

Electroweak Structure of Nuclei

Saori Pastore



Electron-Nucleus Scattering Cross Section



Energy and momentum transferred (ω ,q)

Current and planned experimental programs rely on theoretical calculations at different kinematics



Strategy

Validate the Nuclear Model against available data for strong and electroweak observables

- Energy Spectra, Electromagnetic Form Factors, Electromagnetic Moments, ...
- Electromagnetic and Beta decay rates, ...
- Muon Capture Rates, ...
- Electron-Nucleus Scattering Cross Sections, ...

Use attained information to make (accurate) predictions for BSM searches and precision tests

- EDMs, Hadronic PV, ...
- BSM searches with beta decay, ...
- Neutrinoless double beta decay, ...
- Neutrino-Nucleus Scattering Cross Sections, ...
- ...

Many-body Nuclear Problem

Nuclear Many-body Hamiltonian

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_A, \mathbf{s}_1, \mathbf{s}_2, ..., \mathbf{s}_A, \mathbf{t}_1, \mathbf{t}_2, ..., \mathbf{t}_A)$$



$$\Psi$$
 are spin-isospin vectors in 3A dimensions with $2^A \times \frac{A!}{Z!(A-Z)!}$ components
⁴He : 96
⁶Li : 1280
⁸Li : 14336
¹²C : 540572

(numerically) exactly or within approximations that are under control the many-body nuclear problem

 $H\Psi = E\Psi$

Current Status



Many-body Nuclear Interactions

Many-body Nuclear Hamiltonian

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

 v_{ij} and V_{ijk} are two- and three-nucleon operators based on experimental data fitting; fitted parameters subsume underlying QCD dynamics



Contact term: short-range Two-pion range: intermediate-range $r\propto (2\,m_\pi)^{-1}$ One-pion range: long-range $r\propto m_\pi^{-1}$





Hideki Yukawa

AV18+UIX; AV18+IL7 Wiringa, Schiavilla, Pieper *et al.*

chiral πNΔ N3LO+N2LO Piarulli *et al.* Norfolk Models

Norfolk Two- and Three-body Potentials



Norfolk Chiral Potentials

NV2: two-body

26 LECs fitted to np and pp Granada database (2700-3700 data points; lab energies up to 125-200 MeV) with a chi-square/datum ~1

NV3: three-body

2 LECs

Piarulli *et al*. PRC91(2015) PRC94(2016)

A-less Additional in A-full LO $(Q/\Lambda_{\chi})^0$ NLO $(Q/\Lambda_{\gamma})^2$ NNLO $(Q/\Lambda_{\chi})^3$ N³LO $(Q/\Lambda_{\chi})^4$ N^4LO $(Q/\Lambda_{\chi})^5$

Chiral 3N Force

Figs. credit Entem and Machleidt Phys.Rept.503(2011)1

8 Models depending on the fitting strategy adopted for the LECs

Energies



Piarulli et al. PRL120(2018)052503



Nucleon-Nucleon Potential



The Deuteron



Constant density surfaces for a polarized deuteron in the $M=\pm 1$ (left) and M=0 (right) states

Carlson and Schiavilla Rev.Mod.Phys.70(1998)743

Two-nucleon correlations & momentum distributions



Tensor correlations lead to large differences in the **np** versus **pp** distributions.

These differences are observed in A(e, e'np) and A(e, e'pp) reactions.

Schiavilla Carlson Wiringa Pieper PRL98(2007) & PRC89(2014)

Many-body Nuclear Electroweak Currents



- Two-body currents are a manifestation of two-nucleon correlations
- Electromagnetic two-body currents are required to satisfy current conservation

$$\mathbf{q} \cdot \mathbf{j} = [H, \rho] = [t_i + v_{ij} + V_{ijk}, \rho]$$

Nuclear Charge Operator

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i < j} \rho_{ij} + \dots$$

Nuclear (Vector) Current Operator



Magnetic Moment: Single Particle Picture

Many-body Currents

• Meson Exchange Currents (MEC)

Constrain the MEC current operators by imposing that the current conservation relation is satisfied with the AV18 two-body potential

