Driven Disordered Systems

Ezequiel Ferrero
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São Paulo School of Advanced Science on Disordered Systems
ICTP - SAIFR, São Paulo, 08/05/2025

Driven Disordered Systems

Agenda

L1. The **depinning** transition of elastic interfaces driven in disordered media and the **creep** motion at low driving

L2. The **yielding** transition of amorphous solids under deformation and associated **criticality** and **avalanche** statistics

L3. **Common framework** for depinning and yielding, analogies and **recent endeavors** in the field

L2. The yielding transition of amorphous solids

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L2. The yielding transition of amorphous solids

Ezequiel Ferrero

*Jean-Louis **Barrat**, Kirsten **Martens**, Chen Liu, Alexandre Nicolas, Kamran Karimi, Francesco Puosi, Elisabeth Agoritsas, Silvia Bonfanti, Stefano Zapperi, Gieberth Rodriguez-López, Alejandro Kolton, Alberto Rosso, Eduardo Jagla, ...*

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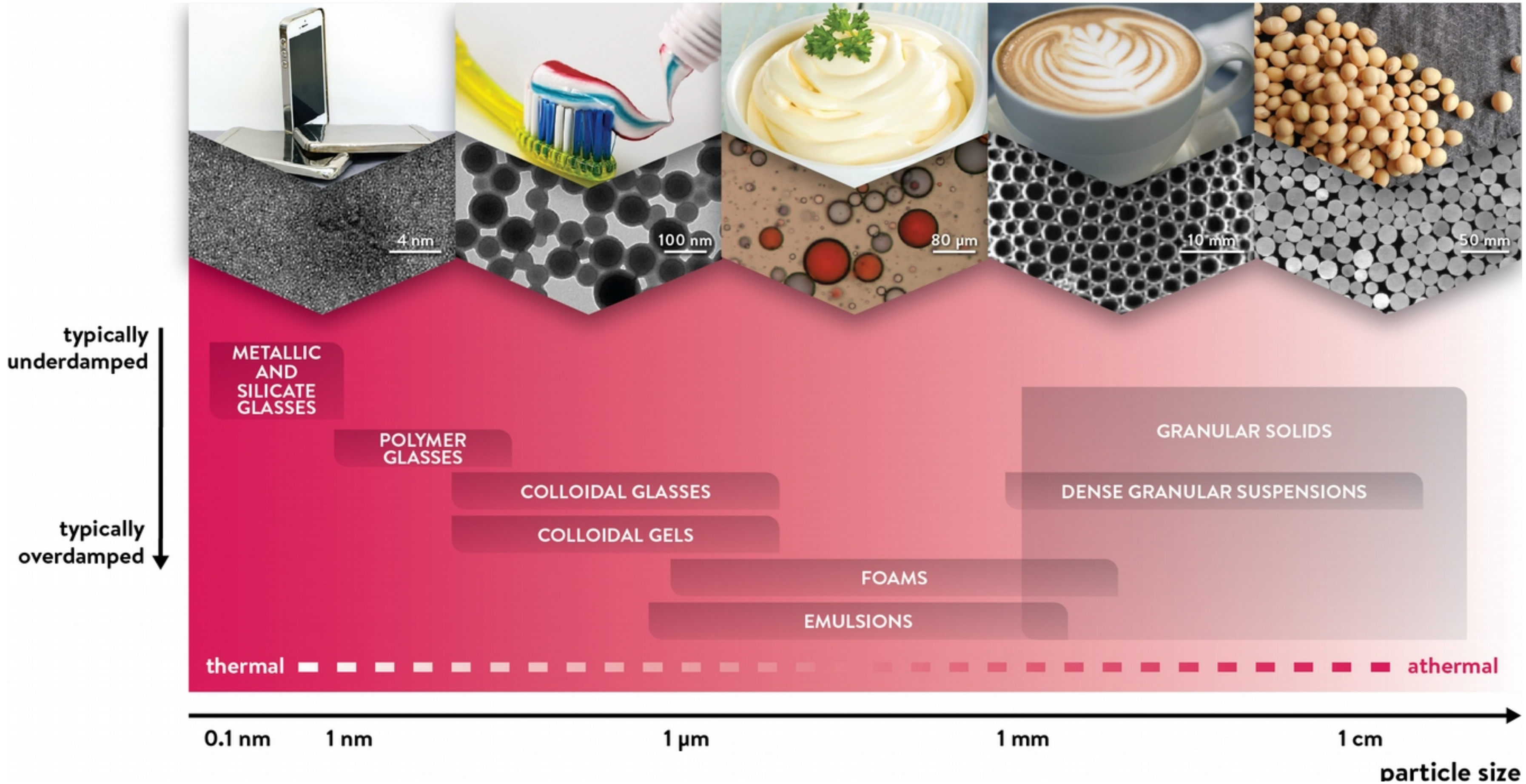
Amorphous materials



very diverse systems... but they share common features

1) Structurally ***disordered***

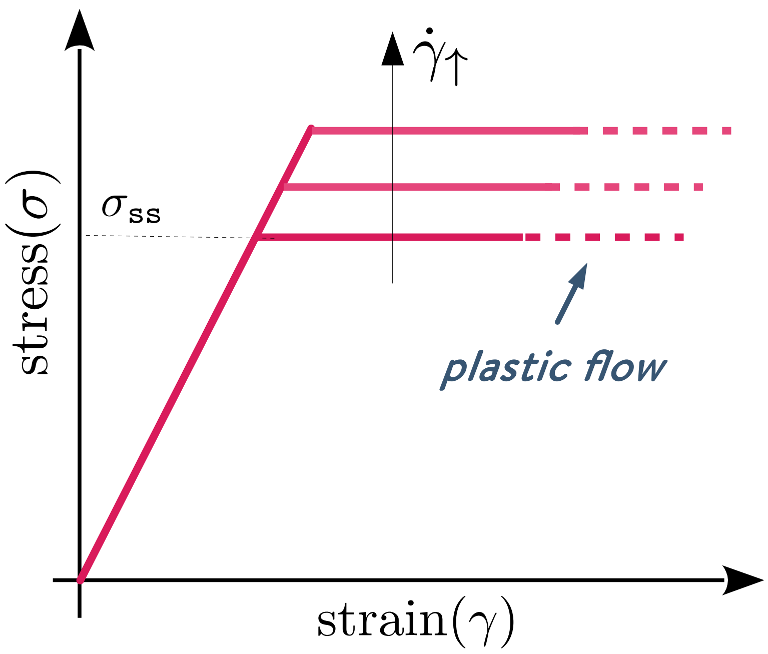
2) Solid-like (***elastic***) behavior below ***yield stress***

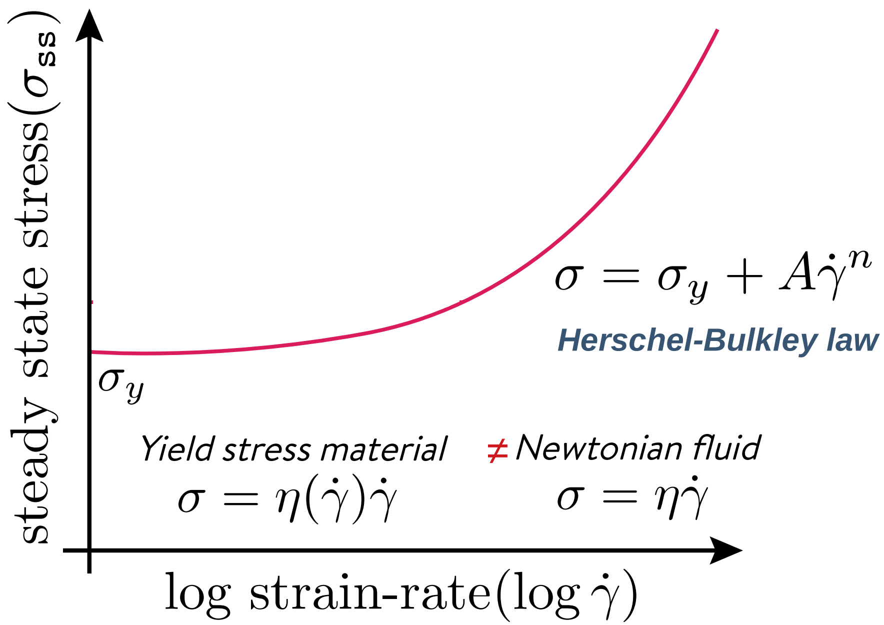
3) ***Flow*** under stress bigger than threshold

D. Bonn *et al.* “*Yield stress materials in soft condensed matter*”, Rev. Mod. Phys. 89, 035005 (2017) ← “experimental”

A. Nicolas, EEF, K. Martens, J.-L. Barrat, Rev. Mod. Phys. 90, 045006 (2018) ← “theoretical”

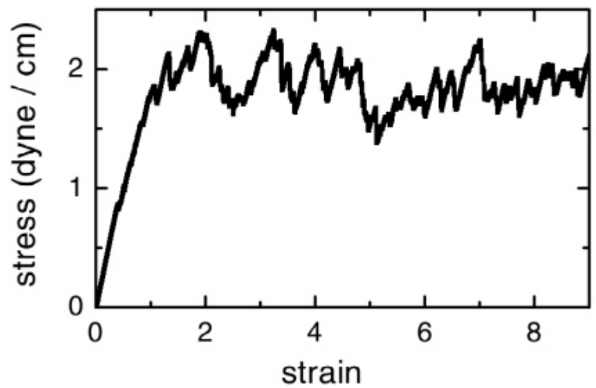
Yield stress systems





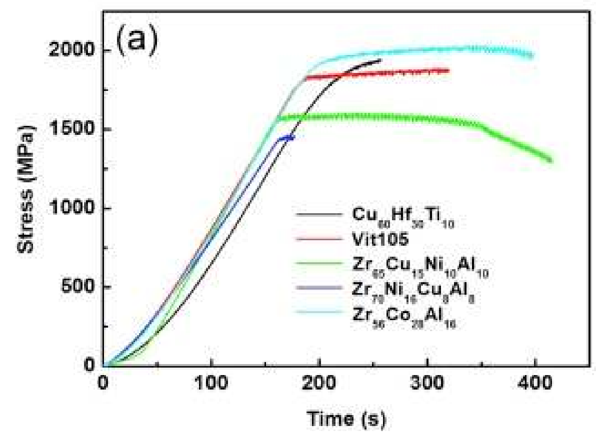
Typical stress-strain and flow curves

Sheared foam



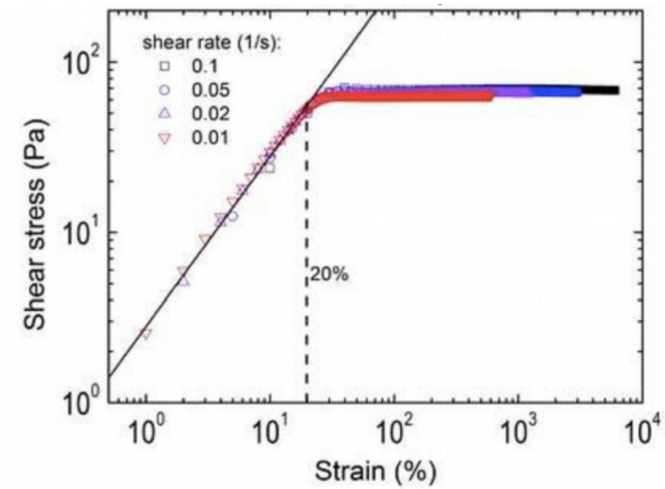
J. Lauridsen et al. PRL **89** 098303 (2002)

Metallic Glasses



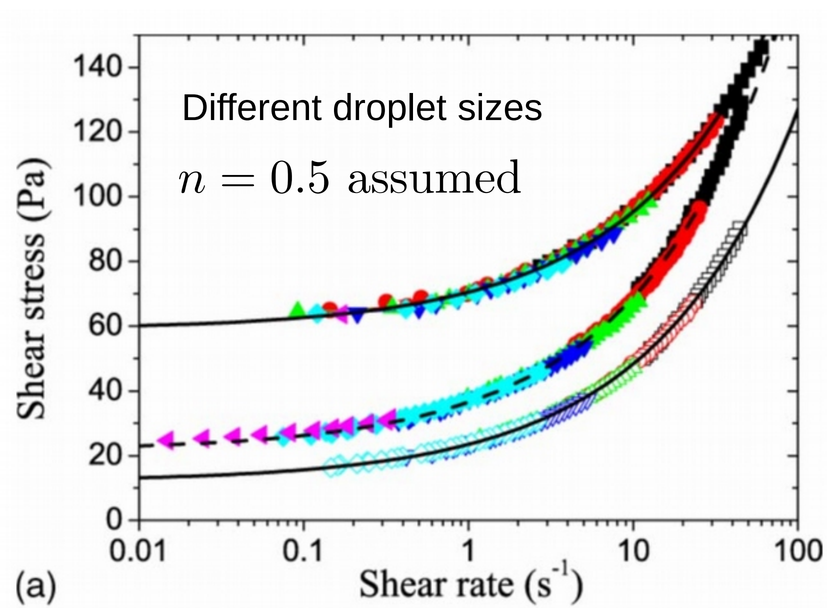
J. Hu et al. Intermetallics **66**, 31 (2015)

Carbopol gel



Dinkgreve et al. Jour. of Rheol. **62**, 773 (2018)

Castor-oil emulsion



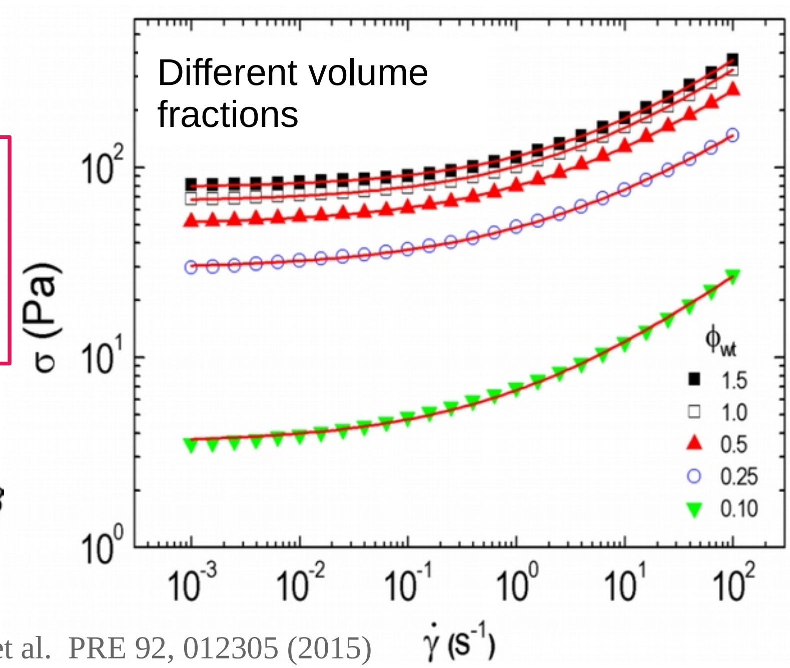
Ovarlez et al. PRE 78, 036307 (2008)

In general:

$$\boxed{\begin{gathered}\sigma = \sigma_y + A\dot{\gamma}^n \\ n \in [0.4, 0.6]\end{gathered}}$$

$$\longrightarrow \beta \simeq 2$$

$$\dot{\gamma} \sim (\sigma - \sigma_c)^\beta$$



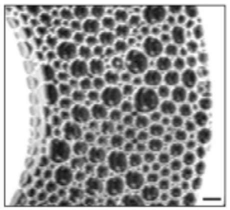
Dinkgreve et al. PRE 92, 012305 (2015)

Stress-strain curves can be more complex

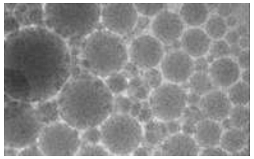
Ductile and brittle materials

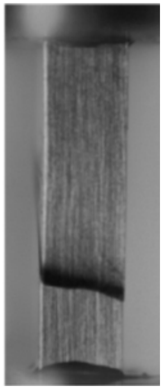
Soap foam
"soft"

E~10-100 Pa



Colloidal glass
"soft"





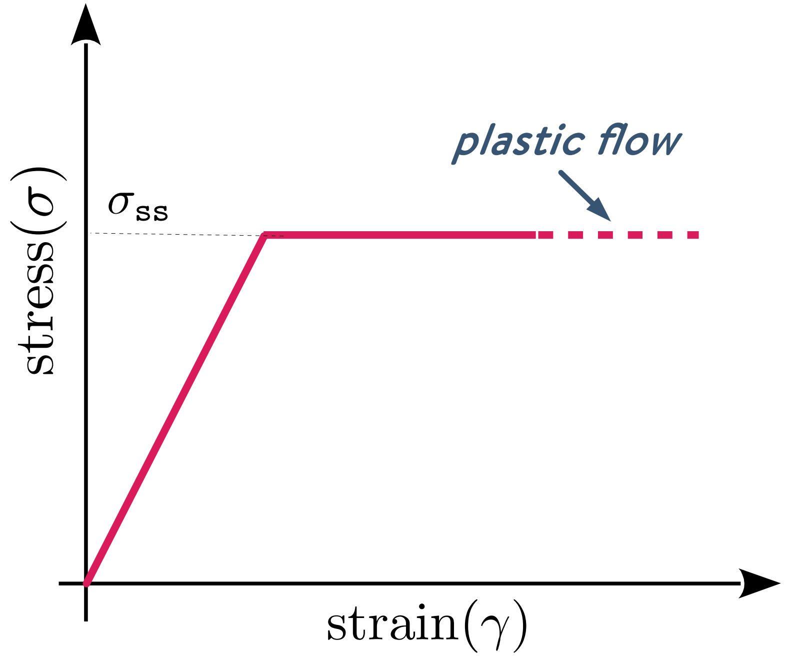
Bulk metallic glass
"hard" somehow ductile

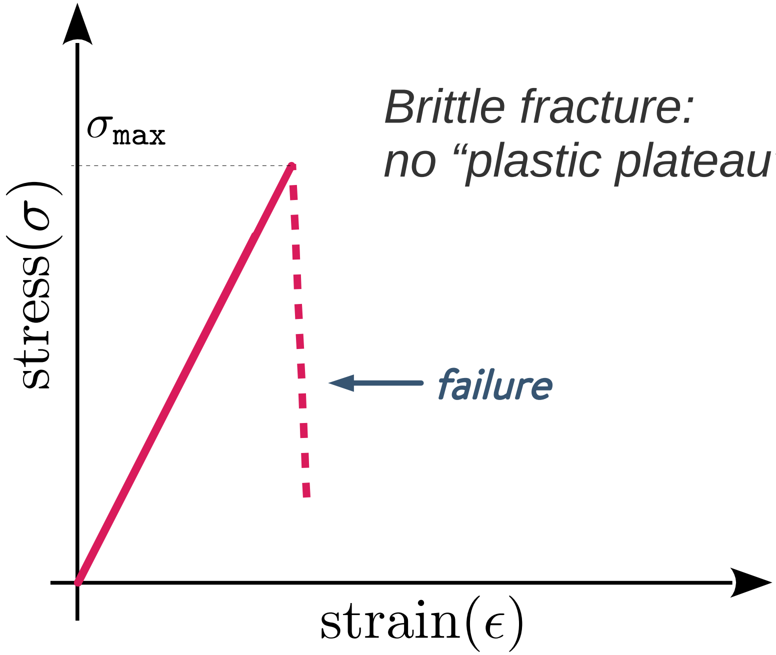
E~100 GPa

Silica glass
hard and brittle

E~70 GPa

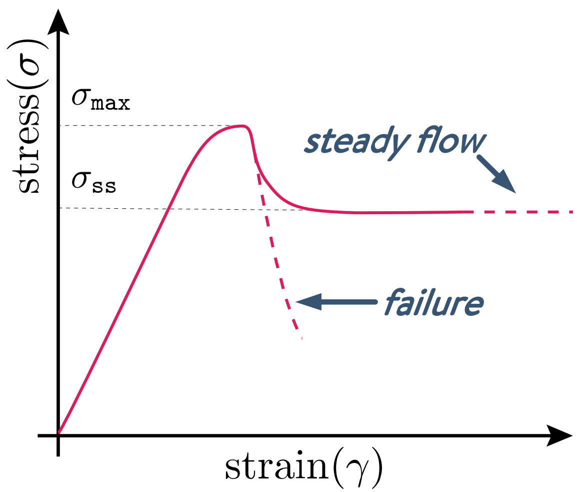
a-SiO_2



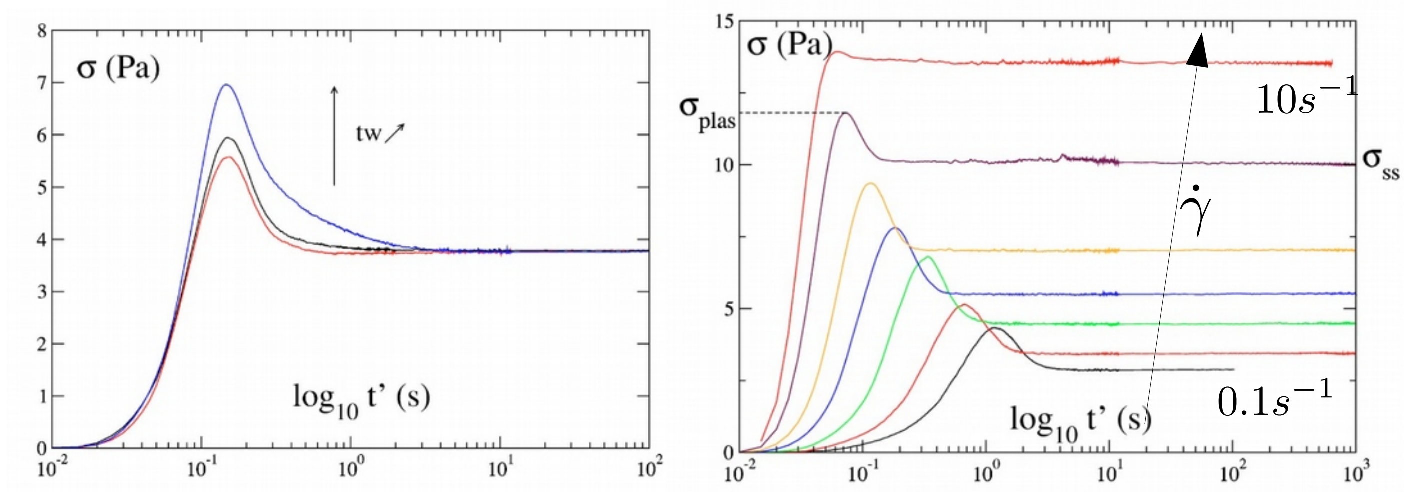


Brittle fracture:
no "plastic plateau"

Further complex stress-strain curves

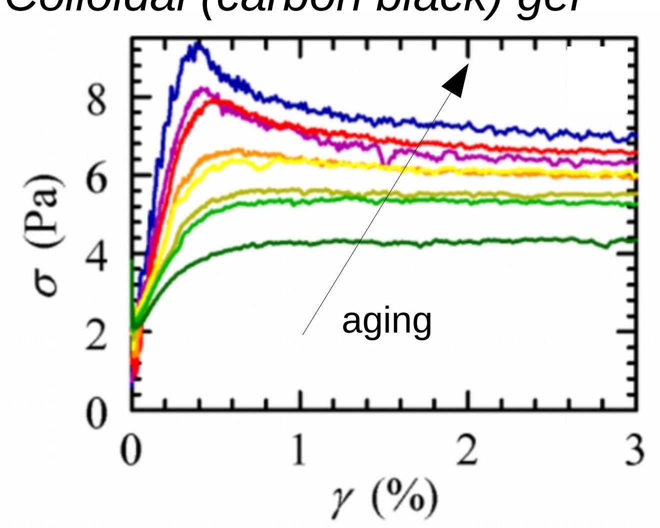


Silica colloids suspensions

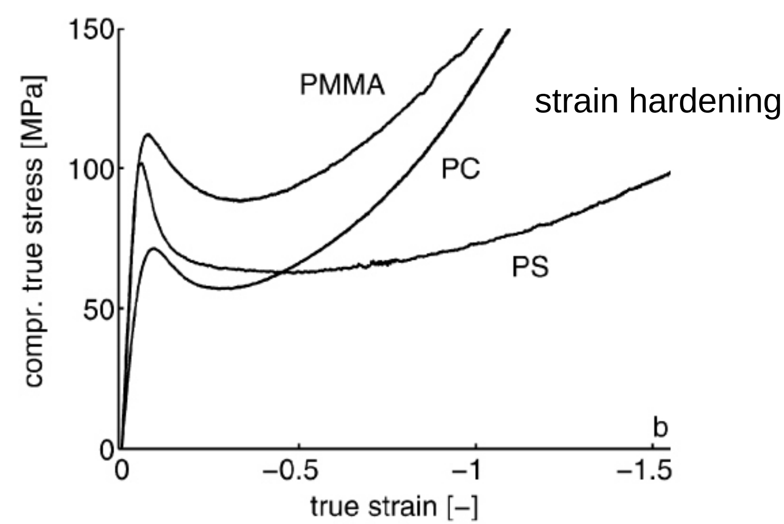


Derec, Ducouret, Adjari, Lequeux PRE 67, 061403 (2003)

