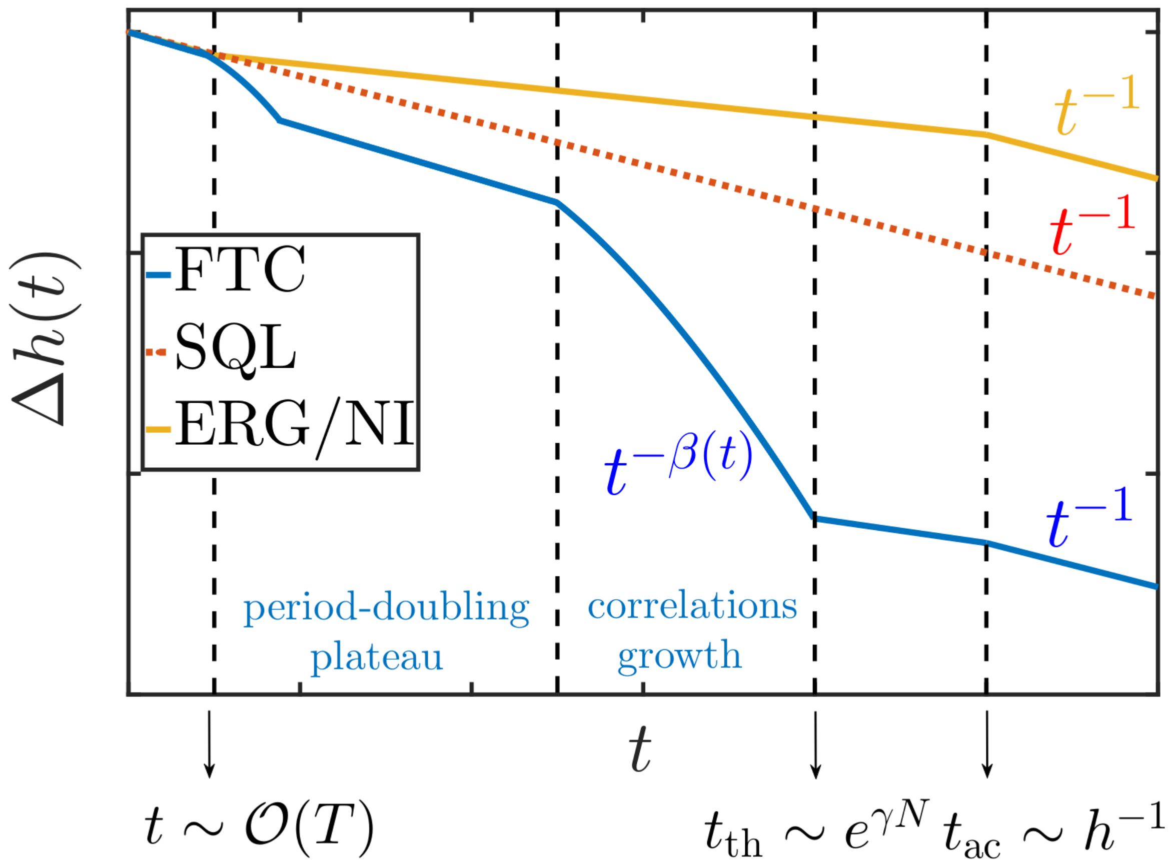


- noisy response;

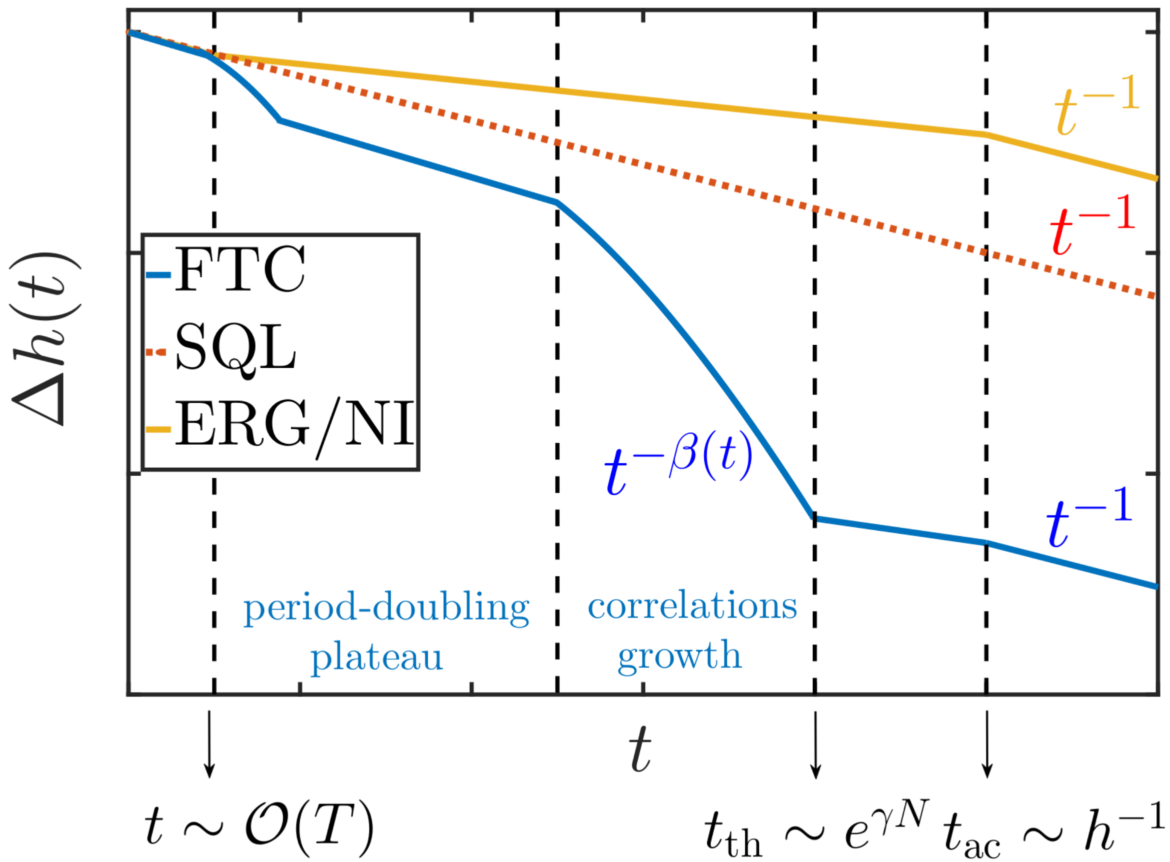
- below standart quantum limit (SQL);

Summary



t^{-1}
t^{-1}
FTC
SQL
ERG/NI
$\Delta h(t)$
$t^{-\beta(t)}$
t^{-1}
period-doubling
plateau
correlations
growth
t
$t \sim \mathcal{O}(T)$
$t_{\mathrm{th}} \sim e^{\gamma N}$
$t_{\mathrm{ac}} \sim h^{-1}$

Summary



Floquet Time Crystal sensors offer several advantages:

