Deformation in one-neutron halo nuclei using halo effective field theory

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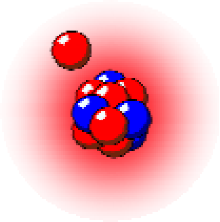
Halo nuclei

- Light, neutron-rich nuclei with large matter radius
- Low $\mathbf{S}_{\mathrm{n}}$ or $\mathbf{S}_{\mathrm{2n}}$: one or two loosely-bound neutrons
- Clusterised structure: neutrons can tunnel far from the core
  $\rightarrow$ halo-nucleus $\equiv$ compact core + valence neutron(s)



- **Our case study** : $^{11}\mathrm{Be} \equiv {}^{10}\mathrm{Be} + \mathrm{n}$
- Short-lived $\rightarrow$ studied via reactions (e.g. **breakup**)
  $\rightarrow$ need of an **effective few-body** model for reaction calculations
  $\rightarrow$ **Halo-EFT**

Halo-EFT description of ^{11}Be

- Halo-structure $\rightarrow$ separation of scales (in energy/distance)
  $\rightarrow$ small parameter $\eta = \sqrt{\frac{\mathrm{S_{1n}}}{\mathrm{E_{2^+}}}}$ or $\frac{\mathrm{R_{core}}}{\mathrm{R_{halo}}} \simeq 0.4 < 1$
  $\rightarrow$ expansion of the core-neutron Hamiltonian along η,
  i.e. reproducing the low-energy (viz. long distance) behaviour of the system
  [Bertulani, Hammer, van Kolck, NPA 712, 37 (2002)]
  Review: **[Hammer, Ji, Phillips, JPG 44, 103002 (2017)]**

- ^{11}Be $=^{10}$Be(0^+)+n [**core has no internal structure**]
  $\rightarrow$ **single-particle description**: $H(\mathbf{r}) = \mathrm{T}_\mathbf{r} + \mathrm{V_{cn}}(\mathbf{r})$
- Effective Gaussian potentials in each partial wave ℓj @NLO ($\ell \leqslant 1$):

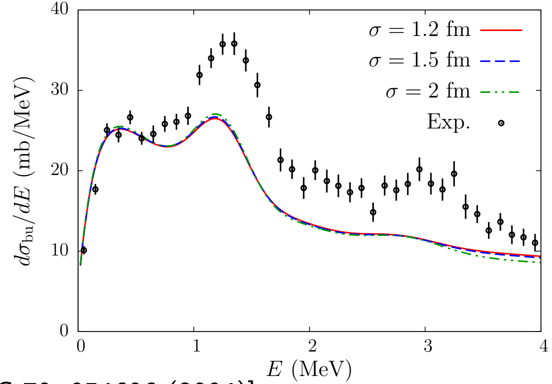
$$\mathrm{V_{cn}(r)} = \mathrm{V}^{(0)}_{\ell \mathrm{j}} \mathrm{e}^{-\frac{\mathrm{r}^2}{2\sigma^2}} + \mathrm{V}^{(2)}_{\ell \mathrm{j}} \mathrm{r}^2 \mathrm{e}^{-\frac{\mathrm{r}^2}{2\sigma^2}}$$

  $\mathrm{V}^{(0)}_{\ell \mathrm{j}}$ and $\mathrm{V}^{(2)}_{\ell \mathrm{j}}$ fitted to reproduce:
  $\rightarrow$ $\mathbf{S}_\mathrm{n}$ & asymptotic normalization coefficient (**ANC**) for bound states
  $\rightarrow$ effective range parameters for continuum states

  $\sigma :=$ cut-off $\rightarrow$ evaluates sensitivity to short-range physics

What is the problem ?