Colloidal (carbon black) gel



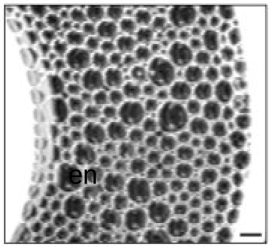
Sprakel et al PRL 106, 248303 (2011)

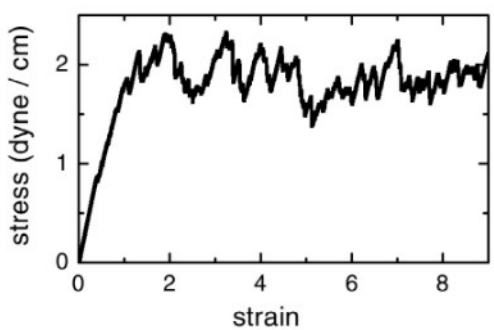


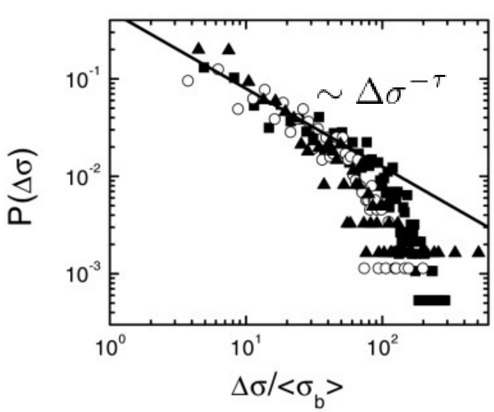
H.G.H. van Melick et al. Polymer **44** 2493 (2003)

Motivation: statistics of “avalanches” of plastic events

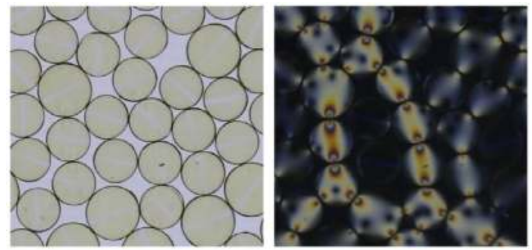
Foams

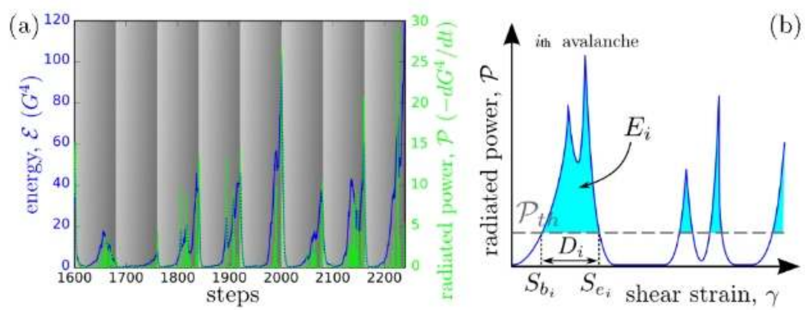


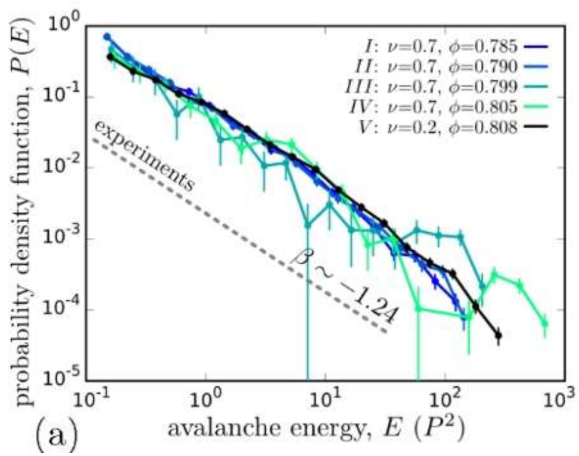




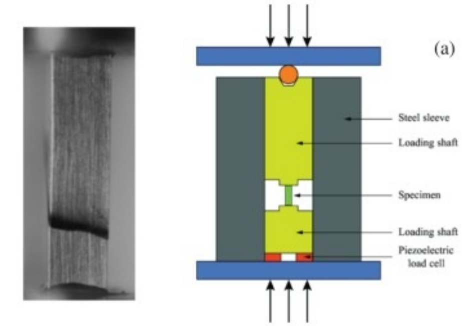
Granular systems

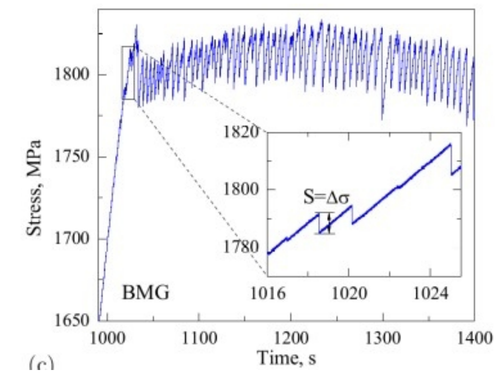


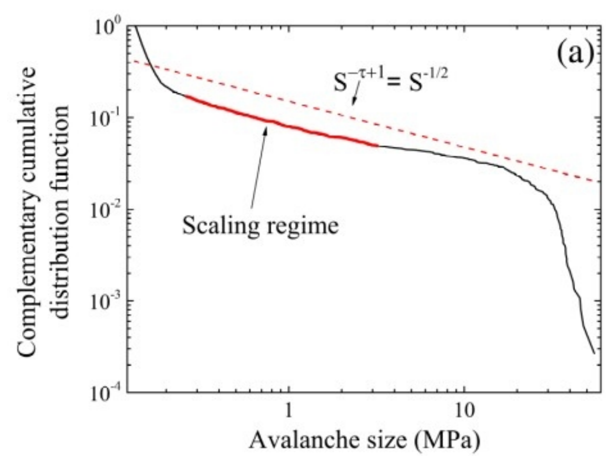




Bulk metallic glasses







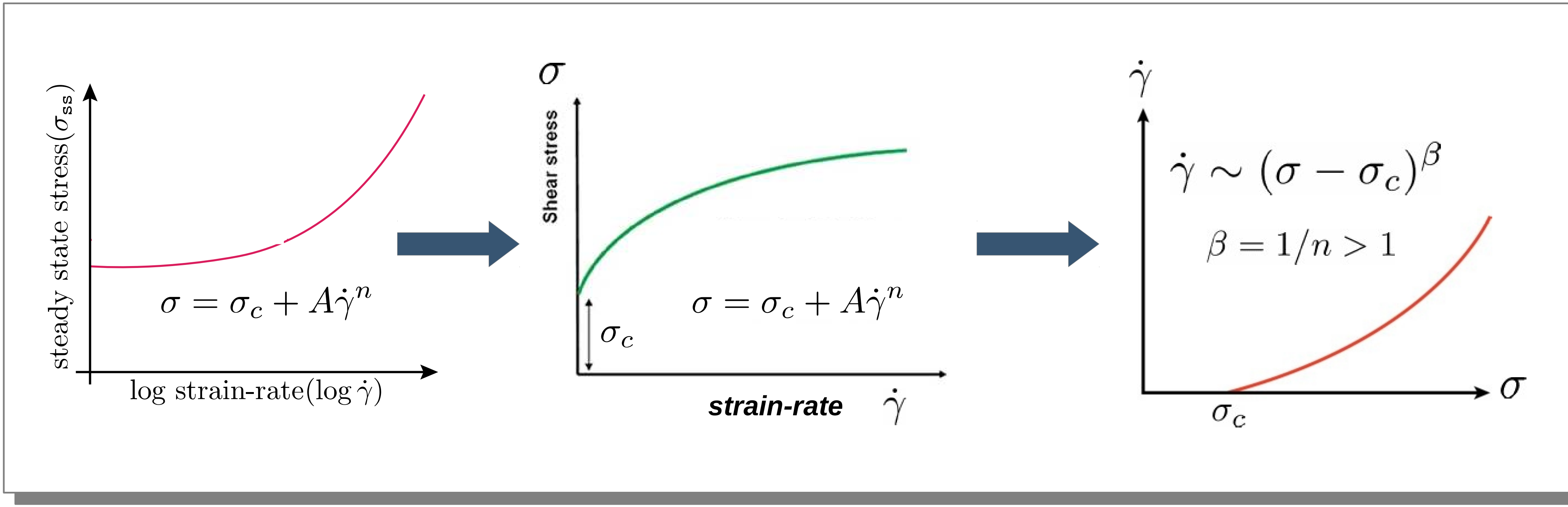
J. Lauridsen et al. PRL **89** 098303 (2002)

J. Barés et al Phys Rev E **96**, 052902 (2017)

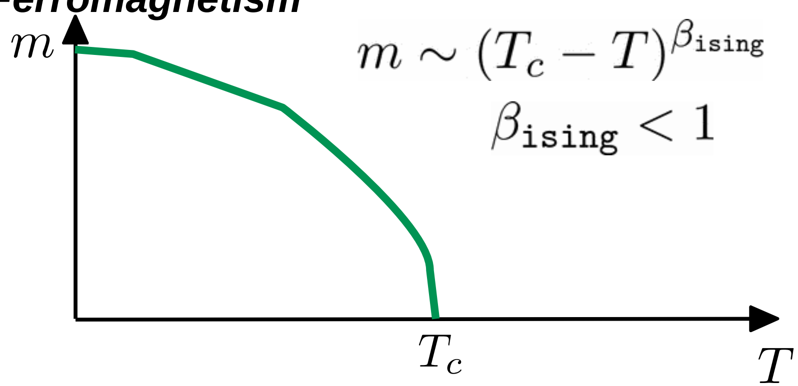
J. Antonaglia et al. PRL **112** 155501 (2014)

The yielding transition

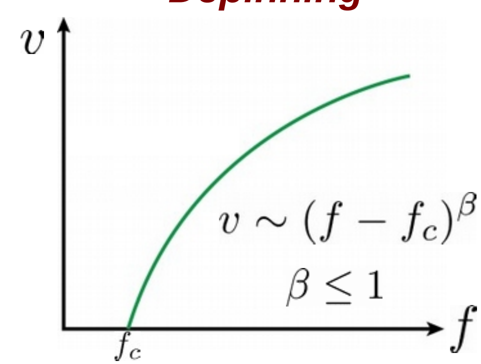
A **dynamical phase transition** between an “arrested” **elastic solid** state and **plastically** “flowing” state when a **critical yield stress** is overcome



Ferromagnetism



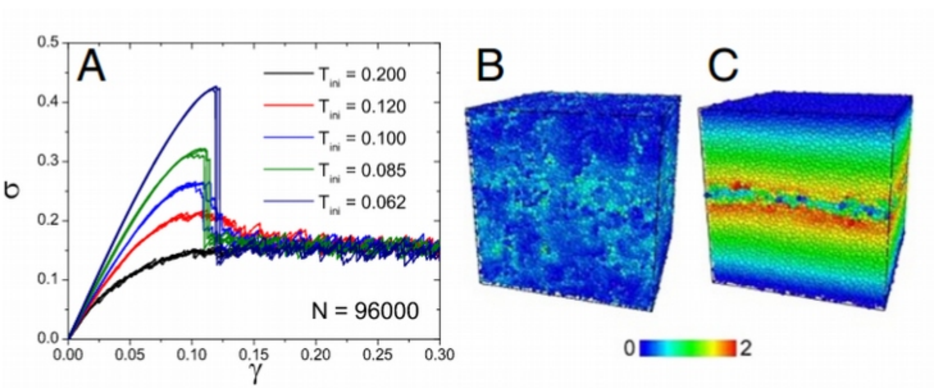
Depinning



Other definitions yielding

The onset of plastic behavior

Yielding onset: distinction among “brittle” and “ductile” transition.

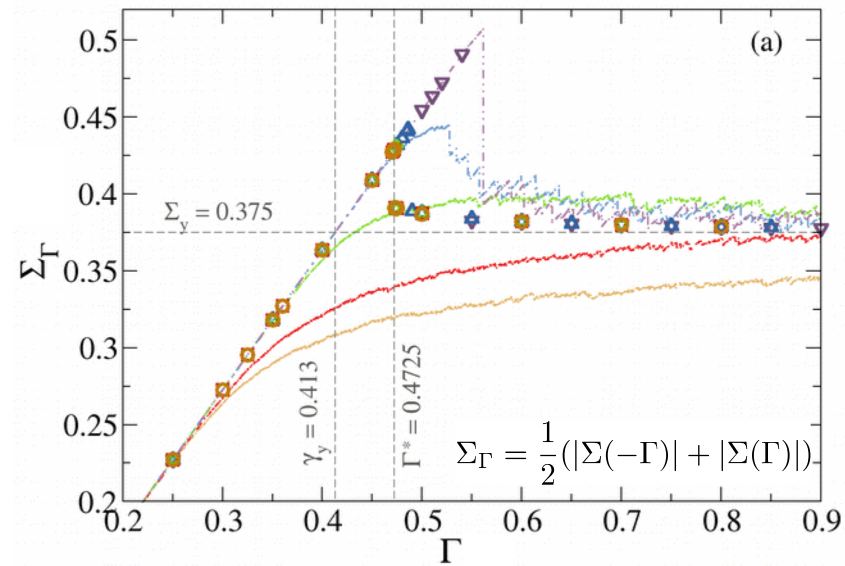


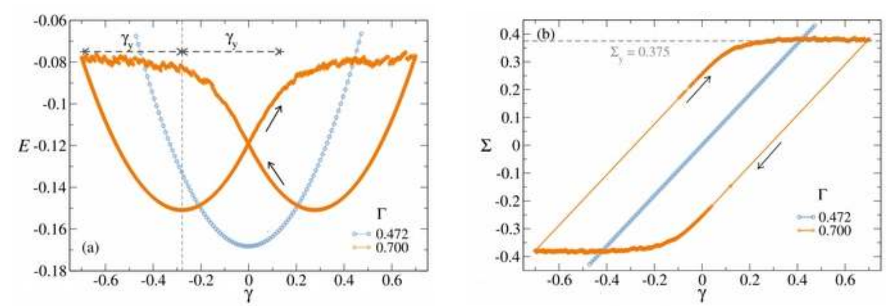
“Yielding transition” **depends on sample preparation:** aging, annealing, etc.

Discontinuous transition, spinodal instability: mapping to Random First-Order Transition

*Ozawa, Berthier, Biroli, Rosso, Tarjus PNAS **115**, 6656 (2018)
Barlow, Cochran, Fielding PRL **125**, 168003 (2020)
Berthier, Biroli, Manning, Zamponi, arXiv:2401.09385

The critical amplitude in oscillatory-shear protocol





P. Leishangthem et al., Nat. Commun. 8, 14653 (2017)
K. Khirallah et al. Phys. Rev. Lett. 126, 218005 (2021)
*C. Liu, EEF, et al. J. Chem. Phys. 156, 104902 (2022)

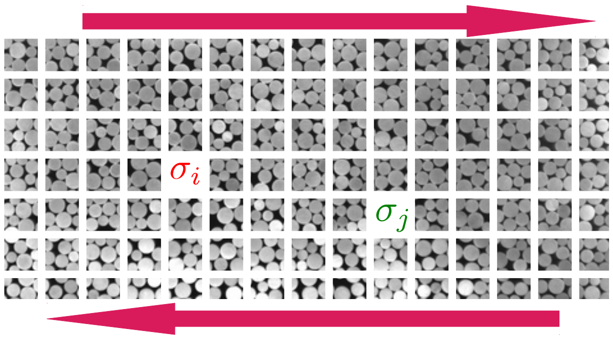
Critical exponents related to yielding

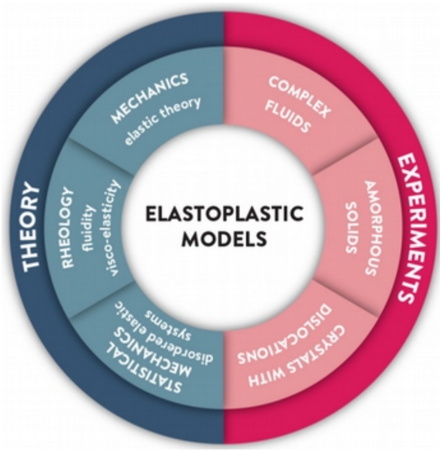
In analogy with equilibrium phenomena and other driven transitions

β	$\dot{\gamma} \sim (\sigma - \sigma_c)^{\beta}$	Flowcurve
$\tau \quad d_f \quad \alpha$	$P(S) \sim S^{-\tau} f(S/S_{\mathtt{cut}})$ $S_{\mathtt{cut}} \sim k^{-\alpha}$ or $S_{\mathtt{cut}} \sim L^{d_f}$	Avalanche size distributions
$\delta \quad z$	$\langle S \rangle \sim \langle T \rangle^{\delta} \quad \delta = d_f / z$	Avalanche mean size and duration
$\phi \quad \theta$	$\langle x_{\mathtt{min}} \rangle \sim N^{-\phi} \quad P(x) \sim x^{\theta}$	Density of shear transformations NEW

...and so on

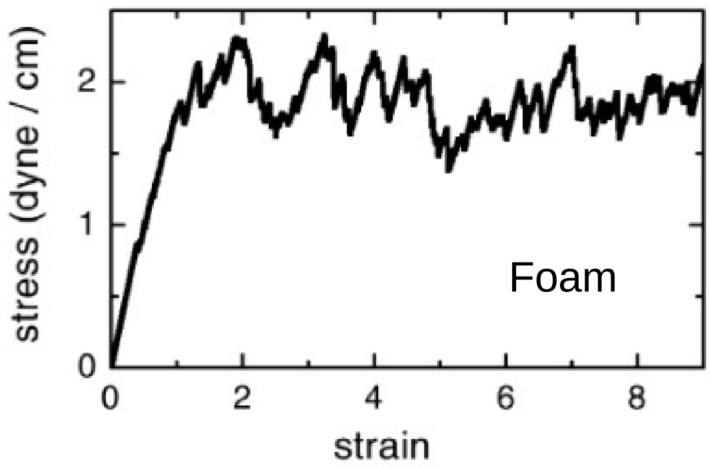
ELASTOPLASTIC MODELS OF AMORPHOUS SOLIDS



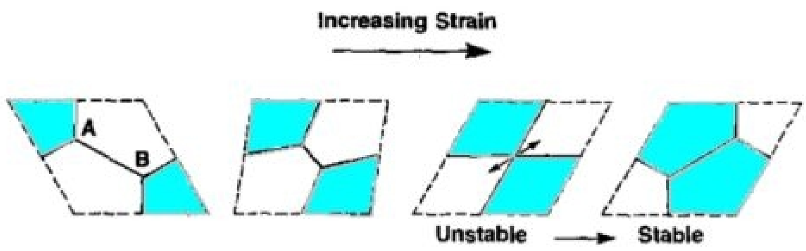


Basic ingredients: 1. Local rearrangements

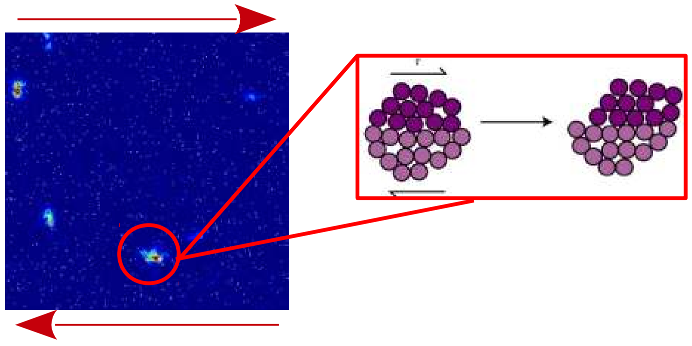
⇒ "jerky" aspect of the stress response

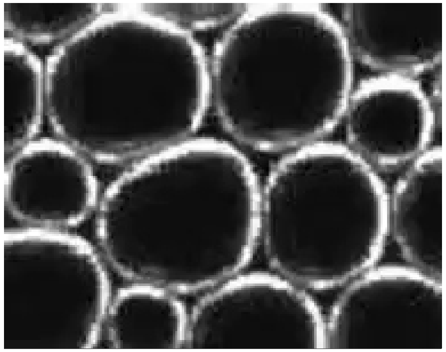


In foams: "T1 event" (4 bubbles)



well identified, localized **elementary plastic events**
or *"shear transformation zones" (STZs)*



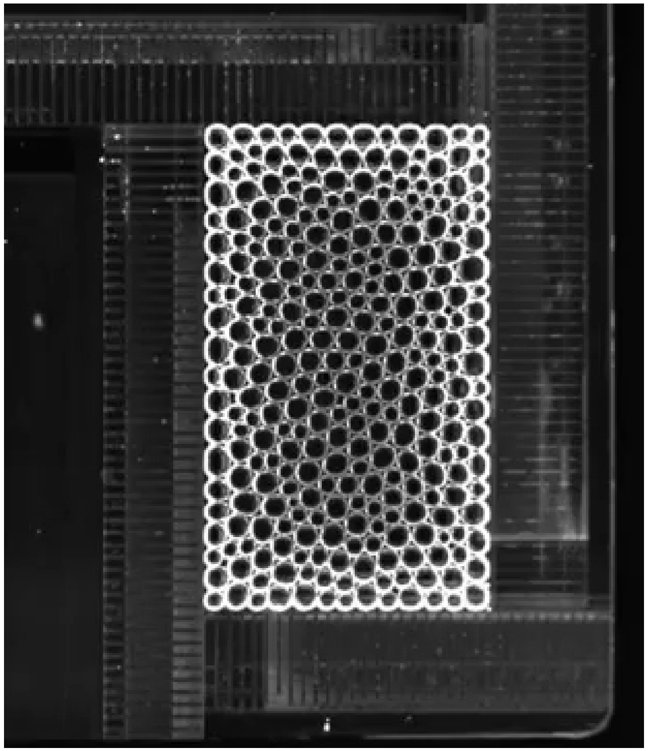


In general: tens/hundreds of particles involved

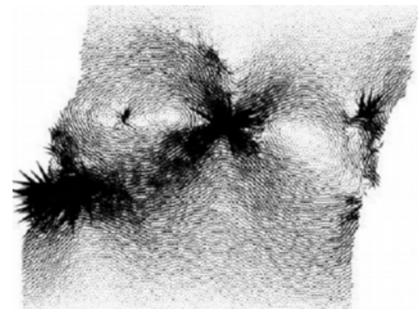
J. Lauridsen et al. PRL (2002)
A. Nicolas et. al EPJE **37** 50 (2014), Argon and Kuo Mat. Sci. Eng. **39** 101 (1979)