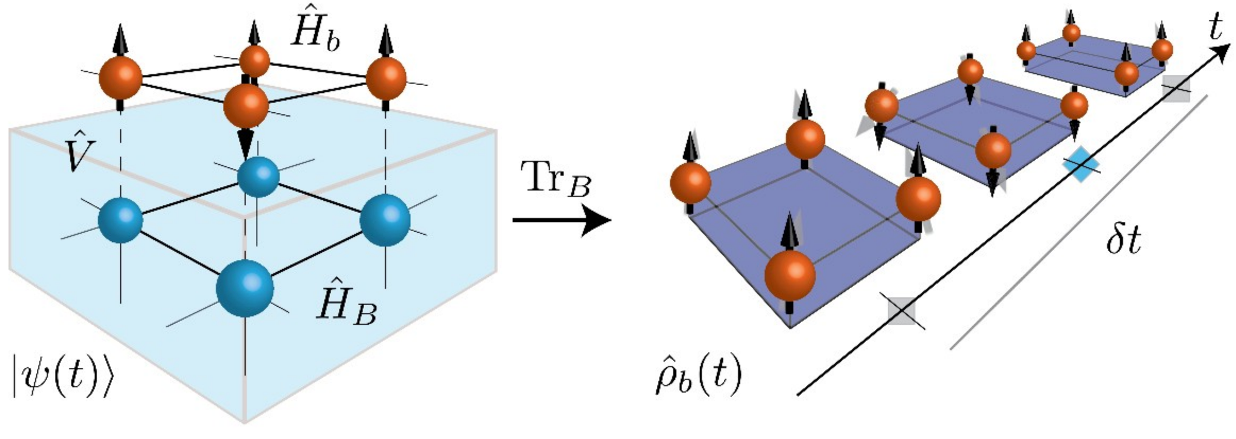
- **slow heating** – exponentially slow with system size N

- useful quantum correlations - **overcome classical limit;**

- **robust** protocol.

Boundary Time Crystal

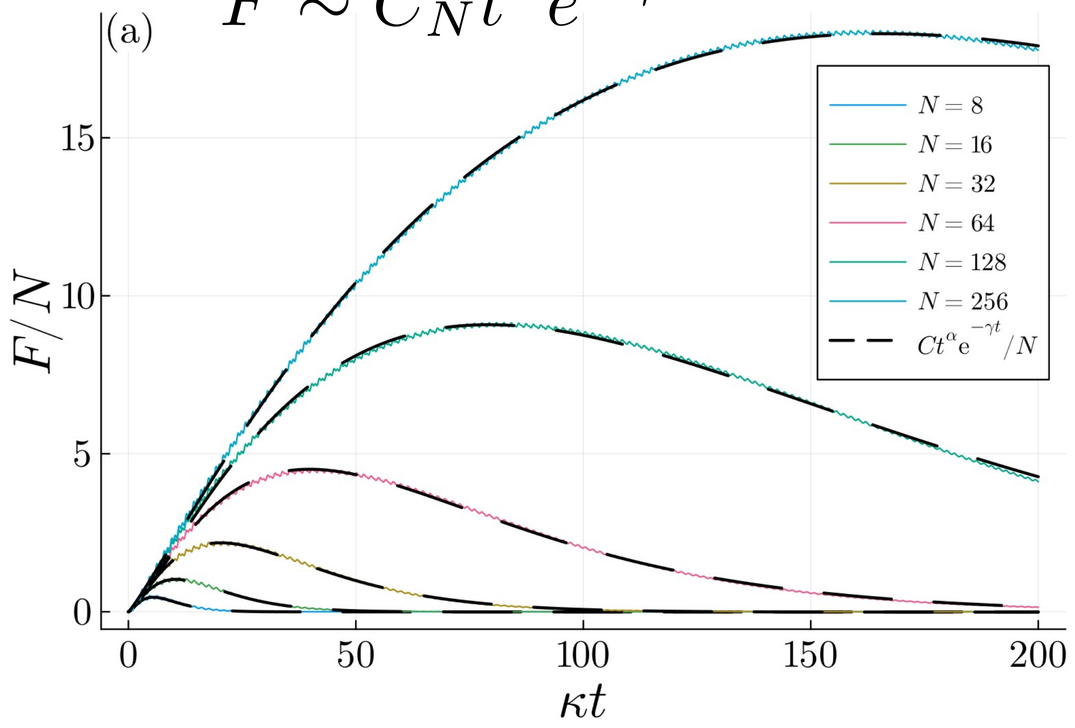
open-system sensors



Dominic Gribben, Anna Sanpera, Rosario Fazio, Jamir Marino, F. Iemini, arXiv:2406.06273 (2024)