Chiral Effective Field Theory Currents

Are constructed consistently with the two-body chiral potential; Unknown parameters, or Low Energy Constants (LECs), need to be determined by either fits to experimental data or by Lattice QCD calculations



Electromagnetic Current Operator

SP *et al.* PRC78(2008)064002, PRC80(2009)034004, PRC84(2011)024001, PRC87(2013)014006 Park *et al.* NPA596(1996)515, Phillips (2005) Kölling *et al.* PRC80(2009)045502 & PRC84(2011)054008

LCQD inputs for neutrino-nucleus scattering



Snowmass WP: Theoretical tools for neutrino scattering: interplay between lattice QCD, EFTs, nuclear physics, phenomenology, and neutrino event generators; arXiv:2203.09030

Building blocks of ab initio nuclear approaches:

Nucleonic form factors Transition form factors Pion production amplitudes Two-nucleon couplings (strong and EW)

Taken from data where available, or from theory

Magnetic Moments of Light Nuclei



Magnetic moment



Chambers-Wall, King, Gnech et al. PRL 2024 2407.03487



Elastic scattering

Cross section

$$\frac{d\sigma}{d\Omega} = 4\pi\sigma_M f_{\rm rec}^{-1} \left[\frac{Q^4}{q^4} F_L^2(q) + \left(\frac{Q^2}{2q^2} + \tan^2\theta_e/2 \right) F_T^2(q) \right]$$

$$F_T^2(q) = F_M^2(q) = \frac{1}{2J_i + 1} \sum_{L=1}^{\infty} |\langle J_f || M_L(q) || J_i \rangle|^2$$
$$F_L^2(q) = \frac{1}{2J_i + 1} \sum_{L=0}^{\infty} |\langle J_f || C_L(q) || J_i \rangle|^2$$

Magnetic and Charge Form Factors $\langle JJ | \mathbf{j}_y(q \hat{\boldsymbol{x}}) | JJ \rangle \ \langle J_f M | \rho^{\dagger}(q) | J_i M \rangle$

Magnetic form factors: comparison with the data





First QMC results for form factors in A>6 systems.

Based on Norfolk interactions and one- and two-body currents.

Error band = truncation error in the ChiEFT expansion.

 $q \, [\text{fm}^{-1}]$ Chambers-Wall, King, Gnech et al. PRL 2024 2407.03487

Magnetic form factors: comparison with the data



 $q \, [\mathrm{fm}^{-1}]$ Chambers-Wall, King, Gnech et al. PRL 2024 2407.03487

Magnetic form factors: predictions





Two-body currents provide 40-60%.

Note the swapping of M1 and M3 in mirror nuclei. Also observed in A=7 nuclei.

It would be interesting to have data for mirror nuclei.

Maybe ⁷Be?

Chambers-Wall, King, Gnech et al. PRL 2024 2407.03487



Charge radii

Extracted from low-momentum transfer behavior of form factor.

$$\frac{1}{Z} \langle JJ | \rho(q\hat{\mathbf{z}}) | JJ \rangle \approx 1 - \frac{1}{6} r_E^2 q^2 + \mathcal{O}(q^4)$$

Accounts for two-body correlations, finite size/nucleon level corrections via nucleonic form factors.



Agreement of $\sim 5\%$ or better.

King et al. submitted to PRC 2025

Magnetic radii

Extracted from low-momentum transfer behavior of form factor.

$$-i\frac{2m}{q\mu}\left\langle JJ|\mathbf{j}_{y}(q\hat{\mathbf{x}})|JJ\right\rangle \approx 1-\frac{1}{6}r_{M}^{2}q^{2}+\mathcal{O}(q^{4})$$

Accounts for two-body currents, finite size/nucleon level corrections via nucleonic form factors.



Limited data, predictions available for A up to 10.