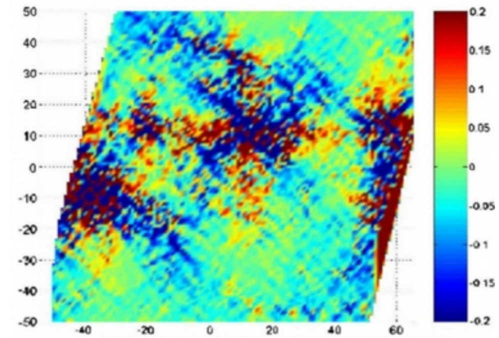
Princen and Kiss, *J Coll. Int. Sci.* **128** 176 (1989)
"T1 event in a densely packed foam" by M. van Hecke, youtube (2014)

Basic ingredients: 2. Medium elastic response



MD simulations:



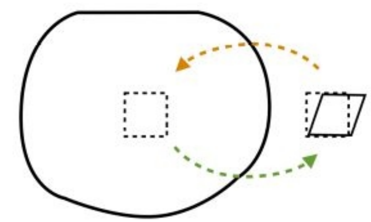


displacement field *stress change*

Maloney and Lemaitre PRL **93**, 195501(2004)

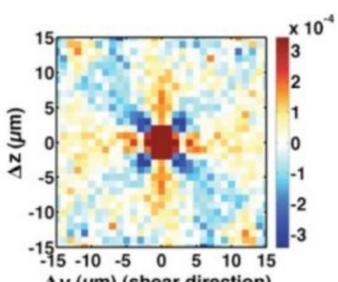
Continuum mechanics:

elastic response to a
deformed inclusion

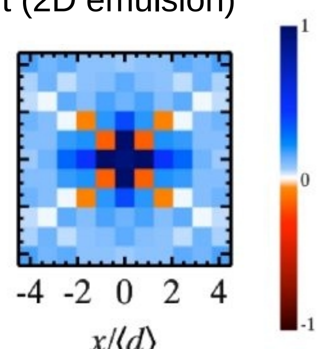


Experimental measurements:

correlations of local strain
(sheared colloidal glass)



average stress change around
an event (2D emulsion)



"Eshelby" propagator for the stress redistribution

$$G(r,\theta) \sim \frac{\cos(4\theta)}{r^d}$$

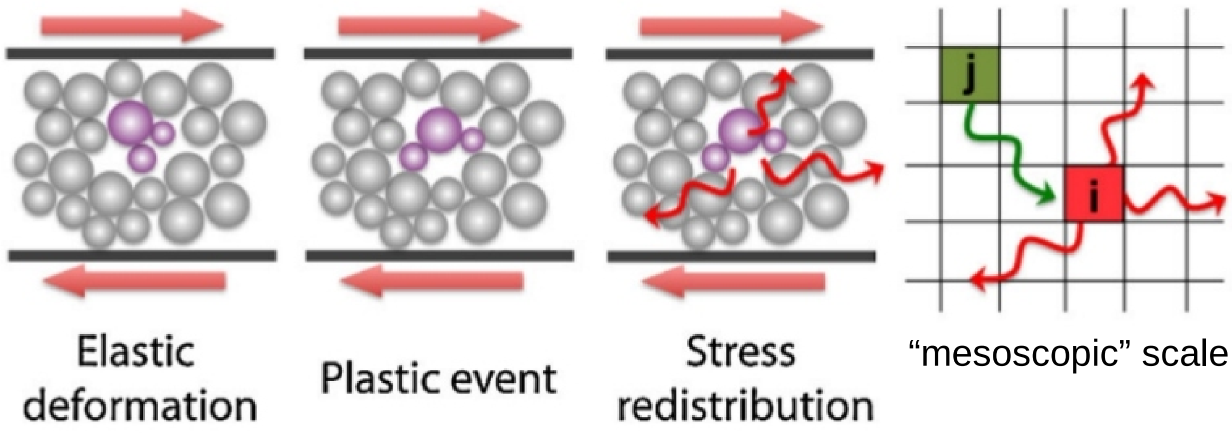
Quadrupolar in symmetry, ***dipolar*** in range

"Shearing a 2D foam" by M. van Hecke, youtube (2014)

Jensen et al, PRE **90**, 042305 (2014)   Desmond and Weeks, PRL **115**, 098302 (2015)

F. Puosi, J. Rottler, J.-L. Barrat PRE **89** 042302 (2014)
J.D. Eshelby Proc. Roy. Soc. A **241** 376 (1957)
Picard *et al*. EPJE **15** 371 (2004)

Phenomenology: description at a **mesoscopic** scale



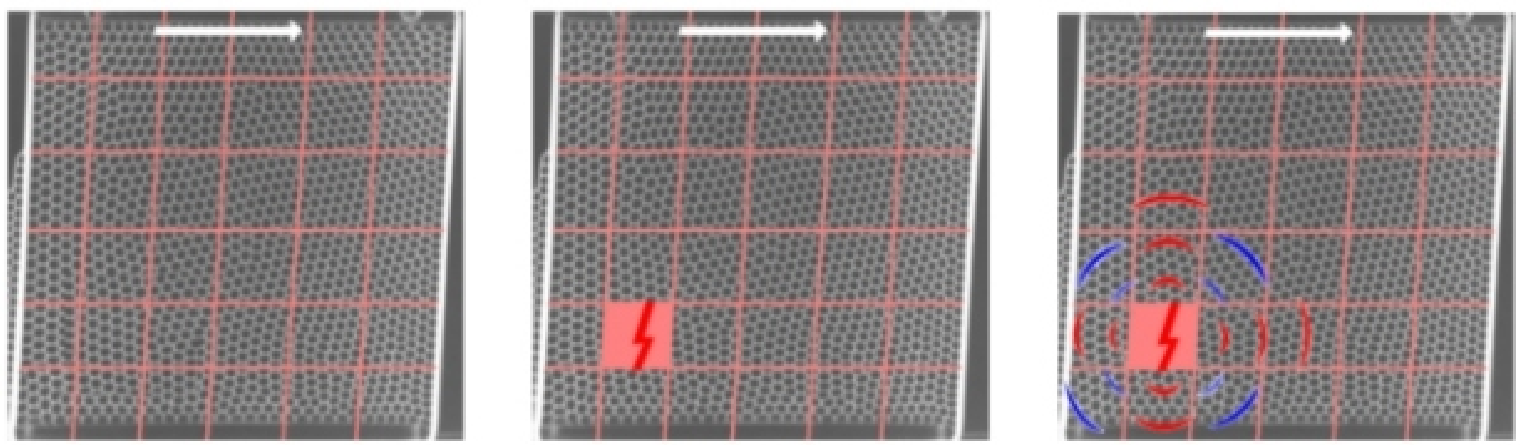
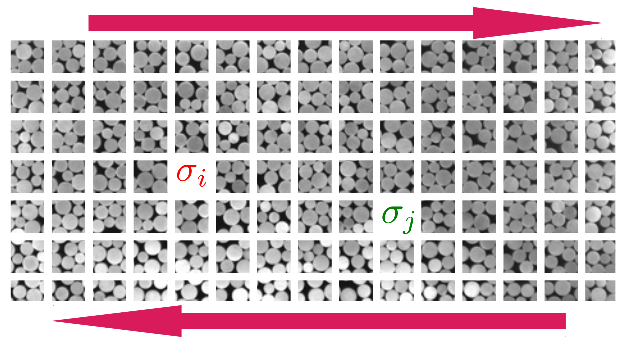


Fig. credit:
Bocquet *et al.* PRL **103**, 036001 (2009)

Coarse-grained Elasto-Plastic Models (EPMs)

Simplifications:
- *Scalar*
- *Athermal*
- *Overdamped*
- *On-lattice*

System:
- Stress configuration $\{\sigma_i\}$
- Elasto-plastic states $\{n_i\}$



Each block:
- locally stable if $\sigma_i < \sigma_{y_i}$
- elastic loading $\sigma_i \to \sigma_i + \mu\dot{\gamma}$
- local yielding & stress redistribution

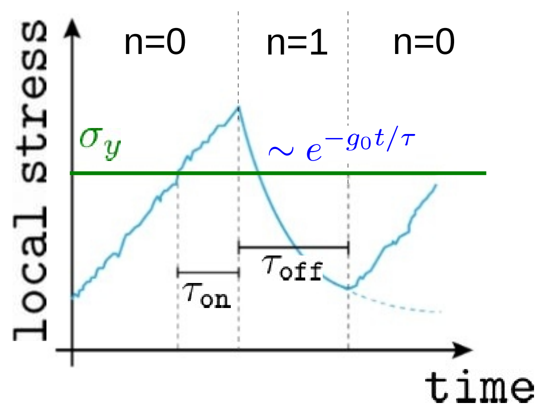
$$\text{if} \quad \sigma_j > \sigma_{y_j} \begin{cases} \sigma_j \to 0 \\ \sigma_i \to \sigma_i + \underline{G_{ij}\sigma_j} \end{cases}$$

$$\partial_t \sigma_i(t) = \underbrace{\mu\dot{\gamma}^{\text{ext}}}_{\substack{\text{global loading} \\ \text{imposed strain rate}}} - \underbrace{g_0 n_i(t)\frac{\sigma_i(t)}{\tau}}_{\substack{\text{exponential stress} \\ \text{decay when fludized}}} + \underbrace{\sum_{j\neq i} G(i,j) n_j(t) \frac{\sigma_j(t)}{\tau}}_{\substack{\text{"mechanical noise" due to} \\ \text{plastic activity elsewhere}}}$$

+ Dynamical **rules** for the local "state" n_i

$$n_i(t) = 0 \quad \text{locally elastic}$$
$$n_i(t) = 1 \quad \text{locally plastic}$$

$$\boxed{n_i : \begin{cases} 0 \to 1 \text{ typically when } \sigma_i > \sigma_{y_i} \\ 1 \to 0 \text{ e.g., after a time } \tau_{\text{off}} \end{cases}}$$



$$g_0 \equiv -G_{ii} > 0$$

$$\boxed{G_{ij}^{2D} \propto \frac{\cos(4\theta_{ij})}{r_{ij}^2}} \quad (i \neq j)$$

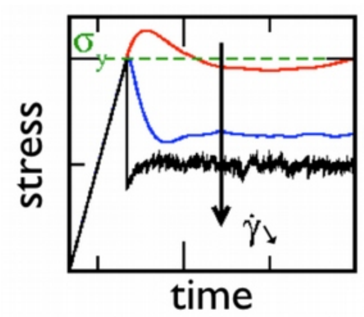
Eshelby **propagator** $G \sim \dfrac{1}{r^d}$

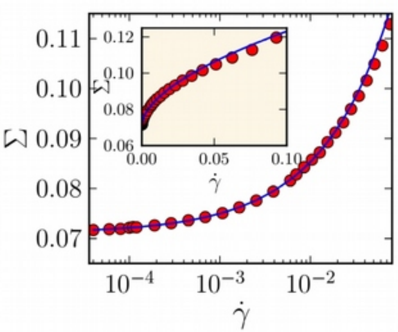
Usefulness of Elasto-Plastic Models

Deformation and flow of amorphous solids: a review of mesoscale elastoplastic models
A. Nicolas, EEF, K. Martens, J.-L. Barrat. *Rev. Mod. Phys.* **90**, 045006 (2018)

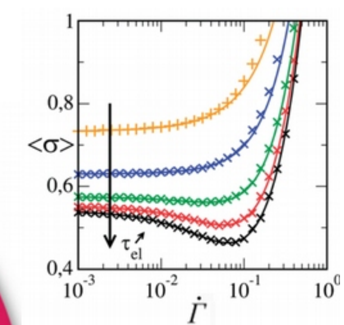
ADVERTISING

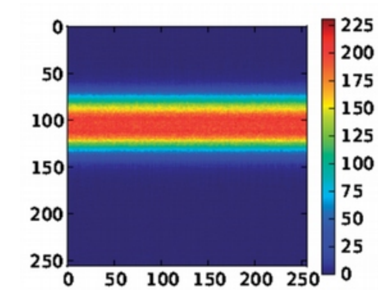
"Quantitative" stress-strain and flowcurves

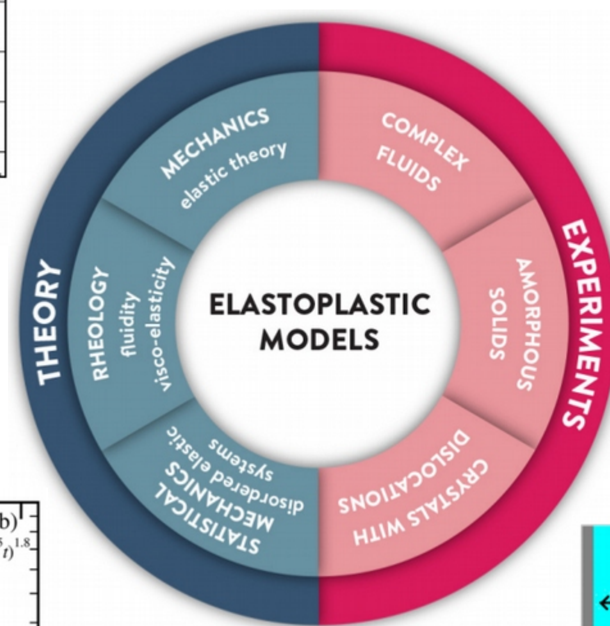




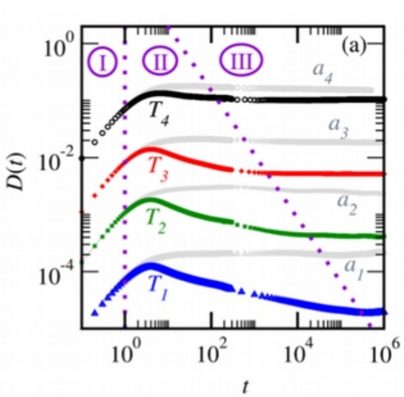
Shear localization

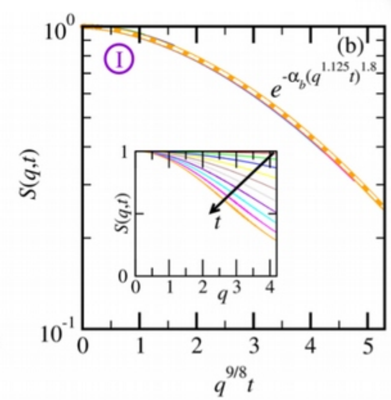




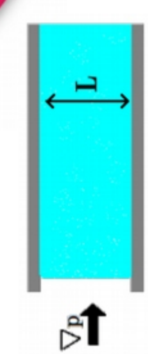


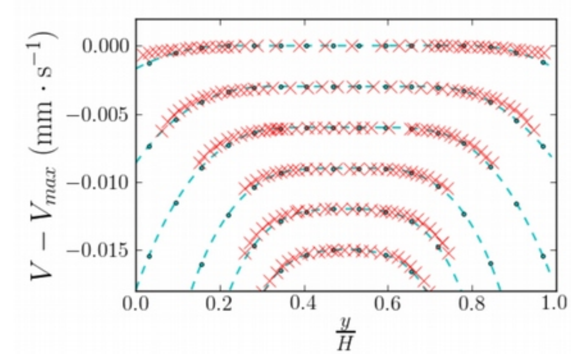
Relaxation in yield-stress systems





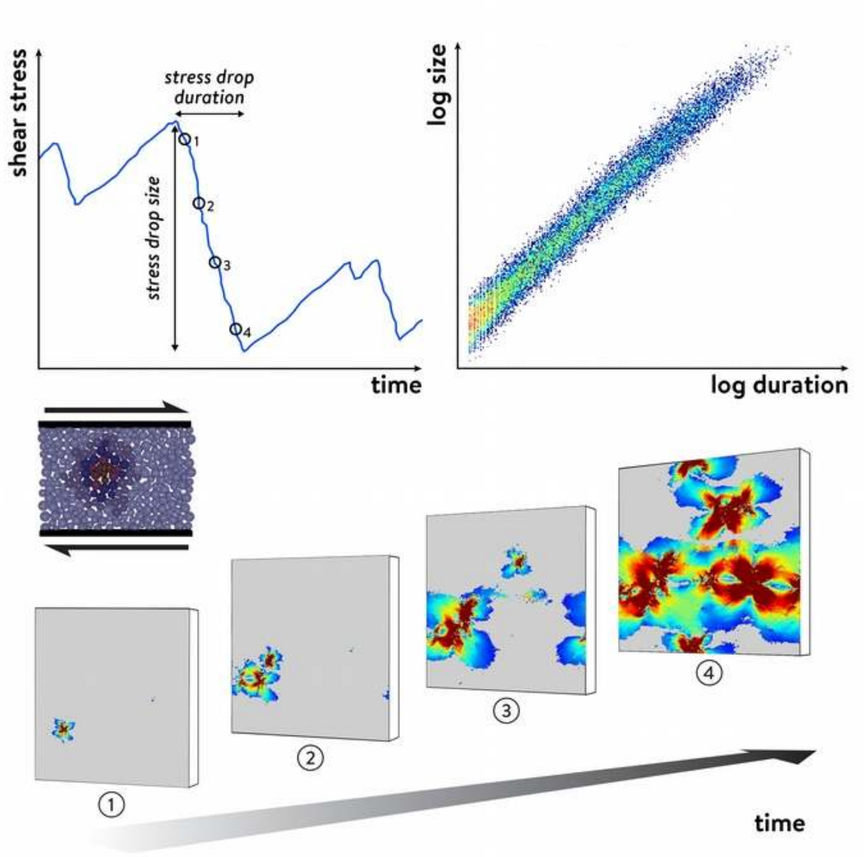
Flow and fluctuations in microchannels





Phenomenological, but also good "toy models" to do theoretical StatMech

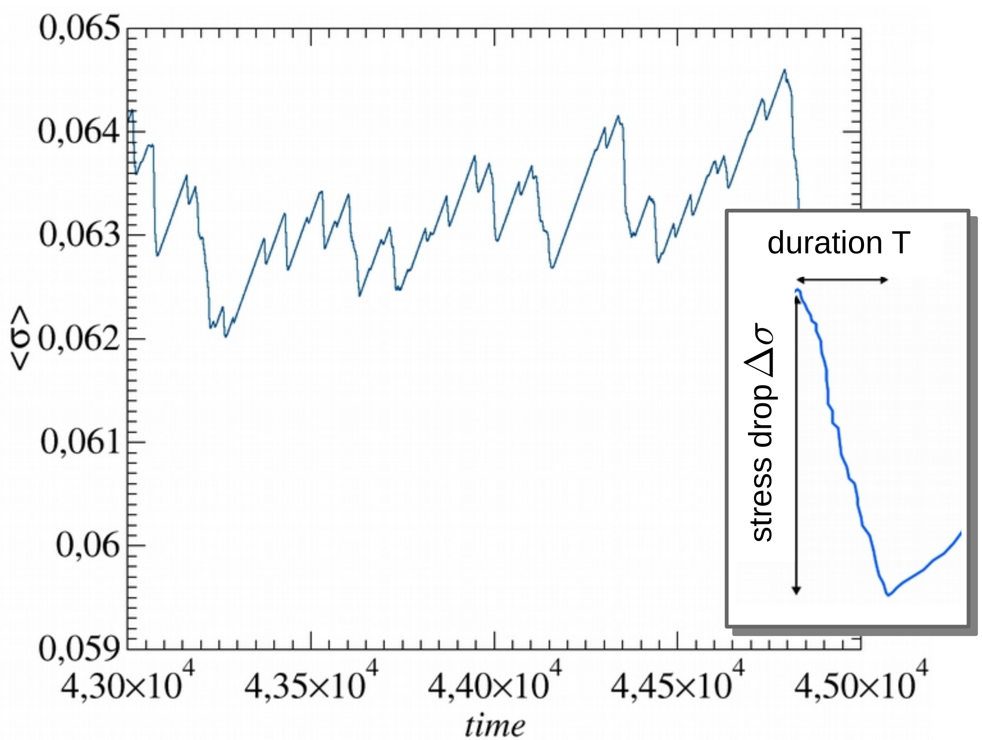
AVALANCHES & RELATED QUANTITIES



Avalanches from EPM simulations

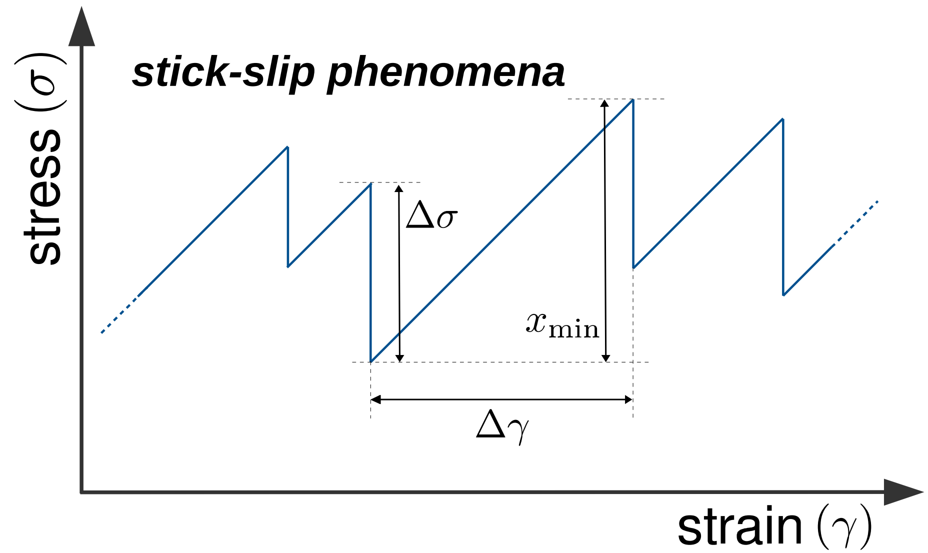
At finite strain-rates

"stress-drops"



Quasistatic

"avalanches"

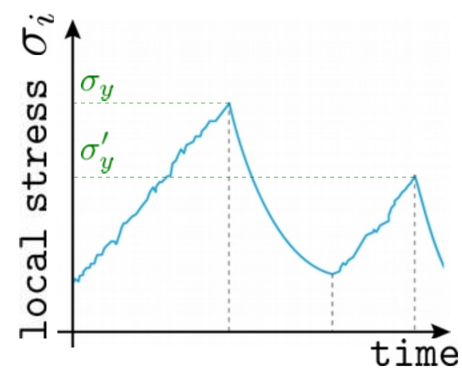


Observables:

duration $T = t_{\text{end}} - t_{\text{start}}$

$\boxed{\textit{size} \quad S \equiv L^d \Delta\sigma}$

shape $\tilde{V}_\sigma(t) = \left[-\frac{d\langle\sigma\rangle(t)}{dt} \right]_{t_{\text{end}}}^{t_{\text{start}}}$



Load needed to trigger next avalanche

local distances to instability

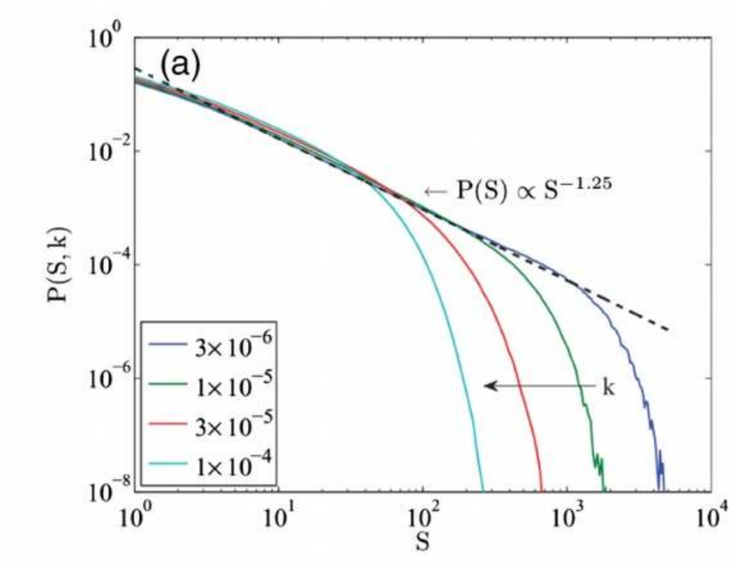
$x_i \equiv \sigma_{y_i} - \sigma_i$

$\Delta\gamma \propto x_{\min} \equiv \min_i \{x_i\}$

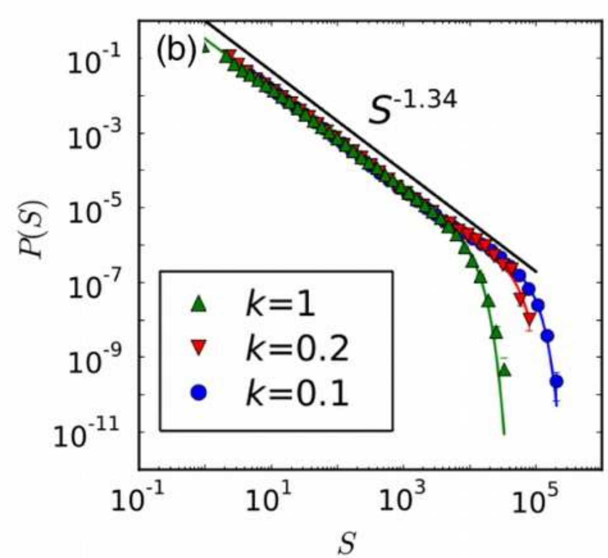
Avalanches in finite dimension (d=2): quasiestatic limit