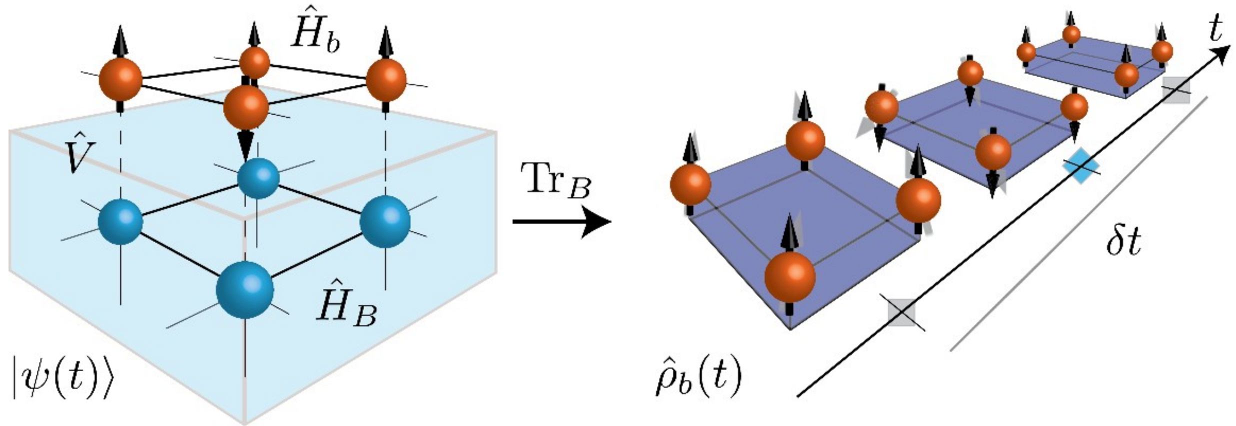
Boundary Time Crystal

$$F \sim C_N t^{\alpha} e^{-\gamma t}$$



*BTC in **resonance** to the AC probing field*

open-system sensors



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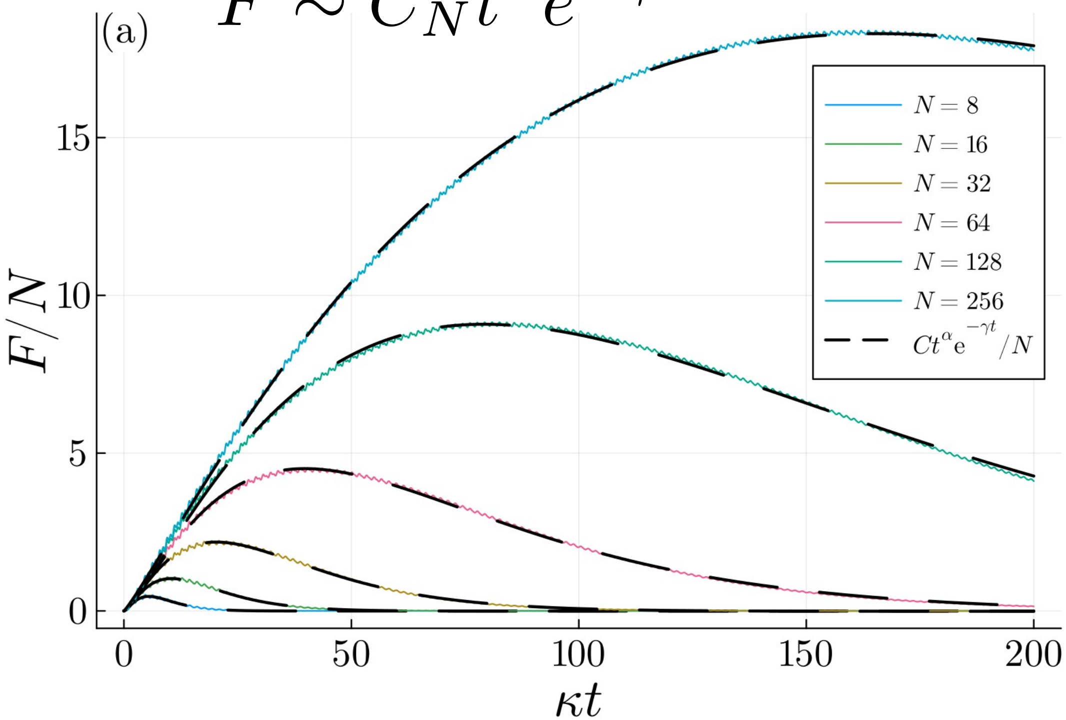

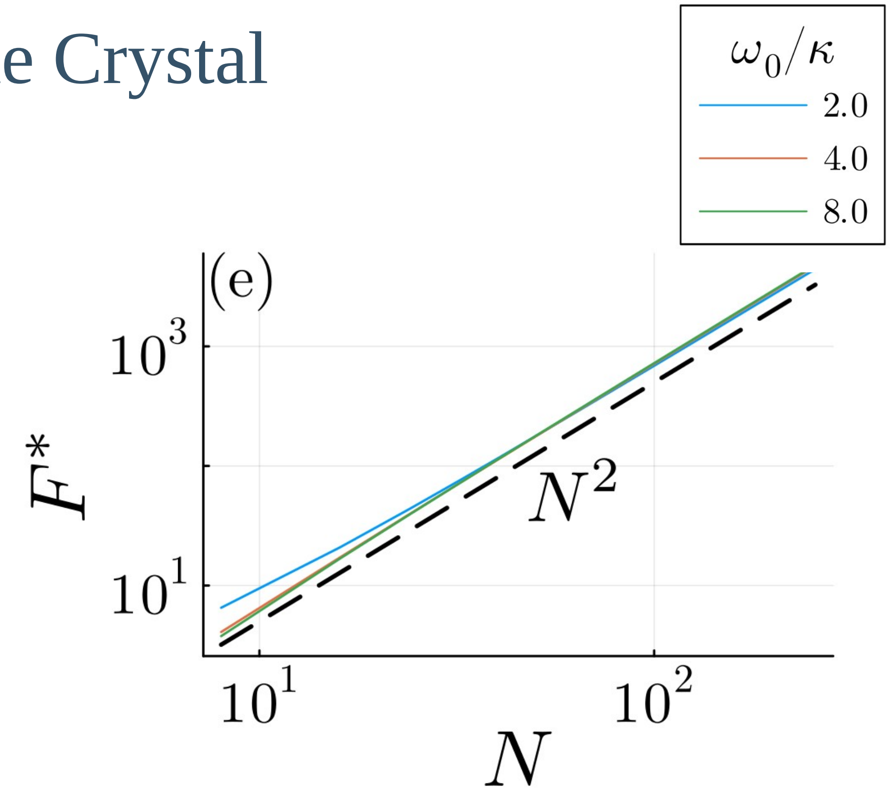

single-shot runs:
collective enhancement

Boundary Time Crystal

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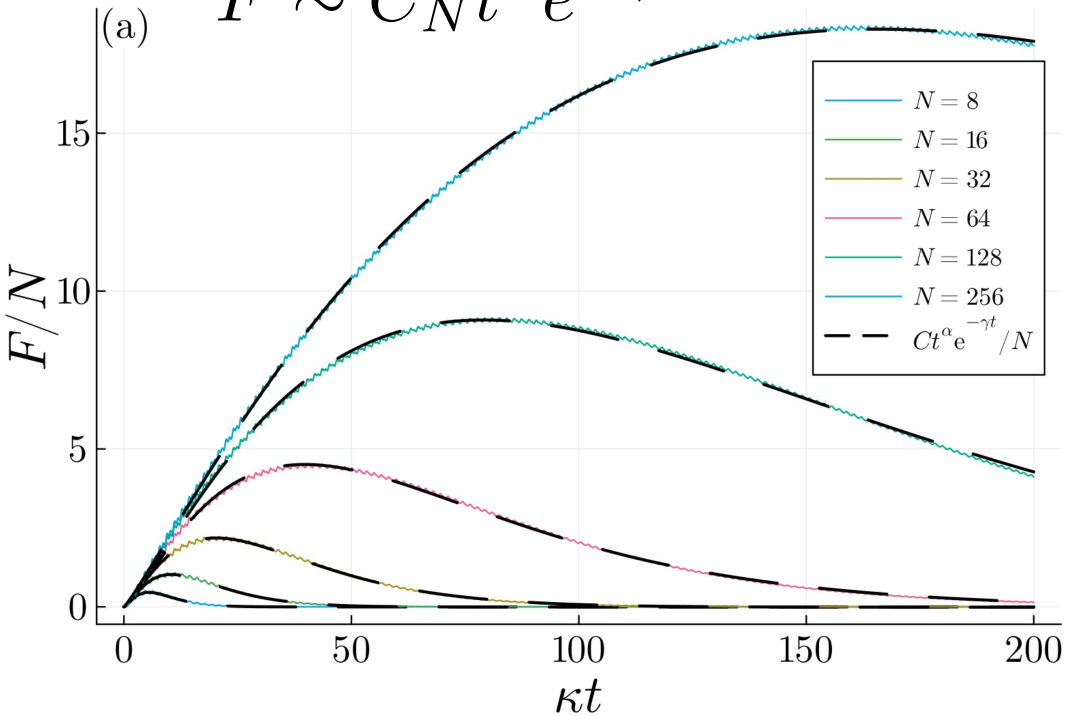

