Evolution of shapes and symmetries along isotopic chains in neutron rich nuclei

Anna E. McCoy

ICTP
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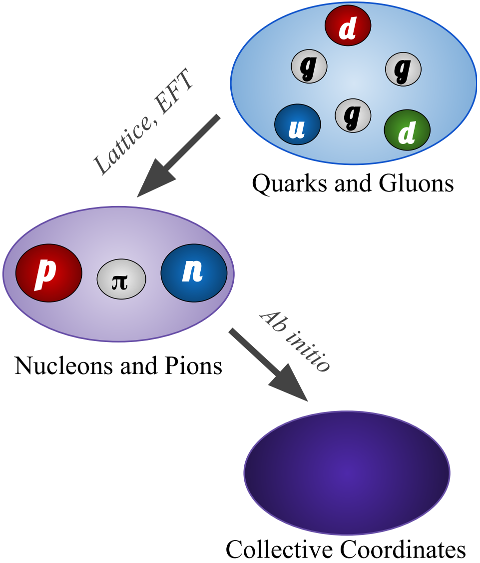
Outline

Goal: Looking for simple pictures that provide a more intuitive understanding of nuclear structure

- Start from an *ab initio* description and mapping back onto simple pictures
  - Effective single particle model
  - Elliott SU(3) model (long range correlations)
- Brief review of *ab initio* no-core shell model
- Two case studies: beryllium and carbon



Neutron rich *p*-shell nuclei

	3	4	5	6	7	8	9	10	11	12	13	14
O 8			(3/2−) ^{13}O	0+ ^{14}O	1/2− ^{15}O	0+ ^{16}O	5/2+ ^{17}O	0+ ^{18}O	5/2+ ^{19}O	0+ ^{20}O	(5/2+) ^{21}O	0+ ^{22}O
N 7			1+ ^{12}N	1/2− ^{13}N	1+ ^{14}N	1/2− ^{15}N	2− ^{16}N	1/2− ^{17}N	1− ^{18}N	^{19}N	2− ^{20}N	(1/2−) ^{21}N
C 6	(3/2−) ^{9}C	0+ ^{10}C	3/2− ^{11}C	0+ ^{12}C	1/2− ^{13}C	0+ ^{14}C	1/2+ ^{15}C	0+ ^{16}C	3/2+ ^{17}C	0+ ^{18}C	1/2+ ^{19}C	0+ ^{20}C
B 5	2+ ^{8}B	3/2− [^{9}B]	3+ ^{10}B	3/2− ^{11}B	1+ ^{12}B	3/2− ^{13}B	2− ^{14}B	^{15}B		(3/2−) ^{17}B		(3/2−) ^{19}B
Be 4	3/2− ^{7}Be	0+ [^{8}Be]	3/2− ^{9}Be	0+ ^{10}Be	1/2+ ^{11}Be	0+ ^{12}Be	(1/2−) [^{13}Be]	0+ ^{14}Be				
Li 3	1+ ^{6}Li	3/2− ^{7}Li	2+ ^{8}Li	3/2− ^{9}Li		3/2− ^{11}Li						

No-core shell model

Solve many-body Schrodinger equation

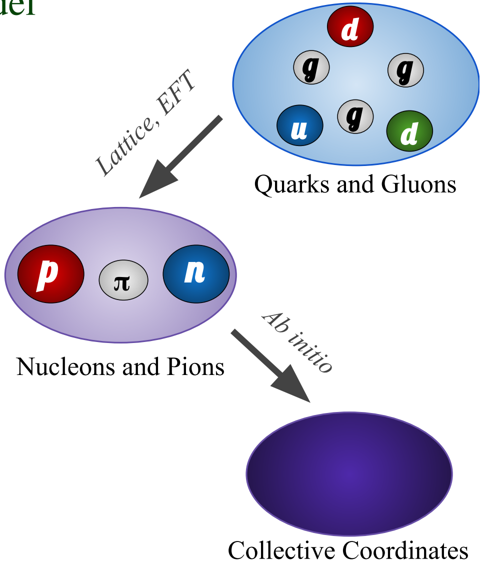
$$\sum_{i}^{A} -\frac{\hbar^2}{2m_i}\nabla_i^2 \Psi + \frac{1}{2}\sum_{i,j=1}^{A} V(|r_i - r_j|)\Psi = E\Psi$$

Expanding wavefunctions in a basis

$$\Psi = \sum_{k=1}^{\infty} a_k \phi_k$$

Reduces to matrix eigenproblem

$$\begin{pmatrix} H_{11} & H_{12} & \dots \\ H_{21} & H_{22} & \dots \\ \vdots & \vdots & \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \\ \vdots \end{pmatrix} = E\begin{pmatrix} a_1 \\ a_2 \\ \vdots \end{pmatrix}$$



Harmonic oscillator basis

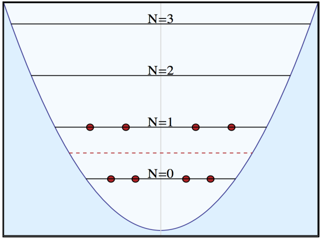
- Basis states are configurations, i.e., distributions of particles over harmonic oscillator shells (*nlj substates*)

- States are organized by total number of oscillator quanta above the lowest Pauli allowed number N_{ex}

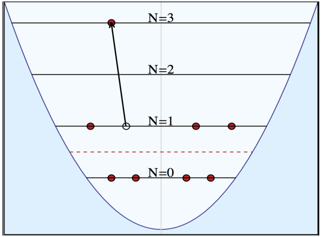
- States with higher N_{ex} contribute less to the wavefunction

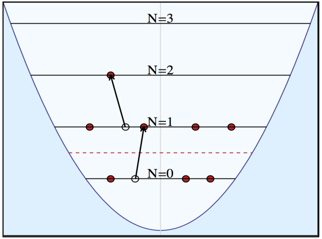
- Basis must be truncated:
  Restrict $N_{ex} \leq N_{max}$

$N = 2n + l$



$N_{ex} = 0$



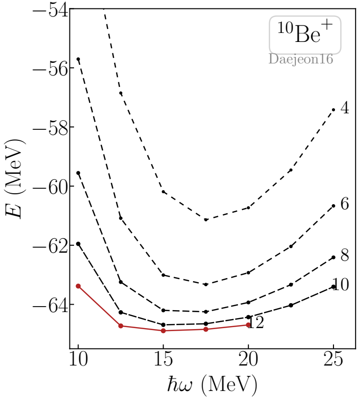


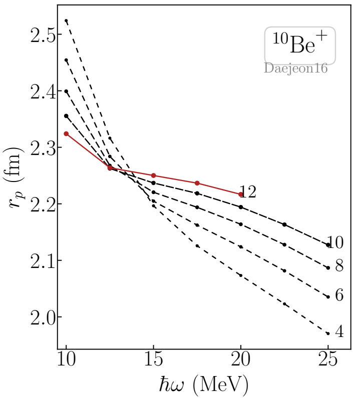
$N_{ex} = 2$

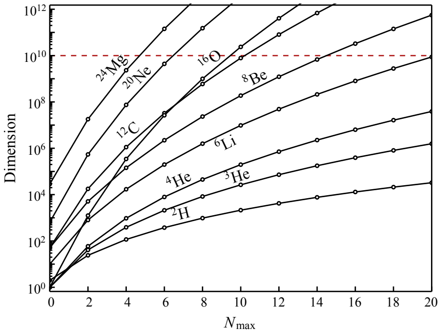
Convergence Challenge

Results for calculations in a finite space depend upon:

- Many-body truncation $N_{\max}$
- Single-particle basis scale $\hbar\omega$



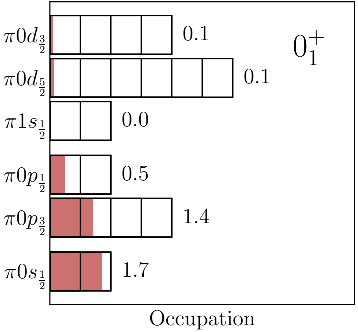


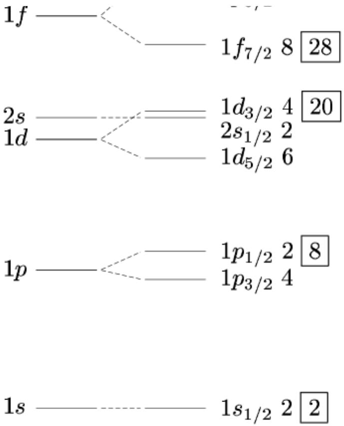


Effective single particle picture

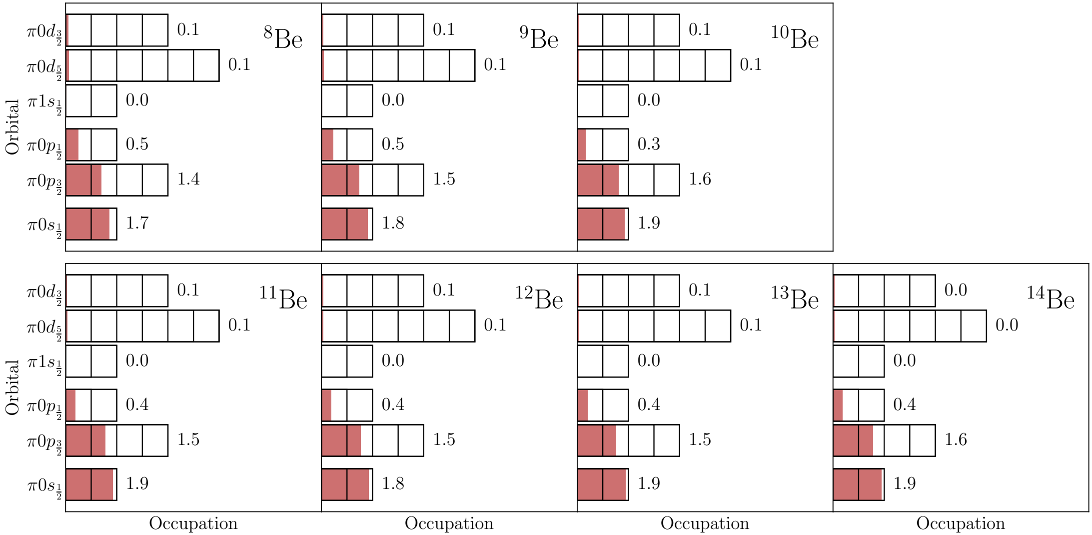
– Natural orbitals maximize occupation of lowest orbitals, by diagonalizing the density matrix

$$\hat{\rho} = \sum_{\alpha\beta} |\alpha\rangle \langle \Psi | a^\dagger_\alpha a_\beta | \Psi \rangle \langle \beta |$$





Natural orbital occupations



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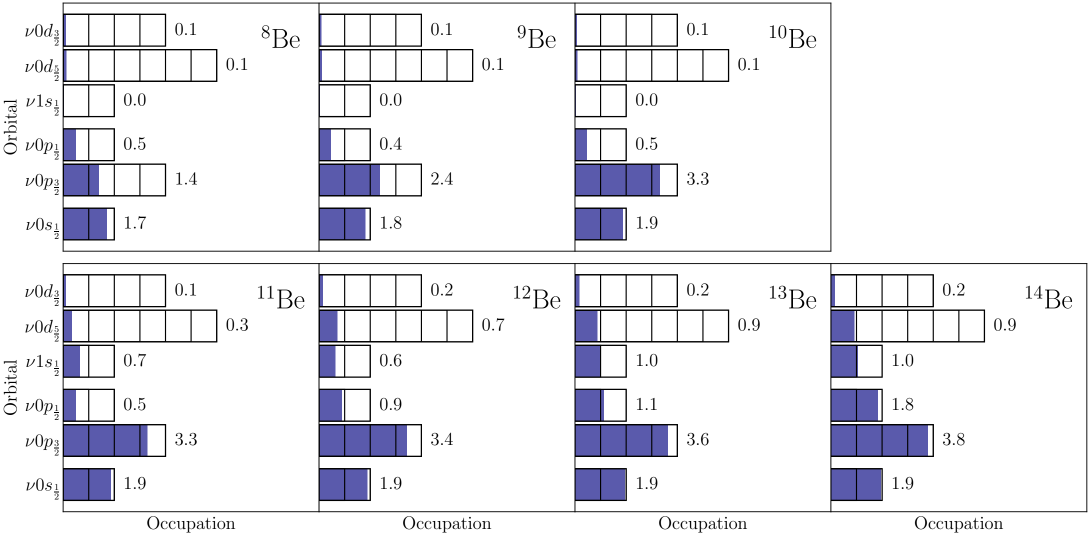

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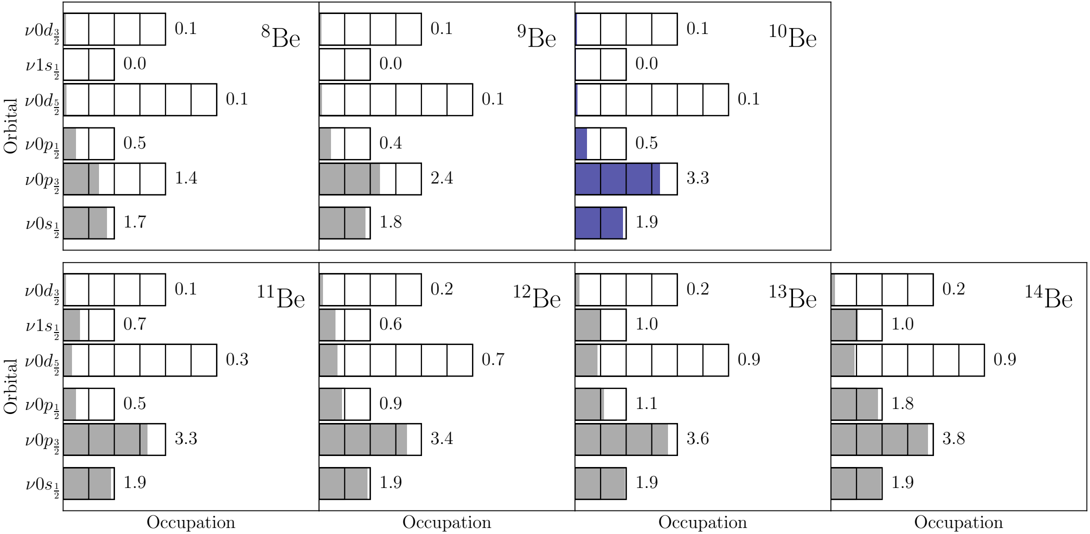
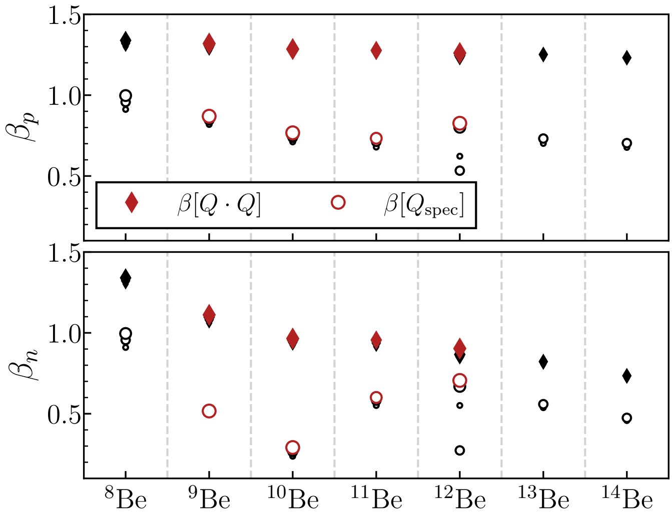


