Many-body Open Quantum Systems - dark states and topological Majorana fermions

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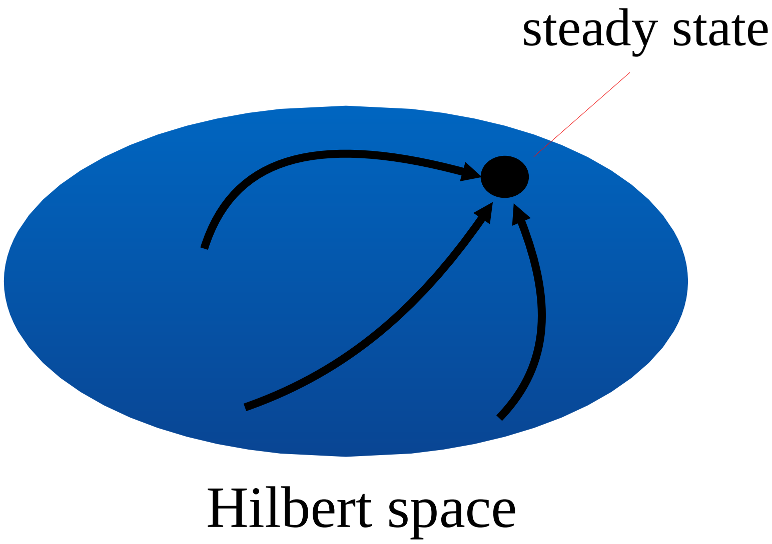
Dissipative State Engineering

- steady state of a many-body system is generically mixed/trivial…



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however, under certain conditions, it is possible to use dissipation in order to engineer:

. ***pure*** steady states;
. **non-trivial** (quantum);

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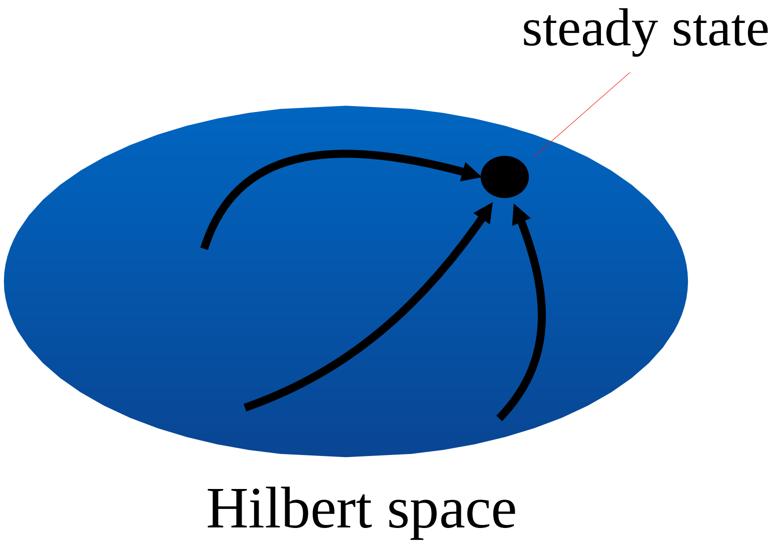

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- we call as ***reservoir engineering***, or ***dissipative state engineering;***

. idea: careful tailoring the coupling of the system to the environment (*synthetic quantum systems*)

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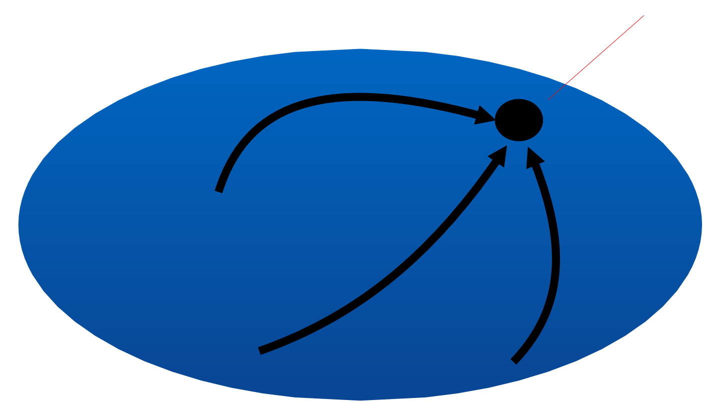

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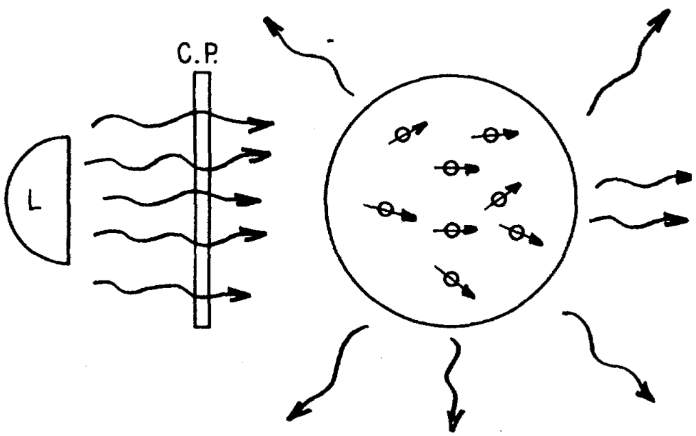
idea dates back to the early studies on laser light coupled to atom systems



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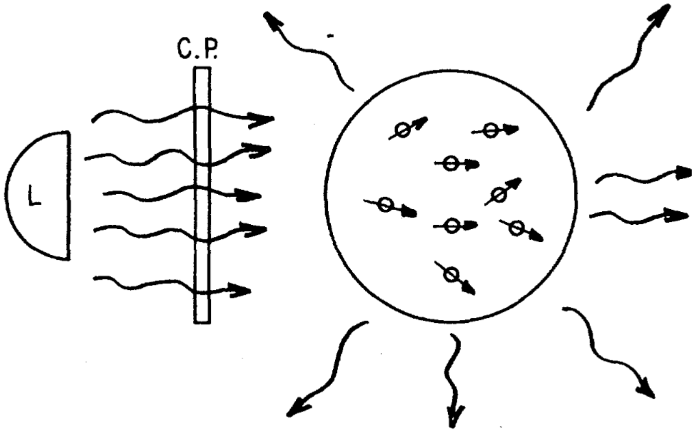

$J = \text{total ang. mom.}$

$m = \text{magnetic mom.}$

$m_e{=}\downarrow$ $m_e{=}\uparrow$

$J_e{=}1/2$

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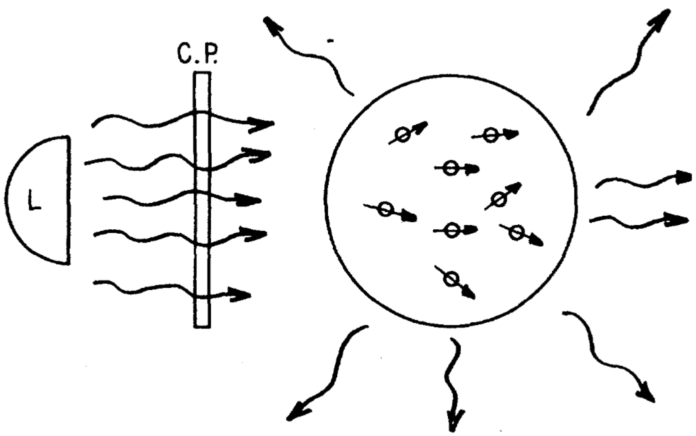

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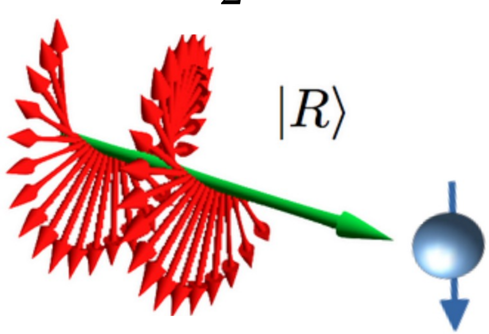
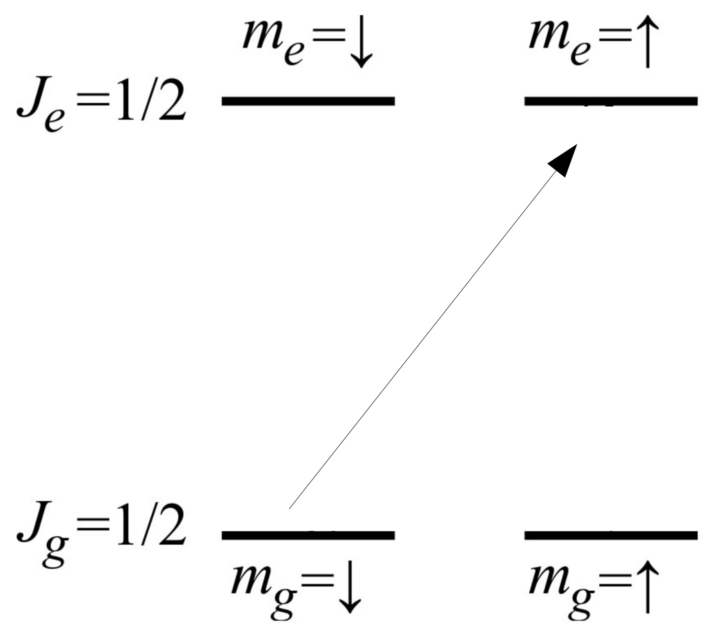
$$J_z = +\frac{1}{2}$$





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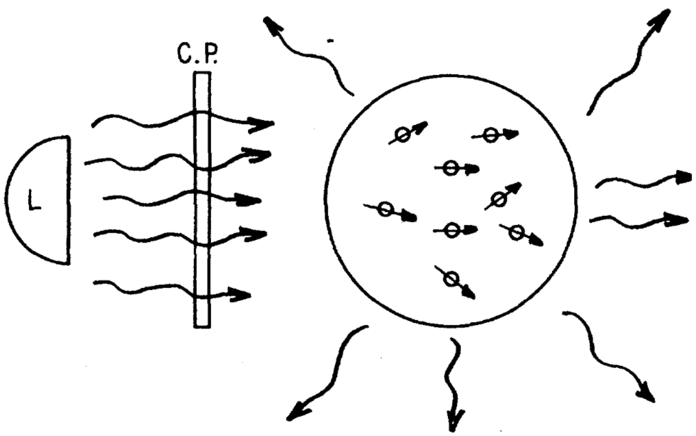

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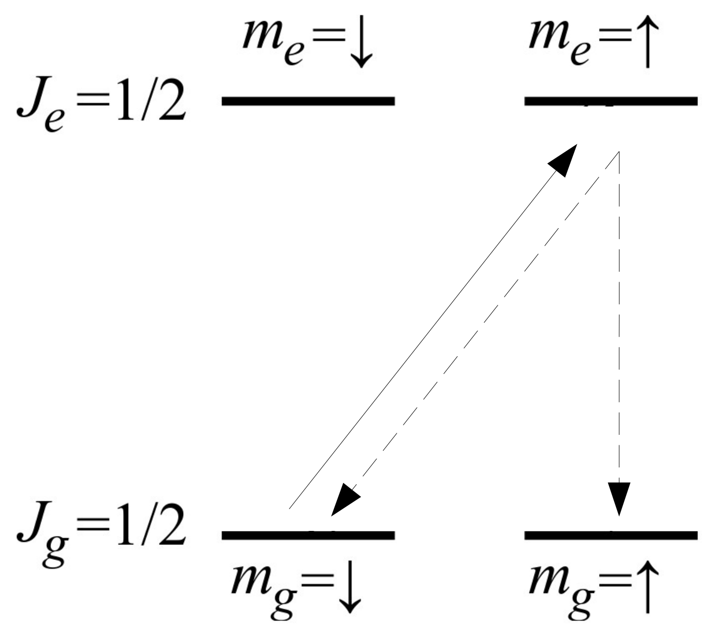

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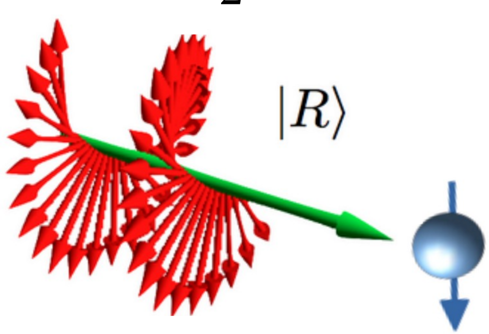

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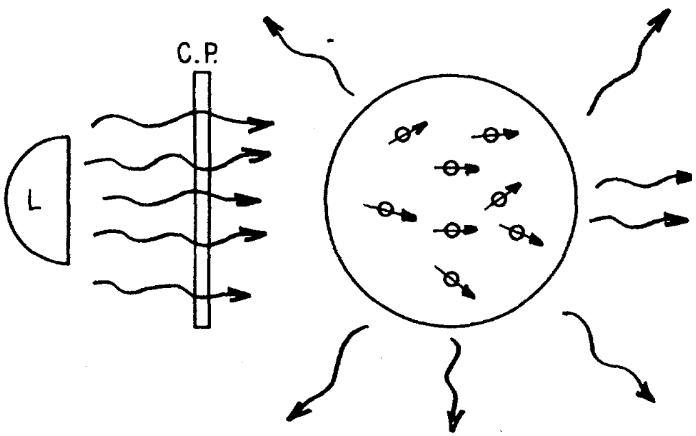

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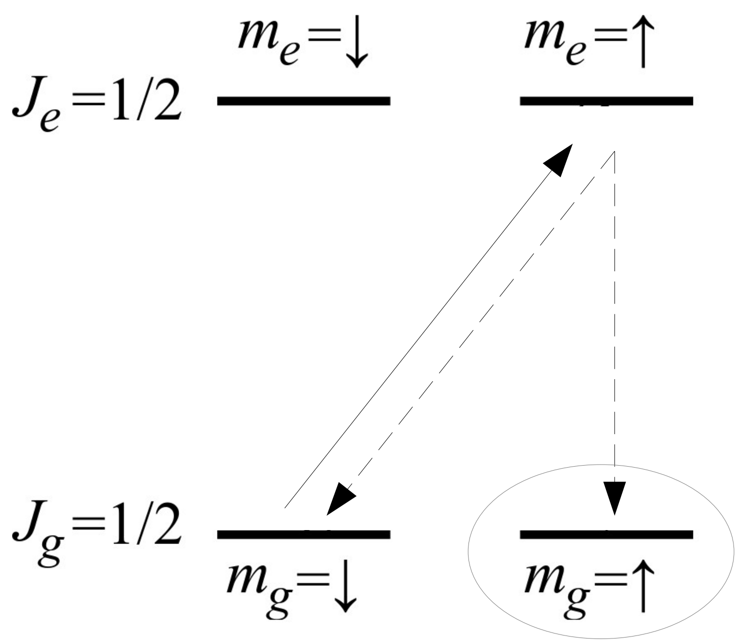

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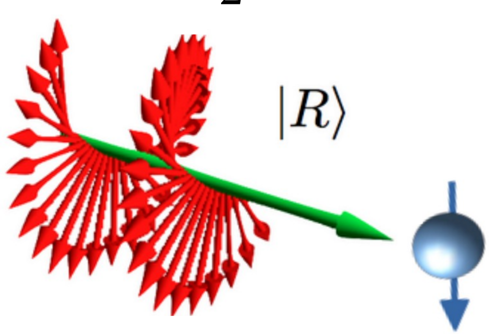
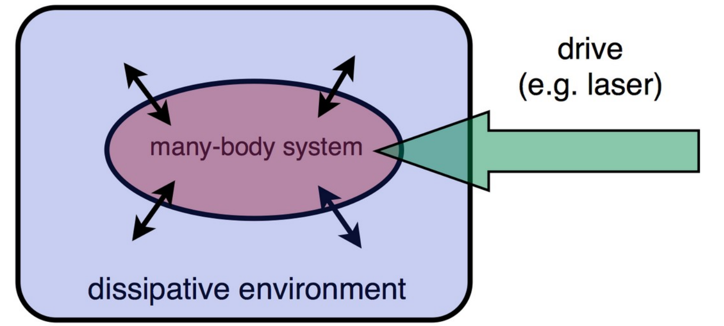


dark states:
cannot absorb nor
emmit photons

- Can we generalize this dark-state physics to the dissipative ***many-body*** scenario?