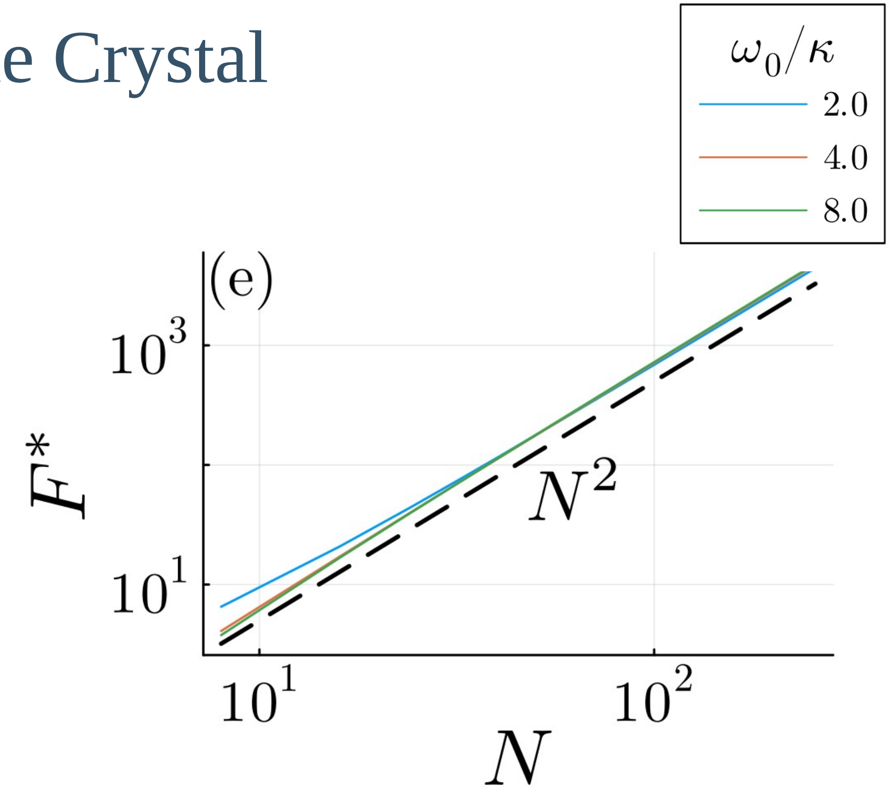

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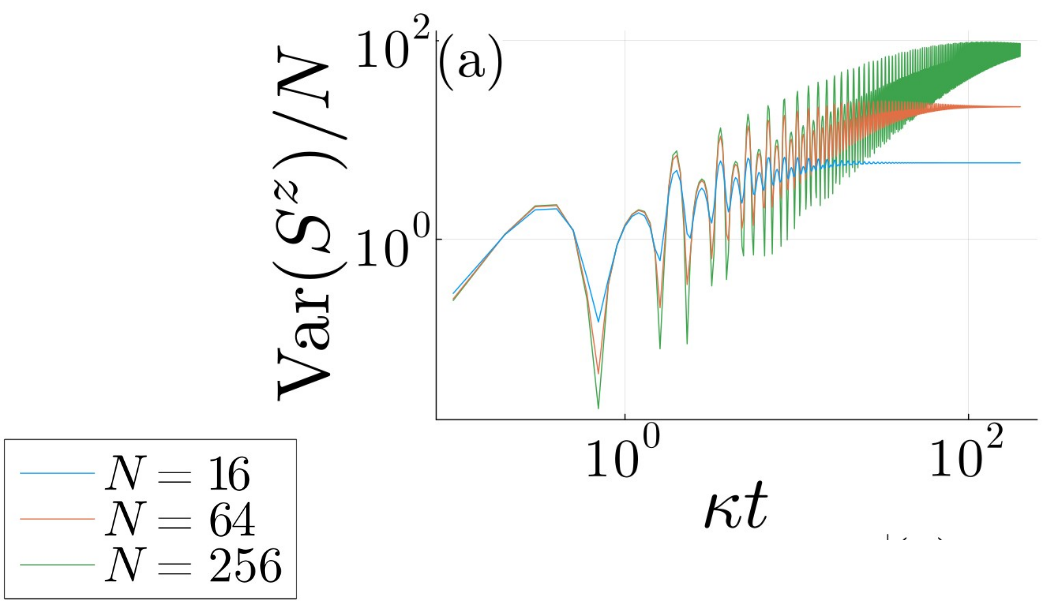
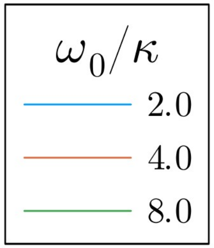


- long coherence time + correlations

Boundary Time Crystal



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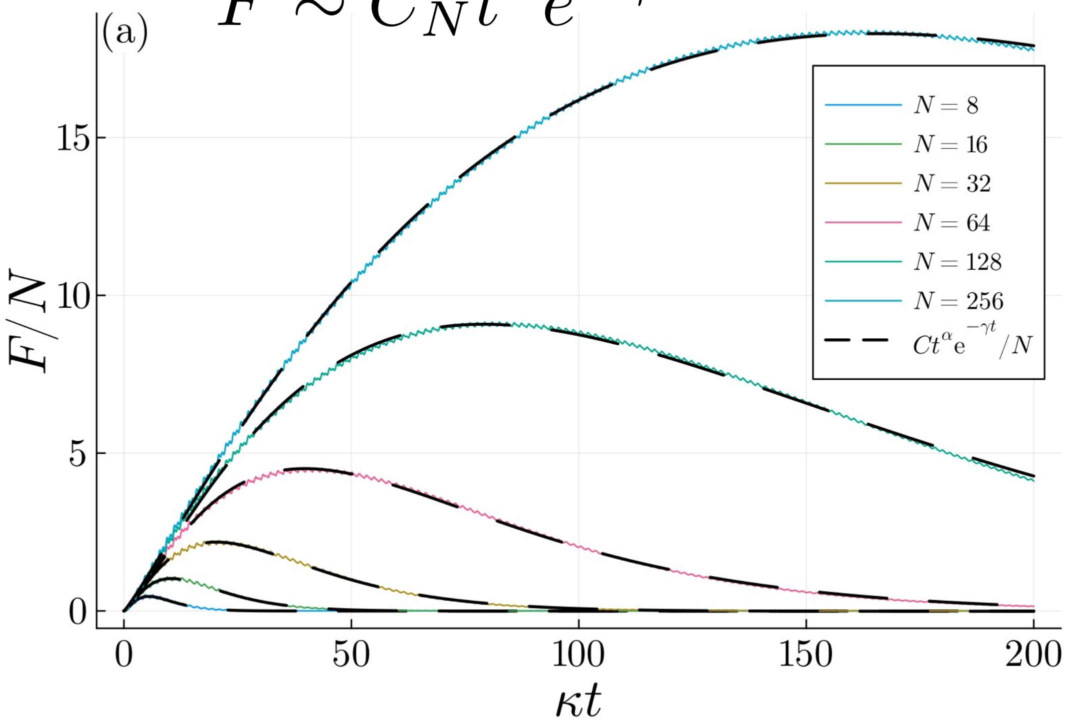
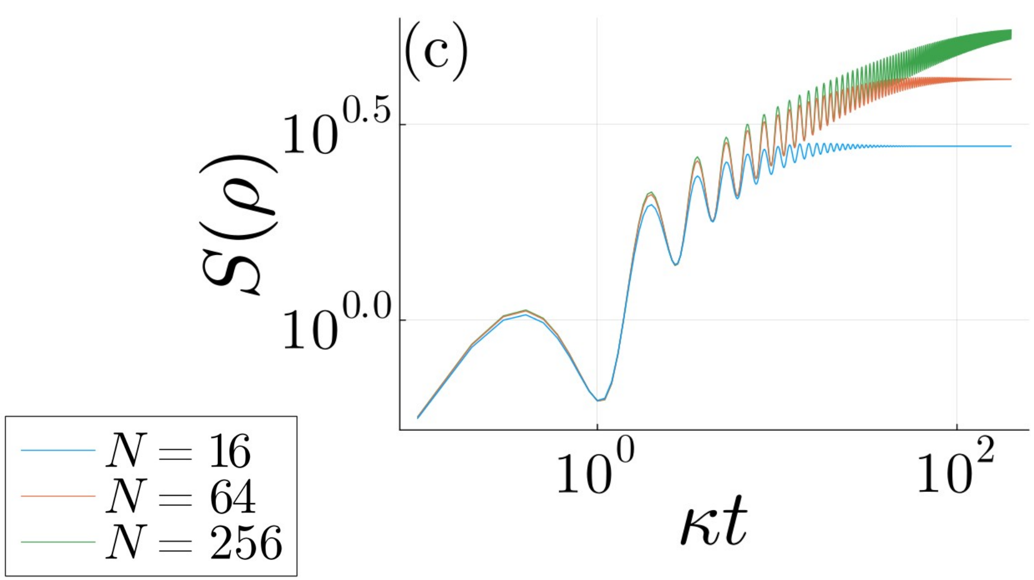