(mini-review of literature)

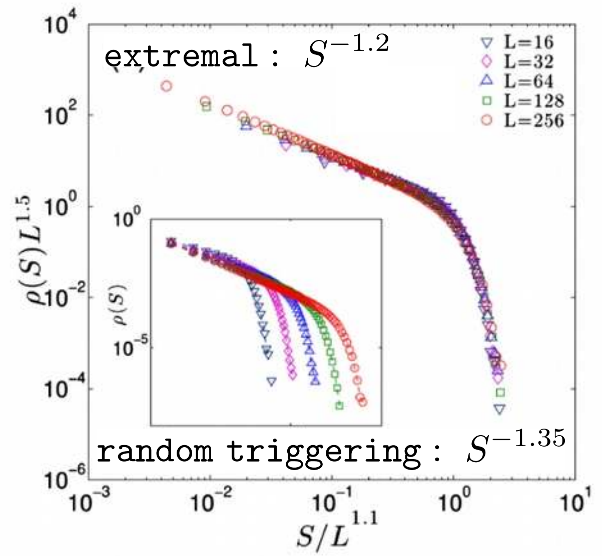
$$P(S) \sim S^{-\tau} f(S/S_{\mathtt{cut}}) \qquad S_{\mathtt{cut}} \sim k^{-\alpha} \quad \text{or} \quad S_{\mathtt{cut}} \sim L^{d_f}$$



Talamali *et al.* PRE 84, 016115 (2011)



Budrikis&Zapperi PRE 88, 062403 (2013)



Lin *et al.* PNAS 111, 14382 (2014)

Strain controlled with a spring of constant k

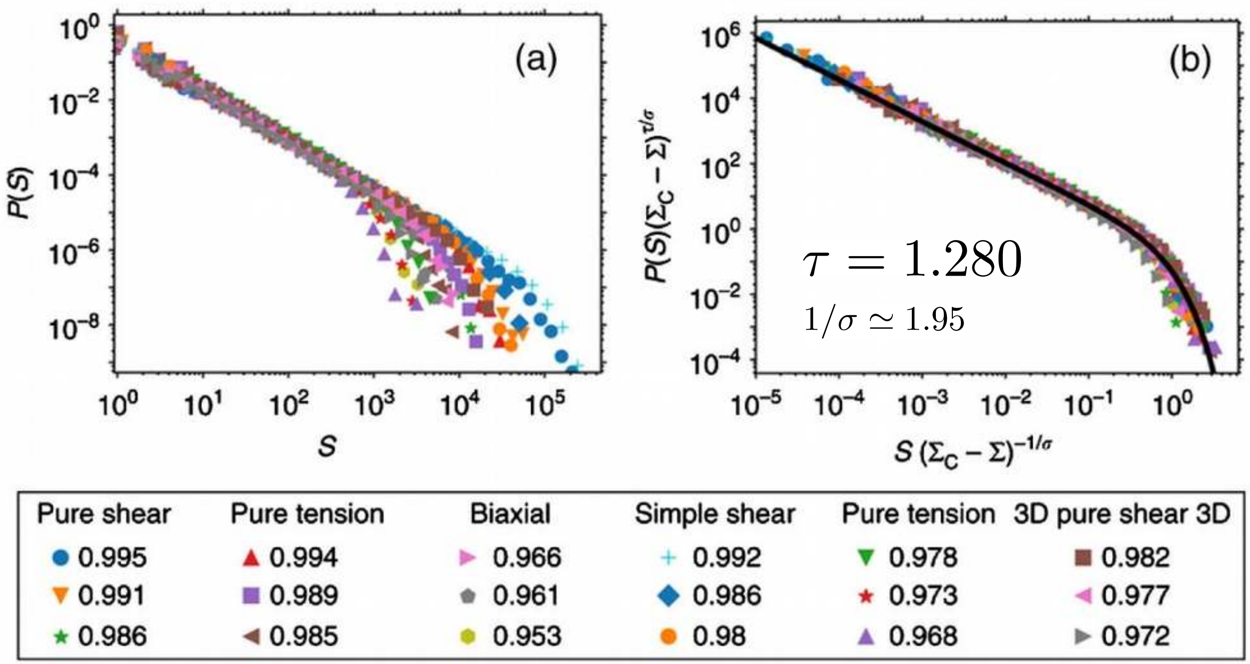
$$F_w = k(u - w)\ , \ w = vt$$

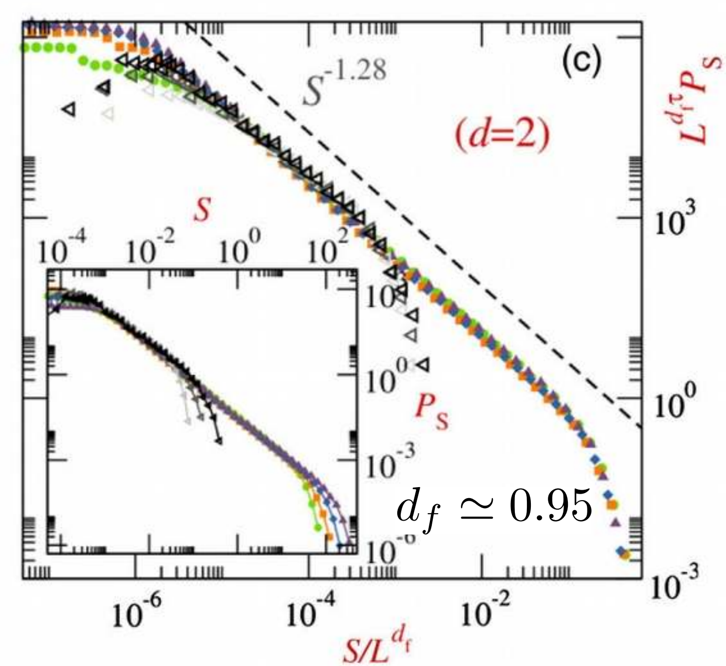
Exponents vary a bit according to the protocol/approach $\quad \tau \sim 1.2 - 1.35$

Avalanches in finite dimension (*d*=2): quasiestatic limit

(mini-review of literature)

$\dot{\gamma} \to 0$





Budrikis et al. Nat. Commun. 8, 15928 (2017)

C. Liu *et al.* PRL 116, 065501 (2016)

$$P(S) = \frac{A}{2\sqrt{\pi}} S^{-\tau} \exp\left(C\sqrt{u} - \frac{B}{4} u^{\delta} \right) \qquad u = S/S_{\mathtt{cut}}$$

“fractal dimension” $d_f \sim 1$

”*Slip-line*” avalanche geometry

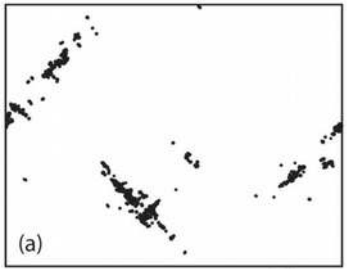
Only τ is a free parameter!

Le Doussal & Wiese, Phys. Rev. E 85, 061102 (2012)

Consensus on:

$$\tau < \tau_{\mathtt{MF}}^{\mathrm{dep}} = 3/2$$

$$d_f^{2\mathrm{D}} \simeq 1$$

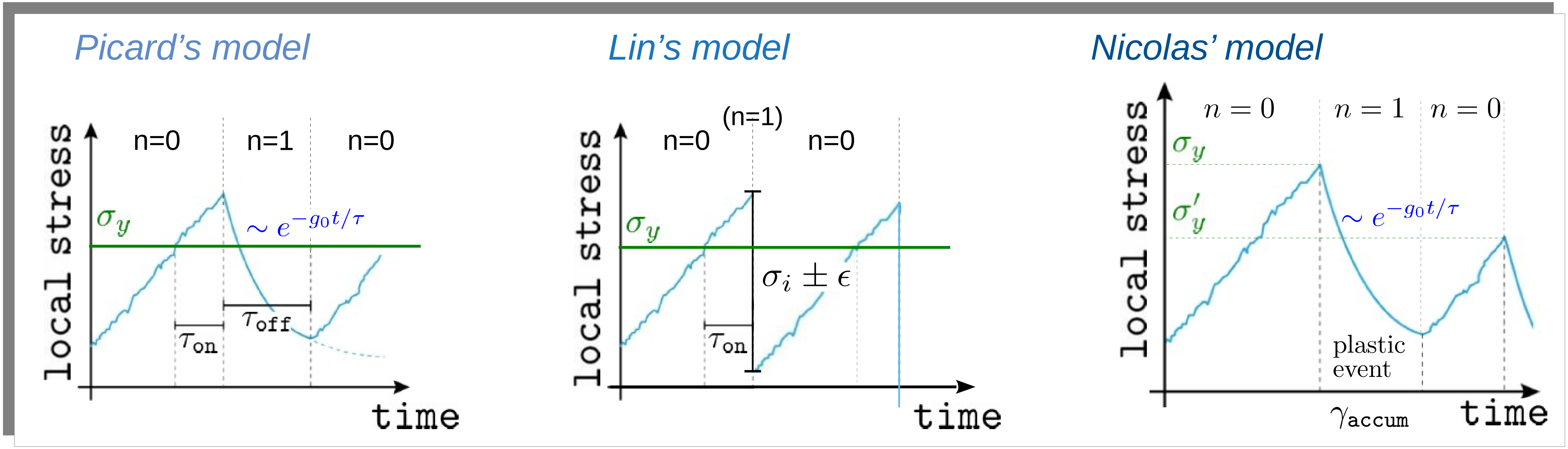


Simulating many variants of EPMs

$$\partial_t \sigma_i(t) = \mu \dot{\gamma}^{\text{ext}} - g_0 n_i(t) \frac{\sigma_i(t)}{\tau} + \sum_{j \neq i} G_{ij} n_j(t) \frac{\sigma_j(t)}{\tau}$$

+ Dynamical **rules** for a local yielding (state variables n_i):

Different rules, different models.



Each one in two flavours (in total 6 models):

Uniform rate $\tau_{\text{on}}^{-1} = \lambda = \text{cst.}$

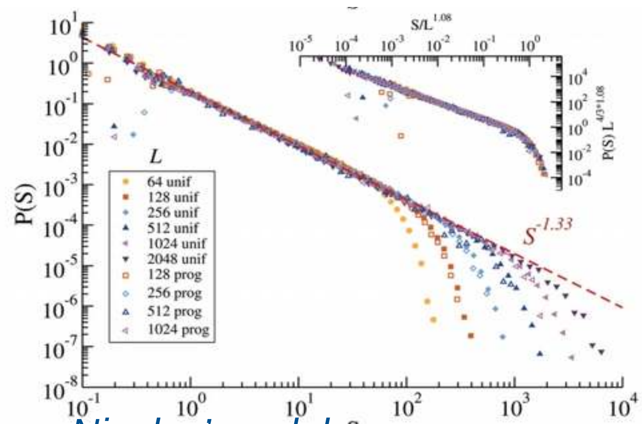
Progressive rate $\tau_{\text{on}}^{-1} = \lambda(\sigma) \propto \sqrt{\sigma - \sigma_y}$

"Universal" quasistatic avalanches size distribution $\boxed{d=2}$

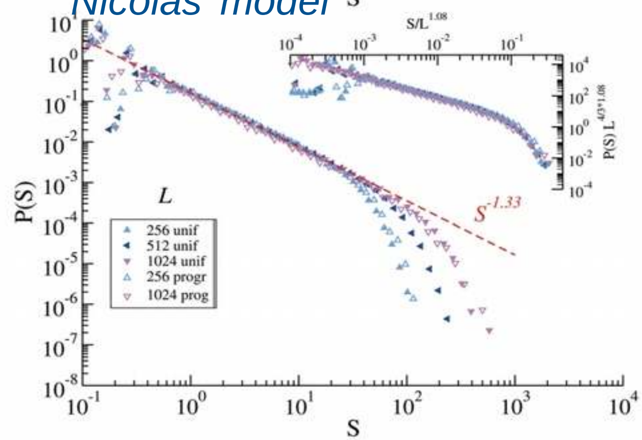
$$\boxed{S \equiv \Delta\sigma L^d = \Delta\sigma N}$$

$N = L^2$

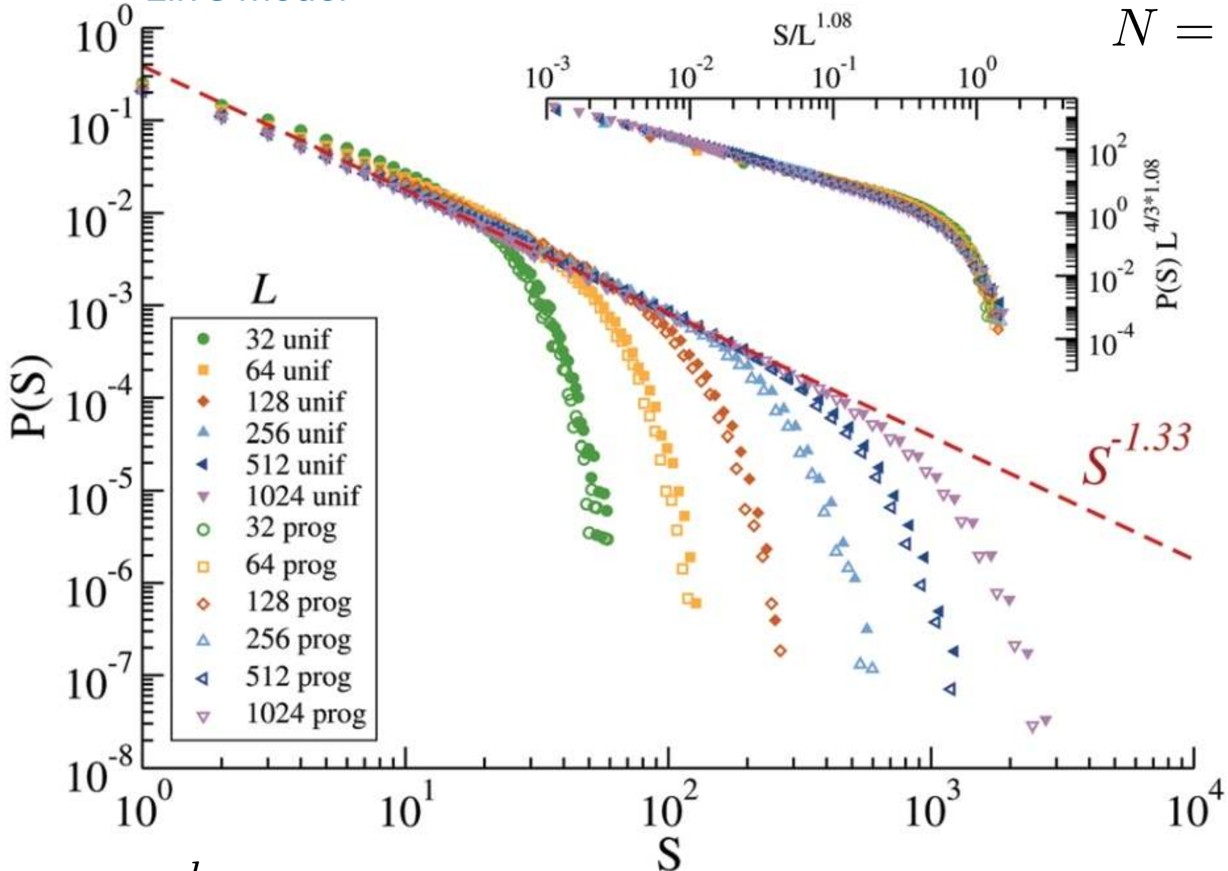
Picard's model



Nicolas' model



Lin's model



$$P(S) \sim S^{-\tau} f(S/S_{\text{cut}}) \qquad S_{\text{cut}} \sim L^{d_f}$$

$$\boxed{\begin{array}{l} \rightarrow \tau \simeq 1.33 \\ \rightarrow d_f \simeq 1.08 \end{array}}$$

- Avalanche **size** statistics is the same on 6 different models

- τ and d_f could be **sensitive** to fit criteria, but are **universal**

EEF & EA Jagla Soft Matter 15, 9041 (2019)

What can modify the estimation of τ ?

$$\frac{\partial \sigma_i(t)}{\partial t} = \mu \dot{\gamma}^{\text{ext}} + \sum_j G_{ij} n_j(t) \frac{\sigma_j(t)}{\tau};$$

$$G_{ii} = -g_0 < 0$$

Usually solved with a pseudo-spectral method: ***G*** is long-range in real space, but local in the Fourier modes.

In real space: $G_{ij}^{2D} = \frac{\cos(4\theta_{ij})}{\pi r_{ij}^2}$

In Fourier: $G_{\mathbf{q}} = -\frac{4 q_x^2 q_y^2}{(q_x^2 + q_y^2)^2}$ (for **q≠0**)

or 45º rotated: $G_{\mathbf{q}} = -\frac{(q_x^2 - q_y^2)^2}{(q_x^2 + q_y^2)^2}$

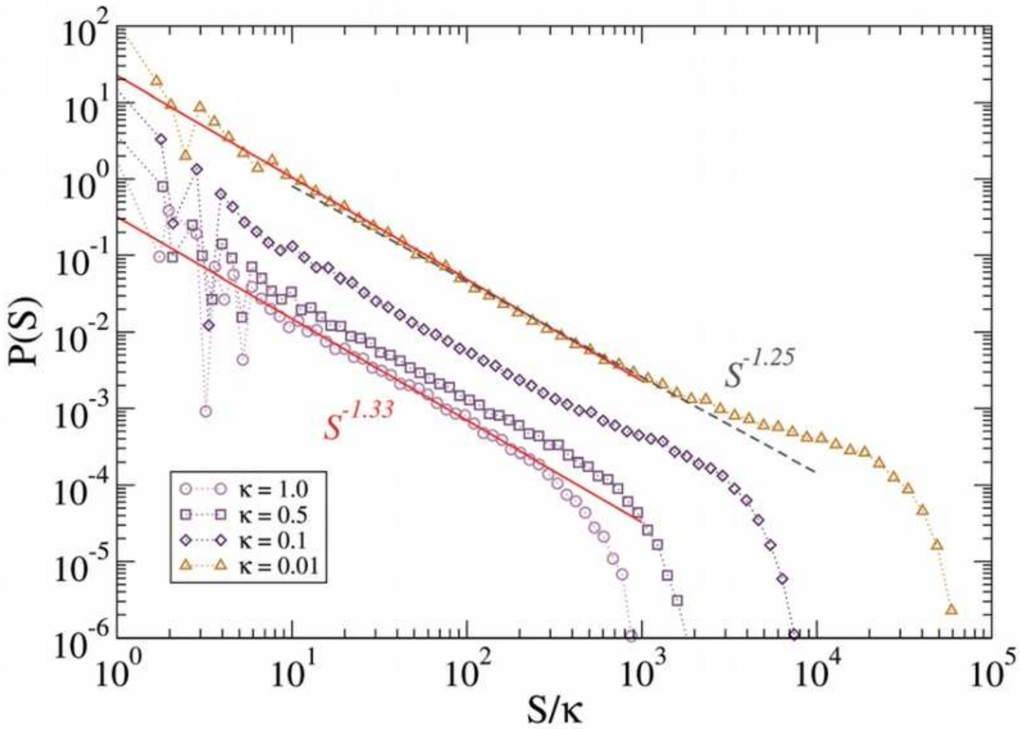
$G_{\mathbf{q}=0} = -\kappa$ (for **q=0**)

Stress conserved dynamics: $\kappa = 0$

Strain controlled dynamics: typically $\kappa = 1$

In general, one can use an arbitrary $\kappa > 0$

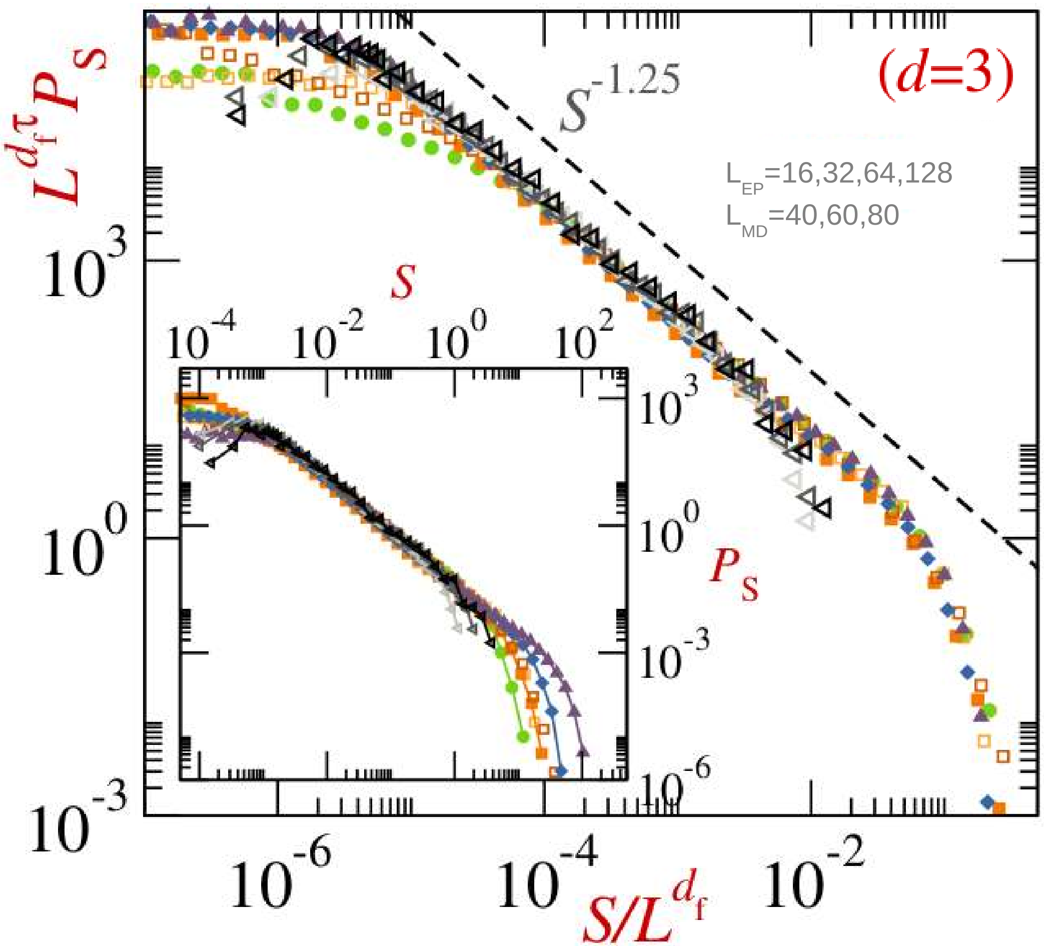
κ: stress non-conservation parameter

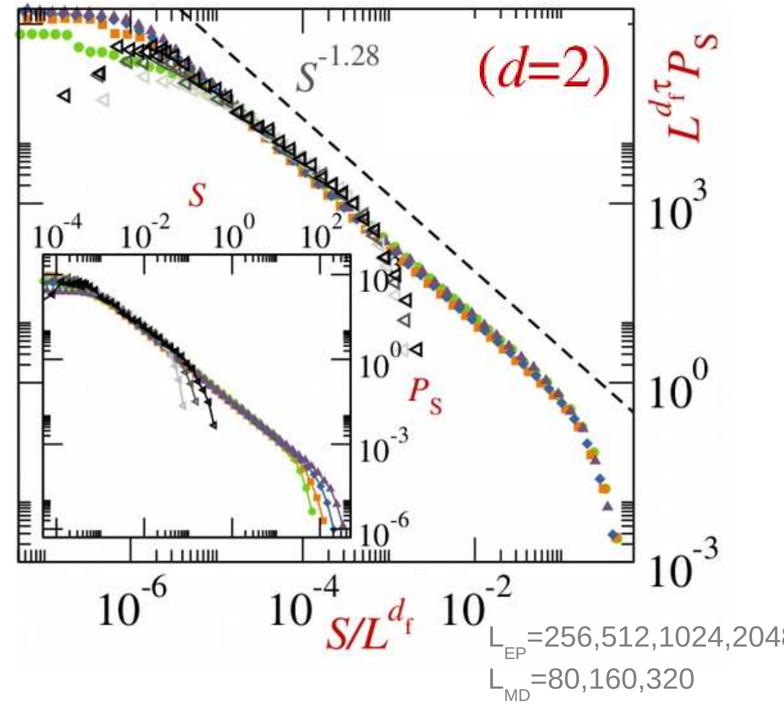


$$\tau \sim 1.25 - 1.33$$

Stress drop size distribution at very low shear rates $\dot{\gamma} \to 0$

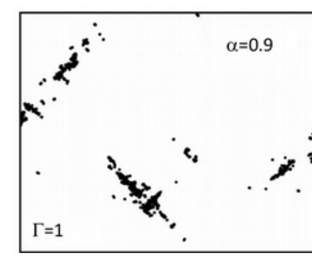
...for different system sizes, **comparing with quasistatic MD** simulations (grayscale triangles)