- Assumption: ^{10}Be remains in its 0^+ ground state still valid ?
  $\rightarrow$ Nuclear breakup: $^{11}\mathrm{Be}+\mathrm{C} \rightarrow {}^{10}\mathrm{Be}+\mathrm{n}+\mathrm{C}$



Exp: [Fukuda *et al.* PRC 70, 054606 (2004)]

Th.: [**LPK** & P. Capel, PRC 111, 054618 (2025)]

$\Rightarrow$ Missing peaks @ $\frac{5}{2}^+$ and $\frac{3}{2}^+$ resonances $\rightarrow$ single-particle picture is not enough

$\Rightarrow$ Missing [^{10}Be(2^+)] degree of freedom **[Moro & Lay, PRL 109, 232502 (2012)]**

Core excitation within Halo-EFT

- **Extension of Halo-EFT to include core excitation**:

$$H(\mathbf{r},\xi) = \mathrm{T}_{\mathbf{r}} + \mathrm{V}_{\mathrm{cn}}(\mathbf{r},\xi) + \mathrm{h}_{\mathrm{c}}(\xi)$$

  $\mathrm{h}_{\mathrm{c}}(\xi)$:= intrinsic Hamiltonian of the core with eigenstates $\chi^{\mathrm{c}}_{\mathrm{I}}(\xi)$

- **Halo-EFT particle-rotor model** [Bohr and Mottelson (1975)]:

$$\mathrm{V}_{\mathrm{cn}}(\mathbf{r},\xi) = \mathrm{V}_{\mathrm{cn}}(\mathrm{r}) + \beta\sigma \mathrm{Y}^0_2(\hat{\mathrm{r}}')\frac{\mathrm{d}}{\mathrm{d}\sigma}\mathrm{V}_{\mathrm{cn}}(\mathrm{r})$$

- Set of radial **coupled-channel** Schrödinger equations:

$$\left[\mathrm{T}^{\ell}_{\mathrm{r}} + \mathrm{V}_{\alpha\alpha}(\mathrm{r}) + \epsilon_\alpha - \mathrm{E}\right]\psi_\alpha(\mathrm{r}) = -\sum_{\alpha'\neq\alpha}\mathrm{V}_{\alpha\alpha'}(\mathrm{r})\psi_{\alpha'}(\mathrm{r})$$

  with $\mathrm{V}_{\alpha\alpha'}(\mathrm{r}) = \langle\mathcal{Y}_\alpha(\hat{\mathrm{r}})\chi_\alpha(\xi)|\mathrm{V}_{\mathrm{cn}}(\mathbf{r},\xi)|\mathcal{Y}_{\alpha'}(\hat{\mathrm{r}})\chi_{\alpha'}(\xi)\rangle$, $\alpha = \{\ell, \mathrm{s}, \mathrm{j}, \mathrm{I}\}$

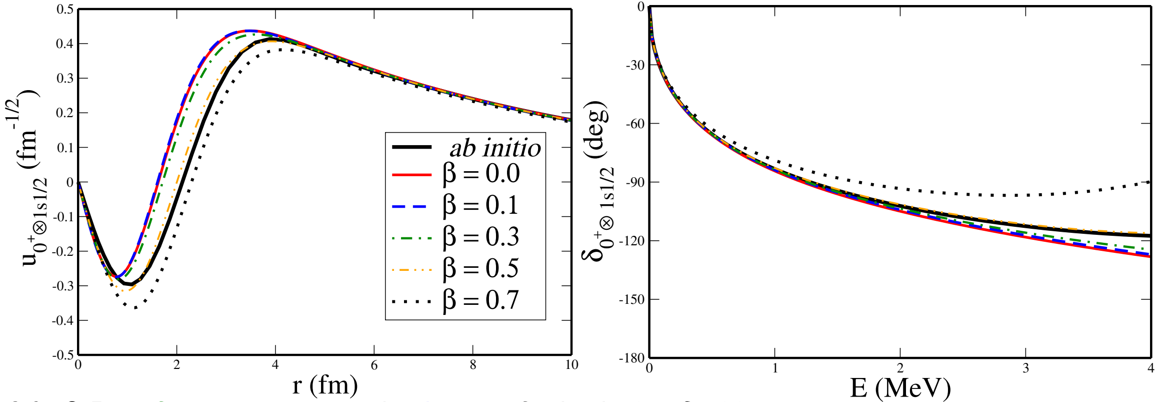
  → solved within the **R-Matrix method** on a Lagrange mesh
  [D. Baye, Phys. Rep. 565 (2015) 1]

