Structure of three-body systems with hard core nucleon-nucleon interactions

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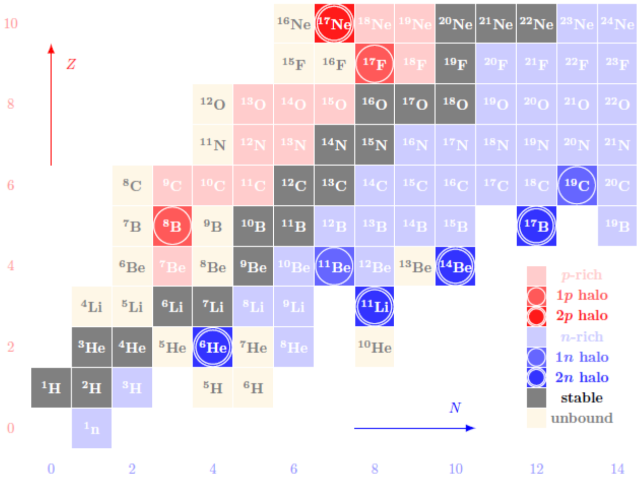
1. **Nuclear matter under extreme condition: Exotic nuclei**
2. **Interplay of *nn* and three-body interactions in three-body systems**
3. **Breakup reactions with various *nn* interactions**
4. **Conclusion**

Nuclear matter under extreme condition:

1. The driplines: limits of the nuclear landscape, where additional nucleons can no longer be kept in the nucleus, they literally drip out.
2. Possible disappearance of normal nuclear shell closures, replaced by new magic numbers.
3. Gradual vanishing of the binding energy of particles (clusters) may give rise to beta-delayed particle emission or even particle radioactivity.
4. Occurrence of nuclear halo states: nucleon(s) outside a saturated core within the same exotic nucleus.


Z
N
10
8
6
4
2
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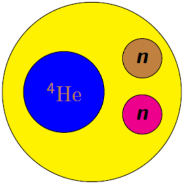
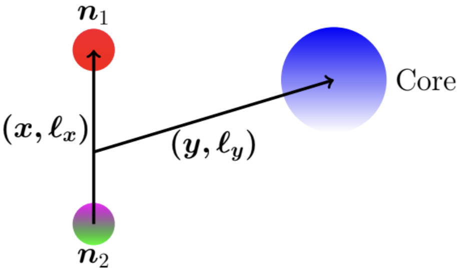
14
^{16}Ne ^{17}Ne ^{18}Ne ^{19}Ne ^{20}Ne ^{21}Ne ^{22}Ne ^{23}Ne ^{24}Ne
^{15}F ^{16}F ^{17}F ^{18}F ^{19}F ^{20}F ^{21}F ^{22}F ^{23}F
^{12}O ^{13}O ^{14}O ^{15}O ^{16}O ^{17}O ^{18}O ^{19}O ^{20}O ^{21}O ^{22}O
^{11}N ^{12}N ^{13}N ^{14}N ^{15}N ^{16}N ^{17}N ^{18}N ^{19}N ^{20}N ^{21}N
^{8}C ^{9}C ^{10}C ^{11}C ^{12}C ^{13}C ^{14}C ^{15}C ^{16}C ^{17}C ^{18}C ^{19}C ^{20}C
^{7}B ^{8}B ^{9}B ^{10}B ^{11}B ^{12}B ^{13}B ^{14}B ^{15}B ^{17}B ^{19}B
^{6}Be ^{7}Be ^{8}Be ^{9}Be ^{10}Be ^{11}Be ^{12}Be ^{13}Be ^{14}Be
^{4}Li ^{5}Li ^{6}Li ^{7}Li ^{8}Li ^{9}Li ^{11}Li
^{3}He ^{4}He ^{5}He ^{6}He ^{7}He ^{8}He ^{10}He
^{1}H ^{2}H ^{3}H ^{5}H ^{6}H
^{1}n
p-rich
$1p$ halo
$2p$ halo
n-rich
$1n$ halo
$2n$ halo
stable
unbound