$\tau_{2D} \simeq 1.28$

$d_f^{2D} \simeq 0.9$



$\tau_{3D} \simeq 1.25 \neq \tau_{\mathtt{MF}}^{\mathrm{dep}}$

$d_f^{3D} \simeq 1.3$

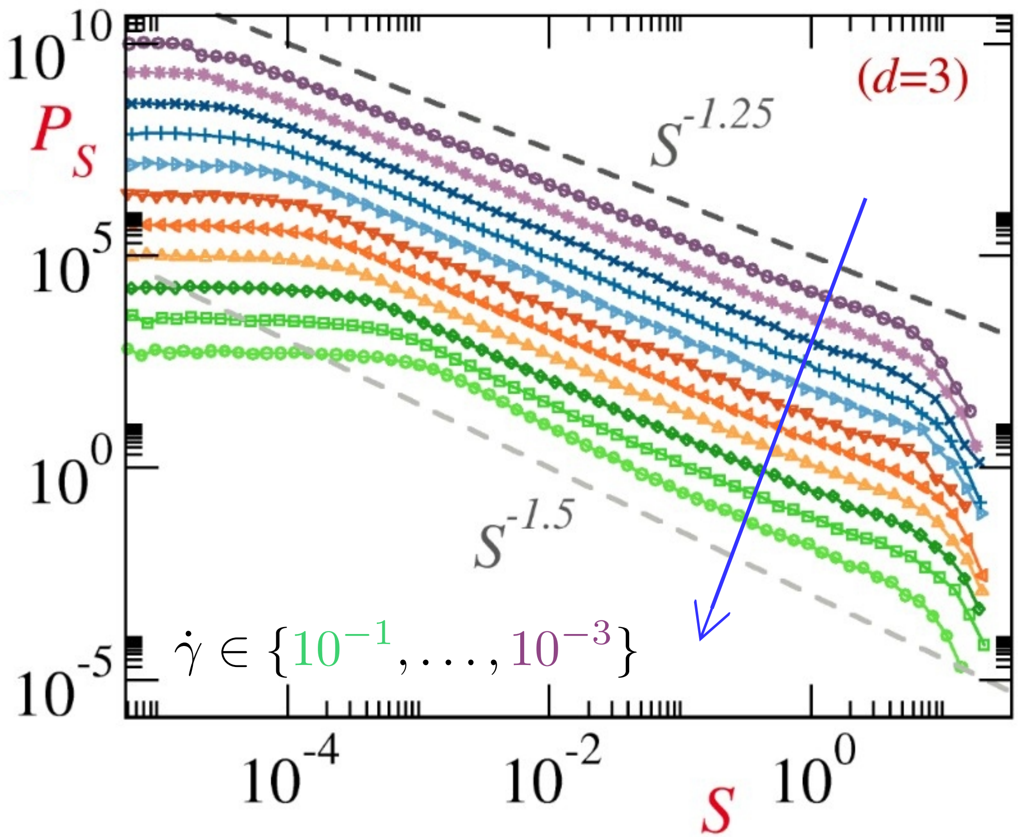
$$P_S \propto S^{-\tau} f(S/S_c) \ , \ S_c \simeq L^{d_f}$$

scaling factor d_f: "fractal dimension". *Slip-line* avalanche geometry

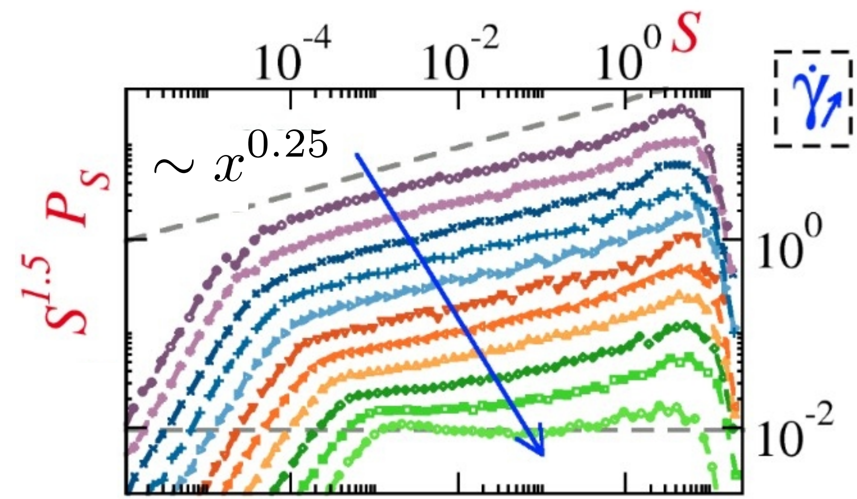
"fractal dimension"

C. Liu, EEF, F. Puosi, J-L Barrat, K Martens *PRL* **116** 065501 (2016)

Stress-drop statistics when increasing strain rate



(curves arbitrarily shifted for clarity)

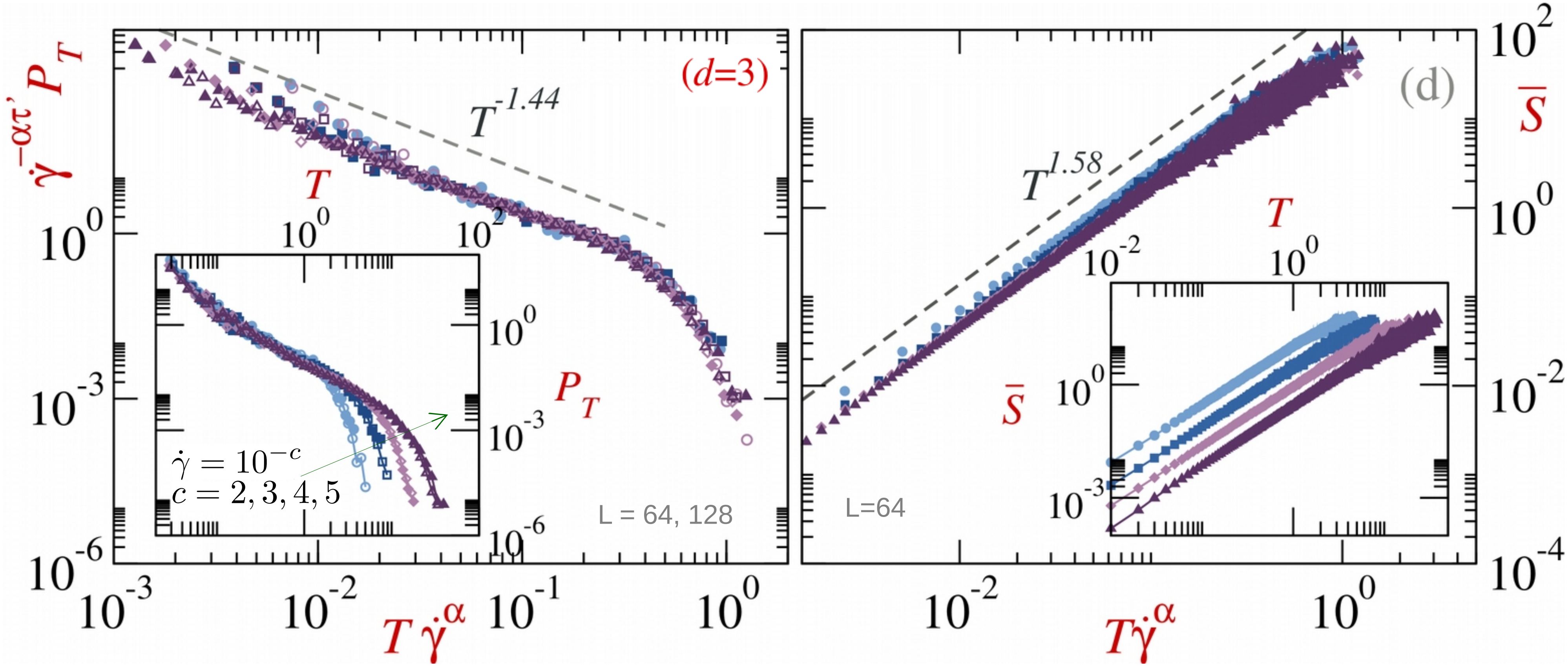


- Large strain-rates produces **avalanche overlapping**, adding up uncorrelated plastic activity in the same stress drop.
- **Crossover** to **larger** exponent when we **go away** from the yielding point.

$$\tau : 1.25 \rightarrow 1.5$$

C. Liu, EEF, F. Puosi, J-L Barrat, K Martens *PRL* **116** 065501 (2016)

Stress drop duration distribution and size–duration scaling



$$P_T \propto S^{-\tau'} g(T/T_c)\ ,\ T_c \simeq \dot{\gamma}^{-\alpha}$$

$$\boxed{\begin{array}{ll} \tau'_{3D} \simeq 1.44 & \alpha_{3D} \simeq 0.30 \\ \tau'_{2D} \simeq 1.41 & \alpha_{2D} \simeq 0.38 \end{array}}$$

We expect: $S \sim T^{\delta}$ $\quad \delta = d_f/z$

We observe: $S \simeq C(L, \dot{\gamma}) T^{\delta}$

$$\frac{S}{L^{d_f}} \sim \left(\frac{T}{\dot{\gamma}^{-\alpha}}\right)^{\delta}$$

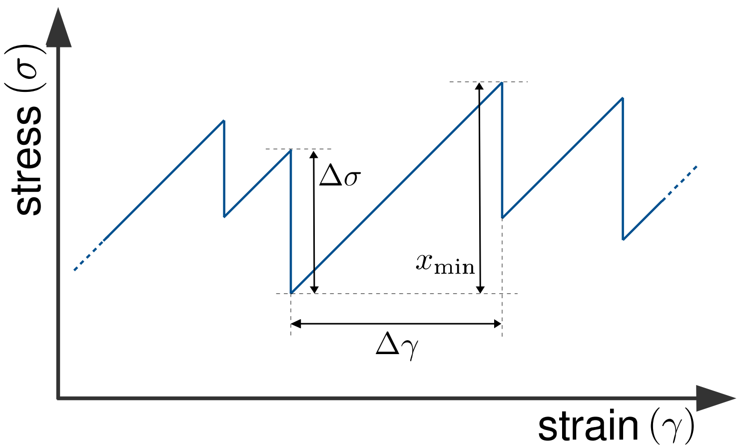
$$\boxed{\delta \simeq 1.58}$$

Who is α?

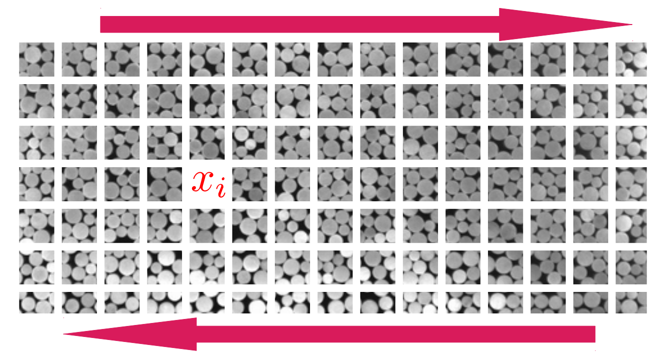
$$T \sim \xi^z \sim \dot{\gamma}^{-\nu z/\beta} \Rightarrow \alpha \equiv \nu z/\beta = \boxed{1 - 1/\beta}$$

$$\xi \sim |\langle\sigma\rangle - \sigma_c|^{-\nu} \qquad T_c \sim \dot{\gamma}^{n-1}$$

Pseudo-gap: an emergent property of a signed propagator



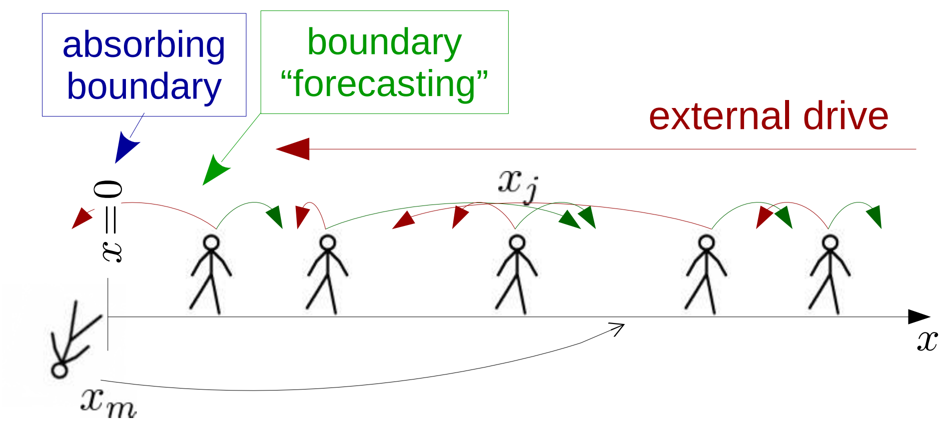
$x_i \equiv \sigma_{yi} - \sigma_i$ "distance" to local instability



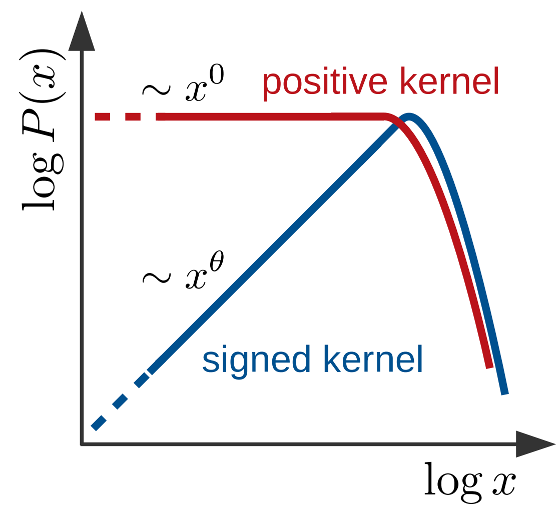
- **Key observation**:
  The rate at which plasticity occurs is **not extensive**

$$\langle \Delta\gamma \rangle \sim \langle x_{\tt min} \rangle \sim 1/N^{\phi} \gg 1/N \ , \ 0 < \phi < 1$$

Maloney&Lemaitre *PRL* **93**, 016001 (2004)



- It can be explained by the emergence of a "**pseudo-gap**" in the **density of shear transformations**



$$P(x) \sim x^{\theta}$$

$$\langle x_{\tt min} \rangle \sim N^{-1/(1+\theta)}$$

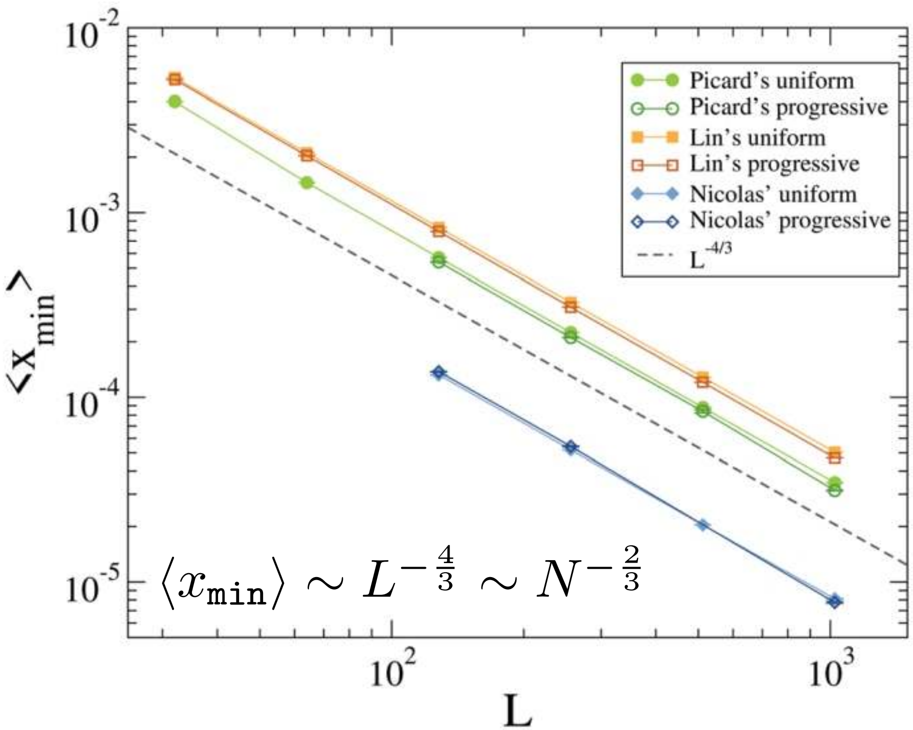
with $\theta > 0$, implying **system-spanning avalanches**

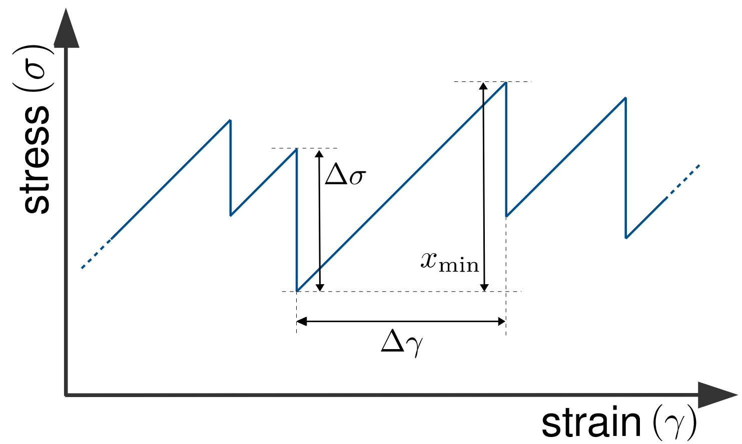
Karmakar, Lerner, Procaccia PRE **82**, 055103R (2010)
Lin, Lerner, Rosso, Wyart PNAS **111** 14382 (2014)

Load needed to trigger new avalanches

d=2

EEF & EA Jagla *Soft Matter* **15**, 9041 (2019)





If stress level is steady $\langle x_{\mathtt{min}} \rangle = \langle \Delta\sigma \rangle$

Recalling $P(S) \sim S^{-\tau} f(S/L^{d_f})$ $1 < \tau < 2$

$$L^{-d\phi} \sim \langle x_{\mathtt{min}} \rangle = \langle \Delta\sigma \rangle \sim \frac{\langle S \rangle}{L^d} \sim L^{d_f(2-\tau)-d}$$

$$\boxed{\tau = 2 - \frac{d}{d_f}(1-\phi)}$$

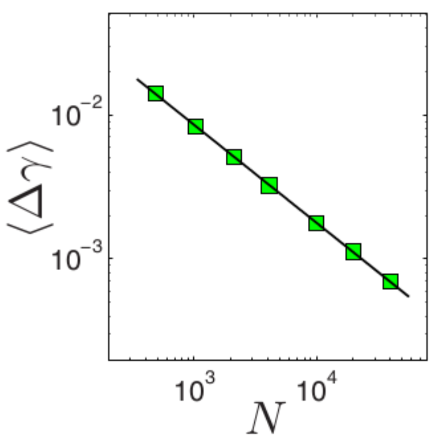
$\rightarrow \tau^{\text{2D}} \simeq 1.33$

$\rightarrow d_f^{\text{2D}} \simeq 1.08$

$\rightarrow \phi^{\text{2D}} \simeq 0.67$

MD simulations in the steady state:

$\langle \Delta\gamma \rangle \sim N^{-2/3}$



τ, d_f, ϕ 'universal' critical exponents

Karmakar, Lerner, Procaccia PRE **82**, 055103R (2010)

Density of shear transformations $P(x)$

$d=2$

$x_i \equiv \sigma_{yi} - \sigma_i$ "distance" to local instability

EEF & EA Jagla, *SM* **15**, 9041 (2019), *JPCM* **33** 124001 (2021)

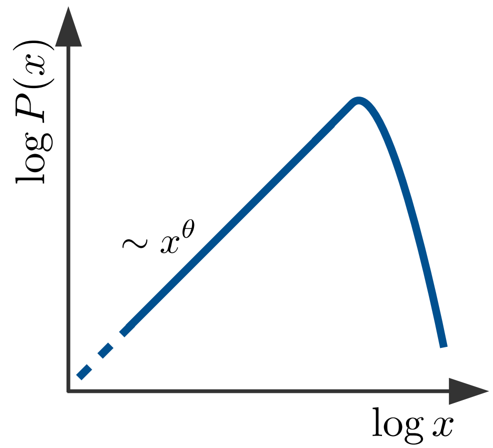
Observation

↓

$\langle x_{\mathtt{min}} \rangle \sim N^{-\phi}$

↓

Ansatz →



If $P(x) \sim x^\theta$, then:

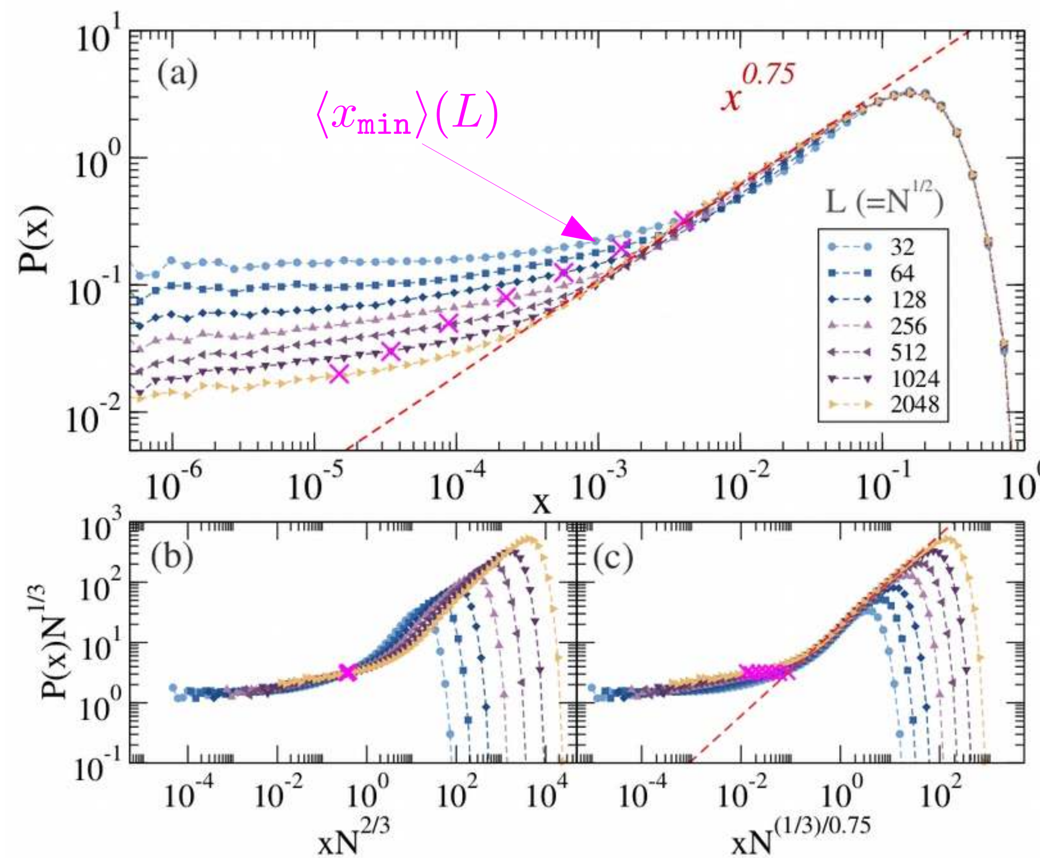
$$P(x_{\mathtt{min}}) \sim x_{\mathtt{min}}^\theta e^{-A x_{\mathtt{min}}^{\theta+1}} \quad \text{(Weibull)}$$

$$\langle x_{\mathtt{min}} \rangle \sim N^{-1/(1+\theta)}$$

but $\langle x_{\mathtt{min}} \rangle \sim N^{-\phi} \not\Rightarrow P(x) \sim x^\theta$

With $\phi \simeq 2/3 \rightarrow \boxed{\theta \simeq 0.5}$ is expected in 2D

(but not really holding in simulations)



Picard's model, but is the same for all

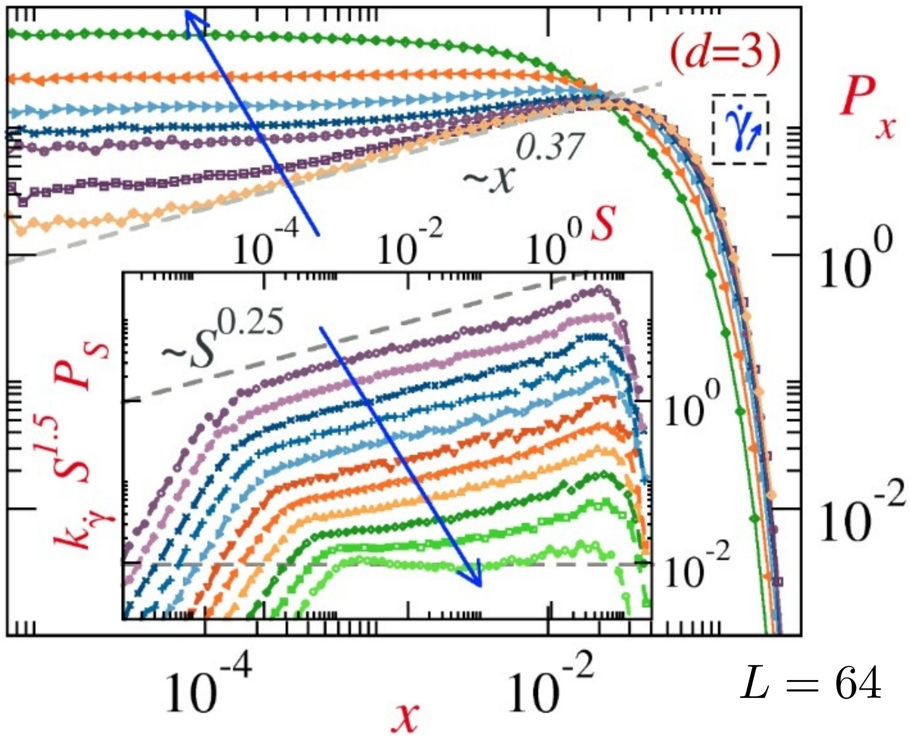
We observe: $$P(x) \simeq P_0 + x^\theta$$

$\boxed{P_0 \sim N^{-1/3}}$ $\boxed{\langle x_{\mathtt{min}} \rangle \sim N^{-2/3}}$

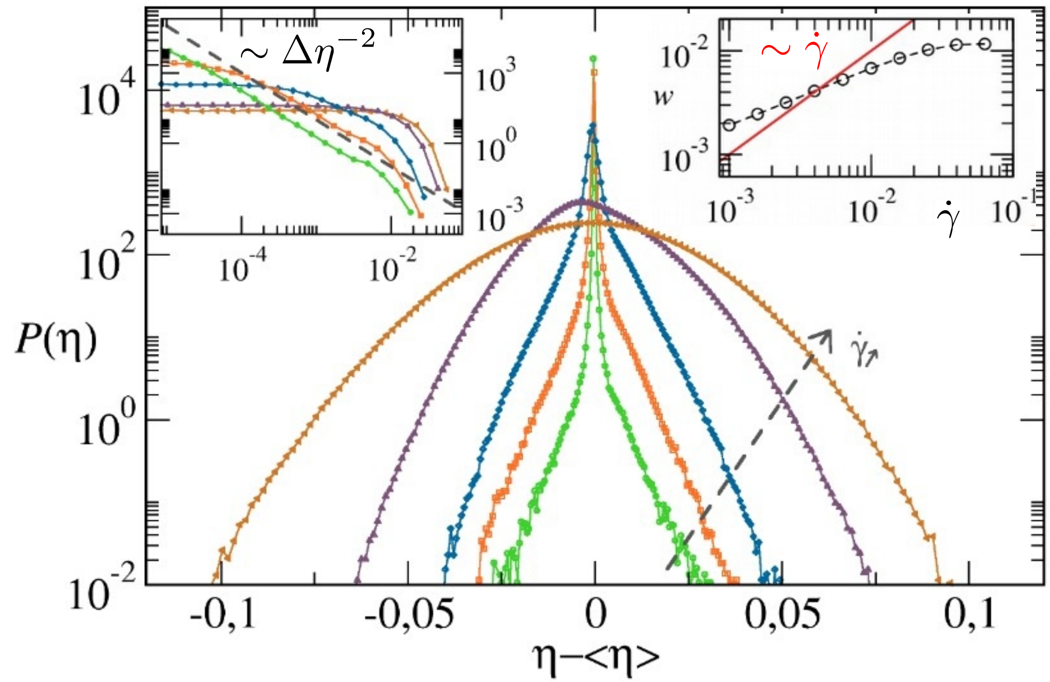
Lerner, Procaccia PRE **79**, 066109 (2009)
Lin, Lerner, Rosso, Wyart PNAS **111** 14382 (2014)

Tyukodi et al. PRE 2019, Ruscher&Rottler SM 2020

Density of shear transformations at increasing strain rate

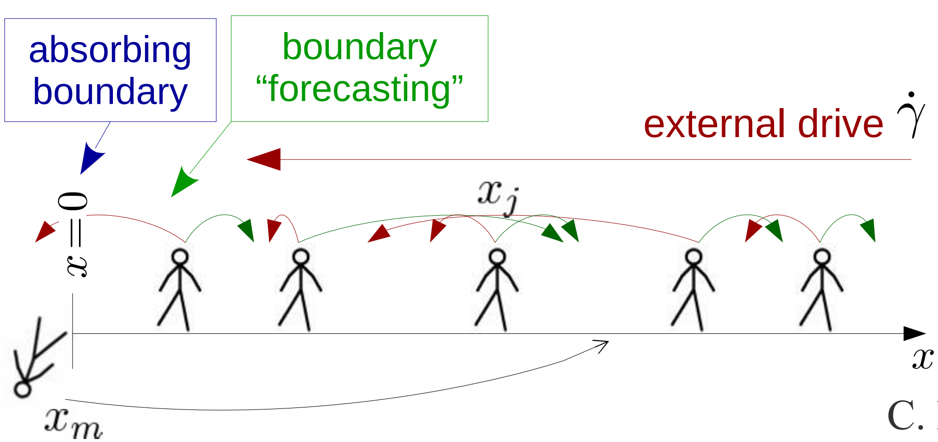


$L = 64$



"Mechanical noise" $\longrightarrow \eta_i$

$$\partial_t \sigma_i = \mu\dot\gamma - g_0 n_i \frac{\sigma_i}{\tau} + \overbrace{\sum_{j\neq i} G_{ij} n_j \frac{\sigma_j}{\tau}}^{\eta_i}$$



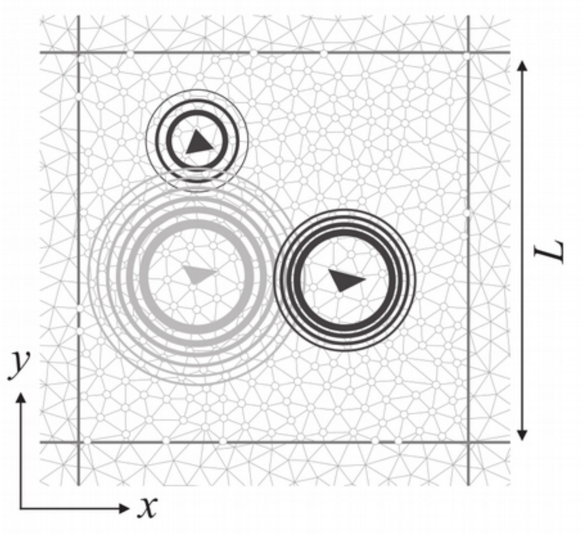
- Variance grow slower than linear with $\dot\gamma$
  → **drift dominates when** $\dot\gamma > \sim 0.004$

For $\dot\gamma \gg 0$ $\qquad \theta \to \theta^{\mathrm{dep}} = 0$

$\tau \to \tau_{\mathrm{MF}}^{\mathrm{dep}} = 3/2$

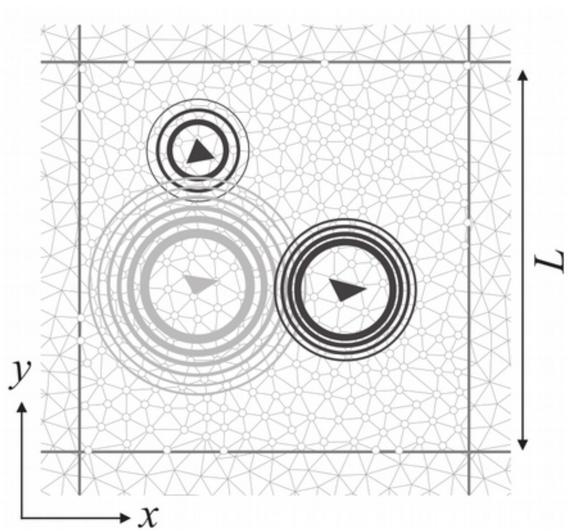
C. Liu, EEF, F. Puosi, J-L Barrat, K Martens *PRL* **116** 065501 (2016)

INERTIAL AVALANCHES

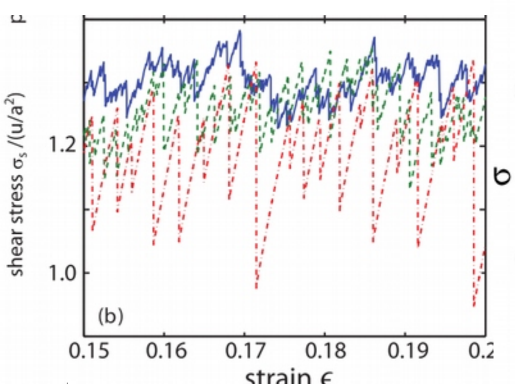


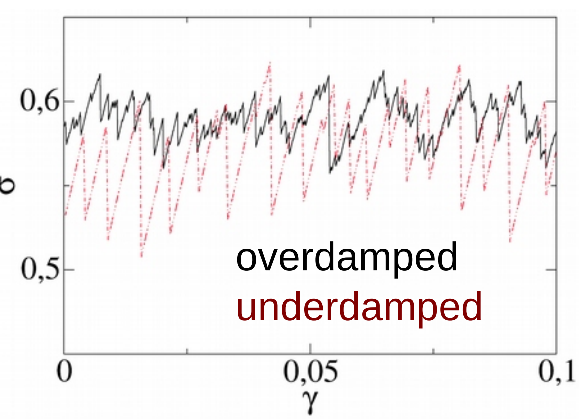
Finite Elements Method to approach the not-overdamped case

Irregular 2d lattice, **tensorial** model



Local yielding
+
Elastic waves





Molecular dynamics

FEM-EPM

Continuum mechanics e.o.m.:

$$\rho\ddot{\mathbf{u}}(\mathbf{r},t) = \boldsymbol{\nabla}.\boldsymbol{\sigma}(\mathbf{r},t)$$

$\mathbf{u}(\mathbf{r},t)$: displacement field $\quad \boldsymbol{\sigma}$: internal stress

$$\boldsymbol{\sigma} = \underbrace{\mu(t)\nabla\mathbf{u}}_{\text{restoring}} + \underbrace{\eta\boldsymbol{\nabla}\dot{\mathbf{u}}}_{\text{dissipative}}$$

$$\Gamma = \frac{\tau_d^{-1}}{\tau_v^{-1}} = \frac{\eta/(\rho a^2)}{\sqrt{\mu/\rho a^2}}$$: dissipation coefficient

Lower η, more inertial

+ EP rules:

$$n : 0 \underset{\text{after elapsed } \tau_{\text{on}}}{\overset{\text{when } \sigma > \sigma_y}{\rightleftarrows}} 1$$

$$\mu(t) = \begin{cases} 0 & \text{while } n = 1 \\ \mu & \text{otherwise} \end{cases}$$

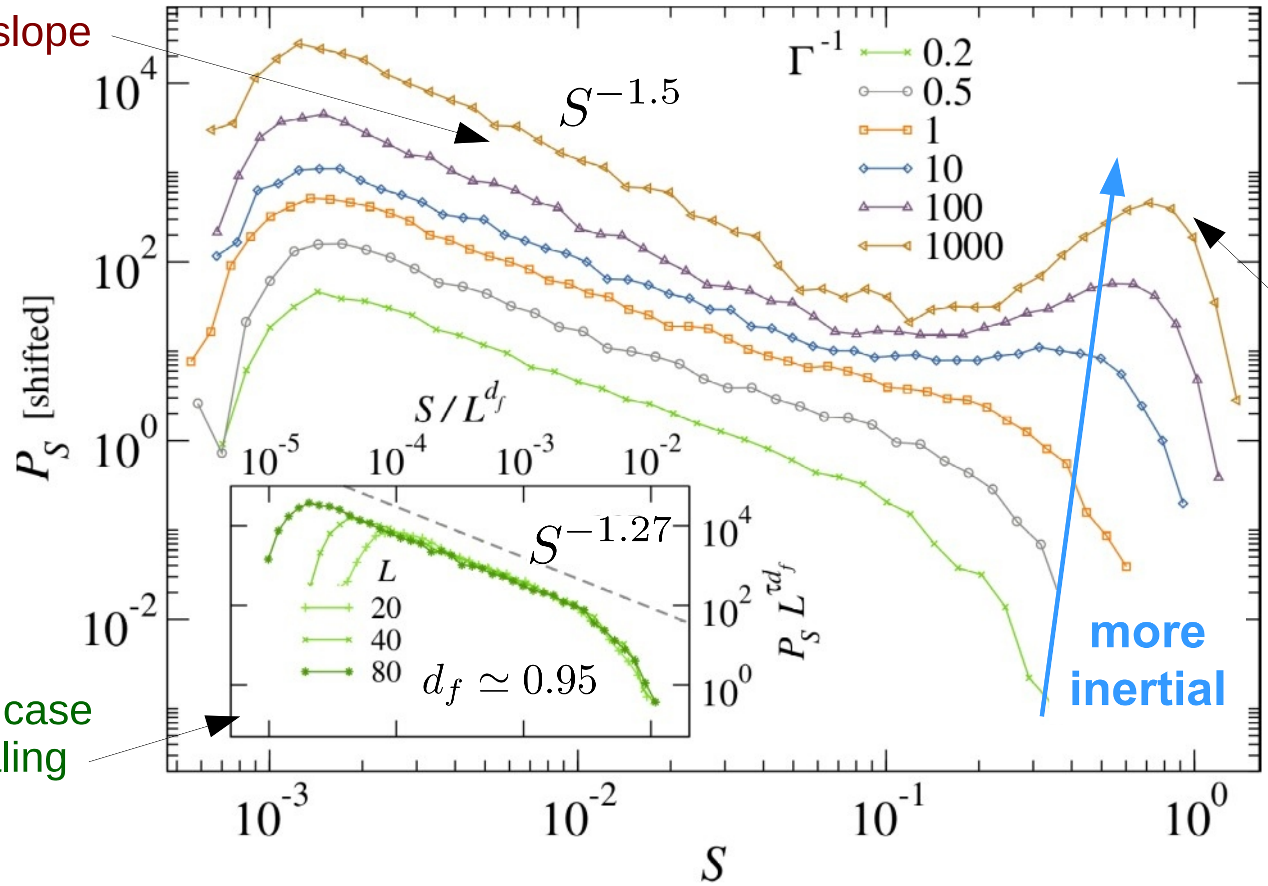
K.M. Salerno & M. Robbins PRE 88, 062206 (2013)

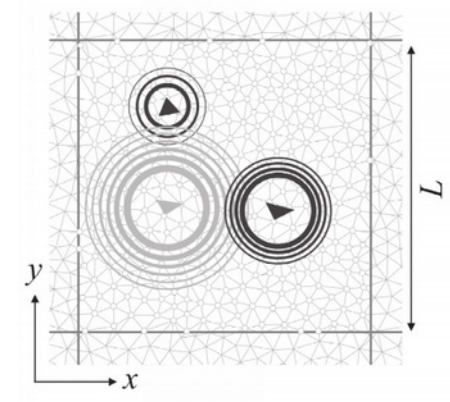
K. Karimi, EEF, J-L Barrat, PRE 95, 013003 (2017)

Inertial avalanches size distributions

Varying damping (dissipation Γ)

steeper slope





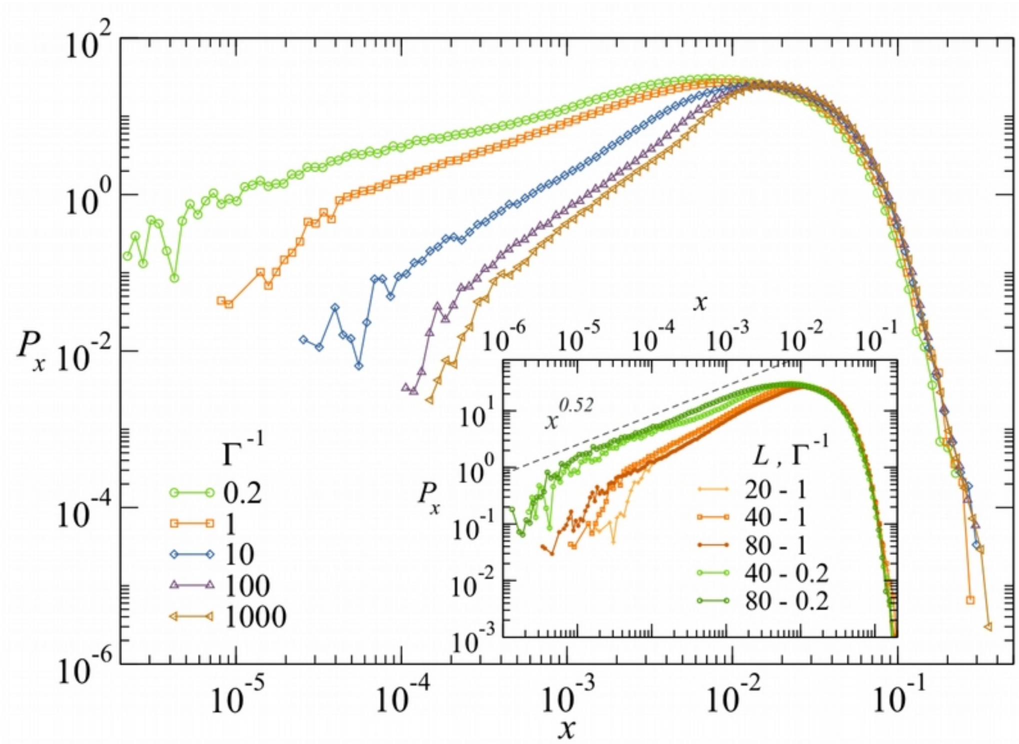
"new" characteristic size

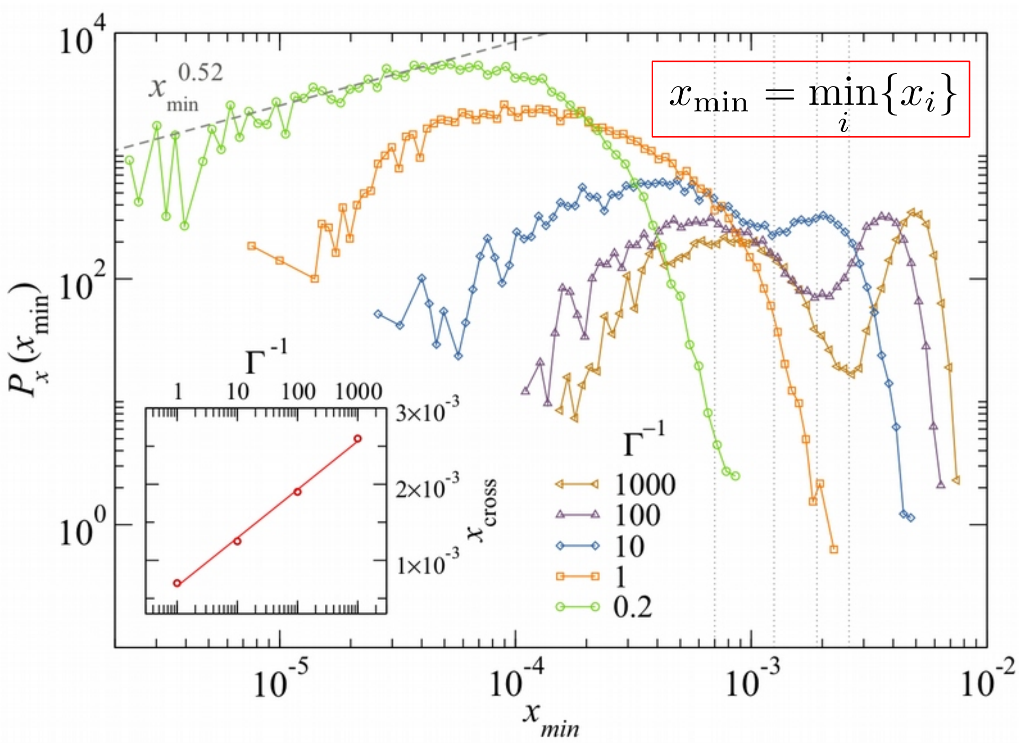
more inertial

Overdamped case
finite size scaling
(idem EPM)