King et al. submitted to PRC 2025

Electromagnetic transitions

Two-body electromagnetic currents bring the theory in agreement with the data

~ 60 – 70% of total two-body current is due to one-pion-exchange currents



Lepton-Nucleus scattering: Inclusive Processes

Electromagnetic Nuclear Response Functions

$$R_{\alpha}(q,\omega) = \sum_{f} \delta\left(\omega + E_0 - E_f\right) |\langle f|O_{\alpha}(\mathbf{q})|0\rangle|^2$$

Longitudinal response induced by the charge operator $O_L = \rho$ Transverse response induced by the current operator $O_T = j$ 5 Responses in neutrino-nucleus scattering

$$\frac{d^2 \sigma}{d \,\omega d \,\Omega} = \sigma_M \left[v_L \, R_L(\mathbf{q}, \omega) + v_T \, R_T(\mathbf{q}, \omega) \right]$$



For a recent review on QMC, SF methods see Rocco Front. In Phys.8 (2020)116

Inclusive Cross Sections with Integral Transforms

Exploit integral properties of the response functions and closure to avoid explicit calculation of the final states (Lorentz Integral Transform **LIT**, **Euclidean**, ...)

$$S(q,\tau) = \int_0^\infty d\omega K(\tau,\omega) R_\alpha(q,\omega)$$



Sobczyk et al, PRL127 (2021)



Lovato et al. PRX10 (2020)

Lepton-Nucleus scattering: Data

5

Transverse Sum Rule

 $S_T(q) \propto \langle 0 | \mathbf{j}^{\dagger} \mathbf{j} | 0 \rangle \propto \langle 0 | \mathbf{j}_{1b}^{\dagger} \mathbf{j}_{1b} | 0 \rangle + \langle 0 | \mathbf{j}_{1b}^{\dagger} \mathbf{j}_{2b} | 0 \rangle + \dots$



Observed transverse enhancement explained by the combined effect of two-body correlations and currents in the interference term

$$\langle \mathbf{j}_{1b}^{\dagger} \ \mathbf{j}_{1b} \rangle > 0$$

Leading one-body term

$$\langle \mathbf{j}_{1b}^{\dagger} \; \mathbf{j}_{2b} \; v_{\pi} \rangle \propto \langle v_{\pi}^2 \rangle > 0$$

Interference term



Transverse/Longitudinal Sum Rule Carlson *et al.* PRC65(2002)024002

Beyond Inclusive: Short-Time-Approximation

Short-Time-Approximation Goals:

- Describe electroweak scattering from A
 > 12 without losing two-body physics
- Account for exclusive processes
- Incorporate relativistic effects



Subedi et al. Science320(2008)1475



Stanford Lab article





Short-Time-Approximation

Short-Time-Approximation:

- Based on Factorization
- Retains two-body physics
- Response functions are given by the scattering from pairs of fully interacting nucleons that propagate into a correlated pair of nucleons
- Allows to retain both two-body correlations and currents at the vertex
- Provides "more" exclusive information in terms of nucleon-pair kinematics via the Response Densities

Response Functions ∞ Cross Sections

$$R_{\alpha}(q,\omega) = \sum_{f} \delta\left(\omega + E_0 - E_f\right) \left|\langle f | O_{\alpha}(\mathbf{q}) | 0 \rangle\right|^2$$

Response *Densities*

$$R(q,\omega) \sim \int \delta \left(\omega + E_0 - E_f\right) dP' dp' \mathcal{D}(p',P';q)$$

P' and *p*' are the CM and relative momenta of the struck nucleon pair

Transverse Response Density: *e*-⁴He scattering

Transverse Density q = 500 MeV/c



SP et al. PRC101(2020)044612

e-⁴He scattering in the back-to-back kinematic



Electromagnetic vertex included

Helium-4: data & model dependence





Benchmark in ⁴He

SP et al. PRC101(2020)044612

¹²C Response Densities



Andreoli et al. Phys. Rev. C 110 (2024) 6, 064004 arXiv:2407.06986

¹²C response functions

$$\frac{d^2 \sigma}{d \,\omega d \,\Omega} = \sigma_M \left[v_L \, R_L(\mathbf{q}, \omega) + v_T \, R_T(\mathbf{q}, \omega) \right]$$

Andreoli et al. Phys. Rev. C 110 (2024) 6, 064004 arXiv:2407.06986





Andreoli *et al. Phys.Rev.C* 110 (2024) 6, 064004 <u>arXiv:2407.06986</u> Data From https://discovery.phys.virginia.edu/research/groups/qes-archive/index.html