Quadrupole deformation



$$\beta_p \propto \frac{Q_{0,p}}{\sqrt{Z}\langle r_p^2 \rangle} \qquad \beta_p \propto \frac{Q_{0,n}}{\sqrt{N}\langle r_n^2 \rangle}$$

From q-invariant (*dynamic deformation*)

$$\langle Q \cdot Q \rangle = \langle Q_0 \cdot Q_0 \rangle$$

If symmetric rotor

$$Q(J) = \frac{\hat{J}}{(1+\delta_{K,0})}(JK20|JK)Q_0$$

D. J. Rowe. Rep. Prog. Phys. **48**(1985) 1419.
D. J. Rowe, Nuclear Collective Motion: Models and Theory (2010).

Nilsson Model

Wood Saxon potential

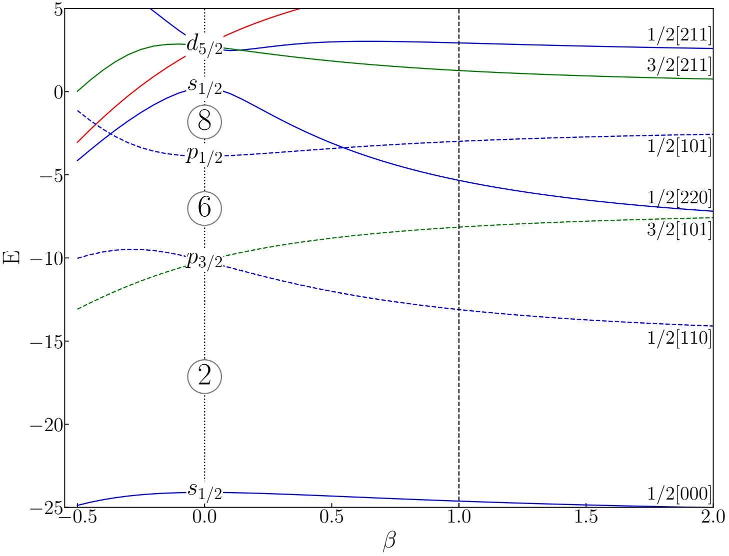
$$V_{\mathrm{WS}} = \frac{-V_0}{1 + e^{(r-R)/a}},$$

$a = 0.67\,\mathrm{fm},\ V_0 = 57\,\mathrm{MeV},$
$R = 1.27(A = 8)^{1/3}\ \mathrm{fm}.$

Nilsson Hamiltonian

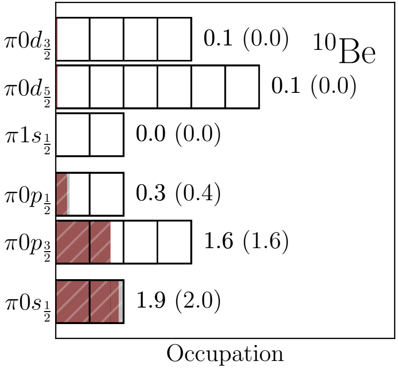
$$H = V_{\mathrm{WS}} + \beta\hbar\omega r^2 Y_{20}$$

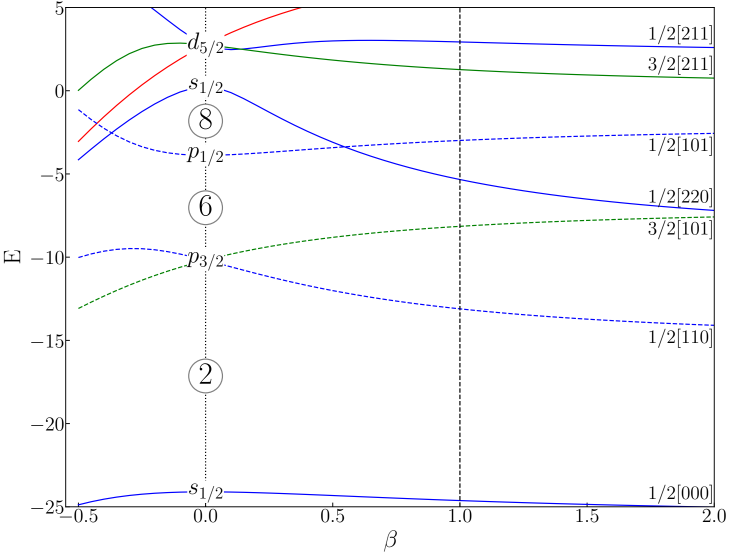
$\hbar\omega = 12.5, \beta = 1$



Wood Saxon parameters: J. Suhonen. From Nucleons to Nuclei
Concepts of Microscopic Nuclear Theory, Chapter 3.

Nilsson Model





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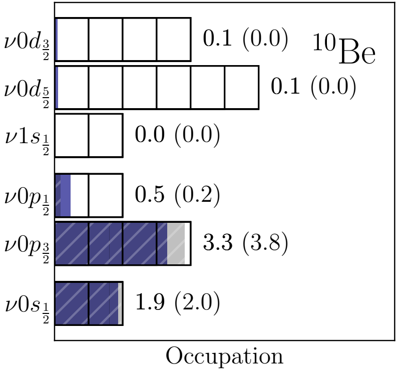
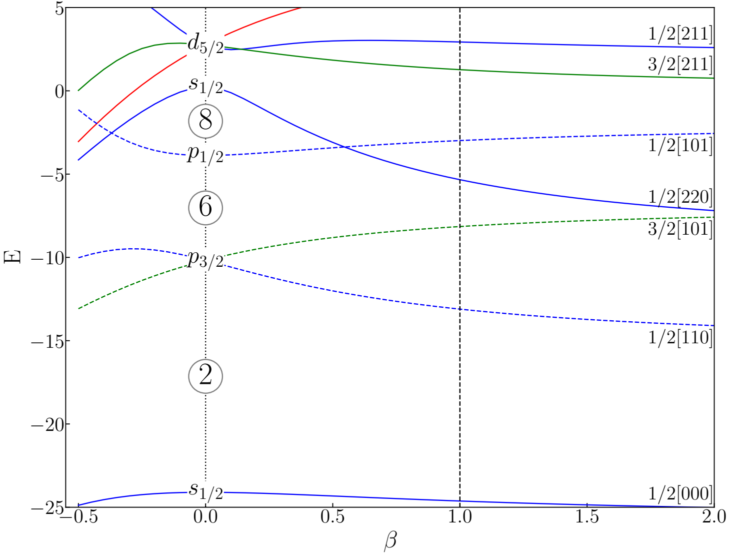
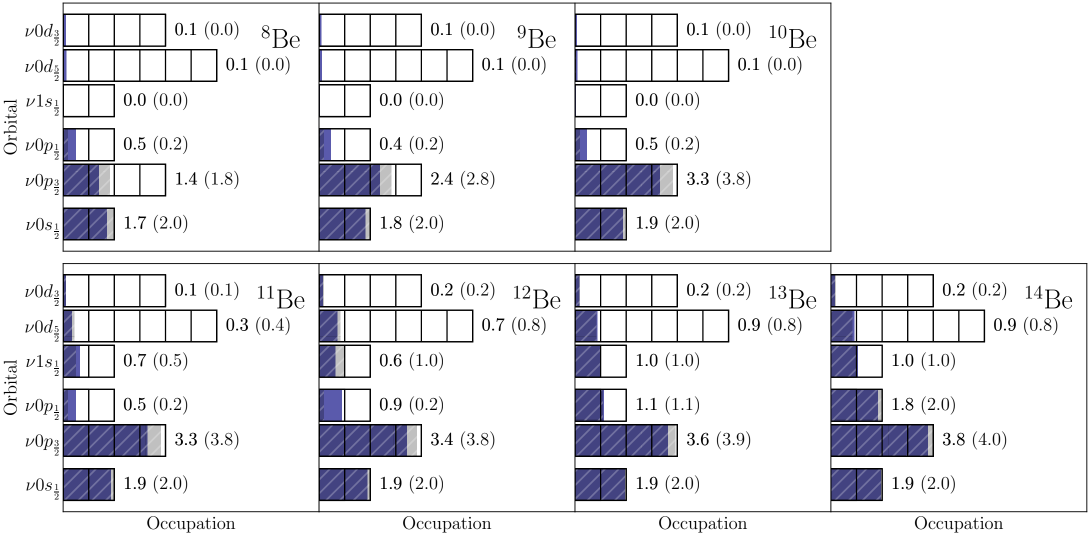