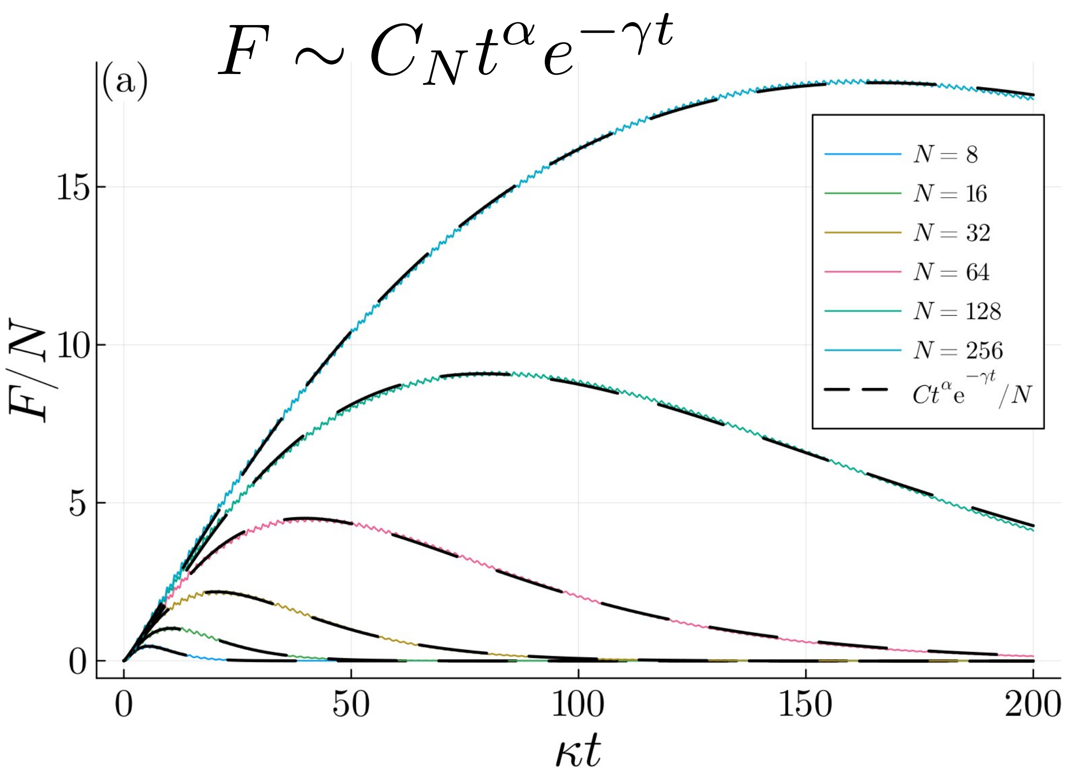


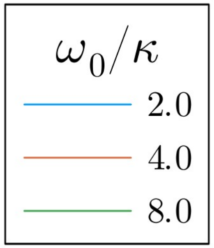
- long coherence time + correlations

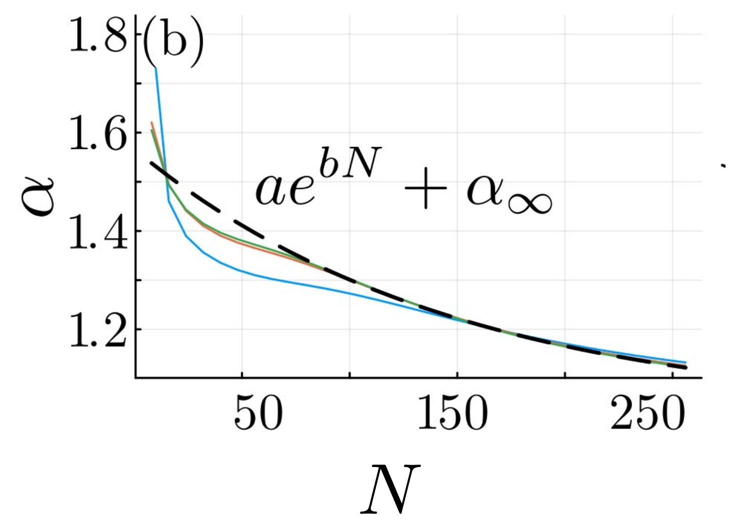
vs

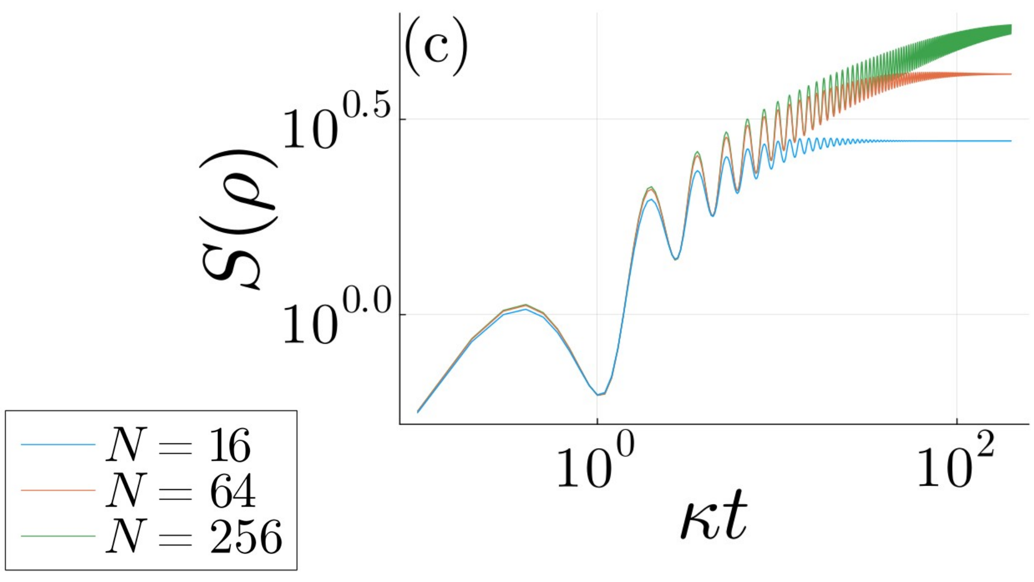
entropic constraints?