  → study impact of core excitation on: ψ_α, δ_α

Core excitation in ^{11}Be $\frac{1}{2}^+$ ground state

Compare to *ab initio* predictions [Calci et al., PRL 117, 242501 (2016)]

- $\Psi_{1/2^+} = \psi_{1s1/2}(\mathbf{r}) \otimes \chi_{0^+}^{^{10}\text{Be}} + \psi_{0d5/2}(\mathbf{r}) \otimes \chi_{2^+}^{^{10}\text{Be}} + \psi_{0d3/2}(\mathbf{r}) \otimes \chi_{2^+}^{^{10}\text{Be}}$
- NLO potentials **fitted to** reproduce $\mathbf{S}_\mathrm{n}$ and *ab initio* **ANC** for $\neq \beta$

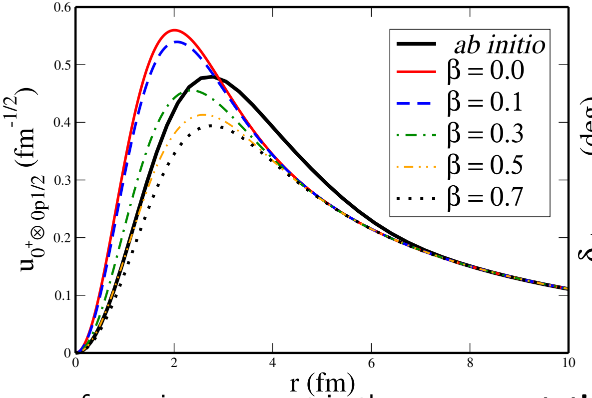


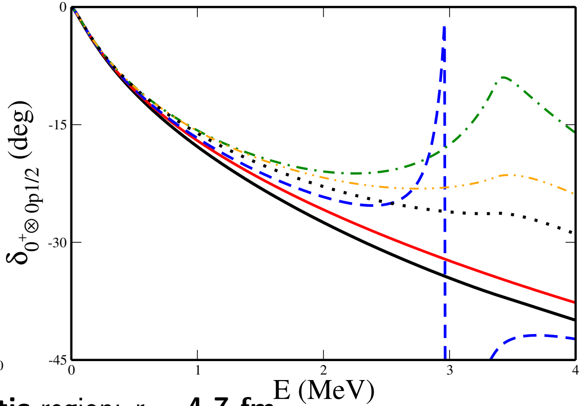
@β=**0.5** perfect agreement with *ab initio* for both ψ_α, δ_α, $\forall\sigma$

$\Rightarrow$ confirms the role of **core excitation** in structure of ^{11}Be g.s

Core excitation in ^{11}Be $\frac{1}{2}^-$ bound excited state

- $\Psi_{1/2^-} = \psi_{0p1/2}(\mathbf{r}) \otimes \chi_{0^+}^{^{10}\mathrm{Be}} + \psi_{0p3/2}(\mathbf{r}) \otimes \chi_{2^+}^{^{10}\mathrm{Be}} + \psi_{0f5/2}(\mathbf{r}) \otimes \chi_{2^+}^{^{10}\mathrm{Be}}$
- NLO potentials **fitted to** reproduce $\mathbf{S}_\mathrm{n}$ and *ab initio* **ANC** for $\neq \beta$





- wfs: no improvement in the **pre-asymptotic** region: r $\sim$ **4-7 fm**
- phase shifts: less good than without core excitation

$\Rightarrow$ No influence of **core excitation** on structure of ^{11}Be e.s.because **shell-model** state ?