Natural orbital occupations



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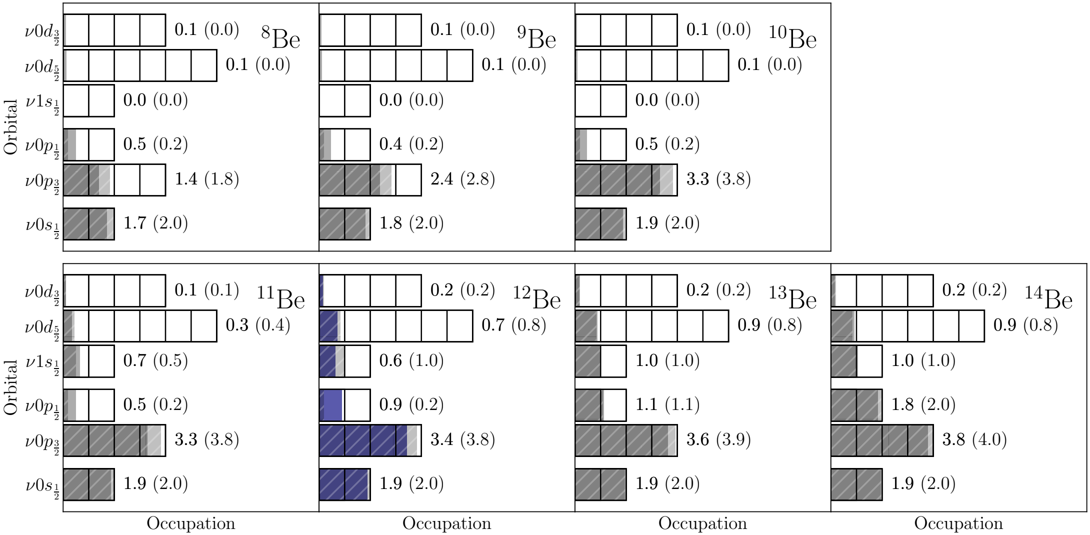
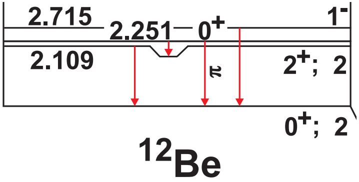
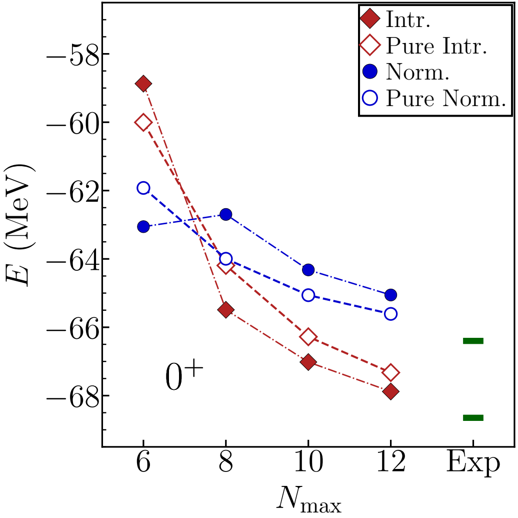


Mixing of intruder and normal states

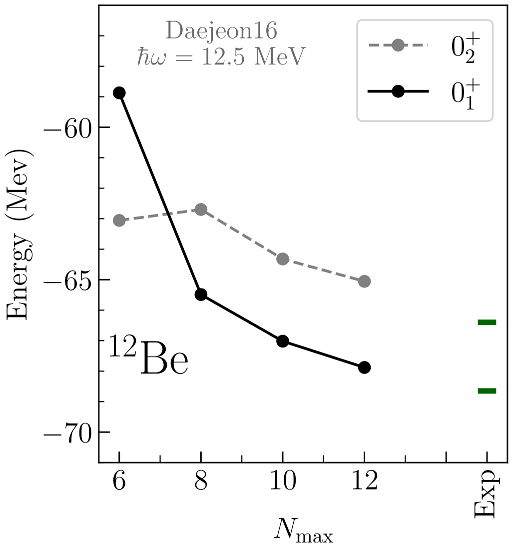
Phys. Lett. B **856** 138870 (2024).

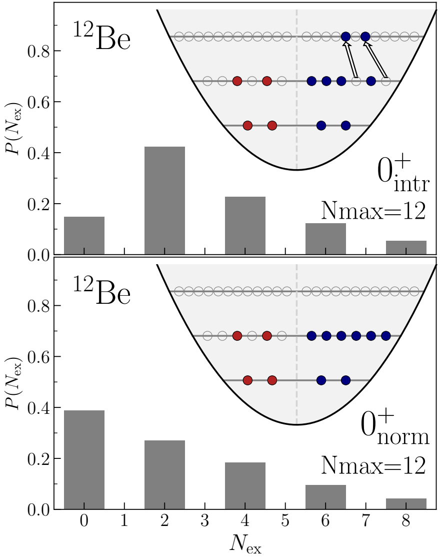
- Mixing between ground state and low lying 0_2^+ state
- Mixing occurs as states cross
- Mixing persists in physical states as indicated by measurable $E0$ transition.





Decomposition by N_{ex} : ^{12}Be





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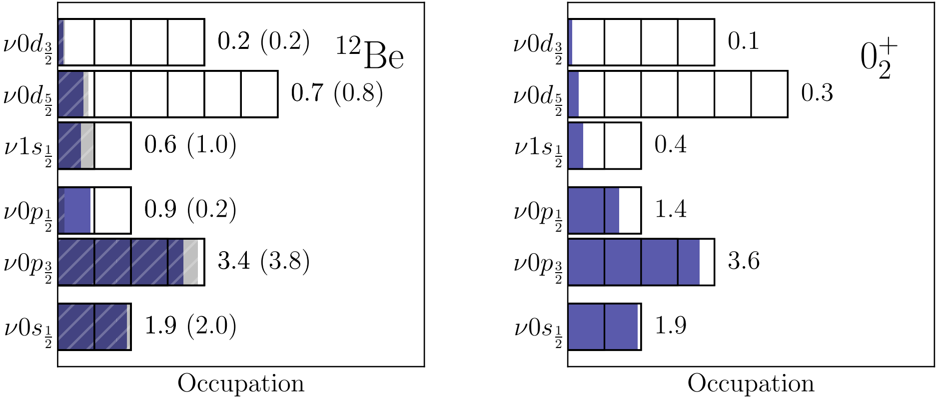