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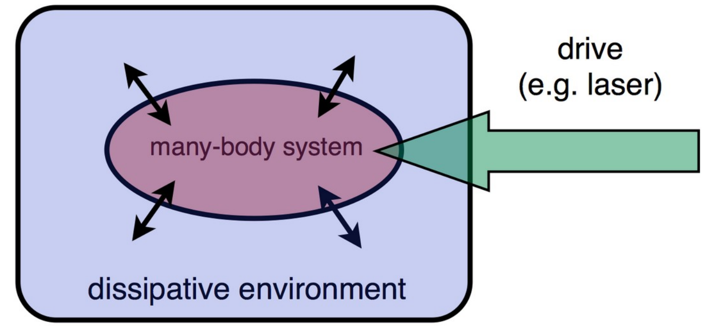

- Mathematical formalism:

. a pure state $|\psi\rangle$ is a dark state if:

$$\frac{d|\psi\rangle\langle\psi|}{dt} = -i[\hat{H}, |\psi\rangle\langle\psi|] + \sum_i \hat{L}_i|\psi\rangle\langle\psi|\hat{L}_i^\dagger - \{\hat{L}_i^\dagger\hat{L}_i, |\psi\rangle\langle\psi|\} = 0$$

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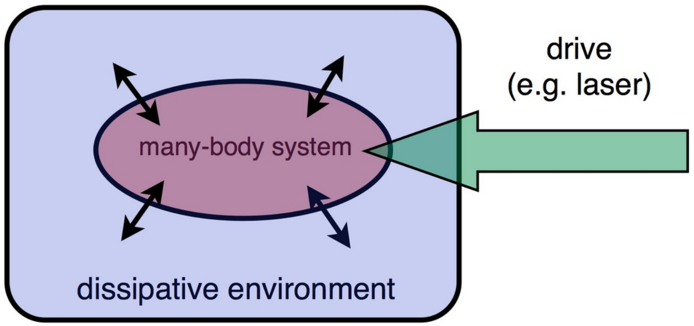

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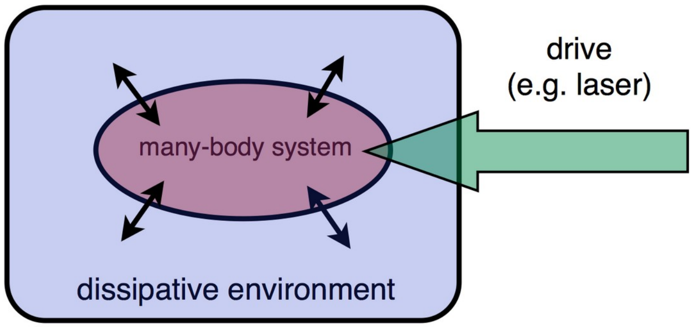

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(ii) $\hat{\mathrm{L}}_{\mathrm{i}}|\psi\rangle = 0,$ kernel of jump operators

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*general conditions are broader than that...

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Topological Majorana fermions

Kitaev chain

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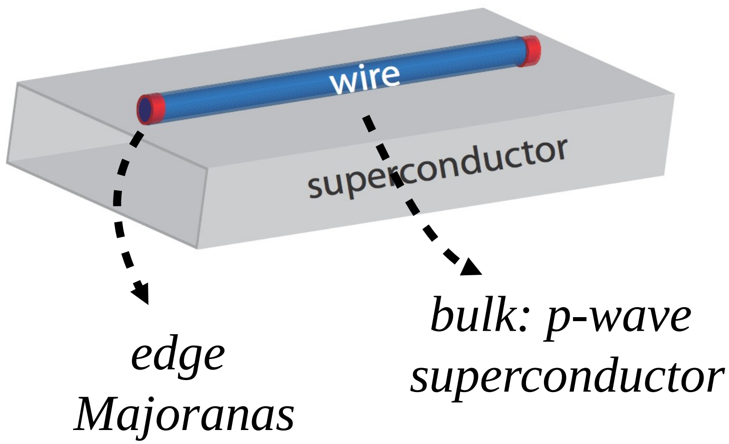

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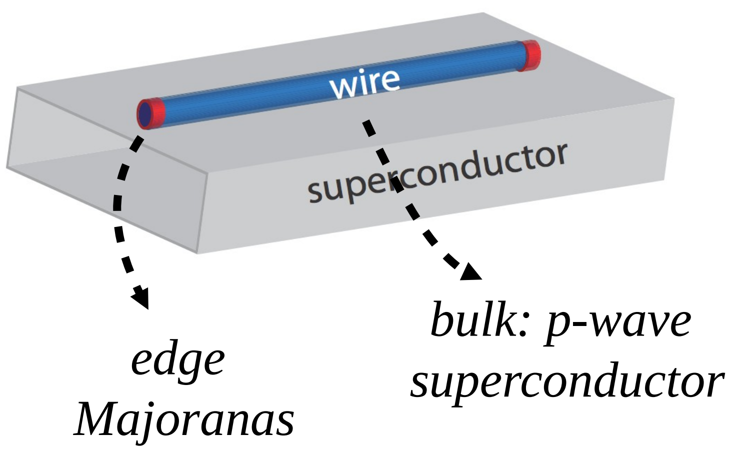
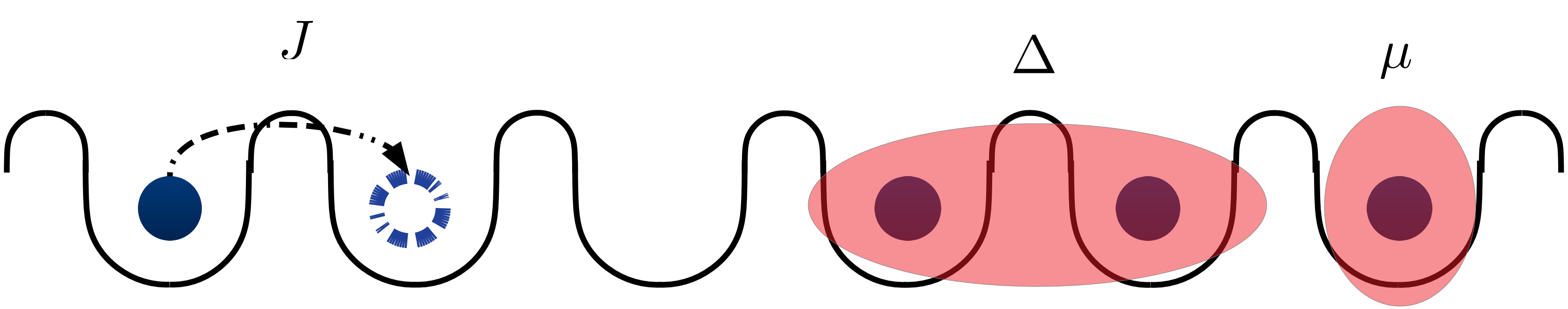


$$\hat{H} = \sum_j \underbrace{-J\hat{a}_j^\dagger \hat{a}_{j+1}}_{\text{hopping}} + \underbrace{\Delta \hat{a}_j \hat{a}_{j+1}}_{\text{pairing coupling}} + \mathrm{H.c} + \underbrace{\mu \hat{a}_j^\dagger \hat{a}_j}_{\text{chemical potential}}$$



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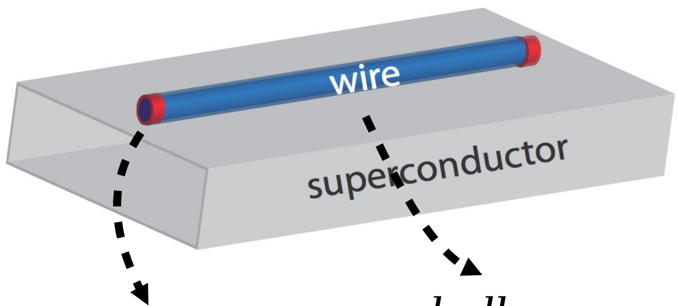

edge Majoranas

bulk: p-wave superconductor

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hopping

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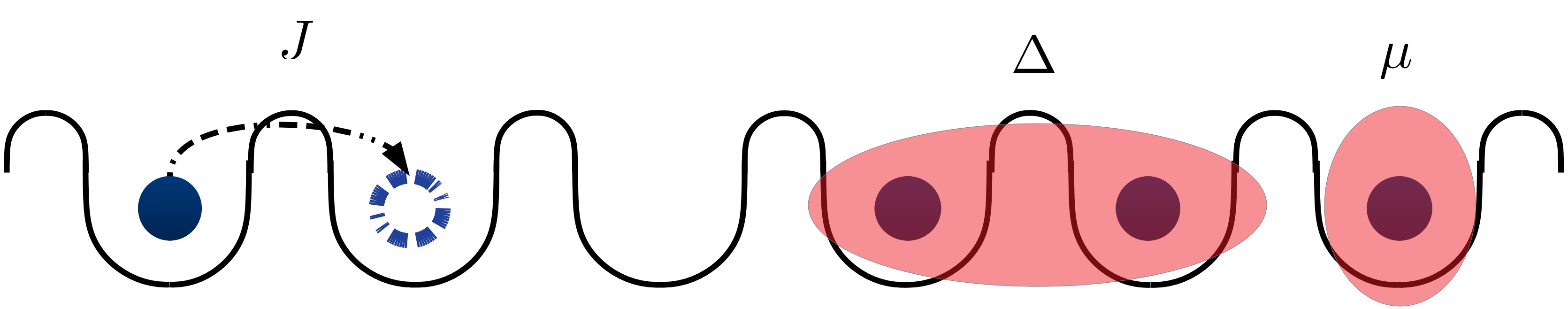
. *quadratic Hamiltonian – solvable...*



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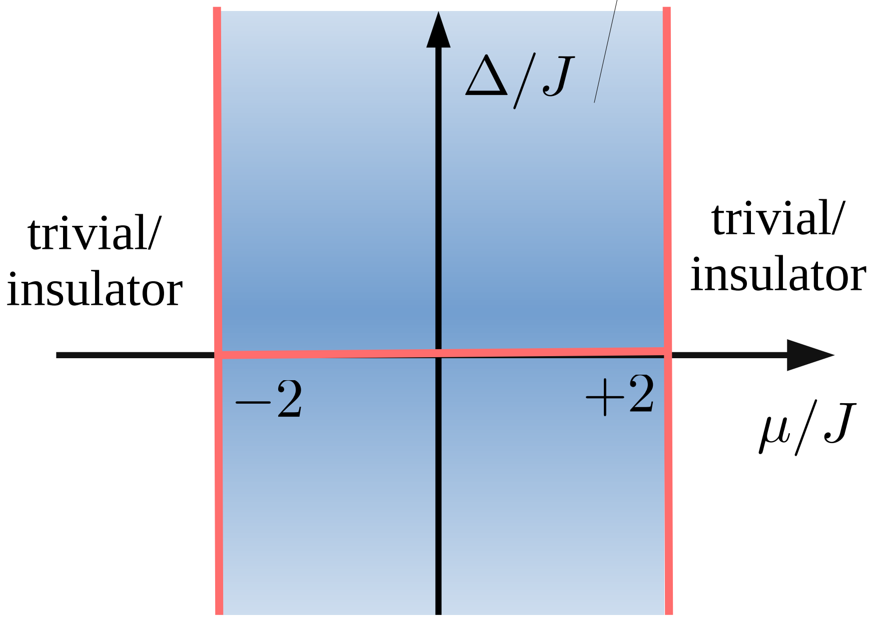