Electric dipole strength: B(E1)

E1 transition from bound state to bound state: $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$

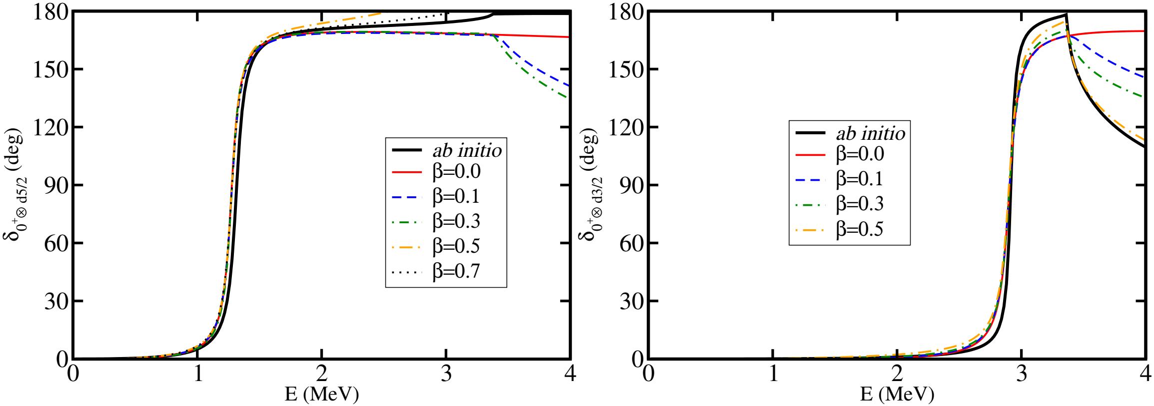
	B(E1) (e^2fm^2)	
σ (fm)	$\beta = 0.5$	$\beta = 0$
1.3	0.104	0.103
1.5	0.106	0.106
1.8	0.109	0.108
2.0	0.110	0.109
ab initio	0.117	
Experiments		
[PRC 28, 497]	0.116(12)	
[PLB 394, 11]	0.099(10)	
[PLB 650, 124]	0.105(12)	
[PLB 732, 210]	0.102(2)	

- **Core excitation** has **no influence** on B(E1)
- Good agreement with exp. data but lower than *ab initio*
- Ab initio **overestimates** exp. B(E1) $\rightarrow$ wrong **pre-asymptotic** region ?

Core excitation in low-energy resonances : $\frac{5}{2}^+$, $\frac{3}{2}^-$, $\frac{3}{2}^+$

Compare to *ab initio* predictions [Calci et al., PRL 117, 242501 (2016)]

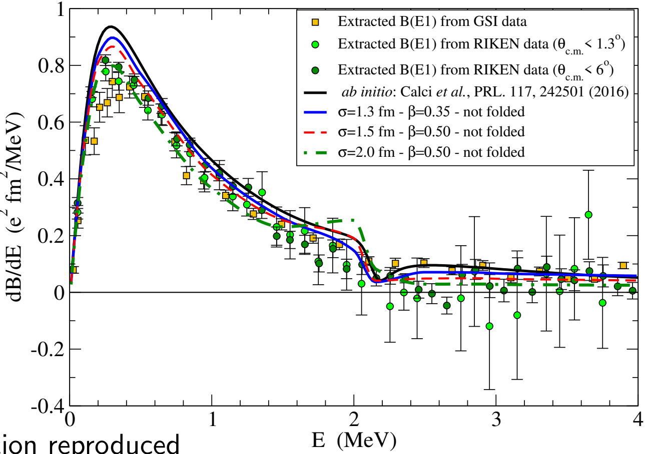
- NLO potentials **fitted to** reproduce exp. $\mathbf{E}_{\rm res}$ and $\Gamma_{\rm res}$ for $\neq \beta$



- Excellent agreement with *ab initio* results → probing **nature of resonances** $[\Gamma_{0^+}, \Gamma_{2^+}]$
- Direct access to scattering wfs, phase shifts → $\frac{dB(E1)}{dE}$, cross sections,...

dB(E1)/dE

E1 transition from $\frac{1}{2}^+$ bound state to the continuum with **final-state interactions**