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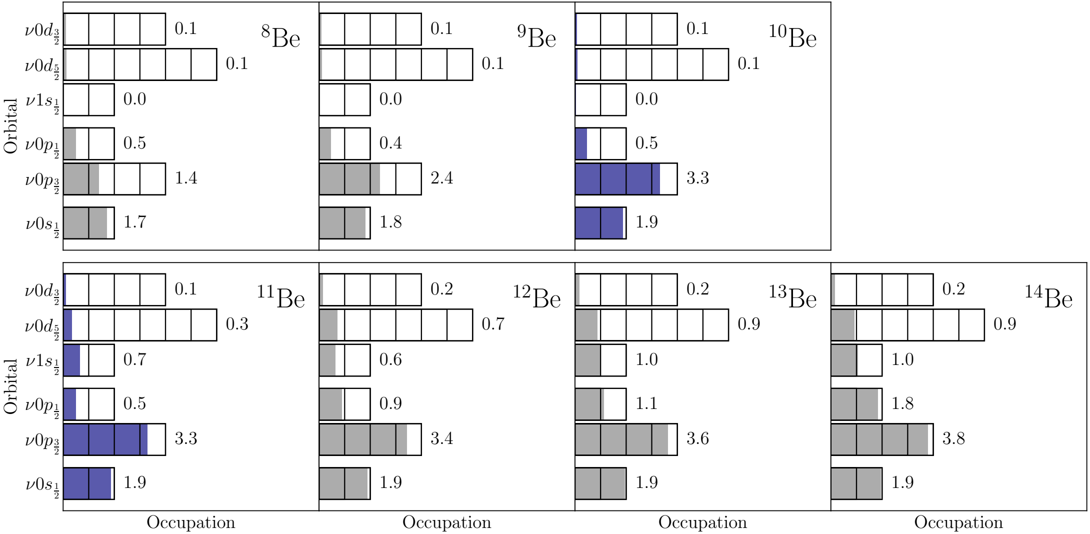
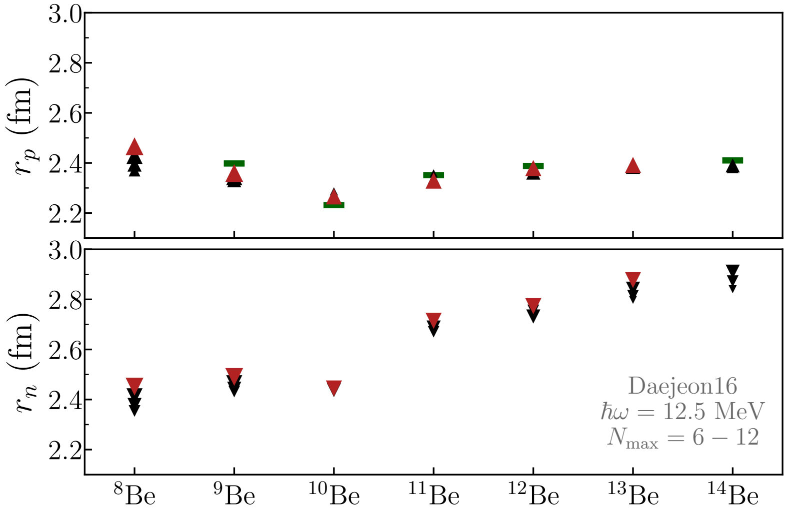


Radii



3.0
2.8
2.6
2.4
2.2
r_p (fm)
r_n (fm)
Daejeon16
$\hbar\omega = 12.5$ MeV
$N_{\max} = 6 - 12$
^{8}Be
^{9}Be
^{10}Be
^{11}Be
^{12}Be
^{13}Be
^{14}Be

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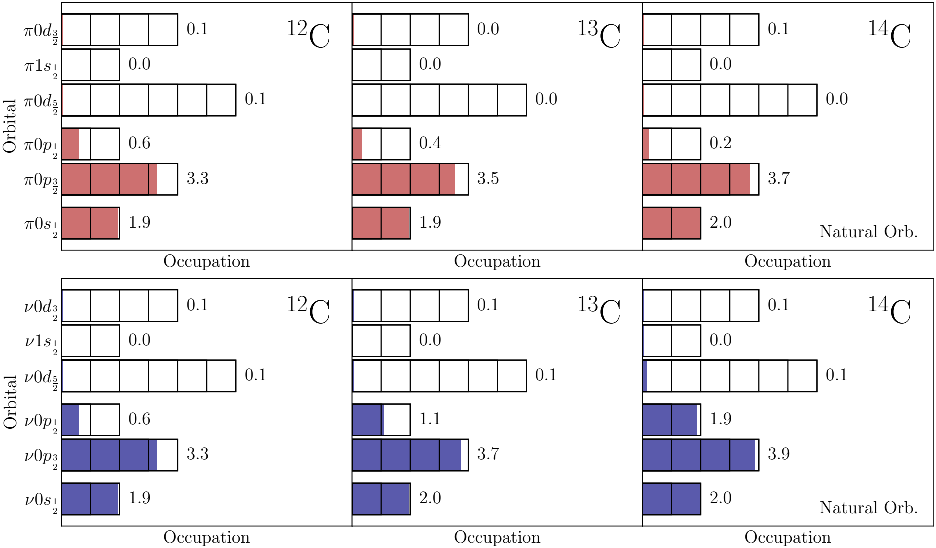
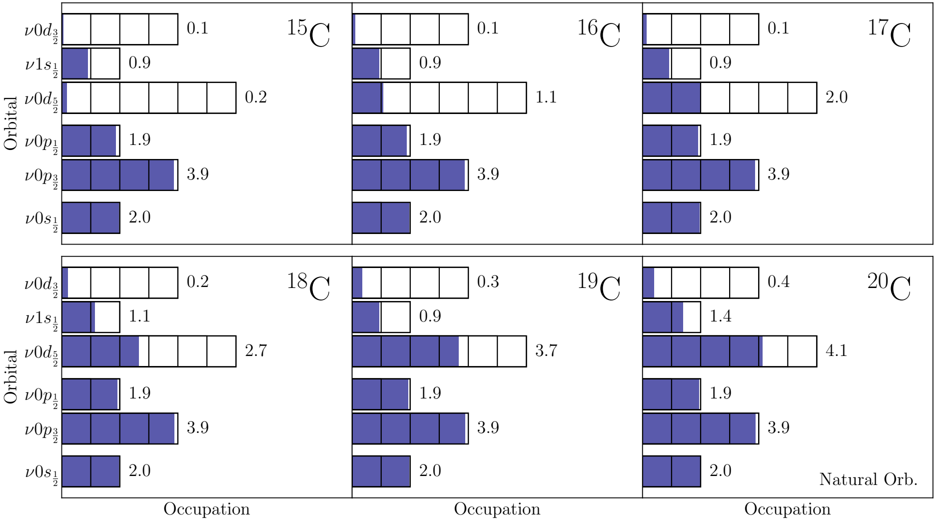
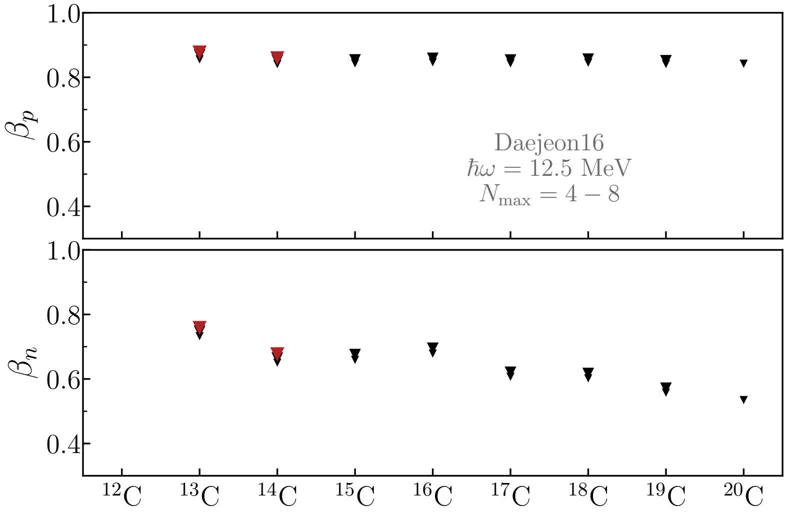
Natural orbital occupations