K. Karimi, EEF, J-L Barrat, *PRE* **95**, 013003 (2017)

Inertial densities of shear transformations





Increasing inertia we observe a **steeper gap**

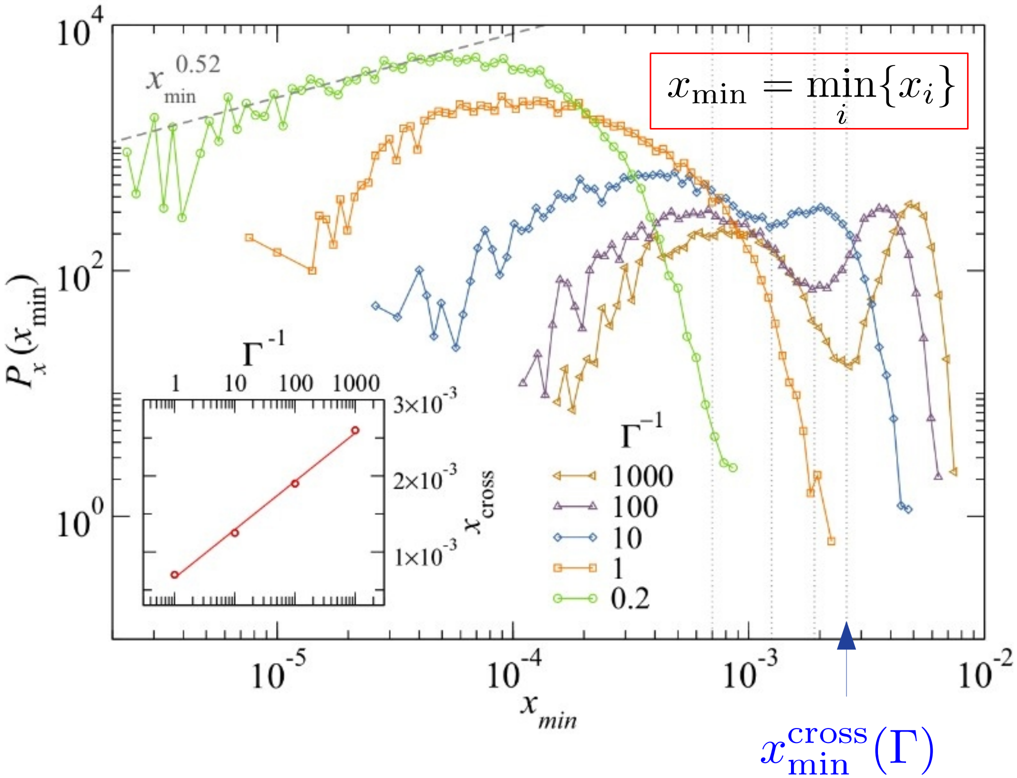
The apparent bigger θ as Γ^{-1} increases is a result of the presence of **two kind of events**

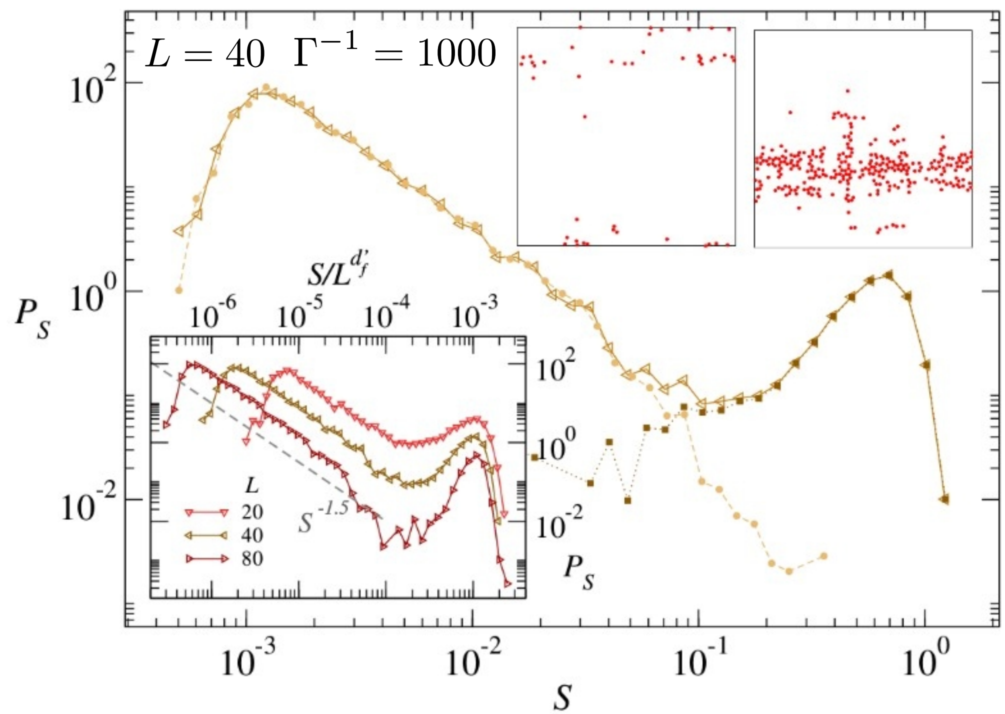
$P(x_{\min})$ displays a **bimodal distribution** for underdamped systems $x_{\min} \lessgtr x_{\min}^{\text{cross}}$

Ansatz:

$x_{\min}^{\text{cross}}$ separates two kind of avalanches

Inertial P(x_{min}) reveals two kind of avalanches





$x_{\min}^{\text{cross}}$ separates two kind of avalanches:

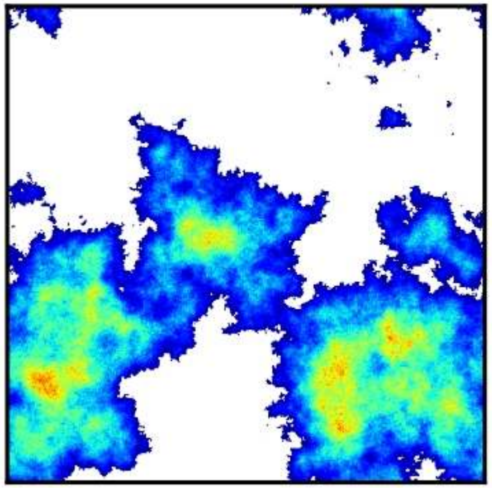
- *massive and **inertial*** (“large” $x_{\min}$)
- *localized and “**overdamped”*** (“small” $x_{\min}$)

The splitting **in two contributions** is clear

The effective “fractal dimension” of the inertial avalanches is now much larger.
e.g.: $d_f' = 1.75 \quad \text{for} \quad \Gamma^{-1} = 1000$

K. Karimi, EEF, J-L Barrat, *PRE* **95**, 013003 (2017)

THERMAL AVALANCHES IN EPMs



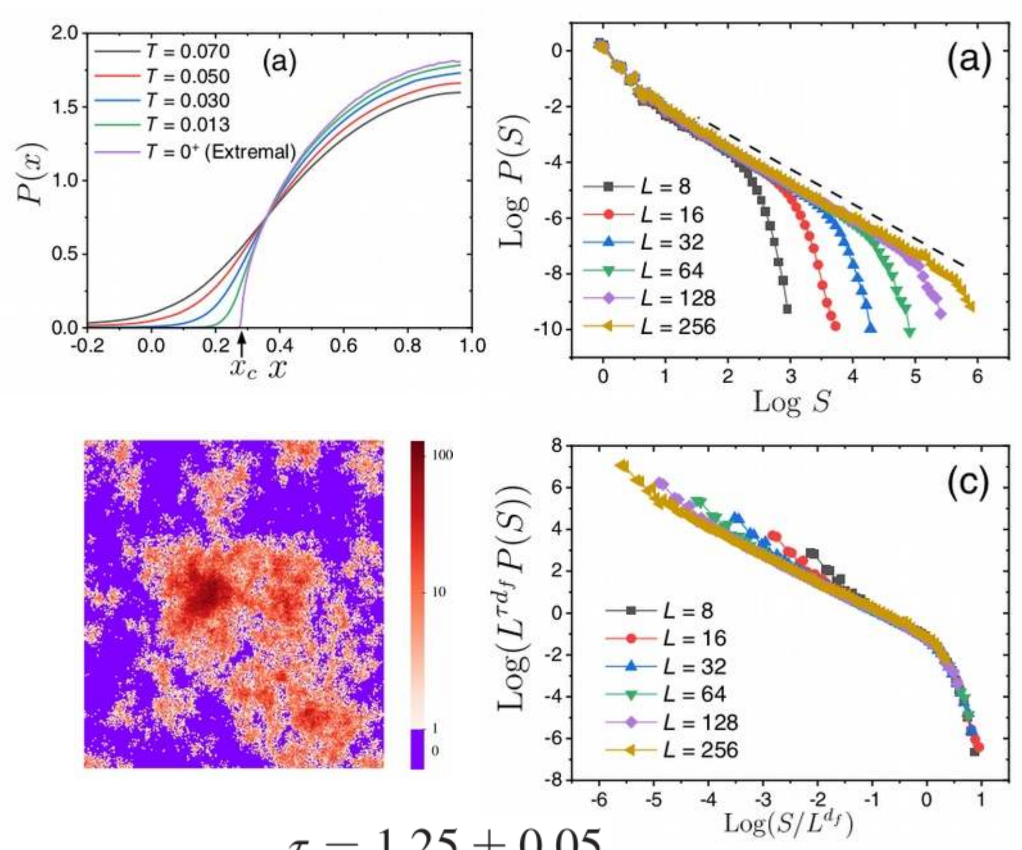
Tom W.J. de Geus et al. arXiv:2401.09830

Avalanches in thermal EPMs at σ=0

A. Tahaei et al PRX 13, 031034 (2023)

Extremal dynamics ($T=0^+$)

Every site below $x_0=x_c$ becomes unstable



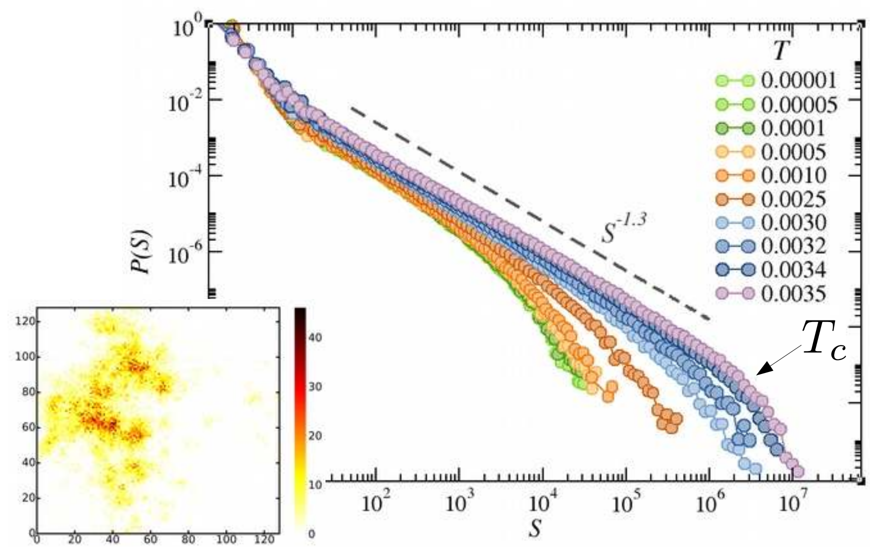
$$\tau = 1.25 \pm 0.05$$
$$d_f = 2.3 \pm 0.1$$

Propagator G is randomized in angle

Work in progress w/Gieberth Rodriguez López

Finite temperature dynamics ($T=0^+$)

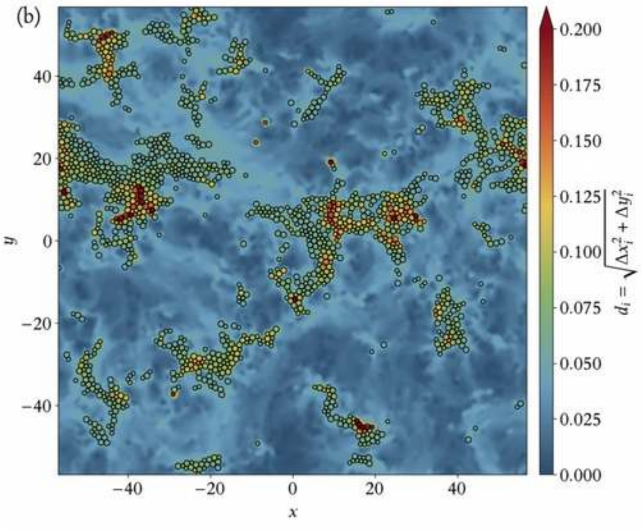
$$n_i = \begin{cases} 0 \to 1 & \text{instantaneously, if } x_i \leq 0, \\ 0 \to 1 & \text{with prob. } e^{-\frac{x_i^\alpha}{k_B T}}, \text{ if } x > 0, \\ 0 \leftarrow 1 & \text{at a rate } \tau_{\text{ev}}^{-1}. \end{cases}$$



We found a size-dependent temperature $T_c(L)$ above which avalanches can self-sustain. $T_c \leftrightarrow x_c$?

Exponents differ a bit from the extremal dynamics

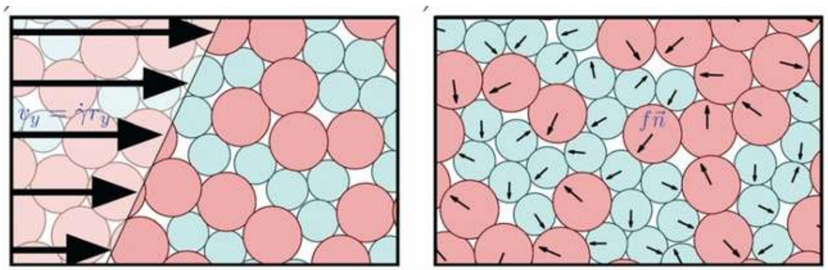
AVALANCHES IN ACTIVE YIELDING



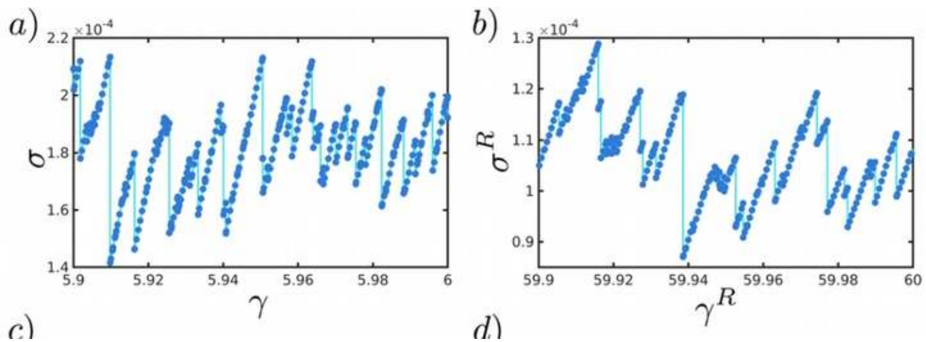
R. Wiese, EEF, D. Levis (unpublished)

Avalanches: from externally deformed glasses to active systems

C. Villaroel, G. During, Soft Matter, 20, 3520 (2024)

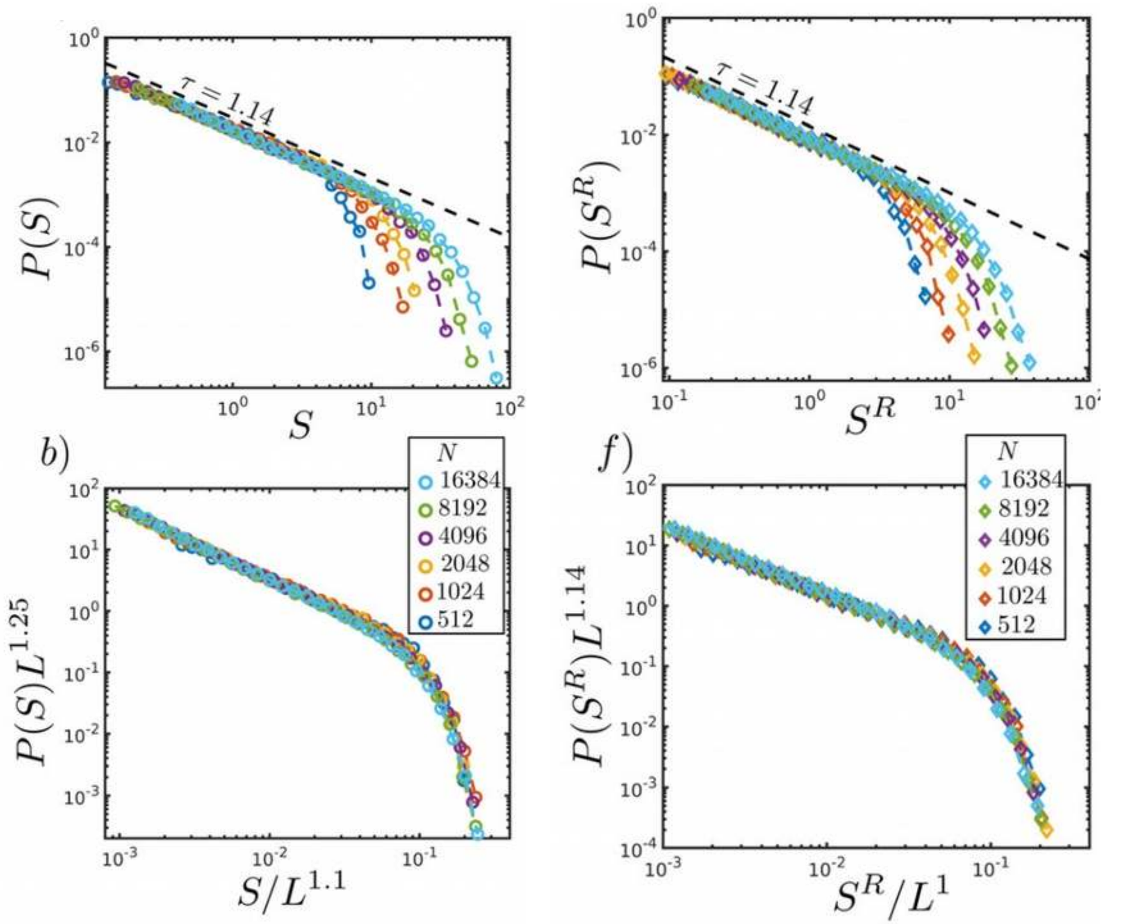


Quasistatic protocols for:
- Simple shear (SS)
- Self-random force (SRF)



$$\text{‘random’ stress } \sigma^R = \frac{1}{L^2}\frac{\mathrm{d}U}{\mathrm{d}\gamma^R} = \frac{1}{2L\sqrt{N}}\sum_{i=1}^{N}\frac{\partial U}{\partial \vec{r}_i}\hat{n}_i^R$$

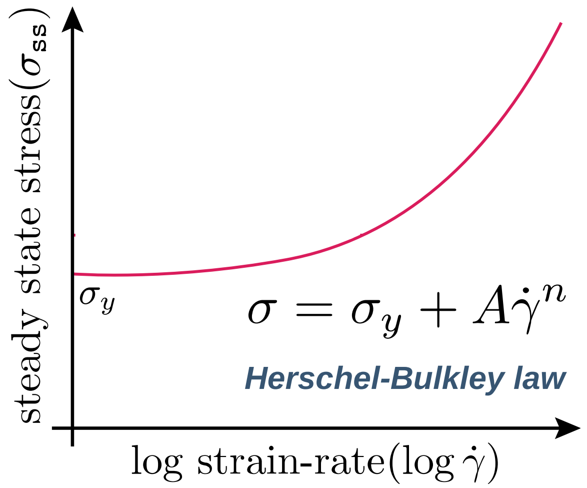
P. K. Morse et al PNAS **118**, 18 (2021)



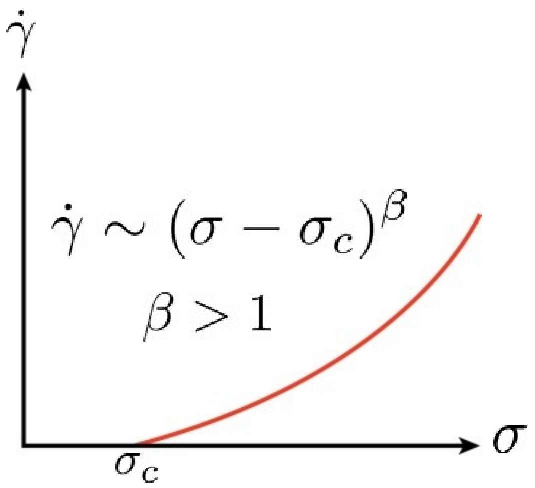
The same avalanche statistics exponents observed for passive or active systems !!

Somehow still to be understood…

FLOWCURVES

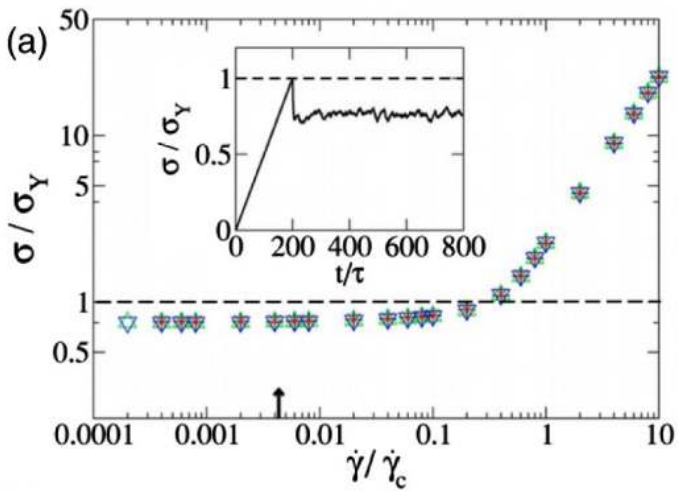


steady state stress(σ_{ss})
σ_y
$\sigma = \sigma_y + A\dot{\gamma}^n$
Herschel-Bulkley law
log strain-rate($\log \dot{\gamma}$)

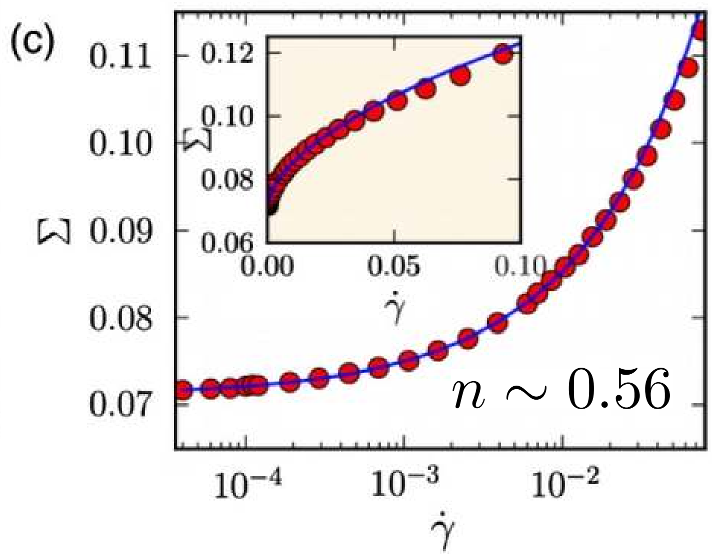


$\dot{\gamma}$
$\dot{\gamma} \sim (\sigma - \sigma_c)^\beta$
$\beta > 1$
σ
σ_c

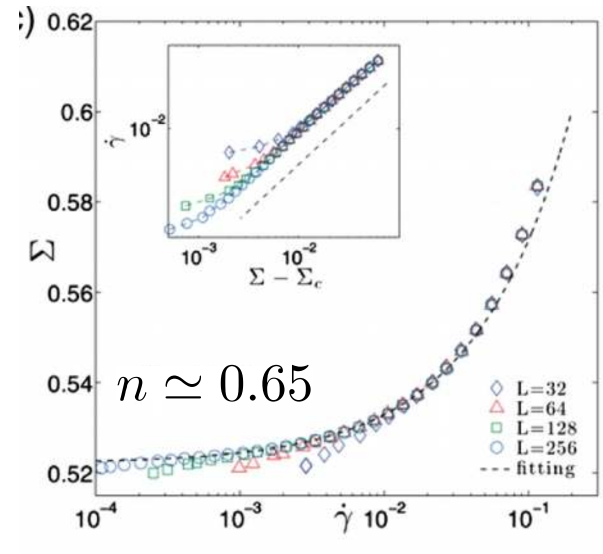
Some flowcurves in the EPM's literature



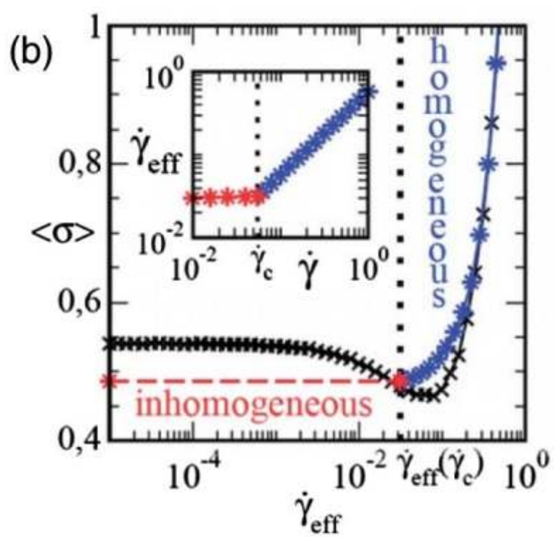
G. Picard et al.
Phys. Rev. E 71, 010501 (2005)