Boundary Time Crystal









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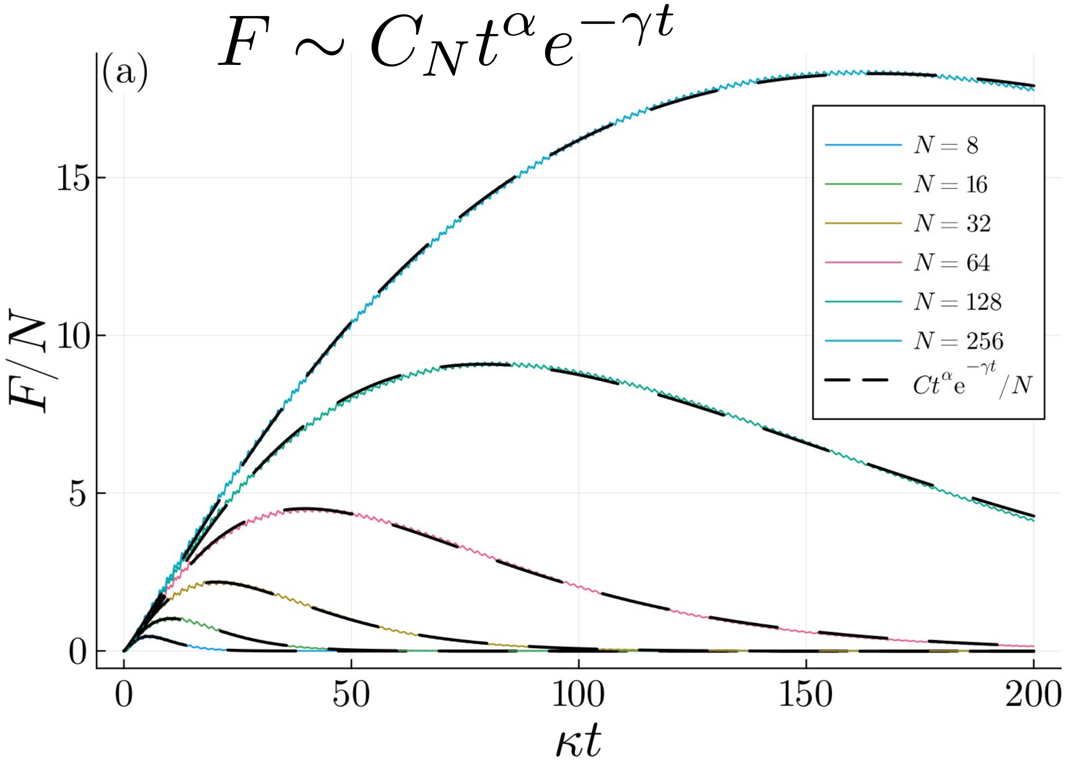
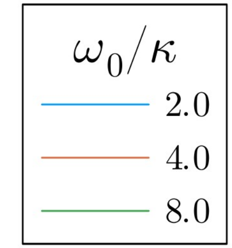
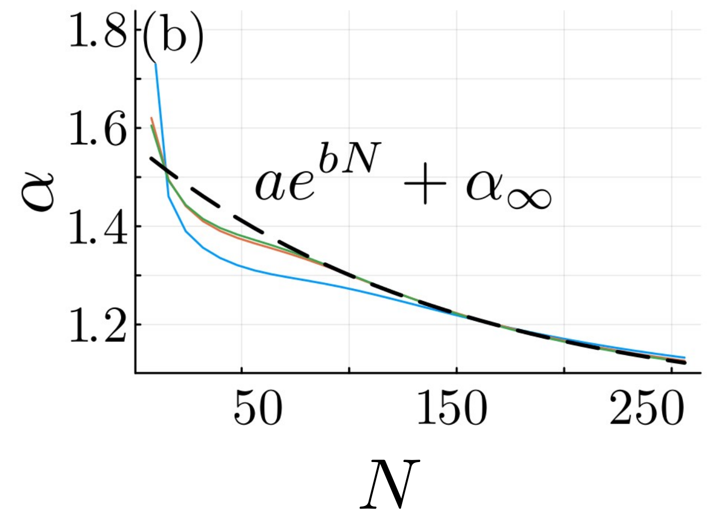
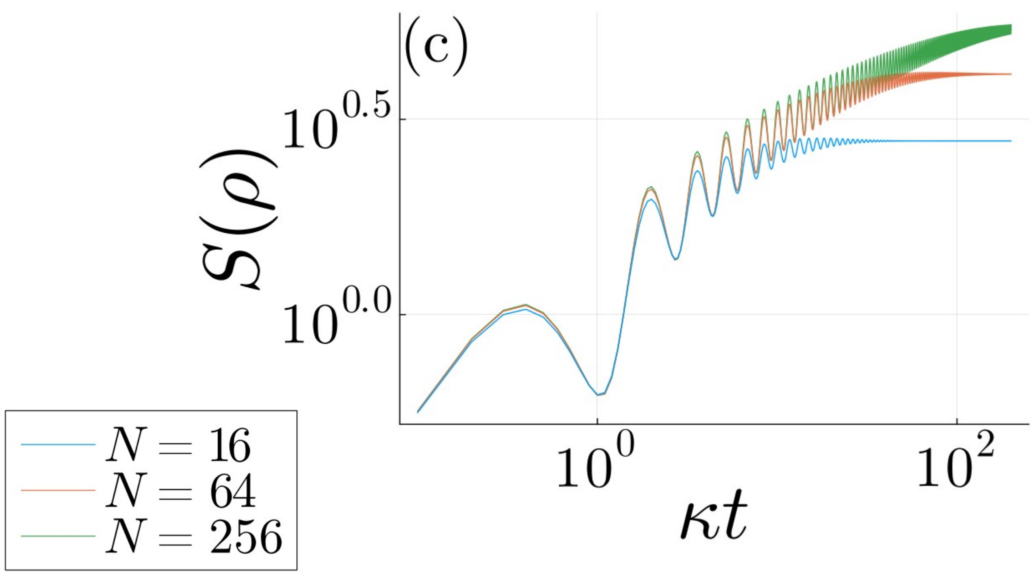
entropic constraints?

Boundary Time Crystal









- long coherence time + correlations

vs

entropic constraints?

- overcome classical bounds
- however, far from Heisenberg

Improve it?

Conclusions

- We discussed about
  . metrology,
  . estimation theory,
  . bounds - classical/quantum CRLB, classical limit, standard quantum limit, Heisenberg limit…
  . Fisher information

- Application of TCs in metrology:

  . improved performance of sensors: correlations + robust long-time sensing protocol;

- Perspectives:

  .other TC's: higher n-tuples TC's, …
  .device measurement protocol
  .enhance correlations