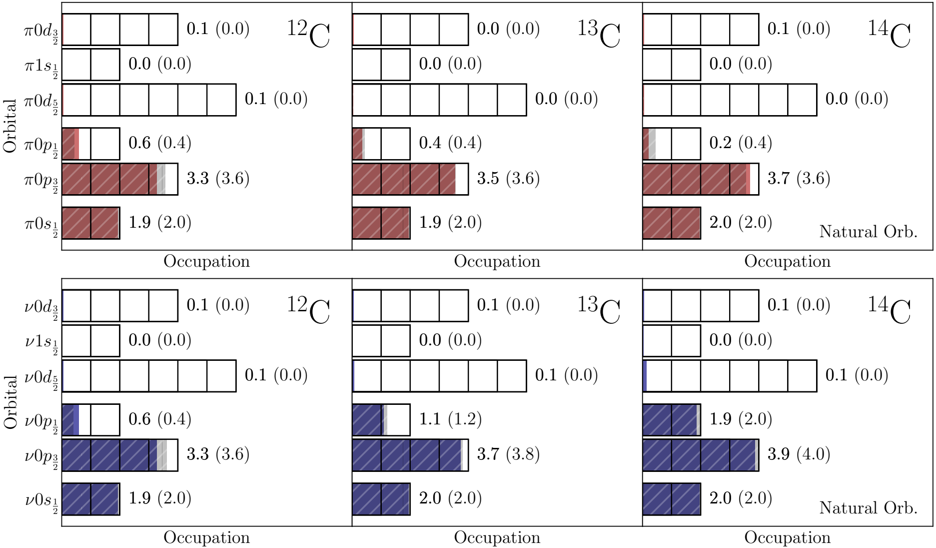
Natural orbital occupations



Deformation of Carbon isotopes



Natural orbital occupations



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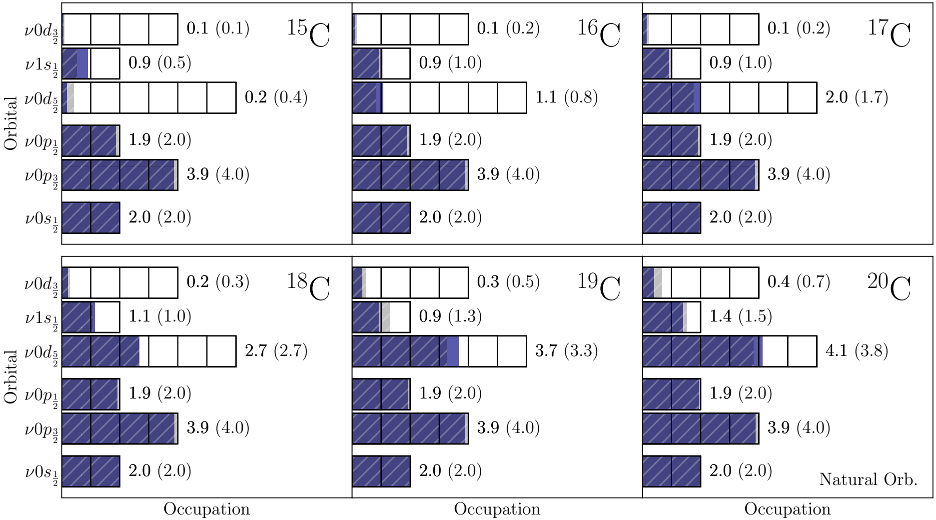
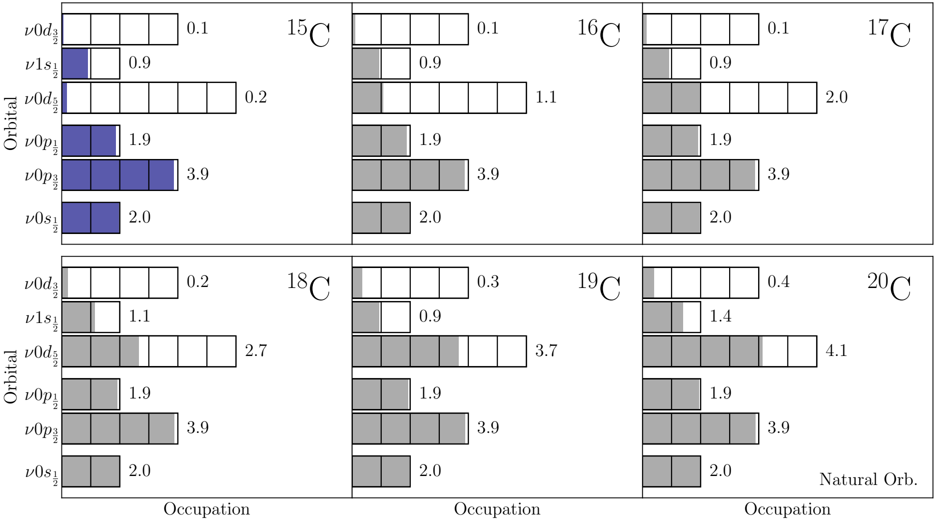
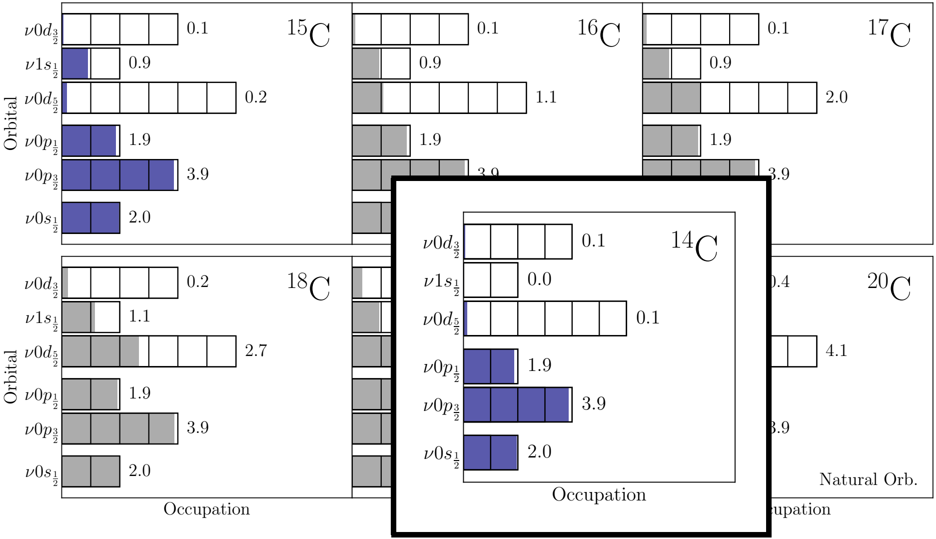