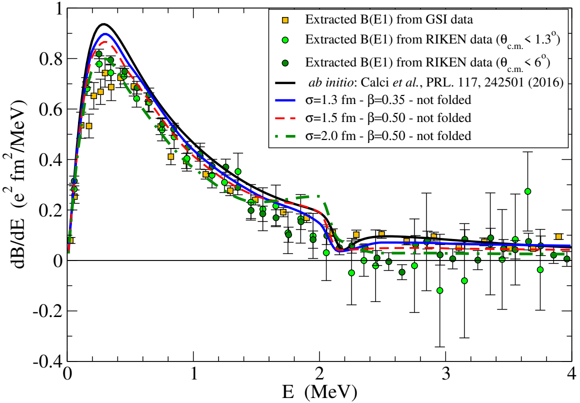


- *Ab initio* prediction reproduced
- **Good agreement** with exp. data reproduced but **overshoot at low** E (like *ab initio*)
- Significant σ-dependency because of $\frac{3}{2}^-$ phaseshift

Coulomb breakup & Equivalent Photon Method

Coulomb breakup: ^{11}Be+Pb → ^{10}Be+n+Pb @69AMeV → **E1**-dominated

$$\frac{\mathrm{d}\sigma}{\mathrm{dE}} = \frac{16\pi^3}{9\hbar c} \mathrm{N_{E1}(E)} \frac{\mathrm{dB(E1)}}{\mathrm{dE}}$$



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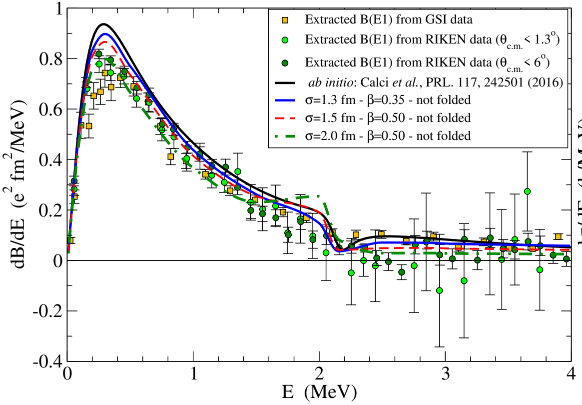
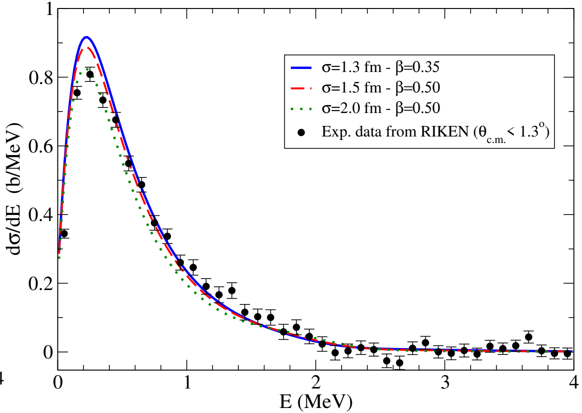




→ B(E1) distribution overshoots reflected on cross-sections (**which are folded**)

Caveats

Simple portable structure model for (breakup) reactions codes, including core deformation
$\rightarrow$ with 2 caveats:

- Power counting not discussed
- @NLO: non zero interactions in channels where $\ell \geq 1$ (mean field vision)

QUESTION: Could we formulate this inclusion of deformation in a more EFT-oriented spirit ?

$\rightarrow$ **Yes. let us discuss it...**

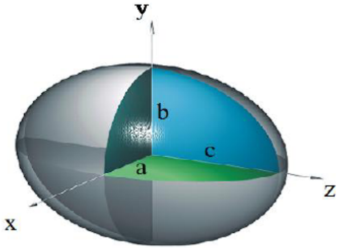
Towards a rotor (Nilsson) Halo-EFT

Goal:=describe key features of the low-energy spectrum of (light) deformed one-neutron nuclei

Key idea:= keep Halo-EFT and add deformation as subleading effect $\rightarrow$ perturbatively

Assumptions on the core:

- axially $[\mathrm{a{=}c{=}R_{core}}]$ symmetric rigid rotor: $\hat{\mathrm{H}}_{\mathrm{core}} = \frac{\hat{\mathrm{I}}^2}{2\theta}$
  $\rightarrow$ rotational spectrum: 0^+g.s. (bandhead) and low-lying 2^+ excited state
- deformed ellipsoid along z-axis (symmetry axis) in intrinsic frame
  $\rightarrow$ stretching parameter ζ directly linked to β for small deformation $[\mathrm{c} = \zeta \mathrm{R_{core}}]$



Operators

-c-n potentials in Halo-EFT (momentum space)

$$\text{@LO} : \mathrm{V}_{\mathrm{LO}} = \mathrm{C}_0$$
$$\text{@NLO} : \mathrm{V}_{\mathrm{NLO}} = \mathrm{C}_0 + \mathrm{C}_2(\mathbf{p}^2 + \mathbf{p}'^2)$$

-Core is a rigid rotor $\rightarrow \hat{\mathrm{H}}_{\mathrm{core}} = \frac{\hat{\mathrm{I}}^2}{2\theta} \sim \mathbf{v}^2$

-**Quadrupole** contribution:

$$\text{@NNLO} : \mathrm{V}_{\mathrm{def}} = \mathrm{C}_{\mathrm{def}} \mathbf{I}.\mathbf{q}.\mathbf{I}.\mathbf{q} - \frac{1}{3}(\mathbf{I}.\mathbf{q})^2 \qquad \text{with} \quad \mathbf{q} = \mathbf{p} - \mathbf{p'}$$

$\rightarrow \mathrm{C}_{\mathrm{def}}$:=free parameter (not an LEC) related to β

-**Hyperfine** contribution:

$$\mathrm{V}_{\mathrm{hf}} = \mathrm{C}_{\mathrm{hf}}\ \mathbf{I}.\mathbf{j} \qquad \text{with} \quad \mathbf{j} = \boldsymbol{\ell} + \boldsymbol{s}$$

$\rightarrow \mathrm{C}_{\mathrm{hf}}$:= additionnal LEC to be fitted

$\rightarrow$ higher order terms suppressed by powers of $\mathbf{v}^2$

Goal: tune C_0, C_2 and C_{hf} to reproduce low-energy spectrum of halo nucleus

Rotor (Nilsson) Halo-EFT

Question: What about the case where $p_{\text{halo}} \sim p_{\text{rotor}}$?

$\rightarrow$ deformation becomes LO $[H_{\text{LO}} := H_{\text{Nilsson}}]$

$$\frac{\gamma^2}{2\mu} \sim \frac{I(I+1)}{2\theta} \tag{1}$$

$$\theta = \theta_{xx} = \theta_{yy} = \frac{A m_N}{5} R_{\text{core}} (1+\zeta^2) \quad \text{and} \quad \mu = \mu_0 m_N \tag{2}$$

$$\frac{\gamma^2}{2\mu_0} \sim \frac{I(I+1)}{\frac{A}{5} R^2_{\text{core}} (1+\zeta^2)} \tag{3}$$

with different regimes:

$\zeta \gg 1$: **prolate** (:=elongation along z-axis); $\zeta \ll 1$: **oblate** (:=flattening); $\zeta = 1$: **spherical**

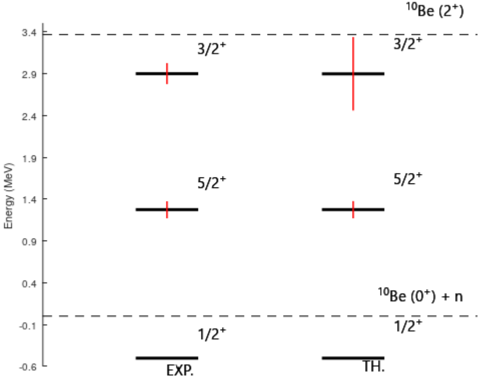
$\rightarrow$ relates the geometry (moment of inertia), binding, nb of nucleons

$\rightarrow$ this scenario could happen for heavier halos (larger nb of nucleons):

eg: ^{17}C, ^{19}C (**sd** shell), ^{31}Ne (**fp** shell)