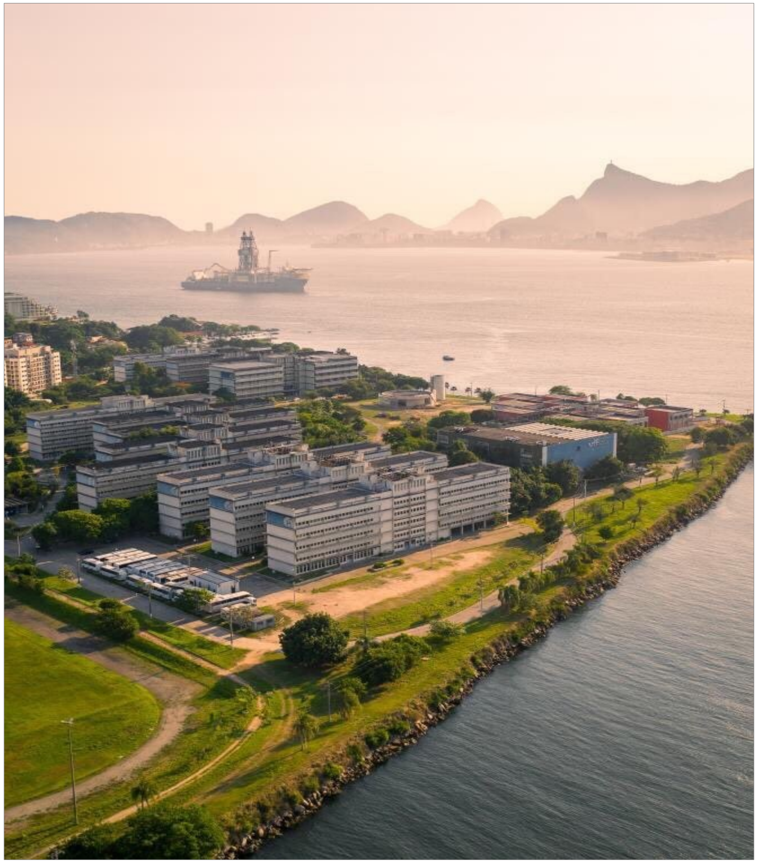
  . *Clocks!* **What are the fundamental limits in our ability to measure time (within the laws of quantum thermodynamics and many-body theory)?**
  (in collaboration with Serrapilheira Institute)

Thanks for your attention!

If you are interested…
contact: fernandoiemini@id.uff.br

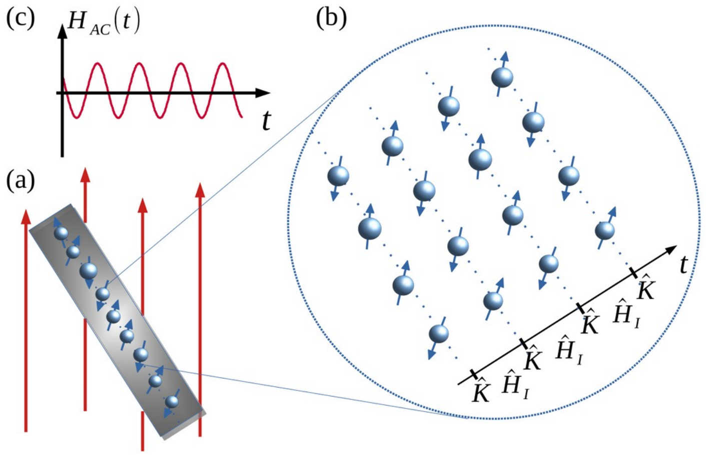
INSTITUTO DE FÍSICA
Universidade Federal Fluminense

serrapilheira



UFF,Niterói,Brazil

Sensing AC fields



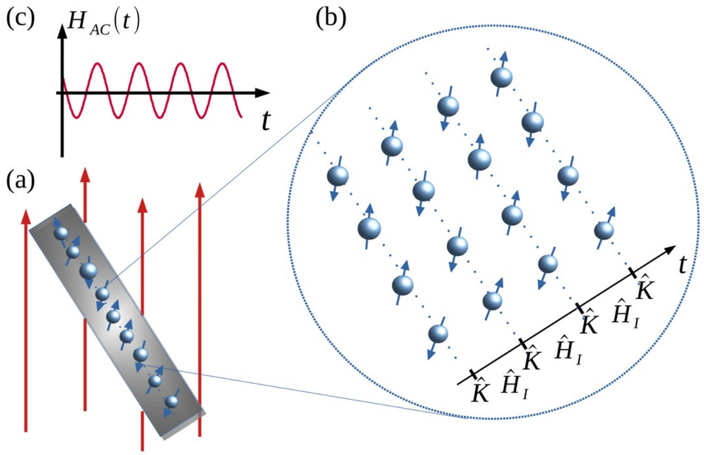
$$\hat{H}_0(t) = \sum_i \left(J_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z + \sum_{\alpha=x,z} h_i^\alpha \hat{\sigma}_i^\alpha \right) + + \sum_{n=1}^{\infty} \delta(t - nT_S)\phi \hat{S}^x,$$

$$\hat{H}_{\mathrm{AC}}(t) = h_{\mathrm{AC}} \sin(\omega_{\mathrm{AC}} t + \theta_{\mathrm{AC}}) \hat{S}_z$$

$$\boldsymbol{Case\ \ with}\ \phi = \pi,\ h_i^x = 0,\ h_{\mathrm{ac}} \to 0$$

. Fisher information: $F_{h_{\mathrm{ac}} \to 0}(t) \sim \varphi(t)^2 \mathrm{var}(\hat{\mathrm{S}}_{\mathrm{z}}(0))$

Sensing AC fields



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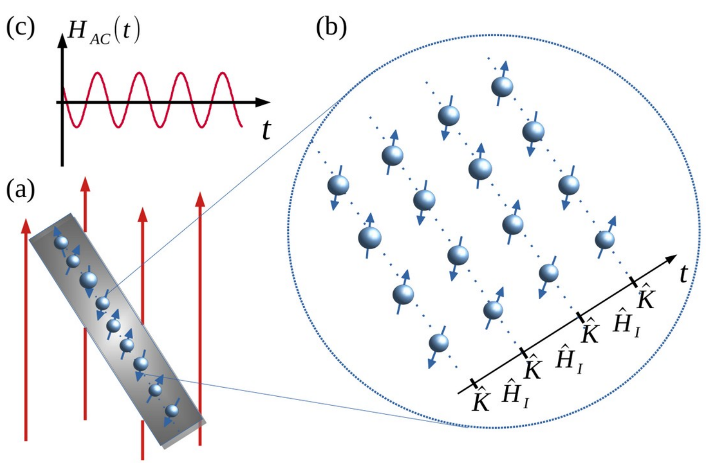

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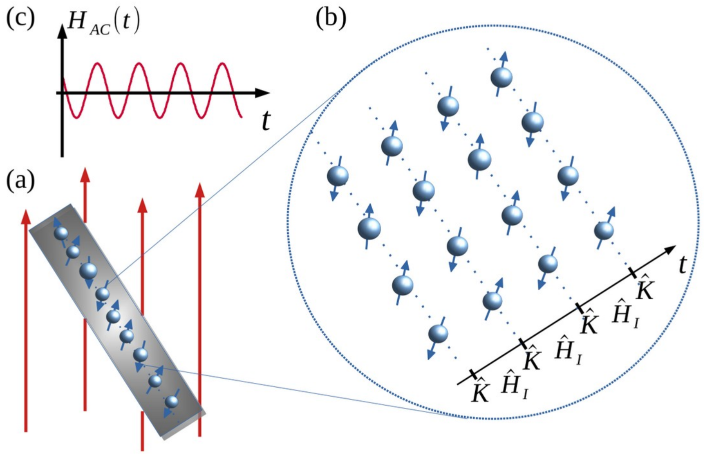
$$\mathrm{var}(\hat{\mathrm{S}}_{\mathrm{z}}(0)) = \langle \hat{\mathrm{S}}_{\mathrm{z}}^2 \rangle_{\psi_{\mathrm{o}}} - \langle \hat{\mathrm{S}}_{\mathrm{z}} \rangle_{\psi_{\mathrm{o}}}^2$$