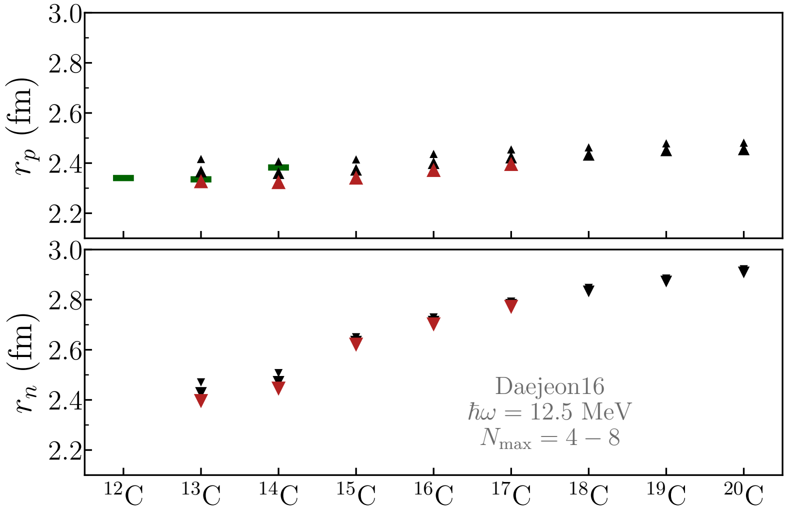
Natural orbital occupations



Natural orbital occupations



Radii



3.0
2.8
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r_p (fm)
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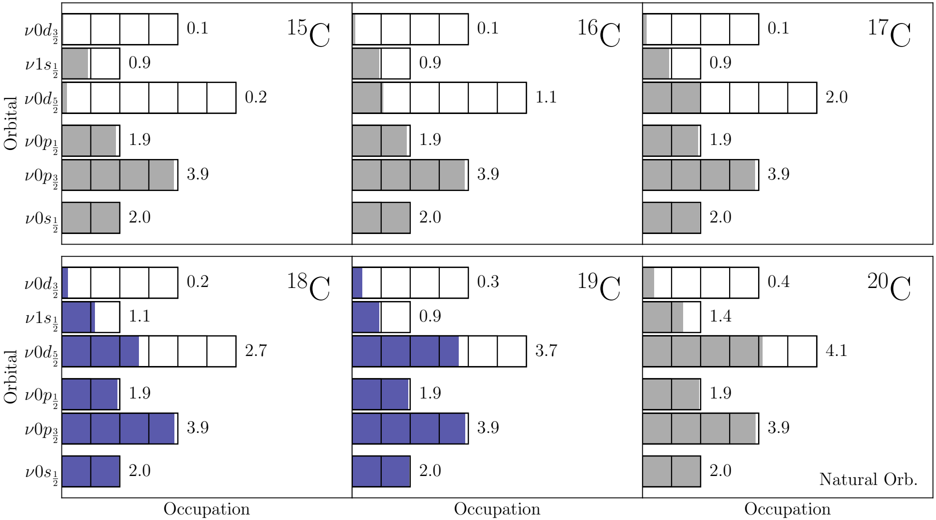
Daejeon16
$\hbar\omega = 12.5$ MeV
$N_{\max} = 4 - 8$
^{12}C
^{13}C
^{14}C
^{15}C
^{16}C
^{17}C
^{18}C
^{19}C
^{20}C

Natural orbital occupations



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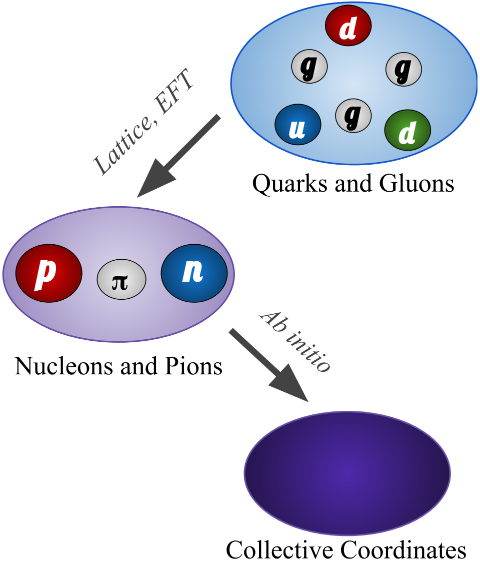
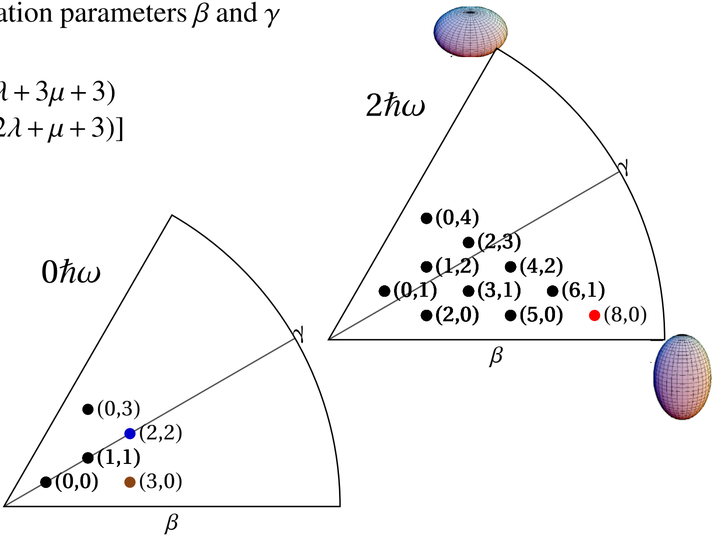


Elliott SU(3)

Labels (λ,μ) associated with deformation parameters β and γ

O. Castanos, J. P. Draayer, Y. Leschber, Z. Phys. A ***329*** *(1988) 3.*

$$\beta^2 \propto (\lambda^2 + \lambda\mu + \mu^2 + 3\lambda + 3\mu + 3)$$
$$\gamma = \tan^{-1}[\sqrt{3}(\mu+1)/(2\lambda+\mu+3)]$$



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$$\beta^2 \propto (\lambda^2 + \lambda\mu + \mu^2 + 3\lambda + 3\mu + 3)$$
$$\gamma = \tan^{-1}[\sqrt{3}(\mu+1)/(2\lambda+\mu+3)]$$

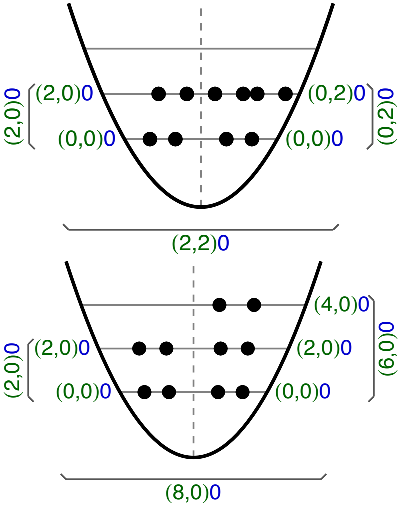
Lowest energies correspond to most deformed state

D. J. Rowe, G. Thiamova, and J. L. Wood. Phys. Rev. Lett. **97** *(2006) 202501.*

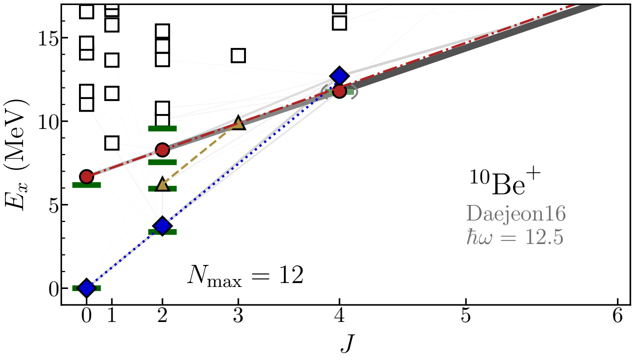
$$H = H_0 - \underbrace{\kappa Q \cdot Q}_{\propto \beta^2 \langle r^2 \rangle^2} + L \cdot S$$