A. Nicolas et al.,
Europhys. Lett. 107, 44003 (2014)



Lin *et al.*
PNAS 111, 14382 (2014)



$$\sigma = \sigma_c + A\dot{\gamma}^n$$

Herschel-Bulkley law

$$\dot{\gamma} \sim (\sigma - \sigma_c)^{\beta} \quad n = 1/\beta$$

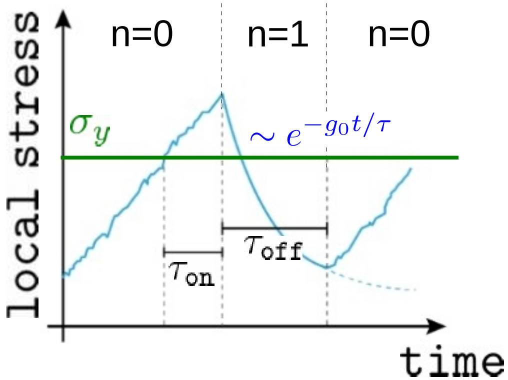
Non-monotonic flowcurve obtained in Picard's model with a long local restructuring time τ_{res}

K. Martens et al.
Soft Matter 8, 4197 (2012)

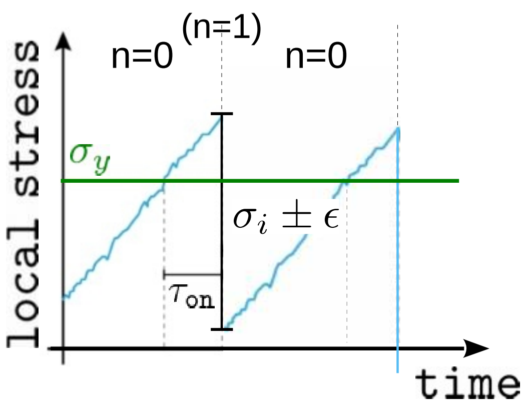
Elasto-Plastic Models (EPM) with stress-dependent rates

EEF & EA Jagla *Soft Matter* **15**, 9041 (2019)

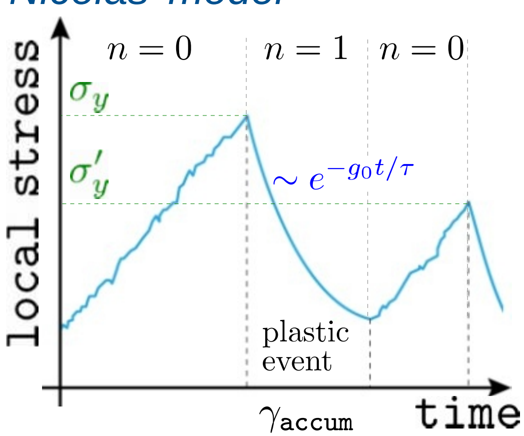
Picard's model



Lin's model



Nicolas' model



$$\partial_t \sigma_i(t) = \mu \dot{\gamma}^{\mathrm{ext}} - g_0 n_i(t) \frac{\sigma_i(t)}{\tau} + \sum_{j \neq i} G_{ij} n_j(t) \frac{\sigma_j(t)}{\tau}$$

$n_i :\ 0 \rightarrow 1$ when $\sigma_i > \sigma_i^y$ at a rate τ_{on}^{-1}

Stochastic rules for local yielding:

(all classic cases) (our proposal)

site is more likely to yield as it is more overloaded

Uniform rate

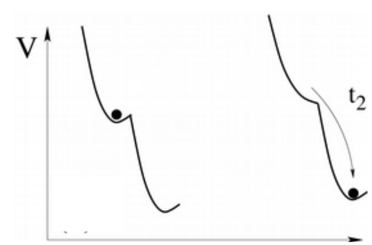
$$\tau_{\mathrm{on}}^{-1} = \lambda = \mathtt{cst.}$$

Progressive rate

$$\tau_{\mathrm{on}}^{-1} = \lambda(\sigma) \propto \sqrt{\sigma - \sigma_y}$$

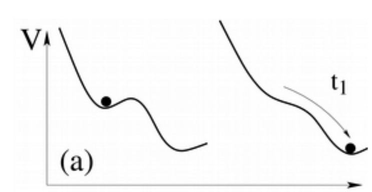
Rationale for the analogy with explicit disorder landscape

cuspy potential



$t_2 \sim \mathtt{cst.}$

smooth potential



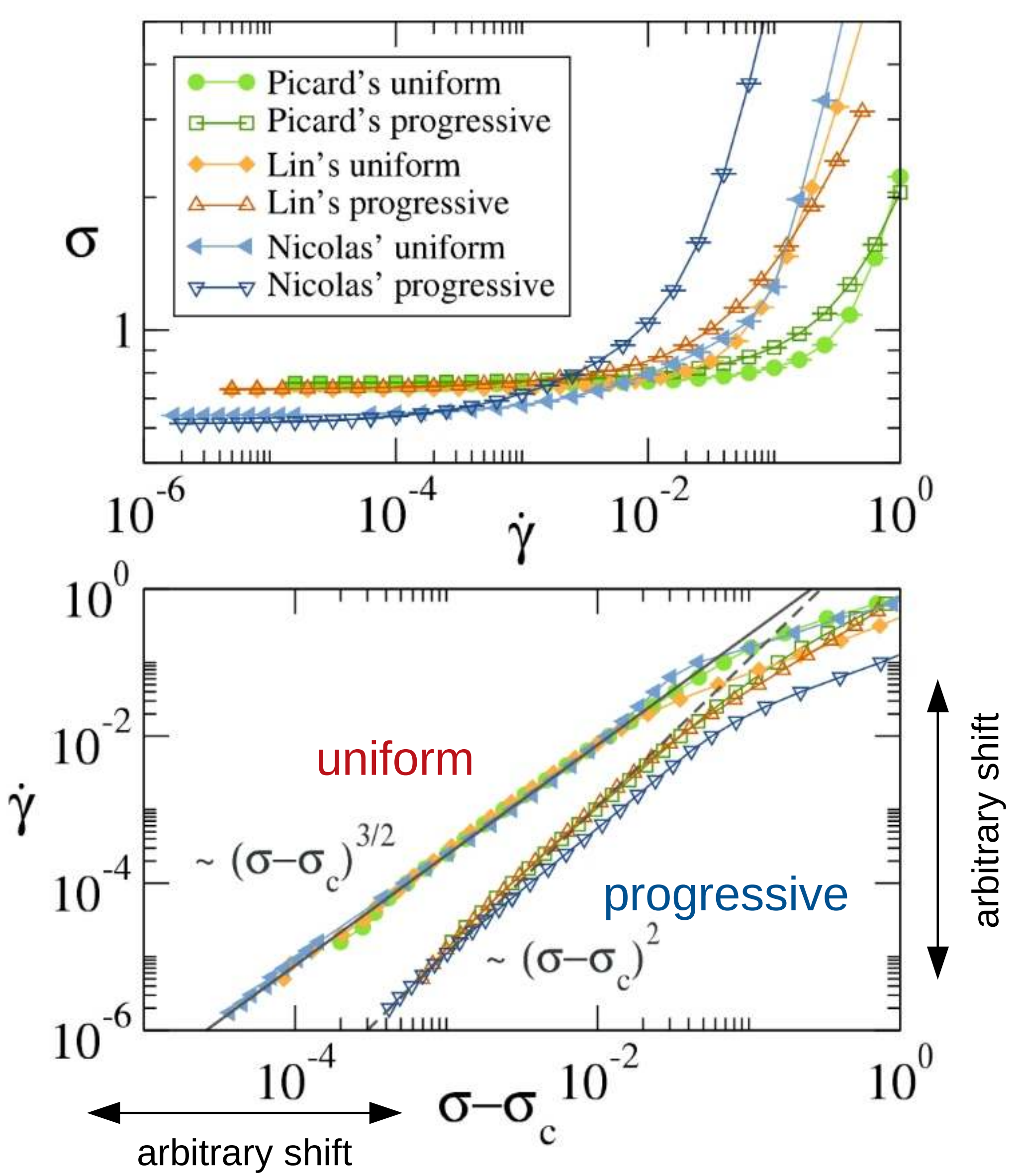
$t_1 \sim \delta\sigma^{-1/2}$

Flowcurves (β exponent)

d=2 *T=0*

We simulate (3x2=)**6 different** EPMs in *d=2*

EEF & EA Jagla *Soft Matter* **15**, 9041 (2019)



$$\dot{\gamma} \propto (\sigma - \sigma_c)^{\beta}$$

Uniform rate

$$\tau_{\mathrm{on}}^{-1} = \lambda = \mathtt{cst.}$$

$$\beta \simeq 1.5$$

Progressive rate

$$\tau_{\mathrm{on}}^{-1} = \lambda(\sigma) \propto \sqrt{\sigma - \sigma_y}$$

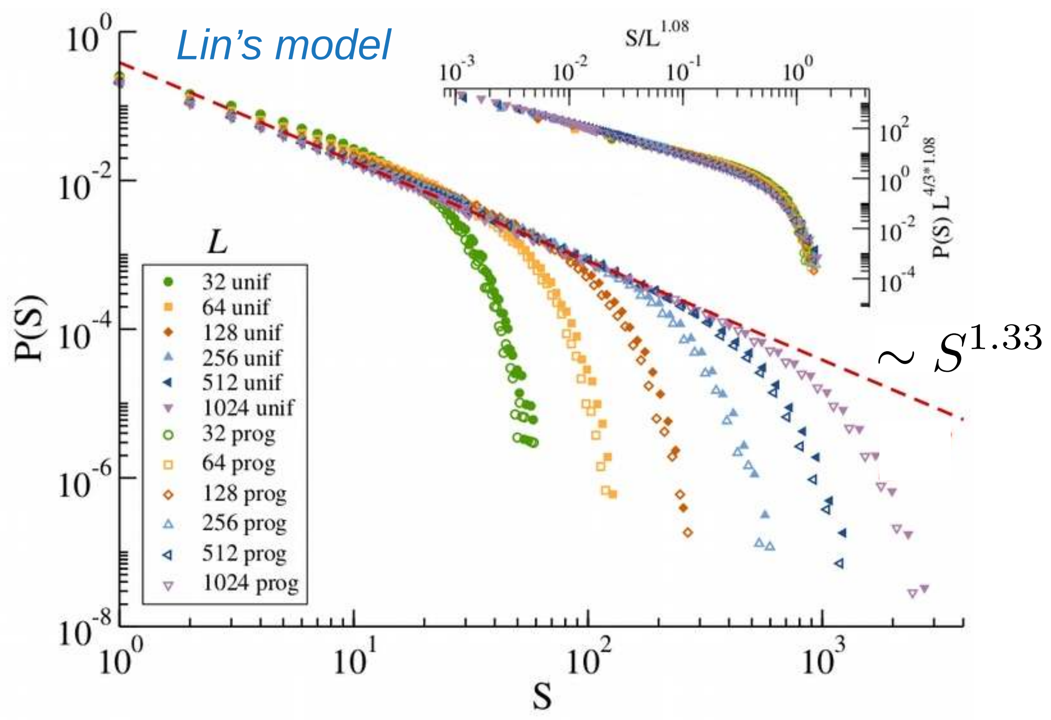
$$\beta \simeq 2$$

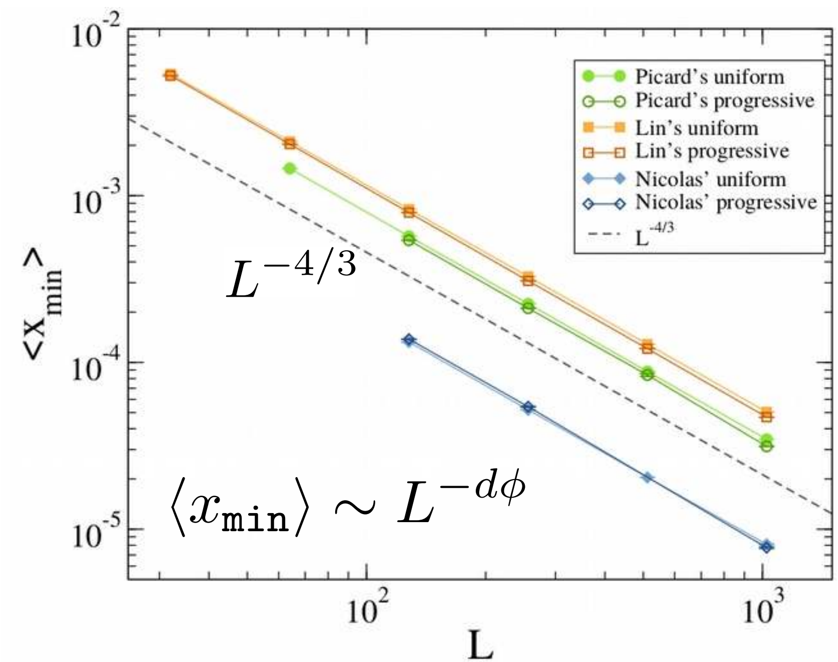
β depends on the local yielding rule only!

Note: progressive rates give an exponent **closer to the measured ones** (*n≈0.5*)

$$\langle \sigma(\dot{\gamma}) \rangle \propto \sigma_c + A\dot{\gamma}^n \qquad n = 1/\beta$$

Universal “static” exponents (τ, d_f, ϕ)





$$P(S) \sim S^{-\tau} f(S/S_{\mathtt{cut}}) \qquad S_{\mathtt{cut}} \sim L^{d_f}$$

$\rightarrow \tau \simeq 1.33$

$\rightarrow d_f \simeq 1.08$

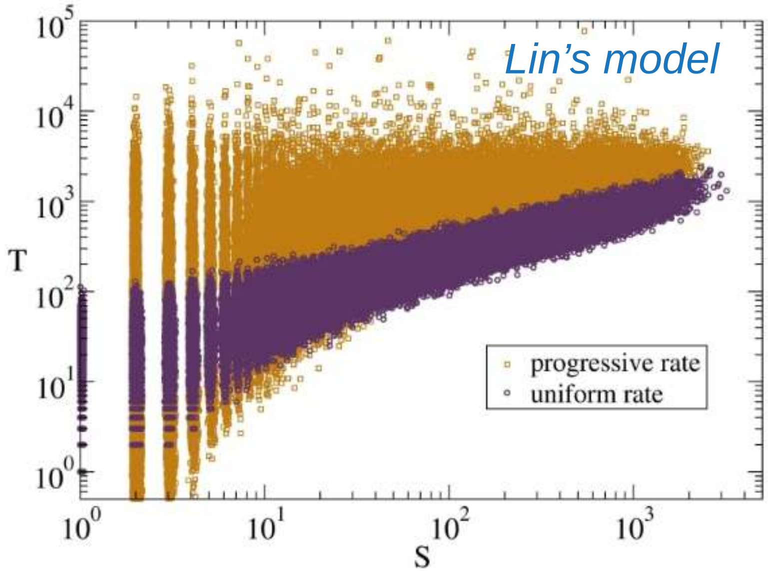
$\rightarrow \phi \simeq 2/3$

$$\boxed{\tau = 2 - \frac{d}{d_f}(1-\phi)}$$

holds

- **Independent** on model and rate rule
- τ might **vary** with the level of non-conserved stress κ defined
- d_f can be **sensitive** to collapse criterion

"Dynamic" exponents (z, β) dependent on rate rule



$$S \sim \ell^{d_f} \qquad T \sim \ell^{z} \qquad T \sim S^{z/d_f}$$

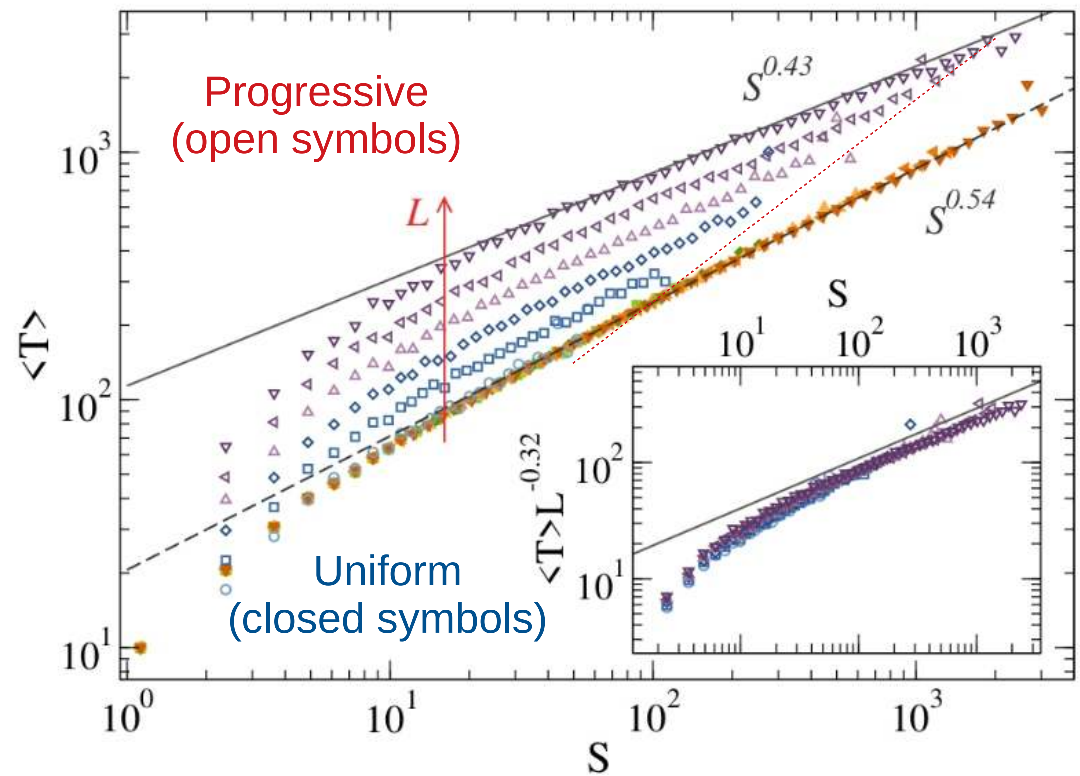
Uniform rate

$$z/d_f \simeq 0.54 \quad z \simeq 0.58$$

Progressive rate

$$z/d_f \simeq 0.43 \quad z \simeq 0.46$$

z depends on the yielding rule! (at least)
Exponents differ, but also the behavior with *L*



New event in the avalanche, time *dT* added

Uniform $\qquad$ *Progressive*

$$dT \simeq \lambda^{-1} \sim 1 \qquad dT \simeq \lambda^{-1} \sim (\sigma - \sigma_Y)^{-1/2}$$

For progressive rates, events that most contribute to the total duration have a small probability of happening but their observation increases with system size

$$T \sim N^{\alpha} S^{(1-\alpha)/2}$$

Take home message

- **Amorphous solids** undergo a ***yielding transition*** characterized by a **power-law** vanishing of the flowcurve at a finite **critical stress**.

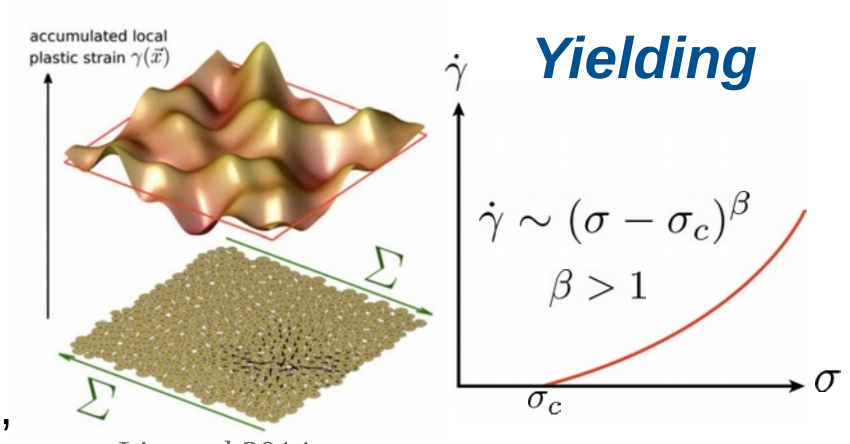
- Associated to it, we find ***avalanche*** statistics and ***critical exponents*. Some consensus** is built in **simulations** about **universality**.

- At **finite strain rate** we can only define **stress-drops**, and those have a **different statistics** as $\dot{\gamma}$ increases.

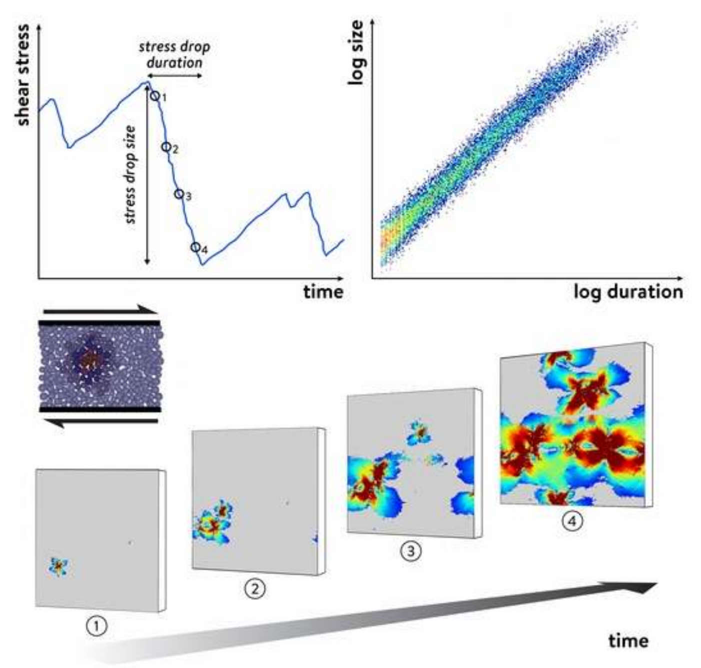
- **Inertia** breaks down the scale-free avalanche statistics and dominates the **large avalanches**.

- A key observable for yielding is the **density of shear transformations *P(x)*** $\sim P_0 + x^{\theta}$ which shows a pseudo-gap, yet **finite-size effects** control x_{min}

- Within EP models, **two coexisting universality classes** exist: **"uniform"** and **"progressive"** yielding rules, with different ***'dynamical'*** exponents (β,z) but the same '***static'*** critical ones (τ, d_f, ϕ).



Lin et al 2014



Liu et al 2016

Hot topics: Thermal avalanches, active yielding avalanches

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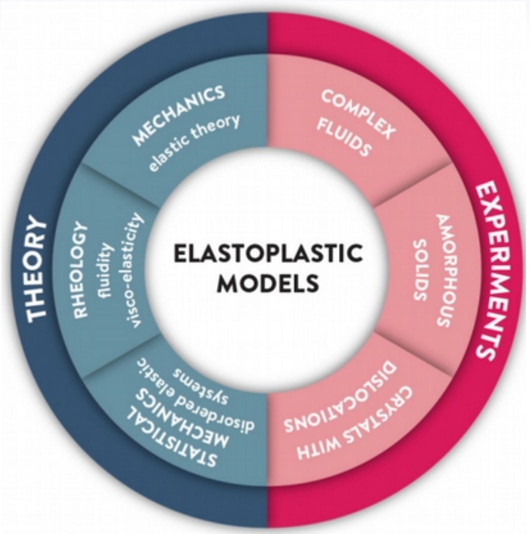

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E. E. Ferrero, E. A. Jagla, arXiv:2501.07782

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Thanks!

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