Theoretical Adv.: Few-Body structure

- $^{6}\mathrm{He} \rightarrow {}^{4}\mathrm{He} + n + n$
- $^{16}\mathrm{Be} \rightarrow {}^{14}\mathrm{Be} + n + n$
- $^{22}\mathrm{C} \rightarrow {}^{20}\mathrm{C} + n + n$

Property: Extended matter radius



One defines:

$\boldsymbol{L} = \boldsymbol{\ell_x} + \boldsymbol{\ell_y}$

$\boldsymbol{S} = \boldsymbol{s}_{n1} + \boldsymbol{s}_{n2}$

$\boldsymbol{J} = L + S$

$\rho = \sqrt{x^2 + y^2}$: **Hyper-radius**

$\alpha = \arctan(y/x)$: **Hyper-angle**

$Y_{\ell_x}^{m_{l_x}}(\Omega_x)$: **spherical harm. associated with** ℓ_x

$Y_{\ell_y}^{m_{l_y}}(\Omega_y)$: **spherical harm. associated with** ℓ_y

The expansion basis is constructed as follows:

$$\Phi_{\beta}^{JM}(\Omega) = [\mathcal{Y}_{KLM_L}(\Omega) \otimes \chi_S]_{JM}$$

$$\mathcal{Y}_{KLM_L}(\Omega) = [Y_{\ell_x}(\Omega_x) \otimes Y_{\ell_y}(\Omega_y)]_{LM_L} \mathcal{P}_K^{\ell_x,\ell_y}(\alpha)$$

$$\mathcal{P}_K^{\ell_x,\ell_y}(\alpha) = N_K^{\ell_x,\ell_y} (\cos\alpha)^{\ell_x} (\sin\alpha)^{\ell_y} P_n^{\ell_x+1/2,\ell_y+1/2}(\cos 2\alpha)$$

$\Omega \equiv (\alpha, \Omega_x, \Omega_y) \equiv (\alpha, \theta_x, \varphi_x, \theta_y, \varphi_y)$

$K = 2n + \ell_x + \ell_y$: **Hyper-momentum**

$P_n^{\ell_x+1/2,\ell_y+1/2}(\cos 2\alpha)$: **Jacobi polynomials**

The WF is expanded as follows:

$$\Psi_{JM}(\rho,\Omega) = \frac{1}{\rho^{5/2}} \sum_{\beta} F_{\beta}^{J}(\rho)\Phi_{\beta}^{JM}(\Omega)$$

- $F_{\beta}^{J}(\rho)$: **radial part of WF**
- $\Phi_{\beta}^{J}(\Omega)$: **angular part off wf.**
- $\beta \equiv (K, L, S, \ell_x, \ell_y)$: **contains the various quantum numbers.**

The substitution of $\Psi_{JM}(\rho,\Omega)$ into Schrödinger eq. gives a set of CDE:

$$\left[-\frac{\hbar^2}{2m_N}\left(\frac{d^2}{d\rho^2} + \frac{\mathcal{L}(\mathcal{L}+1)}{\rho^2}\right) - \varepsilon_{3b}\right]F^J_\beta(\rho) + \sum_{\beta'\neq\beta} V_{\beta'\beta}(\rho)F^J_{\beta'}(\rho) = 0$$

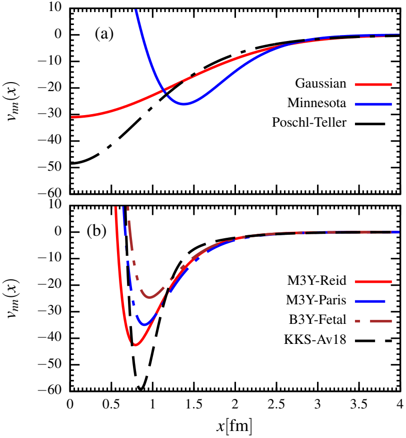
$V_{\beta'\beta}(\rho)$: **Coupling matrix elements**