SU(3) symmetry of a configuration

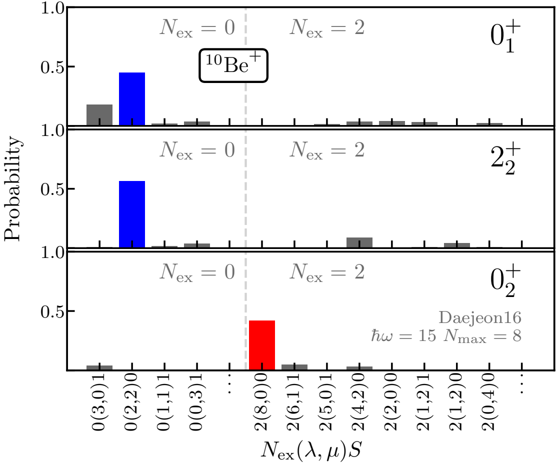
- Each particle has SU(3) symmetry $(N, 0)$, $N = 2n + \ell$
- Allowed spins dictated by antisymmetry constraints
- Final quantum numbers are $N_{\text{ex}}(\lambda\mu)S$.



Elliott rotational bands: ^{10}Be

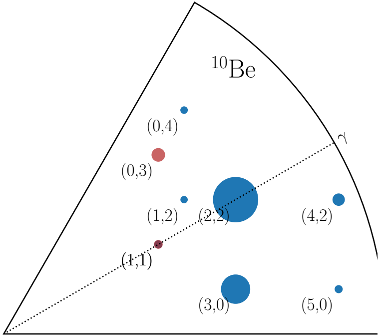


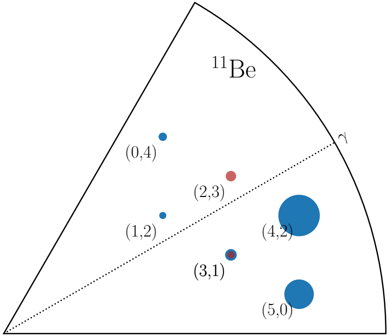
Approximate dynamical symmetry

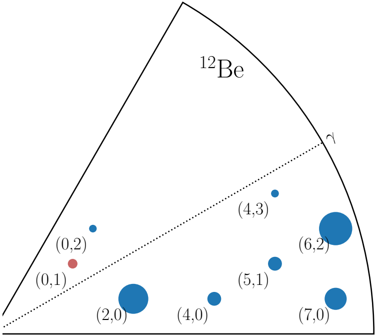


- Ground state band: $N_{\mathrm{ex}}(\lambda\mu)S = 0(2,2)0$
- Intruder band: $N_{\mathrm{ex}}(\lambda\mu)S = 2(8,0)0$

SU(3) and S_p decomposition







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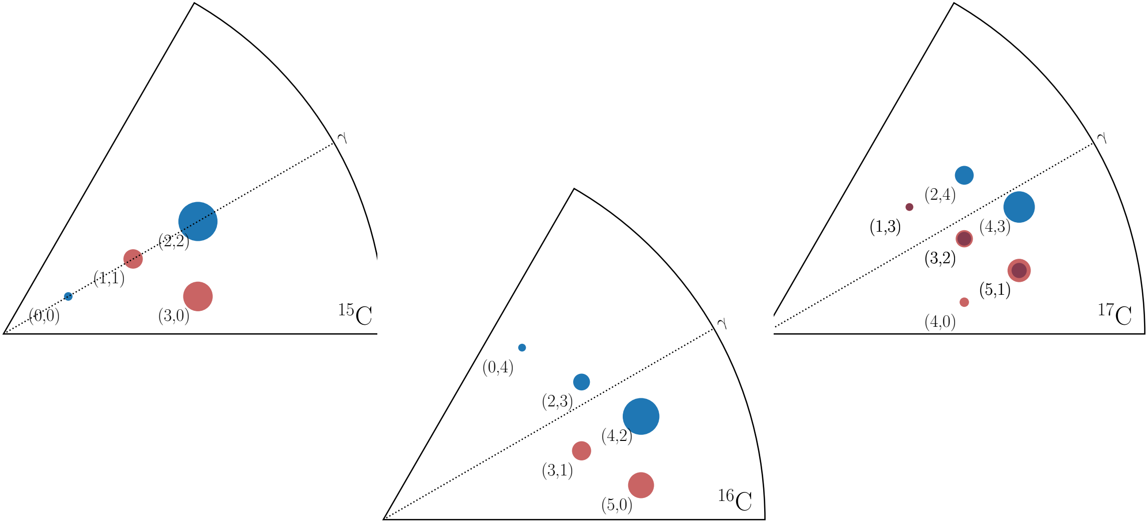
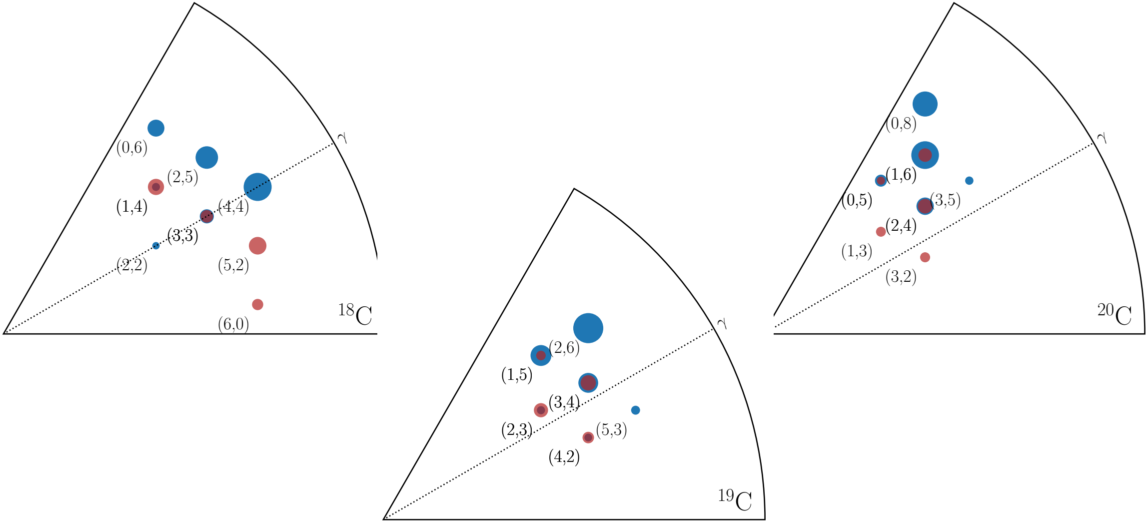


SU(3) and S_p decomposition



Conclusions

– A deformed single particle picture provides an simple approximate description for (light) neutron rich nuclei
Incorporating deformation in theory important → deformed coupled cluster

– Breakdown of $N = 8$ shell closure melts in ^{12}Be, but not in ^{14}C
Also don't see $N = 6$ or $N = 14$ subshell closure

– Occupations hint core of ^{19}C halo not quite ^{18}C
Caveat: Carbon results only with low N_{max}. Intruder states could impact ground state structure.

– SU(3) picture suggests ^{10}Be and carbon isotopes triaxial

– Spin decompositions indicate breakdown of 3α structure in carbon isotopes