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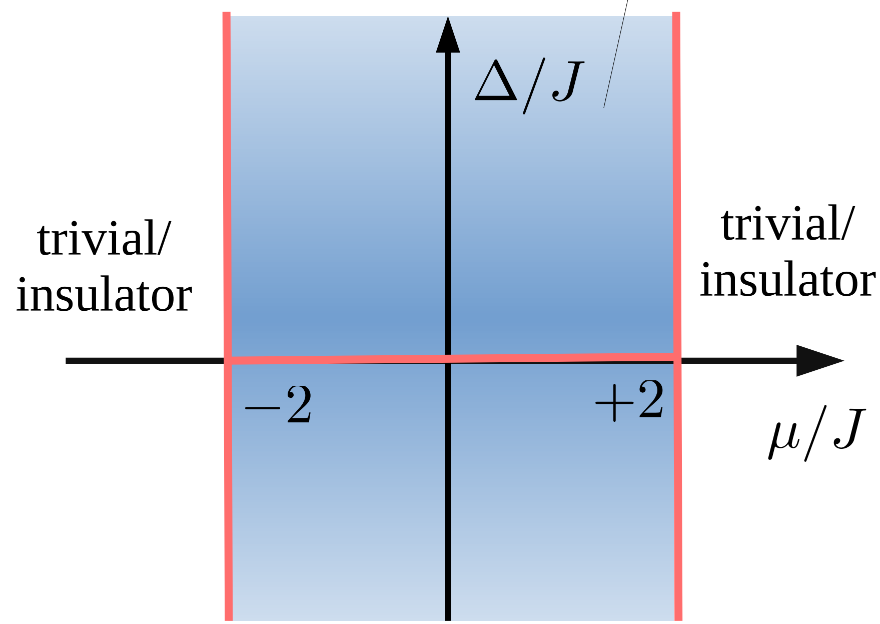

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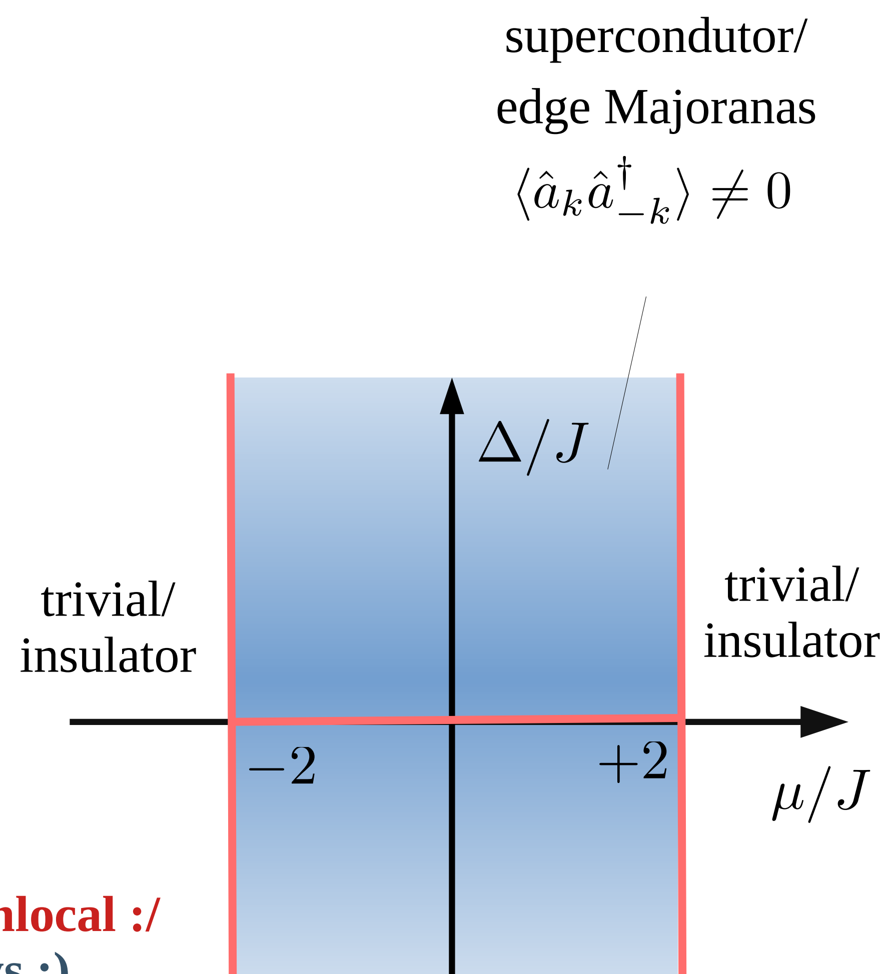

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usually highly nonlocal :/
but not always :)

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(complex) fermion

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(real) Majorana fermion

$$\gamma_j^\dagger = \gamma_j$$

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- "half" fermions;
- particles are their own anti particles;

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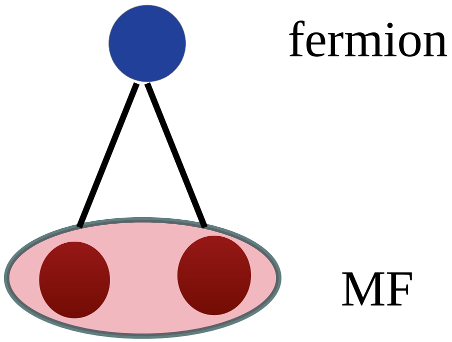

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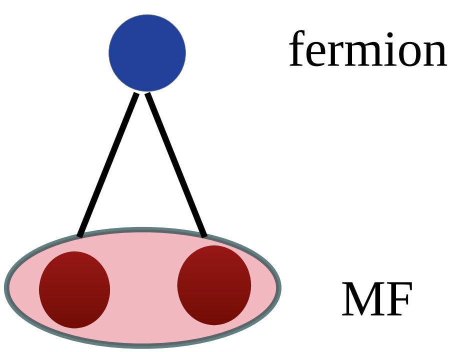
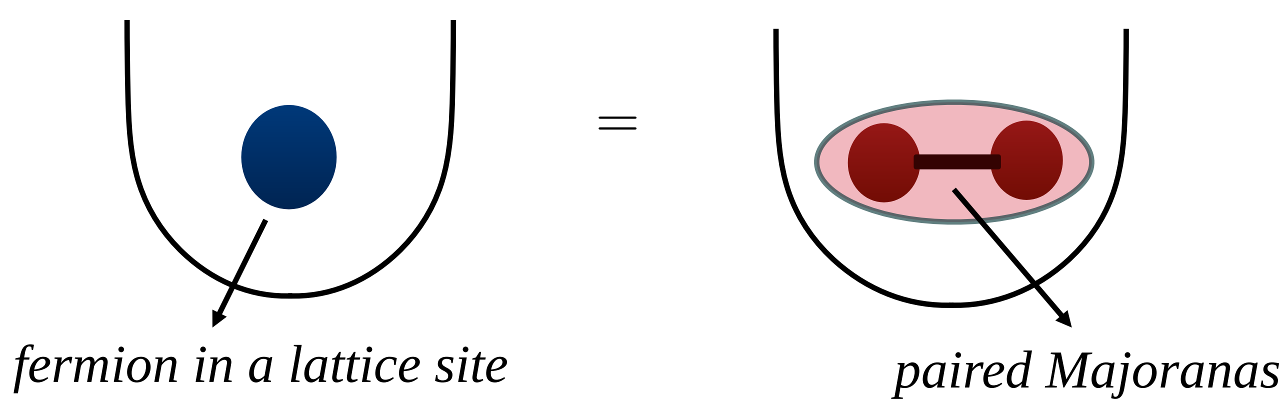


$$\hat{H} = \left(\hat{a}_1^\dagger \hat{a}_1 - \frac{1}{2}\right) = \frac{i}{2}\hat{\gamma}_1\hat{\gamma}_2$$



- substituting in the Hamiltonian:

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$$= \sum_{j=1}^{L-1} [(\Delta + J)\hat{\gamma}_{2j}\hat{\gamma}_{2j+1} + (\Delta - J)\hat{\gamma}_{2j-1}\hat{\gamma}_{2j+2}] - \sum_{j=1}^{L}(\mu i \hat{\gamma}_{2j-1}\hat{\gamma}_{2j} + \frac{1}{2})$$

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- trivial phase:

$J = \Delta = 0$

$$\hat{H} = -\sum_{j=1}^{L}(\mu i \hat{\gamma}_{2j-1}\hat{\gamma}_{2j} + \frac{1}{2})$$

- substituting in the Hamiltonian:

$$\hat{H} = \sum_j -J\hat{a}_j^\dagger \hat{a}_{j+1} + \Delta \hat{a}_j \hat{a}_{j+1} + \text{H.c.} + \mu \sum_j \hat{a}_j^\dagger \hat{a}_j$$

$$= \sum_{j=1}^{L-1} [\cancel{(\Delta + J)\hat{\gamma}_{2j}\hat{\gamma}_{2j+1}} + \cancel{(\Delta - J)\hat{\gamma}_{2j-1}\hat{\gamma}_{2j+2}}] - \sum_{j=1}^{L} (\mu i \hat{\gamma}_{2j-1}\hat{\gamma}_{2j} + \frac{1}{2})$$

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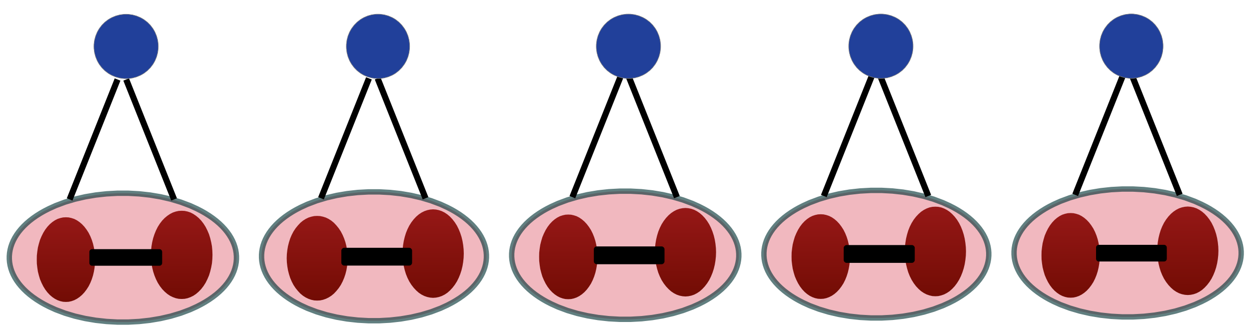

ground state just minimizes each maj. pair / physical fermion site $\longrightarrow$ insulator

- substituting in the Hamiltonian:

$$\hat{H} = \sum_j -J\hat{a}_j^\dagger \hat{a}_{j+1} + \Delta \hat{a}_j \hat{a}_{j+1} + \text{H.c.} + \mu \sum_j \hat{a}_j^\dagger \hat{a}_j$$

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"sweet point":

$$\mu = 0,\ J = \Delta$$

. topological phase;

. local interacting MF's = frustration-free terms

- top. phase ("sweet point"):

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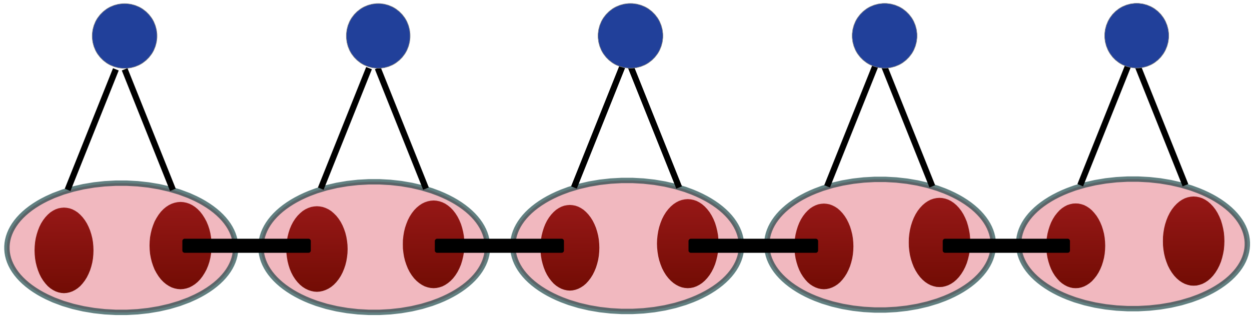
$$\hat{H} = \sum_{j=1}^{L-1} (\Delta + J)\hat{\gamma}_{2j}\hat{\gamma}_{2j+1}$$



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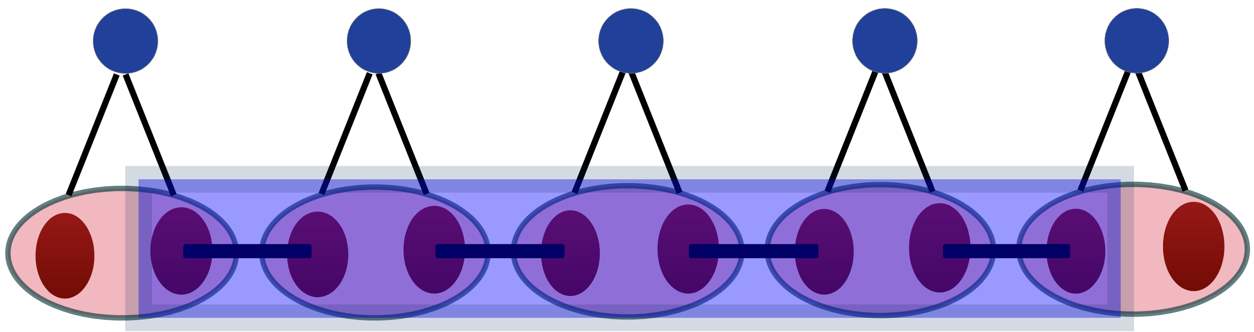
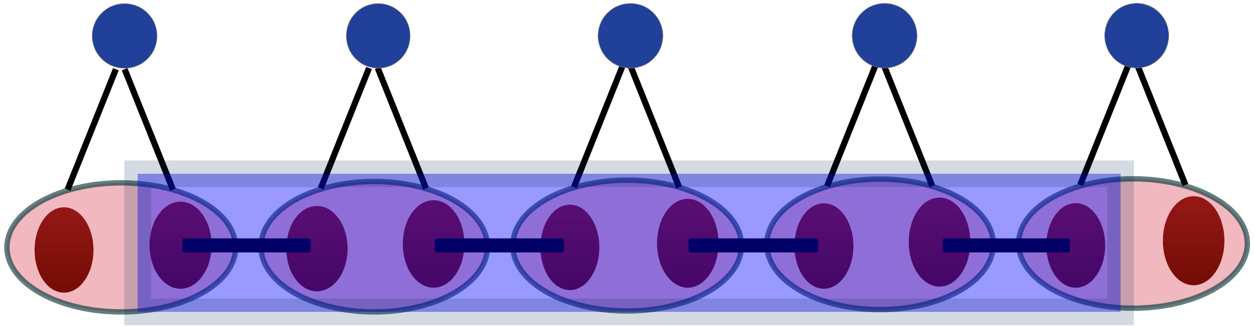


bulk
superconduting

- top. phase ("sweet point"):

$$\mu = 0, \ J = \Delta$$

$$\hat{H} = \sum_{j=1}^{L-1} (\Delta + J) \hat{\gamma}_{2j} \hat{\gamma}_{2j+1} \qquad \sim \sum_{j=1}^{L-1} \hat{\eta}_j^\dagger \hat{\eta}_j$$



bulk
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Bogoliuobov
excitations:

$$\hat{\eta}_j = \hat{a}_j^\dagger + \hat{a}_{j+1}^\dagger + \hat{a}_j - \hat{a}_{j+1}$$