(i) enhancement due to the correlations;

Sensing AC fields



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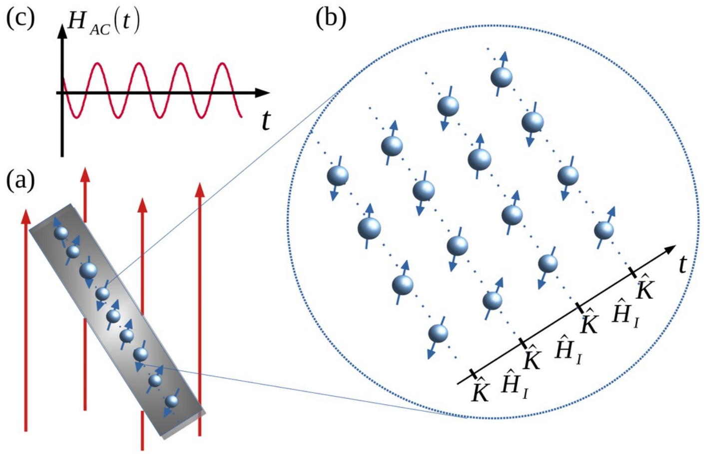
$\varphi\big|_{\omega_S = 2\omega_{\mathrm{AC}}} \sim t$ (ii) saturates SQL in time,

Sensing AC fields



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(i) enhancement due to the correlations;

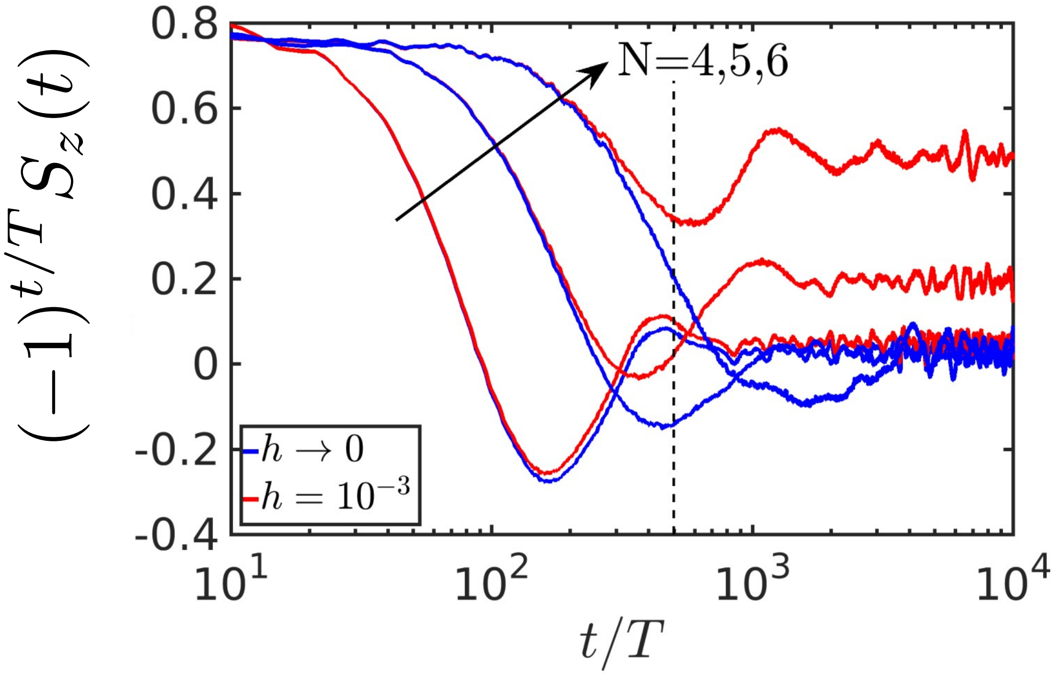
$$\varphi|_{\omega_S = 2\omega_{\text{AC}}} \sim t$$

(ii) saturates SQL in time,

$$\varphi|_{\omega_S = 2\omega_{\text{AC}} + \epsilon} : \quad t^* \sim \theta_{AC}/\epsilon$$

(iii) off-resonance time scale for the optimal growth of Fisher information.

beyond linear response: $h_{\mathrm{AC}} \neq 0, \;\; \omega_{AC} = \omega_S/2$

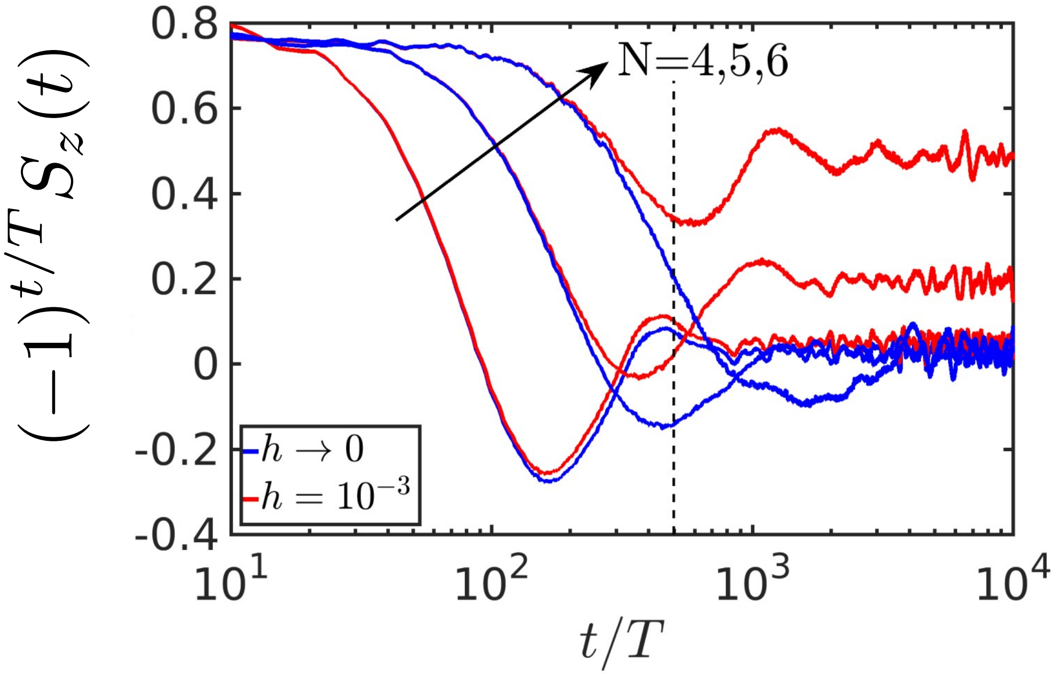


- sets a characteristic time scale,

$$t_{AC} \sim h_{\mathrm{ac}}^{-1}$$

(dominant AC fields over dynamics)

beyond linear response: $h_{\mathrm{AC}} \neq 0, \ \ \omega_{AC} = \omega_S/2$



- sets a characteristic time scale,

$$t_{AC} \sim h_{\mathrm{ac}}^{-1}$$

(dominant AC fields over dynamics)

- recovers Fisher quadratic growth,

$$F_{h_{\mathrm{ac}}}(t) \sim N t^2$$