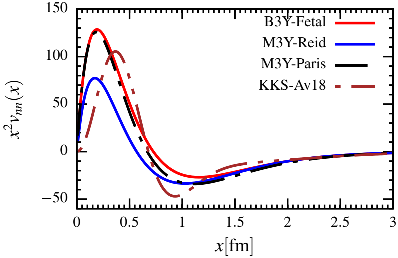
$$V_{\beta'\beta}(\rho) = \langle\Phi^{JM}_{\beta'}(\Omega)|V_{cn}(x) + V_{nc}(x) + v_{nn}(x)|\Phi^{JM}_{\beta}(\Omega)\rangle + V_{3b}(\rho)\delta_{\beta\beta'}$$

$V_{cn_1}(x) = V_{n_1c}(x)$: **core-neutron interaction**

$v_{nn}(x)$: **neutron-neutron interaction**

Seven different *nn* interactions are used:

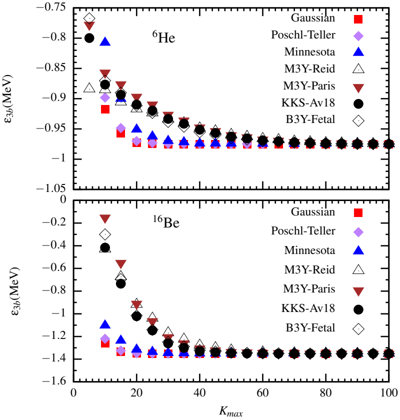


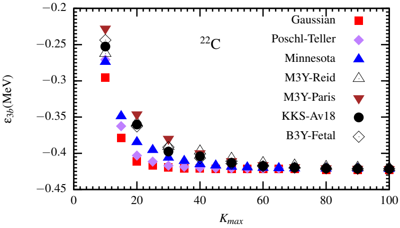


B3Y-Fetal: Shallower, but more repulsive at $x \to 0$

KKS-Av18: Deeper, but more repulsive at $x \in [0.4 : 0.6]$

Convergence of ε_{3b} in terms of K_{max}:





Covergence is obtained in all 7 cases.

Convergence is faster for soft core *nn* interactions.

Table: **Root-mean-square radii, matter radius and the weight of $W_{40}(\%)$ and $W_{42}(\%)$ on the total WF for ^{16}Be**

Interaction	$\sqrt{\langle\rho^2\rangle}$(fm)	R_{mat}(fm)	$W_{40}(\%)$	$W_{42}(\%)$
Gaussian	5.484	3.140	64	25
Minnesota	5.615	3.155	62	27
Pöschl-Tell	5.433	3.134	65	24
M3Y-Reid	5.535	3.146	55	34
M3Y-Paris	5.604	3.153	55	36
KKS-Av18	5.611	3.154	52	37
B3Y-Fetal	5.651	3.158	50	38

Table: **Root-mean-square radii, matter radius and the weight of $W_{00}(\%)$ on the total WF for ^{22}C**

Interaction	$\sqrt{\langle \rho^2 \rangle}$	R_{mat}	$W_{00}(\%)$
Gaussian	10.867	3.667	90
Minnesota	10.877	3.668	90
Pöschl-Tell	10.864	3.664	91
M3Y-Reid	11.003	3.685	97
M3Y-Paris	11.036	3.689	97
KKS-Av18	11.062	3.693	97
B3Y-Fetal	11.092	3.697	97

$$W_{KL} = \int_0^\infty d\rho |F_\beta^J(\rho)|^2$$

$$\sqrt{\langle \rho^2 \rangle} = \left[\int_0^\infty d\rho \rho^2 |F_\beta^J(\rho)|^2 \right]$$