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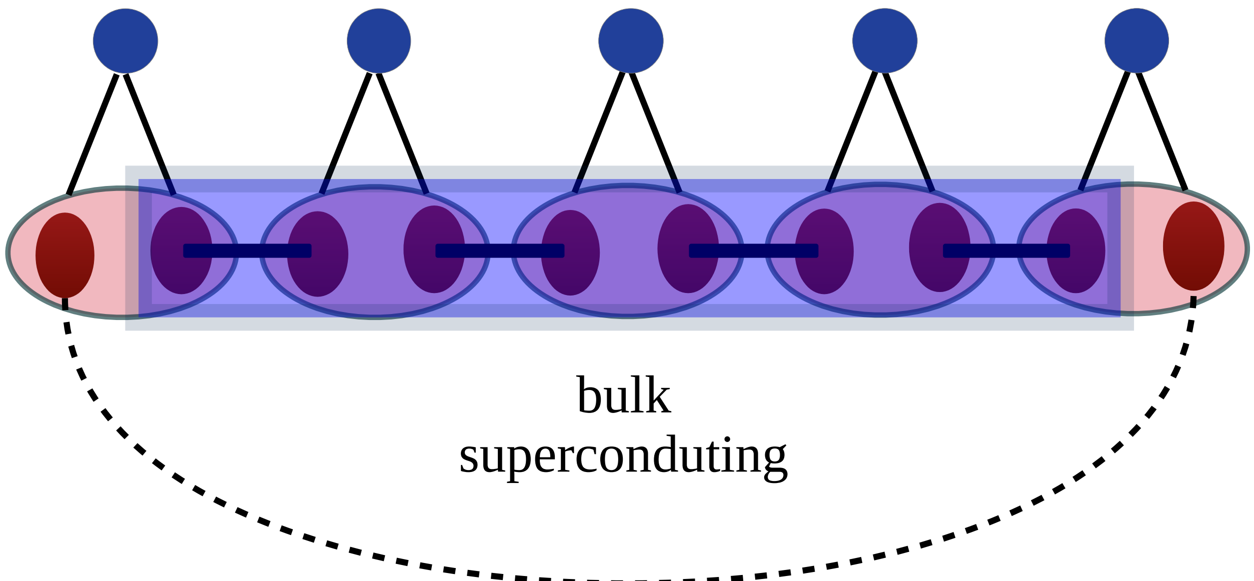

bulk
superconduting

$\hat{\gamma}_1,\ \hat{\gamma}_{2L}$

zero energy edge
Majorana fermions!

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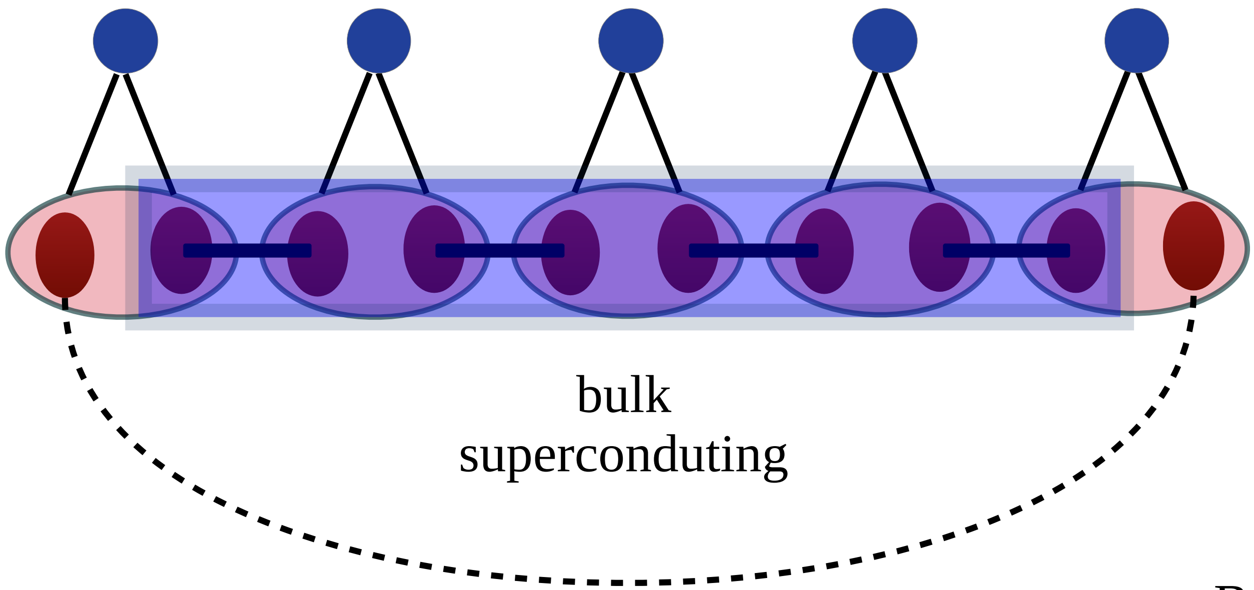

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zero energy
Bog. excitation

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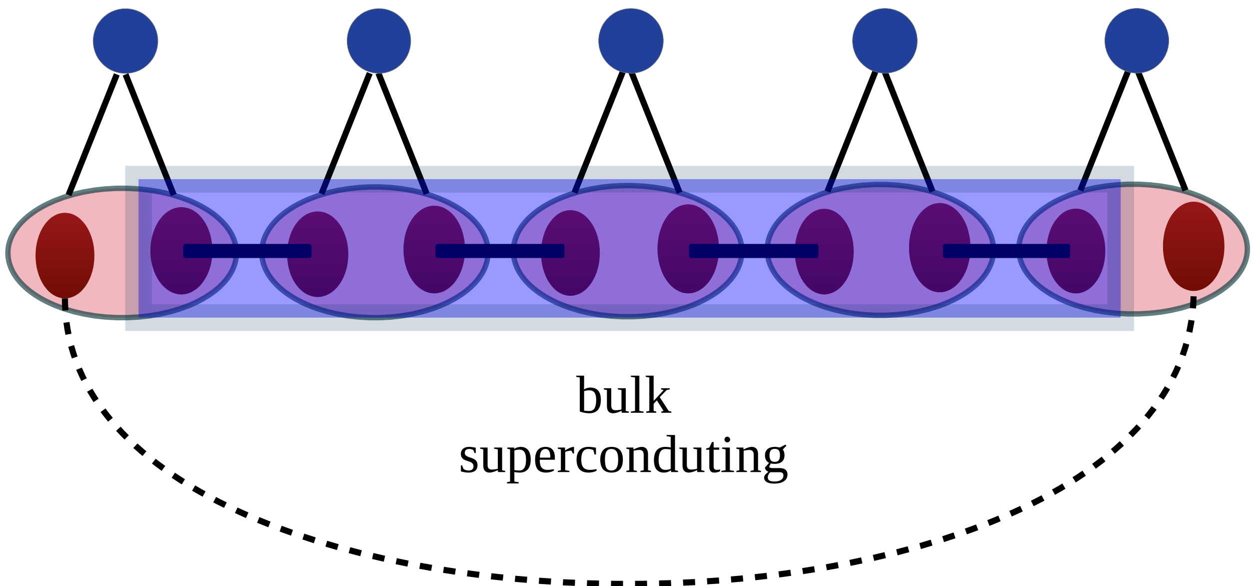

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zero energy
Bog. excitation

$\longrightarrow$ two-fold ground
state degeneracy

$$\{|\mathrm{gs}\rangle, \; \hat{\eta}|\mathrm{gs}\rangle\}$$

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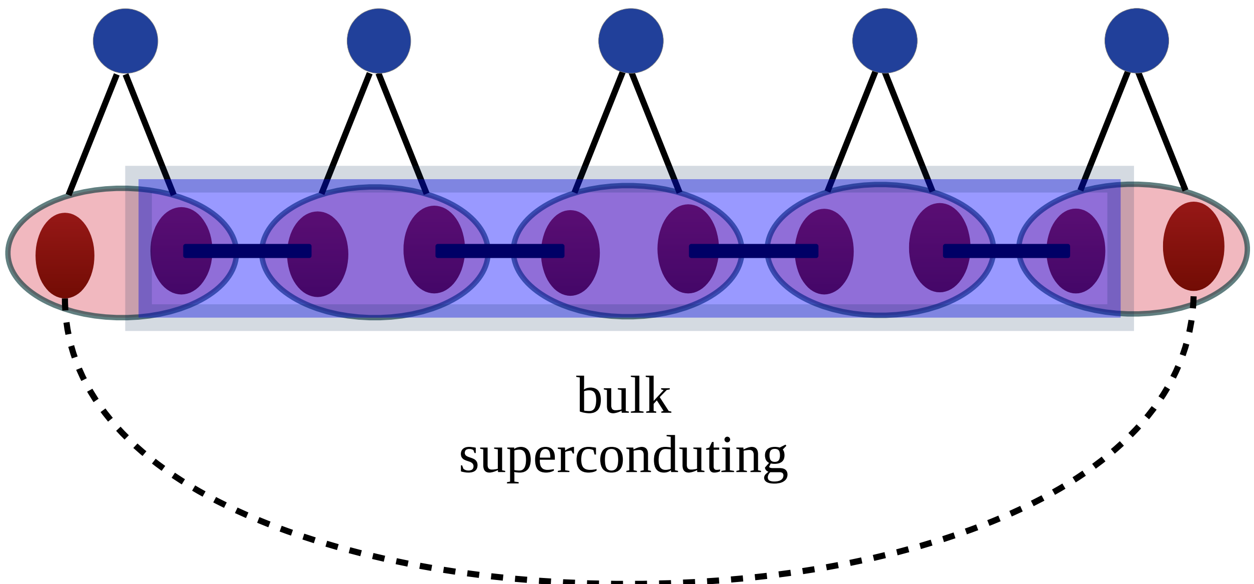

$\hat{\gamma}_1,\ \hat{\gamma}_{2L}$

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zero energy
Bog. excitation

$\longrightarrow$

two-fold ground
state degeneracy

$$\{|\mathrm{gs}\rangle,\ \hat{\eta}|\mathrm{gs}\rangle\}$$

$$\sim \{|0\rangle, |1\rangle\}$$

. qubit
. robust (topological)
. fault-tolerant
computing

- How to generate these states ***dissipatively*** (H~0)?

$$\hat{L}_j \equiv \hat{\eta}_j = \hat{a}_j^\dagger + \hat{a}_{j+1}^\dagger + \hat{a}_j - \hat{a}_{j+1}$$

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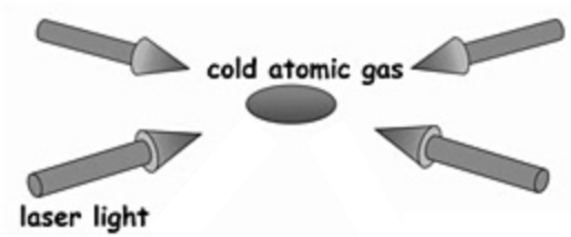

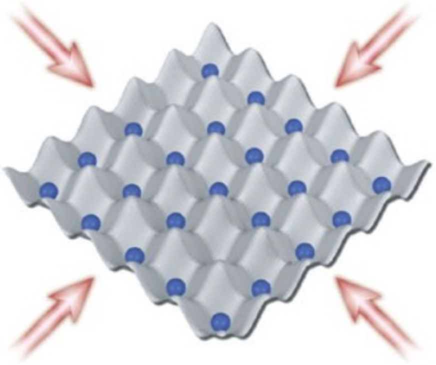

S. Diehl et al., Nature Physics 7(12), 971 (2011).

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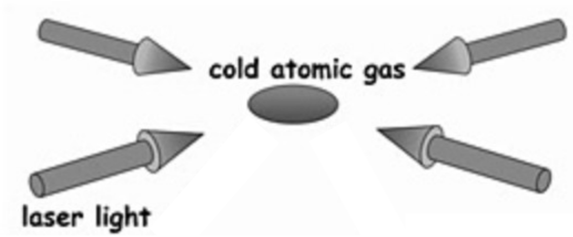