Relativistic effects in e-³H scattering



Andreoli et al. Phys. Rev. C 105 (2022) 1, 014002



Relativistic effects in e-¹²C scattering



Andreoli et al. Phys. Rev. C 110 (2024) 6, 064004 arXiv:2407.06986

Relativistic corrections

Traditional non relativistic expansion of the covariant single nucleon electromagnetic current assumes initial and final nucleon momentum small.

$$egin{aligned} j^{\mu} &= ear{u}ig(m{p}'s'ig)ig(e_N\gamma^{\mu}+rac{i\kappa_N}{2m_N}\sigma^{\mu
u}q_{
u}ig)u(m{p}s)\ m{p}' &=m{p}+m{q} \end{aligned}$$

New paradigme where the relativistic correction is obtained expanding the covariant one-nucleon current for high values of momentum transfer, and small values of initial nucleon momentum p. This changed:

- 1. Expression of the one-body operator
- 2. Energy conserving delta function



With Ronen Weiss and Lorenzo Andreoli

Ronen Weiss Ed Jaynes Fellow at WashU

Implementation single nucleon current

1000

800

600



Response density vs relative and c.m. energy of the struck pair ⁴He Transverse response density at q = 700 MeV/c





Response density vs momenta of individual nucleons

Application to e-³H scattering



Three-body densities







Ronen Weiss, and Stefano Gandolfi *Phys.Rev.C* 108 (2023) 2, L021301

With AFDMC



⁴He Three-Body Momentum Distribution



Summary

Ab initio calculations of light nuclei yield a picture of $G_{\mathcal{H}}^{(m)}$ nuclear structure and dynamics where many-body effects play an essential role to explain available data.



 $E_{\rm cm}$) [MeV⁻²] 2,0001,000 0 200 200150100100500 MeV] $e \, [MeV]$ Close collaborations between NP, LQCD, Pheno, Hep, ⁷Li Comp, Expt, ... are required to progress e.g., NP is represented in the Snowmass process ^{10}B It's a very exciting time!



Graham Chambers-Wall (WashU GS)



Garrett King (LANL PD)



Lorenzo Andreoli (ODU/JLab PD)

King *et al.* <u>PRC 110</u> (2024) 5, 054325; <u>Ann.Rev.Nucl.Part.Sci. 74</u> (2024) 343 Chambers-Wall, Gnech, King *et al.* <u>PRL 133</u> (2024) 21, 212501; <u>PRC 110</u> (2024) 5, 054316 Andreoli *et al.* <u>PRC 110</u> (2024) 6, 064004

Collaborators

WashU: Bub Chambers-Wall Flores Novario Piarulli Weiss

LANL: Carlson Gandolfi Hayes **King** Mereghetti JLab+ODU: **Andreoli Gnech** Schiavilla ANL: McCoy Lovato Wiringa UW/INT: Cirigliano Dekens Pisa U/INFN: Kievsky Marcucci Viviani Salento U: Girlanda Huzhou U: Dong Wang Fermilab: Gardiner Betancourt Rocco MIT: Barrow









Nuclear Theory for New Physics NP&HEP TC

Nuclear Theory for New Physics

About Us

- Commitment to Diversity
- Funding Acknowledgement



Snowmass:

Topical groups and Frontier Reports, Whitepapers, ...

LRP: White papers, 2301.03975, FSNN,

. . .

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Transverse Response Density: two-body physics



q=500

STA: regime of validity

The typical (conservative estimate) energy (time) scale in a nucleus with A correlated nucleons in pairs is

$$\epsilon_{pair} \sim 20 \text{ MeV}$$
 (t ~ 1/ ϵ_{pair})

This sets a natural expansion parameter in the QE region characterized by ω_{QE}

 ϵ_{pair} / ω_{QE}

The STA neglects terms of order $O((\epsilon_{pair} / \omega_{QE})^2)$



Short-Time-Approximation

Short-Time-Approximation:

- Based on Factorization
- Retains two-body physics
- Correctly accounts for interference



$$R(q,\boldsymbol{\omega}) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} \,\mathrm{e}^{i(\boldsymbol{\omega}+E_0)t} \,\langle 0|O^{\dagger}\,\mathrm{e}^{-iHt}\,O|0\rangle$$

$$O_i^{\dagger} e^{-iHt} O_i + O_i^{\dagger} e^{-iHt} O_j + O_i^{\dagger} e^{-iHt} O_{ij} + O_{ij}^{\dagger} e^{-iHt} O_{ij}$$

$$H \sim \sum_i t_i + \sum_{i < j} v_{ij}$$

GFMC SF STA: Benchmark & error estimate in A=3



Andreoli et al. Phys. Rev. C 105 (2022) 1, 014002



GENIE validation using e-scattering

Z = 2, A = 4, Beam Energy = 0.64 GeV, Angle = $60^{\circ} \pm 0.25^{\circ}$



- STA responses used to build the cross sections
- Cross sections are used to generate events in GENIE (a Monte Carlo neutrino event generator)
- Here, we use electromagnetic processes (for which data are available) to validate the generator

$$\frac{d^2 \sigma}{d \,\omega d \,\Omega} = \sigma_M \left[v_L \, R_L(\mathbf{q}, \omega) + v_T \, R_T(\mathbf{q}, \omega) \right]$$

Barrow, Gardiner, SP et al. PRD 103 (2021) 5, 052001

Correlated pairs vs uncorrelated pairs



Scattering from uncorrelated vs correlated nucleon pairs

¹²C cross sections: interpolation scheme

We have coarse grid in q for ¹²C. We use an interpolation scheme tested on He4.



exact

interpolated

¹²C comparison with the data



Christy, Bodek et al (2025)

One-body interference in M1 vs M3



M1 vs M3; spin magnetization (solid line) vs orbital (dashed line)

One-body magnetic density



 $\mu^{1b} \propto \int \rho_M^{1b}(r) dr$

r single particle coordinate from the c.m.

$$1b = \mu_N \sum_i [(L_i + g_p S_i)(1 + \tau_{i,z})/2 + g_n S_i(1 - \tau_{i,z})/2]$$

Two-body magnetic densities



$$\mu^{2b} = \int dr_{ij} 4\pi r_{ij}^2 \rho_M^{2b}(r_{ij})$$

Cluster effects suppress the two-body contribution for A=9,T=1/2



Axial currents with Δ at tree-level



Two body currents of one pion range (red and blue) with $c_3 c_4$ from Krebs *et al.* Eur.Phys.J.(2007)A32

Contact current involves the LEC c_p



P. Gysbers Nature Phys. 15 (2019)

Three-body Force and the Axial Contact Current





Three-body force

Axial two-body contact current

LECs c_{D} and c_{E} are fitted to:

- trinucleon B.E. and nd doublet scattering length in NV2+3-la
- trinucleon B.E. and Gamow-Teller matrix element of tritium NV2+3-la*

Baroni *et al.* PRC98(2018)044003

Energies A=8-10 slightly better with non-starred models

Scaled two-body transition densities



Different fitting procedures lead to different short range behaviours.

Garrett King et al. PRC102(2020)025501

Axial Two-body Transition Density



NV2+3-la; NV2+3-la*

enhanced contribution from contact current in the starred model gives rise to nodes in the two-body transition density

Two-body axial currents



long-range at N2LO and N3LO



contact current at N3LO

Scaling & Universality of Short-Range Dynamics



NV2+3-Ia empty circles; NV2+3-Ia* stars Different colors refer to different transitions

Quantum Monte Carlo Methods

Minimize the expectation value of the nuclear Hamiltonian: $H = T + v_{ii} + V_{iik}$

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \ge E_V$$

using the trial wave function:

$$|\Psi_V\rangle = \left[\mathcal{S}\prod_{i< j} (1 + U_{ij} + \sum_{k\neq i,j} U_{ijk})\right] \left[\prod_{i< j} f_c(r_{ij})\right] |\Phi_A(JMTT_3)\rangle$$

Further improve the trial wave function by eliminating spurious contaminations via a Green's Function Monte Carlo propagation in imaginary time

$$\Psi(\tau) = \exp[-(H - E_0)\tau]\Psi_V = \sum_n \exp[-(E_n - E_0)\tau]a_n\psi_n$$
$$\Psi(\tau \to \infty) = a_0\psi_0$$

Carlson, Wiringa, Pieper et al.