$$R_{mat} = \sqrt{\frac{1}{A}\left(A_c R_c^2 + \langle \rho^2 \rangle \right)}$$

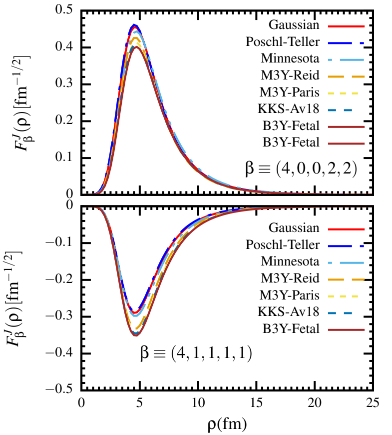
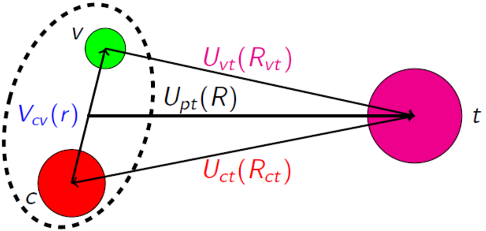


Table: **Depth of 3B adjusted to obtain the experimental ε_{3b}**

Interaction	$v_{3b}(^{6}\mathrm{He})$	$v_{3b}(^{16}\mathrm{Be})$	$v_{3b}(^{22}\mathrm{C})$
Gaussian	−1.7330	−1.0310	−3.938
Pöschl-Teller	−1.308	−0.4590	−3.963
Minnesota	−2.267	−2.419	−4.082
M3Y-Reid	−27.4870	−5.0177	−6.935
M3Y-Paris	−27.6975	−6.0447	−7.180
KKS-Av18	−27.8390	−6.3315	−7.339
B3Y-Fetal	−27.9850	−6.9201	−7.498



The off-diagonal p-t interaction is:

$$U(\boldsymbol{r},\boldsymbol{R}) = U^{Coul}(\boldsymbol{r},\boldsymbol{R}) + U^{Nucl}(\boldsymbol{r},\boldsymbol{R})$$

$$= U_{ct}^{coul}(\boldsymbol{R}_{ct}) + U_{vt}^{Coul}(\boldsymbol{R}_{vt}) + U_{ct}^{Nucl}(\boldsymbol{R}_{ct}) + U_{vt}^{Nucl}(\boldsymbol{R}_{vt})$$

Double folding procedure used for $U_{ct}(R_{ct})$:

$$U_{ct} = \iint \rho_t(\boldsymbol{r}_t)\rho_c(\boldsymbol{r}_c)v_{nn}(\boldsymbol{s})d\boldsymbol{r}_t d\boldsymbol{r}_p$$

- ρ_t: **target nucleus density**
- ρ_c: **core nucleus density**
- $v_{nn}(s)$: **nucleon-nucleon interaction**
- $\boldsymbol{s} = \boldsymbol{R}_{ct} - \boldsymbol{r}_t + \boldsymbol{r}_c$

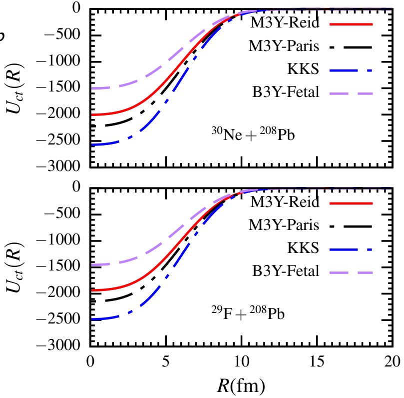


Table: **Calculated Coulomb barrier heights compared with SPP potential**

Interaction	$^{29}\mathrm{F} + {}^{208}\mathrm{Pb}$	$^{30}\mathrm{Ne} + {}^{208}\mathrm{Pb}$
M3Y-Reid	81.13	90.35
M3Y-Paris	80.65	89.82
KKS-Av18	82.06	91.40
B3Y-Fetal	80.37	89.51
SPP	80.35	89.47