^{11}Be: positive parity states **[PRELIMINARY]**

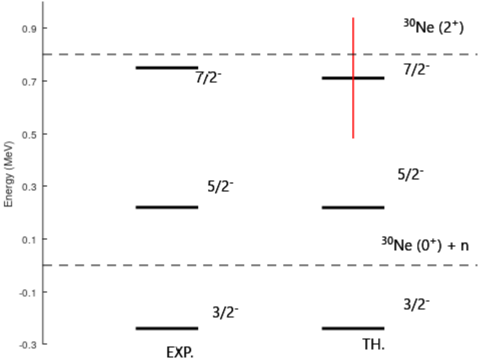
- $\frac{1}{2}^+$ g.s.; $\mathrm{S_{1n}}$=0.5 MeV; $\mathrm{E_{2^+}}(^{10}\mathrm{Be})$=3.368 MeV $\rightarrow$ $\mathrm{p_{halo}} \gg \mathrm{p_{rotor}}$
- Tune $\mathrm{C_0, C_2}$ and $\mathrm{C_{hf}}$ against $\mathrm{S_{1n}}$, positions of the resonances of ^{11}Be



- We reproduce the position of each state
- **Unprecised widths** for resonances $\rightarrow$ higher order effect

^{31}Ne: deformed p-wave halo [PRELIMINARY]

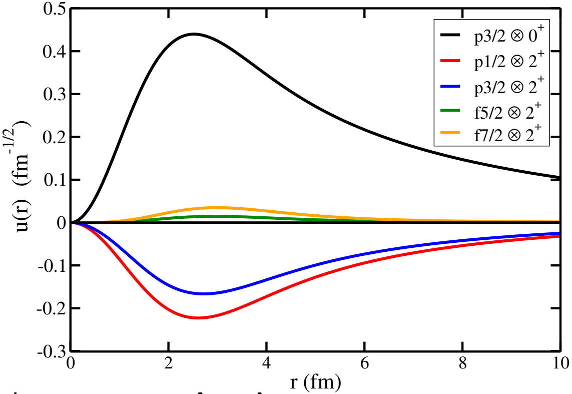
- $\frac{3}{2}^-$ g.s.; S_{1n}=0.24 MeV; $E_{2^+}(^{30}Ne)$=0.801 MeV $\rightarrow$ $p_{halo} \sim p_{rotor}$
- Tune C_0 and C_{hf} against S_{1n}, positions of the resonances, for β=0.56, we get:



- We reproduce the position of each state
- **No scattering data** to compare to (no **exp. widths**)

^{31}Ne: $\frac{3}{2}^-$ ground state **[PRELIMINARY]**

Wave functions in each channel for β=0.56:



Other models available...but **no scattering data**:

- Urata, *et al.* PRC 83, 041303(R) (2011); Minomo, *et al.* PRL 108, 052503 (2012);
- Hong, Bertulani, Kruppa, PRC 96 064603 (2017)...

Outlook: E1-dissociation/Coulomb breakup **[Elkamhawy, Hammer JPG 50 02510 (2023)]**

Conclusion

We want to study reactions involving **one-neutron halo nuclei** :

- need of a **realistic few-body** model for reaction calculations
  $\rightarrow$ Halo-EFT

Our model of one-neutron halo nuclei [^{11}Be] provides:

- explicit inclusion of **core excitation within Halo-EFT**
- realistic description of both bound and low-lying resonant states in deformed halos [^{11}Be]
- portable structure model including deformation for reaction codes

[L.-P. Kubushishi and P. Capel, (2025), (in preparation)]

Outlook:

- same formalism to study structure and breakup of ^{17}C, ^{19}C (sd-shell), ^{37}Mg
- include our model in reaction codes (**nuclear** breakup, knock-out,...)
- Nilsson Halo-EFT for light deformed nuclei
  [L.-P. Kubushishi and D. R. Phillips, (2025), (in preparation)]
  $\rightarrow$ sd-shell nuclei: ^{17}C, ^{19}C

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- Daniel R. Phillips (Ohio University)
- Hans-Werner Hammer (TU Darmstadt)