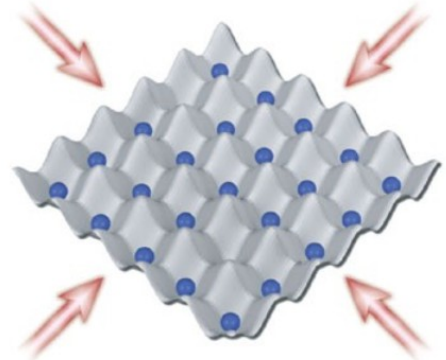

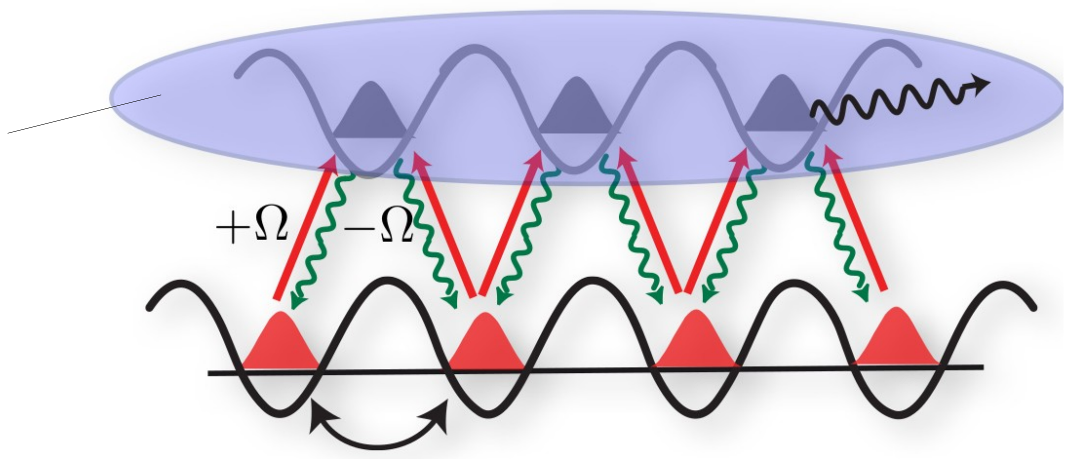

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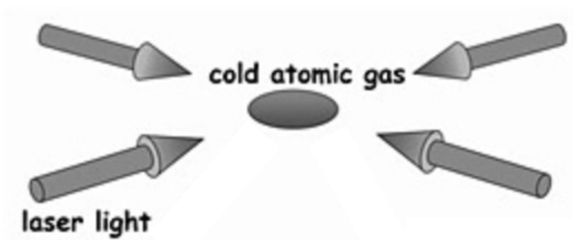
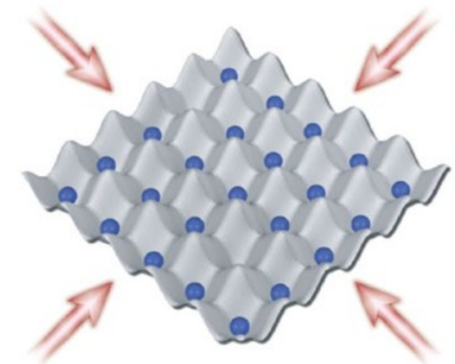
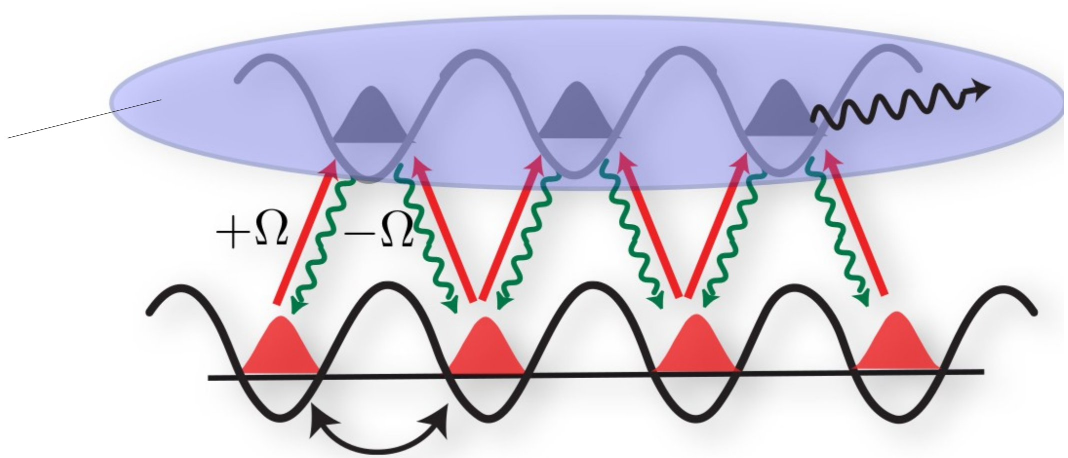




$$\hat{\ell}_j = \qquad\qquad\qquad (\hat{a}_j - \hat{a}_{j+1})$$



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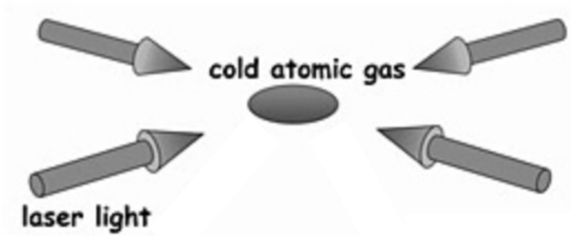

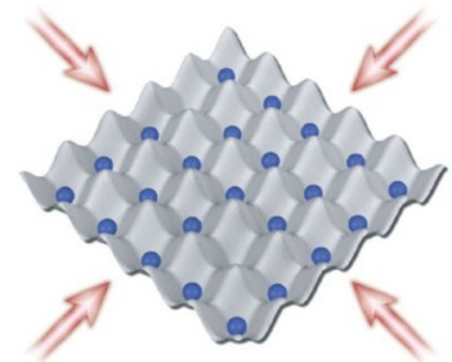

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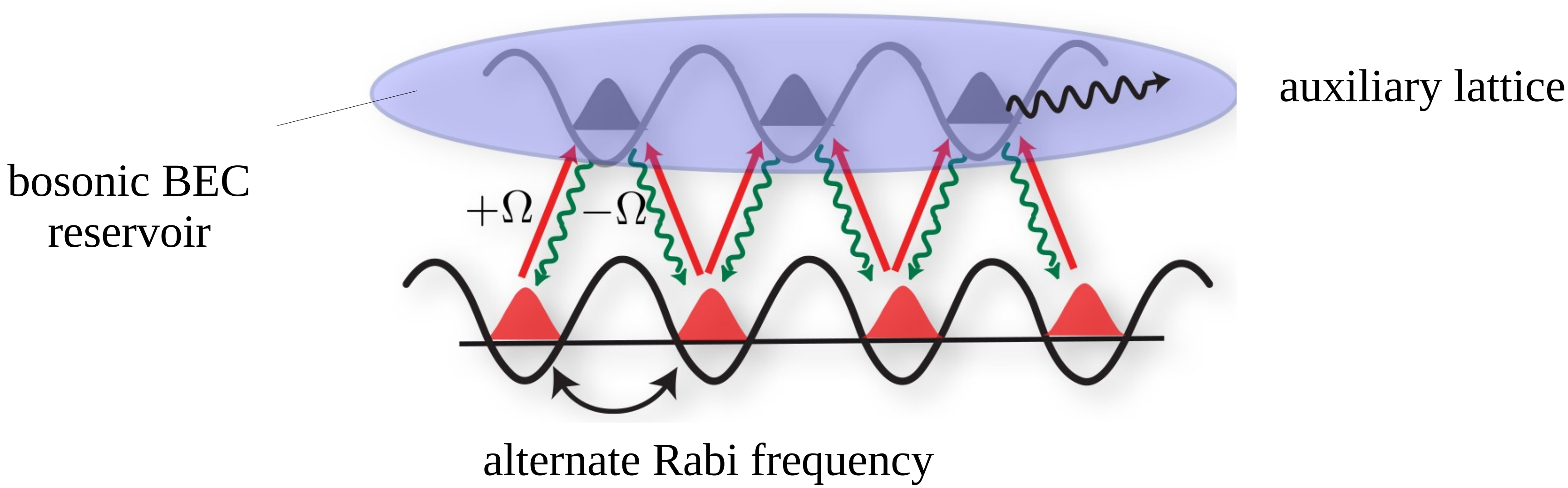

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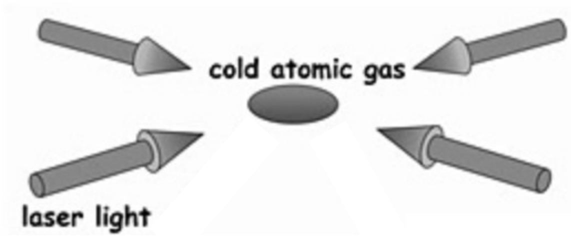
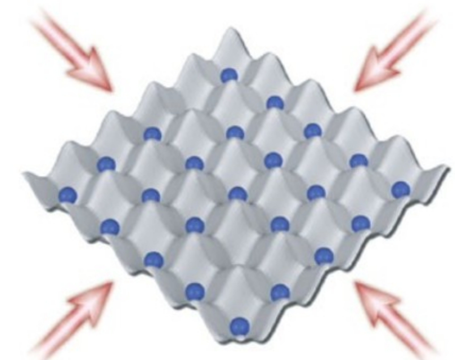
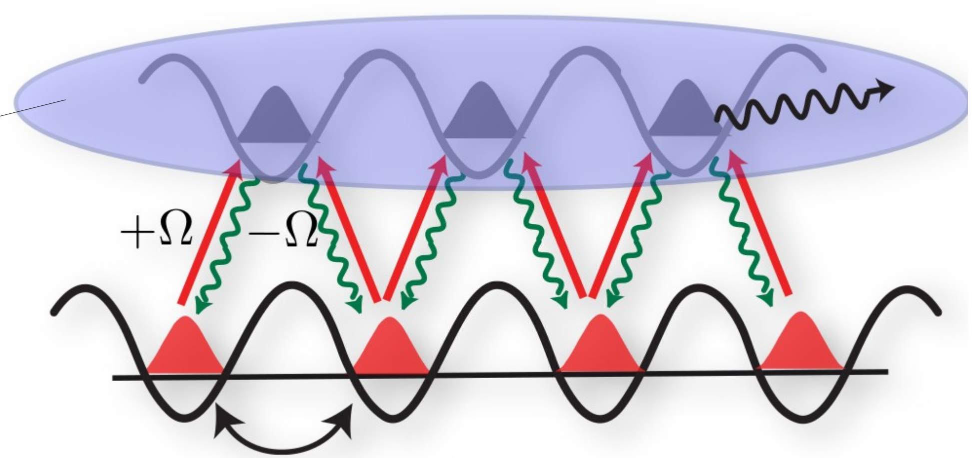


mean-field
approx



$$\hat{\ell}_j = (\hat{a}_j^\dagger + \hat{a}_{j+1}^\dagger)(\hat{a}_j - \hat{a}_{j+1})$$

bosonic BEC
reservoir



auxiliary lattice

alternate Rabi frequency

S. Diehl et al., Nature Physics 7(12), 971 (2011).

- not so simple...

$$\hat{L}_j \equiv \hat{\eta}_j = \hat{a}_j^\dagger + \hat{a}_{j+1}^\dagger + \hat{a}_j - \hat{a}_{j+1}$$

~~mean-field~~
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$$\hat{\ell}_j = (\hat{a}_j^\dagger + \hat{a}_{j+1}^\dagger)(\hat{a}_j - \hat{a}_{j+1})$$

- Conservation of particle number, a ***strong constraint:***

F. Iemini et al., Phys. Rev. B 93, 115113 (2016)

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~~mean-field~~
approx

$$\hat{\ell}_j = (\hat{a}_j^\dagger + \hat{a}_{j+1}^\dagger)(\hat{a}_j - \hat{a}_{j+1})$$

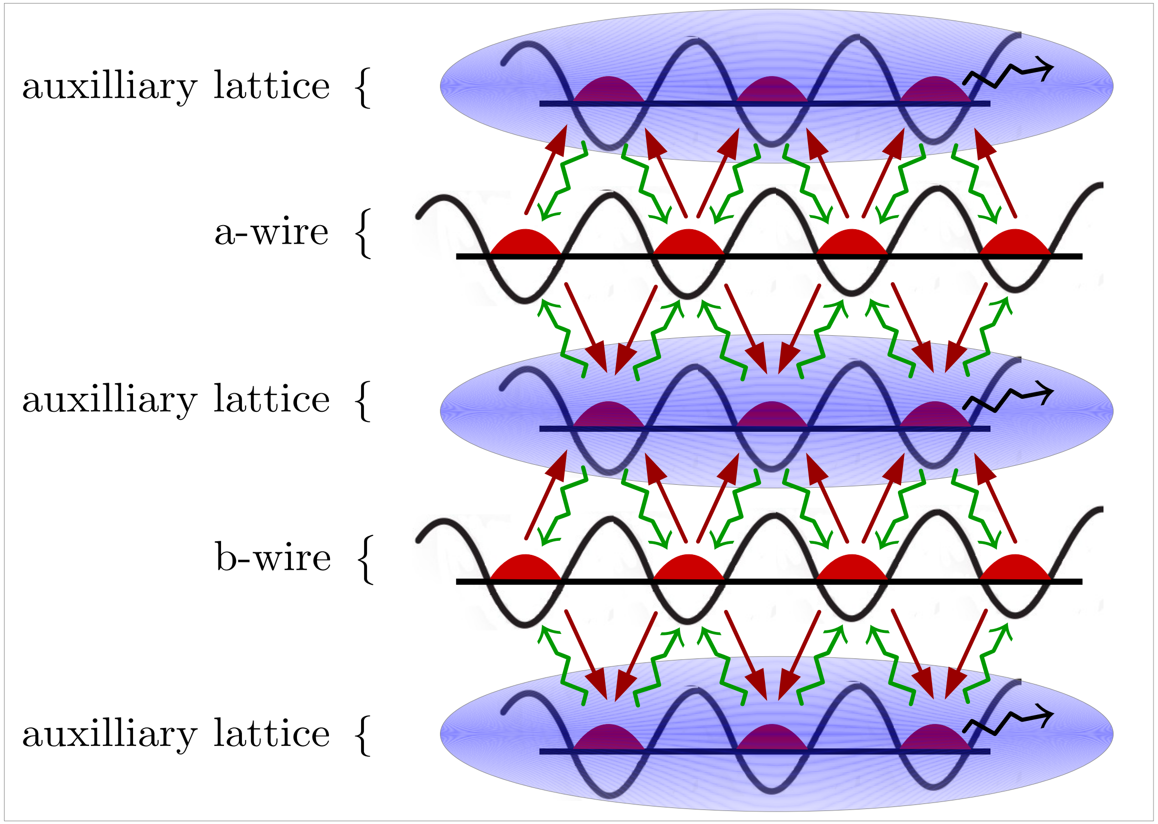
- Conservation of particle number, a ***strong constraint:***

  . despite generating superconduting properties;

  . unique steady states… no topological edge Majorana states;

Phys. Rev. B 93, 115113 (2016)

- a way to circumvent...

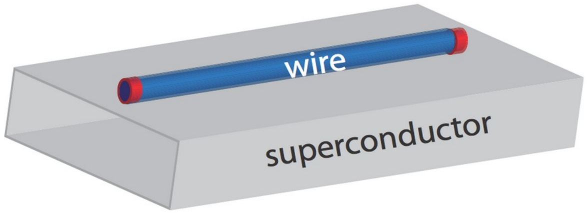


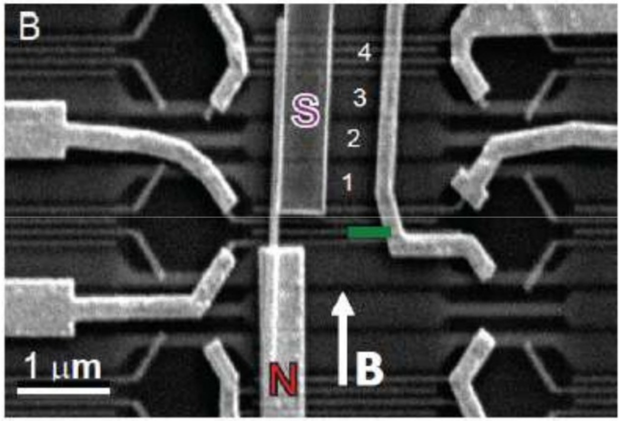
- conservation of particle number: ok
- superconducting: ok
- degenerated steady states (edge Mfs): ok
- physical implementation/feasibility: ?