We solve the following CDE obtained with CDCC formalism

$$[T_R + U^{LJ}_{\alpha_b\alpha_b}(R) + \varepsilon_\alpha - E_{cm}]F^{LJ}_\alpha(R) - \sum_{\alpha\neq\alpha'} i^{L-L'} U^{LL'J}_{\alpha\alpha'}(R) F^{L'J}_{\alpha'}(R) = 0$$

T_R: **Kinetic energy term**

$U^{LJ}_{\alpha_b\alpha_b}(R)$: **Interaction in the elastic scattering channel**

ε_α: **excitation energies**

E_{cm}: **Incident energy in the c.m.**

$U^{LL'J}_{\alpha\alpha'}(R)$: **CME**

Solve CDE to obtain reaction observables

- **BC at $R \to \infty$:**

$$F_{\alpha}^{LJ}(R) \to \frac{i}{2}\left[H_{\alpha}(K_{\alpha}R)\delta_{\alpha_b\alpha} - H_{\alpha}^{+}(K_{\alpha}R)S_{\alpha\alpha'}^{J}(K_{\alpha'})\right]$$

- $H_{\alpha}(K_{\alpha}R)$: **incoming Coulomb wave**
- $H_{\alpha}(K_{\alpha}R)$: **outgoing Coulomb wave**

Obtain scattering matrix $S_{\alpha\alpha'}^{J}(K_{\alpha'})$ and construct breakup observables

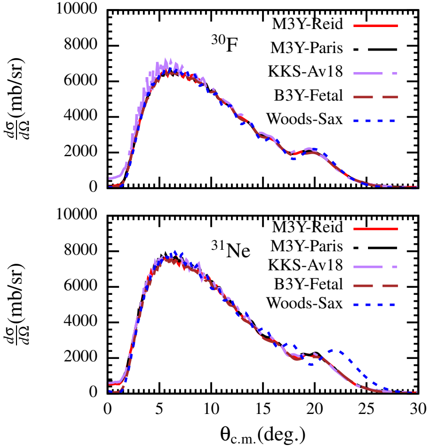
$$^{30}\mathrm{F} + {}^{208}\mathrm{Pb} \rightarrow {}^{29}\mathrm{F} + n + {}^{208}\mathrm{Pb}$$

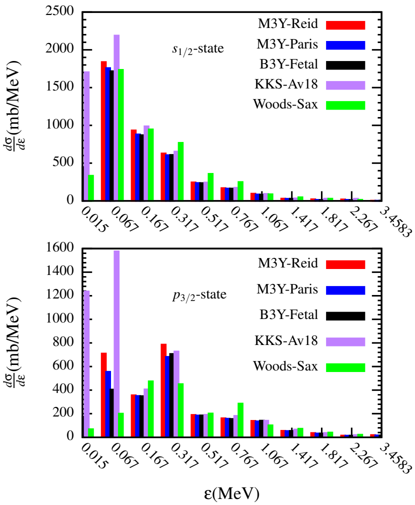
$$^{31}\mathrm{Ne} + {}^{208}\mathrm{Pb} \rightarrow {}^{30}\mathrm{Ne} + n + {}^{208}\mathrm{Pb}$$

Same $S_n = -0.472\,\mathrm{MeV}$ is considered in both cases.

Observation:

Folded potential account for similar results in both cases, similar to a 3B system.





Observation:

At $\varepsilon \ll 1$, KKS-Av18 corresponds to higher peaks ($p_{3/2}$). It also corresponds to larger $min(V_{ct}(R_{ct})$

In conclusion:

- The various *nn* interactions are found to provide equivalent description of the internal structures of three-body neutron-rich nuclei, irrespective of whether they contains strongly repulsive hard cores or not.
- For a light system (with fewer neutrons), a strongly attractive three-body interaction is needed in the case of a *nn* interaction with a strongly repulsive core.
- A weak 3B interactions when the number of neutrons increases, can be an indication that the role of the 3B interaction is being compensated by *nn* correlations.
- These *nn* interactions are also found to describe the breakup cross sections.