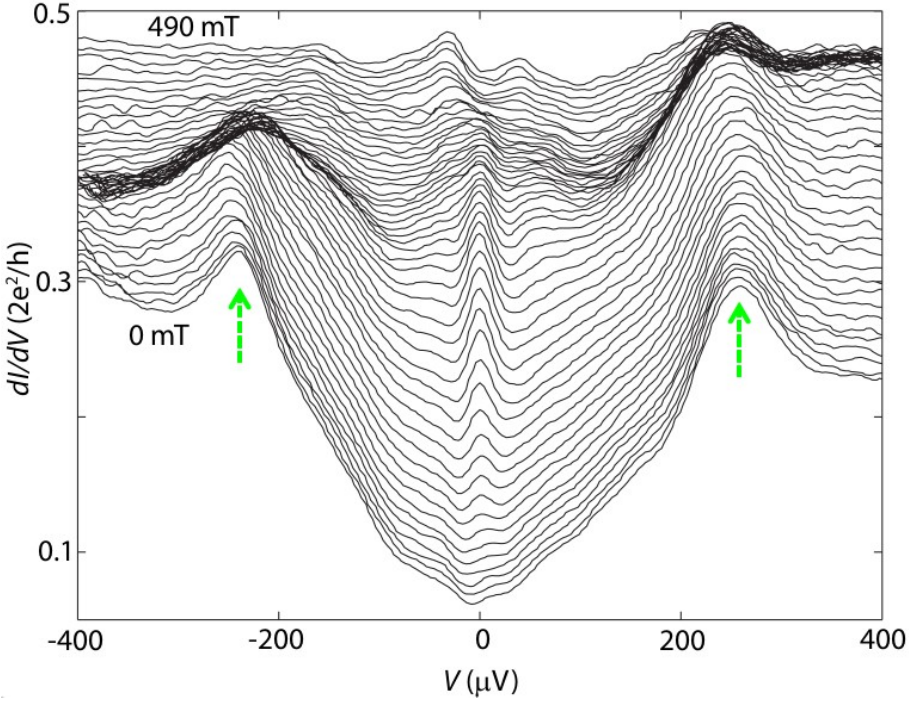
Phys. Rev. B 93, 115113 (2016)

Experimental observations

In fact, the quest for edge Majorana fermions is still open!







Spectroscopy
(zero-bias conductance peaks)

V. Mourik et al., **Science** 336, 1003 (2012);
M. T. Deng et al., **Nano Lett**. 12, 6414 (2012);
A. Das, Y. Ronen et al., **Nature Physics** 8, 887 (2012);
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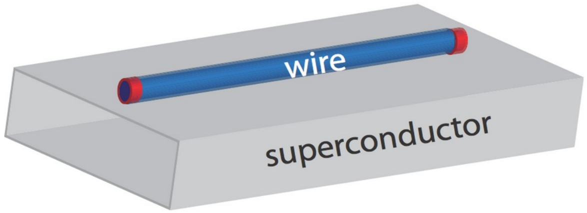
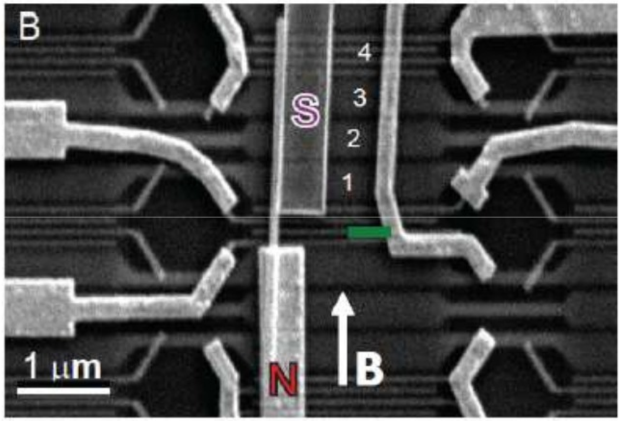
S. Nadj-Perge et al., **Science** 346, 602 (2014);
S. M. Albrecht et al., **Nature** 531, 206 (2016);
Zhang, H. et al Nature 556, 74–79 (2018)
...

much debate, no consensus,
retractions/conflicting data...

Experimental observations

In fact, the quest for edge Majorana fermions is still open!





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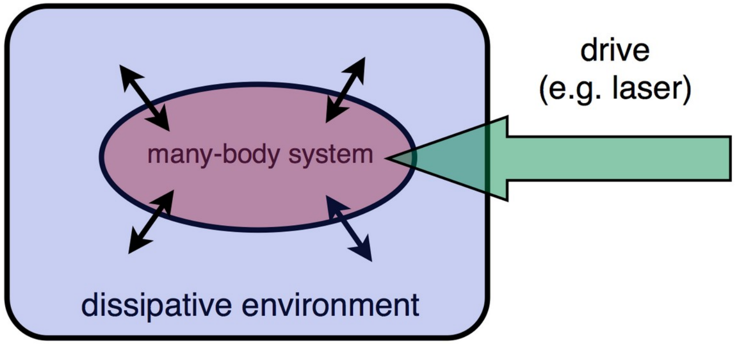
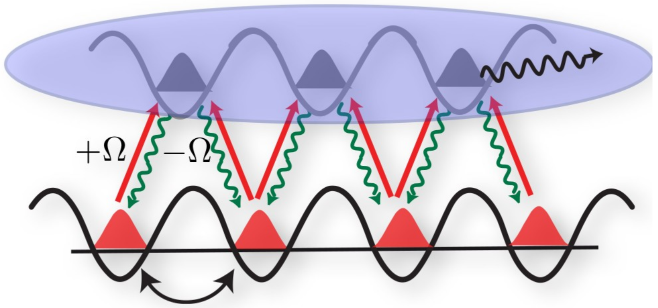


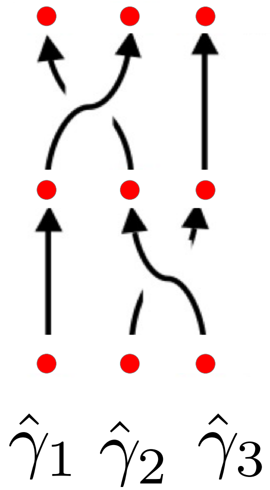
- “radically” approaches may be shed some light into alternative solutions?

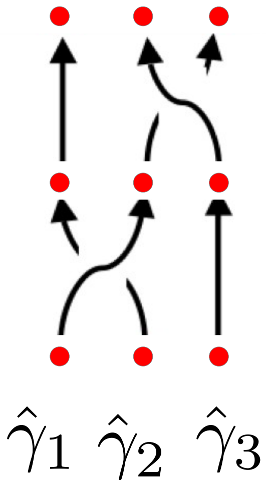


Braiding

- recall... these are “first” steps of many to come in the experimental domain…

- Ultimate goal: ***braiding!***





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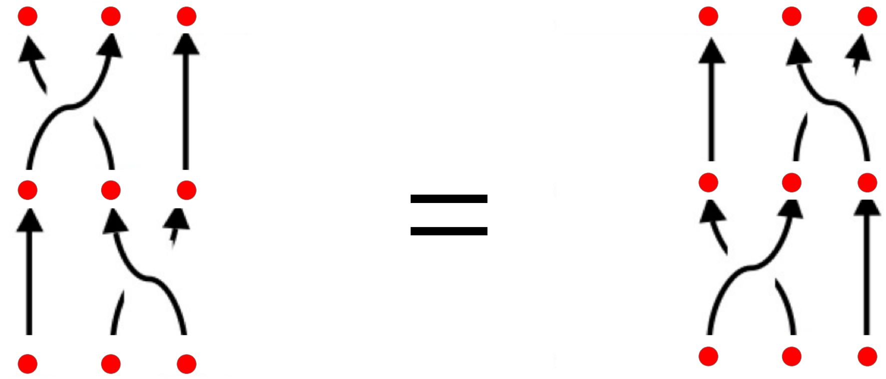

$\hat{\gamma}_1 \ \hat{\gamma}_2 \ \hat{\gamma}_3$ $\qquad\qquad$ $\hat{\gamma}_1 \ \hat{\gamma}_2 \ \hat{\gamma}_3$

$\hat{B}_{ij} =$ (unitary) braiding matrix
for i,j particles

$$\hat{B}_{12}\hat{B}_{23} = \hat{B}_{23}\hat{B}_{12}$$

(abelian statistics)

Braiding

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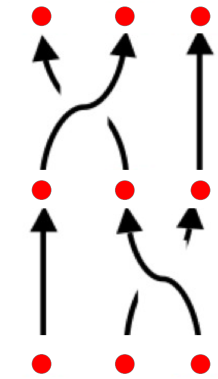
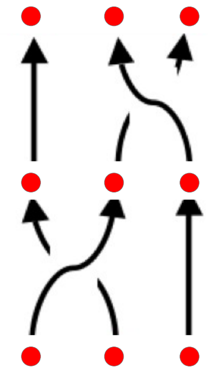


$\hat{\gamma}_1 \; \hat{\gamma}_2 \; \hat{\gamma}_3$

$\neq$



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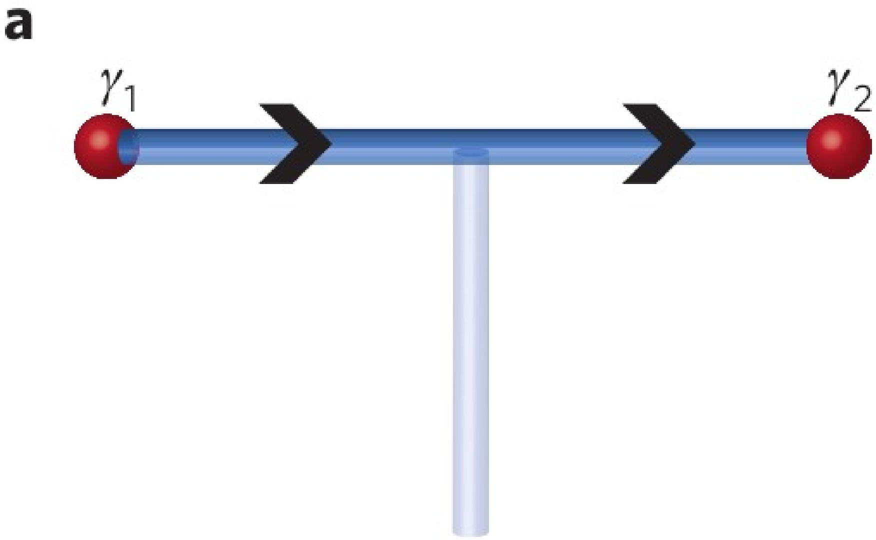
(abelian statistics)

$$\hat{B}_{12}\hat{B}_{23} \neq \hat{B}_{23}\hat{B}_{12}$$

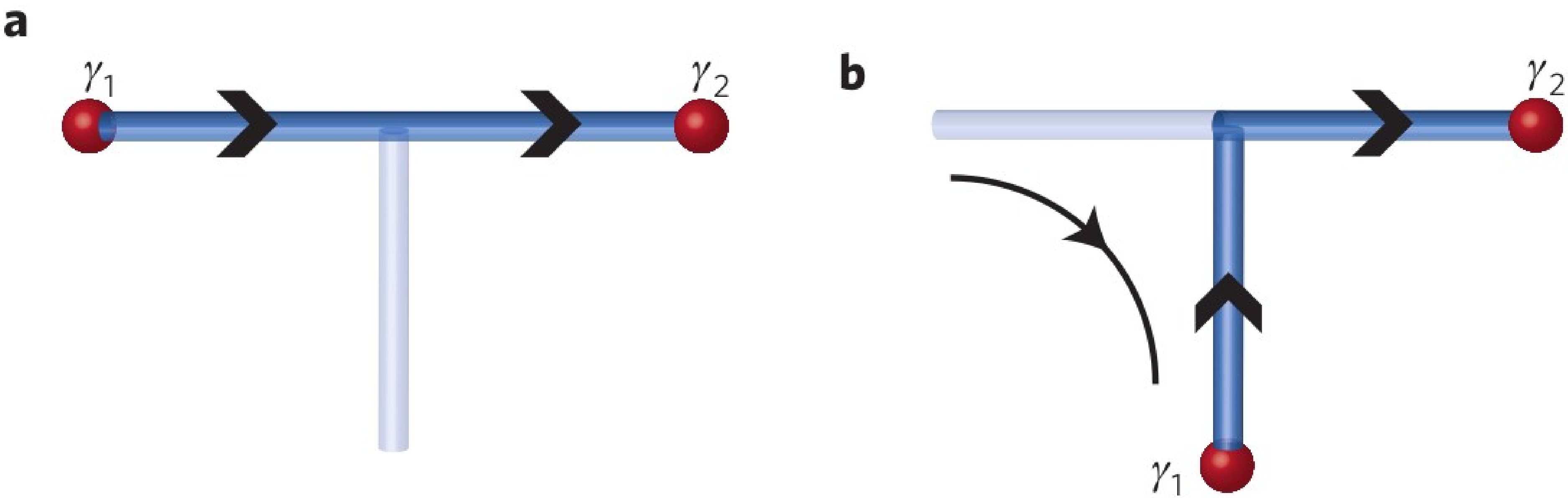
(non-abelian statistics)

B_{ij} : nontrivial action on
quantum subspaces

Braiding: T-junction



Braiding: T-junction



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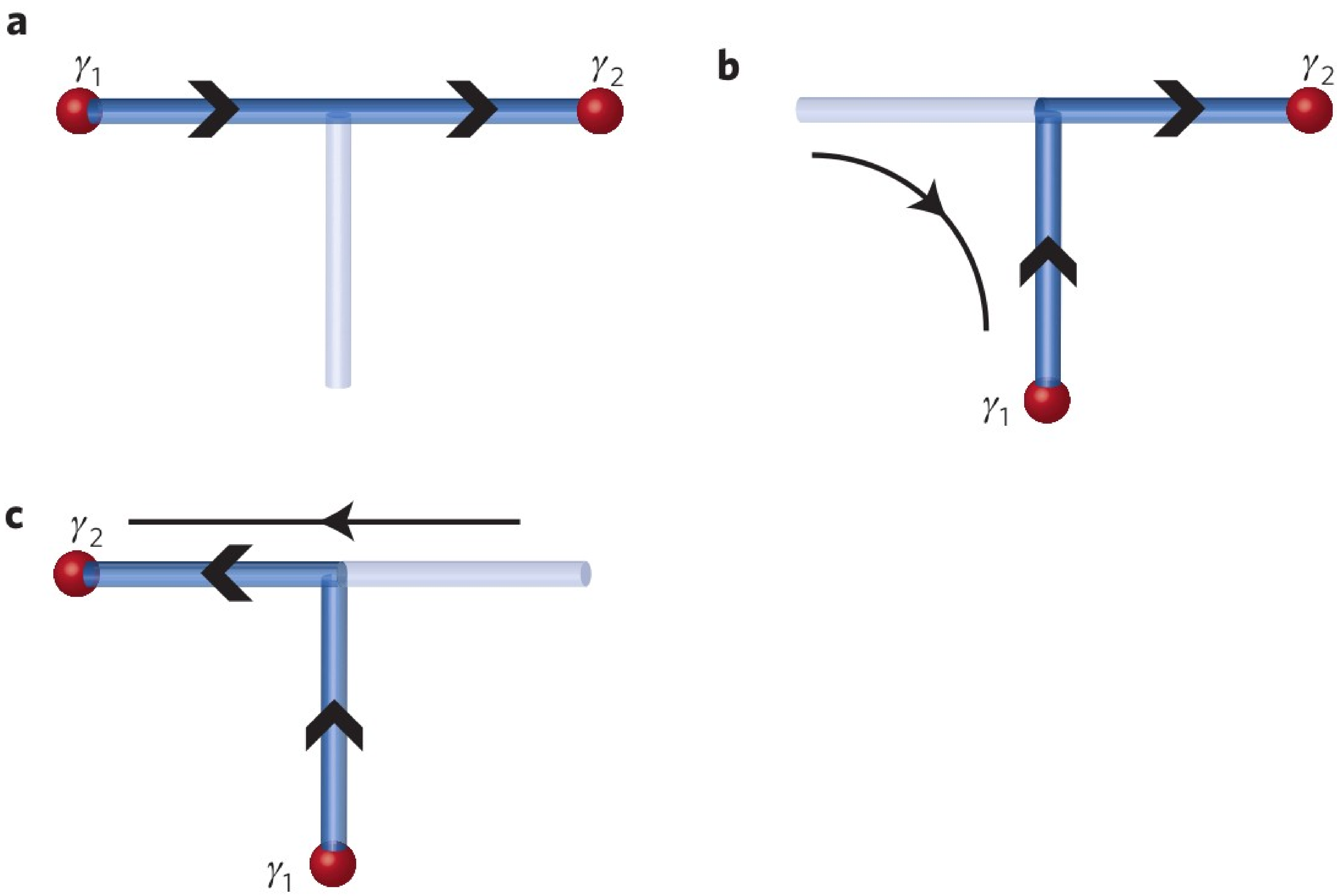

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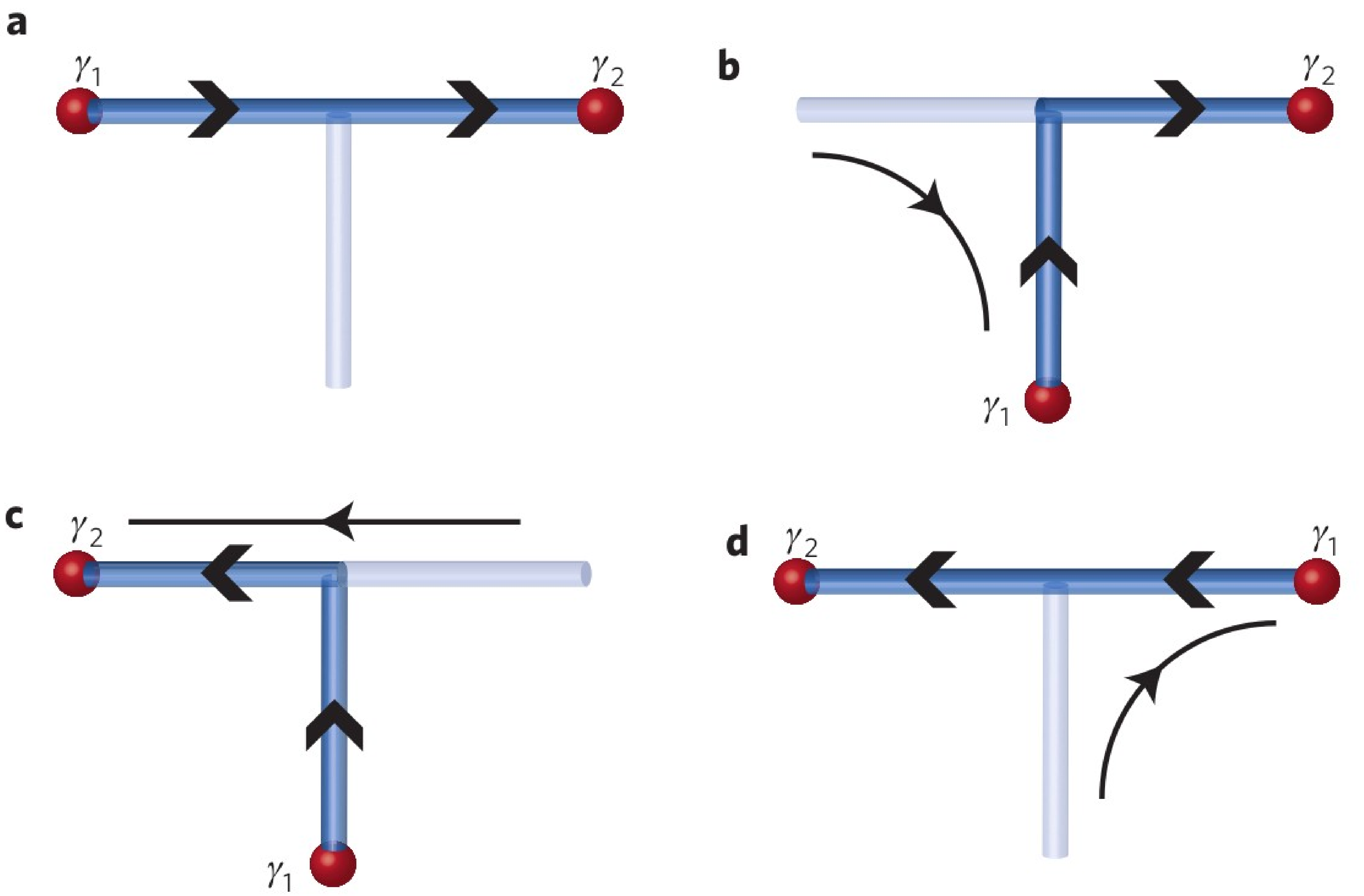
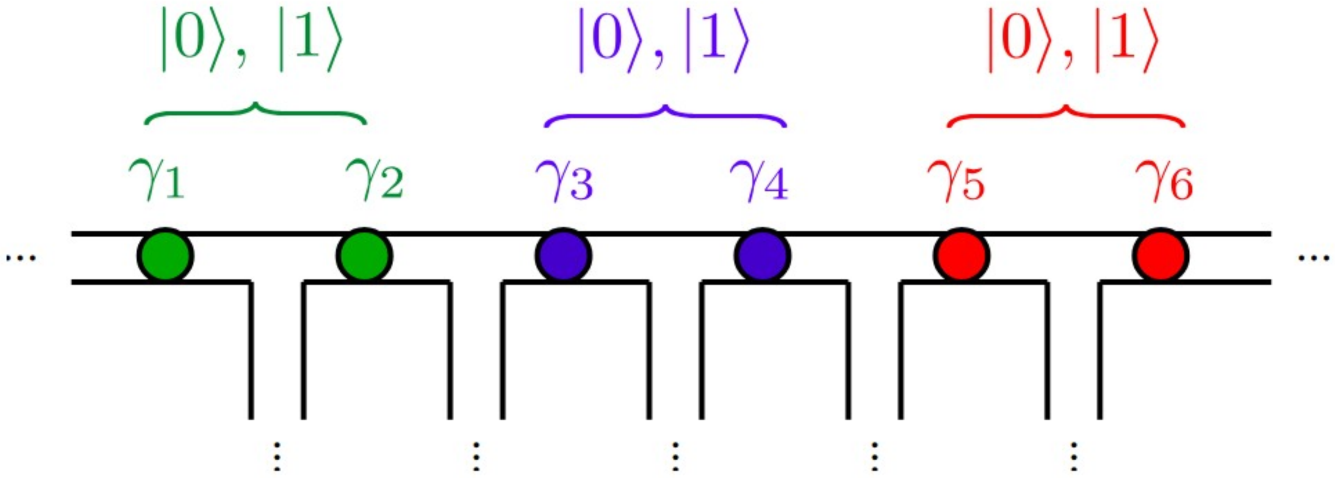
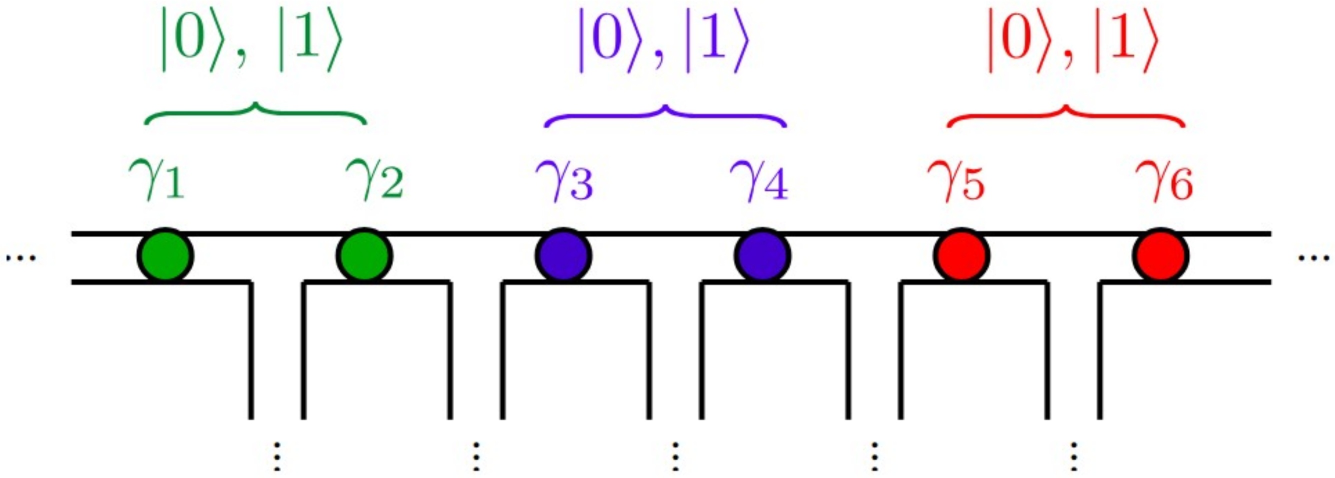


Braiding



$$\hat{B}_{ij}|\mathrm{gs}\rangle = ???$$

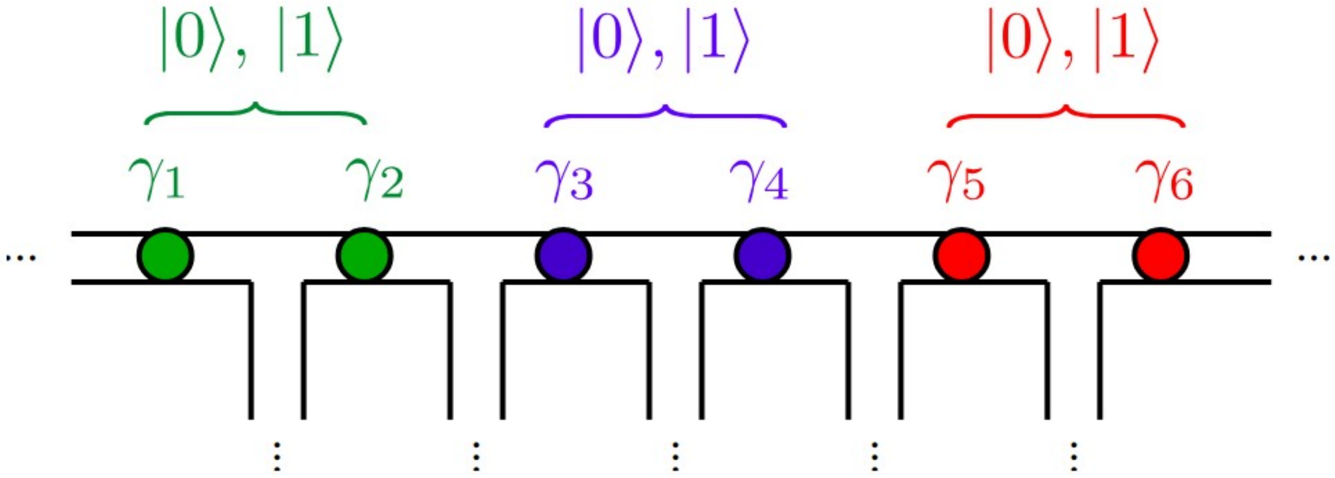
Braiding



$$\hat{B}_{ij}|\mathrm{gs}\rangle = ???$$

- acts in the gs subspace, described by those MF's defining the subspace: $\{\hat{\gamma}_i, \hat{\gamma}_j\}$

Braiding



$$\hat{B}_{ij}|\mathrm{gs}\rangle = ???$$

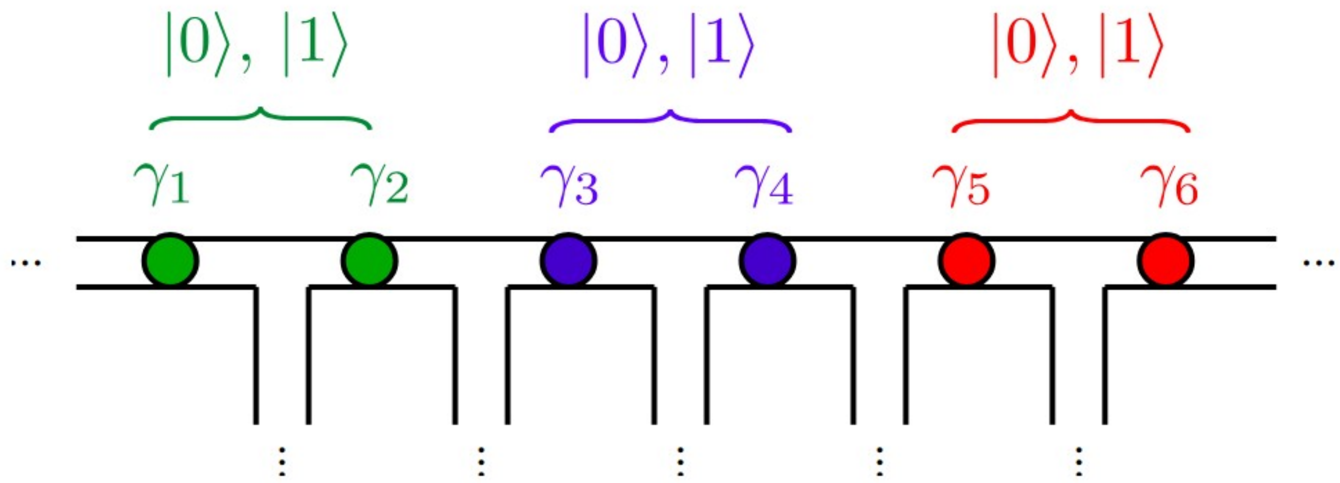
- acts in the gs subspace, described by those MF's defining the subspace: $\{\hat{\gamma}_i, \hat{\gamma}_j\}$

- braiding do not create/anihilate particles, ***conserves parity:***

. $c_i\hat{\gamma}_i + c_j\hat{\gamma}_j$ forbiden;

. $i\hat{\gamma}_i\hat{\gamma}_j$ allowed.

Braiding



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- it is unitary (exponential of "i" times Hermitian)

Braiding

Therefore,

$$\hat{B}_{ij} = e^{-i\hat{h}} = e^{-i(i\beta\hat{\gamma}_i\hat{\gamma}_j)} = e^{\beta\hat{\gamma}_i\hat{\gamma}_j} = \cos(\beta)\mathbb{I} + \sin(\beta)\hat{\gamma}_i\hat{\gamma}_j$$

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which acting on particles i,j:

$$\gamma_i \longrightarrow \hat{B}_{ij}\hat{\gamma}_i\hat{B}_{ij}^{\dagger} = \cos(2\beta)\hat{\gamma}_i - \sin(2\beta)\hat{\gamma}_j$$

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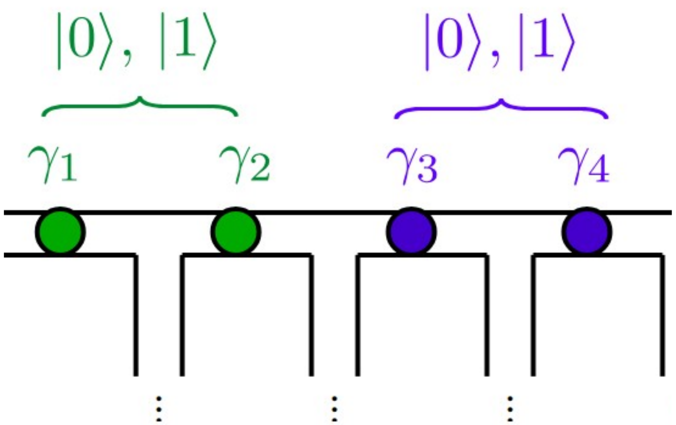
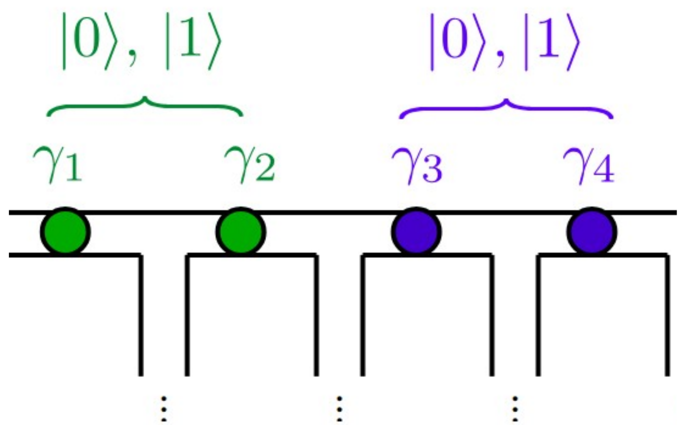
if $\beta = \pi/4$

$$\hat{B}_{ij} = e^{\frac{\pi}{4}\hat{\gamma}_i\hat{\gamma}_j}$$



$$|00\rangle = |\mathrm{gs}\rangle$$

$$|11\rangle = \underbrace{(\hat{\gamma}_1 + i\hat{\gamma}_2)}_{\hat{\eta}_1^\dagger}\underbrace{(\hat{\gamma}_3 + i\hat{\gamma}_4)}_{\hat{\eta}_2^\dagger}|\mathrm{gs}\rangle$$



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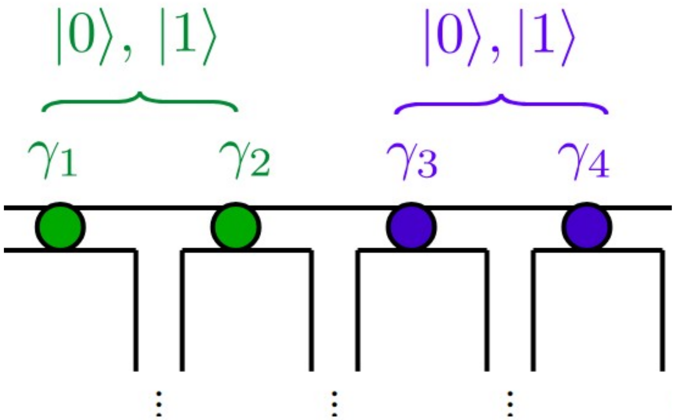
$$\hat{B}_{23}|11\rangle = \frac{1}{\sqrt{2}}(|11\rangle - i|00\rangle)$$



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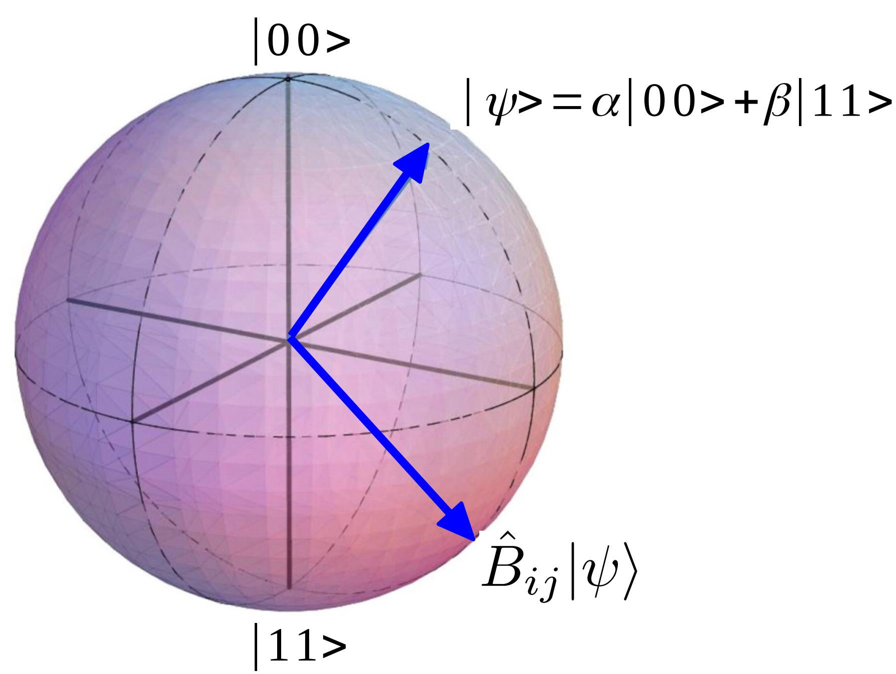
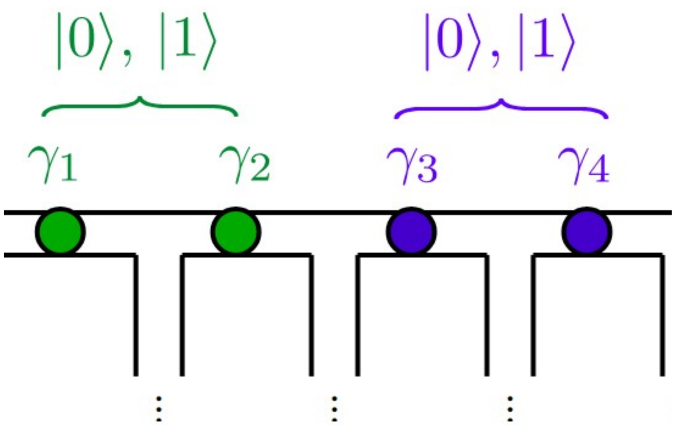
$$\hat{B}_{23}|11\rangle = \underbrace{\frac{1}{\sqrt{2}}(|11\rangle - i|00\rangle)}_{\substack{\textbf{\textit{qubit rotation}}\\ \text{(not only global phases...)}}}$$





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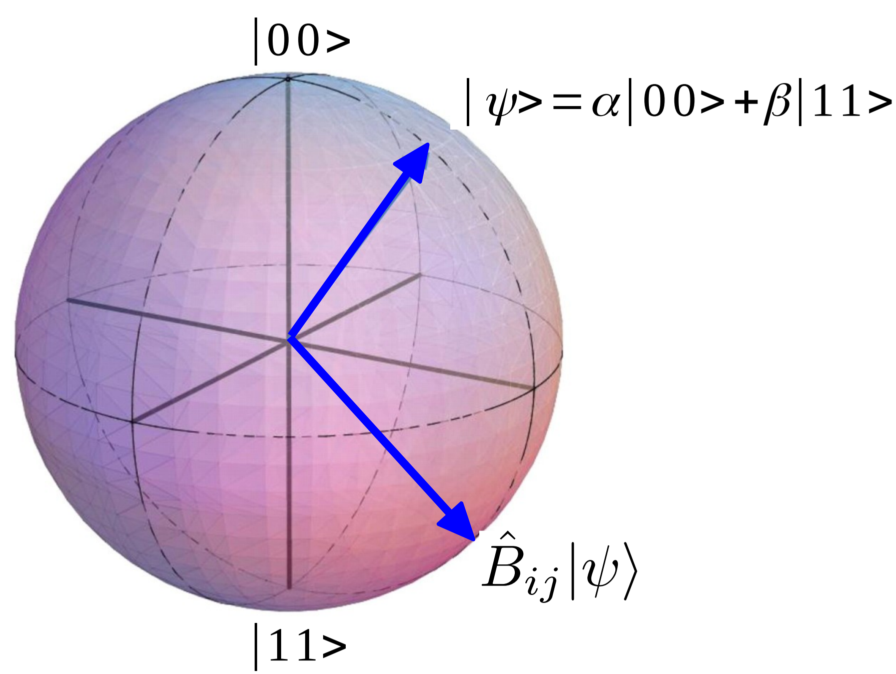

- fault-tolerant
(non-abelian statistics)

Topological Quantum
Computation

Conclusions

- Dissipation not always detrimental;

- Through careful tuning of system/environment coupling one can *engineer* nontrivial dissipative steady states:

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    . dark states - pure

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Conclusions

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- Alternative to usual methods on state preparation

- Quantum computing with dissipation;

Thanks for your attention!

If you are interested…
contact: fernandoiemini@id.uff.br

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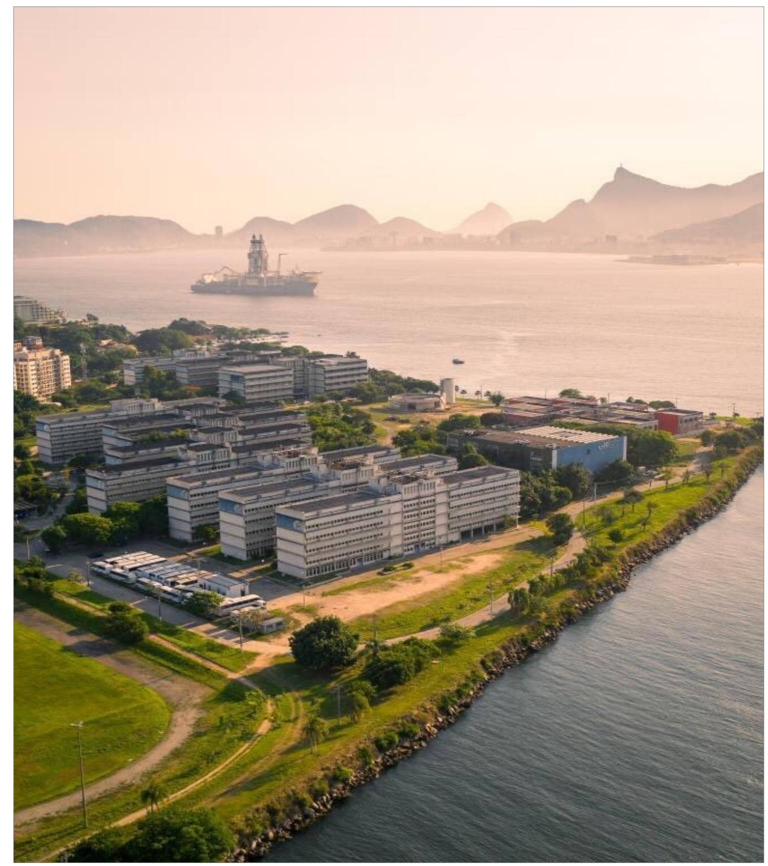
serrapilheira



UFF, Niterói, Brazil